

## 06. Maximum and Minimum Values

Finding the maximum and/or minimum is a frequent query.

- Find the maximum value, minimum value in an array
- Find the  $k$ -th maximum/minimum value in an array
- Find the median value in an array

Exercise 1: Max and min values

Exercise 2: Max work weeks

Exercise 3: Max value at location

Exercise 4: Water container

Exercise 5: Photoshoot II (Feb 2022)

Exercise 6: Counting liars (Mar 2022)

## Max and min values

Given an array of integers, find the max and min values.

Input (from terminal / stdin)

- The first line contains integer  $N$ ,  $1 \leq N \leq 1000$ .
- The next line contains the  $N$  integers, all in the range  $[0, 1000]$ .

Output (to terminal / stdout)

- Report the max and min values, separated by a white space.

Sample input

```
5
6 2 4 7 1
```

Sample output

```
7 1
```

====

Given  $N$  points on the  $xy$ -plane, find the max and min distances (squared) to the origin.

Input (from terminal / stdin)

- The first line contains integer  $N$ ,  $1 \leq N \leq 1000$ .
- Each of the next  $N$  lines contain the  $x$ - and  $y$ -coordinates of a point, all coordinates are in the range  $[0, 1000]$ .

Output (to terminal / stdout)

- Report the max and min distances (squared), separated by a white space.

Sample input

```
5
6 1
2 3
4 5
7 0
1 4
```

Sample output

```
49 13
```

T&D: How do you find the max and min distance between any two points?

## Max work weeks

There are  $N$  projects numbered from 0 to  $N - 1$ . You are given an integer array  $M$  where each  $M_i$  denotes the number of milestones the  $i$ -th project has.

You can work on the projects following these two rules:

- Every week, you will finish exactly one milestone of one project. You must work every week.
- You cannot work on two milestones from the same project for two consecutive weeks.

Once all the milestones of all the projects are finished, or if the only milestones that you can work on will cause you to violate the above rules, you will stop working. Note that you may not be able to finish every project's milestones due to these constraints.

Report the maximum number of weeks you would be able to work on the projects without violating the rules mentioned above.

Input (from terminal / stdin)

- The first line contains integer  $N$ ,  $1 \leq N \leq 1000$ .
- The next line contains the  $N$  integers in array  $M$ , all in the range  $[0, 1e6]$ .

Output (to terminal / stdout)

- Report the max number of weeks you can work.

Sample input

```
3
1 2 3
```

Sample output

```
6
```

During the 1st week, you will work on a milestone of project 0.  
During the 2nd week, you will work on a milestone of project 2.  
During the 3rd week, you will work on a milestone of project 1.  
During the 4th week, you will work on a milestone of project 2.  
During the 5th week, you will work on a milestone of project 1.  
During the 6th week, you will work on a milestone of project 2.  
The total number of weeks is 6.

## Max value at location

You are given three positive integers  $N$ ,  $K$  and  $S$ . You want to construct an array  $A$  (0-indexed) that satisfies the following conditions:

- There are  $N$  positive integers in  $A$ .
- The absolute difference between two adjacent integers in  $A$  is at most 1.
- The sum of all the elements of  $A$  does not exceed  $S$ .
- The integer  $A_K$  is maximized.

Return  $A_K$  of the constructed array.

Input (from terminal / stdin)

- The only line contains integers  $N, K, S$ ,  $0 \leq K < N \leq S < 1e6$ .

Output (to terminal / stdout)

- Report the integer  $A_K$  satisfying the restrictions.

Sample input

4 2 6

Sample output

2

The array  $A$  can be  $\{1, 1, 2, 1\}$  or  $\{1, 2, 2, 1\}$ . The max value  $A_2$  is 2.

## Water container

You are given an integer array  $height$  of length  $N$ . There are  $N$  vertical lines drawn such that the two endpoints of the  $i$ -th line are  $(i, 0)$  and  $(i, H_i)$ .

Find two lines that together with the x-axis form a container, such that the container contains the most water.

Report the maximum amount of water a container can store.

Input (from terminal / stdin)

- The first line contains integer  $N$ ,  $1 \leq N \leq 1000$ .
- The next line contains the  $N$  heights, all in the range  $[0, 1000]$ .

Output (to terminal / stdout)

- Report the max amount of water a container can store.

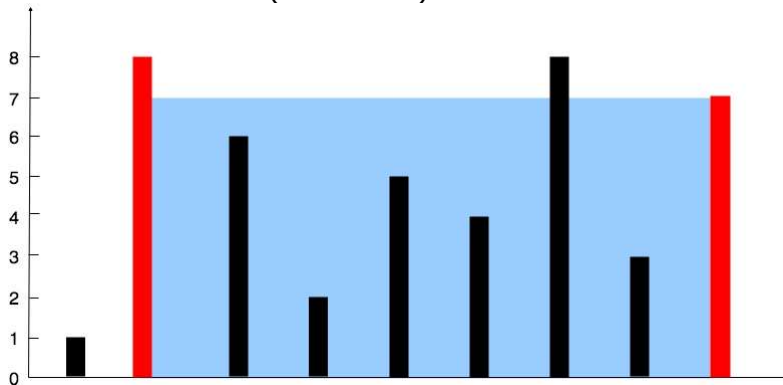
Sample input

```
9
1 8 6 2 5 4 8 3 7
```

Sample output

```
49
```

The max area of water (blue section) the container can contain is 49.



## Photo shoot II (Feb 2022)

In what seems to be a familiar occurrence, Farmer John is lining up his  $N$  cows ( $1 \leq N \leq 10^5$ ), conveniently numbered  $1 \dots N$ , for a photograph.

Initially, the cows are lined up in the order  $a_1, a_2, \dots, a_N$  from left to right. Farmer John's goal is to line up the cows in the order  $b_1, \dots, b_N$  from left to right. To accomplish this, he may perform a series of modifications to the ordering. Each modification consists of choosing a single cow and moving it some number of positions to the left.

Please count the minimum number of modifications required in order for Farmer John to line up his cows in the desired order.

INPUT FORMAT (input arrives from the terminal / stdin):

- The first line of input contains  $N$ . The second line contains  $a_1, a_2, \dots, a_N$ . The third line contains  $b_1, b_2, \dots, b_N$ .

OUTPUT FORMAT (print output to the terminal / stdout):

- Print the minimum number of modifications required to produce Farmer John's desired ordering.

SAMPLE INPUT:

```
5
1 2 3 4 5
1 2 3 4 5
```

SAMPLE OUTPUT:

```
0
```

In this example, the cows are already in the desired order, so no modifications are required.

SAMPLE INPUT:

```
5
5 1 3 2 4
4 5 2 1 3
```

SAMPLE OUTPUT:

```
2
```

In this example, two modifications suffice. Here is one way Farmer John can rearrange his cows:

Choose cow 4 and move it four positions to the left.

Choose cow 2 and move it two positions to the left.

```
5 1 3 2 4
-> 4 5 1 3 2
-> 4 5 2 1 3
```

SCORING:

Test cases 3-6 satisfy  $N \leq 100$ .

Test cases 7-10 satisfy  $N \leq 5000$ .

Test cases 11-14 satisfy no additional constraints.

## Counting liars (Mar 2022)

Bessie the cow is hiding somewhere along the number line. Each of Farmer John's  $N$  other cows ( $1 \leq N \leq 1000$ ) have a piece of information to share: the  $i$ -th cow either says that Bessie is hiding at some location less than or equal to  $p_i$ , or that Bessie is hiding at some location greater than or equal to  $p_i$  ( $0 \leq p_i \leq 1e9$ ).

Unfortunately, it is possible that no hiding location is consistent with the answers of all of the cows, meaning that not all of the cows are telling the truth. Count the minimum number of cows that must be lying.

INPUT FORMAT (input arrives from the terminal / stdin):

- The first line contains  $N$ .
- The next  $N$  lines each contain either L or G, followed by an integer  $p_i$ . L means that the  $i$ -th cow says that Bessie's hiding location is less than or equal to  $p_i$ , and G means that  $i$ -th cow says that Bessie's hiding location is greater than or equal to  $p_i$ .

OUTPUT FORMAT (print output to the terminal / stdout):

- The minimum number of cows that must be lying.

SAMPLE INPUT:

```
2
G 3
L 5
```

SAMPLE OUTPUT:

```
0
It is possible that no cow is lying.
```

SAMPLE INPUT:

```
2
G 3
L 2
```

SAMPLE OUTPUT:

```
1
At least one of the cows must be lying.
```