# Learning to Play Trajectory Games Against Opponents with Unknown Objectives

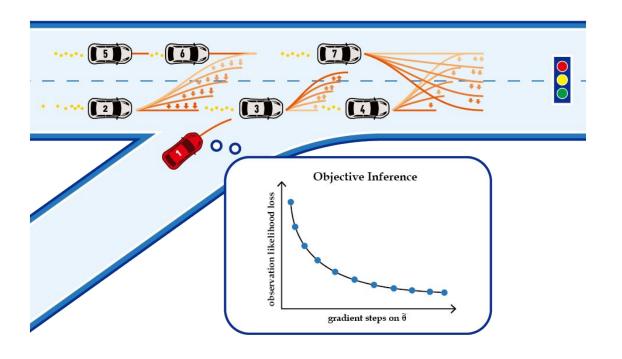
Xinjie Liu

Cognitive Robotics (CoR)

x.liu-47@student.tudelft.nl



#### **Motivation**



- Autonomous driving: robots need to reason about interactions
- Our perspective: dynamic games (explicit modeling of interactions, simultaneous predicting and planning)
- Alternative: predict-then-plan



#### "Forward" Dynamic Games

An N-player open-loop Nash game as coupled trajectory optimization:

$$\forall i \in [N] \begin{cases} \min_{X^i,U^i} & J^i(\mathbf{X},U^i;\theta^i) & \text{cost function} \\ \text{s.t.} & x^i_{t+1} = f^i(x^i_t,u^i_t), \forall t \in [T-1] & \text{system dynamics} \\ & x^i_1 = \hat{x}^i_1 & \text{initial states} \\ & {}^pg^i(X^i,U^i) \geq 0 & \text{private inequalities} \\ & {}^sg(\mathbf{X},\mathbf{U}) \geq 0 & \text{shared inequalities} \end{cases}$$

Solution: generalized Nash equilibrium (GNE)

$$J^i(\mathbf{X}^*, U^{i*}; \theta^i) \leq J^i((X^i, \mathbf{X}^{\neg i*}), U^i; \theta^i)$$



No unilateral change of controls can reduce a player's costs.

#### "Forward" Dynamic Games

An N-player open-loop Nash game as coupled trajectory optimization:

$$\forall i \in [N] \begin{cases} \min_{X^i,U^i} & J^i(\mathbf{X},U^i; \pmb{\theta^i}) & \text{cost function} \\ \text{s.t.} & x^i_{t+1} = f^i(x^i_t,u^i_t), \forall t \in [T-1] & \text{system dynamics} \\ & x^i_1 = \hat{x}^i_1 & \text{initial states} \\ & ^pg^i(X^i,U^i) \geq 0 & \text{private inequalities} \\ & ^sg(\mathbf{X},\mathbf{U}) \geq 0 & \text{shared inequalities} \end{cases}$$

Solution: generalized Nash equilibrium (GNE)

$$J^i(\mathbf{X}^*, U^{i*}; \theta^i) \leq J^i((X^i, \mathbf{X}^{\neg i*}), U^i; \theta^i)$$



No unilateral change of controls can reduce a player's costs.

#### "Forward" Dynamic Games

An N-player open-loop Nash game as coupled trajectory optimization:

Partially observable stochastic game (POSG), generally intractable!!

$$\forall i \in [N] \begin{cases} \min_{X^i,U^i} & J^i(\mathbf{X},U^i; \pmb{\theta^i}) & \text{cost function} \\ \text{s.t.} & x_{t+1}^i = f^i(x_t^i,u_t^i), \forall t \in [T-1] & \text{system dynamics} \\ & x_1^i = \hat{x}_1^i & \text{initial states} \\ & {}^pg^i(X^i,U^i) \geq 0 & \text{private inequalities} \\ & {}^sg(\mathbf{X},\mathbf{U}) \geq 0 & \text{shared inequalities} \end{cases}$$

Solution: generalized Nash equilibrium (GNE)

$$J^i(\mathbf{X}^*, U^{i*}; \theta^i) \leq J^i((X^i, \mathbf{X}^{\neg i*}), U^i; \theta^i)$$



No unilateral change of controls can reduce a player's costs.

#### **Inverse Games**

$$\max_{\boldsymbol{\theta}, \mathbf{X}, \mathbf{U}} \quad p(\mathbf{\hat{Y}} \mid \mathbf{X}, \mathbf{U})$$
 s.t.  $(\mathbf{X}, \mathbf{U})$  is a GNE of the game  $\Gamma(\boldsymbol{\theta})$ 

**Applications** (w.r.t. explicit modeling of the interactions):

- Online interaction with other agents (POSG approximation)
- Trajectory prediction
- Tuning of the ego-agent's controller to match desired behavior (similar to inverse RL)



#### **Inverse Games**

$$\max_{\boldsymbol{\theta}, \mathbf{X}, \mathbf{U}} \begin{array}{c} \text{observation} \\ p(\mathbf{\hat{Y}} \mid \mathbf{X}, \mathbf{U}) \\ \text{s.t.} \end{array}$$
 s.t. 
$$(\mathbf{X}, \mathbf{U}) \text{ is a GNE of the game } \Gamma(\boldsymbol{\theta})$$

optimality conditions of a forward game

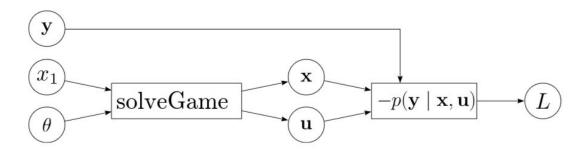
**Challenge:** how to efficiently encode the equilibrium constraints?

- Highly nonlinear
- Naive encoding violates constraint qualification ( $\lambda^{\top} g(\mathbf{X}, \mathbf{U}) = 0$ )
- Real-time computation



### **Approach**

#### **The Forward Computation Graph**



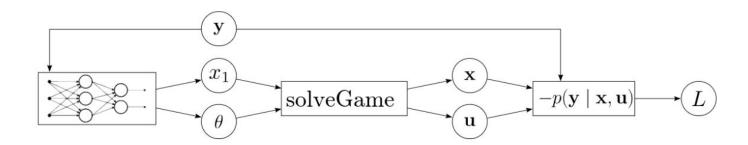
This entire computation graph can be made differentiable!

 $\Rightarrow$  We can update estimates of  $\theta$  and x1 via gradient descent on the loss function.



### **Approach**

#### **Differentiable Games | Extensions**



The gradient signal can be back-propagated to a neural network, which learns to predict the game parameters.



#### Approach: "Forward" Games as Mixed Complementarity Problems (MCPs)

$$z_j^* = \ell_j, F_j(z^*) \ge 0$$
  
$$\ell_j < z_j^* < u_j, F_j(z^*) = 0$$
  
$$z_j^* = u_j, F_j(z^*) \le 0.$$



#### Approach: "Forward" Games as Mixed Complementarity Problems (MCPs)

In a *Mixed Complementarity Problem*, we have decision variable  $z \in \mathbb{R}^n$  and problem data  $F(z): \mathbb{R}^n \mapsto \mathbb{R}^n, \ell_j \in \mathbb{R} \cup \{-\infty\}, u_j \in \mathbb{R} \cup \{\infty\}, j \in [n]$ . At the solution  $z^*$ , one of the following cases holds:

$$z_j^* = \ell_j, F_j(z^*) \ge 0$$
  
$$\ell_j < z_j^* < u_j, F_j(z^*) = 0$$
  
$$z_j^* = u_j, F_j(z^*) \le 0.$$

Optimality conditions of a game can be cast as the solution to an equivalent MCP!

$$\forall i \in [N] \begin{cases} \nabla_{(X^{i},U^{i})} \mathcal{L}^{i}(\mathbf{X}, \mathbf{U}, \mu^{i}, {}^{p}\lambda^{i}, {}^{s}\lambda; \theta) = 0 \\ 0 \leq {}^{p}g^{i}(X^{i}, U^{i}) \perp {}^{p}\lambda^{i} \geq 0 \\ h(\mathbf{X}, \mathbf{U}; \hat{\mathbf{x}}_{1}) = 0 \\ 0 \leq {}^{s}g(\mathbf{X}, \mathbf{U}) \perp {}^{s}\lambda \geq 0, \end{cases} \qquad z = \begin{bmatrix} \begin{pmatrix} X^{i} \\ U^{1} \\ \vdots \\ X^{N} \\ U^{N} \\ \mu \\ p_{\lambda^{1}} \\ \vdots \\ p_{\lambda^{N} \\ s\lambda} \end{bmatrix} F(z; \theta) = \begin{bmatrix} \nabla_{(X^{1},U^{1})} \mathcal{L}^{i}(\mathbf{X}, \mathbf{U}, \mu^{i}, p_{\lambda^{1}}, s_{\lambda}; \theta) \\ \vdots \\ \nabla_{(X^{N},U^{N})} \mathcal{L}^{N}(\mathbf{X}, \mathbf{U}, \mu^{i}, p_{\lambda^{N}}, s_{\lambda}; \theta) \\ h(\mathbf{X}, \mathbf{U}; \hat{\mathbf{x}}_{1}) \\ pg^{1}(X^{1}, U^{1}) \\ \vdots \\ 0 \\ 0 \end{bmatrix} u = \begin{bmatrix} -\infty \\ \vdots \\ -\infty \\ -\infty \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$





$$\begin{aligned} z_j^*(\theta) &= \ell_j, F_j(z^*(\theta); \theta) \ge 0 \\ \ell_j &< z_j^*(\theta) < u_j, F_j(z^*(\theta); \theta) = 0 \\ z_j^*(\theta) &= u_j, F_j(z^*(\theta); \theta) \le 0. \end{aligned}$$



$$\theta$$
 — SolveMCP —  $z^*$ 

#### Assumption: strong complementarity

$$z_j^*(\theta) = \ell_j, F_j(z^*(\theta); \theta) \ge 0$$
  
$$\ell_j < z_j^*(\theta) < u_j, F_j(z^*(\theta); \theta) = 0$$
  
$$z_j^*(\theta) = u_j, F_j(z^*(\theta); \theta) \le 0.$$



$$\theta$$
 — SolveMCP —  $z^*$ 

#### Assumption: strong complementarity

$$z_{j}^{*}(\theta) = \ell_{j}, F_{j}(z^{*}(\theta); \theta) > 0$$
  

$$\ell_{j} < z_{j}^{*}(\theta) < u_{j}, F_{j}(z^{*}(\theta); \theta) = 0$$
  

$$z_{j}^{*}(\theta) = u_{j}, F_{j}(z^{*}(\theta); \theta) < 0.$$



$$\theta$$
 — SolveMCP —  $z^*$ 

Assumption: strong complementarity

$$z_j^*(\theta) = \ell_j, F_j(z^*(\theta); \theta) > 0$$

$$\ell_j < z_j^*(\theta) < u_j, F_j(z^*(\theta); \theta) = 0$$

$$z_j^*(\theta) = u_j, F_j(z^*(\theta); \theta) < 0.$$

$$\nabla_{\theta} \tilde{z}^* = 0$$



$$\theta$$
 — SolveMCP —  $z^*$ 

Assumption: strong complementarity



$$\theta$$
 — SolveMCP —  $z^*$ 

#### Assumption: strong complementarity

In a *Mixed Complementarity Problem*, we have decision variable  $z \in \mathbb{R}^n$  and problem data  $F(z): \mathbb{R}^n \mapsto \mathbb{R}^n, \ell_j \in \mathbb{R} \cup \{-\infty\}, u_j \in \mathbb{R} \cup \{\infty\}, j \in [n]$ . At the solution  $z^*$ , one of the following cases holds:

$$z_j^*(\theta) = \ell_j, F_j(z^*(\theta); \theta) > 0$$

$$\ell_j < z_j^*(\theta) < u_j, F_j(z^*(\theta); \theta) = 0$$

$$z_j^*(\theta) = u_j, F_j(z^*(\theta); \theta) < 0.$$

$$\nabla_{\theta} \tilde{z}^* = 0$$

Implicit function theorem:

$$0 = \nabla_{\theta} \left[ \bar{F}(z^{*}(\theta), \theta) \right] = \\ \nabla_{\theta} \bar{F} + (\nabla_{\bar{z}^{*}} \bar{F})(\nabla_{\theta} \bar{z}^{*}) + (\nabla_{\bar{z}^{*}} \bar{F}) \underbrace{(\nabla_{\theta} \tilde{z}^{*})}_{\equiv 0} \longrightarrow \nabla_{\theta} \bar{z}^{*} = -(\nabla_{\bar{z}^{*}} \bar{F})^{-1} (\nabla_{\theta} \bar{F})$$



$$\theta$$
 — SolveMCP —  $z^*$ 

Weak complementarity: subgradient Invertibility: least-square solution

In a *Mixed Complementarity Problem*, we have decision variable  $z \in \mathbb{R}^n$  and problem data  $F(z): \mathbb{R}^n \mapsto \mathbb{R}^n, \ell_j \in \mathbb{R} \cup \{-\infty\}, u_j \in \mathbb{R} \cup \{\infty\}, j \in [n]$ . At the solution  $z^*$ , one of the following cases holds:

$$z_j^*(\theta) = \ell_j, F_j(z^*(\theta); \theta) > 0$$

$$\ell_j < z_j^*(\theta) < u_j, F_j(z^*(\theta); \theta) = 0$$

$$z_j^*(\theta) = u_j, F_j(z^*(\theta); \theta) < 0.$$

$$\nabla_{\theta} \tilde{z}^* = 0$$

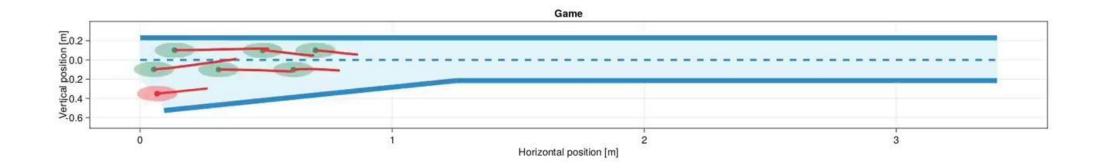
Implicit function theorem:

$$0 = \nabla_{\theta} \left[ \bar{F}(z^{*}(\theta), \theta) \right] =$$

$$\nabla_{\theta} \bar{F} + (\nabla_{\bar{z}^{*}} \bar{F})(\nabla_{\theta} \bar{z}^{*}) + (\nabla_{\bar{z}^{*}} \bar{F}) \underbrace{(\nabla_{\theta} \tilde{z}^{*})}_{\equiv 0} \longrightarrow \nabla_{\theta} \bar{z}^{*} = -(\nabla_{\bar{z}^{*}} \bar{F})^{-1} (\nabla_{\theta} \bar{F})$$

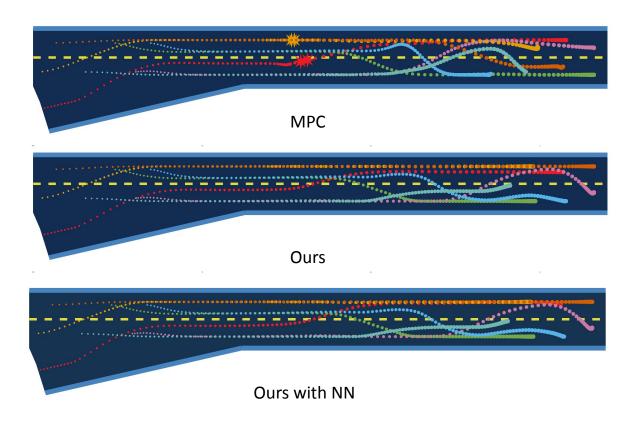


## **Example: 7-player Highway Driving**





## **Example: 7-player Highway Driving**





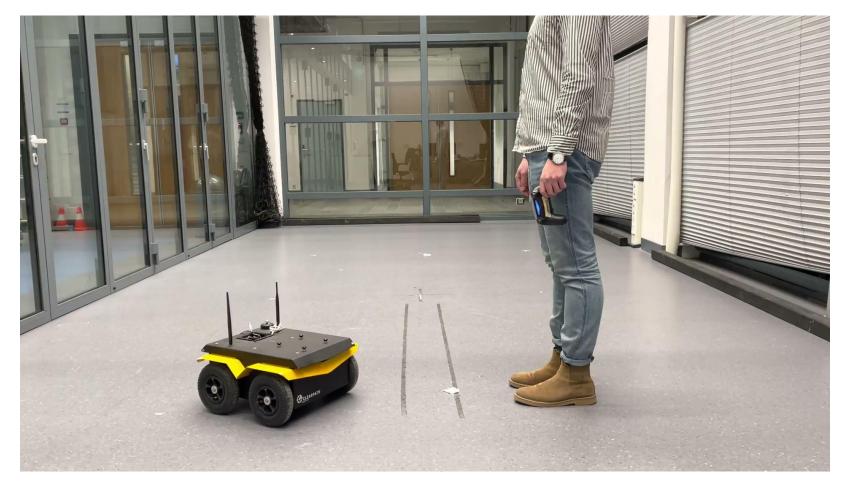
## **Example: Guiding-Tracking Game on Jackals**





## **Example: Human-Robot Interaction**

**1**x





#### **Future Work**

- Integrated end-to-end planning with perception module (picking up additional visual cues, such as gaze or body language, for inference)
- Robustness against uncertainty (reasoning about the inference confidence)



#### **Collaborators**



**Lasse Peters** 



Javier Alonso-Mora



## Thanks for your attention! :-D

