

# Team Reference Document

Heltion

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## 1 Template

### 1.1 .clang-format

```
1 BasedOnStyle: Google
2 IndentWidth: 4
3 ColumnLimit: 160
```

### 1.2 debug.cpp

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 template <class T, size_t size = tuple_size<T>::value>
6 string to_debug(T, string s = "") requires(not ranges::range<T>);
7
8 template<class T>
9 concept check = requires(T x, ostream &os) {
10     os << x;
11 };
12
13 template<check T>
```

```
14 string to_debug(T x) {
15     return static_cast<ostringstream>(ostringstream() << x).str();
16 }
17
18 string to_debug(ranges::range auto x, string s = "") requires(not is_same_v<
19     decltype(x), string>) {
20     for (auto xi : x) {
21         s += ", " + to_debug(xi);
22     }
23     return "[" + s.substr(s.empty() ? 0 : 2) + "]";
24 }
25
26 template <class T, size_t size>
27 string to_debug(T x, string s) requires(not ranges::range<T>) {
28     [&<size_t... I>(index_sequence<I...>) {
29         ((s += ", " + to_debug(get<I>(x))), ...);
30     }(make_index_sequence<size>());
31     return "{" + s.substr(s.empty() ? 0 : 2) + "}";
32 }
33
34 #define debug(...) [](auto... $){ ((cout << to_debug($) << " "), ...); cout
35     << endl; }("[" , __VA_ARGS__ , "]:", __VA_ARGS__)
```

### 1.3 gen.py

```
1 from random import *
2 n = 10000
3 s = 'qwertyuiopasdfghjklzxcvbnm'
4 for i in range(n):
5     print(choice(s), end = '')
6 print()
7 print(randint(0, 1), randint(1, n))
```

### 1.4 head.cpp

```
1 #pragma GCC optimize("Ofast", "inline", "unroll-loops")
2 #include <bits/stdc++.h>
3 using namespace std;
4 #define rep(i, a, n) for (int i = a; i <= n; i++)
5 #define per(i, a, n) for (int i = a; i >= n; i--)
6 #define pb push_back
7 #define eb emplace_back
8 #define all(x) (x).begin(), (x).end()
```

```

9  #define bit(x) (1ll << (x))
10 #define fi first
11 #define se second
12 #define SZ(x) ((int)(x).size())
13 using VI = vector<int>;
14 using PII = pair<int, int>;
15 using ll = long long;
16 using ull = unsigned long long;
17 using db = double;
18 using ldb = long double;
19 mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
20 // head
21
22 #ifdef DEBUG
23 #include "debug.cpp"
24 #else
25 #define debug(...) 42
26 #endif
27
28 void solve() {}
29 int main() {
30     cin.tie(nullptr)->sync_with_stdio(false);
31     cout << fixed << setprecision(16);
32     int tt = 1;
33     cin >> tt;
34     while (tt--) {
35         solve();
36     }
37 }

```

## 1.5 head-apiadu.cpp

---

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  #define rep(i,a,n) for (int i=a;i<n;i++)
4  #define per(i,a,n) for (int i=n-1;i>=a;i--)
5  #define pb push_back
6  #define eb emplace_back
7  #define mp make_pair
8  #define all(x) (x).begin(),(x).end()
9  #define fi first
10 #define se second
11 #define SZ(x) ((int)(x).size())

```

```

12 typedef vector<int> VI;
13 typedef basic_string<int> BI;
14 typedef long long ll;
15 typedef pair<int,int> PII;
16 typedef double db;
17 mt19937 mrand(random_device{}());
18 const ll mod=1000000007;
19 int rnd(int x) { return mrand() % x;}
20 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b>=>=1){if(b&1)
    res=res*a%mod;a=a*a%mod;}return res;}
21 ll gcd(ll a,ll b) { return b?gcd(b,a%b):a;}
22 // head

```

## 1.6 Makefile

---

```

1  %: %.cpp
2      g++-13 $< -o $$@ -std=gnu++20 -g -O2 -Wall -Wextra -DDEBUG -
        D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC

```

## 1.7 pai.py

---

```

1  import os
2  tt = 0
3  while True:
4      os.system('python\gen.py>\A.in')
5      os.system('./a\<\A.in>\a.out')
6      os.system('./b\<\A.in>\b.out')
7      # diff for linux or macos, fc for windows
8      if os.system('diff\<a.out\b.out'):
9          print("WA")
10         exit(0)
11     else:
12         tt += 1
13         print("AC:", tt)

```

## 1.8 settings.json

---

```

1  {
2      "editor.formatOnSave": true,
3      "files.autoSave": "afterDelay",
4      "files.autoSaveDelay": 350,
5      "C_Cpp.default.cppStandard": "gnu++20"
6  }

```

## 2 Data

### 2.1 01trie.cpp

---

```
1 struct node {
2     int son[2];
3     int end;
4     int sz;
5 } seg[maxn << 2];
6 int root, tot;
7 int n, m;
8
9 void insert(ll x) {
10     int cnt = root;
11     for (int i = 62; i >= 0; i--) {
12         int w = (x >> i) & 1;
13         if (seg[cnt].son[w] == 0) seg[cnt].son[w] = ++tot;
14         cnt = seg[cnt].son[w];
15         seg[cnt].sz++;
16     }
17     seg[cnt].end++;
18 }
19
20 ll query(ll x, ll k) {
21     ll res = 0;
22     int cnt = root;
23     for (int i = 62; i >= 0; i--) {
24         int w = (x >> i) & 1;
25         if (seg[seg[cnt].son[w]].sz >= k) cnt = seg[cnt].son[w];
26         else {
27             k -= seg[seg[cnt].son[w]].sz;
28             cnt = seg[cnt].son[abs(w - 1)];
29             res += bit(i);
30         }
31     }
32     return res;
33 }
```

### 2.2 2dtree(bqi343).cpp

---

```
1 const int SZ = 1.1e5;
2 template <class T>
3 struct node {
```

```
4     T val = 0;
5     node<T>* c[2];
6     node() { c[0] = c[1] = NULL; }
7     void upd(int ind, T v, int L = 0, int R = SZ - 1) { // add v
8         if (L == ind && R == ind) {
9             val += v;
10            return;
11        }
12        int M = (L + R) / 2;
13        if (ind <= M) {
14            if (!c[0]) c[0] = new node();
15            c[0]->upd(ind, v, L, M);
16        } else {
17            if (!c[1]) c[1] = new node();
18            c[1]->upd(ind, v, M + 1, R);
19        }
20        val = 0;
21        rep(i, 0, 1) if (c[i]) val += c[i]->val;
22    }
23    T query(int lo, int hi, int L = 0, int R = SZ - 1) { // query sum of
24        segment
25        if (hi < L || R < lo) return 0;
26        if (lo <= L && R <= hi) return val;
27        int M = (L + R) / 2;
28        T res = 0;
29        if (c[0]) res += c[0]->query(lo, hi, L, M);
30        if (c[1]) res += c[1]->query(lo, hi, M + 1, R);
31        return res;
32    }
33    void UPD(int ind, node* c0, node* c1, int L = 0, int R = SZ - 1) { //
34        for 2D segtree
35        if (L != R) {
36            int M = (L + R) / 2;
37            if (ind <= M) {
38                if (!c[0]) c[0] = new node();
39                c[0]->UPD(ind, c0 ? c0->c[0] : NULL, c1 ? c1->c[0] : NULL, L
40                    , M);
41            } else {
42                if (!c[1]) c[1] = new node();
43                c[1]->UPD(ind, c0 ? c0->c[1] : NULL, c1 ? c1->c[1] : NULL, M
44                    + 1, R);
45            }
46        }
47    }
```

```

43     val = (c0 ? c0->val : 0) + (c1 ? c1->val : 0);
44 }
45 };
46
47 /**
48  * Description: BIT of SegTrees. $x\in (0,SZ), y\in [0,SZ)$.
49  * Memory:  $O(N\log^2 N)$ 
50  * Source: USACO Mowing the Field
51  * Verification:
52  * USACO Mowing the Field
53  * http://www.usaco.org/index.php?page=viewproblem2&cpid=722 (13/15, 15/15
    and 1857ms with BumpAllocator)
54 */
55
56 #include "../1DRangeQueries/9.2/SparseSeg/9.2.h"
57
58 template <class T>
59 struct BITseg {
60     node<T> seg[SZ];
61     BITseg() { fill(seg, seg + SZ, node<T>()); }
62     void upd(int x, int y, int v) { // add v
63         for (; x < SZ; x += x & -x) seg[x].upd(y, v);
64     }
65     T query(int x, int yl, int yr) {
66         T res = 0;
67         for (; x; x -= x & -x) res += seg[x].query(yl, yr);
68         return res;
69     }
70     T query(int xl, int xr, int yl, int yr) { // query sum of rectangle
71         return query(xr, yl, yr) - query(xl - 1, yl, yr);
72     }
73 };
74
75 /**
76  * Description: SegTree of SegTrees. $x,y\in [0,SZ)$.
77  * Memory:  $O(N\log^2 N)$ 
78  * Source: USACO Mowing the Field
79  * Verification:
80  * http://www.usaco.org/index.php?page=viewproblem2&cpid=722 (9/15 w/
    BumpAllocator)
81  * http://www.usaco.org/index.php?page=viewproblem2&cpid=601 (4238 ms, 2907
    ms w/ BumpAllocator)
82 */

```

```

83
84 #include "../1DRangeQueries/9.2/SparseSeg/9.2.h"
85
86 template <class T>
87 struct Node {
88     node<T> seg;
89     Node* c[2];
90     Node() { c[0] = c[1] = NULL; }
91     void upd(int x, int y, T v, int L = 0, int R = SZ - 1) { // add v
92         if (L == x && R == x) {
93             seg.upd(y, v);
94             return;
95         }
96         int M = (L + R) / 2;
97         if (x <= M) {
98             if (!c[0]) c[0] = new Node();
99             c[0]->upd(x, y, v, L, M);
100         } else {
101             if (!c[1]) c[1] = new Node();
102             c[1]->upd(x, y, v, M + 1, R);
103         }
104         seg.upd(y, v); // only for addition
105         // seg.UPD(y, c[0]?&c[0]->seg:NULL, c[1]?&c[1]->seg:NULL);
106     }
107     T query(int x1, int x2, int y1, int y2, int L = 0, int R = SZ - 1) { //
        query sum of rectangle
108         if (x1 <= L && R <= x2) return seg.query(y1, y2);
109         if (x2 < L || R < x1) return 0;
110         int M = (L + R) / 2;
111         T res = 0;
112         if (c[0]) res += c[0]->query(x1, x2, y1, y2, L, M);
113         if (c[1]) res += c[1]->query(x1, x2, y1, y2, M + 1, R);
114         return res;
115     }
116 };

```

## 2.3 cdq.cpp

```

1 int ans[maxn], lev[maxn];
2 array<int, 5> v[maxn], tmp[maxn];
3
4 struct BIT {
5     ...

```

```

6 } c;
7
8 void solve(int l, int r) {
9     if (l >= r) return;
10    int mid = (l + r) / 2;
11    solve(l, mid), solve(mid + 1, r);
12    int i = l, j = mid + 1;
13    int piv = l;
14    while (i <= mid || j <= r) {
15        if (i <= mid && (j > r || mp(v[i][1], v[i][2]) <= mp(v[j][1], v[j]
16            ] [2]))) {
17            c.modify(v[i][2], v[i][3]);
18            tmp[piv++] = v[i++];
19        } else {
20            v[j][4] += c.query(v[j][2]);
21            tmp[piv++] = v[j++];
22        }
23    }
24    rep(i, l, mid) c.modify(v[i][2], -v[i][3]);
25    rep(i, l, r) v[i] = tmp[i];
26 }
27 void solve() {
28     cin >> n >> k;
29     c.resize(k);
30     rep(i, 1, n) {
31         int s, c, m;
32         cin >> s >> c >> m;
33         v[i] = {s, c, m, 1, 0};
34     }
35     v[0][0] = -1;
36     sort(v + 1, v + n + 1);
37     int cnt = 0;
38     rep(i, 1, n) {
39         if (v[i][0] == v[cnt][0] && v[i][1] == v[cnt][1] && v[i][2] == v[cnt]
40             ] [2]) v[cnt][3]++;
41         else v[++cnt] = v[i];
42     }
43     solve(1, cnt);
44     rep(i, 1, cnt) {
45         ans[v[i][4] + v[i][3] - 1] += v[i][3];
46     }
47     rep(i, 0, n - 1) cout << ans[i] << '\n';

```

```

47 }

```

## 2.4 compact.cpp

```

1 namespace compact {
2     const int LOGN=18;
3     int l[N],r[N],tot,p[N][20],n;
4     map<int,int> cv;
5     int lca(int u,int v) {
6         if (dep[u]>dep[v]) swap(u,v);
7         per(i,LOGN-1,0) if (dep[p[v][i]]>=dep[u]) v=p[v][i];
8         if (u==v) return u;
9         per(i,LOGN-1,0) if (p[v][i]!=p[u][i]) u=p[u][i],v=p[v][i];
10        return p[u][0];
11    }
12    void dfs(int u,int f) {
13        l[u]=++tot; dep[u]=dep[f]+1; p[u][0]=f;
14        vec[dep[u]].pb(u);
15        for (auto v:vE[u]) {
16            if (v==f) continue;
17            dfs(v,u);
18        }
19        r[u]=tot;
20    }
21    void build(int _n) {
22        n=_n; tot=0;
23        dfs(1,0);
24        rep(j,1,LOGN-1) rep(i,1,n) p[i][j]=p[p[i][j-1]][j-1];
25    }
26
27    bool cmp(int u,int v) { return l[u]<l[v]; }
28    vector<PII> compact(VI v) {
29        int m=SZ(v);
30        vector<PII> E;
31        sort(all(v),cmp);
32        rep(i,0,m-2) {
33            int w=lca(v[i],v[i+1]);
34            v.pb(w);
35        }
36        v.pb(0);
37        v.pb(1);
38        sort(all(v),cmp);
39        v.erase(unique(all(v)),v.end());

```

```

40     cv.clear();
41     per(i,SZ(v)-1,1) {
42         int u=v[i];
43         while (1) {
44             auto it=cv.lower_bound(l[u]);
45             if (it==cv.end()||it->fi>r[u]) break;
46             E.pb(mp(u,v[it->se]));
47             cv.erase(it);
48         }
49         cv[l[u]]=i;
50     }
51     return E;
52 }
53 };

```

## 2.5 dominator.cpp

---

```

1 void solve(int u, int S) {
2     int best = -1, cnt = S + 1;
3     auto find_best = [&](auto &find_best, int u, int par) -> void {
4         sz[u] = 1, sdom[u] = 0;
5         for (auto v : e[u]) {
6             if (v == par || del[v]) continue;
7             find_best(find_best, v, u);
8             sz[u] += sz[v];
9             sdom[u] = max(sdom[u], sz[v]);
10        }
11        sdom[u] = max(sdom[u], S - sz[u]);
12        if (sdom[u] < cnt) {
13            cnt = sdom[u], best = u;
14        }
15    };
16    find_best(find_best, u, 0);
17    int id1 = tot++, dep1 = 0;
18    int id2, dep2;
19    auto dfs = [&](auto &dfs, int u, int par, int dep) -> void {
20        dep1 = max(dep1, dep);
21        dep2 = max(dep2, dep);
22        Q[u].pb({id1, 1, dep});
23        Q[u].pb({id2, -1, dep});
24        for (auto v : e[u]) {
25            if (v == par || del[v]) continue;
26            dfs(dfs, v, u, dep + 1);

```

```

27        }
28    };
29    Q[best].pb({id1, 1, 0});
30    for (auto v : e[best]) {
31        if (del[v]) continue;
32        id2 = tot++, dep2 = 0;
33        dfs(dfs, v, best, 1);
34        fenw[id2] = BIT<ll>(dep2 + 1);
35    }
36    fenw[id1] = BIT<ll>(dep1 + 1);
37    del[best] = 1;
38    for (auto v : e[best]) {
39        if (!del[v]) solve(v, sz[v]);
40    }
41 }

```

## 2.6 dsu.cpp

---

```

1 class dsu {
2     public:
3         vector<int> fa;
4         vector<ll> dist;
5         int n;
6
7         dsu(int _n) : n(_n) {
8             fa.resize(n);
9             dist.assign(n, 0);
10            iota(fa.begin(), fa.end(), 0);
11        }
12
13        int find(int x) {
14            if (fa[x] == x) return x;
15            int par = fa[x];
16            fa[x] = find(fa[x]);
17            dist[x] += dist[par];
18            return fa[x];
19        }
20
21        void unite(int x, int y, ll v) {
22            int px = find(x);
23            int py = find(y);
24            fa[py] = px;
25            dist[py] = dist[x] - dist[y] - v;

```

```

26     }
27 };

```

## 2.7 dsu-on-tree.cpp

```

1 void dfs(int x, int fa) {
2     hs[x] = -1, w[x] = 1;
3     l[x] = ++tot;
4     id[tot] = x;
5     for (auto y : g[x]) if (y != fa) {
6         dfs(y, x);
7         w[x] += w[y];
8         if (hs[x] == -1 || w[y] > w[hs[x]])
9             hs[x] = y;
10    }
11    r[x] = tot;
12 }
13
14 void dsu(int x, int fa, int keep) {
15     for (auto y : g[x]) {
16         if (y != hs[x] && y != fa) {
17             dsu(y, x, 0);
18         }
19     }
20     if (hs[x] != -1) dsu(hs[x], x, 1);
21
22     for (auto y : g[x]) {
23         if (y != hs[x] && y != fa) {
24             for (int i = l[y]; i <= r[y]; i++) {
25
26             }
27         }
28     }
29     // add current node
30
31     ans[x] = cnt;
32
33     if (!keep) {
34         // clear
35     }
36 }

```

## 2.8 fenwick.cpp

```

1 template <typename T>
2 struct BIT {
3     vector<T> fenw;
4     int n, pw;
5
6     BIT(int n_ = 0) : n(n_) {
7         fenw.assign(n + 1, 0);
8         pw = bit_floor(unsigned(n));
9     }
10
11     void Modify(int x, T v) {
12         if (x <= 0) return; // assert(0 <= x && x < n);
13         while (x <= n) { // x < n
14             fenw[x] += v;
15             x += (x & -x); // x /= x + 1;
16         }
17     }
18
19     T Query(int x) {
20         // assert(0 <= x && x <= n);
21         T v{};
22         while (x > 0) {
23             v += fenw[x]; // fenw[x - 1];
24             x -= (x & -x); // x /= x - 1;
25         }
26         return v;
27     }
28
29     // Returns the length of the longest prefix with sum <= c
30     int MaxPrefix(T c) {
31         T v{};
32         int at = 0;
33         for (int i = 20; i >= 0; i--) {
34             if (at + bit(i) <= n && v + fenw[at + bit(i)] <= c) {
35                 v += fenw[at + bit(i)];
36                 at += bit(i);
37             }
38         }
39         /**
40          * for (int len = pw; len > 0; len >>= 1) {
41          *     if (at + len <= n) {
42          *         auto nv = v;

```



```

43     *      nv += fenw[at + len - 1];
44     *      if (!(c < nv)) {
45     *          v = nv;
46     *          at += len;
47     *      }
48     *  }
49     *  }
50     *  assert(0 <= at es at <= n);
51     */
52     return at;
53 }
54 };

```

## 2.9 fenwick2d.cpp

---

```

1  template <typename T>
2  class fenwick2d {
3  public:
4      vector<vector<T>> fenw;
5      int n, m;
6
7      fenwick2d(int _n, int _m) : n(_n), m(_m) {
8          fenw.resize(n);
9          for (int i = 0; i < n; i++) {
10             fenw[i].resize(m);
11         }
12     }
13
14     inline void modify(int i, int j, T v) {
15         int x = i;
16         while (x < n) {
17             int y = j;
18             while (y < m) {
19                 fenw[x][y] += v;
20                 y |= (y + 1);
21             }
22             x |= (x + 1);
23         }
24     }
25
26     inline T get(int i, int j) {
27         T v{};
28         int x = i;

```

```

29         while (x >= 0) {
30             int y = j;
31             while (y >= 0) {
32                 v += fenw[x][y];
33                 y = (y & (y + 1)) - 1;
34             }
35             x = (x & (x + 1)) - 1;
36         }
37         return v;
38     }
39 };
40
41 struct node {
42     int a = ...; // don't forget to set default value
43
44     inline void operator+=(node &other) { ... }
45 };

```

## 2.10 hash-table.cpp

---

```

1  struct Hash_table {
2      static const int V = 1000003;
3      int fst[V], nxt[V];
4      int ctm, ptm[V], T;
5      int val[V];
6      ll key[V];
7      void init() {T = 0, ctm++;}
8      void insert(ll k, int v) {
9          int s = k % V;
10         if (ptm[s] != ctm) ptm[s] = ctm, fst[s] = -1;
11         for (int i = fst[s]; i != -1; i = nxt[i]) if (key[i] == k) {
12             return;
13         }
14         nxt[T] = fst[s], fst[s] = T, key[T] = k, val[T] = v;
15         T++;
16     }
17     int query(ll k) {
18         int s = k % V;
19         if (ptm[s] != ctm) return -1;
20         for (int i = fst[s]; i != -1; i = nxt[i]) {
21             if (key[i] == k) return val[i];
22         }
23         return -1;

```

```

24     }
25 };

```

## 2.11 hilbertOrder.cpp

```

1  template <class M, bool val_on_edge = false>
2  class Mo_Tree {
3  private:
4      static inline int64_t hilbertOrder(int x, int y, int pow, int rotate) {
5          if (pow == 0)
6              return 0;
7          int hpow = 1 << (pow - 1);
8          int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y < hpow) ? 1 : 2);
9          seg = (seg + rotate) & 3;
10         const int rotateDelta[4] = {3, 0, 0, 1};
11         int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
12         int nrot = (rotate + rotateDelta[seg]) & 3;
13         int64_t subSquareSize = int64_t(1) << (2 * pow - 2);
14         int64_t ordd = seg * subSquareSize;
15         int64_t add = hilbertOrder(nx, ny, pow - 1, nrot);
16         ordd += (seg == 1 || seg == 2) ? add : (subSquareSize - add - 1);
17         return ordd;
18     }
19
20     struct query {
21         int l, r, q_idx, lca;
22         int64_t ord;
23     };
24
25     int q;
26     vector<int> depth, flat, st, en, node_freq;
27     vector<vector<int>>> anc;
28     vector<M> par_v;
29     vector<query> Q;
30
31     void add_node(int node) {
32         if (par_v[node] < N) {
33             aux[par_v[node]]++;
34             w.reset(par_v[node]);
35         }
36     }
37
38     void remove_node(int node) {

```

```

39         if (par_v[node] < N) {
40             if (--aux[par_v[node]] == 0) {
41                 w.set(par_v[node]);
42             }
43         }
44     }
45
46     long long calc(...) {
47         return w._Find_first();
48     }
49
50     void add(int ind) {
51         node_freq[flat[ind]]++;
52         if (node_freq[flat[ind]] == 1) {
53             add_node(flat[ind]);
54         } else {
55             remove_node(flat[ind]);
56         }
57     }
58
59     void remove(int ind) {
60         node_freq[flat[ind]]--;
61         if (node_freq[flat[ind]] == 1) {
62             add_node(flat[ind]);
63         } else {
64             remove_node(flat[ind]);
65         }
66     }
67
68 public:
69     Mo_Tree(vector<vector<int>>> &g, const vector<M> &v) {
70         q = 0;
71         this->par_v = v;
72         int n = (int)g.size() - 1;
73         depth.resize(n + 1), st.resize(n + 1);
74         en.resize(n + 1), node_freq.resize(n + 1);
75         anc.assign(n + 1, vector<int>(__lg(n) + 2, 0));
76
77         auto dfs = [&](auto &dfs, int node, int p, int d) -> void {
78             anc[node][0] = p;
79             depth[node] = d;
80             st[node] = flat.size();
81             flat.push_back(node);

```

```

82         for (auto it : g[node]) {
83             if (it != p) {
84                 dfs(dfs, it, node, d + 1);
85             }
86         }
87         en[node] = flat.size();
88         flat.push_back(node);
89     };
90     dfs(dfs, 1, 0, 0);
91
92     for (int j = 1; j <= __lg(n); ++j) {
93         for (int i = 1; i <= n; ++i) {
94             anc[i][j] = anc[anc[i][j - 1]][j - 1];
95         }
96     }
97 }
98
99 int kth_anc(int node, int k) {
100     int ret = node;
101     for (int bit = (int)anc[ret].size() - 1; ~bit; --bit) {
102         if (k & (1 << bit)) {
103             ret = anc[ret][bit];
104         }
105     }
106     return ret;
107 }
108
109 int LCA(int u, int v) {
110     if (depth[u] < depth[v]) {
111         swap(u, v);
112     }
113     u = kth_anc(u, depth[u] - depth[v]);
114     if (u == v) {
115         return u;
116     }
117     for (int bit = anc[0].size() - 1; ~bit; --bit) {
118         if (anc[u][bit] != anc[v][bit]) {
119             u = anc[u][bit];
120             v = anc[v][bit];
121         }
122     }
123     return anc[u][0];
124 }

```

```

125
126 template <class... T>
127 void add_query(int u, int v, T &...x) {
128     if (st[u] > st[v]) {
129         swap(u, v);
130     }
131     int lca = LCA(u, v), l, r, q_lca = -1;
132     if (lca == u || lca == v) {
133         l = st[u] + val_on_edge, r = st[v];
134     } else {
135         l = en[u], r = st[v], q_lca = lca;
136     }
137     Q.push_back({l, r, q++, q_lca, hilbertOrder(l, r, __lg(flat.size())
138         + 1, 0), x...});
139 }
140
141 vector<int> Mo() {
142     vector<int> ans(q);
143     sort(Q.begin(), Q.end(), [&](const query &a, const query &b) {
144         return a.ord < b.ord;
145     });
146     int l = 0, r = -1;
147     for (auto [L, R, q_idx, lca, ord] : Q) {
148         while (l > L)
149             add(--l);
150         while (r < R)
151             add(++r);
152         while (r > R)
153             remove(r--);
154         while (l < L)
155             remove(l++);
156
157         if (~lca && !val_on_edge)
158             add_node(lca);
159         ans[q_idx] = calc();
160         if (~lca && !val_on_edge)
161             remove_node(lca);
162     }
163     return ans;
164 };

```

## 2.12 HLD.cpp

```
1 struct HLD {
2     int n;
3     std::vector<int> siz, top, dep, parent, in, out, seq;
4     std::vector<std::vector<int>>> adj;
5     int cur;
6
7     HLD() {}
8     HLD(int n) {
9         init(n);
10    }
11    void init(int n) {
12        this->n = n;
13        siz.resize(n);
14        top.resize(n);
15        dep.resize(n);
16        parent.resize(n);
17        in.resize(n);
18        out.resize(n);
19        seq.resize(n);
20        cur = 0;
21        adj.assign(n, {});
22    }
23    void addEdge(int u, int v) {
24        adj[u].push_back(v);
25        adj[v].push_back(u);
26    }
27    void work(int root = 0) {
28        top[root] = root;
29        dep[root] = 0;
30        parent[root] = -1;
31        dfs1(root);
32        dfs2(root);
33    }
34    void dfs1(int u) {
35        if (parent[u] != -1) {
36            adj[u].erase(std::find(adj[u].begin(), adj[u].end(), parent[u]));
37        }
38
39        siz[u] = 1;
40        for (auto &v : adj[u]) {
41            parent[v] = u;
```

```
42            dep[v] = dep[u] + 1;
43            dfs1(v);
44            siz[u] += siz[v];
45            if (siz[v] > siz[adj[u][0]]) {
46                std::swap(v, adj[u][0]);
47            }
48        }
49    }
50    void dfs2(int u) {
51        in[u] = cur++;
52        seq[in[u]] = u;
53        for (auto v : adj[u]) {
54            top[v] = v == adj[u][0] ? top[u] : v;
55            dfs2(v);
56        }
57        out[u] = cur;
58    }
59    int lca(int u, int v) {
60        while (top[u] != top[v]) {
61            if (dep[top[u]] > dep[top[v]]) {
62                u = parent[top[u]];
63            } else {
64                v = parent[top[v]];
65            }
66        }
67        return dep[u] < dep[v] ? u : v;
68    }
69
70    int dist(int u, int v) {
71        return dep[u] + dep[v] - 2 * dep[lca(u, v)];
72    }
73
74    int jump(int u, int k) {
75        if (dep[u] < k) {
76            return -1;
77        }
78
79        int d = dep[u] - k;
80
81        while (dep[top[u]] > d) {
82            u = parent[top[u]];
83        }
84    }
```

```

85     return seq[in[u] - dep[u] + d];
86 }
87
88 bool isAncestor(int u, int v) {
89     return in[u] <= in[v] && in[v] < out[u];
90 }
91
92 int rootedParent(int u, int v) {
93     std::swap(u, v);
94     if (u == v) {
95         return u;
96     }
97     if (!isAncestor(u, v)) {
98         return parent[u];
99     }
100     auto it = std::upper_bound(adj[u].begin(), adj[u].end(), v, [&](int
        x, int y) {
101         return in[x] < in[y];
102     }) - 1;
103     return *it;
104 }
105
106 int rootedSize(int u, int v) {
107     if (u == v) {
108         return n;
109     }
110     if (!isAncestor(v, u)) {
111         return siz[v];
112     }
113     return n - siz[rootedParent(u, v)];
114 }
115
116 int rootedLca(int a, int b, int c) {
117     return lca(a, b) ^ lca(b, c) ^ lca(c, a);
118 }
119 };

```

## 2.13 kdtree.cpp

```

1 namespace kd {
2     const int K = 2, N = 2.1e5;
3     template <typename T>
4     using P = array<T, K>;

```

```

5     template <typename T>
6     struct node {
7         P<T> pt, mx, mn;
8         ll val, sum;
9         node *l, *r, *p;
10        int id;
11        node(const P<T> &_pt = P<T>(), ll _val = 0, int _id = 0)
12            : pt(_pt), val(_val), sum(_val), id(_id) {
13            mx = mn = pt;
14            p = l = r = nullptr;
15        }
16    };
17    node<ll> *ptr[N];
18    template <typename T>
19    void pull(node<T> *u) {
20        if (not u) return;
21        u->sum = u->val;
22        rep(i, 0, K - 1) u->mx[i] = u->mn[i] = u->pt[i];
23        if (u->l) {
24            u->sum += u->l->sum;
25            u->l->p = u;
26        }
27        if (u->r) {
28            u->sum += u->r->sum;
29            u->r->p = u;
30        }
31        rep(i, 0, K - 1) {
32            if (u->l) {
33                u->mx[i] = max(u->mx[i], u->l->mx[i]);
34                u->mn[i] = min(u->mn[i], u->l->mn[i]);
35            }
36            if (u->r) {
37                u->mx[i] = max(u->mx[i], u->r->mx[i]);
38                u->mn[i] = min(u->mn[i], u->r->mn[i]);
39            }
40        }
41    }
42
43    template <typename T>
44    node<T> *build(vector<node<T>> &a, int l, int r, int d = 0) {
45        if (d == K) d = 0;
46        if (l >= r) {
47            return nullptr;

```

```

48 } else {
49     int md = (l + r) >> 1;
50     nth_element(a.begin() + l, a.begin() + md, a.begin() + r,
51         [&](node<T> &x, node<T> &y) { return x.pt[d] < y.pt[d]; });
52     node<T> *p = new node<T>(a[md]);
53     ptr[p->id] = p;
54     p->l = build(a, l, md, d + 1);
55     p->r = build(a, md + 1, r, d + 1);
56     pull(p);
57     return p;
58 }
59 }
60
61 template <typename T>
62 node<T> *search(node<T> *u, P<T> p, int d = 0) {
63     if (d == K) d = 0;
64     if (not u) return nullptr;
65     if (u->pt == p) return u;
66     if (p[d] < u->pt[d]) {
67         return search(u->l, p, d + 1);
68     } else if (p[d] > u->pt[d]) {
69         return search(u->r, p, d + 1);
70     } else {
71         auto tmp = search(u->l, p, d + 1);
72         if (tmp) return tmp;
73         return search(u->r, p, d + 1);
74     }
75 }
76
77 template <typename T>
78 void modify(node<T> *u, ll v) {
79     if (not u) return;
80     u->val = v;
81     for (auto cur = u; cur; cur = cur->p) {
82         pull(cur);
83     }
84 }
85
86 template <typename T>
87 bool inside(node<T> *nd, P<T> p, ll c) {
88     int cc = 0;
89     if (nd->mx[0] * p[0] + nd->mx[1] * p[1] >= c) cc++;
90     if (nd->mn[0] * p[0] + nd->mn[1] * p[1] >= c) cc++;

```

```

91     if (nd->mx[0] * p[0] + nd->mn[1] * p[1] >= c) cc++;
92     if (nd->mn[0] * p[0] + nd->mx[1] * p[1] >= c) cc++;
93     return cc == 0;
94 }
95
96 template <typename T>
97 bool outside(node<T> *nd, P<T> p, ll c) {
98     int cc = 0;
99     if (nd->mx[0] * p[0] + nd->mx[1] * p[1] >= c) cc++;
100    if (nd->mn[0] * p[0] + nd->mn[1] * p[1] >= c) cc++;
101    if (nd->mx[0] * p[0] + nd->mn[1] * p[1] >= c) cc++;
102    if (nd->mn[0] * p[0] + nd->mx[1] * p[1] >= c) cc++;
103    return cc == 4;
104 }
105
106 template <typename T>
107 ll query(node<T> *u, P<T> p, ll c) {
108     if (inside(u, p, c)) return u->sum;
109     if (outside(u, p, c)) return 0;
110     ll s = 0;
111     if (u->pt[0] * p[0] + u->pt[1] * p[1] < c) {
112         s += u->val;
113     }
114     if (u->l) s += query(u->l, p, c);
115     if (u->r) s += query(u->r, p, c);
116     return s;
117 }
118
119 template <typename T>
120 T eval_min(node<T> *nd,
121     P<T> p) { // 通过估价函数进行启发式搜索，根据当前结果对搜索剪枝
122     if (not nd) return numeric_limits<T>::max() / 4;
123     ll s = 0;
124     rep(i, 0, K - 1) {
125         if (p[i] <= nd->mn[i]) s += nd->mn[i] - p[i];
126         if (p[i] >= nd->mx[i]) s += p[i] - nd->mx[i];
127     }
128     return s;
129 }
130
131 template <typename T>
132 ll mindist(node<T> *u, P<T> p) {
133     ll s = numeric_limits<T>::max() / 4;

```

```

134     if (u->pt != p) {
135         s = min(s, abs(u->pt[0] - p[0]) + abs(u->pt[1] - p[1]));
136     }
137     ll best1 = eval_min(u->l, p), best2 = eval_min(u->r, p);
138     if (best1 < best2) {
139         if (u->l) s = min(s, mindist(u->l, p));
140         if (u->r and best2 < s) s = min(s, mindist(u->r, p));
141         return s;
142     } else {
143         if (u->r) s = min(s, mindist(u->r, p));
144         if (u->l and best1 < s) s = min(s, mindist(u->l, p));
145         return s;
146     }
147 }
148
149 template <typename T>
150 T eval_max(node<T> *nd,
151             P<T> p) { // 通过估价函数进行启发式搜索，根据当前结果对搜索剪枝
152     if (not nd) return 0;
153     ll s = 0;
154     rep(i, 0, K - 1) s += max(abs(nd->mx[i] - p[i]), abs(nd->mn[i] - p[i]));
155     return s;
156 }
157
158 template <typename T>
159 ll maxdist(node<T> *u, P<T> p) {
160     ll s = 0;
161     if (u->pt != p) {
162         s = max(s, abs(u->pt[0] - p[0]) + abs(u->pt[1] - p[1]));
163     }
164     ll best1 = eval_max(u->l, p), best2 = eval_max(u->r, p);
165     if (best1 > best2) {
166         if (u->l) s = max(s, maxdist(u->l, p));
167         if (u->r and best2 > s) s = max(s, maxdist(u->r, p));
168         return s;
169     } else {
170         if (u->r) s = max(s, maxdist(u->r, p));
171         if (u->l and best1 > s) s = max(s, maxdist(u->l, p));
172         return s;
173     }
174 }
175 } // namespace kd

```

## 2.14 LCT.cpp

```

1 namespace linkCutTree {
2 struct node {
3     node *child[2], *parent, *max;
4     int id;
5     ll sum, val, sz, weight, rev;
6     node(ll val, ll weight, int id) : child {nullptr, nullptr}, parent(
7         nullptr), max(this), sum(val), val(val), sz(weight), weight(weight),
8         id(id), rev(false) {}
9 };
10
11 bool isRoot(node *p) { return p->parent == nullptr || (p->parent->child[0]
12     != p && p->parent->child[1] != p); }
13
14 int side(node *p) { return p->parent->child[1] == p; }
15
16 ll sum(node *p) { return p == nullptr ? 0 : p->sum; }
17
18 ll sz(node *p) { return p == nullptr ? 0 : p->sz; }
19
20 node *max(node *p) { return p == nullptr ? nullptr : p->max; }
21
22 node *max(node *p, node *q) {
23     if (p == nullptr) return q;
24     if (q == nullptr) return p;
25     return p->weight > q->weight ? p : q;
26 }
27
28 void reverse(node *p) {
29     if (p == nullptr) return;
30     swap(p->child[0], p->child[1]);
31     p->rev ^= 1;
32 }
33
34 void push(node *p) {
35     if (p->rev == 0) return;
36     p->rev = 0;
37     reverse(p->child[0]);
38     reverse(p->child[1]);
39 }

```

```

40 }
41
42 void pull(node *p) {
43     p->sum = sum(p->child[0]) + sum(p->child[1]) + p->val;
44     p->max = max(max(max(p->child[0]), max(p->child[1])), p);
45     p->sz = p->weight + sz(p->child[0]) + sz(p->child[1]);
46 }
47
48 void connect(node *p, node *q, int side) {
49     q->child[side] = p;
50     if (p != nullptr)
51         p->parent = q;
52 }
53
54 void rotate(node *p) {
55     auto q = p->parent;
56     int dir = side(p) ^ 1;
57     connect(p->child[dir], q, dir ^ 1);
58     if (!isRoot(q))
59         connect(p, q->parent, side(q));
60     else
61         p->parent = q->parent;
62     connect(q, p, dir);
63     pull(q);
64 }
65
66 void splay(node *p) {
67     vector<node*> stk;
68     for (auto i = p; !isRoot(i); i = i->parent)
69         stk.push_back(i->parent);
70     while (!stk.empty()) {
71         push(stk.back());
72         stk.pop_back();
73     }
74     push(p);
75     while (!isRoot(p)) {
76         auto q = p->parent;
77         if (!isRoot(q))
78             rotate(side(p) == side(q) ? q : p);
79         rotate(p);
80     }
81     pull(p);
82 }

```

```

83
84 node *access(node *p) {
85     node *j = nullptr;
86     for (node *i = p; i != nullptr; j = i, i = i->parent) {
87         splay(i);
88         i->val -= sum(j);
89         i->val += sum(i->child[1]);
90         i->child[1] = j;
91         pull(i);
92     }
93     splay(p);
94     return j;
95 }
96
97 void makeRoot(node *p) {
98     access(p);
99     reverse(p);
100 }
101
102 void link(node *p, node *q) {
103     makeRoot(p);
104     access(q);
105     p->parent = q;
106     q->val += sum(p);
107 }
108
109 void cut(node *p, node *q) {
110     makeRoot(p);
111     access(q);
112     p->parent = q->child[0] = nullptr;
113 }
114
115 node *pathMax(node *p, node *q) {
116     makeRoot(p);
117     access(q);
118     return max(q);
119 }
120
121 ll pathSize(node *p, node *q) {
122     makeRoot(p);
123     access(q);
124     return sz(q);
125 }

```



```

126
127 ll rootedSum(node *p) {
128     makeRoot(p);
129     return sum(p);
130 }
131
132 ll getSubtree(node *rt, node *v) {
133     makeRoot(rt);
134     access(v);
135     return v->val;
136 }
137
138 bool connected(node *p, node *q) {
139     access(p);
140     access(q);
141     return p->parent != nullptr;
142 }
143
144 void fix(node *p, ll v) {
145     access(p);
146     push(p);
147     // modify ...
148     p->val += v;
149     pull(p);
150 }
151
152 node *lca(node *z, node *x, node *y) {
153     makeRoot(z);
154     access(x);
155     return access(y);
156 }
157 } // namespace linkCutTree
158 using namespace linkCutTree;

```

## 2.15 lichao-tree.cpp

```

1 struct Line {
2     i64 k, b;
3     i64 operator()(i64 x) const { return k * x + b; }
4 };
5 template <i64 L, i64 R>
6 struct Segments {
7     struct Node {

```

```

8         optional<Line> s;
9         Node *l, *r;
10    };
11    Node *root;
12    Segments() : root(nullptr) {}
13    void add(i64 l, i64 r, i64 k, i64 b) {
14        auto rec = [&](auto &rec, Node *&p, i64 tl, i64 tr, Line s) -> void {
15            if (p == nullptr) p = new Node();
16            i64 tm = midpoint(tl, tr);
17            if (tl >= l and tr <= r) {
18                if (not p->s) return p->s = s, void();
19                auto t = p->s.value();
20                if (t(tl) >= s(tl)) {
21                    if (t(tr) >= s(tr)) return;
22                    if (t(tm) >= s(tm)) return rec(rec, p->r, tm + 1, tr, s);
23                    return p->s = s, rec(rec, p->l, tl, tm, t);
24                }
25                if (t(tr) <= s(tr)) return p->s = s, void();
26                if (t(tm) <= s(tm)) return p->s = s, rec(rec, p->r, tm + 1, tr, t);
27                return rec(rec, p->l, tl, tm, s);
28            }
29            if (l <= tm) rec(rec, p->l, tl, tm, s);
30            if (r > tm) rec(rec, p->r, tm + 1, tr, s);
31        };
32        rec(rec, root, L, R, {k, b});
33    }
34    optional<i64> get(i64 x) {
35        optional<i64> res = {};
36        auto rec = [&](auto &rec, Node *p, i64 tl, i64 tr) -> void {
37            if (p == nullptr) return;
38            i64 tm = midpoint(tl, tr);
39            if (p->s) {
40                i64 y = p->s.value()(x);
41                if (not res or res.value() < y) res = y;
42            }
43            if (x <= tm)
44                rec(rec, p->l, tl, tm);
45            else
46                rec(rec, p->r, tm + 1, tr);
47        };
48        rec(rec, root, L, R);
49        return res;
50    }

```

```
51 };
```

## 2.16 Mo.cpp

```
1 /**
2  * #define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
3  *      iota(all(s), 0);
4  *      sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
5  */
6
7 VI Mo(const vector<array<int, 3>> &Q) {
8     const int blk = 350;
9     vector<int> s(SZ(Q)), res = s;
10    iota(all(s), 0);
11    sort(all(s), [&](int i, int j) {
12        int u = Q[i][0] / blk, v = Q[j][0] / blk;
13        return u == v ? u % 2 ? Q[i][1] > Q[j][1] : Q[i][1] < Q[j][1] : u <
14            v;
15    });
16    int L = 1, R = 0;
17    for (int qi : s) {
18        while (R < Q[qi][1]) R++, add(R);
19        while (L > Q[qi][0]) L--, add(L);
20        while (R > Q[qi][1]) del(R), R--;
21        while (L < Q[qi][0]) del(L), L++;
22        res[qi] = calc(Q[qi][2]);
23    }
24    return res;
25 }
```

## 2.17 moTree.cpp

```
1 /**
2  * Author: Simon Lindholm
3  * Date: 2019-12-28
4  * License: CCO
5  * Source: https://github.com/hoke-t/tamu-kactl/blob/master/content/data-structures/MoQueries.h
6  * Description: Answer interval or tree path queries by finding an
7  *               approximate TSP through the queries,
8  *               and moving from one query to the next by adding/removing points at the
9  *               ends.
```

```
8  * If values are on tree edges, change \texttt{step} to add/remove the edge
9  *   $(a, c)$ and remove the initial \texttt{add} call (but keep \texttt{in}
10  *   }).
11  *
12  * Time:  $O(N \sqrt{Q})$ 
13  * Status: stress-tested
14  */
15
16 void add(int ind, int end) { ... } // add a [ ind ] (end = 0 or 1)
17 void del(int ind, int end) { ... } // remove a [ ind ]
18 int calc() { ... } // compute current answer
19
20 vi mo(vector<pii> Q) {
21     int L = 0, R = 0, blk = 350; //  $N/\sqrt{Q}$ 
22     vi s(SZ(Q)), res = s;
23     #define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
24     iota(all(s), 0);
25     sort(all(s), [&](int s, int t) { return K(Q[s]) < K(Q[t]); });
26     for (int qi : s) {
27         pii q = Q[qi];
28         while (L > q.first) add(--L, 0);
29         while (R < q.second) add(R++, 1);
30         while (L < q.first) del(L++, 0);
31         while (R > q.second) del(--R, 1);
32         res[qi] = calc();
33     }
34     return res;
35 }
36
37 vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root = 0) {
38     int N = SZ(ed), pos[2] = {}, blk = 350; //  $N/\sqrt{Q}$ 
39     vi s(SZ(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
40     add(0, 0), in[0] = 1;
41     auto dfs = [&](int x, int p, int dep, auto & f) -> void {
42         par[x] = p;
43         L[x] = N;
44         if (dep) I[x] = N++;
45         for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
46         if (!dep) I[x] = N++;
47         R[x] = N;
48     };
49     dfs(root, -1, 0, dfs);
50     #define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
51     iota(all(s), 0);
52     sort(all(s), [&](int s, int t) { return K(Q[s]) < K(Q[t]); });
53     for (int qi : s) rep(end, 0, 2) {
```

```

49     int &a = pos[end], b = Q[qi][end], i = 0;
50 #define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
51 else { add(c, end); in[c] = 1; } a = c; }
52     while (!(L[b] <= L[a] && R[a] <= R[b]))
53         I[i++] = b, b = par[b];
54     while (a != b) step(par[a]);
55     while (i--) step(I[i]);
56     if (end) res[qi] = calc();
57 }
58 return res;
59 }

```

## 2.18 MSTMo.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 #define rep(i,a,n) for (int i=a;i<n;i++)
4 #define per(i,a,n) for (int i=n-1;i>=a;i--)
5 #define pb push_back
6 #define mp make_pair
7 #define all(x) (x).begin(),(x).end()
8 #define fi first
9 #define se second
10 #define SZ(x) ((int)(x).size())
11 typedef vector<int> VI;
12 typedef long long ll;
13 typedef pair<int,int> PII;
14 typedef double db;
15 mt19937 mrand(random_device{}());
16 const ll mod=1000000007;
17 int rnd(int x) { return mrand() % x;}
18 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b>=>1){if(b&1)
    res=res*a%mod;a=a%mod;}return res;}
19 ll gcd(ll a,ll b) { return b?gcd(b,a%b):a;}
20 // head
21
22 const int N=1010000;
23 int a[N];
24 namespace Mo {
25     int Q,l[N],r[N],f[N],l0,r0,ans[N],n;
26     VI ne[N];
27     struct point {
28         int x, y, o;

```

```

29         point(int a, int b, int c): x(a), y(b), o(c) {}
30     };
31     inline bool operator<(const point &a, const point &b) {
32         if (a.x != b.x) return a.x > b.x;
33         else return a.y < b.y;
34     }
35     vector<point> p;
36     struct edge {
37         int s, t, d;
38         edge(const point &a, const point &b): s(a.o), t(b.o),
39             d(abs(a.x - b.x) + abs(a.y - b.y)) {}
40     };
41     inline bool operator<(const edge &a, const edge &b) {return a.d < b.d;}
42     vector<edge> e;
43     int g[N],z[N];
44     int cc,cnt[101000];
45     void addedge() {
46         sort(all(p));
47         memset(g,0,sizeof(g));
48         z[0]=N;
49         rep(i,0,SZ(p)) z[i+1]=p[i].x-p[i].y;
50         rep(i,0,SZ(p)) {
51             int k = 0, t = p[i].x + p[i].y;
52             for (int j = t; j; j -= j & -j)
53                 if (z[g[j]] < z[k]) k = g[j];
54             if (k) e.pb(edge(p[i], p[k - 1]));
55             k = z[i + 1];
56             for (int j = t; j < N; j += j & -j)
57                 if (k < z[g[j]]) g[j] = i + 1;
58         }
59     }
60     void updata(int i, bool j,bool k=0) {
61         // j=1 insert j=0 delete
62         // k=0 left k=1 right
63         if (j==1) {
64             cnt[a[i]]++;
65             if (cnt[a[i]]%2==0) cc++;
66         } else {
67             if (cnt[a[i]]%2==0) cc--;
68             cnt[a[i]]--;
69         }
70     }
71     void init(int l,int r) {

```

```

72     for (int i=1;i<=r;i++) {
73         cnt[a[i]]++;
74         if (cnt[a[i]]%2==0) cc++;
75     }
76 }
77 inline int query() {
78     return cc;
79 }
80 int find(int x) { if (f[x] != x) f[x] = find(f[x]); return f[x];}
81 void dfs(int i,int p) {
82     int l1 = l[i], r1 = r[i];
83     per(j,l1,l0) updata(j,1,0);
84     rep(j,r0+1,r1+1) updata(j,1,1);
85     rep(j,l0,l1) updata(j,0,0);
86     per(j,r1+1,r0+1) updata(j,0,1);
87     ans[i]=query();l0=l1;r0=r1;
88     rep(j,0,SZ(ne[i])) if (ne[i][j]!=p) dfs(ne[i][j],i);
89 }
90 void solve() {
91     p.clear();e.clear();
92     rep(i,1,Q+1) ans[i]=0;
93     rep(i,1,Q+1) p.pb(point(l[i],r[i],i));
94     addedge();
95     rep(i,0,SZ(p)) p[i].y =n-p[i].y+1;
96     addedge();
97     rep(i,0,SZ(p)) {
98         int j =n-p[i].x+1;
99         p[i].x = p[i].y; p[i].y = j;
100     }
101     addedge();
102     rep(i,0,SZ(p)) p[i].x=n-p[i].x+1;
103     addedge();
104     sort(all(e));
105     rep(i,1,Q+1) ne[i].clear(),f[i]=i;
106     rep(i,0,SZ(e)) {
107         int j=e[i].s,k=e[i].t;
108         if (find(j)!=find(k)) f[f[j]]=f[k],ne[j].pb(k),ne[k].pb(j);
109     }
110     l0=l[1];r0=r[1];
111     init(l0,r0);
112     dfs(1,0);
113 }
114 }

```

```

115
116 int main() {
117     scanf("%d",&Mo::n);
118     for (int i=1;i<=Mo::n;i++) scanf("%d",a+i);
119     scanf("%d",&Mo::Q);
120     rep(i,1,Mo::Q+1) scanf("%d%d",&Mo::l[i],&Mo::r[i]);
121     Mo::solve();
122     rep(i,1,Mo::Q+1) printf("%d\n",Mo::ans[i]);
123 }

```

## 2.19 pseg.cpp

```

1 struct node {
2     node *l, *r;
3     ll val, sz, add;
4 };
5
6 void pull(node *u) {
7     u->sz = 0, u->val = 0;
8     if (u->l) u->sz += u->l->sz, u->val += u->l->val;
9     if (u->r) u->sz += u->r->sz, u->val += u->r->val;
10 }
11
12 void push(node *u) {
13     if (u->add) {
14         if (u->l) {
15             node *p = new node();
16             *p = *u->l;
17             u->l = p;
18             p->add += u->add;
19             p->val += p->sz * u->add;
20         }
21         if (u->r) {
22             node *p = new node();
23             *p = *u->r;
24             u->r = p;
25             p->add += u->add;
26             p->val += p->sz * u->add;
27         }
28         u->add = 0;
29     }
30 }
31

```

```

32 node *build(int l, int r) {
33     node *p = new node();
34     p->add = 0;
35     if (l == r) {
36         p->l = p->r = nullptr;
37         p->val = a[l];
38         p->sz = 1;
39     } else {
40         int mid = (l + r) >> 1;
41         p->l = build(l, mid);
42         p->r = build(mid + 1, r);
43         pull(p);
44     }
45     return p;
46 }
47
48 ll query(node *v, int l, int r, int ql, int qr) {
49     if (ql == l && qr == r) {
50         return v->val;
51     } else {
52         push(v);
53         int mid = (l + r) >> 1;
54         if (qr <= mid)
55             return query(v->l, l, mid, ql, qr);
56         else if (ql > mid)
57             return query(v->r, mid + 1, r, ql, qr);
58         else
59             return query(v->l, l, mid, ql, mid) +
60                    query(v->r, mid + 1, r, mid + 1, qr);
61     }
62 }
63
64 node *modify(node *v, int l, int r, int ql, int qr, ll x) {
65     if (ql == l && qr == r) {
66         node *p = new node();
67         *p = *v;
68         p->add += x;
69         p->val += p->sz * x;
70         return p;
71     } else {
72         push(v);
73         int mid = (l + r) >> 1;
74         node *p = new node();

```

```

75         *p = *v;
76         if (qr <= mid)
77             p->l = modify(v->l, l, mid, ql, qr, x);
78         else if (ql > mid)
79             p->r = modify(v->r, mid + 1, r, ql, qr, x);
80         else
81             p->l = modify(v->l, l, mid, ql, mid, x),
82             p->r = modify(v->r, mid + 1, r, mid + 1, qr, x);
83         pull(p);
84         return p;
85     }
86 }

```

## 2.20 rollbackMo.cpp

```

1 VI rollbackMo(const vector<array<int, 3>> &Q) {
2     const int blk = 350;
3     vector<VI> s(SZ(Q));
4     vector<int> BF, res(SZ(Q));
5     for (int i = 0; i < SZ(Q); i++) {
6         int u = Q[i][0] / blk, v = Q[i][1] / blk;
7         if (u == v) BF.push_back(i);
8         else s[u].push_back(i);
9     }
10    for (int i = 0; i < SZ(Q); i++)
11        sort(all(s[i]), [&](int i, int j) { return Q[i][1] < Q[j][1]; });
12    for (int qi : BF) {
13        for (int i = Q[qi][0]; i <= Q[qi][1]; i++)
14            add(i);
15        res[qi] = calc(Q[qi][2]);
16        for (int i = Q[qi][0]; i <= Q[qi][1]; i++)
17            del(i);
18    }
19    for (const auto &v : s) {
20        if (v.empty()) continue;
21        int next_blk = (Q[v.back()][0] / blk + 1) * blk;
22        int L = next_blk, R = next_blk - 1;
23        for (int qi : v) {
24            while (R < Q[qi][1]) R++, add(R);
25            while (L > Q[qi][0]) L--, add(L);
26            res[qi] = calc(Q[qi][2]);
27            while (L < next_blk) del(L), L++;
28        }

```

```

29     for (int i = next_blk; i <= R; i++)
30         del(i);
31     }
32     return res;
33 }

```

## 2.21 segtree.cpp

```

1 struct info {
2     ll sum;
3     int sz;
4     friend info operator+(const info &a, const info &b) {
5         return {(a.sum + b.sum) % mod, a.sz + b.sz};
6     }
7 };
8
9 struct tag {
10     ll add, mul;
11     friend tag operator+(const tag &a, const tag &b) {
12         tag res = {(a.add * b.mul + b.add) % mod, a.mul * b.mul % mod};
13         return res;
14     }
15 };
16 info operator+(const info &a, const tag &b) {
17     return {(a.sum * b.mul + a.sz * b.add) % mod, a.sz};
18 }
19
20 struct node {
21     info val;
22     tag t;
23 } seg[maxn << 2];
24
25 void update(int id) {
26     seg[id].val = seg[id * 2].val + seg[id * 2 + 1].val;
27 }
28 void settag(int id, tag t) {
29     seg[id].val = seg[id].val + t;
30     seg[id].t = seg[id].t + t;
31 }
32 void pushdown(int id) {
33     if (seg[id].t.mul == 1 and seg[id].t.add == 0) return;
34     settag(id * 2, seg[id].t);
35     settag(id * 2 + 1, seg[id].t);

```

```

36     seg[id].t.mul = 1;
37     seg[id].t.add = 0;
38 }
39 void build(int l, int r, int id) {
40     seg[id].t = {0, 1};
41     if (l == r) {
42         seg[id].val = {a[l], 1};
43     } else {
44         int mid = (l + r) >> 1;
45         build(l, mid, id * 2);
46         build(mid + 1, r, id * 2 + 1);
47         update(id);
48     }
49 }
50 void change(int l, int r, int id, int ql, int qr, tag t) {
51     if (l == ql && r == qr) {
52         settag(id, t);
53     } else {
54         int mid = (l + r) >> 1;
55         pushdown(id);
56         if (qr <= mid) {
57             change(l, mid, id * 2, ql, qr, t);
58         } else if (ql > mid) {
59             change(mid + 1, r, id * 2 + 1, ql, qr, t);
60         } else {
61             change(l, mid, id * 2, ql, mid, t);
62             change(mid + 1, r, id * 2 + 1, mid + 1, qr, t);
63         }
64         update(id);
65     }
66 }
67 info query(int l, int r, int id, int ql, int qr) {
68     if (l == ql && r == qr) {
69         return seg[id].val;
70     } else {
71         int mid = (l + r) >> 1;
72         pushdown(id);
73         if (qr <= mid)
74             return query(l, mid, id * 2, ql, qr);
75         else if (ql > mid)
76             return query(mid + 1, r, id * 2 + 1, ql, qr);
77         else
78             return query(l, mid, id * 2, ql, mid) +

```

```

79         query(mid + 1, r, id * 2 + 1, mid + 1, qr);
80     }
81 }
82 ll search(int l, int r, int id, int ql, int qr, int d) {
83     if (ql == l && qr == r) {
84         int mid = (l + r) / 2;
85         // if (l != r) pushdown(id); ...
86         if (seg[id].val < d)
87             return -1;
88         else {
89             if (l == r)
90                 return l;
91             else if (seg[id * 2].val >= d)
92                 return search(l, mid, id * 2, ql, mid, d);
93             else
94                 return search(mid + 1, r, id * 2 + 1, mid + 1, qr, d);
95         }
96     } else {
97         int mid = (l + r) >> 1;
98         // pushdown(id); ...
99         if (qr <= mid)
100             return search(l, mid, id * 2, ql, qr, d);
101         else if (ql > mid)
102             return search(mid + 1, r, id * 2 + 1, ql, qr, d);
103         else {
104             int tmp = search(l, mid, id * 2, ql, mid, d);
105             if (tmp != -1)
106                 return tmp;
107             else
108                 return search(mid + 1, r, id * 2 + 1, mid + 1, qr, d);
109         }
110     }
111 }

```

## 2.22 segtreefast.cpp

```

1 /**
2  * Author: Lucian Bicsi
3  * Description: Very fast and quick segment tree.
4  * Only useful for easy invariants. 0-indexed.
5  * Range queries are half-open.
6  */
7 #pragma once

```

```

8
9 struct SegmTree {
10     vector<int> T; int n;
11     SegmTree(int n) : T(2 * n, (int)2e9), n(n) {}
12
13     void Update(int pos, int val) {
14         for (T[pos += n] = val; pos > 1; pos /= 2)
15             T[pos / 2] = min(T[pos], T[pos ^ 1]);
16     }
17
18     int Query(int b, int e) {
19         int res = (int)2e9;
20         for (b += n, e += n; b < e; b /= 2, e /= 2) {
21             if (b % 2) res = min(res, T[b++]);
22             if (e % 2) res = min(res, T[--e]);
23         }
24         return res;
25     }
26 };

```

## 2.23 SparseTable.cpp

```

1 template <typename T, class F = function<T(const T&, const T&>>>
2 class SparseTable {
3 public:
4     int n;
5     vector<vector<T>> mat;
6     F func;
7
8     SparseTable(const vector<T>& a, const F& f) : func(f) {
9         n = static_cast<int>(a.size());
10         int max_log = 32 - __builtin_clz(n);
11         mat.resize(max_log);
12         mat[0] = a;
13         for (int j = 1; j < max_log; j++) {
14             mat[j].resize(n - (1 << j) + 1);
15             for (int i = 0; i <= n - (1 << j); i++) {
16                 mat[j][i] = func(mat[j - 1][i], mat[j - 1][i + (1 << (j - 1))]);
17             }
18         }
19     }
20
21     T get(int from, int to) const {

```

```

22     assert(0 <= from && from <= to && to <= n - 1);
23     int lg = 32 - __builtin_clz(to - from + 1) - 1;
24     return func(mat[lg][from], mat[lg][to - (1 << lg) + 1]);
25 }
26 };

```

## 2.24 SparseTable2D.cpp

```

1  // lg[1] = 0;
2  // rep(i, 2, N - 1) {
3  //     lg[i] = lg[i / 2] + 1;
4  // }
5  // int k = log2(r - l + 1); very slow!!!
6  // int k = __lg(r - l + 1);
7  // int k = lg[r - l + 1];
8  // int k = 32 - __builtin_clz(r - l + 1) - 1;
9  vector<vector<int>> sparse[12];
10
11 int query(int x, int y, int d) {
12     int k = __lg(d);
13     int s = d - bit(k);
14     return min({sparse[k][x][y], sparse[k][x + s][y], sparse[k][x][y + s],
15                 sparse[k][x + s][y + s]});
16 }
17 void build() {
18     rep(i, 1, n) rep(j, 1, m) sparse[0][i][j] = mat[i][j];
19     rep(k, 1, 11) rep(i, 1, n) rep(j, 1, m) {
20         int d = bit(k - 1);
21         if (i + d > n || j + d > m) continue;
22         sparse[k][i][j] = min({sparse[k - 1][i][j], sparse[k - 1][i + d][j],
23                                 sparse[k - 1][i][j + d], sparse[k - 1][i + d][j + d]});
24     }
25 }

```

## 2.25 treap.cpp

```

1  /**
2   *   author:   tourist
3   *   created:  07.10.2022 20:32:03
4   */
5  #include <bits/stdc++.h>
6

```

```

7  using namespace std;
8
9  #ifdef LOCAL
10 #include "algo/debug.h"
11 #else
12 #define debug(...) 42
13 #endif
14
15 mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
16
17 class node {
18 public:
19     int id;
20     node* l;
21     node* r;
22     node* p;
23     bool rev;
24     int sz;
25     // declare extra variables:
26     long long P;
27     long long add;
28     long long x;
29
30     node(int _id, long long _x) {
31         id = _id;
32         l = r = p = nullptr;
33         rev = false;
34         sz = 1;
35         // init extra variables:
36         P = rng();
37         add = 0;
38         x = _x;
39     }
40
41     // push everything else:
42     void push_stuff() {
43         if (add != 0) {
44             if (l != nullptr) {
45                 l->unsafe_apply(add);
46             }
47             if (r != nullptr) {
48                 r->unsafe_apply(add);
49             }

```



```

50     add = 0;
51 }
52 }
53
54 void unsafe_reverse() {
55     push_stuff();
56     rev ^= 1;
57     swap(l, r);
58     pull();
59 }
60
61 // apply changes:
62 void unsafe_apply(long long delta) {
63     add += delta;
64     x += delta;
65 }
66
67 void push() {
68     if (rev) {
69         if (l != nullptr) {
70             l->unsafe_reverse();
71         }
72         if (r != nullptr) {
73             r->unsafe_reverse();
74         }
75         rev = 0;
76     }
77     push_stuff();
78 }
79
80 void pull() {
81     sz = 1;
82     if (l != nullptr) {
83         l->p = this;
84         sz += l->sz;
85     }
86     if (r != nullptr) {
87         r->p = this;
88         sz += r->sz;
89     }
90 }
91 };
92

```

```

93 void debug_node(node* v, string pref = "") {
94 #ifdef LOCAL
95     if (v != nullptr) {
96         debug_node(v->r, pref + "└");
97         cerr << pref << "-" << "└" << v->id << '\n';
98         debug_node(v->l, pref + "└");
99     } else {
100         cerr << pref << "-" << "└" << "nullptr" << '\n';
101     }
102 #endif
103 }
104
105 namespace treap {
106
107 pair<node*, int> find(node* v, const function<int(node*)>> &go_to) {
108     // go_to returns: 0 -- found; -1 -- go left; 1 -- go right
109     // find returns the last vertex on the descent and its go_to
110     if (v == nullptr) {
111         return {nullptr, 0};
112     }
113     int dir;
114     while (true) {
115         v->push();
116         dir = go_to(v);
117         if (dir == 0) {
118             break;
119         }
120         node* u = (dir == -1 ? v->l : v->r);
121         if (u == nullptr) {
122             break;
123         }
124         v = u;
125     }
126     return {v, dir};
127 }
128
129 node* get_leftmost(node* v) {
130     return find(v, [&](node*) { return -1; }).first;
131 }
132
133 node* get_rightmost(node* v) {
134     return find(v, [&](node*) { return 1; }).first;
135 }

```

```

136
137 node* get_kth(node* v, int k) { // 0-indexed
138     pair<node*, int> p = find(v, [&](node * u) {
139         if (u->l != nullptr) {
140             if (u->l->sz > k) {
141                 return -1;
142             }
143             k -= u->l->sz;
144         }
145         if (k == 0) {
146             return 0;
147         }
148         k--;
149         return 1;
150     });
151     return (p.second == 0 ? p.first : nullptr);
152 }
153
154 int get_pos(node* v) { // 0-indexed
155     int k = (v->l != nullptr ? v->l->sz : 0);
156     while (v->p != nullptr) {
157         if (v == v->p->r) {
158             k++;
159             if (v->p->l != nullptr) {
160                 k += v->p->l->sz;
161             }
162         }
163         v = v->p;
164     }
165     return k;
166 }
167
168 node* get_root(node* v) {
169     while (v->p != nullptr) {
170         v = v->p;
171     }
172     return v;
173 }
174
175 pair<node*, node*> split(node* v, const function<bool(node*)> &is_right) {
176     if (v == nullptr) {
177         return {nullptr, nullptr};
178     }

```

```

179     v->push();
180     if (is_right(v)) {
181         pair<node*, node*> p = split(v->l, is_right);
182         if (p.first != nullptr) {
183             p.first->p = nullptr;
184         }
185         v->l = p.second;
186         v->pull();
187         return {p.first, v};
188     } else {
189         pair<node*, node*> p = split(v->r, is_right);
190         v->r = p.first;
191         if (p.second != nullptr) {
192             p.second->p = nullptr;
193         }
194         v->pull();
195         return {v, p.second};
196     }
197 }
198
199 pair<node*, node*> split_cnt(node* v, int k) {
200     if (v == nullptr) {
201         return {nullptr, nullptr};
202     }
203     v->push();
204     int left_and_me = (v->l != nullptr ? v->l->sz : 0) + 1;
205     if (k < left_and_me) {
206         pair<node*, node*> p = split_cnt(v->l, k);
207         if (p.first != nullptr) {
208             p.first->p = nullptr;
209         }
210         v->l = p.second;
211         v->pull();
212         return {p.first, v};
213     } else {
214         pair<node*, node*> p = split_cnt(v->r, k - left_and_me);
215         v->r = p.first;
216         if (p.second != nullptr) {
217             p.second->p = nullptr;
218         }
219         v->pull();
220         return {v, p.second};
221     }

```

```

222 }
223
224 node* merge(node* v, node* u) {
225     if (v == nullptr) {
226         return u;
227     }
228     if (u == nullptr) {
229         return v;
230     }
231     if (v->P > u->P) {
232         // if (rng() % (v->sz + u->sz) < (unsigned int) v->sz) {
233             v->push();
234             v->r = merge(v->r, u);
235             v->pull();
236             return v;
237         } else {
238             u->push();
239             u->l = merge(v, u->l);
240             u->pull();
241             return u;
242         }
243     }
244
245 int count_left(node* v, const function<bool(node*)> &is_right) {
246     if (v == nullptr) {
247         return 0;
248     }
249     v->push();
250     if (is_right(v)) {
251         return count_left(v->l, is_right);
252     }
253     return (v->l != nullptr ? v->l->sz : 0) + 1 + count_left(v->r, is_right);
254 }
255
256 int count_less(node* v, long long val) {
257     int res = 0;
258     while (v != nullptr) {
259         v->push();
260         if (v->x >= val) {
261             v = v->l;
262         } else {
263             res += (v->l != nullptr ? v->l->sz : 0) + 1;
264             v = v->r;

```

```

265     }
266 }
267 return res;
268 }
269
270 node* add(node* r, node* v, const function<bool(node*)> &go_left) {
271     pair<node*, node*> p = split(r, go_left);
272     return merge(p.first, merge(v, p.second));
273 }
274
275 node* remove(node* v) { // returns the new root
276     v->push();
277     node* x = v->l;
278     node* y = v->r;
279     node* p = v->p;
280     v->l = v->r = v->p = nullptr;
281     v->push();
282     v->pull(); // now v might be reusable...
283     node* z = merge(x, y);
284     if (p == nullptr) {
285         if (z != nullptr) {
286             z->p = nullptr;
287         }
288         return z;
289     }
290     if (p->l == v) {
291         p->l = z;
292     }
293     if (p->r == v) {
294         p->r = z;
295     }
296     while (true) {
297         p->push();
298         p->pull();
299         if (p->p == nullptr) {
300             break;
301         }
302         p = p->p;
303     }
304     return p;
305 }
306
307 node* next(node* v) {

```

```

308     if (v->r == nullptr) {
309         while (v->p != nullptr && v->p->r == v) {
310             v = v->p;
311         }
312         return v->p;
313     }
314     v->push();
315     v = v->r;
316     while (v->l != nullptr) {
317         v->push();
318         v = v->l;
319     }
320     return v;
321 }
322
323 node* prev(node* v) {
324     if (v->l == nullptr) {
325         while (v->p != nullptr && v->p->l == v) {
326             v = v->p;
327         }
328         return v->p;
329     }
330     v->push();
331     v = v->l;
332     while (v->r != nullptr) {
333         v->push();
334         v = v->r;
335     }
336     return v;
337 }
338
339 int get_size(node* v) {
340     return (v != nullptr ? v->sz : 0);
341 }
342
343 template<typename... T>
344 void Apply(node* v, T... args) {
345     v->unsafe_apply(args...);
346 }
347
348 void reverse(node* v) {
349     v->unsafe_reverse();
350 }

```

```

351
352 // extra of mine
353 long long lower(node* u, long long x) {
354     if (u == nullptr)
355         return numeric_limits<long long>::min();
356     else if (x <= u->x)
357         return lower(u->l, x);
358     else
359         return max(u->x, lower(u->r, x));
360 }
361
362 long long upper(node* u, long long x) {
363     if (u == nullptr)
364         return numeric_limits<long long>::max();
365     else if (u->x <= x)
366         return upper(u->r, x);
367     else
368         return min(u->x, upper(u->l, x));
369 }
370
371 } // namespace treap
372
373 using namespace treap;
374
375 int n;
376
377 int main() {
378     ios::sync_with_stdio(false);
379     cin.tie(0);
380     node* root = nullptr;
381     cin >> n;
382     for (int i = 1; i <= n; i++) {
383         int op;
384         long long x;
385         cin >> op >> x;
386         switch (op) {
387             case 1: {
388                 root = add(root, new node(x, x), [&](node * u) {
389                     return x < u->x;
390                 });
391                 break;
392             }
393             case 2: {

```

```

394     auto [pt, w] = find(root, [&](node * u) {
395         if (x < u->x) return -1;
396         else if (x == u->x) return 0;
397         else return 1;
398     });
399     assert(w == 0);
400     root = remove(pt);
401     break;
402 }
403 case 3: {
404     cout << count_less(root, x) + 1 << '\n';
405     break;
406 }
407 case 4: {
408     cout << get_kth(root, x - 1)->x << '\n';
409     break;
410 }
411 case 5: {
412     cout << lower(root, x) << '\n';
413     break;
414 }
415 case 6: {
416     cout << upper(root, x) << '\n';
417     break;
418 }
419 }
420 }
421 }

```

## 2.26 UnionFindRollback.cpp

```

1  vector<int> fa(n);
2  iota(all(fa), 0);
3  vector<int> sz(n, 1);
4  vector<pair<int, int>> ops;
5
6  auto Get = [&](int i) {
7      while (i != fa[i]) {
8          i = fa[i];
9      }
10     return i;
11 };
12 auto Unite = [&](int i, int j) {

```

```

13     i = Get(i), j = Get(j);
14     if (i == j) {
15         return;
16     }
17     if (sz[i] > sz[j]) {
18         swap(i, j);
19     }
20     ops.emplace_back(i, fa[i]);
21     fa[i] = j;
22     ops.emplace_back(~j, sz[j]);
23     sz[j] += sz[i];
24 };
25 auto RollBack = [&](int T) {
26     while (SZ(ops) > T) {
27         auto [i, j] = ops.back();
28         ops.pop_back();
29         if (i >= 0) {
30             fa[i] = j;
31         } else {
32             sz[~i] = j;
33         }
34     }
35 };
36
37 ll ans = 0;
38 auto Dfs = [&](auto &&Dfs, int l, int r) -> void {
39     if (l == r) {
40         for (auto [x, y] : g[l]) {
41             x = Get(x);
42             y = Get(y);
43             ans += 1ll * sz[x] * sz[y];
44         }
45     } else {
46         int mid = midpoint(l, r);
47         {
48             int save = SZ(ops);
49             for (int i = mid + 1; i <= r; i++) {
50                 for (auto [x, y] : g[i]) {
51                     Unite(x, y);
52                 }
53             }
54             Dfs(Dfs, l, mid);
55             RollBack(save);

```

```

56     }
57     {
58         int save = SZ(ops);
59         for (int i = 1; i <= mid; i++) {
60             for (auto [x, y] : g[i]) {
61                 Unite(x, y);
62             }
63         }
64         Dfs(Dfs, mid + 1, r);
65         RollBack(save);
66     }
67 }
68 };
69 Dfs(Dfs, 0, n - 1);

```

## 2.27 树哈希.cpp

```

1  basic_string<int> e[maxn];
2  ull hashv[maxn];
3  ull seed1, seed2, seed3, seed4;
4
5  ull f(ull x) { return x * x * x * seed1 + x * seed2; }
6  ull h(ull x) { return f(x) ^ ((x & seed3) >> 31) ^ ((x & seed4) << 31); }
7
8  void dfs1(int u, int fa) {
9      hashv[u] = 1;
10     for (auto v : e[u]) if (v != fa) {
11         dfs1(v, u);
12         hashv[u] += h(hashv[v]);
13     }
14 }
15
16 void dfs2(int u, int fa, ull fv) {
17     // for each root
18     hashv[u] += fv;
19     for (auto v : e[u]) if (v != fa) {
20         dfs2(v, u, h(hashv[u] - h(hashv[v])));
21     }
22 }
23
24 void solve() {
25     seed1 = rng(), seed2 = rng();
26     seed3 = rng(), seed4 = rng();

```

```

27     cin >> n;
28     rep(i, 2, n) {
29         int u, v;
30         cin >> u >> v;
31         e[u].pb(v);
32         e[v].pb(u);
33     }
34     dfs1(1, 0);
35     sort(hashv + 1, hashv + n + 1);
36     n = unique(hashv + 1, hashv + n + 1) - hashv - 1;
37     cout << n << '\n';
38 }

```

## 2.28 树链剖分 segtree.cpp

```

1  int n, m, a[N];
2  vector<int> e[N];
3  int l[N], r[N], idx[N];
4  int sz[N], hs[N], tot, top[N], dep[N], fa[N];
5
6  struct info {
7      int maxv, sum;
8  };
9
10 info operator + (const info &l, const info &r) {
11     return (info){max(l.maxv, r.maxv), l.sum + r.sum};
12 }
13
14 struct node {
15     info val;
16 } seg[N * 4];
17
18 // [l, r]
19
20 void update(int id) {
21     seg[id].val = seg[id * 2].val + seg[id * 2 + 1].val;
22 }
23
24 void build(int id, int l, int r) {
25     if (l == r) {
26         // l号点, DFS序中第l个点
27         seg[id].val = {a[idx[l]], a[idx[l]]};
28     } else {

```

```

29     int mid = (l + r) / 2;
30     build(id * 2, l, mid);
31     build(id * 2 + 1, mid + 1, r);
32     update(id);
33 }
34 }
35
36 void change(int id, int l, int r, int pos, int val) {
37     if (l == r) {
38         seg[id].val = {val, val};
39     } else {
40         int mid = (l + r) / 2;
41         if (pos <= mid) change(id * 2, l, mid, pos, val);
42         else change(id * 2 + 1, mid + 1, r, pos, val);
43         update(id);
44     }
45 }
46
47 info query(int id, int l, int r, int ql, int qr) {
48     if (l == ql && r == qr) return seg[id].val;
49     int mid = (l + r) / 2;
50     if (qr <= mid) return query(id * 2, l, mid, ql, qr);
51     else if (ql > mid) return query(id * 2 + 1, mid + 1, r, ql, qr);
52     else {
53         return query(id * 2, l, mid, ql, mid) +
54             query(id * 2 + 1, mid + 1, r, mid + 1, qr);
55     }
56 }
57
58 // 第一遍DFS, 子树大小, 重儿子, 父亲, 深度
59 void dfs1(int u, int f) {
60     sz[u] = 1;
61     hs[u] = -1;
62     fa[u] = f;
63     dep[u] = dep[f] + 1;
64     for (auto v : e[u]) {
65         if (v == f) continue;
66         dfs1(v, u);
67         sz[u] += sz[v];
68         if (hs[u] == -1 || sz[v] > sz[hs[u]])
69             hs[u] = v;
70     }
71 }

```

```

72
73 // 第二遍DFS, 每个点DFS序, 重链上的链头的元素。
74 void dfs2(int u, int t) {
75     top[u] = t;
76     l[u] = ++tot;
77     idx[tot] = u;
78     if (hs[u] != -1) {
79         dfs2(hs[u], t);
80     }
81     for (auto v : e[u]) {
82         if (v != fa[u] && v != hs[u]) {
83             dfs2(v, v);
84         }
85     }
86     r[u] = tot;
87 }
88
89 int LCA(int u, int v) {
90     while (top[u] != top[v]) {
91         if (dep[top[u]] < dep[top[v]]) v = fa[top[v]];
92         else u = fa[top[u]];
93     }
94     if (dep[u] < dep[v]) return u;
95     else return v;
96 }
97
98 info query(int u, int v) {
99     info ans{(int)-1e9, 0};
100     while (top[u] != top[v]) {
101         if (dep[top[u]] < dep[top[v]]) {
102             ans = ans + query(1, 1, n, l[top[v]], l[v]);
103             v = fa[top[v]];
104         } else {
105             ans = ans + query(1, 1, n, l[top[u]], l[u]);
106             u = fa[top[u]];
107         }
108     }
109     if (dep[u] <= dep[v]) ans = ans + query(1, 1, n, l[u], l[v]);
110     else ans = ans + query(1, 1, n, l[v], l[u]);
111     return ans;
112 }

```

## 2.29 笛卡尔树.cpp

```
1 int a[maxn], l[maxn], r[maxn], root;
2 int ans[maxn], tot;
3
4 void build() {
5     stack<int> stk;
6     for (int i = 1; i <= n; i++) {
7         int last = 0;
8         while (!stk.empty() && a[stk.top()] > a[i]) {
9             last = stk.top();
10            stk.pop();
11        }
12        if (stk.empty())
13            root = i;
14        else
15            r[stk.top()] = i;
16        l[i] = last;
17        stk.push(i);
18    }
19 }
20
21 void dfs(int c, int L, int R) {
22     ans[c] = ++tot;
23     if (l[c]) dfs(l[c], L, c - 1);
24     if (r[c]) dfs(r[c], c + 1, R);
25 }
```

## 2.30 线段树合并.cpp

```
1 struct node {
2     int sz, sum;
3     node *l, *r;
4     node() : sz(0), sum(0), l(nullptr), r(nullptr) {}
5 } pool[N * 20], *cur = pool;
6
7 node *newnode() {
8     return cur++;
9 }
10
11 void upd(node *rt) {
12     if (not rt) return;
13     rt->sum = rt->sz > 0;
14     if (rt->l) rt->sum += rt->l->sum;
```

```
15     if (rt->r) rt->sum += rt->r->sum;
16 }
17
18 node *modify(node *rt, int l, int r, int pos, int d) {
19     if (not rt) rt = newnode();
20     if (l == r) {
21         rt->sz += d;
22         upd(rt);
23         return rt;
24     } else {
25         int md = (l + r) >> 1;
26         if (pos <= md)
27             rt->l = modify(rt->l, l, md, pos, d);
28         else
29             rt->r = modify(rt->r, md + 1, r, pos, d);
30         upd(rt);
31         return rt;
32     }
33 }
34
35 node *merge(node *u, node *v, int l, int r) {
36     if (not u) return v;
37     if (not v) return u;
38     if (l == r) {
39         u->sz += v->sz;
40         upd(u);
41         return u;
42     } else {
43         int md = (l + r) >> 1;
44         u->l = merge(u->l, v->l, l, md);
45         u->r = merge(u->r, v->r, md + 1, r);
46         upd(u);
47         return u;
48     }
49 }
50
51 ll query(node *rt, int l, int r) {
52     if (not rt) return 0;
53     return rt->sum;
54 }
55
56 pair<node *, node *> split(node *rt, int l, int r, int ql, int qr) {
57     if (not rt) return {nullptr, nullptr};
```



```

58     if (ql == 1 && qr == r) {
59         return {nullptr, rt};
60     } else {
61         int md = (l + r) >> 1;
62         if (qr <= md) {
63             auto [p1, p2] = split(rt->l, l, md, ql, qr);
64             rt->l = p1;
65             upd(rt);
66             if (not p2) return {rt, nullptr};
67             node *u = newnode();
68             u->l = p2;
69             upd(u);
70             return {rt, u};
71         } else if (ql > md) {
72             auto [p1, p2] = split(rt->r, md + 1, r, ql, qr);
73             rt->r = p1;
74             upd(rt);
75             if (not p2) return {rt, nullptr};
76             node *u = newnode();
77             u->r = p2;
78             upd(u);
79             return {rt, u};
80         } else {
81             auto [p1, p2] = split(rt->l, l, md, ql, md);
82             auto [p3, p4] = split(rt->r, md + 1, r, md + 1, qr);
83             rt->l = p1, rt->r = p3;
84             upd(rt);
85             if (not p2 and not p4) return {rt, nullptr};
86             node *u = newnode();
87             u->l = p2, u->r = p4;
88             upd(u);
89             return {rt, u};
90         }
91     }
92 }

```

## 3 DP

### 3.1 Convex hull optimization.cpp

```

1 array<ll, 3> a[maxn];
2 int q[maxn];

```

```

3 ll ans[maxn];
4
5 ll X(int p) {
6     return 2ll * a[p][0];
7 }
8 ll Y(int p) {
9     return a[p][0] * a[p][0] + a[p][1];
10 }
11 ldb slope(int x, int y) {
12     return (ldb)(Y(y) - Y(x)) / (X(y) - X(x));
13 }
14 void solve() {
15     cin >> n;
16     int head = 1, rear = 0;
17     rep(i, 1, n) {
18         cin >> a[i][0] >> a[i][1];
19         a[i][2] = i;
20     }
21     sort(a + 1, a + n + 1);
22
23     rep(i, 1, n) {
24         while (head < rear && slope(q[rear], i) <= slope(q[rear], q[rear -
25             1])) rear--;
26         q[++rear] = i;
27     }
28     rep(i, 1, n) {
29         ll k = -a[i][0];
30         while (head < rear && slope(q[head], q[head + 1]) <= k) head++;
31         ans[a[i][2]] = (a[i][0] + a[q[head]][0]) * (a[i][0] + a[q[head]][0]
32             + a[i][1] + a[q[head]][1]);
33     }
34     rep(i, 1, n) cout << ans[i] << '\n';
35 }

```

### 3.2 DivideAndConquerDP.cpp

```

1 ll w[N][N], sum[N][N], opt[N], dp[805][N];
2
3 ll calc(int i, int j) { return sum[j][j] - sum[j][i] - sum[i][j] + sum[i][i]; }
4
5 void rec(int d, int l, int r, int optl, int opr) {
6     if (l > r) return;
7     int md = (l + r) >> 1;

```

```

8     rep(i,optl,opttr) if (dp[d-1][i]+calc(i,md)<dp[d][md]) {
9         dp[d][md]=dp[d-1][i]+calc(i,md);
10        opt[md]=i;
11    }
12    rec(d,l,md-1,optl,opt[md]);
13    rec(d,md+1,r,opt[md],optr);
14 }

```

### 3.3 有依赖决策单调.cpp

```

1 pair<int, int> stk[N];
2 auto calc = [&](int i, int j) { ... } // dp[j] -> dp[i]
3 int h = 0, t = 0;
4 stk[t++] = {1, 0}; // {left, opt}
5
6 for (int i = 1; i <= n; i++) {
7     if (h < t && stk[h].first < i) stk[h].first++;
8     if (h + 1 < t && stk[h].first >= stk[h + 1].first) ++h;
9     dp[i] = calc(i, stk[h].second);
10    while (h < t && calc(stk[t - 1].first, stk[t - 1].second) >= calc(stk[t
        - 1].first, i))
11        --t;
12    if (h < t) {
13        int l = stk[t - 1].first, r = n + 1;
14        while (l + 1 < r) {
15            int md = (l + r) >> 1;
16            if (calc(md, stk[t - 1].second) < calc(md, i)) l = md; else r =
                md;
17        }
18        if (r <= n) stk[t++] = {r, i};
19    } else stk[t++] = {i, i};
20 }

```

## 4 Geometry

### 4.1 1 (1).cpp

```

1 typedef double db;
2 const db EPS = 1e-9;
3
4 inline int sign(db a) { return a < -EPS ? -1 : a > EPS; }
5

```

```

6 inline int cmp(db a, db b) { return sign(a - b); }
7
8 struct P {
9     db x, y;
10    P() {}
11    P(db _x, db _y) : x(_x), y(_y) {}
12    P operator+(P p) { return {x + p.x, y + p.y}; }
13    P operator-(P p) { return {x - p.x, y - p.y}; }
14    P operator*(db d) { return {x * d, y * d}; }
15    P operator/(db d) { return {x / d, y / d}; }
16
17    bool operator<(P p) const {
18        int c = cmp(x, p.x);
19        if (c) return c == -1;
20        return cmp(y, p.y) == -1;
21    }
22
23    bool operator==(P o) const {
24        return cmp(x, o.x) == 0 && cmp(y, o.y) == 0;
25    }
26
27    db dot(P p) { return x * p.x + y * p.y; }
28    db det(P p) { return x * p.y - y * p.x; }
29
30    db distTo(P p) { return (*this - p).abs(); }
31    db alpha() { return atan2(y, x); }
32    void read() { cin >> x >> y; }
33    void write() { cout << "(" << x << ", " << y << ")" << endl; }
34    db abs() { return sqrt(abs2()); }
35    db abs2() { return x * x + y * y; }
36    P rot90() { return P(-y, x); }
37    P unit() { return *this / abs(); }
38    int quad() const { return sign(y) == 1 || (sign(y) == 0 && sign(x) >= 0)
        ; }
39    P rot(db an) { return {x * cos(an) - y * sin(an), x * sin(an) + y * cos(
        an)}; }
40 };
41
42 #define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
43 #define cross0p(p1,p2,p3) sign(cross(p1,p2,p3))
44
45 // 直线 p1p2, q1q2 是否恰有一个交点
46 bool chkLL(P p1, P p2, P q1, P q2) {

```

```

47     db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
48     return sign(a1 + a2) != 0;
49 }
50
51 // 求直线  $p_1p_2$ ,  $q_1q_2$  的交点
52 P isLL(P p1, P p2, P q1, P q2) {
53     db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
54     return (p1 * a2 + p2 * a1) / (a1 + a2);
55 }
56
57 // 判断区间  $[l1, r1]$ ,  $[l2, r2]$  是否相交
58 bool intersect(db l1, db r1, db l2, db r2) {
59     if (l1 > r1) swap(l1, r1); if (l2 > r2) swap(l2, r2);
60     return !(cmp(r1, l2) == -1 || cmp(r2, l1) == -1);
61 }
62
63 // 线段  $p_1p_2$ ,  $q_1q_2$  相交
64 bool isSS(P p1, P p2, P q1, P q2) {
65     return intersect(p1.x, p2.x, q1.x, q2.x) && intersect(p1.y, p2.y, q1.y,
66         q2.y) &&
67         crossOp(p1, p2, q1) * crossOp(p1, p2, q2) <= 0 && crossOp(q1, q2,
68             p1)
69         * crossOp(q1, q2, p2) <= 0;
70 }
71
72 // 线段  $p_1p_2$ ,  $q_1q_2$  严格相交
73 bool isSS_strict(P p1, P p2, P q1, P q2) {
74     return crossOp(p1, p2, q1) * crossOp(p1, p2, q2) < 0 && crossOp(q1, q2,
75         p1)
76         * crossOp(q1, q2, p2) < 0;
77 }
78
79 //  $m$  在  $a$  和  $b$  之间
80 bool isMiddle(db a, db m, db b) {
81     /*if (a > b) swap(a, b);
82     return cmp(a, m) <= 0 && cmp(m, b) <= 0;*/
83     return sign(a - m) == 0 || sign(b - m) == 0 || (a < m != b < m);
84 }
85
86 bool isMiddle(P a, P m, P b) {
87     return isMiddle(a.x, m.x, b.x) && isMiddle(a.y, m.y, b.y);
88 }

```

```

87 // 点  $p$  在线段  $p_1p_2$  上
88 bool onSeg(P p1, P p2, P q) {
89     return crossOp(p1, p2, q) == 0 && isMiddle(p1, q, p2);
90 }
91 //  $q_1q_2$  和  $p_1p_2$  的交点 在  $p_1p_2$  上?
92
93 // 点  $p$  严格在  $p_1p_2$  上
94 bool onSeg_strict(P p1, P p2, P q) {
95     return crossOp(p1, p2, q) == 0 && sign((q - p1).dot(p1 - p2)) * sign((q
96         - p2).dot(p1 - p2)) < 0;
97 }
98 // 求  $q$  到 直线 $p_1p_2$  的投影 (垂足) :  $p_1 \neq p_2$ 
99 P proj(P p1, P p2, P q) {
100     P dir = p2 - p1;
101     return p1 + dir * (dir.dot(q - p1) / dir.abs2());
102 }
103
104 // 求  $q$  以 直线 $p_1p_2$  为轴的反射
105 P reflect(P p1, P p2, P q) {
106     return proj(p1, p2, q) * 2 - q;
107 }
108
109 // 求  $q$  到 线段 $p_1p_2$  的最小距离
110 db nearest(P p1, P p2, P q) {
111     if (p1 == p2) return p1.distTo(q);
112     P h = proj(p1, p2, q);
113     if (isMiddle(p1, h, p2))
114         return q.distTo(h);
115     return min(p1.distTo(q), p2.distTo(q));
116 }
117
118 // 求 线段 $p_1p_2$  与 线段 $q_1q_2$  的距离
119 db disSS(P p1, P p2, P q1, P q2) {
120     if (isSS(p1, p2, q1, q2)) return 0;
121     return min(min(nearest(p1, p2, q1), nearest(p1, p2, q2)), min(nearest(q1
122         , q2, p1), nearest(q1, q2, p2)));
123 }
124
125 // 极角排序
126 sort(p, p + n, [&](P a, P b) {
127     int qa = a.quad(), qb = b.quad();
128     if (qa != qb) return qa < qb;

```

```

128     else return sign(a.det(b)) > 0;
129 });

```

## 4.2 1 (2).cpp

```

1 db area(vector<P> ps){
2     db ret = 0; rep(i,0,ps.size()) ret += ps[i].det(ps[(i+1)%ps.size()]);
3     return ret/2;
4 }
5
6 int contain(vector<P> ps, P p){ //2:inside,1:on_seg,0:outside
7     int n = ps.size(), ret = 0;
8     rep(i,0,n){
9         P u=ps[i],v=ps[(i+1)%n];
10        if(onSeg(u,v,p)) return 1;
11        if(cmp(u.y,v.y)<=0) swap(u,v);
12        if(cmp(p.y,u.y) > 0 || cmp(p.y,v.y) <= 0) continue;
13        ret ^= crossOp(p,u,v) > 0;
14    }
15    return ret*2;
16 }
17
18 vector<P> convexHull(vector<P> ps) {
19     int n = ps.size(); if(n <= 1) return ps;
20     sort(ps.begin(), ps.end());
21     vector<P> qs(n * 2); int k = 0;
22     for (int i = 0; i < n; qs[k++] = ps[i++])
23         while (k > 1 && crossOp(qs[k - 2], qs[k - 1], ps[i]) <= 0) --k;
24     for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
25         while (k > t && crossOp(qs[k - 2], qs[k - 1], ps[i]) <= 0) --k;
26     qs.resize(k - 1);
27     return qs;
28 }
29
30 vector<P> convexHullNonStrict(vector<P> ps) {
31     //caution: need to unique the Ps first
32     int n = ps.size(); if(n <= 1) return ps;
33     sort(ps.begin(), ps.end());
34     vector<P> qs(n * 2); int k = 0;
35     for (int i = 0; i < n; qs[k++] = ps[i++])
36         while (k > 1 && crossOp(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;
37     for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
38         while (k > t && crossOp(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;

```

```

39     qs.resize(k - 1);
40     return qs;
41 }
42
43 db convexDiameter(vector<P> ps){
44     int n = ps.size(); if(n <= 1) return 0;
45     int is = 0, js = 0; rep(k,1,n) is = ps[k]<ps[is]?k:is, js = ps[js] < ps[
46         k]?k:js;
47     int i = is, j = js;
48     db ret = ps[i].distTo(ps[j]);
49     do{
50         if((ps[(i+1)%n]-ps[i]).det(ps[(j+1)%n]-ps[j]) >= 0)
51             (++j)%=n;
52         else
53             (++i)%=n;
54         ret = max(ret,ps[i].distTo(ps[j]));
55     }while(i!=is || j!=js);
56     return ret;
57 }
58 vector<P> convexCut(const vector<P>&ps, P q1, P q2) {
59     vector<P> qs;
60     int n = ps.size();
61     rep(i,0,n){
62         P p1 = ps[i], p2 = ps[(i+1)%n];
63         int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
64         if(d1 >= 0) qs.push_back(p1);
65         if(d1 * d2 < 0) qs.push_back(isLL(p1,p2,q1,q2));
66     }
67     return qs;
68 }
69
70 void reorderPolygon(vector<P> &ps) {
71     size_t pos = 0;
72     for(size_t i = 1; i < ps.size(); i++){
73         if(ps[i].y < ps[pos].y || (ps[i].y == ps[pos].y && ps[i].x < ps[pos
74             ].x))
75             pos = i;
76     }
77     rotate(ps.begin(), ps.begin() + pos, ps.end());
78 }
79 vector<P> minkowski(vector<P> p, vector<P> q){

```

```

80     if(p.empty()) return q;
81     // the first vertex must be the lowest
82     reorderPolygon(p);
83     reorderPolygon(q);
84     // must ensure cyclic indexing
85     p.push_back(p[0]);
86     p.push_back(p[1]);
87     q.push_back(q[0]);
88     q.push_back(q[1]);
89     // main part
90     vector<P> result;
91     size_t i = 0, j = 0;
92     while(i < p.size() - 2 || j < q.size() - 2){
93         result.push_back(p[i] + q[j]);
94         auto cross = (p[i + 1] - p[i]).det(q[j + 1] - q[j]);
95         if(cross >= 0 && i < SZ(p) - 2)
96             ++i;
97         if(cross <= 0 && j < SZ(q) - 2)
98             ++j;
99     }
100     return result;
101 }
102
103 bool convexContain(const vector<P> &l, P p, bool strict = true) {
104     int a = 1, b = l.size() - 1, r = !strict;
105     if (l.size() < 3) return r && onSeg(l[0], l.back(), p);
106     if (crossOp(l[0], l[a], l[b]) > 0) swap(a, b);
107     if (crossOp(l[0], l[a], p) >= r || crossOp(l[0], l[b], p) <= -r)
108         return false;
109     while (abs(a - b) > 1) {
110         int c = (a + b) / 2;
111         (crossOp(l[0], l[c], p) > 0 ? b : a) = c;
112     }
113     return sign(cross(l[a], l[b], p)) < r;
114 }

```

### 4.3 1 (3).cpp

```

1 int type(P o1,db r1,P o2,db r2){
2     db d = o1.distTo(o2);
3     if(cmp(d,r1+r2) == 1) return 4;
4     if(cmp(d,r1+r2) == 0) return 3;
5     if(cmp(d,abs(r1-r2)) == 1) return 2;

```

```

6     if(cmp(d,abs(r1-r2)) == 0) return 1;
7     return 0;
8 }
9
10 vector<P> isCL(P o,db r,P p1,P p2){
11     if (cmp(abs((o-p1).det(p2-p1)/p1.distTo(p2)),r)>0) return {};
12     db x = (p1-o).dot(p2-p1), y = (p2-p1).abs2(), d = x * x - y * ((p1-o).
13         abs2() - r*r);
14     d = max(d,(db)0.0); P m = p1 - (p2-p1)*(x/y), dr = (p2-p1)*(sqrt(d)/y);
15     return {m-dr,m+dr}; //along dir: p1->p2
16 }
17 vector<P> isCC(P o1, db r1, P o2, db r2) { //need to check whether two
18     circles are the same
19     db d = o1.distTo(o2);
20     if (cmp(d, r1 + r2) == 1) return {};
21     if (cmp(d,abs(r1-r2))==-1) return {};
22     d = min(d, r1 + r2);
23     db y = (r1 * r1 + d * d - r2 * r2) / (2 * d), x = sqrt(r1 * r1 - y * y);
24     P dr = (o2 - o1).unit();
25     P q1 = o1 + dr * y, q2 = dr.rot90() * x;
26     return {q1-q2,q1+q2}; //along circle 1
27 }
28 // extanCC, intanCC : -r2, tanCP : r2 = 0
29 vector<pair<P, P>> tanCC(P o1, db r1, P o2, db r2) {
30     P d = o2 - o1;
31     db dr = r1 - r2, d2 = d.abs2(), h2 = d2 - dr * dr;
32     if (sign(d2) == 0 || sign(h2) < 0) return {};
33     h2 = max((db)0.0, h2);
34     vector<pair<P, P>> ret;
35     for (db sign : {-1, 1}) {
36         P v = (d * dr + d.rot90() * sqrt(h2) * sign) / d2;
37         ret.push_back({o1 + v * r1, o2 + v * r2});
38     }
39     if (sign(h2) == 0) ret.pop_back();
40     return ret;
41 }
42
43 db rad(P p1,P p2){
44     return atan2l(p1.det(p2),p1.dot(p2));
45 }
46

```

```

47 db areaCT(db r, P p1, P p2){
48     vector<P> is = isCL(P(0,0),r,p1,p2);
49     if(is.empty()) return r*r*rad(p1,p2)/2;
50     bool b1 = cmp(p1.abs2(),r*r) == 1, b2 = cmp(p2.abs2(), r*r) == 1;
51     if(b1 && b2){
52         P md=(is[0]+is[1])/2;
53         if(sign((p1-md).dot(p2-md)) <= 0)
54             return r*r*(rad(p1,is[0]) + rad(is[1],p2))/2 + is[0].det(is[1])
                    /2;
55         else return r*r*rad(p1,p2)/2;
56     }
57     if(b1) return (r*r*rad(p1,is[0]) + is[0].det(p2))/2;
58     if(b2) return (p1.det(is[1]) + r*r*rad(is[1],p2))/2;
59     return p1.det(p2)/2;
60 }
61
62 P inCenter(P A, P B, P C) {
63     double a = (B - C).abs(), b = (C - A).abs(), c = (A - B).abs();
64     return (A * a + B * b + C * c) / (a + b + c);
65 }
66
67 P circumCenter(P a, P b, P c) {
68     P bb = b - a, cc = c - a;
69     double db = bb.abs2(), dc = cc.abs2(), d = 2 * bb.det(cc);
70     return a - P(bb.y * dc - cc.y * db, cc.x * db - bb.x * dc) / d;
71 }
72
73 P othroCenter(P a, P b, P c) {
74     P ba = b - a, ca = c - a, bc = b - c;
75     double Y = ba.y * ca.y * bc.y,
76     A = ca.x * ba.y - ba.x * ca.y,
77     x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
78     y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
79     return {x0, y0};
80 }
81
82 pair<P,db> min_circle(vector<P> ps){
83     random_shuffle(ps.begin(), ps.end());
84     int n = ps.size();
85     P o = ps[0]; db r = 0;
86     rep(i,1,n) if(o.distTo(ps[i]) > r + EPS){
87         o = ps[i], r = 0;
88         rep(j,0,i) if(o.distTo(ps[j]) > r + EPS){

```

```

89             o = (ps[i] + ps[j]) / 2; r = o.distTo(ps[i]);
90             rep(k,0,j) if(o.distTo(ps[k]) > r + EPS){
91                 o = circumCenter(ps[i],ps[j],ps[k]);
92                 r = o.distTo(ps[i]);
93             }
94         }
95     }
96     return {o,r};
97 }

```

## 4.4 all.cpp

```

1  typedef double db;
2  const db EPS = 1e-9;
3
4  inline int sign(db a) { return a < -EPS ? -1 : a > EPS; }
5
6  inline int cmp(db a, db b){ return sign(a-b); }
7
8  struct P {
9      db x, y;
10     P() {}
11     P(db _x, db _y) : x(_x), y(_y) {}
12     P operator+(P p) { return {x + p.x, y + p.y}; }
13     P operator-(P p) { return {x - p.x, y - p.y}; }
14     P operator*(db d) { return {x * d, y * d}; }
15     P operator/(db d) { return {x / d, y / d}; }
16
17     bool operator<(P p) const {
18         int c = cmp(x, p.x);
19         if (c) return c == -1;
20         return cmp(y, p.y) == -1;
21     }
22
23     bool operator==(P o) const{
24         return cmp(x,o.x) == 0 && cmp(y,o.y) == 0;
25     }
26
27     db dot(P p) { return x * p.x + y * p.y; }
28     db det(P p) { return x * p.y - y * p.x; }
29
30     db distTo(P p) { return (*this-p).abs(); }
31     db alpha() { return atan2(y, x); }

```

```

32 void read() { cin>>x>>y; }
33 void write() {cout<<"("<<x<<","<<y<<")"<<endl;}
34 db abs() { return sqrt(abs2());}
35 db abs2() { return x * x + y * y; }
36 P rot90() { return P(-y,x);}
37 P unit() { return *this/abs(); }
38 int quad() const { return sign(y) == 1 || (sign(y) == 0 && sign(x) >= 0)
    ; }
39 P rot(db an){ return {x*cos(an)-y*sin(an),x*sin(an) + y*cos(an)}; }
40 };
41
42 struct L{ //ps[0] -> ps[1]
43     P ps[2];
44     P& operator[](int i) { return ps[i]; }
45     P dir() { return ps[1] - ps[0]; }
46     bool include(P p) { return sign((ps[1] - ps[0]).det(p - ps[0])) > 0; }
47     L push(){ // push eps outward
48         const double eps = 1e-6;
49         P delta = (ps[1] - ps[0]).rot90().unit() * eps;
50         return {{ps[0] - delta, ps[1] - delta}};
51     }
52 };
53
54 #define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
55 #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
56
57 bool chkLL(P p1, P p2, P q1, P q2) {
58     db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
59     return sign(a1+a2) != 0;
60 }
61
62 P isLL(P p1, P p2, P q1, P q2) {
63     db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
64     return (p1 * a2 + p2 * a1) / (a1 + a2);
65 }
66
67 P isLL(L l1,L l2){ return isLL(l1[0],l1[1],l2[0],l2[1]); }
68
69 bool intersect(db l1,db r1,db l2,db r2){
70     if(l1>r1) swap(l1,r1); if(l2>r2) swap(l2,r2);
71     return !( cmp(r1,l2) == -1 || cmp(r2,l1) == -1 );
72 }
73

```

```

74 bool isSS(P p1, P p2, P q1, P q2){
75     return intersect(p1.x,p2.x,q1.x,q2.x) && intersect(p1.y,p2.y,q1.y,q2.y)
    &&
76     crossOp(p1,p2,q1) * crossOp(p1,p2,q2) <= 0 && crossOp(q1,q2,p1)
    * crossOp(q1,q2,p2) <= 0;
78 }
79
80 bool isSS_strict(P p1, P p2, P q1, P q2){
81     return crossOp(p1,p2,q1) * crossOp(p1,p2,q2) < 0 && crossOp(q1,q2,p1)
    * crossOp(q1,q2,p2) < 0;
83 }
84
85 bool isMiddle(db a, db m, db b) {
86     return sign(a - m) == 0 || sign(b - m) == 0 || (a < m != b < m);
87 }
88
89 bool isMiddle(P a, P m, P b) {
90     return isMiddle(a.x, m.x, b.x) && isMiddle(a.y, m.y, b.y);
91 }
92
93 bool onSeg(P p1, P p2, P q){
94     return crossOp(p1,p2,q) == 0 && isMiddle(p1, q, p2);
95 }
96
97 bool onSeg_strict(P p1, P p2, P q){
98     return crossOp(p1,p2,q) == 0 && sign((q-p1).dot(p1-p2)) * sign((q-p2).
    dot(p1-p2)) < 0;
99 }
100
101 P proj(P p1, P p2, P q) {
102     P dir = p2 - p1;
103     return p1 + dir * (dir.dot(q - p1) / dir.abs2());
104 }
105
106 P reflect(P p1, P p2, P q){
107     return proj(p1,p2,q) * 2 - q;
108 }
109
110 db nearest(P p1,P p2,P q){
111     P h = proj(p1,p2,q);
112     if(isMiddle(p1,h,p2))
113         return q.distTo(h);
114     return min(p1.distTo(q),p2.distTo(q));

```

```

115 }
116
117 db disSS(P p1, P p2, P q1, P q2){
118     if(isSS(p1,p2,q1,q2)) return 0;
119     return min(min(nearest(p1,p2,q1),nearest(p1,p2,q2)), min(nearest(q1,q2,
        p1),nearest(q1,q2,p2)));
120 }
121
122 db rad(P p1,P p2){
123     return atan2l(p1.det(p2),p1.dot(p2));
124 }
125
126 db incircle(P p1, P p2, P p3){
127     db A = p1.distTo(p2);
128     db B = p2.distTo(p3);
129     db C = p3.distTo(p1);
130     return sqrtl(A*B*C/(A+B+C));
131 }
132
133 //polygon
134
135 db area(vector<P> ps){
136     db ret = 0; rep(i,0,ps.size()) ret += ps[i].det(ps[(i+1)%ps.size()]);
137     return ret/2;
138 }
139
140 int contain(vector<P> ps, P p){ //2:inside,1:on_seg,0:outside
141     int n = ps.size(), ret = 0;
142     rep(i,0,n){
143         P u=ps[i],v=ps[(i+1)%n];
144         if(onSeg(u,v,p)) return 1;
145         if(cmp(u.y,v.y)<=0) swap(u,v);
146         if(cmp(p.y,u.y) >0 || cmp(p.y,v.y) <= 0) continue;
147         ret ^= crossOp(p,u,v) > 0;
148     }
149     return ret*2;
150 }
151
152 vector<P> convexHull(vector<P> ps) {
153     int n = ps.size(); if(n <= 1) return ps;
154     sort(ps.begin(), ps.end());
155     vector<P> qs(n * 2); int k = 0;
156     for (int i = 0; i < n; qs[k++] = ps[i++])

```

```

157         while (k > 1 && crossOp(qs[k - 2], qs[k - 1], ps[i]) <= 0) --k;
158     for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
159         while (k > t && crossOp(qs[k - 2], qs[k - 1], ps[i]) <= 0) --k;
160     qs.resize(k - 1);
161     return qs;
162 }
163
164 vector<P> convexHullNonStrict(vector<P> ps) {
165     //caution: need to unique the Ps first
166     int n = ps.size(); if(n <= 1) return ps;
167     sort(ps.begin(), ps.end());
168     vector<P> qs(n * 2); int k = 0;
169     for (int i = 0; i < n; qs[k++] = ps[i++])
170         while (k > 1 && crossOp(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;
171     for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
172         while (k > t && crossOp(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;
173     qs.resize(k - 1);
174     return qs;
175 }
176
177 db convexDiameter(vector<P> ps){
178     int n = ps.size(); if(n <= 1) return 0;
179     int is = 0, js = 0; rep(k,1,n) is = ps[k]<ps[is]?k:is, js = ps[js] < ps[
        k]?k:js;
180     int i = is, j = js;
181     db ret = ps[i].distTo(ps[j]);
182     do{
183         if((ps[(i+1)%n]-ps[i]).det(ps[(j+1)%n]-ps[j]) >= 0)
184             (++j)%=n;
185         else
186             (++i)%=n;
187         ret = max(ret,ps[i].distTo(ps[j]));
188     }while(i!=is || j!=js);
189     return ret;
190 }
191
192 vector<P> convexCut(const vector<P>&ps, P q1, P q2) {
193     vector<P> qs;
194     int n = ps.size();
195     rep(i,0,n){
196         P p1 = ps[i], p2 = ps[(i+1)%n];
197         int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
198         if(d1 >= 0) qs.pb(p1);

```



```

199         if(d1 * d2 < 0) qs.pb(isLL(p1,p2,q1,q2));
200     }
201     return qs;
202 }
203
204 //min_dist
205
206 db min_dist(vector<P>&ps,int l,int r){
207     if(r-l<=5){
208         db ret = 1e100;
209         rep(i,l,r) rep(j,l,i) ret = min(ret,ps[i].distTo(ps[j]));
210         return ret;
211     }
212     int m = (l+r)>>1;
213     db ret = min(min_dist(ps,l,m),min_dist(ps,m,r));
214     vector<P> qs; rep(i,l,r) if(abs(ps[i].x-ps[m].x)<= ret) qs.pb(ps[i]);
215     sort(qs.begin(), qs.end(),[](P a,P b) -> bool {return a.y<b.y; });
216     rep(i,1,qs.size()) for(int j=i-1;j>=0&&qs[j].y>=qs[i].y-ret;--j)
217         ret = min(ret,qs[i].distTo(qs[j]));
218     return ret;
219 }
220
221 int type(P o1,db r1,P o2,db r2){
222     db d = o1.distTo(o2);
223     if(cmp(d,r1+r2) == 1) return 4;
224     if(cmp(d,r1+r2) == 0) return 3;
225     if(cmp(d,abs(r1-r2)) == 1) return 2;
226     if(cmp(d,abs(r1-r2)) == 0) return 1;
227     return 0;
228 }
229
230 vector<P> isCL(P o,db r,P p1,P p2){
231     db x = (p1-o).dot(p2-p1), y = (p2-p1).abs2(), d = x * x - y * ((p1-o).
        abs2() - r*r);
232     if(sign(d) < 0) return {};
233     d = max(d,0.0); P m = p1 - (p2-p1)*(x/y), dr = (p2-p1)*(sqrt(d)/y);
234     return {m-dr,m+dr}; //along dir: p1->p2
235 }
236
237 vector<P> isCC(P o1, db r1, P o2, db r2) { //need to check whether two
        circles are the same
238     db d = o1.distTo(o2);
239     if (cmp(d, r1 + r2) == 1) return {};

```

```

240     d = min(d, r1 + r2);
241     db y = (r1 * r1 + d * d - r2 * r2) / (2 * d), x = sqrt(r1 * r1 - y * y);
242     P dr = (o2 - o1).unit();
243     P q1 = o1 + dr * y, q2 = dr.rot90() * x;
244     return {q1-q2,q1+q2}; //along circle 1
245 }
246
247 vector<P> tanCP(P o, db r, P p) {
248     db x = (p - o).abs2(), d = x - r * r;
249     if (sign(d) <= 0) return {}; // on circle => no tangent
250     P q1 = o + (p - o) * (r * r / x);
251     P q2 = (p - o).rot90() * (r * sqrt(d) / x);
252     return {q1-q2,q1+q2}; //counter clock-wise
253 }
254
255 vector<L> extanCC(P o1, db r1, P o2, db r2) {
256     vector<L> ret;
257     if (cmp(r1, r2) == 0) {
258         P dr = (o2 - o1).unit().rot90() * r1;
259         ret.pb({{o1 + dr, o2 + dr}}), ret.pb({{o1 - dr, o2 - dr}});
260     } else {
261         P p = (o2 * r1 - o1 * r2) / (r1 - r2);
262         vector<P> ps = tanCP(o1, r1, p), qs = tanCP(o2, r2, p);
263         rep(i,0,min(ps.size(),qs.size())) ret.pb({{ps[i], qs[i]}}); //c1
        counter-clock wise
264     }
265 }
266
267 return ret;
268
269 vector<L> intanCC(P o1, db r1, P o2, db r2) {
270     vector<L> ret;
271     P p = (o1 * r2 + o2 * r1) / (r1 + r2);
272     vector<P> ps = tanCP(o1,r1,p), qs = tanCP(o2,r2,p);
273     rep(i,0,min(ps.size(),qs.size())) ret.pb({{ps[i], qs[i]}}); //c1 counter
        -clock wise
274     return ret;
275 }
276
277 db areaCT(db r, P p1, P p2){
278     vector<P> is = isCL(P(0,0),r,p1,p2);
279     if(is.empty()) return r*r*rad(p1,p2)/2;
280     bool b1 = cmp(p1.abs2(),r*r) == 1, b2 = cmp(p2.abs2(), r*r) == 1;

```

```

281     if(b1 && b2){
282         if(sign((p1-is[0]).dot(p2-is[0])) <= 0 &&
283             sign((p1-is[0]).dot(p2-is[0])) <= 0)
284             return r*r*(rad(p1,is[0]) + rad(is[1],p2))/2 + is[0].det(is[1])/2;
285         else return r*r*rad(p1,p2)/2;
286     }
287     if(b1) return (r*r*rad(p1,is[0]) + is[0].det(p2))/2;
288     if(b2) return (p1.det(is[1]) + r*r*rad(is[1],p2))/2;
289     return p1.det(p2)/2;
290 }
291
292 bool parallel(L l0, L l1) { return sign( l0.dir().det( l1.dir() ) ) == 0; }
293
294 bool sameDir(L l0, L l1) { return parallel(l0, l1) && sign(l0.dir().dot(l1.
    dir())) == 1; }
295
296 bool cmp (P a, P b) {
297     if (a.quad() != b.quad()) {
298         return a.quad() < b.quad();
299     } else {
300         return sign( a.det(b) ) > 0;
301     }
302 }
303
304 bool operator < (L l0, L l1) {
305     if (sameDir(l0, l1)) {
306         return l1.include(l0[0]);
307     } else {
308         return cmp( l0.dir(), l1.dir() );
309     }
310 }
311
312 bool check(L u, L v, L w) {
313     return w.include(isLL(u,v));
314 }
315
316 vector<P> halfPlaneIS(vector<L> &l) {
317     sort(l.begin(), l.end());
318     deque<L> q;
319     for (int i = 0; i < (int)l.size(); ++i) {
320         if (i && sameDir(l[i], l[i - 1])) continue;
321         while (q.size() > 1 && !check(q[q.size() - 2], q[q.size() - 1], l[i
            ])) q.pop_back();

```

```

322         while (q.size() > 1 && !check(q[1], q[0], l[i])) q.pop_front();
323         q.push_back(l[i]);
324     }
325     while (q.size() > 2 && !check(q[q.size() - 2], q[q.size() - 1], q[0])) q
        .pop_back();
326     while (q.size() > 2 && !check(q[1], q[0], q[q.size() - 1])) q.pop_front
        ();
327     vector<P> ret;
328     for (int i = 0; i < (int)q.size(); ++i) ret.push_back(isLL(q[i], q[(i +
        1) % q.size()]));
329     return ret;
330 }
331
332 P inCenter(P A, P B, P C) {
333     double a = (B - C).abs(), b = (C - A).abs(), c = (A - B).abs();
334     return (A * a + B * b + C * c) / (a + b + c);
335 }
336
337 P circumCenter(P a, P b, P c) {
338     P bb = b - a, cc = c - a;
339     double db = bb.abs2(), dc = cc.abs2(), d = 2 * bb.det(cc);
340     return a - P(bb.y * dc - cc.y * db, cc.x * db - bb.x * dc) / d;
341 }
342
343 P othroCenter(P a, P b, P c) {
344     P ba = b - a, ca = c - a, bc = b - c;
345     double Y = ba.y * ca.y * bc.y,
346     A = ca.x * ba.y - ba.x * ca.y,
347     x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
348     y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
349     return {x0, y0};
350 }

```

## 4.5 圆面积并.cpp

```

1 db intergal(db x,db y,db r,db L,db R){
2     return r*r*(R-L) + x*r*(sinl(R) - sinl(L)) + y*r*(-cosl(R) + cosl(L));
3 }
4
5 db calc_area_circle(P c,db r,db L,db R){
6     return intergal(c.x,c.y,r,L,R) / 2;
7 }
8

```

```

9  db norm(db x){
10      while(x < 0) x += 2 * PI;
11      while(x > 2 * PI) x -= 2 * PI;
12      return x;
13  }
14
15  P cs[N]; db rs[N];
16
17  void work(){
18      vector<int> cand = {};
19      rep(i,0,m){
20          bool ok = 1;
21          rep(j,0,m) if(i!=j){
22              if(rs[j] > rs[i] + EPS && rs[i] + cs[i].distTo(cs[j]) <= rs[j] +
                EPS){
23                  ok = 0; break;
24              }
25              if(cs[i] == cs[j] && cmp(rs[i],rs[j]) == 0 && j < i){
26                  ok = 0; break;
27              }
28          }
29          if(ok) cand.pb(i);
30      }
31
32      rep(i,0,cand.size()) cs[i] = cs[cand[i]], rs[i] = rs[cand[i]];
33      m = cand.size();
34
35      db area = 0;
36
37      //work
38      rep(i,0,m){
39          vector<pair<db,int>> ev = {{0,0},{2*PI,0}};
40
41          int cur = 0;
42
43          rep(j,0,m) if(j!=i){
44              auto ret = isCC(cs[i],rs[i],cs[j],rs[j]);
45              if(!ret.empty()){
46                  db l = (ret[0] - cs[i]).alpha();
47                  db r = (ret[1] - cs[i]).alpha();
48                  l = norm(l); r = norm(r);
49                  ev.pb({l,1});ev.pb({r,-1});
50                  if(l > r) ++cur;

```

```

51      }
52      }
53
54      sort(ev.begin(), ev.end());
55      rep(j,0,ev.size() - 1){
56          cur += ev[j].se;
57          if(cur == 0){
58              area += calc_area_circle(cs[i],rs[i],ev[j].fi,ev[j+1].fi);
59          }
60      }
61      }
62  }

```

## 5 Graph

### 5.1 bellmanford.cpp

```

1  vector<PII> e[N];
2
3  template <typename T>
4  void add(int u, int v, T w) {
5      e[u].eb(v, w);
6  }
7
8  template <typename T>
9  vector<T> bellmanford(vector<pair<int, T>> *g, int start) {
10      // assert(0 <= start && start < g.n);
11      // maybe use inf = numeric_limits<T>::max() / 4
12      const T inf = numeric_limits<T>::max() / 4;
13      vector<T> dist(n, inf);
14      dist[start] = 0;
15      int cnt = 0;
16      while (true) {
17          bool upd = 0;
18          cnt++;
19          for (int i = 0; i < n; i++) {
20              for (auto [to, cost] : e[i]) {
21                  if (dist[to] > dist[i] + cost) {
22                      upd = 1;
23                      dist[to] = dist[i] + cost;
24                  }
25              }

```

```

26     }
27     if (!upd || cnt == n) {
28         break;
29     }
30 }
31 return dist;
32 // returns inf if there's no path
33 }

```

## 5.2 BlockCutTree.cpp

---

```

1 struct BlockCutTree {
2     int n;
3     std::vector<std::vector<int>>> adj;
4     std::vector<int> dfn, low, stk;
5     int cnt, cur;
6     std::vector<std::pair<int, int>>> edges;
7
8     BlockCutTree() {}
9     BlockCutTree(int n) {
10         init(n);
11     }
12
13     void init(int n) {
14         this->n = n;
15         adj.assign(n, {});
16         dfn.assign(n, -1);
17         low.resize(n);
18         stk.clear();
19         cnt = cur = 0;
20         edges.clear();
21     }
22
23     void addEdge(int u, int v) {
24         adj[u].push_back(v);
25         adj[v].push_back(u);
26     }
27
28     void dfs(int x) {
29         stk.push_back(x);
30         dfn[x] = low[x] = cur++;
31
32         for (auto y : adj[x]) {

```

```

33             if (dfn[y] == -1) {
34                 dfs(y);
35                 low[x] = std::min(low[x], low[y]);
36                 if (low[y] == dfn[x]) {
37                     int v;
38                     do {
39                         v = stk.back();
40                         stk.pop_back();
41                         edges.emplace_back(n + cnt, v);
42                     } while (v != y);
43                     edges.emplace_back(x, n + cnt);
44                     cnt++;
45                 }
46             } else {
47                 low[x] = std::min(low[x], dfn[y]);
48             }
49         }
50     }
51
52     std::pair<int, std::vector<std::pair<int, int>>>> work() {
53         for (int i = 0; i < n; i++) {
54             if (dfn[i] == -1) {
55                 stk.clear();
56                 dfs(i);
57             }
58         }
59         return {cnt, edges};
60     }
61 };

```

## 5.3 boruvka.cpp

---

```

1 /**
2  * while component > 1:
3  *     for each component:
4  *         find select[i]
5  *     for each component:
6  *         if select[i] != i:
7  *             merge(i, select[i])
8  *         component--
9  */
10
11 ll ans = 0, cnt = n;

```

```

12 while (cnt > 1) {
13     fill(select + 1, select + n + 1, -1);
14     vector<int> cand;
15     for (int i = 1; i <= n; i++) {
16         cand.push_back(col[i]);
17     }
18     ranges::sort(cand);
19     cand.erase(unique(all(cand)), cand.end());
20
21     for (auto id : cand) {
22         for (auto x : S[id]) remove(x);
23         for (auto x : S[id]) {
24             auto [opt, w] = get(x);
25             if (select[id] == -1 || w < mn[id]) {
26                 select[id] = opt, mn[id] = w;
27             }
28         }
29         for (auto x : S[id]) insert(x);
30     }
31
32     for (int i = 1; i <= n; i++) if (col[i] == i) {
33         int j = col[select[i]];
34         if (i == j) continue;
35         ans += mn[i];
36         merge(i, j);
37         cnt--;
38     }
39 }

```

## 5.4 dijkstra.cpp

```

1 vector<PII> e[N];
2
3 template <typename T>
4 void add(int u, int v, T w) {
5     e[u].eb(v, w);
6 }
7
8 template <typename T>
9 vector<T> dijkstra(vector<pair<int, T>> *g, int start) {
10     // assert(0 <= start && start < g.n);
11     // maybe use inf = numeric_limits<T>::max() / 4
12     vector<T> dist(n, numeric_limits<T>::max());

```

```

13     priority_queue<pair<T, int>, vector<pair<T, int>>, greater<pair<T, int>
14         >>> s;
15     dist[start] = 0;
16     s.emplace(dist[start], start);
17     while (!s.empty()) {
18         T expected = s.top().first;
19         int i = s.top().second;
20         s.pop();
21         if (dist[i] != expected) {
22             continue;
23         }
24         for (auto [to, cost] : g[i]) {
25             if (dist[i] + cost < dist[to]) {
26                 dist[to] = dist[i] + cost;
27                 s.emplace(dist[to], to);
28             }
29         }
30     }
31     return dist;
32     // returns numeric_limits<T>::max() if there's no path
33 }

```

## 5.5 dijkstra.cpp

```

1 vector<PII> e[N];
2
3 template <typename T>
4 void add(int u, int v, T w) {
5     e[u].eb(v, w);
6 }
7
8 template <typename T>
9 vector<T> dijkstra(vector<pair<int, T>> *g, int start) {
10     // assert(0 <= start && start < g.n);
11     // maybe use inf = numeric_limits<T>::max() / 4
12     const T inf = numeric_limits<T>::max();
13     vector<T> dist(n, inf);
14     vector<int> was(n, 0);
15     dist[start] = 0;
16     while (true) {
17         int cur = -1;
18         for (int i = 0; i < n; i++) {
19             if (was[i] || dist[i] == inf) continue;

```

```

20         if (cur == -1 || dist[i] < dist[cur]) {
21             cur = i;
22         }
23     }
24     if (cur == -1 || dist[cur] == inf) {
25         break;
26     }
27     was[cur] = 1;
28     for (auto [to, cost] : g[cur]) {
29         dist[to] = min(dist[to], dist[cur] + cost);
30     }
31 }
32 return dist;
33 // returns inf if there's no path
34 }

```

## 5.6 dinic.cpp

```

1  template<typename T>
2  struct FlowGraph {
3      static const int V = 1015;
4      static const int E = 100015;
5      int s, t, vtot;
6      int head[V], etot;
7      int dis[V], cur[V];
8      struct edge {
9          int v, nxt;
10         T f;
11     } e[E * 2];
12     void addedge(int u, int v, T f) {
13         e[etot] = {v, head[u], f};
14         head[u] = etot++;
15         e[etot] = {u, head[v], 0};
16         head[v] = etot++;
17     }
18     bool bfs() {
19         for (int i = 1; i <= vtot; i++) {
20             dis[i] = 0;
21             cur[i] = head[i];
22         }
23         queue<int> q;
24         q.push(s); dis[s] = 1;
25         while (!q.empty()) {

```

```

26             int u = q.front(); q.pop();
27             for (int i = head[u]; i != -1; i = e[i].nxt) {
28                 if (e[i].f && !dis[e[i].v]) {
29                     int v = e[i].v;
30                     dis[v] = dis[u] + 1;
31                     if (v == t) return true;
32                     q.push(v);
33                 }
34             }
35         }
36         return false;
37     }
38     T dfs(int u, T m) {
39         if (u == t) return m;
40         T flow = 0;
41         for (int i = cur[u]; i != -1; cur[u] = i = e[i].nxt) {
42             if (e[i].f && dis[e[i].v] == dis[u] + 1) {
43                 T f = dfs(e[i].v, min(m, e[i].f));
44                 e[i].f -= f;
45                 e[i ^ 1].f += f;
46                 m -= f;
47                 flow += f;
48                 if (!m) break;
49             }
50         }
51         if (!flow) dis[u] = -1;
52         return flow;
53     }
54     T dinic() {
55         T flow = 0;
56         while (bfs()) flow += dfs(s, numeric_limits<T>::max());
57         return flow;
58     }
59     void init(int _s, int _t, int _vtot) {
60         s = _s;
61         t = _t;
62         vtot = _vtot;
63         etot = 0;
64         for (int i = 1; i <= vtot; i++) head[i] = -1;
65     }
66 };

```

## 5.7 dinic-tourist.cpp

```
1  template <typename T>
2  class flow_graph {
3  public:
4      static constexpr T eps = (T)1e-9;
5
6      struct edge {
7          int from;
8          int to;
9          T c;
10         T f;
11     };
12
13     vector<vector<int>>> g;
14     vector<edge> edges;
15     int n;
16     int st;
17     int fin;
18     T flow;
19
20     flow_graph(int _n, int _st, int _fin) : n(_n), st(_st), fin(_fin) {
21         assert(0 <= st && st < n && 0 <= fin && fin < n && st != fin);
22         g.resize(n);
23         flow = 0;
24     }
25
26     void clear_flow() {
27         for (const edge &e : edges) {
28             e.f = 0;
29         }
30         flow = 0;
31     }
32
33     int add(int from, int to, T forward_cap, T backward_cap) {
34         assert(0 <= from && from < n && 0 <= to && to < n);
35         int id = (int)edges.size();
36         g[from].push_back(id);
37         edges.push_back({from, to, forward_cap, 0});
38         g[to].push_back(id + 1);
39         edges.push_back({to, from, backward_cap, 0});
40         return id;
41     }
42 };
```

```
43
44 template <typename T>
45 class dinic {
46 public:
47     flow_graph<T> &g;
48
49     vector<int> ptr;
50     vector<int> d;
51     vector<int> q;
52
53     dinic(flow_graph<T> &_g) : g(_g) {
54         ptr.resize(g.n);
55         d.resize(g.n);
56         q.resize(g.n);
57     }
58
59     bool expath() {
60         fill(d.begin(), d.end(), -1);
61         q[0] = g.fin;
62         d[g.fin] = 0;
63         int beg = 0, end = 1;
64         while (beg < end) {
65             int i = q[beg++];
66             for (int id : g.g[i]) {
67                 const auto &e = g.edges[id];
68                 const auto &back = g.edges[id ^ 1];
69                 if (back.c - back.f > g.eps && d[e.to] == -1) {
70                     d[e.to] = d[i] + 1;
71                     if (e.to == g.st) {
72                         return true;
73                     }
74                     q[end++] = e.to;
75                 }
76             }
77         }
78         return false;
79     }
80
81     T dfs(int v, T w) {
82         if (v == g.fin) {
83             return w;
84         }
85         int &j = ptr[v];
```

```

86     while (j >= 0) {
87         int id = g.g[v][j];
88         const auto &e = g.edges[id];
89         if (e.c - e.f > g.eps && d[e.to] == d[v] - 1) {
90             T t = dfs(e.to, min(e.c - e.f, w));
91             if (t > g.eps) {
92                 g.edges[id].f += t;
93                 g.edges[id ^ 1].f -= t;
94                 return t;
95             }
96         }
97         j--;
98     }
99     return 0;
100 }
101
102 T max_flow() {
103     while (expath()) {
104         for (int i = 0; i < g.n; i++) {
105             ptr[i] = (int)g.g[i].size() - 1;
106         }
107         T big_add = 0;
108         while (true) {
109             T add = dfs(g.st, numeric_limits<T>::max());
110             if (add <= g.eps) {
111                 break;
112             }
113             big_add += add;
114         }
115         if (big_add <= g.eps) {
116             break;
117         }
118         g.flow += big_add;
119     }
120     return g.flow;
121 }
122
123 vector<bool> min_cut() {
124     max_flow();
125     vector<bool> ret(g.n);
126     for (int i = 0; i < g.n; i++) {
127         ret[i] = (d[i] != -1);
128     }

```

```

129     return ret;
130 }
131 };

```

## 5.8 eulerian-digraph.cpp

```

1 optional<vector<int>>> eulerian_path(int n, const vector<PII> &E) {
2     vector<int> res;
3     if (E.empty()) return res;
4     vector<VI> adj(n + 1);
5     vector<int> in(n + 1);
6     for (int i = 0; i < ssize(E); i++) {
7         auto [u, v] = E[i];
8         adj[u].push_back(i);
9         in[v] += 1;
10    }
11
12    int s = -1, hi = 0, lo = 0;
13    for (int i = 1; i <= n; i++) {
14        if (SZ(adj[i]) == in[i]) continue;
15        if (abs(SZ(adj[i]) - in[i]) > 1) return {};
16        if (SZ(adj[i]) > in[i]) {
17            hi++, s = i;
18        } else {
19            lo++;
20        }
21    }
22    if (!(hi == 0 && lo == 0) && !(hi == 1 && lo == 1)) {
23        return {};
24    }
25    for (int i = 1; s == -1 && i <= n; i++)
26        if (!adj[i].empty()) s = i;
27
28    auto Dfs = [&](auto &Dfs, int u) -> void {
29        while (!adj[u].empty()) {
30            auto id = adj[u].back();
31            adj[u].pop_back();
32            int v = E[id].second;
33            Dfs(Dfs, v);
34            res.push_back(v);
35        }
36    };
37    Dfs(Dfs, s);

```



```

38     if (SZ(res) != SZ(E)) return {};
39     ranges::reverse(res);
40     return res;
41 }

```

## 5.9 eulerian-undigraph.cpp

```

1 optional<vector<int>> eulerian_path(int n, const vector<PII> &E) {
2     vector<int> res;
3     if (E.empty()) return res;
4     vector<VI> adj(n + 1);
5     for (int i = 0; i < ssize(E); i++) {
6         auto [u, v] = E[i];
7         adj[u].push_back(i);
8         adj[v].push_back(i);
9     }
10
11     int s = -1, odd = 0;
12     for (int i = 1; i <= n; i++) {
13         if (ssize(adj[i]) % 2 == 0) continue;
14         if (++odd > 2) return {};
15         s = i;
16     }
17     for (int i = 1; s == -1 && i <= n; i++)
18         if (!adj[i].empty()) s = i;
19
20     vector<int> vis(ssize(E));
21     auto Dfs = [&](auto &Dfs, int u) -> void {
22         while (!adj[u].empty()) {
23             auto id = adj[u].back();
24             adj[u].pop_back();
25             if (vis[id]) continue;
26             vis[id] = 1;
27             int v = u ^ E[id].fi ^ E[id].se;
28             Dfs(Dfs, v);
29             res.push_back(v);
30         }
31     };
32     Dfs(Dfs, s);
33     if (SZ(res) != SZ(E)) return {};
34     ranges::reverse(res);
35     return res;
36 }

```

## 5.10 hungarian.cpp

```

1 vector<int> g[maxn];
2 int idx;
3 int a[N][N], use[N][N], p[maxn], vis[maxn];
4
5 bool find(int x) {
6     vis[x] = 1;
7     for (auto y : g[x]) {
8         if (!p[y] || (!vis[p[y]] && find(p[y]))) {
9             p[y] = x;
10            return true;
11        }
12    }
13    return false;
14 }
15
16 int match() {
17     int res = 0;
18     fill(p + 1, p + idx + 1, 0);
19     for (int i = 1; i <= idx; i++) {
20         fill(vis + 1, vis + idx + 1, 0);
21         if (find(i)) res++;
22     }
23     return res;
24 }

```

## 5.11 KM.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 using ll = long long;
4
5 // L <= R, 左边完全匹配
6 // 最小权完备匹配
7
8 // 带权匹配: 使得该二分图的权值和最大 (或最小) 的匹配。
9 // 最大匹配: 使得该二分图边数最多的匹配。
10 // 完备匹配: 使得点数较小的点集中每个点都被匹配的匹配。
11 // 完美匹配: 所有点都被匹配的匹配。
12 // 定理1: 最大匹配数 = 最小点覆盖数 (Konig 定理)
13 // 定理2: 最大匹配数 = 最大独立数
14 // 定理3: 最小路径覆盖数 = 顶点数 - 最大匹配数
15

```

```

16 // 二分图的最小点覆盖
17 // 定义：在二分图中，求最少的点集，使得每一条边至少都有端点在这个点集中。
18 // 二分图的最小点覆盖 = 二分图的最大匹配
19
20 // 二分图的最少边覆盖
21 // 定义：在二分图中，求最少的边，使得他们覆盖所有的点，并且每一个点只被一条
    边覆盖。
22 // 二分图的最少边覆盖 = 点数 - 二分图的最大匹配
23
24 // 二分图的最大独立集
25 // 定义：在二分图中，选最多的点，使得任意两个点之间没有直接边连接。
26 // 二分图的最大独立集 = 点数 - 二分图的最大匹配
27
28 template<class T>
29 pair<T, vector<int>> hungarian(const vector<vector<T>> &a) {
30     if (a.empty()) return {0, {}};
31     int n = a.size() + 1, m = a[0].size() + 1;
32     vector<T> u(n), v(m); // 顶标
33     vector<int> p(m), ans(n - 1);
34     for (int i = 1; i < n; i++) {
35         p[0] = i;
36         int j0 = 0;
37         vector<T> dist(m, numeric_limits<T>::max());
38         vector<int> pre(m, -1);
39         vector<bool> done(m + 1);
40         do { // dijkstra
41             done[j0] = true;
42             int i0 = p[j0], j1;
43             T delta = numeric_limits<T>::max();
44             for (int j = 1; j < m; j++) if (!done[j]) {
45                 auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
46                 if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
47                 if (dist[j] < delta) delta = dist[j], j1 = j;
48             }
49             for (int j = 0; j < m; j++) {
50                 if (done[j]) u[p[j]] += delta, v[j] -= delta;
51                 else dist[j] -= delta;
52             }
53             j0 = j1;
54         } while (p[j0]);
55         while (j0) { // update alternating path
56             int j1 = pre[j0];
57             p[j0] = p[j1], j0 = j1;

```

```

58         }
59     }
60     for (int j = 1; j < m; j++) {
61         if (p[j]) ans[p[j] - 1] = j - 1;
62     }
63     return {-v[0], ans}; // min cost
64 }
65
66 int L, R, m;
67 int main() {
68     scanf("%d%d%d", &L, &R, &m);
69     R = max(L, R);
70     auto a = vector<vector<ll>>(L, vector<ll>(R, 0));
71     for (int i = 0; i < m; i++) {
72         int u, v, w;
73         scanf("%d%d%d", &u, &v, &w);
74         --u; --v;
75         a[u][v] = -w;
76     }
77     auto [val, ans] = hungarian(a);
78     printf("%lld\n", -val);
79     for (int i = 0; i < L; i++) {
80         if (a[i][ans[i]] >= 0) ans[i] = -1;
81         printf("%d%c", ans[i] + 1, "\n"[i == L - 1]);
82     }
83 }

```

## 5.12 kosaraju.cpp

```

1 vector<int> e[maxn], erev[maxn];
2 vector<int> c, out;
3 vector<vector<int>> scc;
4 int vis[maxn];
5 void dfs(int u) {
6     vis[u] = 1;
7     for (auto v : e[u]) if (!vis[v]) dfs(v);
8     out.pb(u);
9 }
10 void dfs_rev(int u) {
11     vis[u] = 1;
12     for (auto v : erev[u]) if (!vis[v]) dfs_rev(v);
13     c.pb(u);
14 }

```

```

15 void solve() {
16     cin >> n >> m;
17     rep(i, 1, m) {
18         int u, v;
19         cin >> u >> v;
20         e[u].pb(v);
21         erev[v].pb(u);
22     }
23     rep(i, 1, n) if (!vis[i]) dfs(i);
24     fill(vis + 1, vis + n + 1, 0);
25     reverse(all(out));
26     for (auto v : out) if (!vis[v]) {
27         c.clear();
28         dfs_rev(v);
29         scc.pb(c);
30     }
31 }

```

### 5.13 kruskal 重构树.cpp

```

1 /**
2  * 构建后是一颗二叉树，如果按最小生成树建立的话是大根堆。
3  * 性质：原图中两个点间所有路径上的边最大权值的最小值=最小生成树上两点简单路
   径的边最大权值
4  * =kruskal 重构树上两点 LCA 的权值。
5  * 重构树中代表原树中的点的节点全是叶子节点，其余节点都代表了一条边的边权。
6  * 利用这个性质可以找到点 P 的简单路径上边权最大值小于 lim 深度最小的节点。
7  * 要求最小权值最大值，可以建最大生成树的重构树从而达到一样的效果。
8  */
9
10 vector<tuple<ll, ll, ll>> E;
11 rep(i, 1, m) {
12     int u, v, w;
13     cin >> u >> v >> w;
14     E.emplace_back(w, u, v);
15 }
16 ranges::sort(E);
17 for (auto [w, u, v] : E) {
18     u = find(u), v = find(v);
19     if (u == v) continue;
20     int p = ++idx;
21     lim[p] = w;
22     fa[u] = p, fa[v] = p;

```

```

23     e[p].push_back(u);
24     e[u].push_back(p);
25     e[p].push_back(v);
26     e[v].push_back(p);
27 }

```

### 5.14 MCMF.cpp

```

1 template<typename T>
2 struct MinCostGraph {
3     static const int V = 20100;
4     static const int E = 201000;
5     int s, t, vtot;
6     int head[V], etot;
7     T dis[V], flow, cost;
8     int pre[V];
9     bool vis[V];
10
11     struct edge {
12         int v, nxt;
13         T f, c;
14     } e[E * 2];
15     void addedge(int u, int v, T f, T c, T f2 = 0) {
16         e[etot] = {v, head[u], f, c}; head[u] = etot++;
17         e[etot] = {u, head[v], f2, -c}; head[v] = etot++;
18     }
19
20     bool spfa() {
21         T inf = numeric_limits<T>::max() / 2;
22         for (int i = 1; i <= vtot; i++) {
23             dis[i] = inf;
24             vis[i] = false;
25             pre[i] = -1;
26         }
27         dis[s] = 0;
28         vis[s] = true;
29         queue<int> q;
30         q.push(s);
31         while (!q.empty()) {
32             int u = q.front();
33             for (int i = head[u]; ~i; i = e[i].nxt) {
34                 int v = e[i].v;
35                 if (e[i].f && dis[v] > dis[u] + e[i].c) {

```

```

36         dis[v] = dis[u] + e[i].c;
37         pre[v] = i;
38         if (!vis[v]) {
39             vis[v] = 1;
40             q.push(v);
41         }
42     }
43 }
44 q.pop();
45 vis[u] = false;
46 }
47 return dis[t] != inf;
48 }
49
50 void augment() {
51     int u = t;
52     T f = numeric_limits<T>::max();
53     while (~pre[u]) {
54         f = min(f, e[pre[u]].f);
55         u = e[pre[u] ^ 1].v;
56     }
57     flow += f;
58     cost += f * dis[t];
59     u = t;
60     while (~pre[u]) {
61         e[pre[u]].f -= f;
62         e[pre[u] ^ 1].f += f;
63         u = e[pre[u] ^ 1].v;
64     }
65 }
66
67 pair<T, T> solve() {
68     flow = 0;
69     cost = 0;
70     while (spfa()) augment();
71     return {flow, cost};
72 }
73
74 void init(int s_, int t_, int vtot_) {
75     s = s_;
76     t = t_;
77     vtot = vtot_;
78     etot = 0;
79     for (int i = 1; i <= vtot; i++) head[i] = -1;

```

```

79     }
80 };

```

## 5.15 MCMFfast.cpp

```

1  template <typename flow_t = int, typename cost_t = long long>
2  struct MCMF_SSPA {
3      int N;
4      vector<vector<int>>> adj;
5      struct edge_t {
6          int dest;
7          flow_t cap;
8          cost_t cost;
9      };
10     vector<edge_t> edges;
11
12     vector<char> seen;
13     vector<cost_t> pi;
14     vector<int> prv;
15
16     explicit MCMF_SSPA(int N_) : N(N_), adj(N), pi(N, 0), prv(N) {}
17
18     void addEdge(int from, int to, flow_t cap, cost_t cost) {
19         assert(cap >= 0);
20         int e = int(edges.size());
21         edges.emplace_back(edge_t{to, cap, cost});
22         edges.emplace_back(edge_t{from, 0, -cost});
23         adj[from].push_back(e);
24         adj[to].push_back(e+1);
25     }
26
27     const cost_t INF_COST = numeric_limits<cost_t>::max() / 4;
28     const flow_t INF_FLOW = numeric_limits<flow_t>::max() / 4;
29     vector<cost_t> dist;
30     __gnu_pbds::priority_queue<pair<cost_t, int>> q;
31     vector<typename decltype(q)::point_iterator> its;
32     void path(int s) {
33         dist.assign(N, INF_COST);
34         dist[s] = 0;
35
36         its.assign(N, q.end());
37         its[s] = q.push({0, s});
38

```

```

39     while (!q.empty()) {
40         int i = q.top().second; q.pop();
41         cost_t d = dist[i];
42         for (int e : adj[i]) {
43             if (edges[e].cap) {
44                 int j = edges[e].dest;
45                 cost_t nd = d + edges[e].cost;
46                 if (nd < dist[j]) {
47                     dist[j] = nd;
48                     prv[j] = e;
49                     if (its[j] == q.end()) {
50                         its[j] = q.push({-(dist[j] - pi[j]), j});
51                     } else {
52                         q.modify(its[j], {-(dist[j] - pi[j]), j});
53                     }
54                 }
55             }
56         }
57     }
58
59     swap(pi, dist);
60 }
61
62 vector<pair<flow_t, cost_t>> maxflow(int s, int t) {
63     assert(s != t);
64     flow_t totFlow = 0; cost_t totCost = 0;
65     vector<pair<flow_t, cost_t>> res;
66     while (path(s), pi[t] < INF_COST) {
67         flow_t curFlow = numeric_limits<flow_t>::max();
68         for (int cur = t; cur != s; ) {
69             int e = prv[cur];
70             int nxt = edges[e^1].dest;
71             curFlow = min(curFlow, edges[e].cap);
72             cur = nxt;
73         }
74         totFlow += curFlow;
75         totCost += pi[t] * curFlow;
76         for (int cur = t; cur != s; ) {
77             int e = prv[cur];
78             int nxt = edges[e^1].dest;
79             edges[e].cap -= curFlow;
80             edges[e^1].cap += curFlow;
81             cur = nxt;

```

```

82         }
83
84         res.emplace_back(totFlow, totCost);
85     }
86     return res;
87 }
88 };

```

## 5.16 MCMFfull.cpp

```

1  template <typename T, typename C>
2  class MCMF {
3  public:
4      static constexpr T eps = (T) 1e-9;
5
6      struct edge {
7          int from;
8          int to;
9          T c;
10         T f;
11         C cost;
12     };
13
14     int n;
15     vector<vector<int>> g;
16     vector<edge> edges;
17     vector<C> d;
18     vector<C> pot;
19     __gnu_pbds::priority_queue<pair<C, int>> q;
20     vector<typename decltype(q)::point_iterator> its;
21     vector<int> pe;
22     const C INF_C = numeric_limits<C>::max() / 2;
23
24     explicit MCMF(int n_) : n(n_), g(n), d(n), pot(n, 0), its(n), pe(n) {}
25
26     int add(int from, int to, T forward_cap, T backward_cap, C edge_cost) {
27         assert(0 <= from && from < n && 0 <= to && to < n);
28         assert(forward_cap >= 0 && backward_cap >= 0);
29         int id = static_cast<int>(edges.size());
30         g[from].push_back(id);
31         edges.push_back({from, to, forward_cap, 0, edge_cost});
32         g[to].push_back(id + 1);
33         edges.push_back({to, from, backward_cap, 0, -edge_cost});

```

```

34     return id;
35 }
36
37 void expath(int st) {
38     fill(d.begin(), d.end(), INF_C);
39     q.clear();
40     fill(its.begin(), its.end(), q.end());
41     its[st] = q.push({pot[st], st});
42     d[st] = 0;
43     while (!q.empty()) {
44         int i = q.top().second;
45         q.pop();
46         its[i] = q.end();
47         for (int id : g[i]) {
48             const edge &e = edges[id];
49             int j = e.to;
50             if (e.c - e.f > eps && d[i] + e.cost < d[j]) {
51                 d[j] = d[i] + e.cost;
52                 pe[j] = id;
53                 if (its[j] == q.end()) {
54                     its[j] = q.push({pot[j] - d[j], j});
55                 } else {
56                     q.modify(its[j], {pot[j] - d[j], j});
57                 }
58             }
59         }
60     }
61     swap(d, pot);
62 }
63
64 pair<T, C> calc(int st, int fin) { // max_flow_min_cost
65     T flow = 0;
66     C cost = 0;
67     bool ok = true;
68     for (auto& e : edges) {
69         if (e.c - e.f > eps && e.cost + pot[e.from] - pot[e.to] < 0) {
70             ok = false;
71             break;
72         }
73     }
74     if (ok) {
75         expath(st);
76     } else {

```

```

77     vector<int> deg(n, 0);
78     for (int i = 0; i < n; i++) {
79         for (int eid : g[i]) {
80             auto& e = edges[eid];
81             if (e.c - e.f > eps) {
82                 deg[e.to] += 1;
83             }
84         }
85     }
86     vector<int> que;
87     for (int i = 0; i < n; i++) {
88         if (deg[i] == 0) {
89             que.push_back(i);
90         }
91     }
92     for (int b = 0; b < (int) que.size(); b++) {
93         for (int eid : g[que[b]]) {
94             auto& e = edges[eid];
95             if (e.c - e.f > eps) {
96                 deg[e.to] -= 1;
97                 if (deg[e.to] == 0) {
98                     que.push_back(e.to);
99                 }
100             }
101         }
102     }
103     fill(pot.begin(), pot.end(), INF_C);
104     pot[st] = 0;
105     if (static_cast<int>(que.size()) == n) {
106         for (int v : que) {
107             if (pot[v] < INF_C) {
108                 for (int eid : g[v]) {
109                     auto& e = edges[eid];
110                     if (e.c - e.f > eps) {
111                         if (pot[v] + e.cost < pot[e.to]) {
112                             pot[e.to] = pot[v] + e.cost;
113                             pe[e.to] = eid;
114                         }
115                     }
116                 }
117             }
118         }
119     } else {

```

```

120     que.assign(1, st);
121     vector<bool> in_queue(n, false);
122     in_queue[st] = true;
123     for (int b = 0; b < (int) que.size(); b++) {
124         int i = que[b];
125         in_queue[i] = false;
126         for (int id : g[i]) {
127             const edge &e = edges[id];
128             if (e.c - e.f > eps && pot[i] + e.cost < pot[e.to]) {
129                 pot[e.to] = pot[i] + e.cost;
130                 pe[e.to] = id;
131                 if (!in_queue[e.to]) {
132                     que.push_back(e.to);
133                     in_queue[e.to] = true;
134                 }
135             }
136         }
137     }
138 }
139 // debug(pot[fin]);
140 while (pot[fin] < INF_C) { // < 0
141     T push = numeric_limits<T>::max();
142     int v = fin;
143     while (v != st) {
144         const edge &e = edges[pe[v]];
145         push = min(push, e.c - e.f);
146         v = e.from;
147     }
148     v = fin;
149     while (v != st) {
150         edge &e = edges[pe[v]];
151         e.f += push;
152         edge &back = edges[pe[v] ^ 1];
153         back.f -= push;
154         v = e.from;
155     }
156     flow += push;
157     cost += push * pot[fin];
158     expath(st);
159 }
160 }
161 return {flow, cost};
162 }

```

```

163 };

```

## 5.17 prim.cpp

```

1  vector<PII> e[N];
2
3  template <typename T>
4  void add(int u, int v, T w) {
5      e[u].eb(v, w);
6  }
7
8  template <typename T>
9  T prim(vector<pair<int, T>> *g, int start) {
10     const T inf = numeric_limits<T>::max() / 4;
11     T res = 0;
12     vector<T> dist(n, inf);
13     dist[start] = 0;
14     priority_queue<pair<T, int>, vector<pair<T, int>>, greater<pair<T, int>>> s;
15     s.emplace(dist[start], start);
16     vector<int> was(n, 0);
17     while (!s.empty()) {
18         T expected = s.top().first;
19         int i = s.top().second;
20         s.pop();
21         if (dist[i] != expected || was[i]) {
22             continue;
23         }
24         was[i] = 1;
25         res += expected;
26         for (auto [to, cost] : g[i]) {
27             if (cost < dist[to]) {
28                 dist[to] = cost;
29                 s.emplace(dist[to], to);
30             }
31         }
32     }
33     return res;
34 }

```

## 5.18 PushRelabel.cpp

```

1  /**

```

```

2  * Author: Simon Lindholm
3  * Date: 2015-02-24
4  * License: CC0
5  * Source: Wikipedia, tinyKACTL
6  * Description: Push-relabel using the highest label selection rule and the
   gap heuristic. Quite fast in practice.
7  * To obtain the actual flow, look at positive values only.
8  * Time:  $O(V^2\sqrt{E})$ 
9  * Status: Tested on Kattis and SPOJ, and stress-tested
10 */
11 #pragma once
12
13 struct PushRelabel {
14     typedef vector<int> vi;
15     struct Edge {
16         int dest, back;
17         ll f, c;
18     };
19     vector<vector<Edge>> g;
20     vector<ll> ec;
21     vector<Edge*> cur;
22     vector<vi> hs; vi H;
23     PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}
24
25     void addEdge(int s, int t, ll cap, ll rcap=0) {
26         if (s == t) return;
27         g[s].push_back({t, SZ(g[t]), 0, cap});
28         g[t].push_back({s, SZ(g[s])-1, 0, rcap});
29     }
30
31     void addFlow(Edge& e, ll f) {
32         Edge &back = g[e.dest][e.back];
33         if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
34         e.f += f; e.c -= f; ec[e.dest] += f;
35         back.f -= f; back.c += f; ec[back.dest] -= f;
36     }
37     ll calc(int s, int t) {
38         int v = SZ(g); H[s] = v; ec[t] = 1;
39         vi co(2*v); co[0] = v-1;
40         rep(i,0,v-1) cur[i] = g[i].data();
41         for (Edge& e : g[s]) addFlow(e, e.c);
42
43         for (int hi = 0;;) {

```

```

44         while (hs[hi].empty()) if (!hi--) return -ec[s];
45         int u = hs[hi].back(); hs[hi].pop_back();
46         while (ec[u] > 0) // discharge u
47             if (cur[u] == g[u].data() + SZ(g[u])) {
48                 H[u] = 1e9;
49                 for (Edge& e : g[u]) if (e.c && H[u] > H[e.dest]+1)
50                     H[u] = H[e.dest]+1, cur[u] = &e;
51                 if (++co[H[u]], !--co[hi] && hi < v)
52                     rep(i,0,v-1) if (hi < H[i] && H[i] < v)
53                         --co[H[i]], H[i] = v + 1;
54                 hi = H[u];
55             } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
56                 addFlow(*cur[u], min(ec[u], cur[u]->c));
57             else ++cur[u];
58         }
59     }
60     bool leftOfMinCut(int a) { return H[a] >= SZ(g); }
61 };

```

## 5.19 tarjan 割点.cpp

```

1  vector<int> g[maxn], ans;
2  stack<int> stk;
3  int dfn[maxn], cut[maxn], low[maxn], idx;
4
5  void dfs(int x, int f) {
6      low[x] = dfn[x] = ++idx;
7      stk.push(x);
8      int ch = 0;
9      for (auto y : g[x]) {
10         if (!dfn[y]) {
11             ch++;
12             dfs(y, x);
13             low[x] = min(low[x], low[y]);
14             if (low[y] >= dfn[x]) cut[x] = 1;
15         } else {
16             if (y != f) low[x] = min(low[x], dfn[y]);
17         }
18     }
19     if (x == 1 && ch <= 1) cut[x] = 0;
20     if (cut[x]) ans.pb(x);
21 }

```



## 5.20 tarjan 割边.cpp

```
1 vector<PII> g[maxn];
2 stack<int> stk;
3 int dfn[maxn], ins[maxn], low[maxn];
4 int idx, tot;
5 VI ans;
6 void dfs(int x, int f) {
7     low[x] = dfn[x] = ++idx;
8     stk.push(x);
9     ins[x] = 1;
10    for (auto [y, id] : g[x]) {
11        if (!dfn[y]) {
12            dfs(y, id);
13            low[x] = min(low[x], low[y]);
14        } else {
15            if (ins[y] && id != f) low[x] = min(low[x], dfn[y]);
16        }
17    }
18    if (low[x] >= dfn[x]) {
19        ++tot;
20        while (true) {
21            int cur = stk.top();
22            stk.pop();
23            ins[cur] = 0;
24            if (cur == x) break;
25        }
26        if (f != 0) ans.pb(f);
27    }
28 }
```

## 5.21 tarjan 强连通分量.cpp

```
1 vector<int> g[maxn];
2 stack<int> stk;
3 int dfn[maxn], ins[maxn], low[maxn], belong[maxn];
4 int idx, tot;
5
6 void dfs(int x) {
7     low[x] = dfn[x] = ++idx;
8     ins[x] = 1;
9     stk.push(x);
10    for (auto y : g[x]) {
11        if (!dfn[y]) {
```

```
12            dfs(y);
13            low[x] = min(low[x], low[y]);
14        } else {
15            if (ins[y]) low[x] = min(low[x], dfn[y]);
16        }
17    }
18    if (low[x] >= dfn[x]) {
19        ++tot;
20        while (true) {
21            int cur = stk.top(); stk.pop();
22            ins[cur] = 0;
23            belong[cur] = tot;
24            if (cur == x) break;
25        }
26    }
27 }
```

## 5.22 tarjan 点双.cpp

```
1 vector<int> g[maxn];
2 stack<int> stk;
3 int dfn[maxn], low[maxn], idx, tot, cut[maxn];
4 vector<int> bcc[maxn];
5
6 void dfs(int x, int f) {
7     low[x] = dfn[x] = ++idx;
8     stk.push(x);
9     int ch = 0;
10    for (auto y : g[x]) {
11        if (!dfn[y]) {
12            ch++;
13            dfs(y, x);
14            low[x] = min(low[x], low[y]);
15            if (low[y] >= dfn[x]) {
16                cut[x] = 1;
17                ++tot;
18                bcc[tot].pb(x);
19                while (true) {
20                    int cur = stk.top();
21                    stk.pop();
22                    bcc[tot].pb(cur);
23                    if (cur == y) break;
24                }
```

```

25         }
26     } else {
27         if (y != f) low[x] = min(low[x], dfn[y]);
28     }
29 }
30 if (x == 1 && ch <= 1) cut[x] = 0;
31 }

```

## 5.23 tarjan 边双.cpp

```

1  vector<PII> g[maxn];
2  stack<int> stk;
3  int dfn[maxn], low[maxn], idx, tot, belong[maxn];
4  vector<int> bcc[maxn];
5
6  void dfs(int x, int f) {
7      low[x] = dfn[x] = ++idx;
8      stk.push(x);
9      for (auto [y, id] : g[x]) {
10         if (!dfn[y]) {
11             dfs(y, id);
12             low[x] = min(low[x], low[y]);
13         } else {
14             if (id != f) low[x] = min(low[x], dfn[y]);
15         }
16     }
17     if (low[x] >= dfn[x]) {
18         ++tot;
19         while (true) {
20             int cur = stk.top();
21             stk.pop();
22             belong[cur] = tot;
23             bcc[tot].pb(cur);
24             if (cur == x) break;
25         }
26     }
27 }

```

## 5.24 twosat.cpp

```

1  class twosat {
2  public:
3      digraph<int> g;

```

```

4      int n;
5
6      twosat(int _n) : g(digraph<int>(2 * _n)), n(_n) {
7      }
8
9      // (v[x] == value_x)
10     inline void add(int x, int value_x) {
11         assert(0 <= x && x < n);
12         assert(0 <= value_x && value_x <= 1);
13         g.add(2 * x + (value_x ^ 1), 2 * x + value_x);
14     }
15
16     // (v[x] == value_x || v[y] == value_y)
17     inline void add(int x, int value_x, int y, int value_y) {
18         assert(0 <= x && x < n && 0 <= y && y < n);
19         assert(0 <= value_x && value_x <= 1 && 0 <= value_y && value_y <= 1);
20
21         g.add(2 * x + (value_x ^ 1), 2 * y + value_y);
22         g.add(2 * y + (value_y ^ 1), 2 * x + value_x);
23     }
24
25     inline vector<int> solve() {
26         int cnt;
27         vector<int> c = find_scc(g, cnt);
28         vector<int> res(n);
29         for (int i = 0; i < n; i++) {
30             if (c[2 * i] == c[2 * i + 1]) {
31                 return vector<int>();
32             }
33             res[i] = (c[2 * i] < c[2 * i + 1]);
34         }
35         return res;
36     };

```

## 5.25 差分约束系统.cpp

```

1  /**
2      Description:
3      求解方程组  $x_u - x_v \leq w_i$ , 求出的  $x_i$  为满足条件的最大值
4      转化为  $x_u \leq x_v + w_i$ 
5      问题等价于求最短路 (bellmanford 或 Floyd)
6      即加一条有向边  $add(u, v, w)$ ,  $dist[v] = \min(dist[v], dist[u] + w)$ 

```

```

7   求最小值（满足条件情况下尽量小）等价于求 $(-x_i)$ 最大（或者转化为求最长路）
8   求非负解只需要添加超级节点 $S$ ， $S$ 向各个点连边 $(S + 0 \leq x_i)$ ，再设 $dist[S]$ 
    = 0
9   */
10  void solve() {
11      cin >> n >> m;
12      vector<int> dist(n, 0);
13      vector<vector<PII>> g(n);
14      rep(i, 0, m - 1) {
15          int u, v, w;
16          cin >> u >> v >> w;
17          u--, v--;
18          g[u].eb(v, -w);
19      }
20      bool ok = 1;
21      rep(i, 1, n) {
22          bool upd = 0;
23          rep(u, 0, n - 1) {
24              for (auto [v, w] : g[u]) {
25                  if (dist[v] < dist[u] + w) {
26                      dist[v] = dist[u] + w;
27                      upd = 1;
28                  }
29              }
30          }
31          if (!upd) break;
32          // 仍然有约束未满足
33          if (i == n && upd) ok = 0;
34      }
35      if (!ok) {
36          return cout << -1 << '\n', void();
37      }
38      rep(i, 0, n - 1) {
39          cout << dist[i] << "␣\n"[i == n - 1];
40      }
41  }

```

## 6 Math

### 6.1 binom.cpp

---

```

1  vector<Mint> fact(1, 1);

```

```

2  vector<Mint> inv_fact(1, 1);
3
4  Mint C(int n, int k) {
5      if (k < 0 || k > n) {
6          return 0;
7      }
8      while ((int)fact.size() < n + 1) {
9          fact.push_back(fact.back() * (int)fact.size());
10         inv_fact.push_back(1 / fact.back());
11     }
12     return fact[n] * inv_fact[k] * inv_fact[n - k];
13 }
14
15 const int mod = 1000000007;
16 const int T = 1000000;
17 ll fact[] = {};
18 ll powmod(ll a, ll b) {
19     ll ret = 1;
20     for (; b; b >>= 1) {
21         if (b & 1) ret = ret * a % mod;
22         a = a * a % mod;
23     }
24     return ret;
25 }
26 ll fac(int n) {
27     ll v = fact[n / T];
28     for (int i = n / T * T + 1; i <= n; i++)
29         v = v * i % mod;
30     return v;
31 }
32 ll binom(int n, int m) {
33     if (m < 0 || m > n) return 0;
34     return fac(n) * powmod(fac(m) * fac(n - m) % mod, mod - 2) % mod;
35 }

```

### 6.2 bsgs.cpp

---

```

1  int bsgs(int a, int b, int m) { //  $a^x \equiv b \pmod m$ 
2      int res = m + 1;
3      int t = sqrt(m) + 2;
4      ll d = powmod(a, t, m);
5      ll cnt = 1;
6      //map<int, int> p;

```

```

7     hs.init();
8     for (int i = 1; i <= t; i++) {
9         cnt = cnt * d % m;
10        //if (!p.count(cnt)) p[cnt] = i;
11        if (hs.query(cnt) == -1) hs.insert(cnt, i);
12    }
13    cnt = b;
14    for (int i = 1; i <= t; i++) {
15        cnt = cnt * a % m;
16        //if (p.count(cnt)) res = min(res, p[cnt] * t - i);
17        int tmp = hs.query(cnt);
18        if (tmp != -1) res = min(res, tmp * t - i);
19    }
20    if (res >= m) res = -1;
21    return res;
22 }

```

### 6.3 cantor.cpp

```

1  ll fac[maxn], A[maxn], w[maxn];
2  void init(int n) {
3      fac[0] = 1;
4      rep(i, 1, n) fac[i] = fac[i - 1] * i % mod;
5  }
6  ll cantor(int w[], int n) {
7      ll ans = 1;
8      for (int i = 1; i <= n; i++) { // can optimize by BIT
9          for (int j = i + 1; j <= n; j++) {
10             if (w[i] > w[j]) A[i]++;
11          }
12      }
13      for (int i = 1; i < n; i++) {
14          ans += A[i] * fac[n - i];
15      }
16      return ans;
17  }
18
19  void decanter(ll x, int n) { // x->rank n->length
20      x--;
21      vector<int> rest(n, 0);
22      iota(rest.begin(), rest.end(), 1); // rest->1,2,3,4...
23      for (int i = 1; i <= n; i++) {
24          A[i] = x / fac[n - i];

```

```

25         x %= fac[n - i];
26     }
27     for (int i = 1; i <= n; i++) {
28         w[i] = rest[A[i]];
29         rest.erase(lower_bound(rest.begin(), rest.end(), w[i]));
30     }
31 }

```

### 6.4 EXCRT modequ exgcd.cpp

```

1  ll exgcd(ll a, ll b, ll &x, ll &y) {
2      if (b == 0) {
3          x = 1, y = 0;
4          return a;
5      }
6      ll d = exgcd(b, a % b, y, x);
7      y -= (a / b) * x;
8      return d;
9  }
10
11  // 求  $a * x = b \pmod m$  的解
12  ll modequ(ll a, ll b, ll m) {
13      ll x, y;
14      ll d = exgcd(a, m, x, y);
15      if (b % d != 0) return -1;
16      m /= d; a /= d; b /= d;
17      x = x * b % m;
18      if (x < 0) x += m;
19      return x;
20  }
21
22  void merge(ll &a, ll &b, ll c, ll d) {
23      if (a == -1 || b == -1) return;
24      ll x, y;
25      ll g = exgcd(b, d, x, y);
26      if ((c - a) % g != 0) {
27          a = -1, b = -1;
28          return;
29      }
30      d /= g;
31      ll t = ((c - a) / g) % d * x % d;
32      if (t < 0) t += d;
33      a = b * t + a;

```

```

34     b = b * d;
35 }

```

## 6.5 factor.cpp

```

1 namespace Factor {
2     const int N=1010000;
3     ll C,fac[10010],n,mul,a[1001000];
4     int T,cnt,i,l,prime[N],p[N],psize,_cnt;
5     ll _e[100],_pr[100];
6     vector<ll> d;
7     inline ll mul(ll a,ll b,ll p) {
8         if (p<=1000000000) return a*b%p;
9         else if (p<=100000000000011) return (((a*(b>>20)%p)<<20)+(a*(b
10             &((1<<20)-1))))%p;
11         else {
12             ll d=(ll)floor(a*(long double)b/p+0.5);
13             ll ret=(a*b-d*p)%p;
14             if (ret<0) ret+=p;
15             return ret;
16         }
17     }
18     void prime_table(){
19         int i,j,tot,t1;
20         for (i=1;i<=psize;i++) p[i]=i;
21         for (i=2,tot=0;i<=psize;i++){
22             if (p[i]==i) prime[++tot]=i;
23             for (j=1;j<=tot && (t1=prime[j]*i)<=psize;j++){
24                 p[t1]=prime[j];
25                 if (i%prime[j]==0) break;
26             }
27         }
28     }
29     void init(int ps) {
30         psize=ps;
31         prime_table();
32     }
33     ll powl(ll a,ll n,ll p) {
34         ll ans=1;
35         for (;n;n>>=1) {
36             if (n&1) ans=mul(ans,a,p);
37             a=mul(a,a,p);
38         }
39     }

```

```

38     return ans;
39 }
40 bool witness(ll a,ll n) {
41     int t=0;
42     ll u=n-1;
43     for (;~u&1;u>>=1) t++;
44     ll x=powl(a,u,n),_x=0;
45     for (;t;t--) {
46         _x=mul(x,x,n);
47         if (_x==1 && x!=1 && x!=n-1) return 1;
48         x=_x;
49     }
50     return _x!=1;
51 }
52 bool miller(ll n) {
53     if (n<2) return 0;
54     if (n<=psize) return p[n]==n;
55     if (~n&1) return 0;
56     for (int j=0;j<=7;j++) if (witness(rng()%(n-1)+1,n)) return 0;
57     return 1;
58 }
59 ll gcd(ll a,ll b) {
60     ll ret=1;
61     while (a!=0) {
62         if ((~a&1) && (~b&1)) ret<<=1,a>>=1,b>>=1;
63         else if (~a&1) a>>=1; else if (~b&1) b>>=1;
64         else {
65             if (a<b) swap(a,b);
66             a-=b;
67         }
68     }
69     return ret*b;
70 }
71 ll rho(ll n) {
72     while (1) {
73         ll X=rng()%n,Y,Z,T=1,*lY=a,*lX=lY;
74         int tmp=20;
75         C=rng()%10+3;
76         X=mul(X,X,n)+C;*(lY++)=X;lX++;
77         Y=mul(X,X,n)+C;*(lY++)=Y;
78         for(;X!=Y;) {
79             ll t=X-Y+n;
80             Z=mul(T,t,n);

```

```

81         if(Z==0) return gcd(T,n);
82         tmp--;
83         if (tmp==0) {
84             tmp=20;
85             Z=gcd(Z,n);
86             if (Z!=1 && Z!=n) return Z;
87         }
88         T=Z;
89         Y*(1Y++)=mul(Y,Y,n)+C;
90         Y*(1Y++)=mul(Y,Y,n)+C;
91         X*(1X++);
92     }
93 }
94
95 void _factor(ll n) {
96     for (int i=0;i<cnt;i++) {
97         if (n%fac[i]==0) n/=fac[i],fac[cnt++]=fac[i];}
98     if (n<=psize) {
99         for (;n!=1;n/=p[n]) fac[cnt++]=p[n];
100        return;
101    }
102    if (miller(n)) fac[cnt++]=n;
103    else {
104        ll x=rho(n);
105        _factor(x);_factor(n/x);
106    }
107 }
108 void dfs(ll x,int dep) {
109     if (dep==_cnt) d.pb(x);
110     else {
111         dfs(x,dep+1);
112         for (int i=1;i<=_e[dep];i++) dfs(x*=_pr[dep],dep+1);
113     }
114 }
115 void norm() {
116     sort(fac,fac+cnt);
117     _cnt=0;
118     rep(i,0,cnt-1) if (i==0||fac[i]!=fac[i-1]) _pr[_cnt]=fac[i],_e[_cnt]
        ++=1;
119     else _e[_cnt-1]++;
120 }
121 vector<ll> getd() {
122     d.clear();

```

```

123     dfs(1,0);
124     return d;
125 }
126 vector<ll> factor(ll n) {
127     cnt=0;
128     _factor(n);
129     norm();
130     return getd();
131 }
132 vector<PLL> factorG(ll n) {
133     cnt=0;
134     _factor(n);
135     norm();
136     vector<PLL> d;
137     rep(i,0,_cnt-1) d.pb(mp(_pr[i],_e[i]));
138     return d;
139 }
140 bool is_primitive(ll a,ll p) {
141     assert(miller(p));
142     vector<PLL> D=factorG(p-1);
143     rep(i,0,SZ(D)-1) if (powl(a,(p-1)/D[i].fi,p)==1) return 0;
144     return 1;
145 }
146 ll phi(ll n) {
147     auto d=factorG(n);
148     for (auto p:d) n=n/p.fi*(p.fi-1);
149     return n;
150 }
151 }

```

## 6.6 fft.cpp

```

1 namespace fft {
2     typedef double dbl;
3
4     struct num {
5         dbl x, y;
6         num() { x = y = 0; }
7         num(dbl x, dbl y) : x(x), y(y) { }
8     };
9
10    inline num operator+(num a, num b) { return num(a.x + b.x, a.y + b.y); }
11    inline num operator-(num a, num b) { return num(a.x - b.x, a.y - b.y); }

```

```

12 inline num operator*(num a, num b) { return num(a.x * b.x - a.y * b.y, a.x
    * b.y + a.y * b.x); }
13 inline num conj(num a) { return num(a.x, -a.y); }
14
15 int base = 1;
16 vector<num> roots = {{0, 0}, {1, 0}};
17 vector<int> rev = {0, 1};
18
19 const dbl PI = acos(-1.0);
20
21 void ensure_base(int nbase) {
22     if (nbase <= base) {
23         return;
24     }
25     rev.resize(1 << nbase);
26     for (int i = 0; i < (1 << nbase); i++) {
27         rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));
28     }
29     roots.resize(1 << nbase);
30     while (base < nbase) {
31         dbl angle = 2 * PI / (1 << (base + 1));
32         // num z(cos(angle), sin(angle));
33         for (int i = 1 << (base - 1); i < (1 << base); i++) {
34             roots[i << 1] = roots[i];
35             // roots[(i << 1) + 1] = roots[i] * z;
36             dbl angle_i = angle * (2 * i + 1 - (1 << base));
37             roots[(i << 1) + 1] = num(cos(angle_i), sin(angle_i));
38         }
39         base++;
40     }
41 }
42
43 void fft(vector<num> &a, int n = -1) {
44     if (n == -1) {
45         n = a.size();
46     }
47     assert((n & (n - 1)) == 0);
48     int zeros = __builtin_ctz(n);
49     ensure_base(zeros);
50     int shift = base - zeros;
51     for (int i = 0; i < n; i++) {
52         if (i < (rev[i] >> shift)) {
53             swap(a[i], a[rev[i] >> shift]);

```

```

54     }
55 }
56 /* for (int k = 1; k < n; k <= 1) {
57     for (int i = 0; i < n; i += 2 * k) {
58         for (int j = 0; j < k; j++) {
59             num z = a[i + j + k] * roots[j + k];
60             a[i + j + k] = a[i + j] - z;
61             a[i + j] = a[i + j] + z;
62         }
63     }
64 }*/
65 for (int len = 1; len < n; len <= 1) {
66     for (int i = 0; i < n; i += 2 * len) {
67         for (int j = i, k = i + len; j < i + len; j++, k++) {
68             num z = a[k] * roots[k - i];
69             a[k] = a[j] - z;
70             a[j] = a[j] + z;
71         }
72     }
73 }
74 }
75
76 vector<num> fa, fb;
77
78 vector<long long> multiply(vector<int> &a, vector<int> &b) {
79     int need = a.size() + b.size() - 1;
80     int nbase = 0;
81     while ((1 << nbase) < need) nbase++;
82     ensure_base(nbase);
83     int sz = 1 << nbase;
84     if (sz > (int) fa.size()) {
85         fa.resize(sz);
86     }
87     for (int i = 0; i < sz; i++) {
88         int x = (i < (int) a.size() ? a[i] : 0);
89         int y = (i < (int) b.size() ? b[i] : 0);
90         fa[i] = num(x, y);
91     }
92     fft(fa, sz);
93     num r(0, -0.25 / sz);
94     for (int i = 0; i <= (sz >> 1); i++) {
95         int j = (sz - i) & (sz - 1);
96         num z = (fa[j] * fa[j] - conj(fa[i] * fa[i])) * r;

```

```

97     if (i != j) {
98         fa[j] = (fa[i] * fa[i] - conj(fa[j] * fa[j])) * r;
99     }
100    fa[i] = z;
101 }
102 fft(fa, sz);
103 vector<long long> res(need);
104 for (int i = 0; i < need; i++) {
105     res[i] = fa[i].x + 0.5;
106 }
107 return res;
108 }
109
110 vector<int> multiply_mod(vector<int> &a, vector<int> &b, int m, int eq =
111     0) {
112     int need = a.size() + b.size() - 1;
113     int nbase = 0;
114     while ((1 << nbase) < need) nbase++;
115     ensure_base(nbase);
116     int sz = 1 << nbase;
117     if (sz > (int) fa.size()) {
118         fa.resize(sz);
119     }
120     for (int i = 0; i < (int) a.size(); i++) {
121         int x = (a[i] % m + m) % m;
122         fa[i] = num(x & ((1 << 15) - 1), x >> 15);
123     }
124     fill(fa.begin() + a.size(), fa.begin() + sz, num {0, 0});
125     fft(fa, sz);
126     if (eq) {
127         copy(fa.begin(), fa.begin() + sz, fb.begin());
128     } else {
129         if (sz > (int) fb.size()) {
130             fb.resize(sz);
131         }
132         for (int i = 0; i < (int) b.size(); i++) {
133             int x = (b[i] % m + m) % m;
134             fb[i] = num(x & ((1 << 15) - 1), x >> 15);
135         }
136         fill(fb.begin() + b.size(), fb.begin() + sz, num {0, 0});
137         fft(fb, sz);
138     }
139     dbl ratio = 0.25 / sz;

```

```

139     num r2(0, -1);
140     num r3(ratio, 0);
141     num r4(0, -ratio);
142     num r5(0, 1);
143     for (int i = 0; i <= (sz >> 1); i++) {
144         int j = (sz - i) & (sz - 1);
145         num a1 = (fa[i] + conj(fa[j]));
146         num a2 = (fa[i] - conj(fa[j])) * r2;
147         num b1 = (fb[i] + conj(fb[j])) * r3;
148         num b2 = (fb[i] - conj(fb[j])) * r4;
149         if (i != j) {
150             num c1 = (fa[j] + conj(fa[i]));
151             num c2 = (fa[j] - conj(fa[i])) * r2;
152             num d1 = (fb[j] + conj(fb[i])) * r3;
153             num d2 = (fb[j] - conj(fb[i])) * r4;
154             fa[i] = c1 * d1 + c2 * d2 * r5;
155             fb[i] = c1 * d2 + c2 * d1;
156         }
157         fa[j] = a1 * b1 + a2 * b2 * r5;
158         fb[j] = a1 * b2 + a2 * b1;
159     }
160     fft(fa, sz);
161     fft(fb, sz);
162     vector<int> res(need);
163     for (int i = 0; i < need; i++) {
164         long long aa = fa[i].x + 0.5;
165         long long bb = fb[i].x + 0.5;
166         long long cc = fa[i].y + 0.5;
167         res[i] = (aa + ((bb % m) << 15) + ((cc % m) << 30)) % m;
168     }
169     return res;
170 }
171
172 vector<int> square_mod(vector<int> &a, int m) {
173     return multiply_mod(a, a, m, 1);
174 }
175
176 // fft::multiply uses dbl, outputs vector<long long> of rounded values
177 // fft::multiply_mod might work for res.size() up to 2^21
178 // typedef long double dbl;          =>          up to 2^25 (but takes a lot
179 //                                     of memory)
180 };

```



## 6.7 fftfast.cpp

```
1 // FFT_MAXN = 2^k
2 // fft_init() to precalc FFT_MAXN-th roots
3
4 typedef long double db;
5 const int FFT_MAXN = 262144;
6 const int N = 3.1e5;
7 const db pi = acos(-1.);
8 struct cp {
9     db a, b;
10     cp operator+(const cp &y) const { return (cp){a + y.a, b + y.b}; }
11     cp operator-(const cp &y) const { return (cp){a - y.a, b - y.b}; }
12     cp operator*(const cp &y) const { return (cp){a * y.a - b * y.b, a * y.b
13         + b * y.a}; }
14     cp operator!() const { return (cp){a, -b}; };
15 } nw[FFT_MAXN + 1];
16 int bitrev[FFT_MAXN];
17 void dft(cp *a, int n, int flag = 1) {
18     int d = 0;
19     while ((1 << d) * n != FFT_MAXN) d++;
20     rep(i, 0, n - 1) if (i < (bitrev[i] >> d)) swap(a[i], a[bitrev[i] >> d]);
21
22     for (int l = 2; l <= n; l <= 1) {
23         int del = FFT_MAXN / l * flag;
24         for (int i = 0; i < n; i += l) {
25             cp *le = a + i, *ri = a + i + (l >> 1), *w = flag == 1 ? nw : nw
26                 + FFT_MAXN;
27             rep(k, 0, l / 2 - 1) {
28                 cp ne = *ri * *w;
29                 *ri = *le - ne, *le = *le + ne;
30                 le++, ri++, w += del;
31             }
32         }
33     }
34     if (flag != 1) rep(i, 0, n - 1) a[i].a /= n, a[i].b /= n;
35 }
36 void fft_init() {
37     int L = 0;
38     while ((1 << L) != FFT_MAXN) L++;
39     bitrev[0] = 0;
40     rep(i, 1, FFT_MAXN - 1) bitrev[i] = bitrev[i >> 1] >> 1 | ((i & 1) << (L
41         - 1));
42     nw[0] = nw[FFT_MAXN] = (cp){1, 0};
```

```
39     rep(i, 0, FFT_MAXN)
40         nw[i] = (cp){cos(2 * pi / FFT_MAXN * i), sin(2 * pi / FFT_MAXN * i)};
41         // very slow
42 }
43 void convo(db *a, int n, db *b, int m, db *c) {
44     static cp f[FFT_MAXN >> 1], g[FFT_MAXN >> 1], t[FFT_MAXN >> 1];
45     int N = 2;
46     while (N <= n + m) N <= 1;
47     rep(i, 0, N - 1) if (i & 1) {
48         f[i >> 1].b = (i <= n) ? a[i] : 0.0;
49         g[i >> 1].b = (i <= m) ? b[i] : 0.0;
50     }
51     else {
52         f[i >> 1].a = (i <= n) ? a[i] : 0.0;
53         g[i >> 1].a = (i <= m) ? b[i] : 0.0;
54     }
55     dft(f, N >> 1);
56     dft(g, N >> 1);
57     int del = FFT_MAXN / (N >> 1);
58     cp qua = (cp){0, 0.25}, one = (cp){1, 0}, four = (cp){4, 0}, *w = nw;
59     rep(i, 0, N / 2 - 1) {
60         int j = i ? (N >> 1) - i : 0;
61         t[i] = (four * !(f[j] * g[j]) - (!f[j] - f[i]) * (!g[j] - g[i]) * (
62             one + *w)) * qua;
63         w += del;
64     }
65     dft(t, N >> 1, -1);
66     rep(i, 0, n + m) c[i] = (i & 1) ? t[i >> 1].a : t[i >> 1].b;
67 }
68 int mo;
69 void mul(int *a, int *b, int n) { // n<=N, 0<=a[i],b[i]<mo
70     static cp f[N], g[N], t[N], r[N];
71     int nn = 2;
72     while (nn <= n + n) nn <= 1;
73     rep(i, 0, nn - 1) {
74         f[i] = (i <= n) ? (cp){(db)(a[i] >> 15), (db)(a[i] & 32767)} : (cp)
75             {0, 0};
76         g[i] = (i <= n) ? (cp){(db)(b[i] >> 15), (db)(b[i] & 32767)} : (cp)
77             {0, 0};
78     }
79     swap(n, nn);
```

```

78     dft(f, n, 1);
79     dft(g, n, 1);
80     rep(i, 0, n - 1) {
81         int j = i ? n - i : 0;
82         t[i] = ((f[i] + !f[j]) * (!g[j] - g[i]) + (!f[j] - f[i]) * (g[i] + !
            g[j])) * (cp){0, 0.25};
83         r[i] = (!f[j] - f[i]) * (!g[j] - g[i]) * (cp){-0.25, 0} + (cp){0,
            0.25} * (f[i] + !f[j]) * (g[i] + !g[j]);
84     }
85     dft(t, n, -1);
86     dft(r, n, -1);
87     rep(i, 0, n - 1)
88     a[i] = ((ll(t[i].a + 0.5) % mo << 15) + ll(r[i].a + 0.5) + (ll(r[i].b +
        0.5) % mo << 30)) % mo;
89 }

```

## 6.8 fftnew.cpp

```

1  namespace fft {
2
3  typedef double dbl;
4
5  struct num {
6      dbl x, y;
7      num() { x = y = 0; }
8      num(dbl x_, dbl y_) : x(x_), y(y_) {}
9  };
10
11  inline num operator+(num a, num b) { return num(a.x + b.x, a.y + b.y); }
12  inline num operator-(num a, num b) { return num(a.x - b.x, a.y - b.y); }
13  inline num operator*(num a, num b) { return num(a.x * b.x - a.y * b.y, a.x *
        b.y + a.y * b.x); }
14  inline num conj(num a) { return num(a.x, -a.y); }
15
16  int base = 1;
17  vector<num> roots = {{0, 0}, {1, 0}};
18  vector<int> rev = {0, 1};
19
20  const dbl PI = static_cast<dbl>(acosl(-1.0));
21
22  void ensure_base(int nbase) {
23      if (nbase <= base) {
24          return;

```

```

25     }
26     rev.resize(1 << nbase);
27     for (int i = 0; i < (1 << nbase); i++) {
28         rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));
29     }
30     roots.resize(1 << nbase);
31     while (base < nbase) {
32         dbl angle = 2 * PI / (1 << (base + 1));
33         //     num z(cos(angle), sin(angle));
34         for (int i = 1 << (base - 1); i < (1 << base); i++) {
35             roots[i << 1] = roots[i];
36             //     roots[(i << 1) + 1] = roots[i] * z;
37             dbl angle_i = angle * (2 * i + 1 - (1 << base));
38             roots[(i << 1) + 1] = num(cos(angle_i), sin(angle_i));
39         }
40         base++;
41     }
42 }
43
44 void fft(vector<num>& a, int n = -1) {
45     if (n == -1) {
46         n = (int) a.size();
47     }
48     assert((n & (n - 1)) == 0);
49     int zeros = __builtin_ctz(n);
50     ensure_base(zeros);
51     int shift = base - zeros;
52     for (int i = 0; i < n; i++) {
53         if (i < (rev[i] >> shift)) {
54             swap(a[i], a[rev[i] >> shift]);
55         }
56     }
57     for (int k = 1; k < n; k <= 1) {
58         for (int i = 0; i < n; i += 2 * k) {
59             for (int j = 0; j < k; j++) {
60                 num z = a[i + j + k] * roots[j + k];
61                 a[i + j + k] = a[i + j] - z;
62                 a[i + j] = a[i + j] + z;
63             }
64         }
65     }
66 }
67

```

```

68 vector<num> fa, fb;
69
70 vector<int64_t> square(const vector<int>& a) {
71     if (a.empty()) {
72         return {};
73     }
74     int need = (int) a.size() + (int) a.size() - 1;
75     int nbase = 1;
76     while ((1 << nbase) < need) nbase++;
77     ensure_base(nbase);
78     int sz = 1 << nbase;
79     if ((sz >> 1) > (int) fa.size()) {
80         fa.resize(sz >> 1);
81     }
82     for (int i = 0; i < (sz >> 1); i++) {
83         int x = (2 * i < (int) a.size() ? a[2 * i] : 0);
84         int y = (2 * i + 1 < (int) a.size() ? a[2 * i + 1] : 0);
85         fa[i] = num(x, y);
86     }
87     fft(fa, sz >> 1);
88     num r(1.0 / (sz >> 1), 0.0);
89     for (int i = 0; i <= (sz >> 2); i++) {
90         int j = ((sz >> 1) - i) & ((sz >> 1) - 1);
91         num fe = (fa[i] + conj(fa[j])) * num(0.5, 0);
92         num fo = (fa[i] - conj(fa[j])) * num(0, -0.5);
93         num aux = fe * fe + fo * fo * roots[(sz >> 1) + i] * roots[(sz >> 1) + i
94             ];
95         num tmp = fe * fo;
96         fa[i] = r * (conj(aux) + num(0, 2) * conj(tmp));
97         fa[j] = r * (aux + num(0, 2) * tmp);
98     }
99     fft(fa, sz >> 1);
100     vector<int64_t> res(need);
101     for (int i = 0; i < need; i++) {
102         res[i] = llround(i % 2 == 0 ? fa[i >> 1].x : fa[i >> 1].y);
103     }
104     return res;
105 }
106
107 vector<int64_t> multiply(const vector<int>& a, const vector<int>& b) {
108     if (a.empty() || b.empty()) {
109         return {};
110     }

```

```

110     if (a == b) {
111         return square(a);
112     }
113     int need = (int) a.size() + (int) b.size() - 1;
114     int nbase = 1;
115     while ((1 << nbase) < need) nbase++;
116     ensure_base(nbase);
117     int sz = 1 << nbase;
118     if (sz > (int) fa.size()) {
119         fa.resize(sz);
120     }
121     for (int i = 0; i < sz; i++) {
122         int x = (i < (int) a.size() ? a[i] : 0);
123         int y = (i < (int) b.size() ? b[i] : 0);
124         fa[i] = num(x, y);
125     }
126     fft(fa, sz);
127     num r(0, -0.25 / (sz >> 1));
128     for (int i = 0; i <= (sz >> 1); i++) {
129         int j = (sz - i) & (sz - 1);
130         num z = (fa[j] * fa[j] - conj(fa[i] * fa[i])) * r;
131         fa[j] = (fa[i] * fa[i] - conj(fa[j] * fa[j])) * r;
132         fa[i] = z;
133     }
134     for (int i = 0; i < (sz >> 1); i++) {
135         num A0 = (fa[i] + fa[i + (sz >> 1)]) * num(0.5, 0);
136         num A1 = (fa[i] - fa[i + (sz >> 1)]) * num(0.5, 0) * roots[(sz >> 1) + i
137             ];
138         fa[i] = A0 + A1 * num(0, 1);
139     }
140     fft(fa, sz >> 1);
141     vector<int64_t> res(need);
142     for (int i = 0; i < need; i++) {
143         res[i] = llround(i % 2 == 0 ? fa[i >> 1].x : fa[i >> 1].y);
144     }
145     return res;
146 }
147
148 vector<int> multiply_mod(const vector<int>& a, const vector<int>& b, int m)
149 {
150     if (a.empty() || b.empty()) {
151         return {};
152     }

```

```

151 int eq = (a.size() == b.size() && a == b);
152 int need = (int) a.size() + (int) b.size() - 1;
153 int nbase = 0;
154 while ((1 << nbase) < need) nbase++;
155 ensure_base(nbase);
156 int sz = 1 << nbase;
157 if (sz > (int) fa.size()) {
158     fa.resize(sz);
159 }
160 for (int i = 0; i < (int) a.size(); i++) {
161     int x = (a[i] % m + m) % m;
162     fa[i] = num(x & ((1 << 15) - 1), x >> 15);
163 }
164 fill(fa.begin() + a.size(), fa.begin() + sz, num {0, 0});
165 fft(fa, sz);
166 if (sz > (int) fb.size()) {
167     fb.resize(sz);
168 }
169 if (eq) {
170     copy(fa.begin(), fa.begin() + sz, fb.begin());
171 } else {
172     for (int i = 0; i < (int) b.size(); i++) {
173         int x = (b[i] % m + m) % m;
174         fb[i] = num(x & ((1 << 15) - 1), x >> 15);
175     }
176     fill(fb.begin() + b.size(), fb.begin() + sz, num {0, 0});
177     fft(fb, sz);
178 }
179 dbl ratio = 0.25 / sz;
180 num r2(0, -1);
181 num r3(ratio, 0);
182 num r4(0, -ratio);
183 num r5(0, 1);
184 for (int i = 0; i <= (sz >> 1); i++) {
185     int j = (sz - i) & (sz - 1);
186     num a1 = (fa[i] + conj(fa[j]));
187     num a2 = (fa[i] - conj(fa[j])) * r2;
188     num b1 = (fb[i] + conj(fb[j])) * r3;
189     num b2 = (fb[i] - conj(fb[j])) * r4;
190     if (i != j) {
191         num c1 = (fa[j] + conj(fa[i]));
192         num c2 = (fa[j] - conj(fa[i])) * r2;
193         num d1 = (fb[j] + conj(fb[i])) * r3;

```

```

194         num d2 = (fb[j] - conj(fb[i])) * r4;
195         fa[i] = c1 * d1 + c2 * d2 * r5;
196         fb[i] = c1 * d2 + c2 * d1;
197     }
198     fa[j] = a1 * b1 + a2 * b2 * r5;
199     fb[j] = a1 * b2 + a2 * b1;
200 }
201 fft(fa, sz);
202 fft(fb, sz);
203 vector<int> res(need);
204 for (int i = 0; i < need; i++) {
205     int64_t aa = llround(fa[i].x);
206     int64_t bb = llround(fb[i].x);
207     int64_t cc = llround(fa[i].y);
208     res[i] = static_cast<int>((aa + ((bb % m) << 15) + ((cc % m) << 30)) % m
209         );
210 }
211 return res;
212 }
213 } // namespace fft

```

## 6.9 FST.cpp

---

```

1 void fst(VI &a, bool inv) {
2     for (int n=SZ(a), step=1; step<n; step*=2) {
3         for (int i=0; i<n; i+=2*step) rep(j, i, i+step-1) {
4             int &u=a[j], &v=a[j+step];
5             tie(u, v)=
6                 inv?PII(v-u, u):PII(v, u+v); // AND
7                 inv?PII(v, u-v):PII(u+v, u); // OR
8                 PII(u+v, u-v); // XOR
9         }
10    }
11    if (inv) for (auto &x : a) x/=SZ(a); // XOR only
12 }
13 VI conv(VI a, VI b) {
14     fst(a, 0), fst(b, 0);
15     rep(i, 0, SZ(a)-1) a[i]=a[i]*b[i];
16     fst(a, 1); return a;
17 }

```

## 6.10 FWT.cpp

```
1 ll f[maxn], g[maxn], h[maxn];
2 int main() {
3     for (int i = 0; i < n; i++) {
4         for (int j = 0; j < bit(n); j++) {
5             if ((j & bit(i)) == 0) {
6                 f[j] += f[j + bit(i)];
7                 g[j] += g[j + bit(i)];
8             }
9         }
10    }
11    for (int i = 0; i < bit(n); i++) {
12        f[i] %= mod;
13        g[i] %= mod;
14        h[i] = f[i] * g[i] % mod;
15    }
16    for (int i = 0; i < n; i++) {
17        for (int j = 0; j < bit(n); j++) {
18            if ((j & bit(i)) == 0)
19                h[j] -= h[j + bit(i)];
20        }
21    }
22    for (int i = 0; i < bit(n); i++) {
23        h[i] %= mod;
24        if (h[i] < 0) h[i] += mod;
25    }
26
27    ll ans = 0;
28    rep(i, 0, bit(n) - 1) ans ^= h[i];
29    cout << ans << '\n';
30 }
```

## 6.11 gauss(合数).cpp

```
1 void gauss(int n) {
2     int ans = 1;
3     //rep(i, 1, n) rep(j, 1, n) p[i][j] %= mod;
4     for (int i = 1; i <= n; i++) {
5         for (int j = i + 1; j <= n; j++) {
6             int x = i, y = j;
7             while (p[x][i]) {
8                 int t = p[y][i] / p[x][i];
9                 for (int k = i; k <= n; k++)
```

```
10                 p[y][k] = (p[y][k] - p[x][k] * t) % mod;
11                 swap(x, y);
12             }
13             if (x == i) {
14                 for (int k = i; k <= n; k++) swap(p[i][k], p[j][k]);
15                 ans = -ans;
16             }
17         }
18     }
19 }
```

## 6.12 gauss.cpp

```
1 ll f[N][N];
2 ll v[N], a[N];
3 void gauss() {
4     for (int i = 1; i <= n; i++) {
5         for (int j = i; j <= n; j++) {
6             if (f[j][i] > f[i][i]) {
7                 swap(v[i], v[j]);
8                 for (int k = 1; k <= n; k++)
9                     swap(f[j][k], f[i][k]);
10            }
11        }
12        for (int j = i + 1; j <= n; j++) {
13            if (f[j][i]) {
14                int delta = f[j][i] * fpow(f[i][i], mod - 2) % mod;
15                for (int k = i; k <= n; k++) {
16                    f[j][k] -= f[i][k] * delta % mod;
17                    if (f[j][k] < 0)
18                        f[j][k] += mod;
19                }
20                v[j] -= v[i] * delta % mod;
21                if (v[j] < mod)
22                    v[j] += mod;
23            }
24        }
25    }
26    for (int j = n; j > 0; j--) {
27        for (int k = j + 1; k <= n; k++) {
28            v[j] -= f[j][k] * a[k] % mod;
29            if (v[j] < 0)
30                v[j] += mod;
```

```

31     }
32     a[j] = v[j] * fpow(f[j][j], mod - 2) % mod;
33 }
34 }

```

### 6.13 linearbasis.cpp

---

```

1 struct linear_base {
2     ll w[64];
3     ll zero = 0;
4     ll tot = -1;
5     void clear() {
6         rep(i, 0, 63) w[i] = 0;
7         zero = 0;
8         tot = -1;
9     }
10    void insert(ll x) {
11        for (int i = 62; i >= 0; i--) {
12            if (x & bit(i))
13                if (!w[i]) {w[i] = x; return;}
14            else x ^= w[i];
15        }
16        zero++;
17    }
18    void build() {
19        rep(i, 0, 63) rep(j, 0, i - 1) {
20            if (w[i]&bit(j)) w[i] ^= w[j];
21        }
22        for (int i = 0; i <= 62; i++) {
23            if (w[i] != 0) w[++tot] = w[i];
24        }
25    }
26    ll qmax() {
27        ll res = 0;
28        for (int i = 62; i >= 0; i--) {
29            res = max(res, res ^ w[i]);
30        }
31        return res;
32    }
33    bool check(ll x) {
34        for (int i = 62; i >= 0; i--) {
35            if (x & bit(i))
36                if (!w[i]) return false;

```

```

37            else x ^= w[i];
38        }
39        return true;
40    }
41    ll query(ll k) {
42        ll res = 0;
43        // if (zero) k-=1;
44        // if (k >= bit(tot)) return -1;
45        for (int i = tot; i >= 0; i--) {
46            if (k & bit(i)) {
47                res = max(res, res ^ w[i]);
48            } else {
49                res = min(res, res ^ w[i]);
50            }
51        }
52        return res;
53    }
54 };

```

### 6.14 lucas.cpp

---

```

1 ll fac[maxn], fnv[maxn];
2
3 ll binom(ll a, ll b) {
4     if (b > a || b < 0) return 0;
5     return fac[a] * fnv[a - b] % p * fnv[b] % p;
6 }
7
8 ll lucas(ll a, ll b, ll p) {
9     ll ans = 1;
10    while (a > 0 || b > 0) {
11        ans = (ans * binom(a % p, b % p)) % p;
12        a /= p, b /= p;
13    }
14    return ans;
15 }
16
17 int main() {
18     cin >> p >> T;
19     fac[0] = 1;
20     rep(i, 1, p - 1) fac[i] = fac[i - 1] * i % p;
21     fnv[p - 1] = powmod(fac[p - 1], p - 2, p);
22     per(i, p - 2, 0) fnv[i] = fnv[i + 1] * (i + 1) % p;

```

```

23     assert(fnv[0] == 1);
24 }

```

## 6.15 mathdiv.cpp

---

```

1 ll floor_div(ll x, ll y) {
2     assert(y != 0);
3     if (y < 0) {
4         y = -y;
5         x = -x;
6     }
7     if (x >= 0) return x / y;
8     return (x + 1) / y - 1;
9 }
10 ll ceil_div(ll x, ll y) {
11     assert(y != 0);
12     if (y < 0) {
13         y = -y;
14         x = -x;
15     }
16     if (x <= 0) return x / y;
17     return (x - 1) / y + 1;
18 }

```

## 6.16 matrix.cpp

---

```

1 template <typename T>
2 vector<vector<T>> operator*(const vector<vector<T>>& a, const vector<vector<
    T>>& b) {
3     if (a.empty() || b.empty()) {
4         return {{}};
5     }
6     vector<vector<T>> c(a.size(), vector<T>(b[0].size()));
7     for (int i = 0; i < static_cast<int>(c.size()); i++) {
8         for (int j = 0; j < static_cast<int>(c[0].size()); j++) {
9             c[i][j] = 0;
10            for (int k = 0; k < static_cast<int>(b.size()); k++) {
11                c[i][j] += a[i][k] * b[k][j];
12            }
13        }
14    }
15    return c;
16 }

```

```

17
18 template <typename T>
19 vector<vector<T>>& operator*=(vector<vector<T>>& a, const vector<vector<T>>&
    b) {
20     return a = a * b;
21 }
22
23 template <typename T, typename U>
24 vector<vector<T>> power(const vector<vector<T>>& a, const U& b) {
25     assert(b >= 0);
26     vector<U> binary;
27     U bb = b;
28     while (bb > 0) {
29         binary.push_back(bb & 1);
30         bb >>= 1;
31     }
32     vector<vector<T>> res(a.size(), vector<T>(a.size()));
33     for (int i = 0; i < static_cast<int>(a.size()); i++) {
34         res[i][i] = 1;
35     }
36     for (int j = (int)binary.size() - 1; j >= 0; j--) {
37         res *= res;
38         if (binary[j] == 1) {
39             res *= a;
40         }
41     }
42     return res;
43 }

```

## 6.17 matrixfast.cpp

---

```

1 Description: Basic operations on square matrices.
2 Usage: Matrix<int, 3> A;
3 A.d = {{{{1, 2, 3}}, {{4, 5, 6}}, {{7, 8, 9}}}};
4 vector<int> vec = {1, 2, 3};
5 vec = (A^N) * vec;
6
7 template<class T, int N> struct Matrix {
8     typedef Matrix M;
9     array<array<T, N>, N> d{};
10    M operator*(const M& m) const {
11        M a;
12        rep(i, 0, N) rep(j, 0, N)

```

```

13         rep(k, 0, N) a.d[i][j] += d[i][k] * m.d[k][j];
14         return a;
15     }
16     vector<T> operator*(const vector<T>& vec) const {
17         vector<T> ret(N);
18         rep(i, 0, N) rep(j, 0, N) ret[i] += d[i][j] * vec[j];
19         return ret;
20     }
21     M operator^(ll p) const {
22         assert(p >= 0);
23         M a, b(*this);
24         rep(i, 0, N) a.d[i][i] = 1;
25         while (p) {
26             if (p & 1) a = a * b;
27             b = b * b;
28             p >>= 1;
29         }
30         return a;
31     }
32 };

```

## 6.18 MillerRabbin pollard modmul.cpp

```

1  /*ModMulLL.h
2  Description: Calculate  $a \cdot b \bmod c$  (or  $a$ 
3   $b \bmod c$ ) for  $0 \leq a, b < c \leq 7.2 \cdot 10^{18}$ 
4  Time:  $O(1)$  for modmul,  $O(\log b)$  for modpow*/
5  /*ull modmul(ull a, ull b, ull M) {
6      ll ret = a * b - M * ull(1.L / M * a * b);
7      return ret + M * (ret < 0) - M * (ret >= (ll)M);
8  }
9  ull modpow(ull b, ull e, ull mod) {
10     ull ans = 1;
11     for (; e; b = modmul(b, b, mod), e /= 2)
12         if (e & 1) ans = modmul(ans, b, mod);
13     return ans;
14 }*/
15 ll modmul(ll a, ll b, ll m) {
16     a %= m, b %= m;
17     ll d = ((ldb)a * b / m);
18     d = a * b - d * m;
19     if (d >= m) d -= m;
20     if (d < 0) d += m;

```

```

21     return d;
22 }
23 ll modpow(ll a, ll b, ll p) {
24     ll ans = 1;
25     while (b) {
26         if (b & 1) ans = modmul(ans, a, p);
27         a = modmul(a, a, p); b >>= 1;
28     } return ans;
29 }
30 /*MillerRabin.h
31 Description: Deterministic Miller-Rabin primality test. Guaranteed to
32 work for numbers up to  $7 \cdot 10^{18}$ ; for larger numbers, use Python and extend A
33 randomly.
34 Time: 7 times the complexity of  $a^b \bmod c$  */
35 bool isPrime(ll n) {
36     if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
37     ll A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
38         s = __builtin_ctzll(n - 1), d = n >> s;
39     for (ll a : A) { // ^ count trailing zeroes
40         ll p = modpow(a % n, d, n), i = s;
41         while (p != 1 && p != n - 1 && a % n && i--)
42             p = modmul(p, p, n);
43         if (p != n - 1 && i != s) return 0;
44     }
45     return 1;
46 }
47 /*Factor.h
48 Description: Pollard-rho randomized factorization algorithm. Returns
49 prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
50 Time:  $O(n^{1/4})$ , less for numbers with small factors.*/
51 ll pollard(ll n) {
52     auto f = [n](ll x) { return modmul(x, x, n) + 1; };
53     ll x = 0, y = 0, t = 30, prd = 2, i = 1, q;
54     while (t++ % 40 || __gcd(prd, n) == 1) {
55         if (x == y) x = ++i, y = f(x);
56         if ((q = modmul(prd, max(x, y) - min(x, y), n))) prd = q;
57         x = f(x), y = f(f(y));
58     }
59     return __gcd(prd, n);
60 }
61 vector<ll> factor(ll n) {
62     if (n == 1) return {};
63     if (isPrime(n)) return {n};

```



```

63     ll x = pollard(n);
64     auto l = factor(x), r = factor(n / x);
65     l.insert(l.end(), all(r));
66     return l;
67 }

```

## 6.19 ntt(polynomial).cpp

---

```

1  #include<bits/stdc++.h>
2  using namespace std;
3
4  const int mod = 998244353;
5
6  inline void add(int &x, int y) {
7      x += y;
8      if (x >= mod) {
9          x -= mod;
10     }
11 }
12
13 inline void sub(int &x, int y) {
14     x -= y;
15     if (x < 0) {
16         x += mod;
17     }
18 }
19
20 inline int mul(int x, int y) {
21     return (long long) x * y % mod;
22 }
23
24 inline int power(int x, int y) {
25     int res = 1;
26     for (; y; y >>= 1, x = mul(x, x)) {
27         if (y & 1) {
28             res = mul(res, x);
29         }
30     }
31     return res;
32 }
33
34 inline int inv(int a) {
35     a %= mod;

```

```

36     if (a < 0) {
37         a += mod;
38     }
39     int b = mod, u = 0, v = 1;
40     while (a) {
41         int t = b / a;
42         b -= t * a;
43         swap(a, b);
44         u -= t * v;
45         swap(u, v);
46     }
47     if (u < 0) {
48         u += mod;
49     }
50     return u;
51 }
52
53 namespace ntt {
54     int base = 1, root = -1, max_base = -1;
55     vector<int> rev = {0, 1}, roots = {0, 1};
56
57     void init() {
58         int temp = mod - 1;
59         max_base = 0;
60         while (temp % 2 == 0) {
61             temp >>= 1;
62             ++max_base;
63         }
64         root = 2;
65         while (true) {
66             if (power(root, 1 << max_base) == 1 && power(root, 1 << (max_base - 1))
67                 != 1) {
68                 break;
69             }
70             ++root;
71         }
72
73         void ensure_base(int nbase) {
74             if (max_base == -1) {
75                 init();
76             }
77             if (nbase <= base) {

```

```

78     return;
79 }
80 assert(nbase <= max_base);
81 rev.resize(1 << nbase);
82 for (int i = 0; i < 1 << nbase; ++i) {
83     rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (nbase - 1));
84 }
85 roots.resize(1 << nbase);
86 while (base < nbase) {
87     int z = power(root, 1 << (max_base - 1 - base));
88     for (int i = 1 << (base - 1); i < 1 << base; ++i) {
89         roots[i << 1] = roots[i];
90         roots[i << 1 | 1] = mul(roots[i], z);
91     }
92     ++base;
93 }
94 }
95
96 void dft(vector<int> &a) {
97     int n = a.size(), zeros = __builtin_ctz(n);
98     ensure_base(zeros);
99     int shift = base - zeros;
100    for (int i = 0; i < n; ++i) {
101        if (i < rev[i] >> shift) {
102            swap(a[i], a[rev[i] >> shift]);
103        }
104    }
105    for (int i = 1; i < n; i <= 1) {
106        for (int j = 0; j < n; j += i << 1) {
107            for (int k = 0; k < i; ++k) {
108                int x = a[j + k], y = mul(a[j + k + i], roots[i + k]);
109                a[j + k] = (x + y) % mod;
110                a[j + k + i] = (x + mod - y) % mod;
111            }
112        }
113    }
114 }
115
116 vector<int> multiply(vector<int> a, vector<int> b) {
117     int need = a.size() + b.size() - 1, nbase = 0;
118     while (1 << nbase < need) {
119         ++nbase;
120     }

```

```

121     ensure_base(nbase);
122     int sz = 1 << nbase;
123     a.resize(sz);
124     b.resize(sz);
125     bool equal = a == b;
126     dft(a);
127     if (equal) {
128         b = a;
129     } else {
130         dft(b);
131     }
132     int inv_sz = inv(sz);
133     for (int i = 0; i < sz; ++i) {
134         a[i] = mul(mul(a[i], b[i]), inv_sz);
135     }
136     reverse(a.begin() + 1, a.end());
137     dft(a);
138     a.resize(need);
139     return a;
140 }
141
142 vector<int> inverse_new(const vector<int> &a) {
143     assert(!a.empty());
144     int n = (int) a.size();
145     vector<int> b = {inv(a[0])};
146     while ((int) b.size() < n) {
147         vector<int> x(a.begin(), a.begin() + min(a.size(), b.size() << 1));
148         x.resize(b.size() << 1);
149         b.resize(b.size() << 1);
150         vector<int> c = b;
151         // NTT<T>::fft(c);
152         // NTT<T>::fft(x);
153         dft(c);
154         dft(x);
155         // Modular<T> inv = 1 / static_cast<Modular<T>>((int) x.size());
156         int inv_sz = inv((int)x.size());
157         for (int i = 0; i < (int) x.size(); i++) {
158             // x[i] *= c[i] * inv;
159             x[i] = mul(x[i], mul(c[i], inv_sz));
160         }
161         reverse(x.begin() + 1, x.end());
162         // NTT<T>::fft(x);
163         dft(x);

```

```

164     rotate(x.begin(), x.begin() + (x.size() >> 1), x.end());
165     fill(x.begin() + (x.size() >> 1), x.end(), 0);
166     // NTT<T>::fft(x);
167     dft(x);
168     for (int i = 0; i < (int) x.size(); i++) {
169         // x[i] *= c[i] * inv;
170         x[i] = mul(x[i], mul(c[i], inv_sz));
171     }
172     reverse(x.begin() + 1, x.end());
173     // NTT<T>::fft(x);
174     dft(x);
175     for (int i = 0; i < ((int) x.size() >> 1); i++) {
176         // b[i + ((int) x.size() >> 1)] = -x[i];
177         int t = 0; sub(t, x[i]);
178         b[i + ((int) x.size() >> 1)] = t;
179     }
180 }
181 b.resize(n);
182 return b;
183 }
184
185 vector<int> inverse(vector<int> a) {
186     int n = a.size(), m = (n + 1) >> 1;
187     if (n == 1) {
188         return vector<int>(1, inv(a[0]));
189     } else {
190         vector<int> b = inverse(vector<int>(a.begin(), a.begin() + m));
191         int need = n << 1, nbase = 0;
192         while (1 << nbase < need) {
193             ++nbase;
194         }
195         ensure_base(nbase);
196         int sz = 1 << nbase;
197         a.resize(sz);
198         b.resize(sz);
199         dft(a);
200         dft(b);
201         int inv_sz = inv(sz);
202         for (int i = 0; i < sz; ++i) {
203             a[i] = mul(mul(mod + 2 - mul(a[i], b[i]), b[i]), inv_sz);
204         }
205         reverse(a.begin() + 1, a.end());
206         dft(a);

```

```

207     a.resize(n);
208     return a;
209 }
210 }
211 }
212
213 using ntt::multiply;
214 using ntt::inverse;
215
216 vector<int>& operator += (vector<int> &a, const vector<int> &b) {
217     if (a.size() < b.size()) {
218         a.resize(b.size());
219     }
220     for (int i = 0; i < b.size(); ++i) {
221         add(a[i], b[i]);
222     }
223     return a;
224 }
225
226 vector<int> operator + (const vector<int> &a, const vector<int> &b) {
227     vector<int> c = a;
228     return c += b;
229 }
230
231 vector<int>& operator -= (vector<int> &a, const vector<int> &b) {
232     if (a.size() < b.size()) {
233         a.resize(b.size());
234     }
235     for (int i = 0; i < b.size(); ++i) {
236         sub(a[i], b[i]);
237     }
238     return a;
239 }
240
241 vector<int> operator - (const vector<int> &a, const vector<int> &b) {
242     vector<int> c = a;
243     return c -= b;
244 }
245
246 vector<int>& operator *= (vector<int> &a, const vector<int> &b) {
247     if (min(a.size(), b.size()) < 128) {
248         vector<int> c = a;
249         a.assign(a.size() + b.size() - 1, 0);

```

```

250     for (int i = 0; i < c.size(); ++i) {
251         for (int j = 0; j < b.size(); ++j) {
252             add(a[i + j], mul(c[i], b[j]));
253         }
254     }
255 } else {
256     a = multiply(a, b);
257 }
258 return a;
259 }
260
261 vector<int> operator * (const vector<int> &a, const vector<int> &b) {
262     vector<int> c = a;
263     return c *= b;
264 }
265
266 vector<int>& operator /= (vector<int> &a, const vector<int> &b) {
267     int n = a.size(), m = b.size();
268     if (n < m) {
269         a.clear();
270     } else {
271         vector<int> c = b;
272         reverse(a.begin(), a.end());
273         reverse(c.begin(), c.end());
274         c.resize(n - m + 1);
275         a *= inverse(c);
276         a.erase(a.begin() + n - m + 1, a.end());
277         reverse(a.begin(), a.end());
278     }
279     return a;
280 }
281
282 vector<int> operator / (const vector<int> &a, const vector<int> &b) {
283     vector<int> c = a;
284     return c /= b;
285 }
286
287 vector<int>& operator %= (vector<int> &a, const vector<int> &b) {
288     int n = a.size(), m = b.size();
289     if (n >= m) {
290         vector<int> c = (a / b) * b;
291         a.resize(m - 1);
292         for (int i = 0; i < m - 1; ++i) {

```

```

293             sub(a[i], c[i]);
294         }
295     }
296     return a;
297 }
298
299 vector<int> operator % (const vector<int> &a, const vector<int> &b) {
300     vector<int> c = a;
301     return c %= b;
302 }
303
304 vector<int> derivative(const vector<int> &a) {
305     int n = a.size();
306     vector<int> b(n - 1);
307     for (int i = 1; i < n; ++i) {
308         b[i - 1] = mul(a[i], i);
309     }
310     return b;
311 }
312
313 vector<int> primitive(const vector<int> &a) {
314     int n = a.size();
315     vector<int> b(n + 1), invs(n + 1);
316     for (int i = 1; i <= n; ++i) {
317         invs[i] = i == 1 ? 1 : mul(mod - mod / i, invs[mod % i]);
318         b[i] = mul(a[i - 1], invs[i]);
319     }
320     return b;
321 }
322
323 vector<int> logarithm(const vector<int> &a) {
324     vector<int> b = primitive(derivative(a) * inverse(a));
325     b.resize(a.size());
326     return b;
327 }
328
329 vector<int> exponent(const vector<int> &a) {
330     vector<int> b(1, 1);
331     while (b.size() < a.size()) {
332         vector<int> c(a.begin(), a.begin() + min(a.size(), b.size() << 1));
333         add(c[0], 1);
334         vector<int> old_b = b;
335         b.resize(b.size() << 1);

```

```

336     c -= logarithm(b);
337     c *= old_b;
338     for (int i = b.size() >> 1; i < b.size(); ++i) {
339         b[i] = c[i];
340     }
341 }
342 b.resize(a.size());
343 return b;
344 }
345
346 vector<int> power(vector<int> a, int m) {
347     int n = a.size(), p = -1;
348     vector<int> b(n);
349     for (int i = 0; i < n; ++i) {
350         if (a[i]) {
351             p = i;
352             break;
353         }
354     }
355     if (p == -1) {
356         b[0] = !m;
357         return b;
358     }
359     if ((long long) m * p >= n) {
360         return b;
361     }
362     int mu = power(a[p], m), di = inv(a[p]);
363     vector<int> c(n - m * p);
364     for (int i = 0; i < n - m * p; ++i) {
365         c[i] = mul(a[i + p], di);
366     }
367     c = logarithm(c);
368     for (int i = 0; i < n - m * p; ++i) {
369         c[i] = mul(c[i], m);
370     }
371     c = exponent(c);
372     for (int i = 0; i < n - m * p; ++i) {
373         b[i + m * p] = mul(c[i], mu);
374     }
375     return b;
376 }
377
378 vector<int> sqrt(const vector<int> &a) {

```

```

379     vector<int> b(1, 1);
380     while (b.size() < a.size()) {
381         vector<int> c(a.begin(), a.begin() + min(a.size(), b.size() << 1));
382         vector<int> old_b = b;
383         b.resize(b.size() << 1);
384         c *= inverse(b);
385         for (int i = b.size() >> 1; i < b.size(); ++i) {
386             b[i] = mul(c[i], (mod + 1) >> 1);
387         }
388     }
389     b.resize(a.size());
390     return b;
391 }
392
393 vector<int> multiply_all(int l, int r, vector<vector<int>> &all) {
394     if (l > r) {
395         return vector<int>();
396     } else if (l == r) {
397         return all[l];
398     } else {
399         int y = (l + r) >> 1;
400         return multiply_all(l, y, all) * multiply_all(y + 1, r, all);
401     }
402 }
403
404 vector<int> evaluate(const vector<int> &f, const vector<int> &x) {
405     int n = x.size();
406     if (!n) {
407         return vector<int>();
408     }
409     vector<vector<int>> up(n * 2);
410     for (int i = 0; i < n; ++i) {
411         up[i + n] = vector<int> {(mod - x[i]) % mod, 1};
412     }
413     for (int i = n - 1; i; --i) {
414         up[i] = up[i << 1] * up[i << 1 | 1];
415     }
416     vector<vector<int>> down(n * 2);
417     down[1] = f % up[1];
418     for (int i = 2; i < n * 2; ++i) {
419         down[i] = down[i >> 1] % up[i];
420     }
421     vector<int> y(n);

```

```

422     for (int i = 0; i < n; ++i) {
423         y[i] = down[i + n][0];
424     }
425     return y;
426 }
427
428 vector<int> interpolate(const vector<int> &x, const vector<int> &y) {
429     int n = x.size();
430     vector<vector<int>> up(n * 2);
431     for (int i = 0; i < n; ++i) {
432         up[i + n] = vector<int> {(mod - x[i]) % mod, 1};
433     }
434     for (int i = n - 1; i; --i) {
435         up[i] = up[i << 1] * up[i << 1 | 1];
436     }
437     vector<int> a = evaluate(derivative(up[1]), x);
438     for (int i = 0; i < n; ++i) {
439         a[i] = mul(y[i], inv(a[i]));
440     }
441     vector<vector<int>> down(n * 2);
442     for (int i = 0; i < n; ++i) {
443         down[i + n] = vector<int>(1, a[i]);
444     }
445     for (int i = n - 1; i; --i) {
446         down[i] = down[i << 1] * up[i << 1 | 1] + down[i << 1 | 1] * up[i << 1];
447     }
448     return down[1];
449 }
450
451 int main() {
452
453 }

```

## 6.20 PrimitiveRoot.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long ll;
4
5 int pf[1010];
6
7 ll powmod(ll a, ll b, ll mod) {
8     ll res = 1;

```

```

9     a %= mod;
10    for (; b; b >>= 1) {
11        if (b & 1) res = res * a % mod;
12        a = a * a % mod;
13    }
14    return res;
15 }
16
17 int main() {
18     int T;
19     scanf("%d", &T);
20     for (int tc = 0; tc < T; tc++) {
21         int p;
22         scanf("%d", &p);
23         int m = p - 1, t = 0;
24         for (int i = 2; i * i <= m; i++) {
25             if (m % i == 0) {
26                 pf[t++] = i;
27                 while (m % i == 0) m /= i;
28             }
29         }
30         if (m != 1) pf[t++] = m;
31         for (int g = 1; g < p; g++) {
32             bool valid = true;
33             for (int i = 0; i < t; i++)
34                 if (powmod(g, (p - 1) / pf[i], p) == 1) {
35                     valid = false;
36                     break;
37                 }
38             if (valid) {
39                 printf("%d\n", g);
40                 break;
41             }
42         }
43     }
44 }

```

## 6.21 simplex.cpp

```

1 /**
2  * Author: Stanford
3  * Source: Stanford Notebook
4  * License: MIT

```

```

5  * Description: Solves a general linear maximization problem: maximize  $x^T$ 
   * subject to  $Ax \leq b$ ,  $x \geq 0$ .
6  * Returns -inf if there is no solution, inf if there are arbitrarily good
   * solutions, or the maximum value of  $x^T$  otherwise.
7  * The input vector is set to an optimal  $x$  (or in the unbounded case, an
   * arbitrary solution fulfilling the constraints).
8  * Numerical stability is not guaranteed. For better performance, define
   * variables such that  $x = 0$  is viable.
9  * Usage:
10 * vvd A = {{1,-1}, {-1,1}, {-1,-2}};
11 * vd b = {1,1,-4}, c = {-1,-1}, x;
12 * T val = LPSolver(A, b, c).solve(x);
13 * Time:  $O(NM * \#pivots)$ , where a pivot may be e.g. an edge relaxation.  $O(2^n)$ 
   * in the general case.
14 * Status: seems to work?
15 */
16
17 typedef long double T; // long double, Rational, double + mod<P>...
18 typedef vector<T> vd;
19 typedef vector<vd> vvd;
20
21 const T eps = 1e-8, inf = 1/.0;
22 #define MP make_pair
23 #define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j
24
25 struct LPSolver {
26     int m, n;
27     vector<int> N, B;
28     vvd D;
29
30     LPSolver(const vvd& A, const vd& b, const vd& c) :
31         m(b.size()), n(c.size()), N(n+1), B(m), D(m+2, vd(n+2)) {
32         for(int i = 0; i < m; i++){
33             for(int j = 0; j < n; j++){
34                 D[i][j] = A[i][j];
35             }
36         }
37         for(int i = 0; i < m; i++){
38             B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];
39         }
40         for(int j = 0; j < n; j++){
41             N[j] = j; D[m][j] = -c[j];
42         }

```

```

43         N[n] = -1; D[m+1][n] = 1;
44     }
45
46 void pivot(int r, int s) {
47     T *a = D[r].data(), inv = 1 / a[s];
48     for(int i = 0; i < m+2; i++){
49         if (i != r && abs(D[i][s]) > eps) {
50             T *b = D[i].data(), inv2 = b[s] * inv;
51             for(int j = 0; j < n+2; j++){
52                 b[j] -= a[j] * inv2;
53             }
54             b[s] = a[s] * inv2;
55         }
56     }
57     for(int j = 0; j < n+2; j++){
58         if (j != s) D[r][j] *= inv;
59     }
60     for(int i = 0; i < m+2; i++){
61         if (i != r) D[i][s] *= -inv;
62     }
63     D[r][s] = inv;
64     swap(B[r], N[s]);
65 }
66
67 bool simplex(int phase) {
68     int x = m + phase - 1;
69     for (;;) {
70         int s = -1;
71         for(int j = 0; j < n+1; j++){
72             if (N[j] != -phase) ltj(D[x]);
73         }
74         if (D[x][s] >= -eps) return true;
75         int r = -1;
76         for(int i = 0; i < m; i++){
77             if (D[i][s] <= eps) continue;
78             if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
79                 < MP(D[r][n+1] / D[r][s], B[r])) r = i;
80         }
81         if (r == -1) return false;
82         pivot(r, s);
83     }
84 }
85

```

```

86     T solve(vd &x) {
87         int r = 0;
88         for(int i = 1; i < m; i++){
89             if (D[i][n+1] < D[r][n+1]) r = i;
90         }
91         if (D[r][n+1] < -eps) {
92             pivot(r, n);
93             if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
94             for(int i = 0; i < m; i++) if (B[i] == -1) {
95                 int s = 0;
96                 for(int j = 1; j < n+1; j++){
97                     ltj(D[i]);
98                 }
99                 pivot(i, s);
100             }
101         }
102         bool ok = simplex(1); x = vd(n);
103         for(int i = 0; i < m; i++){
104             if (B[i] < n) x[B[i]] = D[i][n+1];
105         }
106         return ok ? D[m][n+1] : inf;
107     }
108 };

```

## 6.22 区间互质.cpp

```

1  int p[N / 5], num;
2  void prime(int n) {
3      num = 0;
4      for (int i = 2; i * i <= n; i++) {
5          if ((n % i) == 0) {
6              p[++num] = i;
7              while ((n % i) == 0) n /= i;
8          }
9      }
10     if (n > 1) p[++num] = n;
11 }
12 ll solve(ll r, int k) {
13     prime(k);
14     ll res = 0;
15     for (int i = 1; i < (1 << num); i++) {
16         int k = 0;
17         ll div = 1;

```

```

18         for (int j = 1; j <= num; j++) {
19             if (i & (1 << (j - 1))) {
20                 k++;
21                 div *= p[j];
22             }
23         }
24         if (k % 2)
25             res += r / div;
26         else
27             res -= r / div;
28     }
29     return r - res;
30 }
31 ll que(ll L, ll R, ll k) {
32     return solve(R, k) - solve(L - 1, k);
33 }

```

## 6.23 幂转下降幂 (求幂和).cpp

```

1  ll comb[N][N];
2  ll s[maxn], inv[maxn], p;
3  //  $1^k + 2^k + \dots + n^k$ 
4  void solve() {
5      cin >> k >> n >> p;
6      rep(i, 0, k + 1) {
7          comb[i][0] = comb[i][i] = 1;
8          rep(j, 1, i - 1) {
9              comb[i][j] = (comb[i - 1][j - 1] + comb[i - 1][j]) % p;
10             }
11         }
12     inv[1] = 1;
13     rep(i, 2, k + 1) inv[i] = (p - p / i) * inv[p % i] % p;
14     assert(inv[k] * k % p == 1);
15
16     ll pw = 1;
17     //  $(k+1) * S[k] = (n+1)^{(k+1)} - [0-k-1](k+1, j) * S[j] - 1$ 
18     rep(i, 0, k) {
19         pw = pw * (n + 1) % p;
20         s[i] = (pw - 1 + p) % p;
21         rep(j, 0, i - 1) {
22             s[i] = (s[i] - comb[i + 1][j] * s[j] % p + p) % p;
23         }
24         s[i] = s[i] * inv[i + 1] % p;

```



```

25     }
26     cout << s[k] << '\n';
27 }

```

## 6.24 扩展欧拉定理.cpp

```

1  // mod {min(b, b % phi + phi)}
2  ll calc(ll p) {
3      if (p == 1) return 0;
4      int phi = p, q = p;
5      for (int i = 2; i * i <= p; i++) {
6          if (q % i == 0) {
7              phi = phi / i * (i - 1);
8              while (q % i == 0) q /= i;
9          }
10     }
11     if (q != 1) phi = phi / q * (q - 1);
12     return powmod(2, calc(phi) + phi, p);
13 }

```

## 6.25 拉格朗日插值.cpp

```

1  // k阶多项式(需要k+1个点)
2  // 求在点n上的值
3  // O(k)
4  ll lagrange(ll n, int k) {
5      vector<ll> x(k + 5), y(k + 5);
6      rep(i, 1, k + 1) {
7          x[i] = i;
8          // y[i]=(y[i-1]+powmod(i,k-1,mod))%mod;
9      }
10     if (n <= k + 1) return y[n];
11
12     vector<ll> fac(k + 5);
13     fac[0] = 1;
14     ll coe = 1;
15     rep(i, 1, k + 4) fac[i] = fac[i - 1] * i % mod;
16     rep(i, 1, k + 1) coe = coe * (n - i + mod) % mod;
17     ll ans = 0;
18     rep(i, 1, k + 1) {
19         ll sgn = (((k + 1 - i) % 2) ? -1 : 1);
20         ll f1 = powmod(fac[i - 1] * fac[k + 1 - i] % mod, mod - 2, mod);
21         ll f2 = powmod(n - i, mod - 2, mod);

```

```

22         ans += sgn * coe * f1 % mod * f2 % mod * y[i] % mod;
23         ans = (ans + mod) % mod;
24     }
25     return ans;
26 }

```

## 6.26 整除分块.cpp

```

1  void solve() {
2      u64 ans = 0;
3      cin >> n;
4      for (ll l = 1; l <= n; l++) {
5          ll d = n / l, r = n / d;
6          ans += (l + r) * (r - l + 1) / 2 * d;
7          l = r;
8      }
9  }

```

## 6.27 枚举子集.cpp

```

1  void solve() {
2      f[0] = 1;
3      for (int i = 1; i < (1ll << n); i++) {
4          int t = i;
5          ll res = 0;
6          while (true) {
7              if (t == 0) break;
8              t = (t - 1) & i;
9              res = (res + f[t]) % mod;
10         }
11         f[i] = res * i;
12     }
13 }

```

## 6.28 枚举超集.cpp

```

1  void solve() {
2      for (int i = 1; i < (1ll << n); i++) {
3          int t = i;
4          while (true) {
5              t = (t + 1) | i;
6              if (t == bit(n) - 1) break;
7          }

```

```

8     }
9 }

```

## 6.29 狄利克雷卷积.cpp

```

1  const int N = 1010000;
2  int p[N], pr[N / 5], n, tot;
3  unsigned int A, B, C, mu[N], f[N], g[N];
4
5  inline unsigned int rng61() {
6      A ^= A << 16;
7      A ^= A >> 5;
8      A ^= A << 1;
9      unsigned int t = A;
10     A = B;
11     B = C;
12     C ^= t ^ A;
13     return C;
14 }
15
16 int main() {
17     scanf("%d%u%u%u", &n, &A, &B, &C);
18     for (int i = 1; i <= n; i++)
19         f[i] = rng61();
20
21     p[1] = 1; mu[1] = 1;
22     for (int i = 2; i <= n; i++) {
23         if (!p[i]) p[i] = i, mu[i] = (uint)-1, pr[++tot] = i;
24         for (int j = 1; j <= tot && pr[j] * i <= n; j++) {
25             p[i * pr[j]] = pr[j];
26             if (p[i] == pr[j]) {
27                 mu[i * pr[j]] = 0;
28                 break;
29             } else {
30                 mu[i * pr[j]] = (uint)-mu[i];
31             }
32         }
33     }
34     for (int d1 = 1; d1 <= n; d1++)
35         for (int d2 = 1; d1 * d2 <= n; d2++)
36             g[d1 * d2] += f[d1] * mu[d2];
37     uint ans = 0;
38     for (int i = 1; i <= n; i++) ans ^= g[i];

```

```

39     printf("%u\n", ans);
40 }

```

## 6.30 线性筛常见积性函数.cpp

```

1  const int N = 20010000;
2  int p[N], pr[N / 5], n, pe[N], tot;
3  uint f[N], a, b, ans;
4
5  void compute(int n, function<void(int)> calcpe) {
6      ans = 0;
7      f[1] = 1;
8      for (int i = 2; i <= n; i++) {
9          if (i == pe[i])
10             calcpe(i);
11         else
12             f[i] = f[pe[i]] * f[i / pe[i]];
13     }
14     for (uint i = 1; i <= n; i++) {
15         ans ^= (a * i * f[i] + b);
16     }
17     printf("%u\n", ans);
18 }
19
20 int main() {
21     scanf("%d%u%u", &n, &a, &b);
22     p[1] = 1;
23     for (int i = 2; i <= n; i++) {
24         if (!p[i]) p[i] = i, pe[i] = i, pr[++tot] = i;
25         for (int j = 1; j <= tot && pr[j] * i <= n; j++) {
26             p[i * pr[j]] = pr[j];
27             if (p[i] == pr[j]) {
28                 pe[i * pr[j]] = pe[i] * pr[j];
29                 break;
30             } else {
31                 pe[i * pr[j]] = pr[j];
32             }
33         }
34     }
35     // 因子个数, 因子和, 欧拉函数, 莫比乌斯函数
36     compute(n, [&](int x) {
37         f[x] = f[x / p[x]] + 1;
38     });

```

```

39
40     compute(n, [&](int x) {
41         f[x] = f[x / p[x]] + x;
42     });
43
44     compute(n, [&](int x) {
45         f[x] = x / p[x] * (p[x] - 1);
46     });
47
48     compute(n, [&](int x) {
49         f[x] = x == p[x] ? -1 : 0;
50     });
51 }

```

### 6.31 莫比乌斯反演 gcd 常见结论.cpp

```

1 // u * 1 = e, phi * 1 = id, phi = id * u
2 const int N = 10100000, M = 10000000;
3 int p[N], pr[N / 5], n, tot;
4 int mu[N], smu[N];
5
6 int main() {
7     p[1] = 1; mu[1] = 1;
8     for (int i = 2; i <= M; i++) {
9         if (!p[i]) p[i] = i, mu[i] = -1, pr[++tot] = i;
10        for (int j = 1; j <= tot && pr[j] * i <= M; j++) {
11            p[i * pr[j]] = pr[j];
12            if (p[i] == pr[j]) {
13                mu[i * pr[j]] = 0;
14                break;
15            } else {
16                mu[i * pr[j]] = -mu[i];
17            }
18        }
19    }
20    for (int i = 1; i <= M; i++)
21        smu[i] = smu[i - 1] + mu[i];
22    int T;
23    scanf("%d", &T);
24    for (int tc = 0; tc < T; tc++) {
25        int n, m;
26        scanf("%d%d", &n, &m);
27        if (n > m) swap(n, m);

```

```

28        ll ans = 0;
29        for (int l = 1; l <= n; l++) {
30            int n1 = n / l, m1 = m / l;
31            int r = min(n / n1, m / m1);
32            // l ... r
33            ans += 1ll * (smu[r] - smu[l - 1]) * n1 * m1;
34            l = r;
35        }
36        printf("%lld\n", ans);
37    }
38 }

```

## 7 String

### 7.1 ACAM.cpp

```

1 const int AC_SIGMA = 26, AC_V = 26, AC_N = 810000;
2 struct AC_automaton {
3     struct node {
4         node *go[AC_V], *fail, *f;
5     } // declare extra variables:
6     pool[AC_N], *cur, *root, *q[AC_N];
7     node* newnode() {
8         node *p = cur++;
9     } // init extra variables:
10    return p;
11 }
12 // CALL init() and CHECK all const variables:
13 void init() { cur = pool; root = newnode(); }
14 node* append(node *p, int w) {
15     if (!p->go[w]) p->go[w] = newnode(), p->go[w]->f = p;
16     return p->go[w];
17 }
18 void build() {
19     int t = 0;
20     q[t++] = root;
21     root->fail = root;
22     rep(i, 0, AC_SIGMA - 1) if (root->go[i]) {
23         q[t++] = root->go[i];
24         root->go[i]->fail = root;
25     } else {
26         root->go[i] = root;

```

```

27     }
28     rep(i, 1, t - 1) {
29         node *u = q[i];
30         rep(j, 0, AC_SIGMA - 1) if (u->go[j]) {
31             u->go[j]->fail = u->fail->go[j];
32             q[t++] = u->go[j];
33         } else {
34             u->go[j] = u->fail->go[j];
35         }
36     }
37 }
38 } ac;
39 typedef AC_automaton::node ACnode;
40
41 const int M = 2, N = 2.1e5;
42 struct node {
43     node *son[M], *go[M], *fail;
44     int cnt, vis, ins;
45 } pool[N], *cur = pool, *q[N], *root;
46
47 node *newnode() { return cur++; }
48 int t, n;
49
50 void build() {
51     t = 0;
52     q[t++] = root;
53     for (int i = 0; i < t; i++) {
54         node *u = q[i];
55         for (int j = 0; j < M; j++) {
56             if (u->son[j]) {
57                 u->go[j] = u->son[j];
58                 if (u != root)
59                     u->go[j]->fail = u->fail->go[j];
60             } else
61                 u->go[j]->fail = root;
62             q[t++] = u->son[j];
63         } else {
64             if (u != root)
65                 u->go[j] = u->fail->go[j];
66             else
67                 u->go[j] = root;
68         }
69     }

```

```

70     }
71 }
72
73 void insert(string &s) {
74     node *cur = root;
75     for (auto c : s) {
76         int w = c - '0';
77         if (!cur->son[w]) {
78             cur->son[w] = newnode();
79         }
80         cur = cur->son[w];
81     }
82     cur->cnt = 1;
83 }

```

## 7.2 hash61.cpp

```

1 struct hash61 {
2     static const uint64_t md = (1LL << 61) - 1;
3     static uint64_t step;
4     static vector<uint64_t> pw;
5
6     uint64_t addmod(uint64_t a, uint64_t b) const {
7         a += b;
8         if (a >= md) a -= md;
9         return a;
10    }
11
12    uint64_t submod(uint64_t a, uint64_t b) const {
13        a += md - b;
14        if (a >= md) a -= md;
15        return a;
16    }
17
18    uint64_t mulmod(uint64_t a, uint64_t b) const {
19        uint64_t l1 = (uint32_t) a, h1 = a >> 32, l2 = (uint32_t) b, h2 = b >>
20            32;
21        uint64_t l = l1 * l2, m = l1 * h2 + l2 * h1, h = h1 * h2;
22        uint64_t ret = (l & md) + (l >> 61) + (h << 3) + (m >> 29) + (m << 35 >>
23            3) + 1;
24        ret = (ret & md) + (ret >> 61);
25        ret = (ret & md) + (ret >> 61);
26        return ret - 1;

```

```

25     }
26
27     void ensure_pw(int sz) {
28         int cur = (int) pw.size();
29         if (cur < sz) {
30             pw.resize(sz);
31             for (int i = cur; i < sz; i++) {
32                 pw[i] = mulmod(pw[i - 1], step);
33             }
34         }
35     }
36
37     vector<uint64_t> pref;
38     int n;
39
40     template<typename T>
41     hash61(const T& s) {
42         n = (int) s.size();
43         ensure_pw(n + 1);
44         pref.resize(n + 1);
45         pref[0] = 1;
46         for (int i = 0; i < n; i++) {
47             pref[i + 1] = addmod(mulmod(pref[i], step), s[i]);
48         }
49     }
50
51     inline uint64_t operator()(const int from, const int to) const {
52         assert(0 <= from && from <= to && to <= n - 1);
53         return submod(pref[to + 1], mulmod(pref[from], pw[to - from + 1]));
54     }
55 };
56
57 uint64_t hash61::step = (md >> 2) + rng() % (md >> 1);
58 vector<uint64_t> hash61::pw = vector<uint64_t>(1, 1);

```

### 7.3 kmp.cpp

```

1 template <typename T>
2 vector<int> kmp_table(int n, const T &s) {
3     vector<int> p(n, 0);
4     int k = 0;
5     for (int i = 1; i < n; i++) {
6         while (k > 0 && !(s[i] == s[k])) {

```

```

7         k = p[k - 1];
8     }
9     if (s[i] == s[k]) {
10         k++;
11     }
12     p[i] = k;
13 }
14 return p;
15 }
16
17 template <typename T>
18 vector<int> kmp_table(const T &s) {
19     return kmp_table((int) s.size(), s);
20 }
21
22 template <typename T>
23 vector<int> kmp_search(int n, const T &s, int m, const T &w, const vector<
24     int> &p) {
25     assert(n >= 1 && (int) p.size() == n);
26     vector<int> res;
27     int k = 0;
28     for (int i = 0; i < m; i++) {
29         while (k > 0 && (k == n || !(w[i] == s[k]))) {
30             k = p[k - 1];
31         }
32         if (w[i] == s[k]) {
33             k++;
34         }
35         if (k == n) {
36             res.push_back(i - n + 1);
37         }
38     }
39     return res;
40 }
41
42 // returns 0-indexed positions of occurrences of s in w
43
44 template <typename T>
45 vector<int> kmp_search(const T &s, const T &w, const vector<int> &p) {
46     return kmp_search((int) s.size(), s, (int) w.size(), w, p);
47 }

```

### 7.4 manacherfast.cpp

---

```

1  template <typename T>
2  vector<int> manacher(int n, const T &s) {
3      if (n == 0) {
4          return vector<int>();
5      }
6      vector<int> res(2 * n - 1, 0);
7      int l = -1, r = -1;
8      for (int z = 0; z < 2 * n - 1; z++) {
9          int i = (z + 1) >> 1;
10         int j = z >> 1;
11         int p = (i >= r ? 0 : min(r - i, res[2 * (l + r) - z]));
12         while (j + p + 1 < n && i - p - 1 >= 0) {
13             if (!(s[j + p + 1] == s[i - p - 1])) {
14                 break;
15             }
16             p++;
17         }
18         if (j + p > r) {
19             l = i - p;
20             r = j + p;
21         }
22         res[z] = p;
23     }
24     return res;
25     // res[2 * i] = odd radius in position i
26     // res[2 * i + 1] = even radius between positions i and i + 1
27     // s = "abaa" -> res = {0, 0, 1, 0, 0, 1, 0}
28     // in other words, for every z from 0 to 2 * n - 2:
29     // calculate i = (z + 1) >> 1 and j = z >> 1
30     // now there is a palindrome from i - res[z] to j + res[z]
31     // (watch out for i > j and res[z] = 0)
32 }
33
34 template <typename T>
35 vector<int> manacher(const T &s) {
36     return manacher((int) s.size(), s);
37 }

```

## 7.5 MinRotation.cpp

---

```

1  Description: Finds the lexicographically smallest rotation of a string.
2  Usage: rotate(v.begin(), v.begin() + minRotation(v), v.end());

```

```

3  Time: O(N)
4
5  int minRotation(string s) {
6      int a = 0, N = sz(s); s += s;
7      rep(b, 0, N) rep(k, 0, N) {
8          if (a + k == b || s[a + k] < s[b + k]) {b += max(0, k - 1); break;}
9          if (s[a + k] > s[b + k]) { a = b; break; }
10     }
11     return a;
12 }

```

## 7.6 PAM.cpp

---

```

1  struct PAM {
2      struct T {
3          array<int, 10> tr;
4          int fail, len, tag;
5          T() : fail(0), len(0), tag(0) {
6              tr.fill(0);
7          }
8      };
9      vector<T> t;
10     vector<int> stk;
11     int newnode(int len) {
12         t.emplace_back();
13         t.back().len = len;
14         return (int)t.size() - 1;
15     }
16     PAM() : t(2) {
17         t[0].fail = 1, t[0].len = 0;
18         t[1].fail = 0, t[1].len = -1;
19         stk.push_back(-1);
20     }
21     int getfail(int v) {
22         while (stk.end()[-2 - t[v].len] != stk.back()) {
23             v = t[v].fail;
24         }
25         return v;
26     }
27     int insert(int lst, int c, int td) {
28         stk.emplace_back(c);
29         int x = getfail(lst);
30         if (!t[x].tr[c]) {

```

```

31         int u = newnode(t[x].len + 2);
32         t[u].fail = t[getfail(t[x].fail)].tr[c];
33         t[x].tr[c] = u;
34     }
35     t[t[x].tr[c]].tag += td;
36     return t[x].tr[c];
37 }
38 int build(int n) {
39     int ans = 0;
40     for (int i = (int)t.size() - 1; i > 1; i--) {
41         t[t[i].fail].tag += t[i].tag;
42         if (t[i].len > n) {
43             continue;
44         }
45         ans = (ans + 1ll * t[i].tag * t[i].tag % M * t[i].len) % M;
46     }
47     return ans;
48 }
49 };

```

## 7.7 rollingHash.cpp

```

1  typedef pair<int,int> hashv;
2  const ll mod1=1000000007;
3  const ll mod2=1000000009;
4
5  // prefixSum trick for high dimensions
6
7  hashv operator + (hashv a,hashv b) {
8      int c1=a.fi+b.fi,c2=a.se+b.se;
9      if (c1>=mod1) c1-=mod1;
10     if (c2>=mod2) c2-=mod2;
11     return mp(c1,c2);
12 }
13
14 hashv operator - (hashv a,hashv b) {
15     int c1=a.fi-b.fi,c2=a.se-b.se;
16     if (c1<0) c1+=mod1;
17     if (c2<0) c2+=mod2;
18     return mp(c1,c2);
19 }
20
21 hashv operator * (hashv a,hashv b) {

```

```

22     return mp(1ll*a.fi*b.fi%mod1,1ll*a.se*b.se%mod2);
23 }

```

## 7.8 SA.cpp

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int N = 101000;
5  char s[N];
6  int sa[N], rk[N], ht[N], n;
7  // 0-based sa 表示第 i 大的为哪个, rk 表示第 i 个后缀第几大
8  // ht 表示 lcp(sa[i], sa[i-1])
9  void buildSA(char *s, int *sa, int *rk, int *ht, int n, int m = 128) {
10     static int x[N], y[N], c[N];
11     s[n] = 0;
12     for (int i = 0; i < m; i++) c[i] = 0;
13     for (int i = 0; i < n; i++) c[x[i] = s[i]]++;
14     for (int i = 1; i < m; i++) c[i] += c[i - 1];
15     for (int i = n - 1; i >= 0; i--) sa[--c[x[i]]] = i;
16     for (int k = 1; k < n; k <= 1) {
17         int p=0;
18         for (int i = n - 1; i >= n - k; i--) y[p++] = i;
19         for (int i = 0; i < n; i++) if (sa[i] >= k) y[p++] = sa[i] - k;
20         for (int i = 0; i < m; i++) c[i] = 0;
21         for (int i = 0; i < n; i++) c[x[y[i]]]++;
22         for (int i = 1; i < m; i++) c[i] += c[i - 1];
23         for (int i = n - 1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
24         swap(x, y);
25         p = 1; x[sa[0]] = 0; y[n] = -1;
26         for (int i = 1; i < n; i++) {
27             if (y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k]
28                 )
29                 x[sa[i]] = p - 1;
30             else
31                 x[sa[i]] = p++;
32         }
33         if (p == n) break;
34         m = p;
35     }
36     for (int i = 0; i < n; i++) rk[sa[i]] = i;
37     int k = 0;
38     for (int i = 0; i < n; i++) {

```

```

38     k = max(k - 1, 0);
39     if (rk[i] == 0) continue;
40     int j = sa[rk[i] - 1];
41     while (s[i + k] == s[j + k]) k++;
42     ht[rk[i]] = k;
43 }
44 }
45
46 int LCP(int u, int v) {
47     if (u == v) return n - u;
48     if (rk[u] > rk[v]) swap(u, v);
49     // RMQ(ht, rk[u] + 1, rk[v])
50 }
51
52 int main() {
53     scanf("%s", s);
54     n = strlen(s);
55     buildSA(s, sa, rk, ht, n);
56     for (int i = 0; i < n; i++) printf("%d_", sa[i] + 1); puts("");
57     for (int i = 1; i < n; i++) printf("%d_", ht[i]); puts("");
58 }

```

## 7.9 SAfast.cpp

```

1  template <typename T>
2  vector<int> suffix_array(int n, const T &s, int char_bound) {
3      vector<int> a(n);
4      if (n == 0) {
5          return a;
6      }
7      if (char_bound != -1) {
8          vector<int> aux(char_bound, 0);
9          for (int i = 0; i < n; i++) {
10             aux[s[i]]++;
11         }
12         int sum = 0;
13         for (int i = 0; i < char_bound; i++) {
14             int add = aux[i];
15             aux[i] = sum;
16             sum += add;
17         }
18         for (int i = 0; i < n; i++) {
19             a[aux[s[i]]++] = i;

```

```

20     }
21 } else {
22     iota(a.begin(), a.end(), 0);
23     sort(a.begin(), a.end(), [&s](int i, int j) { return s[i] < s[j]; });
24 }
25 vector<int> sorted_by_second(n);
26 vector<int> ptr_group(n);
27 vector<int> new_group(n);
28 vector<int> group(n);
29 group[a[0]] = 0;
30 for (int i = 1; i < n; i++) {
31     group[a[i]] = group[a[i - 1]] + (!s[a[i]] == s[a[i - 1]]));
32 }
33 int cnt = group[a[n - 1]] + 1;
34 int step = 1;
35 while (cnt < n) {
36     int at = 0;
37     for (int i = n - step; i < n; i++) {
38         sorted_by_second[at++] = i;
39     }
40     for (int i = 0; i < n; i++) {
41         if (a[i] - step >= 0) {
42             sorted_by_second[at++] = a[i] - step;
43         }
44     }
45     for (int i = n - 1; i >= 0; i--) {
46         ptr_group[group[a[i]]] = i;
47     }
48     for (int i = 0; i < n; i++) {
49         int x = sorted_by_second[i];
50         a[ptr_group[group[x]]++] = x;
51     }
52     new_group[a[0]] = 0;
53     for (int i = 1; i < n; i++) {
54         if (group[a[i]] != group[a[i - 1]]) {
55             new_group[a[i]] = new_group[a[i - 1]] + 1;
56         } else {
57             int pre = (a[i - 1] + step >= n ? -1 : group[a[i - 1] + step]);
58             int cur = (a[i] + step >= n ? -1 : group[a[i] + step]);
59             new_group[a[i]] = new_group[a[i - 1]] + (pre != cur);
60         }
61     }
62     swap(group, new_group);

```



```

63     cnt = group[a[n - 1]] + 1;
64     step <= 1;
65 }
66 return a;
67 }
68
69 template <typename T>
70 vector<int> suffix_array(const T &s, int char_bound) {
71     return suffix_array((int) s.size(), s, char_bound);
72 }
73
74 template <typename T>
75 vector<int> build_lcp(int n, const T &s, const vector<int> &sa) {
76     assert((int) sa.size() == n);
77     vector<int> pos(n);
78     for (int i = 0; i < n; i++) {
79         pos[sa[i]] = i;
80     }
81     vector<int> lcp(max(n - 1, 0));
82     int k = 0;
83     for (int i = 0; i < n; i++) {
84         k = max(k - 1, 0);
85         if (pos[i] == n - 1) {
86             k = 0;
87         } else {
88             int j = sa[pos[i] + 1];
89             while (i + k < n && j + k < n && s[i + k] == s[j + k]) {
90                 k++;
91             }
92             lcp[pos[i]] = k;
93         }
94     }
95     return lcp;
96 }
97
98 template <typename T>
99 vector<int> build_lcp(const T &s, const vector<int> &sa) {
100     return build_lcp((int) s.size(), s, sa);
101 }

```

## 7.10 SAM.cpp

```

1 struct SAM {

```

```

2     static constexpr int ALPHABET_SIZE = 26;
3     struct Node {
4         int len;
5         int link;
6         std::array<int, ALPHABET_SIZE> next;
7         Node() : len{}, link{}, next{} {}
8     };
9     std::vector<Node> t;
10    SAM() {
11        init();
12    }
13    void init() {
14        t.assign(2, Node());
15        t[0].next.fill(1);
16        t[0].len = -1;
17    }
18    int newNode() {
19        t.emplace_back();
20        return t.size() - 1;
21    }
22    int extend(int p, int c) {
23        if (t[p].next[c]) {
24            int q = t[p].next[c];
25            if (t[q].len == t[p].len + 1) {
26                return q;
27            }
28            int r = newNode();
29            t[r].len = t[p].len + 1;
30            t[r].link = t[q].link;
31            t[r].next = t[q].next;
32            t[q].link = r;
33            while (t[p].next[c] == q) {
34                t[p].next[c] = r;
35                p = t[p].link;
36            }
37            return r;
38        }
39        int cur = newNode();
40        t[cur].len = t[p].len + 1;
41        while (!t[p].next[c]) {
42            t[p].next[c] = cur;
43            p = t[p].link;
44        }

```

```

45         t[cur].link = extend(p, c);
46         return cur;
47     }
48 };

```

## 7.11 SA-IS.cpp

```

1  /*
2   * Time Complexity: Suffix Array:  $O(N + \text{Character\_Set\_Size})$  time and space
3   *
4   * 128 --- ASCII
5   *
6   * LCP:  $O(N)$  time and space
7   *
8   * Usage:
9   *
10  * 1. Suffix Array (returns s.size() elements, NOT considering
11  * 0-length/empty suffix)
12  *
13  * auto sa = suffix_array(s); // s is the input string with
14  * ASCII
15  * characters
16  *
17  * auto sa_wide_char = suffix_array(s, LIM); // LIM = max(s[i])
18  * + 2,
19  * s is the string with arbitrary big characters.
20  *
21  * 2. LCP:
22  *
23  * auto lcp = LCP(s, suffix_array(s)); // returns s.size()
24  * elements,
25  * where lcp[i]=LCP(sa[i], sa[i+1])
26  *
27  * Status: Tested (DMOJ: ccc03s4, SPOJ: SARRAY (100pts), Yosupo's: Suffix
28  * Array
29  *
30  * & Number of Substrings, CodeForces EDU
31  */
32
33 // Based on: Rickypon, https://judge.yosupo.jp/submission/10105
34 void induced_sort(const std::vector<int>& vec, int val_range,
35                  std::vector<int>& SA, const std::vector<bool>& sl,
36                  const std::vector<int>& lms_idx) {
37     std::vector<int> l(val_range, 0), r(val_range, 0);
38     for (int c : vec) {
39         if (c + 1 < val_range) ++l[c + 1];
40         ++r[c];
41     }
42     std::partial_sum(l.begin(), l.end(), l.begin());
43     std::partial_sum(r.begin(), r.end(), r.begin());
44     std::fill(SA.begin(), SA.end(), -1);
45     for (int i = (int)lms_idx.size() - 1; i >= 0; --i)
46         SA[--r[vec[lms_idx[i]]]] = lms_idx[i];

```

```

32     for (int i : SA)
33         if (i >= 1 && sl[i - 1]) SA[l[vec[i - 1]]++] = i - 1;
34     std::fill(r.begin(), r.end(), 0);
35     for (int c : vec) ++r[c];
36     std::partial_sum(r.begin(), r.end(), r.begin());
37     for (int k = (int)SA.size() - 1, i = SA[k]; k >= 1; --k, i = SA[k])
38         if (i >= 1 && !sl[i - 1]) {
39             SA[--r[vec[i - 1]]] = i - 1;
40         }
41 }
42
43 std::vector<int> SA_IS(const std::vector<int>& vec, int val_range) {
44     const int n = vec.size();
45     std::vector<int> SA(n), lms_idx;
46     std::vector<bool> sl(n);
47     sl[n - 1] = false;
48     for (int i = n - 2; i >= 0; --i) {
49         sl[i] = (vec[i] > vec[i + 1] || (vec[i] == vec[i + 1] && sl[i + 1]));
50         ;
51         if (sl[i] && !sl[i + 1]) lms_idx.push_back(i + 1);
52     }
53     std::reverse(lms_idx.begin(), lms_idx.end());
54     induced_sort(vec, val_range, SA, sl, lms_idx);
55     std::vector<int> new_lms_idx(lms_idx.size()), lms_vec(lms_idx.size());
56     for (int i = 0, k = 0; i < n; ++i)
57         if (!sl[SA[i]] && SA[i] >= 1 && sl[SA[i] - 1]) {
58             new_lms_idx[k++] = SA[i];
59         }
60     int cur = 0;
61     SA[n - 1] = cur;
62     for (size_t k = 1; k < new_lms_idx.size(); ++k) {
63         int i = new_lms_idx[k - 1], j = new_lms_idx[k];
64         if (vec[i] != vec[j]) {
65             SA[j] = ++cur;
66             continue;
67         }
68         bool flag = false;
69         for (int a = i + 1, b = j + 1;; ++a, ++b) {
70             if (vec[a] != vec[b]) {
71                 flag = true;
72                 break;
73             }
74             if ((!sl[a] && sl[a - 1]) || (!sl[b] && sl[b - 1])) {

```

```

74         flag = !((!sl[a] && sl[a - 1]) && (!sl[b] && sl[b - 1]));
75         break;
76     }
77 }
78 SA[j] = (flag ? ++cur : cur);
79 }
80 for (size_t i = 0; i < lms_idx.size(); ++i) lms_vec[i] = SA[lms_idx[i]];
81 if (cur + 1 < (int)lms_idx.size()) {
82     auto lms_SA = SA_IS(lms_vec, cur + 1);
83     for (size_t i = 0; i < lms_idx.size(); ++i) {
84         new_lms_idx[i] = lms_idx[lms_SA[i]];
85     }
86 }
87 induced_sort(vec, val_range, SA, sl, new_lms_idx);
88 return SA;
89 }
90
91 std::vector<int> suffix_array(const std::string& s, const char first = 'a',
92                             const char last = 'z') {
93     std::vector<int> vec(s.size() + 1);
94     std::copy(std::begin(s), std::end(s), std::begin(vec));
95     for (auto& x : vec) x -= (int)first - 1;
96     vec.back() = 0;
97     auto ret = SA_IS(vec, (int)last - (int)first + 2);
98     ret.erase(ret.begin());
99     return ret;
100 }
101 // Author: https://codeforces.com/blog/entry/12796?#comment-175287
102 // Uses kasai's algorithm linear in time and space
103 std::vector<int> LCP(const std::string& s, const std::vector<int>& sa) {
104     int n = s.size(), k = 0;
105     std::vector<int> lcp(n), rank(n);
106     for (int i = 0; i < n; i++) rank[sa[i]] = i;
107     for (int i = 0; i < n; i++, k ? k-- : 0) {
108         if (rank[i] == n - 1) {
109             k = 0;
110             continue;
111         }
112         int j = sa[rank[i] + 1];
113         while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
114         lcp[rank[i]] = k;
115     }
116     lcp[n - 1] = 0;

```

```

117     return lcp;
118 }
119
120 template <typename T, class F = function<T(const T&, const T&)>>
121 class SparseTable {
122 public:
123     int n;
124     vector<vector<T>> mat;
125     F func;
126
127     SparseTable(const vector<T>& a, const F& f) : func(f) {
128         n = static_cast<int>(a.size());
129         int max_log = 32 - __builtin_clz(n);
130         mat.resize(max_log);
131         mat[0] = a;
132         for (int j = 1; j < max_log; j++) {
133             mat[j].resize(n - (1 << j) + 1);
134             for (int i = 0; i <= n - (1 << j); i++) {
135                 mat[j][i] = func(mat[j - 1][i], mat[j - 1][i + (1 << (j - 1))]);
136             }
137         }
138     }
139
140     T get(int from, int to) const {
141         assert(0 <= from && from <= to && to <= n - 1);
142         int lg = 32 - __builtin_clz(to - from + 1) - 1;
143         return func(mat[lg][from], mat[lg][to - (1 << lg) + 1]);
144     }
145 };

```

## 7.12 Z.cpp

```

1 template <typename T>
2 vector<int> z_function(int n, const T &s) {
3     vector<int> z(n, n);
4     int l = 0, r = 0;
5     for (int i = 1; i < n; i++) {
6         z[i] = (i > r ? 0 : min(r - i + 1, z[i - l]));
7         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
8             z[i]++;
9         }
10        if (i + z[i] - 1 > r) {
11            l = i;

```

```

12     r = i + z[i] - 1;
13 }
14 }
15 return z;
16 }
17
18 template <typename T>
19 vector<int> z_function(const T &s) {
20     return z_function((int) s.size(), s);
21 }

```

## 8 Basic

### 8.1 AST.py

---

```

1 class Solution:
2     def calculate(self, s: str) -> int:
3         sign = ['+', '-', '*', '/', '(', ')']
4         v = []
5         num = ''
6         for c in s:
7             if c in sign:
8                 if num:
9                     v.append(num)
10                    num = ''
11                if c == '-' and (not v or v[-1] == '('):
12                    v.append('0')
13                v.append(c)
14            elif c.isnumeric():
15                num += c
16        if num:
17            v.append(num)
18
19        stk0 = []
20        stk1 = []
21        for e in v:
22            if e.isnumeric():
23                stk0.append(e)
24            elif e in ['+', '-']:
25                while stk1 and stk1[-1] in ['*', '/', '+', '-']:
26                    stk0.append(stk1.pop())
27                stk1.append(e)

```

```

28            elif e in ['*', '/', '(':
29                stk1.append(e)
30            else:
31                while stk1 and stk1[-1] != '(':
32                    stk0.append(stk1.pop())
33                    stk1.pop()
34            while stk1:
35                stk0.append(stk1.pop())
36
37        res = []
38        for e in stk0:
39            if e.isnumeric():
40                res.append(int(e))
41            else:
42                v = res.pop()
43                u = res.pop()
44                if e == '+':
45                    res.append(u + v)
46                if e == '-':
47                    res.append(u - v)
48                if e == '*':
49                    res.append(u * v)
50                if e == '/':
51                    res.append(u // v)
52        return res[0]

```

### 8.2 bitset.cpp

---

```

1 template <int len = 1>
2 void solve(int n) {
3     if (n > len) {
4         solve<std::min(len*2, MAXLEN)>(n);
5         return;
6     }
7     // solution using bitset<len>
8 }
9
10 struct Bitset {
11     vector<ull> b;
12     int n;
13     Bitset(int x = 0) {
14         n = x;
15         b.resize((n + 63) / 64, 0);

```

```

16     }
17
18     int get(int x) {
19         return (b[x >> 6] >> (x & 63)) & 1;
20     }
21
22     void set(int x, int y) {
23         b[x >> 6] |= 1ULL << (x & 63);
24         if (!y) b[x >> 6] ^= 1ULL << (x & 63);
25     }
26
27     Bitset &operator&=(const Bitset &another) {
28         rep(i, 0, min(SZ(b), SZ(another.b)) - 1) {
29             b[i] &= another.b[i];
30         }
31         return (*this);
32     }
33
34     Bitset operator&(const Bitset &another) const {
35         return (Bitset(*this) &= another);
36     }
37
38     Bitset &operator|=(const Bitset &another) {
39         rep(i, 0, min(SZ(b), SZ(another.b)) - 1) {
40             b[i] |= another.b[i];
41         }
42         return (*this);
43     }
44
45     Bitset operator|(const Bitset &another) const {
46         return (Bitset(*this) |= another);
47     }
48
49     Bitset &operator^=(const Bitset &another) {
50         rep(i, 0, min(SZ(b), SZ(another.b)) - 1) {
51             b[i] ^= another.b[i];
52         }
53         return (*this);
54     }
55
56     Bitset operator^(const Bitset &another) const {
57         return (Bitset(*this) ^= another);
58     }

```

```

59
60     Bitset &operator>>=(int x) {
61         if (x & 63) {
62             rep(i, 0, SZ(b) - 2) {
63                 b[i] >>= (x & 63);
64                 b[i] ^= (b[i + 1] << (64 - (x & 63)));
65             }
66             b.back() >>= (x & 63);
67         }
68
69         x >>= 6;
70         rep(i, 0, SZ(b) - 1) {
71             if (i + x < SZ(b)) b[i] = b[i + x];
72             else b[i] = 0;
73         }
74         return (*this);
75     }
76
77     Bitset operator>>(int x) const {
78         return (Bitset(*this) >>= x);
79     }
80
81     Bitset &operator<<=(int x) {
82         if (x & 63) {
83             for (int i = SZ(b) - 1; i >= 1; i--) {
84                 b[i] <<= (x & 63);
85                 b[i] ^= b[i - 1] >> (64 - (x & 63));
86             }
87             b[0] <<= x & 63;
88         }
89
90         x >>= 6;
91         for (int i = SZ(b) - 1; i >= 0; i--) {
92             if (i - x >= 0) b[i] = b[i - x];
93             else b[i] = 0;
94         }
95         return (*this);
96     }
97
98     Bitset operator<<(int x) const {
99         return (Bitset(*this) <<= x);
100     }
101 };

```

## 8.3 fastIO.cpp

```
1 static struct FastInput {
2     static constexpr int BUF_SIZE = 1 << 20;
3     char buf[BUF_SIZE];
4     size_t chars_read = 0;
5     size_t buf_pos = 0;
6     FILE *in = stdin;
7     char cur = 0;
8
9     inline char get_char() {
10         if (buf_pos >= chars_read) {
11             chars_read = fread(buf, 1, BUF_SIZE, in);
12             buf_pos = 0;
13             buf[0] = (chars_read == 0 ? -1 : buf[0]);
14         }
15         return cur = buf[buf_pos++];
16     }
17
18     template <typename T>
19     inline void tie(T) {}
20
21     inline explicit operator bool() {
22         return cur != -1;
23     }
24
25     inline static bool is_blank(char c) {
26         return c <= ' ';
27     }
28
29     inline bool skip_blanks() {
30         while (is_blank(cur) && cur != -1) {
31             get_char();
32         }
33         return cur != -1;
34     }
35
36     inline FastInput& operator>>(char& c) {
37         skip_blanks();
38         c = cur;
39         get_char();
40         return *this;
41     }
42
```

```
43 inline FastInput& operator>>(string& s) {
44     if (skip_blanks()) {
45         s.clear();
46         do {
47             s += cur;
48         } while (!is_blank(get_char()));
49     }
50     return *this;
51 }
52
53 template <typename T>
54 inline FastInput& read_integer(T& n) {
55     // unsafe, doesn't check that characters are actually digits
56     n = 0;
57     if (skip_blanks()) {
58         int sign = +1;
59         if (cur == '-') {
60             sign = -1;
61             get_char();
62         }
63         do {
64             n += n + (n << 3) + cur - '0';
65         } while (!is_blank(get_char()));
66         n *= sign;
67     }
68     return *this;
69 }
70
71 template <typename T>
72 inline typename enable_if<is_integral<T>::value, FastInput&>::type
73     operator>>(T& n) {
74     return read_integer(n);
75 }
76
77 #if !defined(_WIN32) || defined(_WIN64)
78 inline FastInput& operator>>(__int128& n) {
79     return read_integer(n);
80 }
81 #endif
82
83 template <typename T>
84 inline typename enable_if<is_floating_point<T>::value, FastInput&>::type
85     operator>>(T& n) {
```

```

84     // not sure if really fast, for compatibility only
85     n = 0;
86     if (skip_blanks()) {
87         string s;
88         (*this) >> s;
89         sscanf(s.c_str(), "%lf", &n);
90     }
91     return *this;
92 }
93 } fast_input;
94
95 #define cin fast_input
96
97 static struct FastOutput {
98     static constexpr int BUF_SIZE = 1 << 20;
99     char buf[BUF_SIZE];
100     size_t buf_pos = 0;
101     static constexpr int TMP_SIZE = 1 << 20;
102     char tmp[TMP_SIZE];
103     FILE *out = stdout;
104
105     inline void put_char(char c) {
106         buf[buf_pos++] = c;
107         if (buf_pos == BUF_SIZE) {
108             fwrite(buf, 1, buf_pos, out);
109             buf_pos = 0;
110         }
111     }
112
113     ~FastOutput() {
114         fwrite(buf, 1, buf_pos, out);
115     }
116
117     inline FastOutput& operator<<(char c) {
118         put_char(c);
119         return *this;
120     }
121
122     inline FastOutput& operator<<(const char* s) {
123         while (*s) {
124             put_char(*s++);
125         }
126         return *this;

```

```

127     }
128
129     inline FastOutput& operator<<(const string& s) {
130         for (int i = 0; i < (int) s.size(); i++) {
131             put_char(s[i]);
132         }
133         return *this;
134     }
135
136     template <typename T>
137     inline char* integer_to_string(T n) {
138         // beware of TMP_SIZE
139         char* p = tmp + TMP_SIZE - 1;
140         if (n == 0) {
141             *--p = '0';
142         } else {
143             bool is_negative = false;
144             if (n < 0) {
145                 is_negative = true;
146                 n = -n;
147             }
148             while (n > 0) {
149                 *--p = (char) ('0' + n % 10);
150                 n /= 10;
151             }
152             if (is_negative) {
153                 *--p = '-';
154             }
155         }
156         return p;
157     }
158
159     template <typename T>
160     inline typename enable_if<is_integral<T>::value, char*>::type stringify(T
        n) {
161         return integer_to_string(n);
162     }
163
164     #if !defined(_WIN32) || defined(_WIN64)
165     inline char* stringify(__int128 n) {
166         return integer_to_string(n);
167     }
168     #endif

```

```

169
170     template <typename T>
171     inline typename enable_if<is_floating_point<T>::value, char*>::type
172         stringify(T n) {
173         sprintf(tmp, "%.17f", n);
174         return tmp;
175     }
176
177     template <typename T>
178     inline FastOutput& operator<<(const T& n) {
179         auto p = stringify(n);
180         for (; *p != 0; p++) {
181             put_char(*p);
182         }
183     }
184 } fast_output;
185
186 #define cout fast_output

```

## 8.4 FastMod.cpp

1 Description: Compute  $a \% b$  about 5 times faster than usual, where  $b$  is  
2 constant but not known at compile time. Returns a value congruent to  $a$   
3  $(\text{mod } b)$  in the range  $[0, 2b)$ .

```

4
5 typedef unsigned long long ull;
6 struct FastMod {
7     ull b, m;
8     FastMod(ull b) : b(b), m(-1ULL / b) {}
9     ull reduce(ull a) { //  $a \% b + (0 \text{ or } b)$ 
10         return a - (ull)((__uint128_t(m) * a) >> 64) * b;
11     }
12 };

```

## 8.5 intervalContainer.cpp

1 Description: Add and remove intervals from a set of disjoint intervals.  
2 Will merge the added interval with any overlapping intervals in the set when  
3 adding. Intervals are [inclusive, exclusive).  
4 Time:  $O(\log N)$   
5  
6 `set<pii>::iterator addInterval(set<pii>& is, int L, int R) {`

```

7     if (L == R) return is.end();
8     auto it = is.lower_bound({L, R}), before = it;
9     while (it != is.end() && it->first <= R) {
10         R = max(R, it->second);
11         before = it = is.erase(it);
12     }
13     if (it != is.begin() && (--it)->second >= L) {
14         L = min(L, it->first);
15         R = max(R, it->second);
16         is.erase(it);
17     }
18     return is.insert(before, {L, R});
19 }
20 void removeInterval(set<pii>& is, int L, int R) {
21     if (L == R) return;
22     auto it = addInterval(is, L, R);
23     auto r2 = it->second;
24     if (it->first == L) is.erase(it);
25     else (int&)it->second = L;
26     if (R != r2) is.emplace(R, r2);
27 }

```

## 8.6 lineContainer.cpp

```

1 /**
2  * Author: Simon Lindholm
3  * Date: 2017-04-20
4  * License: CC0
5  * Source: own work
6  * Description: Container where you can add lines of the form  $kx+m$ , and
7  *               query maximum values at points  $x$ .
8  * Useful for dynamic programming ('`convex hull trick`').
9  * Time:  $O(\log N)$ 
10  * Status: stress-tested
11  */
12 #pragma once
13
14 struct Line {
15     mutable ll k, m, p;
16     bool operator<(const Line& o) const { return k < o.k; }
17     bool operator<(ll x) const { return p < x; }
18 };

```



```

19 struct LineContainer : multiset<Line, less<>> {
20     // (for doubles, use inf = 1/.0, div(a,b) = a/b)
21     static const ll inf = LLONG_MAX;
22     ll div(ll a, ll b) { // floored division
23         return a / b - ((a ^ b) < 0 && a % b); }
24     bool isect(iterator x, iterator y) {
25         if (y == end()) return x->p = inf, 0;
26         if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
27         else x->p = div(y->m - x->m, x->k - y->k);
28         return x->p >= y->p;
29     }
30     void add(ll k, ll m) {
31         auto z = insert({k, m, 0}), y = z++, x = y;
32         while (isect(y, z)) z = erase(z);
33         if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
34         while ((y = x) != begin() && (--x)->p >= y->p)
35             isect(x, erase(y));
36     }
37     ll query(ll x) {
38         assert(!empty());
39         auto l = *lower_bound(x);
40         return l.k * x + l.m;
41     }
42 };

```

## 8.7 mint.cpp

```

1 template<int MOD, int RT> struct mint {
2     static const int mod = MOD;
3     static constexpr mint rt() { return RT; } // primitive root for FFT
4     int v; explicit operator int() const { return v; } // explicit -> don't
        silently convert to int
5     mint():v(0) {}
6     mint(ll _v) { v = int((-MOD < _v && _v < MOD) ? _v : _v % MOD);
7         if (v < 0) v += MOD; }
8     bool operator==(const mint& o) const {
9         return v == o.v; }
10    friend bool operator!=(const mint& a, const mint& b) {
11        return !(a == b); }
12    friend bool operator<(const mint& a, const mint& b) {
13        return a.v < b.v; }
14
15    mint& operator+=(const mint& o) {

```

```

16        if ((v += o.v) >= MOD) v -= MOD;
17        return *this; }
18    mint& operator-=(const mint& o) {
19        if ((v -= o.v) < 0) v += MOD;
20        return *this; }
21    mint& operator*=(const mint& o) {
22        v = int((ll)v*o.v%MOD); return *this; }
23    mint& operator/=(const mint& o) { return (*this) *= inv(o); }
24    friend mint pow(mint a, ll p) {
25        mint ans = 1; assert(p >= 0);
26        for (; p; p /= 2, a *= a) if (p&1) ans *= a;
27        return ans; }
28    friend mint inv(const mint& a) { assert(a.v != 0);
29        return pow(a,MOD-2); }
30
31    mint operator-() const { return mint(-v); }
32    mint& operator++() { return *this += 1; }
33    mint& operator--() { return *this -= 1; }
34    friend mint operator+(mint a, const mint& b) { return a += b; }
35    friend mint operator-(mint a, const mint& b) { return a -= b; }
36    friend mint operator*(mint a, const mint& b) { return a *= b; }
37    friend mint operator/(mint a, const mint& b) { return a /= b; }
38 };
39
40 const int MOD=998244353;
41 using mi = mint<MOD,5>; // 5 is primitive root for both common mods
42
43 namespace simp {
44     vector<mi> fac,ifac,invn;
45     void check(int x) {
46         if (fac.empty()) {
47             fac={mi(1),mi(1)};
48             ifac={mi(1),mi(1)};
49             invn={mi(0),mi(1)};
50         }
51         while (SZ(fac)<=x) {
52             int n=SZ(fac),m=SZ(fac)*2;
53             fac.resize(m);
54             ifac.resize(m);
55             invn.resize(m);
56             for (int i=n;i<m;i++) {
57                 fac[i]=fac[i-1]*mi(i);
58                 invn[i]=mi(MOD-MOD/i)*invn[MOD%i];

```

```

59         ifac[i]=ifac[i-1]*invn[i];
60     }
61 }
62 }
63 mi gfac(int x) {
64     check(x); return fac[x];
65 }
66 mi ginv(int x) {
67     check(x); return invn[x];
68 }
69 mi gifac(int x) {
70     check(x); return ifac[x];
71 }
72 mi binom(int n,int m) {
73     if (m < 0 || m > n) return mi(0);
74     return gfac(n)*gifac(m)*gifac(n - m);
75 }
76 }

```

## 8.8 pbds.cpp

```

1 #include <bits/extc++.h>
2 using namespace __gnu_cxx;
3 using namespace __gnu_pbds;
4
5 #include<ext/pb_ds/assoc_container.hpp>
6 #include<ext/pb_ds/tree_policy.hpp>
7 #include<ext/pb_ds/hash_policy.hpp>
8 #include<ext/pb_ds/trie_policy.hpp>
9 #include<ext/pb_ds/priority_queue.hpp>
10
11 pairing_heap_tag: 配对堆
12 thin_heap_tag: 斐波那契堆
13 binomial_heap_tag: 二项堆
14 binary_heap_tag: 二叉堆
15
16 __gnu_pbds::priority_queue<PII, greater<PII>, pairing_heap_tag> q;
17 __gnu_pbds::priority_queue<PII, greater<PII>, pairing_heap_tag>::
    point_iterator its[N];
18
19 its[v] = q.push({dis[v], v});
20 q.modify(its[v], {dis[v], v});
21

```

22 可以将两个优先队列中的元素合并（无任何约束）  
23 使用方法为 `a.join(b)`  
24 此时优先队列 `b` 内所有元素就被合并进优先队列 `a` 中，且优先队列 `b` 被清空  
25  
26 `cc_hash_table<string, int>` mp1 拉链法  
27 `gp_hash_table<string, int>` mp2 查探法

## 8.9 simu.cpp

```

1 db rnd(db l, db r) {
2     static uniform_real_distribution<db> u(0, 1);
3     static default_random_engine e(rng());
4     return l + (r - l) * u(e); // u(rng);
5 }
6
7 db eval(pair<db, db> x) { ... }
8
9 void simulate_anneal() {
10     pair<db, db> cur(rnd(0, 10000), rnd(0, 10000));
11     for (double k = 10000; k > 1e-5; k *= 0.99) {
12         // [start, end, step]
13         pair<db, db> nxt(cur.fi + rnd(-k, k), cur.se + rnd(-k, k));
14         db delta = eval(nxt) - eval(cur);
15         if (exp(-delta / k) > rnd(0, 1)) {
16             cur = nxt;
17         }
18     }
19 }
20
21 /**
22  * https://codeforces.com/gym/104813/submission/234982955
23  * The 9th CCPC (Harbin) 2023
24  * Author: QwertyPi
25  */
26 LD Prob() {
27     static uniform_real_distribution<> dist(0.0, 1.0);
28     return dist(rng);
29 }
30 LD Sigma(LD x) { return 1 / (1 + exp(-x)); }
31
32 LD overall_max_score = 0;
33 for (int main_loop = 0; main_loop < 5; main_loop++) {
34     vector<LD> e(n, (LD)1 / n);

```

```

35     for (int tr = 0; tr < 1000; tr++) {
36         vector<LD> ne(n);
37         for (int i = 0; i < n; i++) {
38             ne[i] = Prob();
39         }
40         LD s = accumulate(all(ne), 0.0L);
41         for (int i = 0; i < n; i++) {
42             ne[i] /= s;
43         }
44         if (eval(ne) > eval(e)) e = ne;
45     }
46     LD t = (LD)0.0002;
47     LD max_score = 0;
48     const LD depr = 0.999995;
49     const int tries = 2E6;
50     const int loop = 1E5;
51
52     LD score_old = eval(e);
53     for (int tr = 0; tr < tries; tr++) {
54 #ifdef LOCAL
55         if (tr % loop == loop - 1) {
56             cout << fixed << setprecision(10) << "current_score=" << max_score
57                 << ", t=" << t << '\n';
58         }
59 #endif
60         int x = rng() % n, y = rng() % n;
61         if (e[x] < t || x == y) {
62             t *= depr;
63             continue;
64         }
65         e[x] -= t;
66         e[y] += t;
67         LD score_new = eval(e);
68         if (score_new > score_old) { // ok
69             ;
70         } else { // revert
71             e[x] += t;
72             e[y] -= t;
73         }
74         score_old = score_new;
75         max_score = max(max_score, score_new);
76         t *= depr;
77     }

```

```

78     overall_max_score = max(overall_max_score, max_score);
79 #ifdef LOCAL
80     cout << "Loop#" << main_loop << ": " << max_score << '\n';
81 #endif
82 }

```

## 8.10 sort.cpp

```

1 void merge_sort(int q[], int l, int r) {
2     if (l >= r) return;
3     int mid = l + r >> 1;
4     merge_sort(q, l, mid);
5     merge_sort(q, mid + 1, r);
6
7     int k = 0, i = l, j = mid + 1;
8     while (i <= mid && j <= r)
9         if (q[i] <= q[j])
10             tmp[k++] = q[i++];
11         else
12             tmp[k++] = q[j++];
13
14     while (i <= mid)
15         tmp[k++] = q[i++];
16     while (j <= r)
17         tmp[k++] = q[j++];
18
19     for (i = l, j = 0; i <= r; i++, j++) q[i] = tmp[j];
20 }
21
22 void quick_sort(int q[], int l, int r) {
23     if (l >= r) return;
24     int i = l - 1, j = r + 1, x = q[l + r >> 1];
25     while (i < j) {
26         do i ++ ; while (q[i] < x);
27         do j -- ; while (q[j] > x);
28         if (i < j) swap(q[i], q[j]);
29     }
30     quick_sort(q, l, j), quick_sort(q, j + 1, r);
31 }
32
33 template<class T>
34 void radixsort(T *a, ll n) {
35     int base = 0;

```

```

36     rep(i, 1, n) sa[i] = i;
37     rep(k, 1, 5) {
38         rep(i, 0, 255) c[i] = 0;
39         rep(i, 1, n) c[(a[i] >> base) & 255]++;
40         rep(i, 1, 255) c[i] += c[i - 1];
41         per(i, n, 1) {
42             rk[sa[i]] = c[(a[sa[i]] >> base) & 255]--;
43         }
44         rep(i, 1, n) sa[rk[i]] = i;
45         base += 7;
46     }
47 }

```

## 8.11 高精度.cpp

```

1 vector<int> add(vector<int> &A, vector<int> &B) {
2     if (A.size() < B.size()) return add(B, A);
3     vector<int> C;
4     int t = 0;
5     for (int i = 0; i < A.size(); i++) {
6         t += A[i];
7         if (i < B.size()) t += B[i];
8         C.push_back(t % 10);
9         t /= 10;
10    }
11    if (t) C.push_back(t);
12    return C;
13 }
14
15 vector<int> sub(vector<int> &A, vector<int> &B) {
16     vector<int> C;
17     for (int i = 0, t = 0; i < A.size(); i++) {
18         t = A[i] - t;

```

```

19         if (i < B.size()) t -= B[i];
20         C.push_back((t + 10) % 10);
21         if (t < 0) t = 1;
22         else t = 0;
23     }
24     while (C.size() > 1 && C.back() == 0) C.pop_back();
25     return C;
26 }
27
28 vector<int> mul(vector<int> &A, int b) {
29     vector<int> C;
30     int t = 0;
31     for (int i = 0; i < A.size() || t; i++) {
32         if (i < A.size()) t += A[i] * b;
33         C.push_back(t % 10);
34         t /= 10;
35     }
36     while (C.size() > 1 && C.back() == 0) C.pop_back();
37     return C;
38 }
39
40 vector<int> div(vector<int> &A, int b, int &r) {
41     vector<int> C;
42     r = 0;
43     for (int i = A.size() - 1; i >= 0; i--) {
44         r = r * 10 + A[i];
45         C.push_back(r / b);
46         r %= b;
47     }
48     reverse(C.begin(), C.end());
49     while (C.size() > 1 && C.back() == 0) C.pop_back();
50     return C;
51 }

```