

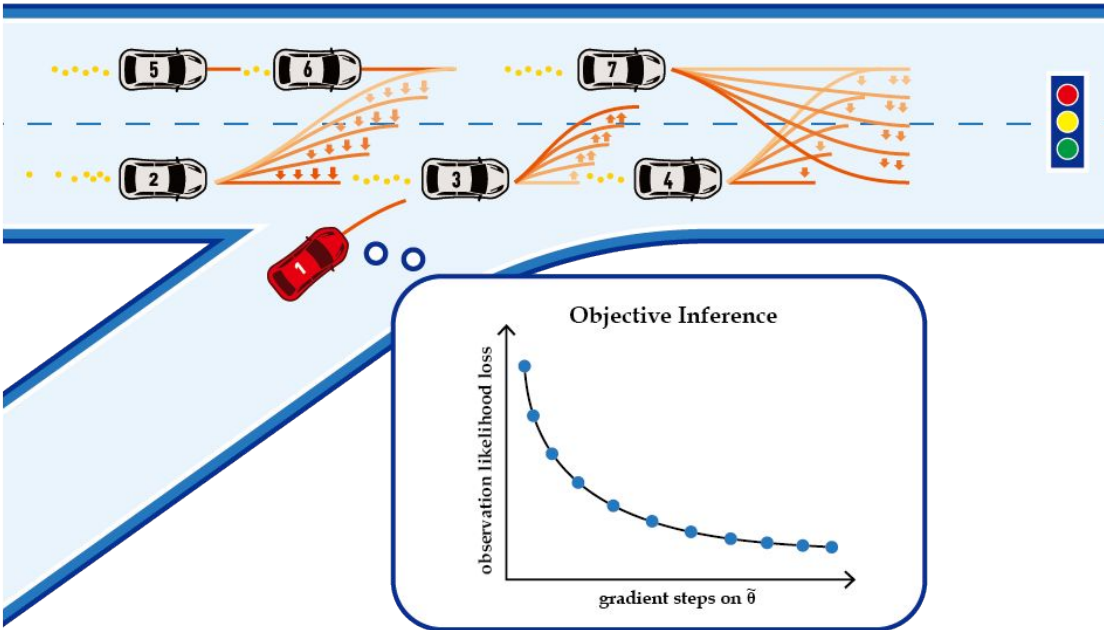
Learning to Play Trajectory Games Against Opponents with Unknown Objectives

Xinjie Liu

Cognitive Robotics (CoR)

x.liu-47@student.tudelft.nl

Motivation



- Autonomous driving: robots need to reason about interactions
- Our perspective: dynamic games (**explicit** modeling of interactions, **simultaneous** predicting and planning)
- Alternative: predict-then-plan

“Forward” Dynamic Games

An N-player open-loop Nash game as coupled trajectory optimization:

$$\forall i \in [N] \left\{ \begin{array}{ll} \min_{X^i, U^i} & J^i(\mathbf{X}, U^i; \theta^i) & \text{cost function} \\ \text{s.t.} & x_{t+1}^i = f^i(x_t^i, u_t^i), \forall t \in [T-1] & \text{system dynamics} \\ & x_1^i = \hat{x}_1^i & \text{initial states} \\ & p g^i(X^i, U^i) \geq 0 & \text{private inequalities} \\ & {}^s g(\mathbf{X}, \mathbf{U}) \geq 0 & \text{shared inequalities} \end{array} \right.$$

Solution: generalized Nash equilibrium (GNE)

$$J^i(\mathbf{X}^*, U^{i*}; \theta^i) \leq J^i((X^i, \mathbf{X}^{-i*}), U^i; \theta^i)$$

No **unilateral** change of controls can reduce a player's costs.

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“Forward” Dynamic Games

An N-player open-loop Nash game as coupled trajectory optimization:

Partially observable stochastic game (POSG), generally **intractable!!**

$$\forall i \in [N] \left\{ \begin{array}{ll} \min_{X^i, U^i} J^i(\mathbf{X}, U^i; \theta^i) & \text{cost function} \\ \text{s.t. } x_{t+1}^i = f^i(x_t^i, u_t^i), \forall t \in [T-1] & \text{system dynamics} \\ x_1^i = \hat{x}_1^i & \text{initial states} \\ p g^i(X^i, U^i) \geq 0 & \text{private inequalities} \\ {}^s g(\mathbf{X}, \mathbf{U}) \geq 0 & \text{shared inequalities} \end{array} \right.$$

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Inverse Games

$$\begin{aligned} & \max_{\theta, \mathbf{X}, \mathbf{U}} \quad \overset{\text{observation}}{p(\overset{\frown}{\mathbf{Y}} \mid \mathbf{X}, \mathbf{U})} \\ & \text{s.t.} \quad (\mathbf{X}, \mathbf{U}) \text{ is a GNE of the game } \Gamma(\theta) \end{aligned}$$

Applications (w.r.t. explicit modeling of the interactions):

- Online interaction with other agents (POSG approximation)
- Trajectory prediction
- Tuning of the ego-agent's controller to match desired behavior (similar to inverse RL)

Inverse Games

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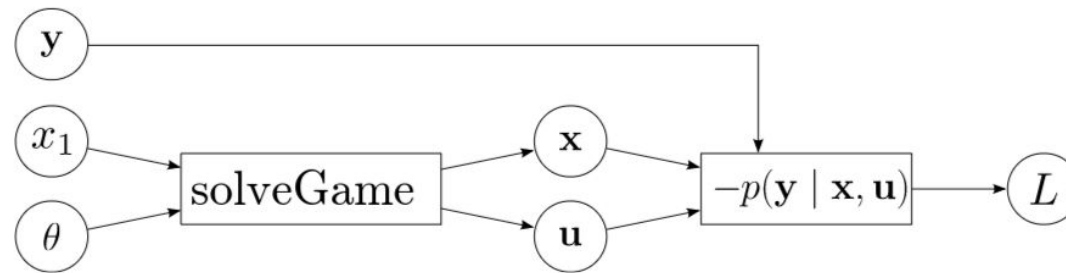
optimality conditions of a forward game

Challenge: how to efficiently encode the equilibrium constraints?

- Highly nonlinear
- Naive encoding violates constraint qualification ($\lambda^\top g(\mathbf{X}, \mathbf{U}) = 0$)
- Real-time computation

Approach

The Forward Computation Graph

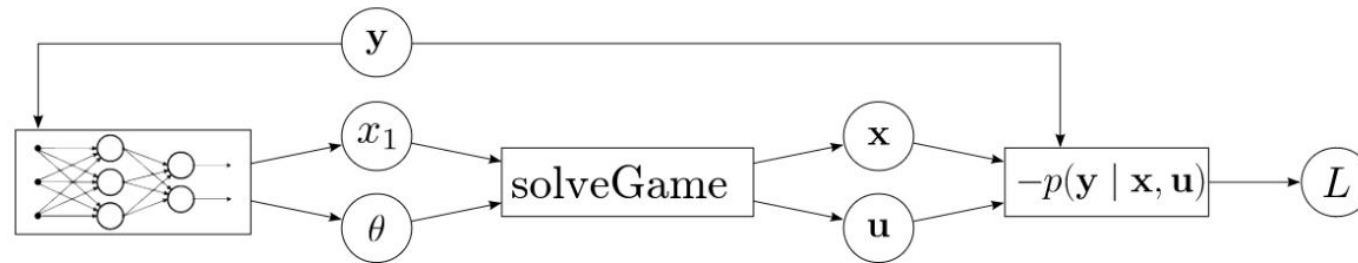


This entire computation graph can be made [differentiable](#)!

⇒ We can update estimates of θ and x_1 via [gradient descent](#) on the loss function.

Approach

Differentiable Games | Extensions



The gradient signal can be back-propagated to a [neural network](#), which learns to [predict the game parameters](#).

Approach: “Forward” Games as Mixed Complementarity Problems (MCPs)

In a *Mixed Complementarity Problem*, we have decision variable $z \in \mathbb{R}^n$ and problem data $F(z) : \mathbb{R}^n \mapsto \mathbb{R}^n, \ell_j \in \mathbb{R} \cup \{-\infty\}, u_j \in \mathbb{R} \cup \{\infty\}, j \in [n]$. At the solution z^* , *one* of the following cases holds:

$$\begin{aligned} z_j^* &= \ell_j, F_j(z^*) \geq 0 \\ \ell_j &< z_j^* < u_j, F_j(z^*) = 0 \\ z_j^* &= u_j, F_j(z^*) \leq 0. \end{aligned}$$

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Optimality conditions of a **game** can be cast as the solution to an equivalent **MCP**!

$$\forall i \in [N] \begin{cases} \nabla_{(X^i, U^i)} \mathcal{L}^i(\mathbf{X}, \mathbf{U}, \mu^i, {}^p\lambda^i, {}^s\lambda; \theta) = 0 \\ 0 \leq {}^p g^i(X^i, U^i) \perp {}^p\lambda^i \geq 0 \\ h(\mathbf{X}, \mathbf{U}; \hat{\mathbf{x}}_1) = 0 \\ 0 \leq {}^s g(\mathbf{X}, \mathbf{U}) \perp {}^s\lambda \geq 0, \end{cases} \longrightarrow z = \begin{bmatrix} X^1 \\ U^1 \\ \vdots \\ X^N \\ U^N \\ \mu \\ {}^p\lambda^1 \\ \vdots \\ {}^p\lambda^N \\ {}^s\lambda \end{bmatrix} F(z; \theta) = \begin{bmatrix} \nabla_{(X^1, U^1)} \mathcal{L}^1(\mathbf{X}, \mathbf{U}, \mu^1, {}^p\lambda^1, {}^s\lambda; \theta) \\ \vdots \\ \nabla_{(X^N, U^N)} \mathcal{L}^N(\mathbf{X}, \mathbf{U}, \mu^N, {}^p\lambda^N, {}^s\lambda; \theta) \\ h(\mathbf{X}, \mathbf{U}; \hat{\mathbf{x}}_1) \\ {}^p g^1(X^1, U^1) \\ \vdots \\ {}^p g^N(X^N, U^N) \\ {}^s g(\mathbf{X}, \mathbf{U}) \end{bmatrix} \ell = \begin{bmatrix} -\infty \\ \vdots \\ -\infty \\ -\infty \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u = \begin{bmatrix} \infty \\ \vdots \\ \infty \\ \infty \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

Approach: Differentiation Through Mixed Complementarity Problems (MCPs)

θ — **SolveMCP** — z^*

In a *Mixed Complementarity Problem*, we have decision variable $z \in \mathbb{R}^n$ and problem data $F(z) : \mathbb{R}^n \mapsto \mathbb{R}^n, \ell_j \in \mathbb{R} \cup \{-\infty\}, u_j \in \mathbb{R} \cup \{\infty\}, j \in [n]$. At the solution z^* , *one* of the following cases holds:

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Implicit function theorem:

$$0 = \nabla_{\theta} [\bar{F}(z^*(\theta), \theta)] = \nabla_{\theta} \bar{F} + \underbrace{(\nabla_{\bar{z}^*} \bar{F})(\nabla_{\theta} \tilde{z}^*)}_{\equiv 0} + (\nabla_{\bar{z}^*} \bar{F})(\nabla_{\theta} \tilde{z}^*) \longrightarrow \nabla_{\theta} \tilde{z}^* = -(\nabla_{\bar{z}^*} \bar{F})^{-1} (\nabla_{\theta} \bar{F})$$

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Weak complementarity: subgradient
Invertibility: least-square solution

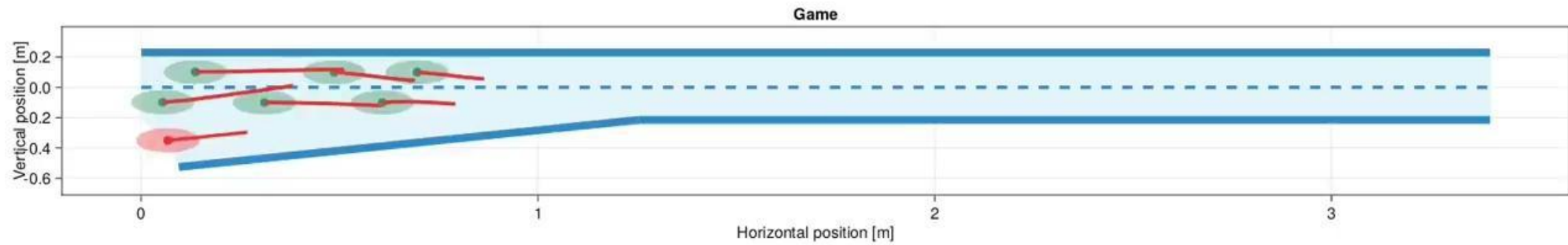
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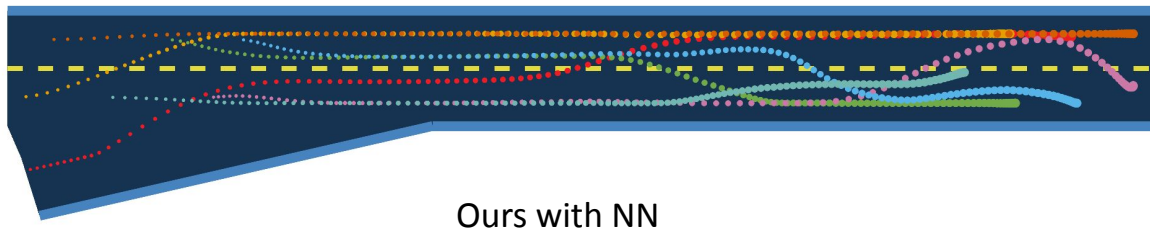
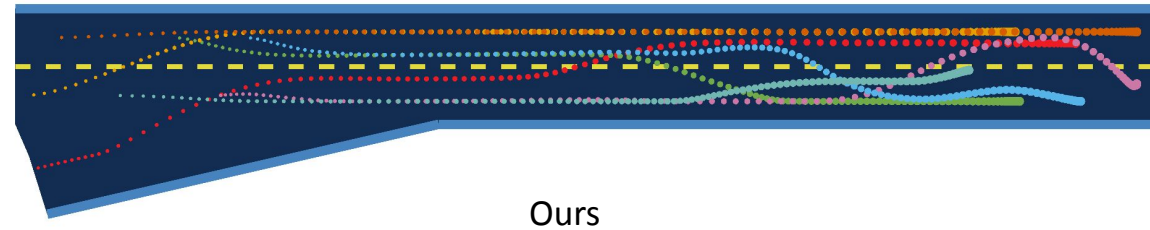
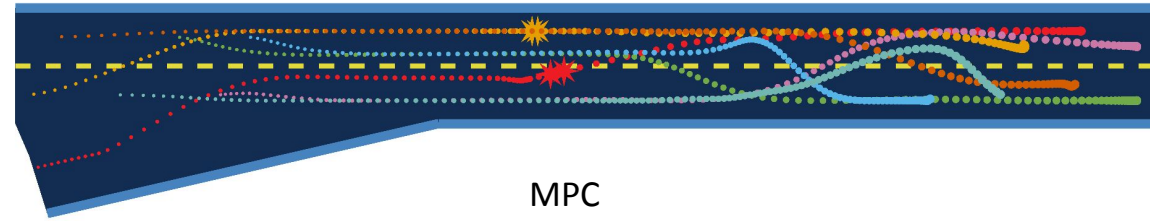
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Example: 7-player Highway Driving

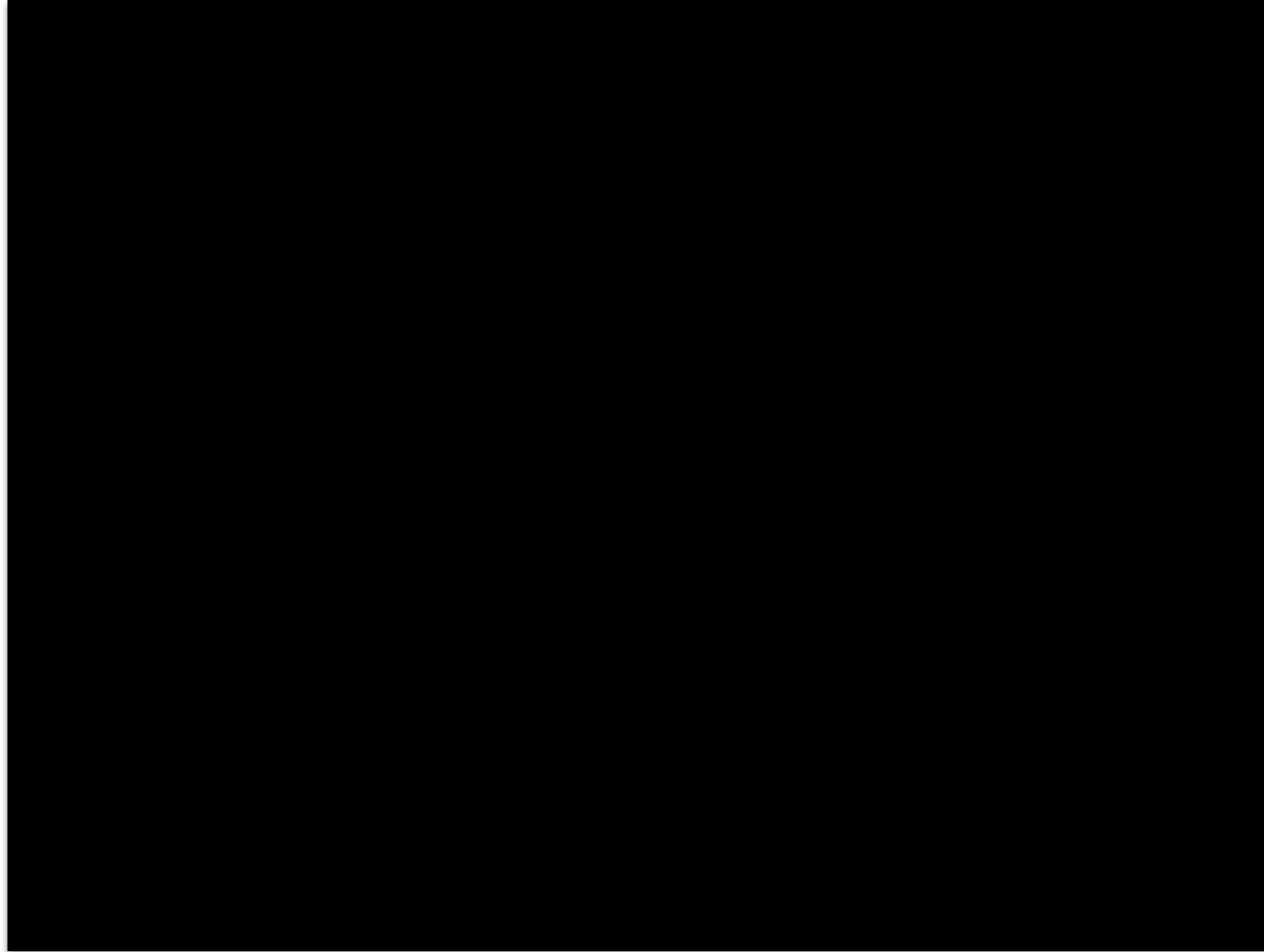


Example: 7-player Highway Driving



Example: Guiding-Tracking Game on Jackals

2x



Example: Human-Robot Interaction

1x



Future Work

- Integrated end-to-end planning with perception module (picking up additional visual cues, such as gaze or body language, for inference)
- Robustness against uncertainty (reasoning about the inference confidence)

Collaborators



Lasse Peters



Javier Alonso-Mora

Thanks for your attention! :-D