SDCA for Regularized Loss Minimization

Shai and Tong

2013 年 6 月 5 日

Xinkai

Main Contritbution

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Proving the convergence rate of duality gap for SDCA.

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We'll skip the proofs.

Idea

Recall SGD

$$\omega := \omega - c \bigtriangledown P_i(\omega),$$

where our problem is: $\min P(\omega)$

Then how about SDCA?

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For direction α_i, we find $\Delta\alpha_i$ that maximizes $D_i(\alpha_i^{t-1} + \Delta\alpha_i)$, where $D(\alpha) = \sum_i D_i(\alpha)$, and t is the loop index.

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Then let's "S" (dca)

Random without Repetition (permutation)

Random with Repetition

Cyclic

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Questions?

Exp.0: Effect of different “S”

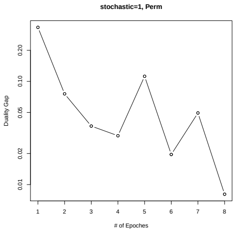

Figure: Perm

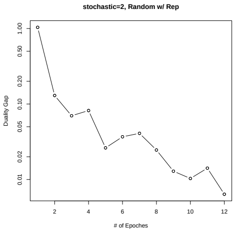

Figure: R w Rep

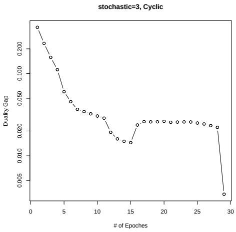

Figure: Cyclic

Exp.1: Convergence Rate of non-smooth Hinge

Result

For non-smooth hinge loss (we saw in class), the duality gap of SDCA should converge sub-linearly ($O(n + \frac{1}{\lambda\epsilon})$).

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We would expect $\cdots$

Conti Exp.1: Convergence Rate of non-smooth Hinge

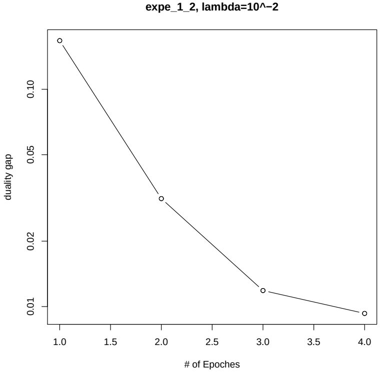

Figure: $\lambda = 10^{-2}$

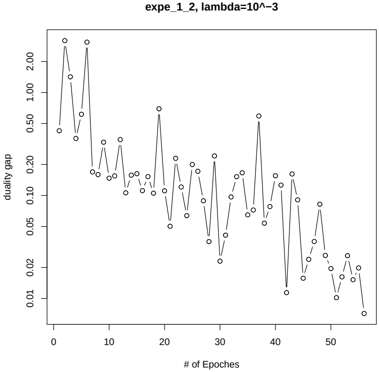

Figure: $\lambda = 10^{-3}$

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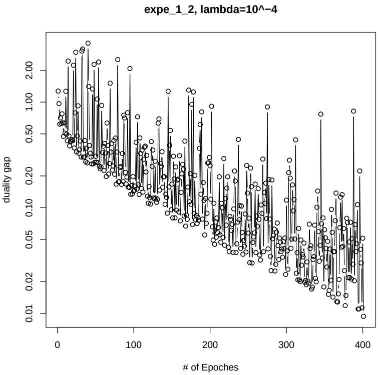

Figure: $\lambda = 10^{-4}$

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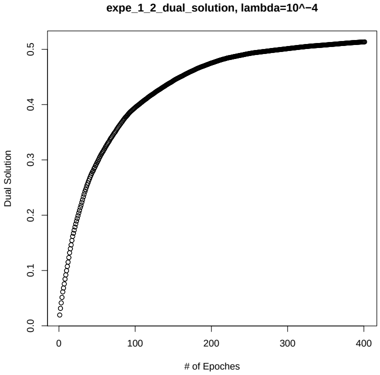

Figure: Dual Sol

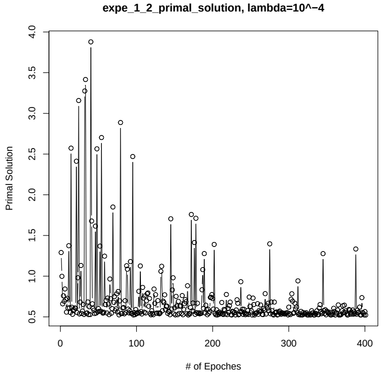

Figure: Primal Sol

Exp.2: Convergence Rate

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For smooth hinge loss, the duality gap of SDCA should converge linearly.

Have a look at code.

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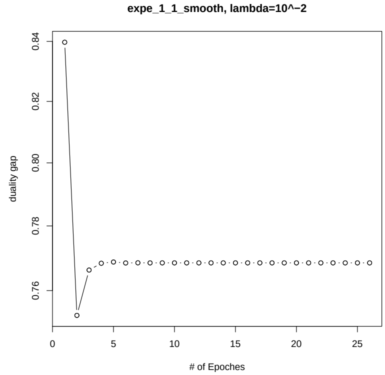

Duality Gap for Smooth Hinge.

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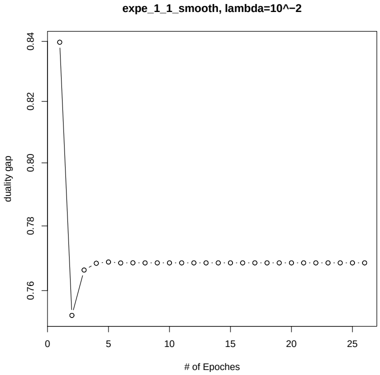

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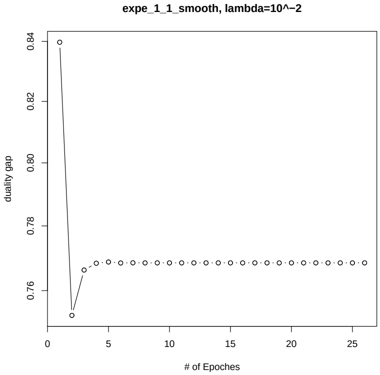

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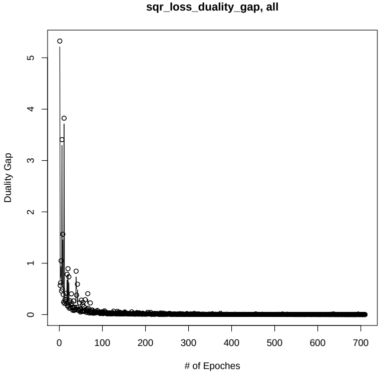
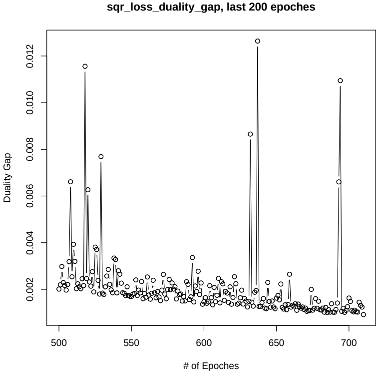


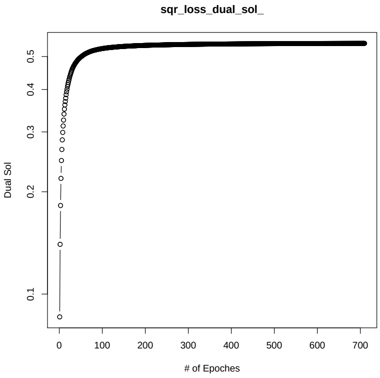
Duality Gap for Squared Loss

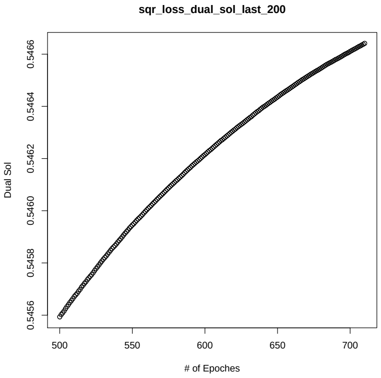
Conti Exp.3: Convergence Rate of squared loss



Telescoping the last 200 epoches

Conti Exp.3: Convergence Rate of squared loss, Dual Sol





Conti Exp.3: Convergence Rate of squared loss, Primal Sol