Team Reference Document

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目录

1	Template	1
2	Data	3
3	DP	32
4	Geometry	33
5	Graph	42
6	Math	58
7	String	82
8	Basic	91

1 Template

1.1 .clang-format

```
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```

1.2 debug.cpp

```
#include <bits/stdc++.h>

using namespace std;

template <class T, size_t size = tuple_size<T>::value>
string to_debug(T, string s = "") requires(not ranges::range<T>);

template<class T>
concept check = requires(T x, ostream &os) {
    os << x;
};

template<check T>
```

```
14 string to_debug(T x) {
      return static_cast<ostringstream>(ostringstream() << x).str();</pre>
16 }
17
18 string to_debug(ranges::range auto x, string s = "") requires(not is_same_v<
        decltype(x), string>) {
     for (auto xi : x) {
20
        s += ", " + to_debug(xi);
21
     return "[__" + s.substr(s.empty() ? 0 : 2) + "__]";
23 }
24
25 template <class T, size_t size>
    string to_debug(T x, string s) requires(not ranges::range<T>) {
      [&] < size_t... I > (index_sequence < I... >) {
       ((s += ", | " + to_debug(get < I > (x))), ...);
     }(make index sequence<size>());
      return "{" + s.substr(s.empty() ? 0 : 2) + "}";
31 }
32
   #define debug(...) [](auto... $){ ((cout << to_debug($) << "□"), ...); cout
        << endl; }("[", #__VA_ARGS__, "]:", __VA_ARGS__)
```

1.3 gen.py

```
from random import *
2  n = 10000
3  s = 'qwertyuiopasdfghjklzxcvbnm'
4  for i in range(n):
5     print(choice(s), end = '')
6  print()
7  print(randint(0, 1), randint(1, n))
```

1.4 head.cpp

```
#pragma GCC optimize("Ofast", "inline", "unroll-loops")
#include <bits/stdc++.h>
using namespace std;
#define rep(i, a, n) for (int i = a; i <= n; i++)
#define per(i, a, n) for (int i = a; i >= n; i--)
#define pb push_back
#define eb emplace_back
#define all(x) (x).begin(), (x).end()
```

```
#define fi first
11 #define se second
12 #define SZ(x) ((int)(x).size())
   using VI = vector<int>;
14 using PII = pair<int, int>;
15 using 11 = long long;
16 using ull = unsigned long long;
17 using db = double;
18 using ldb = long double;
   mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
21
   #ifdef DEBUG
   #include "debug.cpp"
   #else
   #define debug(...) 42
   #endif
27
28 void solve() {}
   int main() {
       cin.tie(nullptr)->sync_with_stdio(false);
30
       cout << fixed << setprecision(16);</pre>
31
32
       int tt = 1;
33
       cin >> tt:
34
       while (tt--) {
            solve();
       }
37 }
```

1.5 head-apiadu.cpp

#define bit(x) (111 << (x))

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 #define rep(i,a,n) for (int i=a;i<n;i++)
4 #define per(i,a,n) for (int i=n-1;i>=a;i--)
5 #define pb push_back
6 #define eb emplace_back
7 #define mp make_pair
8 #define all(x) (x).begin(),(x).end()
9 #define fi first
10 #define se second
11 #define SZ(x) ((int)(x).size())
```

```
12 typedef vector<int> VI;
13 typedef basic_string<int> BI;
14 typedef long long ll;
15 typedef pair<int,int> PII;
16 typedef double db;
17 mt19937 mrand(random_device{}());
18 const ll mod=10000000007;
19 int rnd(int x) { return mrand() % x;}
20 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b;b>>=1){if(b&1) res=res*a%mod;a=a*a%mod;}return res;}
21 ll gcd(ll a,ll b) { return b?gcd(b,a%b):a;}
22 // head
```

1.6 Makefile

```
1 %: %.cpp
2 g++-13 $< -o $@ -std=gnu++20 -g -02 -Wall -Wextra -DDEBUG -
D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC
```

1.7 pai.py

```
1 import os
2 \text{ tt = 0}
3 while True:
        os.system('python_gen.py_>_A.in')
        os.system('./a,<,A.in,>,a.out')
        os.system('./b_{\parallel} < A.in_{\parallel} > b.out')
        # diff for linux or macos, fc for windows
        if os.system('diffua.outub.out'):
             print("WA")
10
             exit(0)
11
        else:
12
             tt += 1
13
             print("AC:", tt)
```

1.8 settings.json

```
1 {
2     "editor.formatOnSave": true,
3     "files.autoSave": "afterDelay",
4     "files.autoSaveDelay": 350,
5     "C_Cpp.default.cppStandard": "gnu++20"
6 }
```

2 Data

2.1 01trie.cpp

```
1 struct node {
       int son[2]:
       int end;
       int sz;
   } seg[maxn << 2];</pre>
    int root, tot;
7 int n, m;
   void insert(ll x) {
10
       int cnt = root;
11
       for (int i = 62; i >= 0; i--) {
            int w = (x >> i) & 1;
12
            if (seg[cnt].son[w] == 0) seg[cnt].son[w] = ++tot;
13
            cnt = seg[cnt].son[w];
14
            seg[cnt].sz++;
15
16
       }
17
        seg[cnt].end++;
18 }
19
   ll query(ll x, ll k) {
       ll res = 0:
21
22
       int cnt = root;
       for (int i = 62; i >= 0; i--) {
23
24
            int w = (x >> i) & 1;
            if (seg[seg[cnt].son[w]].sz >= k) cnt = seg[cnt].son[w];
25
26
            else {
                k -= seg[seg[cnt].son[w]].sz;
27
                cnt = seg[cnt].son[abs(w - 1)];
28
29
                res += bit(i);
           }
30
       }
31
       return res;
33 }
```

2.2 2dtree(bqi343).cpp

```
1 const int SZ = 1.1e5;
2 template <class T>
3 struct node {
```

```
T val = 0;
        node<T>* c[2];
6
        node() { c[0] = c[1] = NULL; }
7
        void upd(int ind, T v, int L = 0, int R = SZ - 1) { // add v
8
            if (L == ind && R == ind) {
9
                 val += v:
10
                 return;
11
            }
12
            int M = (L + R) / 2;
            if (ind <= M) {
13
14
                 if (!c[0]) c[0] = new node();
15
                 c[0]->upd(ind, v, L, M);
            } else {
16
17
                 if (!c[1]) c[1] = new node();
18
                 c[1] \rightarrow upd(ind, v, M + 1, R);
            }
19
20
            val = 0;
21
            rep(i, 0, 1) if (c[i]) val += c[i] -> val;
22
23
        T query(int lo, int hi, int L = 0, int R = SZ - 1) { // query sum of
             segment
24
            if (hi < L || R < lo) return 0:
25
            if (lo <= L && R <= hi) return val;
26
            int M = (L + R) / 2;
            T res = 0;
27
28
            if (c[0]) res += c[0]->query(lo, hi, L, M);
29
            if (c[1]) res += c[1]->query(lo, hi, M + 1, R);
30
            return res;
31
        }
32
        void UPD(int ind, node* c0, node* c1, int L = 0, int R = SZ - 1) { //
             for 2D seatree
33
            if (L != R) {
34
                 int M = (L + R) / 2;
                 if (ind <= M) {
36
                     if (!c[0]) c[0] = new node();
37
                     c[0] \rightarrow UPD(ind, c0 ? c0 \rightarrow c[0] : NULL, c1 ? c1 \rightarrow c[0] : NULL, L
                          , M);
                 } else {
39
                     if (!c[1]) c[1] = new node();
40
                     c[1] \rightarrow UPD(ind, c0 ? c0 \rightarrow c[1] : NULL, c1 ? c1 \rightarrow c[1] : NULL, M
                           + 1, R);
41
                 }
42
            }
```

```
val = (c0 ? c0 -> val : 0) + (c1 ? c1 -> val : 0):
       }
44
45 }:
46
47
    * Description: BIT of SeqTrees. x\in (0,SZ), y\in [0,SZ).
    * Memory: O(N\log^2 N)
    * Source: USACO Mowing the Field
    * Verification:
    * USACO Mowing the Field
    * http://www.usaco.org/index.php?page=viewproblem2&cpid=722 (13/15, 15/15
         and 1857ms with BumpAllocator)
    */
54
55
   #include "../1D_{\sqcup}Range_{\sqcup}Queries_{\sqcup}(9.2)/SparseSeg_{\sqcup}(9.2).h"
57
    template <class T>
    struct BITseg {
        node<T> seg[SZ];
60
        BITseg() { fill(seg, seg + SZ, node<T>()); }
61
62
        void upd(int x, int y, int v) { // add v
            for (; x < SZ; x += x & -x) seg[x].upd(y, v);
63
       }
64
65
       T query(int x, int yl, int yr) {
66
            T res = 0:
            for (; x; x \rightarrow x \& x) res += seg[x].query(yl, yr);
69
       }
70
       T query(int xl, int xr, int yl, int yr) { // query sum of rectangle
71
            return query(xr, yl, yr) - query(xl - 1, yl, yr);
72
       }
73 }:
74
75
    * Description: SeqTree of SeqTrees. x,y \in [0,SZ).
76
    * Memory: O(N\log^2 N)
    * Source: USACO Mowing the Field
   * Verification:
    * http://www.usaco.org/index.php?page=viewproblem2&cpid=722 (9/15 w/
         BumpAllocator)
    * http://www.usaco.org/index.php?page=viewproblem2&cpid=601 (4238 ms, 2907
         ms w/ BumpAllocator)
82
```

```
83
    #include "../1D,Range,Queries,(9.2)/SparseSeg,(9.2).h"
    template <class T>
     struct Node {
         node<T> seg;
         Node* c[2];
         Node() \{ c[0] = c[1] = NULL: \}
 91
         void upd(int x, int y, T v, int L = 0, int R = SZ - 1) { // add v
 92
             if (L == x && R == x) {
 93
                 seg.upd(y, v);
 94
                 return;
             }
 95
 96
             int M = (L + R) / 2;
 97
             if (x <= M) {
                 if (!c[0]) c[0] = new Node();
 99
                 c[0] \rightarrow upd(x, v, v, L, M);
             } else {
100
                 if (!c[1]) c[1] = new Node();
101
102
                 c[1] - \sup (x, y, v, M + 1, R);
103
104
             seg.upd(y, v); // only for addition
105
             // sea.UPD(u.c[0]?&c[0]->sea:NULL.c[1]?&c[1]->sea:NULL):
106
         }
107
         T query(int x1, int x2, int y1, int y2, int L = 0, int R = SZ - 1) { //
               query sum of rectangle
108
             if (x1 <= L && R <= x2) return seg.query(y1, y2);
             if (x2 < L \mid | R < x1) return 0:
110
             int M = (L + R) / 2;
111
             T res = 0:
112
             if (c[0]) res += c[0]->query(x1, x2, y1, y2, L, M);
113
             if (c[1]) res += c[1]->query(x1, x2, y1, y2, M + 1, R);
114
             return res:
115
         }
116 };
```

2.3 cdq.cpp

```
int ans[maxn], lev[maxn];
array<int, 5> v[maxn], tmp[maxn];

struct BIT {

...
```

```
6 } c:
   void solve(int 1. int r) {
9
        if (1 >= r) return;
        int mid = (1 + r) / 2:
10
        solve(1, mid), solve(mid + 1, r);
11
        int i = 1, j = mid + 1;
12
        int piv = 1;
13
        while (i <= mid || j <= r) {
14
            if (i <= mid && (j > r || mp(v[i][1], v[i][2]) <= mp(v[j][1], v[j
15
                ][2]))) {
                c.modify(v[i][2], v[i][3]);
16
                tmp[piv++] = v[i++];
17
18
            } else {
                v[j][4] += c.query(v[j][2]);
19
                tmp[piv++] = v[j++];
20
            }
21
        }
22
        rep(i, 1, mid) c.modify(v[i][2], -v[i][3]);
23
        rep(i, 1, r) v[i] = tmp[i];
24
25 }
   void solve() {
27
        cin >> n >> k;
28
29
        c.resize(k):
        rep(i, 1, n) {
30
            int s, c, m;
            cin >> s >> c >> m;
32
33
            v[i] = {s, c, m, 1, 0};
        }
34
        v[0][0] = -1;
35
36
        sort(v + 1, v + n + 1);
        int cnt = 0;
37
38
        rep(i, 1, n) {
39
            if (v[i][0] == v[cnt][0] \&\& v[i][1] == v[cnt][1] \&\& v[i][2] == v[cnt]
                ][2]) v[cnt][3]++;
            else v[++cnt] = v[i];
40
        }
41
        solve(1, cnt);
42
        rep(i, 1, cnt) {
43
            ans[v[i][4] + v[i][3] - 1] += v[i][3];
44
45
        }
        rep(i, 0, n - 1) cout << ans[i] << '\n';
```

47 }

2.4 compact.cpp

```
1 namespace compact {
2 const int LOGN=18;
3 int 1[N],r[N],tot,p[N][20],n;
4 map<int,int> cv;
5 int lca(int u,int v) {
       if (dep[u]>dep[v]) swap(u,v);
7
       per(i,LOGN-1,0) if (dep[p[v][i]]>=dep[u]) v=p[v][i];
       if (u==v) return u;
       per(i,LOGN-1,0) if (p[v][i]!=p[u][i]) u=p[u][i],v=p[v][i];
10
       return p[u][0];
11 }
12 void dfs(int u,int f) {
       l[u]=++tot; dep[u]=dep[f]+1; p[u][0]=f;
       vec[dep[u]].pb(u);
14
15
       for (auto v:vE[u]) {
16
           if (v==f) continue;
17
           dfs(v.u):
18
       }
       r[u]=tot:
19
20 }
21 void build(int n) {
       n=_n; tot=0;
       dfs(1,0);
24
       rep(j,1,LOGN-1) rep(i,1,n) p[i][j]=p[p[i][j-1]][j-1];
25 }
26
27 bool cmp(int u,int v) { return l[u]<l[v]; }
   vector<PII> compact(VI v) {
29
       int m=SZ(v):
30
       vector<PII> E;
31
       sort(all(v),cmp);
32
       rep(i,0,m-2) {
           int w=lca(v[i],v[i+1]);
34
           v.pb(w);
35
       }
       v.pb(0);
37
       v.pb(1);
38
       sort(all(v),cmp);
39
       v.erase(unique(all(v)), v.end());
```

```
cv.clear();
40
        per(i,SZ(v)-1,1) {
41
            int u=v[i]:
43
            while (1) {
                auto it=cv.lower_bound(1[u]);
44
                if (it==cv.end()||it->fi>r[u]) break;
45
                E.pb(mp(u,v[it->se]));
                cv.erase(it);
47
            }
            cv[l[u]]=i;
49
50
        }
51
        return E;
53 };
```

2.5 dominator.cpp

```
void solve(int u. int S) {
      int best = -1, cnt = S + 1;
      auto find best = [&](auto &find best, int u, int par) -> void {
        sz[u] = 1, sdom[u] = 0;
       for (auto v : e[u]) {
         if (v == par || del[v]) continue;
         find best(find best, v, u);
         sz[u] += sz[v];
         sdom[u] = max(sdom[u], sz[v]);
10
11
       sdom[u] = max(sdom[u], S - sz[u]);
       if (sdom[u] < cnt) {</pre>
12
          cnt = sdom[u]. best = u:
13
14
       }
     };
15
     find_best(find_best, u, 0);
16
17
      int id1 = tot++, dep1 = 0;
      int id2, dep2;
18
19
      auto dfs = [&](auto &dfs, int u, int par, int dep) -> void {
20
       dep1 = max(dep1, dep);
21
       dep2 = max(dep2, dep);
       Q[u].pb({id1, 1, dep});
22
23
       Q[u].pb({id2, -1, dep});
       for (auto v : e[u]) {
24
25
         if (v == par || del[v]) continue;
26
          dfs(dfs, v, u, dep + 1);
```

```
27
       }
     };
     Q[best].pb({id1, 1, 0});
     for (auto v : e[best]) {
31
       if (del[v]) continue:
       id2 = tot++, dep2 = 0;
32
       dfs(dfs, v, best, 1);
       fenw[id2] = BIT<11>(dep2 + 1);
35
     fenw[id1] = BIT<11>(dep1 + 1);
     del[best] = 1;
     for (auto v : e[best]) {
       if (!del[v]) solve(v, sz[v]);
40
    }
41 }
```

2.6 dsu.cpp

```
1 class dsu {
    public:
     vector<int> fa;
     vector<ll> dist;
5
     int n:
6
     dsu(int n) : n( n) {
       fa.resize(n):
       dist.assign(n, 0);
10
       iota(fa.begin(), fa.end(), 0);
11
12
     int find(int x) {
14
       if (fa[x] == x) return x;
       int par = fa[x];
15
       fa[x] = find(fa[x]);
16
17
       dist[x] += dist[par]:
       return fa[x];
18
19
     }
20
21
      void unite(int x, int y, ll v) {
22
       int px = find(x);
23
       int py = find(y);
24
       fa[py] = px;
25
        dist[py] = dist[x] - dist[y] - v;
```

```
26 }
27 };
```

2.7 dsu-on-tree.cpp

```
1 void dfs(int x, int fa) {
       hs[x] = -1, w[x] = 1:
       l[x] = ++tot;
       id[tot] = x:
       for (auto y : g[x]) if (y != fa) {
                dfs(y, x);
               w[x] += w[y];
                if (hs[x] == -1 || w[y] > w[hs[x]])
                   hs[x] = y;
           }
11
       r[x] = tot;
13
   void dsu(int x, int fa, int keep) {
       for (auto y : g[x]) {
15
16
           if (y != hs[x] && y != fa) {
                dsu(y, x, 0);
18
           }
19
       if (hs[x] != -1) dsu(hs[x], x, 1);
21
22
       for (auto y : g[x]) {
           if (y != hs[x] && y != fa) {
23
               for (int i = l[v]; i <= r[v]; i++) {
               }
27
           }
28
       // add current node
30
31
       ans[x] = cnt;
32
       if (!keep) {
34
           // clear
```

2.8 fenwick.cpp

```
1 template <typename T>
2 struct BIT {
     vector<T> fenw;
     int n, pw;
     BIT(int n = 0) : n(n) {
     fenw.assign(n + 1, 0);
       pw = bit_floor(unsigned(n));
10
11
   void Modify(int x, T v) {
    if (x \le 0) return; // assert (0 \le x \& x \le n);
    while (x \le n) \{ // x < n \}
       fenw[x] += v;
         x += (x \& -x); // x /= x + 1;
16
       }
17
    }
    T Query(int x) {
    // assert(0 <= x && x <= n);
21
    T v{};
    while (x > 0) {
         v += fenw[x]; // fenw[x - 1];
         x = (x \& -x); // x \& = x - 1;
25
26
       return v;
27
28
     // Returns the length of the longest prefix with sum <= c
     int MaxPrefix(T c) {
31
     T v{};
      int at = 0:
       for (int i = 20; i >= 0; i--) {
         if (at + bit(i) <= n && v + fenw[at + bit(i)] <= c) {</pre>
           v += fenw[at + bit(i)];
           at += bit(i);
         }
       }
       /**
        * for (int len = pw; len > 0; len >>= 1) {
41
         * if (at + len <= n) {
42
              auto nv = v:
```

```
43  * nv += fenw[at + len - 1];

44  * if (!(c < nv)) {

45  * v = nv;

46  * at += len;

47  * }

48  * }

49  * }

50  * assert(0 <= at && at <= n);

51  */

52  return at;

53 }

54 };
```

2.9 fenwick2d.cpp

```
1 template <typename T>
   class fenwick2d {
    public:
     vector<vector<T>> fenw;
     int n, m;
     fenwick2d(int n, int m) : n(n), m(m) {
       fenw.resize(n);
       for (int i = 0; i < n; i++) {
         fenw[i].resize(m);
       }
11
12
     }
13
     inline void modify(int i, int j, T v) {
14
       int x = i:
15
16
       while (x < n) {
17
        int y = j;
         while (y < m) {
18
19
           fenw[x][y] += v;
           y = (y + 1);
20
21
         x = (x + 1);
23
24
     }
25
26
     inline T get(int i, int j) {
27
       T v{};
28
       int x = i:
```

```
29
        while (x \ge 0) {
         int y = j;
31
         while (y \ge 0) {
32
           v += fenw[x][v];
           y = (y & (y + 1)) - 1;
34
          x = (x & (x + 1)) - 1;
37
       return v;
39 };
40
41 struct node {
     int a = ...; // don't forget to set default value
43
44
     inline void operator+=(node &other) { ... }
45 };
```

2.10 hash-table.cpp

```
1 struct Hash table {
        static const int V = 1000003;
       int fst[V], nxt[V];
       int ctm, ptm[V], T;
       int val[V];
       ll key[V];
       void init() \{T = 0, ctm++;\}
       void insert(ll k, int v) {
9
           int s = k \% V;
10
            if (ptm[s] != ctm) ptm[s] = ctm, fst[s] = -1;
            for (int i = fst[s]; i != -1; i = nxt[i]) if (key[i] == k) {
11
12
                    return;
13
            nxt[T] = fst[s], fst[s] = T, key[T] = k, val[T] = v;
14
15
           T++:
16
       }
17
       int query(ll k) {
18
            int s = k \% V:
19
            if (ptm[s] != ctm) return -1;
            for (int i = fst[s]: i != -1: i = nxt[i]) {
21
                if (key[i] == k) return val[i];
22
           }
23
            return -1:
```

```
24 }
25 };
```

2.11 hilbertOrder.cpp

```
1 template <class M, bool val_on_edge = false>
2 class Mo Tree {
   private:
        static inline int64_t hilbertOrder(int x, int y, int pow, int rotate) {
            if (pow == 0)
                return 0:
            int hpow = 1 << (pow - 1);
            int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y < hpow) ? 1 : 2);
            seg = (seg + rotate) & 3;
            const int rotateDelta[4] = \{3, 0, 0, 1\};
            int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
11
12
            int nrot = (rotate + rotateDelta[seg]) & 3;
            int64_t subSquareSize = int64_t(1) << (2 * pow - 2);
13
14
            int64 t ordd = seg * subSquareSize;
15
            int64_t add = hilbertOrder(nx, ny, pow - 1, nrot);
            ordd += (seg == 1 || seg == 2) ? add : (subSquareSize - add - 1);
16
17
            return ordd;
       }
18
19
20
        struct query {
21
            int l, r, q_indx, lca;
22
            int64 t ord;
23
       }:
24
25
26
       vector<int> depth, flat, st, en, node_freq;
27
       vector<vector<int>> anc;
28
       vector < M > par_v;
29
        vector<query> Q;
30
31
       void add node(int node) {
            if (par_v[node] < N) {</pre>
32
33
                aux[par_v[node]]++;
34
                w.reset(par_v[node]);
35
            }
36
       }
37
38
       void remove node(int node) {
```

```
if (par_v[node] < N) {</pre>
                if (--aux[par v[node]] == 0) {
40
                    w.set(par_v[node]);
41
42
                }
            }
43
        }
44
45
46
        long long calc(...) {
47
            return w. Find first();
48
        }
49
50
        void add(int ind) {
51
            node_freq[flat[ind]]++;
52
            if (node freg[flat[ind]] == 1) {
53
                add_node(flat[ind]);
54
            } else {
                remove node(flat[ind]);
56
57
        }
58
59
        void remove(int ind) {
60
            node freg[flat[ind]]--:
61
            if (node_freq[flat[ind]] == 1) {
62
                add_node(flat[ind]);
63
            } else {
64
                remove_node(flat[ind]);
        }
67
        Mo_Tree(vector<vector<int>> &g, const vector<M> &v) {
70
            q = 0;
71
            this->par_v = v;
72
            int n = (int)g.size() - 1;
73
            depth.resize(n + 1), st.resize(n + 1);
74
            en.resize(n + 1), node_freq.resize(n + 1);
            anc.assign(n + 1, vector<int>(__lg(n) + 2, 0));
76
77
            auto dfs = [&](auto &dfs, int node, int p, int d) -> void {
78
                anc[node][0] = p;
79
                depth[node] = d;
                st[node] = flat.size();
81
                flat.push_back(node);
```

```
82
                 for (auto it : g[node]) {
                     if (it != p) {
 83
                         dfs(dfs, it, node, d + 1);
 84
 85
                     }
                 }
 86
                 en[node] = flat.size();
 87
                 flat.push back(node);
 89
             }:
 90
             dfs(dfs, 1, 0, 0);
 91
 92
             for (int j = 1; j \le _lg(n); ++j) {
                 for (int i = 1; i <= n; ++i) {
 93
                     anc[i][j] = anc[anc[i][j - 1]][j - 1];
 94
 95
                }
 96
             }
         }
 97
 98
         int kth anc(int node, int k) {
100
             int ret = node;
             for (int bit = (int)anc[ret].size() - 1: ~bit: --bit) {
101
                 if (k & (1 << bit)) {
102
                     ret = anc[ret][bit]:
103
                }
104
105
             }
106
             return ret:
         }
107
108
         int LCA(int u, int v) {
109
110
             if (depth[u] < depth[v]) {</pre>
111
                 swap(u, v);
112
113
             u = kth_anc(u, depth[u] - depth[v]);
             if (u == v) {
114
115
                 return u;
116
117
             for (int bit = anc[0].size() - 1; ~bit; --bit) {
118
                 if (anc[u][bit] != anc[v][bit]) {
                     u = anc[u][bit];
119
                     v = anc[v][bit];
120
121
                }
             }
122
123
             return anc[u][0];
124
         }
```

```
125
126
         template <class... T>
127
         void add_query(int u, int v, T &...x) {
128
             if (st[u] > st[v]) {
129
                 swap(u, v);
130
             }
131
             int lca = LCA(u, v), l, r, q lca = -1;
132
             if (lca == u || lca == v) {
133
                 l = st[u] + val on edge, r = st[v];
134
             } else {
135
                 1 = en[u], r = st[v], q_lca = lca;
136
137
             Q.push_back({1, r, q++, q_lca, hilbertOrder(1, r, __lg(flat.size())
                 + 1, 0), x...});
138
        }
139
140
         vector<int> Mo() {
141
             vector<int> ans(q):
142
             sort(Q.begin(), Q.end(), [&](const query &a, const query &b) {
143
                 return a.ord < b.ord:
144
             });
145
             int 1 = 0, r = -1:
146
             for (auto [L, R, q_indx, lca, ord] : Q) {
147
                 while (1 > L)
148
                     add(--1):
149
                 while (r < R)
150
                     add(++r):
151
                 while (r > R)
152
                     remove(r--);
153
                 while (1 < L)
154
                     remove(1++);
155
156
                 if (~lca && !val on edge)
157
                     add_node(lca);
158
                 ans[q_indx] = calc();
159
                 if (~lca && !val on edge)
160
                     remove_node(lca);
161
162
             return ans;
163
164 }:
```

2.12 HLD.cpp

```
struct HLD {
2
        int n;
        std::vector<int> siz, top, dep, parent, in, out, seq;
        std::vector<std::vector<int>> adj;
        int cur;
       HLD() {}
        HLD(int n) {
9
            init(n);
10
       }
11
        void init(int n) {
            this \rightarrow n = n;
12
13
            siz.resize(n);
            top.resize(n);
14
            dep.resize(n);
15
16
            parent.resize(n);
17
            in.resize(n);
18
            out.resize(n);
19
            seq.resize(n);
            cur = 0:
20
21
            adj.assign(n, {});
       }
22
23
        void addEdge(int u, int v) {
24
            adj[u].push_back(v);
            adj[v].push_back(u);
25
26
       }
27
        void work(int root = 0) {
            top[root] = root;
28
            dep[root] = 0;
29
            parent[root] = -1;
30
            dfs1(root);
31
            dfs2(root);
32
       }
33
34
        void dfs1(int u) {
            if (parent[u] != -1) {
35
36
                adj[u].erase(std::find(adj[u].begin(), adj[u].end(), parent[u]))
            }
37
38
39
            siz[u] = 1;
40
            for (auto &v : adj[u]) \{
41
                parent[v] = u;
```

```
42
                dep[v] = dep[u] + 1;
43
                dfs1(v);
44
                siz[u] += siz[v];
45
                if (siz[v] > siz[adj[u][0]]) {
                    std::swap(v, adj[u][0]);
46
47
                }
48
            }
49
        }
        void dfs2(int u) {
50
51
            in[u] = cur++:
52
            seq[in[u]] = u;
53
            for (auto v : adj[u]) {
                top[v] = v == adj[u][0] ? top[u] : v;
54
55
                dfs2(v);
56
            }
57
            out[u] = cur;
        }
58
        int lca(int u, int v) {
59
            while (top[u] != top[v]) {
                if (dep[top[u]] > dep[top[v]]) {
61
62
                    u = parent[top[u]];
63
                } else {
64
                    v = parent[top[v]];
65
                }
66
            }
67
            return dep[u] < dep[v] ? u : v;</pre>
       }
69
70
        int dist(int u, int v) {
71
            return dep[u] + dep[v] - 2 * dep[lca(u, v)];
72
       }
73
74
        int jump(int u, int k) {
75
            if (dep[u] < k) {
76
                return -1;
77
            }
78
79
            int d = dep[u] - k;
80
81
            while (dep[top[u]] > d) {
82
                u = parent[top[u]];
            }
83
84
```

```
return seq[in[u] - dep[u] + d];
 85
         }
 86
 87
 88
         bool isAncester(int u, int v) {
             return in[u] <= in[v] && in[v] < out[u];
 89
         }
 90
 91
         int rootedParent(int u, int v) {
 92
 93
             std::swap(u, v);
             if (u == v) {
 94
                 return u;
 95
             }
             if (!isAncester(u, v)) {
 97
 98
                 return parent[u];
             }
 99
             auto it = std::upper_bound(adj[u].begin(), adj[u].end(), v, [&](int
100
                 x, int y) {
                 return in[x] < in[y];</pre>
101
102
             }) - 1;
             return *it;
103
104
         }
105
         int rootedSize(int u. int v) {
106
             if (u == v) {
107
108
                 return n:
109
             if (!isAncester(v, u)) {
110
111
                 return siz[v];
112
             return n - siz[rootedParent(u, v)]:
113
         }
114
115
         int rootedLca(int a, int b, int c) {
116
             return lca(a, b) ^ lca(b, c) ^ lca(c, a);
117
118
         }
119 };
```

2.13 kdtree.cpp

```
1 namespace kd {
2 const int K = 2, N = 2.1e5;
3 template <typename T>
4 using P = array<T, K>;
```

```
5 template <typename T>
   struct node {
      P<T> pt, mx, mn;
      ll val, sum;
      node *1, *r, *p;
10
      int id;
      node(const P<T> & pt = P<T>(), 11 val = 0, int id = 0)
          : pt(_pt), val(_val), sum(_val), id(_id) {
13
        mx = mn = pt;
        p = 1 = r = nullptr;
15
16 };
17 node<11> *ptr[N];
   template <typename T>
    void pull(node<T> *u) {
      if (not u) return;
21
      u \rightarrow sum = u \rightarrow val;
      rep(i, 0, K - 1) u \rightarrow mx[i] = u \rightarrow mn[i] = u \rightarrow pt[i];
23
      if (u->1) {
24
       u->sum += u->l->sum:
25
        u - > 1 - > p = u;
27
     if (u->r) {
        u->sum += u->r->sum;
29
        u \rightarrow r \rightarrow p = u;
30
31
      rep(i, 0, K - 1) {
        if (u->1) {
          u - mx[i] = max(u - mx[i], u - l - mx[i]);
           u->mn[i] = min(u->mn[i], u->l->mn[i]);
34
      }
35
        if (u->r) {
           u - mx[i] = max(u - mx[i], u - r - mx[i]);
           u \rightarrow mn[i] = min(u \rightarrow mn[i], u \rightarrow r \rightarrow mn[i]);
39
        }
40
     }
41 }
42
43 template <typename T>
44 node<T> *build(vector<node<T>> &a, int 1, int r, int d = 0) {
      if (d == K) d = 0;
     if (1 >= r) {
46
47
        return nullptr;
```

```
} else {
48
        int md = (1 + r) >> 1;
49
        nth_element(a.begin() + 1, a.begin() + md, a.begin() + r,
50
51
                     [&](node<T> &x, node<T> &y) { return x.pt[d] < y.pt[d]; });
52
        node < T > *p = new node < T > (a[md]):
53
        ptr[p->id] = p;
54
        p->1 = build(a, 1, md, d + 1);
        p->r = build(a, md + 1, r, d + 1);
55
        pull(p);
56
57
        return p;
58
59 }
60
    template <typename T>
61
    node<T> *search(node<T> *u, P<T> p, int d = 0) {
      if (d == K) d = 0;
      if (not u) return nullptr;
64
65
     if (u->pt == p) return u;
     if (p[d] < u->pt[d]) {
66
        return search(u->1, p, d + 1);
67
     } else if (p[d] > u->pt[d]) {
        return search(u->r, p, d + 1);
69
     } else {
70
71
        auto tmp = search(u->1, p, d + 1);
72
        if (tmp) return tmp;
73
        return search(u \rightarrow r, p, d + 1);
     }
75 }
76
   template <typename T>
77
    void modify(node<T> *u, ll v) {
79
      if (not u) return;
     u \rightarrow val = v:
80
     for (auto cur = u; cur; cur = cur->p) {
81
        pull(cur);
82
83
     }
84 }
85
    template <typename T>
    bool inside(node<T> *nd, P<T> p, 11 c) {
      int cc = 0:
     if (nd->mx[0] * p[0] + nd->mx[1] * p[1] >= c) cc++;
     if (nd-mn[0] * p[0] + nd-mn[1] * p[1] >= c) cc++;
```

```
if (nd->mx[0] * p[0] + nd->mn[1] * p[1] >= c) cc++;
      if (nd-mn[0] * p[0] + nd-mx[1] * p[1] >= c) cc++;
 93
      return cc == 0:
 94 }
 95
     template <typename T>
     bool outside(node<T> *nd, P<T> p, ll c) {
       int cc = 0:
      if (nd-mx[0] * p[0] + nd-mx[1] * p[1] >= c) cc++;
      if (nd-mn[0] * p[0] + nd-mn[1] * p[1] >= c) cc++;
100
101
      if (nd->mx[0] * p[0] + nd->mn[1] * p[1] >= c) cc++;
102
      if (nd-mn[0] * p[0] + nd-mx[1] * p[1] >= c) cc++;
103
      return cc == 4:
104 }
105
     template <typename T>
    ll query(nodeT> *u, PT> p, ll c) {
      if (inside(u, p, c)) return u->sum;
      if (outside(u, p, c)) return 0;
110
      11 s = 0:
111
      if (u->pt[0] * p[0] + u->pt[1] * p[1] < c) {
112
        s += u->val:
113
     }
114
      if (u\rightarrow 1) s += query(u\rightarrow 1, p, c);
      if (u->r) s += query(u->r, p, c);
115
116
      return s;
117 }
118
     template <typename T>
    T eval_min(node<T> *nd,
121
               P<T>p) { // 通过估价函数进行启发式搜索,根据当前结果对搜索剪枝
122
      if (not nd) return numeric_limits<T>::max() / 4;
123
      11 s = 0:
124
      rep(i, 0, K - 1) {
125
        if (p[i] <= nd->mn[i]) s += nd->mn[i] - p[i];
126
        if (p[i] >= nd->mx[i]) s += p[i] - nd->mx[i];
127
128
      return s;
129 }
130
131 template <typename T>
132 ll mindist(node<T> *u, P<T> p) {
      11 s = numeric limits<T>::max() / 4;
```

```
if (u->pt != p) {
134
         s = min(s, abs(u->pt[0] - p[0]) + abs(u->pt[1] - p[1]));
135
136
       ll best1 = eval min(u->1, p), best2 = eval min(u->r, p);
137
       if (best1 < best2) {
138
         if (u->1) s = min(s, mindist(u->1, p));
139
         if (u->r) and best2 < s) s = min(s, mindist(u->r, p));
140
141
         return s:
       } else {
142
         if (u->r) s = min(s, mindist(u->r, p)):
143
         if (u\rightarrow 1 \text{ and best} 1 < s) s = min(s, mindist(u\rightarrow 1, p));
144
145
         return s;
146
147 }
148
     template <typename T>
149
     T eval max(node<T> *nd,
150
                P<T> p) { // 通过估价函数进行启发式搜索,根据当前结果对搜索剪枝
151
152
       if (not nd) return 0;
      11 s = 0:
153
154
       rep(i, 0, K - 1) s += max(abs(nd->mx[i] - p[i]), abs(nd->mn[i] - p[i]));
155
156 }
157
     template <typename T>
158
     11 maxdist(node<T> *u, P<T> p) {
159
       11 s = 0:
160
161
       if (u->pt != p) {
         s = max(s, abs(u->pt[0] - p[0]) + abs(u->pt[1] - p[1]));
162
163
       11 best1 = eval max(u->1, p), best2 = eval max(u->r, p);
164
165
       if (best1 > best2) {
         if (u->1) s = max(s, maxdist(u->1, p));
166
167
         if (u->r \text{ and best2} > s) s = max(s, maxdist(u->r, p));
168
         return s;
       } else {
169
         if (u->r) s = max(s, maxdist(u->r, p));
170
         if (u\rightarrow 1 \text{ and best} 1 \rightarrow s) s = max(s, maxdist(u\rightarrow 1, p));
171
172
         return s:
      }
173
174 }
175 } // namespace kd
```

2.14 LCT.cpp

```
1 namespace linkCutTree {
   struct node {
        node *child[2], *parent, *max;
        int id;
       ll sum, val, sz, weight, rev;
       node(ll val, ll weight, int id) : child {nullptr, nullptr}, parent(
            nullptr), max(this), sum(val), val(val), sz(weight), weight(weight),
             id(id), rev(false) {}
7 };
9 bool isRoot(node *p) { return p->parent == nullptr || (p->parent->child[0]
        != p && p->parent->child[1] != p); }
10
   int side(node *p) { return p->parent->child[1] == p; }
12
13 ll sum(node *p) { return p == nullptr ? 0 : p->sum; }
14
15
   11 sz(node *p) { return p == nullptr ? 0 : p->sz; }
16
   node *max(node *p) { return p == nullptr ? nullptr : p->max; }
17
18
19
    node *max(node *p, node *q) {
20
        if (p == nullptr)
21
            return q;
22
        if (q == nullptr)
23
            return p;
24
        return p->weight > q->weight ? p : q;
25 }
26
   void reverse(node *p) {
28
       if (p == nullptr)
29
            return:
30
        swap(p->child[0], p->child[1]);
        p->rev ^= 1;
32 }
34 void push(node *p) {
        if (p\rightarrow rev == 0)
            return:
37
       p \rightarrow rev = 0;
38
       reverse(p->child[0]);
39
        reverse(p->child[1]);
```

```
40 }
41
42 void pull(node *p) {
43
        p->sum = sum(p->child[0]) + sum(p->child[1]) + p->val;
        p\rightarrow max = max(max(max(p\rightarrow child[0]), max(p\rightarrow child[1])), p);
44
        p->sz = p->weight + sz(p->child[0]) + sz(p->child[1]);
45
46 }
47
    void connect(node *p, node *q, int side) {
        q->child[side] = p;
49
        if (p != nullptr)
50
51
            p->parent = q;
52 }
53
    void rotate(node *p) {
54
55
        auto q = p->parent;
56
        int dir = side(p) ^ 1;
57
        connect(p->child[dir], q, dir ^ 1);
58
        if (!isRoot(q))
59
            connect(p, q->parent, side(q));
60
61
            p->parent = q->parent;
62
        connect(q, p, dir);
63
        pull(q);
64 }
65
    void splay(node *p) {
66
67
        vector<node *> stk;
68
        for (auto i = p; !isRoot(i); i = i->parent)
69
            stk.push_back(i->parent);
70
        while (!stk.empty()) {
71
            push(stk.back());
72
            stk.pop_back();
        }
73
74
        push(p);
75
        while (!isRoot(p)) {
76
            auto q = p->parent;
77
            if (!isRoot(q))
78
                 rotate(side(p) == side(q) ? q : p);
79
            rotate(p);
80
        }
        pull(p);
81
82 }
```

```
83
 84 node *access(node *p) {
 85
         node *j = nullptr;
 86
         for (node *i = p; i != nullptr; j = i, i = i -> parent) {
 87
             splay(i);
 88
             i->val -= sum(j);
 89
             i->val += sum(i->child[1]);
 90
             i->child[1] = j;
 91
             pull(i);
 92
         }
 93
         splay(p);
 94
         return j;
 95 }
 96
    void makeRoot(node *p) {
 98
         access(p);
 99
         reverse(p);
100 }
101
102 void link(node *p, node *q) {
103
         makeRoot(p);
104
         access(q);
105
         p->parent = q;
106
         q->val += sum(p);
107 }
108
109
    void cut(node *p, node *q) {
110
         makeRoot(p);
111
         access(q);
112
         p->parent = q->child[0] = nullptr;
113 }
114
115 node *pathMax(node *p, node *q) {
116
         makeRoot(p);
117
         access(q);
118
         return max(q);
119 }
120
121 11 pathSize(node *p, node *q) {
122
         makeRoot(p);
123
         access(q);
124
         return sz(q);
125 }
```

```
126
    11 rootedSum(node *p) {
127
         makeRoot(p):
128
129
         return sum(p);
130 }
131
    11 getSubtree(node *rt, node *v) {
132
         makeRoot(rt):
133
134
         access(v);
         return v->val:
135
136 }
137
    bool connected(node *p, node *q) {
138
139
         access(p);
140
         access(q);
         return p->parent != nullptr;
141
142 }
143
    void fix(node *p, ll v) {
144
         access(p);
145
146
         push(p);
         // modify ...
147
148
         p \rightarrow val += v;
149
         pull(p);
150 }
151
    node *lca(node *z,node *x,node *y) {
152
         makeRoot(z):
153
154
         access(x);
         return access(y);
155
156 }
157 } // namespace linkCutTree
158 using namespace linkCutTree;
```

2.15 lichao-tree.cpp

```
struct Line {
  i64 k. b:
  i64 operator()(i64 x) const { return k * x + b; }
template <i64 L, i64 R>
struct Segments {
  struct Node {
```

```
8
        optional < Line > s;
9
        Node *1, *r;
10
     }:
11
      Node *root;
      Segments() : root(nullptr) {}
12
      void add(i64 1, i64 r, i64 k, i64 b) {
13
        auto rec = [&](auto &rec, Node *&p, i64 tl, i64 tr, Line s) -> void {
14
          if (p == nullptr) p = new Node();
15
16
          i64 tm = midpoint(tl, tr);
          if (t1 >= 1 and tr <= r) {
17
18
            if (not p->s) return p->s = s, void();
19
            auto t = p->s.value();
            if (t(t1) >= s(t1)) {
20
21
              if (t(tr) >= s(tr)) return;
22
              if (t(tm) \ge s(tm)) return rec(rec, p->r, tm + 1, tr, s);
23
              return p \rightarrow s = s, rec(rec, p \rightarrow l, tl, tm, t);
24
            if (t(tr) \le s(tr)) return p->s = s, void();
            if (t(tm) \le s(tm)) return p->s = s, rec(rec, p->r, tm + 1, tr, t);
27
            return rec(rec, p->1, t1, tm, s);
28
          if (1 <= tm) rec(rec, p->1, t1, tm, s):
          if (r > tm) rec(rec. p->r. tm + 1. tr. s):
31
        };
32
        rec(rec. root, L. R. {k, b}):
33
34
      optional < i64 > get(i64 x) {
35
        optional < i64> res = {};
36
        auto rec = [&](auto &rec, Node *p, i64 tl, i64 tr) -> void {
37
          if (p == nullptr) return;
38
          i64 tm = midpoint(tl, tr);
39
          if (p\rightarrow s) {
            i64 y = p -> s.value()(x);
40
41
            if (not res or res.value() < y) res = y;</pre>
42
          }
43
          if (x <= tm)
44
            rec(rec, p->1, t1, tm);
45
          else
46
            rec(rec, p->r, tm + 1, tr);
47
        };
48
        rec(rec, root, L, R);
        return res:
50
     }
```

```
51 };
```

2.16 Mo.cpp

```
1 /**
    * #define K(x) pii(x.first/blk, x.second ^-(x.first/blk & 1))
            iota(all(s), 0):
            sort(all(s), [@](int s, int t) \{ return K(Q[s]) < K(Q[t]); \});
   VI Mo(const vector < array < int, 3>> &Q) {
        const int blk = 350;
       vector<int> s(SZ(Q)), res = s;
9
       iota(all(s), 0);
10
11
       sort(all(s), [&](int i, int j) {
12
           int u = Q[i][0] / blk, v = Q[j][0] / blk;
           return u == v ? u % 2 ? Q[i][1] > Q[j][1] : Q[i][1] < Q[j][1] : u <
13
                v;
       });
14
       int L = 1. R = 0:
15
       for (int qi : s) {
17
            while (R < Q[qi][1]) R++, add(R);
18
            while (L > Q[qi][0]) L--, add(L);
            while (R > Q[qi][1]) del(R), R--;
19
            while (L < Q[qi][0]) del(L), L++;
20
21
           res[qi] = calc(Q[qi][2]);
22
       }
23
        return res;
24 }
```

2.17 moTree.cpp

1 /**

```
2 * Author: Simon Lindholm
3 * Date: 2019-12-28
4 * License: CCO
5 * Source: https://github.com/hoke-t/tamu-kactl/blob/master/content/data-structures/MoQueries.h
6 * Description: Answer interval or tree path queries by finding an approximate TSP through the queries,
7 * and moving from one query to the next by adding/removing points at the ends.
```

```
* If values are on tree edges, change \texttt{step} to add/remove the edge
         $(a, c)$ and remove the initial \texttt{add} call (but keep \texttt{in
   * Time: O(N \sqrt Q)
    * Status: stress-tested
11
12
13 void add(int ind, int end) \{ \dots \} // add \ a \ [ind ] \ (end = 0 \ or \ 1)
   void del(int ind, int end) { ... } // remove a [ ind ]
15 int calc() { ... } // compute current answer
16 vi mo(vector<pii>Q) {
    int L = 0, R = 0, blk = 350; // N/sqrt(Q)
    vi s(sz(Q)), res = s:
19 #define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
20
     iota(all(s), 0):
21
     sort(all(s), [\&](int s, int t) \{ return K(Q[s]) < K(Q[t]); \});
     for (int qi : s) {
       pii q = Q[qi];
24
       while (L > q.first) add(--L, 0);
       while (R < q.second) add(R++, 1);
25
26
       while (L < q.first) del(L++, 0);
       while (R > q.second) del(--R, 1):
       res[gi] = calc():
29
    }
30
     return res:
31 }
32 vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root = 0) {
     int N = sz(ed), pos[2] = {}, blk = 350; // N/sqrt (Q)
     vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
     add(0, 0), in[0] = 1:
     auto dfs = [&](int x, int p, int dep, auto & f) -> void {
37
       par[x] = p;
       L[x] = N;
     if (dep) I[x] = N++;
       for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
41
       if (!dep) I[x] = N++;
42
       R[x] = N:
43
    };
     dfs(root, -1, 0, dfs);
   #define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
46
     iota(all(s), 0);
47
     sort(all(s), [\&](int s, int t) { return K(Q[s]) < K(Q[t]); });
48
     for (int qi : s) rep(end, 0, 2) {
```

```
int &a = pos[end], b = Q[qi][end], i = 0;
    #define step(c) { if (in[c]) { del(a, end); in[a] = 0; }
    else { add(c, end); in[c] = 1; } a = c; }
52
        while (!(L[b] \le L[a] \&\& R[a] \le R[b]))
         I[i++] = b, b = par[b];
53
       while (a != b) step(par[a]);
54
       while (i--) step(I[i]);
       if (end) res[qi] = calc();
56
57
     }
58
     return res:
59 }
```

2.18 MSTMo.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 #define rep(i,a,n) for (int i=a;i<n;i++)</pre>
4 #define per(i,a,n) for (int i=n-1;i>=a;i--)
5 #define pb push back
6 #define mp make_pair
7 #define all(x) (x).begin(),(x).end()
8 #define fi first
9 #define se second
10 #define SZ(x) ((int)(x).size())
11 typedef vector <int> VI;
12 typedef long long 11;
13 typedef pair<int,int> PII;
14 typedef double db;
15 mt19937 mrand(random device{}());
16 const ll mod=1000000007:
17 int rnd(int x) { return mrand() % x:}
18 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b;b>>=1){if(b&1)
        res=res*a%mod:a=a*a%mod:}return res:}
19  11 gcd(11 a,11 b) { return b?gcd(b,a%b):a;}
20 // head
21
22 const int N=1010000;
23 int a[N]:
   namespace Mo {
     int Q,1[N],r[N],f[N],10,r0,ans[N],n;
26
     VI ne[N];
27
     struct point {
       int x, y, o;
```

```
29
        point(int a, int b, int c): x(a), y(b), o(c) {}
     };
30
31
      inline bool operator < (const point &a. const point &b) {
32
       if (a.x != b.x) return a.x > b.x;
33
        else return a.y < b.y;</pre>
34
     }
35
      vector<point> p;
      struct edge {
37
       int s, t, d;
       edge(const point &a, const point &b): s(a.o), t(b.o),
39
          d(abs(a.x - b.x) + abs(a.y - b.y)) {}
40
     };
41
      inline bool operator < (const edge &a. const edge &b) {return a.d < b.d:}
      vector<edge> e;
43
      int g[N],z[N];
      int cc, cnt[101000];
44
45
      void addedge() {
       sort(all(p));
47
          memset(g,0,sizeof(g));
         z[0]=N:
48
49
       rep(i,0,SZ(p)) z[i+1]=p[i].x-p[i].y;
       rep(i,0,SZ(p)) {
51
              int k = 0, t = p[i].x + p[i].y;
52
              for (int j = t; j; j -= j & -j)
53
                  if (z[g[j]] < z[k]) k = g[j];
54
              if (k) e.pb(edge(p[i], p[k - 1]));
              k = z[i + 1]:
              for (int j = t; j < N; j += j & -j)
57
                  if (k < z[g[j]]) g[j] = i + 1;
58
         }
     }
      void updata(int i, bool j,bool k=0) {
61
       // j=1 insert j=0 delete
       // k=0 left k=1 right
63
       if (j==1) {
          cnt[a[i]]++;
          if (cnt[a[i]]%2==0) cc++;
       } else {
          if (cnt[a[i]]%2==0) cc--;
          cnt[a[i]]--;
69
       }
70
71
      void init(int 1,int r) {
```

```
72
         for (int i=1:i<=r:i++) {
 73
           cnt[a[i]]++;
           if (cnt[a[i]]%2==0) cc++;
 74
 75
        }
      }
 76
       inline int query() {
 77
         return cc;
 78
 79
       int find(int x) { if (f[x] != x) f[x] = find(f[x]); return f[x];}
 80
       void dfs(int i,int p) {
 81
 82
        int l1 = l[i], r1 = r[i];
         per(j,11,10) updata(j,1,0);
 83
         rep(j,r0+1,r1+1) updata(j,1,1);
 84
         rep(j,10,11) updata(j,0,0);
 85
 86
         per(j,r1+1,r0+1) updata(j,0,1);
         ans[i]=query();10=11;r0=r1;
 87
         rep(j,0,SZ(ne[i])) if (ne[i][j]!=p) dfs(ne[i][j],i);
 88
      }
 89
       void solve() {
 90
         p.clear();e.clear();
 91
         rep(i,1,Q+1) ans[i]=0;
 92
         rep(i,1,Q+1) p.pb(point(l[i],r[i],i));
 93
         addedge();
 94
         rep(i,0,SZ(p)) p[i].y =n-p[i].y+1;
 95
 96
         addedge();
         rep(i,0,SZ(p)) {
 97
          int j =n-p[i].x+1;
          p[i].x = p[i].y; p[i].y = j;
 99
100
        }
101
         addedge():
         rep(i,0,SZ(p)) p[i].x=n-p[i].x+1;
102
103
         addedge();
         sort(all(e));
104
         rep(i,1,Q+1) ne[i].clear(),f[i]=i;
105
106
         rep(i,0,SZ(e)) {
107
          int j=e[i].s,k=e[i].t;
          if (find(j)!=find(k)) f[f[j]]=f[k],ne[j].pb(k),ne[k].pb(j);
108
        }
109
        10=1[1];r0=r[1];
110
111
        init(10,r0);
112
         dfs(1,0);
      }
113
114 }
```

```
115
116 int main() {
117     scanf("%d",&Mo::n);
118     for (int i=1;i<=Mo::n;i++) scanf("%d",a+i);
119     scanf("%d",&Mo::Q);
120     rep(i,1,Mo::Q+1) scanf("%d%d",&Mo::l[i],&Mo::r[i]);
121     Mo::solve();
122     rep(i,1,Mo::Q+1) printf("%d\n",Mo::ans[i]);
123 }
```

2.19 psegt.cpp

```
1 struct node {
      node *1. *r:
      ll val, sz, add;
4 }:
5
6 void pull(node *u) {
      u -> sz = 0, u -> val = 0;
     if (u->1) u->sz += u->1->sz, u->val += u->1->val;
      if (u\rightarrow r) u\rightarrow sz += u\rightarrow r\rightarrow sz, u\rightarrow val += u\rightarrow r\rightarrow val:
10 }
11
12 void push(node *u) {
      if (u->add) {
14
        if (u->1) {
15
           node *p = new node();
16
          *p = *u ->1;
17
           u ->1 = p;
           p->add += u->add:
           p->val += p->sz * u->add;
20
        }
21
        if (u->r) {
22
           node *p = new node();
23
           *p = *u -> r:
24
           u->r = p;
25
           p->add += u->add;
           p->val += p->sz * u->add;
26
27
        }
        u->add = 0:
28
29
     }
30 }
31
```

```
32 node *build(int 1, int r) {
      node *p = new node();
     p->add = 0;
34
35
     if (1 == r) {
       p->1 = p->r = nullptr;
36
37
       p->val = a[1];
       p->sz = 1;
     } else {
40
       int mid = (1 + r) >> 1;
       p->1 = build(1, mid);
41
       p \rightarrow r = build(mid + 1, r);
42
       pull(p);
43
44
45
      return p;
46 }
47
    ll query(node *v, int l, int r, int ql, int qr) {
      if (ql == 1 && qr == r) {
50
       return v->val;
     } else {
51
52
       push(v);
       int mid = (1 + r) >> 1:
53
       if (ar <= mid)
54
55
         return query(v->1, 1, mid, q1, qr);
56
       else if (ql > mid)
         return query(v->r, mid + 1, r, ql, qr);
          return query(v->1, 1, mid, q1, mid) +
59
60
                 query(v->r, mid + 1, r, mid + 1, qr);
61
62 }
   node *modify(node *v, int 1, int r, int q1, int qr, l1 x) {
65
      if (ql == 1 && qr == r) {
       node *p = new node();
66
       *p = *v;
       p->add += x;
       p->val += p->sz * x;
69
70
       return p;
     } else {
71
72
       push(v);
73
       int mid = (1 + r) >> 1;
74
       node *p = new node();
```

```
75
        *p = *v;
        if (qr <= mid)
77
          p->1 = modify(v->1, 1, mid, ql, qr, x);
78
        else if (ql > mid)
          p->r = modify(v->r, mid + 1, r, ql, qr, x);
80
        else
          p->1 = modify(v->1, 1, mid, ql, mid, x),
          p \rightarrow r = modify(v \rightarrow r, mid + 1, r, mid + 1, qr, x);
83
        pull(p);
        return p;
86 }
```

2.20 rollbackMo.cpp

```
1 VI rollbackMo(const vector<array<int, 3>> &Q) {
        const int blk = 350;
       vector < VI > s(SZ(Q)):
       vector<int> BF, res(SZ(Q));
       for (int i = 0; i < SZ(Q); i++) {
            int u = Q[i][0] / blk, v = Q[i][1] / blk;
7
            if (u == v) BF.push back(i);
8
            else s[u].push_back(i);
9
10
        for (int i = 0; i < SZ(Q); i++)
11
            sort(all(s[i]), [&](int i, int j) { return Q[i][1] < Q[j][1]; });</pre>
12
       for (int qi : BF) {
13
            for (int i = Q[qi][0]; i \le Q[qi][1]; i++)
14
                add(i);
15
            res[qi] = calc(Q[qi][2]);
            for (int i = Q[qi][0]; i <= Q[qi][1]; i++)
16
17
                del(i);
       }
18
        for (const auto &v : s) {
19
20
            if (v.empty()) continue;
21
            int next blk = (Q[v.back()][0] / blk + 1) * blk;
            int L = next_blk, R = next_blk - 1;
23
            for (int qi : v) {
24
                while (R < Q[qi][1]) R++, add(R);
25
                while (L > Q[qi][0]) L--, add(L);
26
                res[qi] = calc(Q[qi][2]);
                while (L < next_blk) del(L), L++;</pre>
27
28
            }
```

2.21 segtree.cpp

```
struct info {
        ll sum;
        int sz:
        friend info operator+(const info &a, const info &b) {
            return {(a.sum + b.sum) % mod, a.sz + b.sz};
        }
 7 };
    struct tag {
        ll add. mul:
10
        friend tag operator+(const tag &a, const tag &b) {
11
            tag res = {(a.add * b.mul + b.add) % mod, a.mul * b.mul % mod};
12
13
            return res:
14
       }
15 }:
    info operator+(const info &a, const tag &b) {
        return {(a.sum * b.mul + a.sz * b.add) % mod, a.sz};
17
18 }
19
    struct node {
21
        info val;
        tag t;
    } seg[maxn << 2];</pre>
24
    void update(int id) {
25
26
        seg[id].val = seg[id * 2].val + seg[id * 2 + 1].val;
27 }
28 void settag(int id, tag t) {
29
        seg[id].val = seg[id].val + t;
30
        seg[id].t = seg[id].t + t;
31 }
32 void pushdown(int id) {
33
        if (seg[id].t.mul == 1 and seg[id].t.add == 0) return;
34
        settag(id * 2, seg[id].t);
35
        settag(id * 2 + 1, seg[id].t);
```

```
36
        seg[id].t.mul = 1;
37
        seg[id].t.add = 0;
38 }
39 void build(int 1, int r, int id) {
        seg[id].t = {0, 1};
41
       if (1 == r) {
42
            seg[id].val = {a[1], 1};
43
       } else {
44
            int mid = (1 + r) >> 1;
45
            build(1, mid, id * 2):
            build(mid + 1, r, id * 2 + 1);
47
            update(id);
       }
48
49 }
   void change(int 1, int r, int id, int ql, int qr, tag t) {
51
        if (1 == ql && r == qr) {
52
            settag(id, t);
53
       } else {
54
            int mid = (1 + r) >> 1;
55
            pushdown(id);
56
            if (qr <= mid) {
57
                change(1, mid, id * 2, ql, qr, t);
           } else if (al > mid) {
58
59
                change(mid + 1, r, id * 2 + 1, ql, qr, t);
60
           } else {
                change(1, mid, id * 2, q1, mid, t);
61
                change(mid + 1, r, id * 2 + 1, mid + 1, qr, t);
            update(id);
65
       }
66 }
67 info query(int 1, int r, int id, int ql, int qr) {
        if (1 == q1 && r == qr) {
69
            return seg[id].val;
70
       } else {
71
            int mid = (1 + r) >> 1;
72
            pushdown(id);
73
            if (qr <= mid)</pre>
74
                return query(1, mid, id * 2, ql, qr);
75
            else if (ql > mid)
76
                return query(mid + 1, r, id * 2 + 1, ql, qr);
77
            else
78
                return query(1, mid, id * 2, q1, mid) +
```

```
80
 81 }
     11 search(int 1, int r, int id, int ql, int qr, int d) {
         if (ql == 1 && qr == r) {
             int mid = (1 + r) / 2;
 84
             // if (l != r) pushdown(id); ...
             if (seg[id].val < d)</pre>
                 return -1;
             else {
                 if (1 == r)
                     return 1;
                 else if (seg[id * 2].val >= d)
 91
 92
                     return search(1, mid, id * 2, ql, mid, d);
 93
                 else
 94
                      return search(mid + 1, r, id * 2 + 1, mid + 1, qr, d);
             }
 95
         } else {
 97
             int mid = (1 + r) >> 1;
             // pushdown(id); ...
 98
 99
             if (qr <= mid)
                 return search(1, mid, id * 2, q1, qr, d);
100
             else if (al > mid)
101
102
                 return search(mid + 1, r, id * 2 + 1, ql, qr, d);
103
             else {
                  int tmp = search(1, mid, id * 2, q1, mid, d);
104
                 if (tmp != -1)
105
106
                     return tmp;
                  else
107
                     return search(mid + 1, r, id * 2 + 1, mid + 1, gr, d):
108
109
110
111 }
```

query(mid + 1, r, id * 2 + 1, mid + 1, qr);

2.22 segtreefast.cpp

79

```
1 /**
2 * Author: Lucian Bicsi
3 * Description: Very fast and quick segment tree.
4 * Only useful for easy invariants. O-indexed.
5 * Range queries are half-open.
6 */
7 #pragma once
```

```
8
9 struct SegmTree {
10
      vector<int> T: int n:
      SegmTree(int n) : T(2 * n, (int)2e9), n(n) {}
11
12
13
      void Update(int pos, int val) {
        for (T[pos += n] = val; pos > 1; pos /= 2)
14
15
          T[pos / 2] = min(T[pos], T[pos ^ 1]);
16
17
18
      int Query(int b, int e) {
19
        int res = (int)2e9;
       for (b += n, e += n; b < e; b /= 2, e /= 2) {
20
21
          if (b \% 2) res = min(res, T[b++]);
22
         if (e \% 2) res = min(res, T[--e]):
23
       }
24
        return res;
26 };
```

2.23 SparseTable.cpp

```
1 template <typename T, class F = function <T(const T&, const T&)>>
2 class SparseTable {
   public:
     int n:
      vector<vector<T>> mat;
     F func:
7
8
      SparseTable(const vector < T > & a, const F & f) : func(f) {
9
       n = static_cast<int>(a.size());
10
       int max log = 32 - builtin clz(n);
11
       mat.resize(max_log);
12
       mat[0] = a;
13
       for (int j = 1; j < max_log; j++) {
14
          mat[j].resize(n - (1 << j) + 1);
15
         for (int i = 0; i \le n - (1 \le j); i++) {
16
           mat[j][i] = func(mat[j-1][i], mat[j-1][i+(1 << (j-1))]);
         }
17
18
       }
19
20
21
     T get(int from, int to) const {
```

```
22 assert(0 <= from && from <= to && to <= n - 1);
23 int lg = 32 - __builtin_clz(to - from + 1) - 1;
24 return func(mat[lg][from], mat[lg][to - (1 << lg) + 1]);
25 }
26 };
```

2.24 SparseTable2D.cpp

```
1 // lq[1] = 0;
2 // rep(i, 2, N-1)  {
3 // lq[i] = lq[i / 2] + 1;
4 // }
5 // int k = log2(r - l + 1); very slow!!!
6 // int k = lq(r - l + 1);
7 // int k = lq[r - l + 1];
8 // int k = 32 - \_builtin_clz(r - l + 1) - 1;
9 vector<vector<int>> sparse[12];
11 int query(int x, int y, int d) {
   int k = __lg(d);
    int s = d - bit(k);
    return min({sparse[k][x][y], sparse[k][x + s][y], sparse[k][x][y + s],
         sparse[k][x + s][y + s]);
15 }
16
17 void build() {
18
     rep(i, 1, n) rep(j, 1, m) sparse[0][i][j] = mat[i][j];
19
    rep(k, 1, 11) rep(i, 1, n) rep(j, 1, m) {
     int d = bit(k - 1);
20
      if (i + d > n | | i + d > m) continue:
21
       sparse[k][i][j] = min({sparse[k - 1][i][j], sparse[k - 1][i + d][j],}
           sparse[k - 1][i][j + d], sparse[k - 1][i + d][j + d]});
    }
23
24 }
```

2.25 treap.cpp

```
1 /**
2 * author: tourist
3 * created: 07.10.2022 20:32:03
4 **/
5 #include <bits/stdc++.h>
6
```

```
using namespace std;
8
9 #ifdef LOCAL
10 #include "algo/debug.h"
11 #else
12 #define debug(...) 42
13 #endif
14
15 mt19937 64 rng(chrono::steady clock::now().time since epoch().count());
16
17 class node {
18 public:
    int id:
    node* 1;
    node* r:
    node* p;
     bool rev;
    int sz:
     // declare extra variables:
26
    long long P;
     long long add;
28
     long long x;
29
30
     node(int _id, long long _x) {
31
     id = _id;
    l = r = p = nullptr;
      rev = false;
34
       sz = 1:
       // init extra variables:
       P = rng();
       add = 0;
       x = _x;
39
40
41
     // push everything else:
42
     void push stuff() {
43
       if (add != 0) {
         if (1 != nullptr) {
44
           1->unsafe_apply(add);
45
46
         }
47
         if (r != nullptr) {
48
           r->unsafe_apply(add);
49
         }
```

```
add = 0;
        }
51
     }
52
53
      void unsafe_reverse() {
54
        push_stuff();
55
        rev ^= 1;
56
        swap(1, r);
57
58
        pull();
     }
59
60
61
      // apply changes:
      void unsafe_apply(long long delta) {
62
63
        add += delta;
        x += delta;
64
     }
65
66
      void push() {
67
        if (rev) {
68
          if (l != nullptr) {
69
70
            1->unsafe_reverse();
71
          if (r != nullptr) {
72
73
            r->unsafe_reverse();
74
         }
75
          rev = 0;
        }
76
77
        push_stuff();
78
     }
79
80
      void pull() {
        sz = 1;
81
        if (1 != nullptr) {
82
83
         1->p = this;
          sz += 1->sz;
84
85
        if (r != nullptr) {
         r->p = this;
87
          sz += r->sz;
     }
91 };
92
```

```
void debug_node(node* v, string pref = "") {
 94 #ifdef LOCAL
      if (v != nullptr) {
        debug_node(v->r, pref + "");
        cerr << pref << "-" << "<sub>||</sub>" << v->id << '\n';
 97
        debug_node(v->1, pref + "");
     } else {
         cerr << pref << "-" << "nullptr" << '\n';
101
102 #endif
103 }
104
    namespace treap {
106
107
     pair<node*, int> find(node* v, const function<int(node*)> &go_to) {
108
      // go_to returns: 0 -- found; -1 -- go left; 1 -- go right
109
      // find returns the last vertex on the descent and its go to
110
      if (v == nullptr) {
111
        return {nullptr, 0};
112
      }
113
      int dir;
114
      while (true) {
115
        v->push();
116
        dir = go_to(v);
117
        if (dir == 0) {
118
           break;
119
        }
120
        node* u = (dir == -1 ? v->1 : v->r);
121
        if (u == nullptr) {
122
           break:
123
        }
124
        v = u;
125
126
      return {v, dir};
127 }
128
    node* get_leftmost(node* v) {
130
      return find(v, [&](node*) { return -1; }).first;
131 }
132
133 node* get_rightmost(node* v) {
      return find(v, [&](node*) { return 1; }).first;
135 }
```

```
136
     node* get kth(node* v, int k) { // O-indexed
137
       pair<node*, int> p = find(v, [&](node * u) {
138
139
         if (u->1 != nullptr) {
           if (u->1->sz > k) {
140
141
             return -1;
           }
142
143
           k = u > 1 > sz:
         }
144
         if (k == 0) {
145
           return 0;
146
         }
147
148
         k--:
149
         return 1;
150
       }):
       return (p.second == 0 ? p.first : nullptr);
151
152 }
153
     int get_pos(node* v) { // 0-indexed
154
       int k = (v->1 != nullptr ? v->1->sz : 0);
155
156
       while (v->p != nullptr) {
         if (v == v -> p -> r) {
157
          k++:
158
159
           if (v->p->l != nullptr) {
160
             k += v->p->1->sz;
          }
161
         }
162
163
         v = v -> p;
164
       return k:
165
166 }
167
     node* get root(node* v) {
168
169
       while (v->p != nullptr) {
170
         v = v - > p;
      }
171
       return v;
172
173 }
174
     pair<node*, node*> split(node* v, const function<bool(node*)> &is_right) {
175
       if (v == nullptr) {
176
         return {nullptr, nullptr};
177
178
      }
```

```
179
       v->push();
180
       if (is right(v)) {
181
         pair<node*, node*> p = split(v->1, is_right);
182
         if (p.first != nullptr) {
183
           p.first->p = nullptr;
184
         }
185
         v \rightarrow 1 = p.second;
         v->pull();
187
         return {p.first, v};
188
       } else {
189
         pair<node*, node*> p = split(v->r, is_right);
190
         v \rightarrow r = p.first;
191
         if (p.second != nullptr) {
192
           p.second->p = nullptr;
193
         }
194
         v->pull();
195
         return {v, p.second};
196
197 }
198
     pair<node*, node*> split_cnt(node* v, int k) {
200
       if (v == nullptr) {
201
         return {nullptr, nullptr};
202
      }
203
       v->push();
204
       int left_and_me = (v->l != nullptr ? v->l->sz : 0) + 1;
205
       if (k < left_and_me) {</pre>
206
         pair < node *, node *> p = split_cnt(v->1, k);
207
         if (p.first != nullptr) {
208
           p.first->p = nullptr;
209
         }
210
         v \rightarrow 1 = p.second;
211
         v->pull();
212
         return {p.first, v};
213
      } else {
214
         pair<node*, node*> p = split_cnt(v->r, k - left_and_me);
215
         v->r = p.first;
216
         if (p.second != nullptr) {
217
           p.second->p = nullptr;
218
         }
219
         v->pull();
220
         return {v, p.second};
221
      }
```

```
222 }
                                                                                                 265
                                                                                                          }
                                                                                                 266
                                                                                                        }
223
     node* merge(node* v, node* u) {
                                                                                                 267
224
225
       if (v == nullptr) {
                                                                                                 268 }
                                                                                                 269
         return u:
226
                                                                                                 270
227
       }
       if (u == nullptr) {
                                                                                                 271
228
         return v:
                                                                                                 272
229
                                                                                                 273 }
230
       }
       if (v->P > u->P) {
                                                                                                 274
231
            if (rnq() \% (v\rightarrow sz + u\rightarrow sz) < (unsigned int) v\rightarrow sz) {
                                                                                                 275
232
         v->push();
                                                                                                 276
233
         v \rightarrow r = merge(v \rightarrow r, u);
                                                                                                 277
234
                                                                                                 278
235
         v->pull();
         return v:
                                                                                                 279
236
                                                                                                 280
       } else {
                                                                                                 281
         u->push();
238
         u \rightarrow 1 = merge(v, u \rightarrow 1);
                                                                                                 282
239
                                                                                                 283
240
         u->pull();
                                                                                                 284
         return u:
241
                                                                                                 285
242
       }
243 }
                                                                                                 286
                                                                                                 287
244
     int count_left(node* v, const function<bool(node*)> &is_right) {
                                                                                                 288
245
       if (v == nullptr) {
                                                                                                 289
                                                                                                        }
246
                                                                                                 290
         return 0;
247
       }
                                                                                                 291
248
       v->push();
                                                                                                 292
249
       if (is_right(v)) {
                                                                                                 293
250
         return count_left(v->1, is_right);
                                                                                                 294
251
                                                                                                 295
252
253
       return (v->l != nullptr ? v->l->sz : 0) + 1 + count_left(v->r, is_right);
                                                                                                 296
                                                                                                 297
254 }
                                                                                                 298
255
     int count_less(node* v, long long val) {
                                                                                                 299
256
257
       int res = 0;
                                                                                                 300
                                                                                                 301
       while (v != nullptr) {
258
         v->push();
                                                                                                 302
259
         if (v->x >= val) {
260
                                                                                                 303
           v = v -> 1:
                                                                                                 304
261
         } else {
                                                                                                 305 }
262
           res += (v->1 != nullptr ? v->1->sz : 0) + 1;
                                                                                                 306
           v = v -> r;
264
```

```
return res:
     node* add(node* r, node* v, const function<bool(node*)> &go_left) {
       pair<node*, node*> p = split(r, go_left);
       return merge(p.first, merge(v, p.second));
     node* remove(node* v) { // returns the new root
      v->push();
      node* x = v->1:
      node* v = v->r;
      node* p = v -> p;
      v->1 = v->r = v->p = nullptr;
      v->push();
      v->pull(); // now v might be reusable...
       node* z = merge(x, y);
      if (p == nullptr) {
        if (z != nullptr) {
           z->p = nullptr;
        }
        return z;
       if (p->1 == v) {
        p->1 = z;
      if (p->r == v) {
        p->r = z;
       while (true) {
        p->push();
        p->pull();
        if (p->p == nullptr) {
          break;
        }
        p = p->p;
       return p;
307 node* next(node* v) {
```

```
if (v->r == nullptr) {
308
         while (v->p != nullptr && v->p->r == v) {
309
310
           v = v -> p;
311
        }
312
         return v->p;
313
      }
314
       v->push();
315
       v = v -> r:
       while (v->1 != nullptr) {
316
317
         v->push();
         v = v -> 1;
318
      }
319
320
       return v;
321 }
322
     node* prev(node* v) {
323
       if (v->1 == nullptr) {
324
325
         while (v->p != nullptr && v->p->l == v) {
326
           v = v -> p;
        }
327
328
         return v->p;
329
330
      v->push();
331
       v = v -> 1;
332
       while (v->r != nullptr) {
         v->push();
333
334
         v = v -> r;
335
      }
336
       return v;
337 }
338
339
     int get_size(node* v) {
       return (v != nullptr ? v->sz : 0);
340
341 }
342
     template < typename . . . T>
343
     void Apply(node* v, T... args) {
344
      v->unsafe_apply(args...);
345
346 }
347
     void reverse(node* v) {
348
       v->unsafe_reverse();
349
350 }
```

```
351
352 // extra of mine
353 long long lower(node* u, long long x) {
354
       if (u == nullptr)
355
         return numeric_limits<long long>::min();
356
       else if (x \le u -> x)
357
         return lower(u->1, x);
358
359
         return max(u->x, lower(u->r, x));
360 }
361
    long long upper(node* u, long long x) {
       if (u == nullptr)
364
        return numeric_limits<long long>::max();
365
       else if (u->x <= x)
366
         return upper(u->r, x);
367
368
         return min(u->x, upper(u->1, x));
369 }
370
    } // namespace treap
    using namespace treap;
374
375
    int n:
376
377 int main() {
      ios::sync_with_stdio(false);
379
      cin.tie(0);
380
      node* root = nullptr;
381
       cin >> n;
382
      for (int i = 1; i <= n; i++) {
383
        int op;
384
        long long x;
385
         cin >> op >> x;
386
         switch (op) {
387
           case 1: {
             root = add(root, new node(x, x), [\&](node * u) {
389
               return x < u->x;
390
             });
391
             break;
392
           }
393
           case 2: {
```

```
394
              auto [pt, w] = find(root, [&](node * u) {
                if (x < u \rightarrow x) return -1;
395
                else if (x == u->x) return 0;
396
397
                else return 1;
              });
398
              assert(w == 0);
399
              root = remove(pt);
400
401
              break:
402
           }
            case 3: {
403
              cout << count_less(root, x) + 1 << '\n';</pre>
404
405
              break;
406
407
            case 4: {
              cout << get_kth(root, x - 1)->x << '\n';</pre>
408
409
              break;
           }
410
411
            case 5: {
412
              cout << lower(root, x) << '\n';</pre>
413
              break;
414
           }
            case 6: {
415
              cout << upper(root, x) << '\n';</pre>
416
              break;
417
           }
418
419
420
421 }
```

2.26 UnionFindRollback.cpp

```
1 vector<int> fa(n);
2 iota(all(fa), 0);
3 vector<int> sz(n, 1);
4 vector<pair<int, int>> ops;
5
6 auto Get = [&](int i) {
7    while (i != fa[i]) {
8         i = fa[i];
9    }
10    return i;
11 };
12 auto Unite = [&](int i, int j) {
```

```
13
       i = Get(i), j = Get(j);
       if (i == j) {
14
15
            return;
16
       }
17
       if (sz[i] > sz[j]) {
18
            swap(i, j);
19
20
       ops.emplace_back(i, fa[i]);
21
       fa[i] = j;
        ops.emplace_back(~j, sz[j]);
23
        sz[j] += sz[i];
24 };
   auto RollBack = [&](int T) {
26
        while (SZ(ops) > T) {
27
            auto [i, j] = ops.back();
28
            ops.pop_back();
29
            if (i >= 0) {
30
                fa[i] = j;
31
            } else {
                sz[~i] = j;
            }
34
       }
35 }:
36
37 11 ans = 0:
    auto Dfs = [&](auto &&Dfs, int 1, int r) -> void {
39
        if (1 == r) {
40
            for (auto [x, y] : g[1]) {
41
                x = Get(x);
42
                y = Get(y);
43
                ans += 111 * sz[x] * sz[y];
44
            }
       } else {
45
46
            int mid = midpoint(1, r);
47
            {
                int save = SZ(ops);
                for (int i = mid + 1; i <= r; i++) {
                    for (auto [x, y] : g[i]) {
50
51
                        Unite(x, y);
52
                    }
53
                }
54
                Dfs(Dfs, 1, mid);
55
                RollBack(save);
```

2.27 树哈希.cpp

```
1 basic_string<int> e[maxn];
   ull hashv[maxn];
   ull seed1, seed2, seed3, seed4;
   ull f(ull x) { return x * x * x * seed1 + x * seed2; }
   ull h(ull x) { return f(x) ^ ((x & seed3) >> 31) ^ ((x & seed4) << 31); }
   void dfs1(int u, int fa) {
       hashv[u] = 1:
10
       for (auto v : e[u]) if (v != fa) {
11
                dfs1(v, u);
12
                hashv[u] += h(hashv[v]);
           }
13
14 }
15
16 void dfs2(int u, int fa, ull fv) {
17 // for each root
       hashv[u] += fv:
18
       for (auto v : e[u]) if (v != fa) {
19
                dfs2(v, u, h(hashv[u] - h(hashv[v])));
21
           }
22 }
   void solve() {
25
       seed1 = rng(), seed2 = rng();
       seed3 = rng(), seed4 = rng();
```

```
27
        cin >> n;
       rep(i, 2, n) {
            int u, v;
30
            cin >> u >> v;
31
            e[u].pb(v);
            e[v].pb(u);
32
       }
       dfs1(1, 0);
       sort(hashv + 1, hashv + n + 1);
       n = unique(hashv + 1, hashv + n + 1) - hashv - 1;
37
       cout << n << '\n';
38 }
```

2.28 树链剖分 segtree.cpp

```
1 int n, m, a[N];
2 vector<int> e[N];
3 int 1[N], r[N], idx[N];
4 int sz[N], hs[N], tot, top[N], dep[N], fa[N];
  struct info {
       int maxv, sum;
8 }:
10 info operator + (const info &1, const info &r) {
       return (info){max(1.maxv, r.maxv), 1.sum + r.sum};
12 }
13
14 struct node {
       info val:
16 } seg[N * 4];
17
18 // [l, r]
20 void update(int id) {
       seg[id].val = seg[id * 2].val + seg[id * 2 + 1].val;
21
22 }
23
24 void build(int id, int 1, int r) {
       if (1 == r) {
           // 1号点, DFS序中第1个点
27
           seg[id].val = {a[idx[1]], a[idx[1]]};
28
       } else {
```

```
29
            int mid = (1 + r) / 2;
30
            build(id * 2, 1, mid);
            build(id * 2 + 1, mid + 1, r);
31
32
            update(id);
       }
33
34 }
35
    void change(int id, int 1, int r, int pos, int val) {
       if (1 == r) {
37
           seg[id].val = {val, val};
38
39
       } else {
           int mid = (1 + r) / 2;
40
            if (pos <= mid) change(id * 2, 1, mid, pos, val);</pre>
41
            else change(id * 2 + 1, mid + 1, r, pos, val);
           update(id):
43
       }
44
45 }
   info query(int id, int 1, int r, int q1, int qr) {
       if (l == ql && r == qr) return seg[id].val;
48
       int mid = (1 + r) / 2;
49
       if (ar <= mid) return querv(id * 2, 1, mid, al, ar):
50
       else if (ql > mid) return query(id * 2 + 1, mid + 1, r, ql,qr);
51
52
       else {
53
           return query(id * 2, 1, mid, ql, mid) +
                query(id * 2 + 1, mid + 1, r, mid + 1, qr);
54
55
56 }
57
   // 第一遍 DFS, 子树大小, 重儿子, 父亲, 深度
    void dfs1(int u,int f) {
        sz[u] = 1:
       hs[u] = -1;
61
62
       fa[u] = f;
63
       dep[u] = dep[f] + 1;
64
       for (auto v : e[u]) {
           if (v == f) continue:
           dfs1(v, u);
67
           sz[u] += sz[v];
68
           if (hs[u] == -1 \mid | sz[v] > sz[hs[u]])
               hs[u] = v:
69
70
       }
71 }
```

```
72
 73 // 第二遍 DFS, 每个点 DFS 序, 重链上的链头的元素。
 74 void dfs2(int u. int t) {
        top[u] = t;
        1[u] = ++tot;
        idx[tot] = u;
 77
        if (hs[u] != -1) {
             dfs2(hs[u], t):
 80
        }
 81
        for (auto v : e[u]) {
             if (v != fa[u] && v != hs[u]) {
                 dfs2(v, v);
 84
            }
        }
        r[u] = tot:
 87 }
 89 int LCA(int u. int v) {
         while (top[u] != top[v]) {
             if (dep[top[u]] < dep[top[v]]) v = fa[top[v]];</pre>
 91
 92
             else u = fa[top[u]];
 93
        }
        if (dep[u] < dep[v]) return u;</pre>
         else return v;
 96 }
 97
    info query(int u,int v) {
         info ans{(int)-1e9, 0};
100
         while (top[u] != top[v]) {
101
             if (dep[top[u]] < dep[top[v]]) {</pre>
102
                 ans = ans + query(1, 1, n, l[top[v]], l[v]);
103
                 v = fa[top[v]];
104
            } else {
105
                 ans = ans + query(1, 1, n, l[top[u]], l[u]);
                 u = fa[top[u]];
106
107
            }
108
        }
         if (dep[u] \le dep[v]) ans = ans + query(1, 1, n, l[u], l[v]);
110
         else ans = ans + query(1, 1, n, 1[v], 1[u]);
111
         return ans:
112 }
```

2.29 笛卡尔树.cpp

```
1 int a[maxn], l[maxn], r[maxn], root;
   int ans[maxn], tot;
   void build() {
       stack<int> stk;
       for (int i = 1; i <= n; i++) {
           int last = 0;
            while (!stk.empty() && a[stk.top()] > a[i]) {
                last = stk.top();
10
                stk.pop();
11
           }
12
           if (stk.empty())
                root = i;
13
14
            else
                r[stk.top()] = i;
15
           1[i] = last;
16
17
            stk.push(i);
       }
18
19 }
20
   void dfs(int c, int L, int R) {
       ans[c] = ++tot;
22
       if (l[c]) dfs(l[c], L, c - 1);
23
       if (r[c]) dfs(r[c], c + 1, R);
24
25 }
```

2.30 线段树合并.cpp

```
1 struct node {
2    int sz, sum;
3    node *1, *r;
4    node() : sz(0), sum(0), l(nullptr), r(nullptr) {}
5    } pool[N * 20], *cur = pool;
6
7    node *newnode() {
8       return cur++;
9    }
10
11    void upd(node *rt) {
12       if (not rt) return;
13       rt->sum = rt->sz > 0;
14       if (rt->l) rt->sum += rt->l->sum;
```

```
if (rt->r) rt->sum += rt->r->sum;
16 }
17
18 node *modify(node *rt, int 1, int r, int pos, int d) {
     if (not rt) rt = newnode();
     if (1 == r) {
     rt->sz += d;
     upd(rt);
23
     return rt;
24
    } else {
       int md = (1 + r) >> 1;
26
    if (pos <= md)
27
       rt \rightarrow 1 = modify(rt \rightarrow 1, 1, md, pos, d);
28
29
         rt->r = modify(rt->r, md + 1, r, pos, d);
       upd(rt);
       return rt;
33 }
34
   node *merge(node *u, node *v, int 1, int r) {
     if (not u) return v:
     if (not v) return u:
    if (1 == r) {
    u->sz += v->sz;
    upd(u);
       return u;
    } else {
    int md = (1 + r) >> 1;
    u->1 = merge(u->1, v->1, 1, md);
    u - r = merge(u - r, v - r, md + 1, r);
46
       upd(u);
47
       return u;
48
49 }
51 ll query(node *rt, int l, int r) {
     if (not rt) return 0;
     return rt->sum;
54 }
   pair<node *, node *> split(node *rt, int l, int r, int ql, int qr) {
     if (not rt) return {nullptr, nullptr};
```

```
if (ql == 1 && qr == r) {
        return {nullptr, rt};
     } else {
60
61
        int md = (1 + r) >> 1;
        if (qr <= md) {
62
          auto [p1, p2] = split(rt->1, 1, md, ql, qr);
          rt->1 = p1;
          upd(rt);
          if (not p2) return {rt, nullptr};
          node *u = newnode();
67
          u ->1 = p2;
          upd(u);
          return {rt, u};
70
71
        } else if (ql > md) {
72
          auto [p1, p2] = split(rt->r, md + 1, r, q1, qr);
73
          rt->r = p1;
74
          upd(rt);
75
          if (not p2) return {rt, nullptr};
          node *u = newnode();
76
          u \rightarrow r = p2;
77
78
          upd(u);
          return {rt. u}:
79
        } else {
80
81
          auto [p1, p2] = split(rt->1, 1, md, q1, md);
82
          auto [p3, p4] = split(rt->r, md + 1, r, md + 1, qr);
          rt->1 = p1, rt->r = p3;
          upd(rt);
84
          if (not p2 and not p4) return {rt, nullptr};
85
          node *u = newnode();
          u -> 1 = p2, u -> r = p4;
87
          upd(u);
          return {rt, u};
90
91
     }
92 }
```

3 DP

3.1 Convex hull optimization.cpp

```
1 array<11, 3> a[maxn];
2 int q[maxn];
```

```
3 ll ans[maxn]:
5 11 X(int p) {
        return 211 * a[p][0];
7 }
8 11 Y(int p) {
       return a[p][0] * a[p][0] + a[p][1];
10 }
11 ldb slope(int x, int y) {
        return (1db)(Y(y) - Y(x)) / (X(y) - X(x));
13 }
14 void solve() {
       cin >> n:
16
       int head = 1, rear = 0;
17
       rep(i, 1, n) {
           cin >> a[i][0] >> a[i][1];
19
           a[i][2] = i;
       }
20
21
       sort(a + 1, a + n + 1);
22
23
       rep(i, 1, n) {
24
            while (head < rear && slope(q[rear], i) <= slope(q[rear], q[rear -
                1])) rear--:
25
           q[++rear] = i;
26
       }
27
       rep(i, 1, n) {
           11 k = -a[i][0];
           while (head < rear && slope(q[head], q[head + 1]) <= k) head++;</pre>
           ans[a[i][2]] = (a[i][0] + a[q[head]][0]) * (a[i][0] + a[q[head]][0])
                 + a[i][1] + a[q[head]][1];
31
       }
       rep(i, 1, n) cout << ans[i] << '\n';
33 }
```

3.2 DivideAndConquerDP.cpp

3.3 有依赖决策单调.cpp

```
1 pair<int, int> stk[N];
2 auto calc = [\&] (int i, int j) \{\ldots\} // dp[j] \rightarrow dp[i]
3 int h = 0, t = 0;
   stk[t++] = \{1, 0\}; // \{left, opt\}
6 for (int i = 1; i \le n; i++) {
        if (h < t && stk[h].first < i) stk[h].first++;</pre>
        if (h + 1 < t && stk[h].first >= stk[h + 1].first) ++h;
        dp[i] = calc(i, stk[h].second);
10
        while (h < t && calc(stk[t - 1].first, stk[t - 1].second) >= calc(stk[t
            - 1].first, i))
            --t:
11
12
        if (h < t) {
13
            int l = stk[t - 1].first, r = n + 1;
            while (1 + 1 < r) {
14
                int md = (1 + r) >> 1;
15
                if (calc(md. stk[t - 1].second) < calc(md. i)) l = md: else r =
16
            }
17
            if (r \le n) stk[t++] = \{r, i\}:
18
19
       } else stk[t++] = {i, i};
20 }
```

4 Geometry

4.1 1 (1).cpp

```
1 typedef double db;
2 const db EPS = 1e-9;
3
4 inline int sign(db a) { return a < -EPS ? -1 : a > EPS; }
```

```
inline int cmp(db a, db b) { return sign(a - b); }
7
   struct P {
        db x, y;
       P() {}
10
11
       P(db _x, db _y) : x(_x), y(_y) {}
       P operator+(P p) { return \{x + p.x, y + p.y\}; }
        P operator-(P p) { return {x - p.x, y - p.y}; }
       P operator*(db d) { return \{x * d, y * d\}; \}
14
        P operator/(db d) { return {x / d, y / d}; }
15
16
17
        bool operator<(P p) const {</pre>
18
            int c = cmp(x, p.x):
19
            if (c) return c == -1;
20
            return cmp(y, p.y) == -1;
21
       }
22
        bool operator == (P o) const {
24
            return cmp(x, o.x) == 0 && cmp(y, o.y) == 0;
       }
25
26
27
        db dot(P p) { return x * p.x + v * p.v: }
28
        db det(P p) { return x * p.y - y * p.x; }
29
30
        db distTo(P p) { return (*this - p).abs(): }
31
        db alpha() { return atan2(v, x); }
32
        void read() { cin >> x >> y; }
        void write() {cout << "(" << x << "," << y << ")" << endl;}</pre>
34
        db abs() { return sqrt(abs2());}
        db abs2() { return x * x + v * v: }
       P rot90() { return P(-v, x);}
37
       P unit() { return *this / abs(); }
        int quad() const { return sign(y) == 1 \mid | (sign(y) == 0 \&\& sign(x) >= 0)
            : }
        P rot(db an) { return \{x * \cos(an) - y * \sin(an), x * \sin(an) + y * \cos(an) \}
            an)}: }
40 };
42 #define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
43 #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
45 // 直线 p1p2, q1q2 是否恰有一个交点
46 bool chkLL(P p1, P p2, P q1, P q2) {
```

```
db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
47
       return sign(a1 + a2) != 0;
48
49 }
50
51 // 求直线 p1p2, q1q2 的交点
52 P isLL(P p1, P p2, P q1, P q2) {
       db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
       return (p1 * a2 + p2 * a1) / (a1 + a2);
54
55 }
56
57 // 判断区间 [l1, r1], [l2, r2] 是否相交
58 bool intersect(db 11, db r1, db 12, db r2) {
       if (11 > r1) swap(11, r1); if (12 > r2) swap(12, r2);
59
60
       return ! ( cmp(r1, 12) == -1 | | cmp(r2, 11) == -1 );
61 }
62
63 // 线段 p1p2, q1q2 相交
64 bool isSS(P p1, P p2, P q1, P q2) {
65
       return intersect(p1.x, p2.x, q1.x, q2.x) && intersect(p1.y, p2.y, q1.y,
           q2.y) &&
              crossOp(p1, p2, q1) * crossOp(p1, p2, q2) <= 0 && crossOp(q1, q2,
67
              * crossOp(q1, q2, p2) <= 0;
68 }
69
70 // 线段 p1p2, q1q2 严格相交
71 bool isSS_strict(P p1, P p2, P q1, P q2) {
72
       return crossOp(p1, p2, q1) * crossOp(p1, p2, q2) < 0 && crossOp(q1, q2,
           p1)
              * crossOp(a1, a2, p2) < 0:
73
74 }
76 // m 在 a 和 b 之间
77 bool isMiddle(db a, db m, db b) {
       /*if (a > b) swap(a, b);
78
       return cmp(a, m) \le 0 && cmp(m, b) \le 0;*/
       return sign(a - m) == 0 \mid \mid sign(b - m) == 0 \mid \mid (a < m != b < m);
81 }
82
83 bool isMiddle(Pa, Pm, Pb) {
       return isMiddle(a.x, m.x, b.x) && isMiddle(a.y, m.y, b.y);
85 }
86
```

```
87 // 点 p 在线段 p1p2 上
   88 bool onSeg(P p1, P p2, P q) {
                      return crossOp(p1, p2, q) == 0 && isMiddle(p1, q, p2);
  90 }
  91 // q1q2 和 p1p2 的交点 在 p1p2 上?
  93 // 点 p 严格在 p1p2 上
  94 bool onSeg_strict(P p1, P p2, P q) {
                      return crossOp(p1, p2, q) == 0 && sign((q - p1).dot(p1 - p2)) * 
                                 - p2).dot(p1 - p2)) < 0;
  96 }
  98 // 求 q 到 直线 p1p2 的投影 (垂足) : p1 != p2
  99 P proj(P p1, P p2, P q) {
100
                      P dir = p2 - p1;
101
                      return p1 + dir * (dir.dot(q - p1) / dir.abs2());
102 }
103
|104 // 求 q 以 直线p1p2 为轴的反射
105 P reflect(P p1, P p2, P q) {
                      return proj(p1, p2, q) * 2 - q;
107 }
108
|109 // 求 q 到 线段p1p2 的最小距离
110 db nearest(P p1, P p2, P q) {
111
                      if (p1 == p2) return p1.distTo(q);
112
                      P h = proj(p1, p2, q);
113
                      if (isMiddle(p1, h, p2))
114
                                return q.distTo(h);
115
                      return min(p1.distTo(q), p2.distTo(q));
116 }
1117
118 // 求 线段p1p2 与 线段q1q2 的距离
119 db disSS(P p1, P p2, P q1, P q2) {
120
                      if (isSS(p1, p2, q1, q2)) return 0;
121
                      return min(min(nearest(p1, p2, q1), nearest(p1, p2, q2)), min(nearest(q1
                                  , q2, p1), nearest(q1, q2, p2)));
122 }
123
124 // 极角排序
125 sort(p, p + n, [&](P a, P b) {
126
                      int qa = a.quad(), qb = b.quad();
127
                      if (qa != qb) return qa < qb;</pre>
```

```
128
         else return sign(a.det(b)) > 0;
129 });
    4.2 1 (2).cpp
 1 db area(vector<P> ps){
         db ret = 0; rep(i,0,ps.size()) ret += ps[i].det(ps[(i+1)%ps.size()]);
         return ret/2;
 4 }
    int contain(vector <P> ps, P p){ //2:inside,1:on seq,0:outside
         int n = ps.size(), ret = 0;
         rep(i,0,n){
             P u=ps[i],v=ps[(i+1)%n];
             if(onSeg(u,v,p)) return 1;
             if (cmp(u.y,v.y) \le 0) swap(u,v);
 11
 12
             if(cmp(p.y,u.y) > 0 \mid | cmp(p.y,v.y) \le 0) continue;
 13
             ret ^= crossOp(p,u,v) > 0;
 14
         }
 15
         return ret*2;
 16 }
17
 18
     vector<P> convexHull(vector<P> ps) {
 19
         int n = ps.size(); if(n <= 1) return ps;</pre>
 20
         sort(ps.begin(), ps.end());
         vector < P > qs(n * 2); int k = 0;
 21
 22
         for (int i = 0; i < n; qs[k++] = ps[i++])
 23
             while (k > 1 \&\& crossOp(qs[k - 2], qs[k - 1], ps[i]) \le 0) --k;
 24
         for (int i = n - 2, t = k; i \ge 0; qs[k++] = ps[i--])
             while (k > t \&\& crossOp(qs[k - 2], qs[k - 1], ps[i]) \le 0) --k;
 25
 26
         qs.resize(k - 1);
 27
         return qs;
 28 }
 29
     vector<P> convexHullNonStrict(vector<P> ps) {
 31
         //caution: need to unique the Ps first
 32
         int n = ps.size(); if(n <= 1) return ps;</pre>
 33
         sort(ps.begin(), ps.end());
 34
         vector < P > qs(n * 2); int k = 0;
         for (int i = 0: i < n: as[k++] = ps[i++])
 35
 36
             while (k > 1 \&\& crossOp(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;
```

for (int i = n - 2, t = k; $i \ge 0$; qs[k++] = ps[i--])

while (k > t && crossOp(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;

37

38

```
39
        qs.resize(k - 1);
40
        return qs;
41 }
42
    db convexDiameter(vector < P > ps) {
44
        int n = ps.size(); if(n <= 1) return 0;</pre>
45
        int is = 0, js = 0; rep(k,1,n) is = ps[k] < ps[is]?k:is, js = ps[js] < ps[is]
            kl?k:is:
46
        int i = is, j = js;
47
        db ret = ps[i].distTo(ps[j]);
48
49
            if((ps[(i+1)\%n]-ps[i]).det(ps[(j+1)\%n]-ps[j]) >= 0)
50
                 (++j)%=n;
51
            else
52
                 (++i)%=n:
53
            ret = max(ret,ps[i].distTo(ps[j]));
54
        }while(i!=is || j!=js);
        return ret:
56 }
57
    vector<P> convexCut(const vector<P>&ps, P q1, P q2) {
59
        vector <P> as:
60
        int n = ps.size();
61
        rep(i,0,n){
62
            P p1 = ps[i], p2 = ps[(i+1)%n];
63
            int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
64
            if(d1 >= 0) qs.push_back(p1);
            if(d1 * d2 < 0) qs.push_back(isLL(p1,p2,q1,q2));</pre>
66
        }
67
        return qs;
68 }
69
   void reorderPolygon(vector<P> &ps) {
71
        size_t pos = 0;
72
        for(size_t i = 1; i < ps.size(); i++){</pre>
73
            if(ps[i].y < ps[pos].y \mid | (ps[i].y == ps[pos].y && ps[i].x < ps[pos]
                1.x))
74
                pos = i;
75
76
        rotate(ps.begin(), ps.begin() + pos, ps.end());
77 }
78
79 vector<P> minkowski(vector<P> p, vector<P> q){
```

```
80
         if(p.empty()) return q;
         // the first vertex must be the lowest
 81
         reorderPolygon(p);
 82
 83
         reorderPolygon(q);
         // must ensure cyclic indexing
 84
 85
         p.push_back(p[0]);
         p.push back(p[1]);
         q.push_back(q[0]);
 87
 88
         q.push back(q[1]);
         // main part
 89
 90
         vector<P> result;
 91
         size t i = 0, j = 0;
         while(i < p.size() - 2 \mid \mid j < q.size() - 2){
 92
 93
             result.push back(p[i] + q[j]);
 94
             auto cross = (p[i + 1] - p[i]).det(q[j + 1] - q[j]);
             if(cross \geq 0 && i \leq SZ(p) - 2)
 95
                 ++i;
 97
             if(cross \leq 0 \&\& j \leq SZ(q) - 2)
 98
                 ++j;
         }
 99
100
         return result;
101 }
102
103
     bool convexContain(const vector<P> &1, P p, bool strict = true) {
104
         int a = 1, b = l.size() - 1, r = !strict:
         if (1.size() < 3) return r && onSeg(1[0], 1.back(), p);</pre>
105
         if (crossOp(1[0], 1[a], 1[b]) > 0) swap(a, b);
106
         if (crossOp(1[0], 1[a], p) >= r || crossOp(1[0], 1[b], p) <= -r)
107
108
             return false;
109
         while (abs(a - b) > 1) {
             int c = (a + b) / 2;
110
             (crossOp(1[0], 1[c], p) > 0 ? b : a) = c;
111
         }
112
113
         return sign(cross(l[a], l[b], p)) < r;</pre>
114 }
     4.3 1 (3).cpp
```

```
1 int type(P o1,db r1,P o2,db r2){
2    db d = o1.distTo(o2);
3    if(cmp(d,r1+r2) == 1) return 4;
4    if(cmp(d,r1+r2) == 0) return 3;
5    if(cmp(d,abs(r1-r2)) == 1) return 2;
```

```
if(cmp(d,abs(r1-r2)) == 0) return 1;
7
       return 0;
8 }
9
10 vector<P> isCL(P o,db r,P p1,P p2){
11
       if (cmp(abs((o-p1).det(p2-p1)/p1.distTo(p2)),r)>0) return {};
12
       db x = (p1-o) \cdot dot(p2-p1), y = (p2-p1) \cdot abs2(), d = x * x - y * ((p1-o))
            abs2() - r*r):
13
       d = max(d,(db)0.0); P = p1 - (p2-p1)*(x/y), dr = (p2-p1)*(sqrt(d)/y);
       return {m-dr,m+dr}; //along dir: p1->p2
15 }
16
17 vector <P isCC(P o1, db r1, P o2, db r2) { //need to check whether two
        circles are the same
18
       db d = o1.distTo(o2):
       if (cmp(d, r1 + r2) == 1) return {};
       if (cmp(d,abs(r1-r2))==-1) return {};
20
       d = min(d, r1 + r2):
       db y = (r1 * r1 + d * d - r2 * r2) / (2 * d), x = sqrt(r1 * r1 - y * y);
       P dr = (o2 - o1).unit():
23
24
       P q1 = o1 + dr * v, q2 = dr.rot90() * x;
25
       return {q1-q2,q1+q2}; //along circle 1
26 }
27
28 // extanCC. intanCC : -r2. tanCP : r2 = 0
29 vector<pair<P, P>> tanCC(P o1, db r1, P o2, db r2) {
       P d = o2 - o1:
       db dr = r1 - r2, d2 = d.abs2(), h2 = d2 - dr * dr;
31
32
       if (sign(d2) == 0|| sign(h2) < 0) return {};
33
       h2 = max((db)0.0, h2):
       vector<pair<P, P>> ret;
34
35
       for (db sign : {-1, 1}) {
           P v = (d * dr + d.rot90() * sqrt(h2) * sign) / d2;
37
           ret.push back(\{01 + v * r1, 02 + v * r2\});
38
39
       if (sign(h2) == 0) ret.pop back();
40
       return ret:
41 }
43 db rad(P p1,P p2){
       return atan21(p1.det(p2),p1.dot(p2));
45 }
46
```

```
47 db areaCT(db r, P p1, P p2){
        vector\langle P \rangle is = isCL(P(0,0),r,p1,p2);
48
        if(is.empty()) return r*r*rad(p1,p2)/2;
49
50
        bool b1 = cmp(p1.abs2(),r*r) == 1, b2 = cmp(p2.abs2(),r*r) == 1;
51
        if(b1 && b2){
            P md=(is[0]+is[1])/2:
52
            if(sign((p1-md).dot(p2-md)) \le 0)
53
                return r*r*(rad(p1,is[0]) + rad(is[1],p2))/2 + is[0].det(is[1])
54
            else return r*r*rad(p1.p2)/2:
55
56
       }
        if(b1) return (r*r*rad(p1,is[0]) + is[0].det(p2))/2;
57
        if(b2) return (p1.det(is[1]) + r*r*rad(is[1],p2))/2;
58
59
        return p1.det(p2)/2;
60 }
61
62 P inCenter(P A, P B, P C) {
        double a = (B - C).abs(), b = (C - A).abs(), c = (A - B).abs();
        return (A * a + B * b + C * c) / (a + b + c);
64
65 }
66
67 P circumCenter(P a. P b. P c) {
        P bb = b - a, cc = c - a:
69
        double db = bb.abs2(), dc = cc.abs2(), d = 2 * bb.det(cc);
70
        return a - P(bb.v * dc - cc.v * db. cc.x * db - bb.x * dc) / d:
71 }
72
73 P othroCenter(P a, P b, P c) {
74
       P ba = b - a, ca = c - a, bc = b - c;
        double Y = ba.y * ca.y * bc.y,
75
76
       A = ca.x * ba.y - ba.x * ca.y,
77
        x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
       y0 = -ba.x * (x0 - c.x) / ba.y + ca.y;
78
79
        return {x0, y0};
80 }
81
   pair < P, db > min_circle(vector < P > ps) {
        random_shuffle(ps.begin(), ps.end());
83
84
        int n = ps.size();
       P \circ = ps[0]; db r = 0;
85
        rep(i,1,n) if (o.distTo(ps[i]) > r + EPS){
86
87
            o = ps[i], r = 0;
88
            rep(j,0,i) if (o.distTo(ps[j]) > r + EPS){
```

```
89
                o = (ps[i] + ps[j]) / 2; r = o.distTo(ps[i]);
90
                rep(k,0,j) if (o.distTo(ps[k]) > r + EPS){
                     o = circumCenter(ps[i],ps[j],ps[k]);
91
92
                     r = o.distTo(ps[i]);
93
               }
94
            }
95
       }
       return {o.r}:
97 }
```

4.4 all.cpp

```
1 typedef double db:
2 const db EPS = 1e-9:
4 inline int sign(db a) { return a < -EPS ? -1 : a > EPS; }
6 inline int cmp(db a, db b){ return sign(a-b); }
8
  struct P {
        db x, y;
10
       P() {}
11
       P(db _x, db _y) : x(_x), y(_y) {}
       P operator+(P p) { return \{x + p.x, y + p.y\}; }
12
        P operator-(P p) { return \{x - p.x, y - p.y\}; }
13
14
        P operator*(db d) { return {x * d, y * d}; }
        P operator/(db d) { return \{x / d, y / d\}; \}
15
16
17
        bool operator<(P p) const {</pre>
18
            int c = cmp(x, p.x):
19
            if (c) return c == -1:
20
            return cmp(v, p.v) == -1;
21
       }
22
23
        bool operator == (P o) const{
24
            return cmp(x,o.x) == 0 && cmp(y,o.y) == 0;
25
       }
26
27
        db dot(P p) { return x * p.x + y * p.y; }
        db det(P p) { return x * p.y - y * p.x; }
28
29
30
        db distTo(P p) { return (*this-p).abs(); }
31
        db alpha() { return atan2(y, x); }
```

```
32
       void read() { cin>>x>>y; }
       void write() {cout<<"("<<x<<","<<y<<")"<<endl;}</pre>
33
        db abs() { return sgrt(abs2()):}
34
       db abs2() { return x * x + y * y; }
35
       P rot90() { return P(-v.x):}
36
       P unit() { return *this/abs(): }
37
       int quad() const { return sign(y) == 1 \mid | (sign(y) == 0 \&\& sign(x) >= 0)
38
39
       P rot(db an) { return \{x*\cos(an)-y*\sin(an),x*\sin(an) + y*\cos(an)\}; }
40 }:
41
    struct L{ //ps[0] -> ps[1]
       P ps[2]:
43
44
       P& operator[](int i) { return ps[i]; }
       P dir() { return ps[1] - ps[0]; }
45
       bool include(P p) { return sign((ps[1] - ps[0]).det(p - ps[0])) > 0; }
       L push(){ // push eps outward
            const double eps = 1e-6:
           P delta = (ps[1] - ps[0]).rot90().unit() * eps;
           return {{ps[0] - delta, ps[1] - delta}};
50
       }
51
52 }:
53
    #define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
55
    #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
56
   bool chkLL(P p1, P p2, P q1, P q2) {
       db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
58
59
       return sign(a1+a2) != 0;
60 }
61
62 P isLL(P p1, P p2, P q1, P q2) {
       db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
63
       return (p1 * a2 + p2 * a1) / (a1 + a2);
64
65 }
66
67 P isLL(L 11,L 12) { return isLL(11[0],11[1],12[0],12[1]); }
68
   bool intersect(db 11,db r1,db 12,db r2){
70
       if (11>r1) swap(11,r1); if (12>r2) swap(12,r2);
       return ! ( cmp(r1,12) == -1 | | cmp(r2,11) == -1 );
71
72. }
73
```

```
74 bool isSS(P p1, P p2, P q1, P q2){
         return intersect(p1.x,p2.x,q1.x,q2.x) && intersect(p1.y,p2.y,q1.y,q2.y)
 76
         crossOp(p1,p2,q1) * crossOp(p1,p2,q2) <= 0 && crossOp(q1,q2,p1)
                 * crossOp(q1,q2,p2) <= 0;
 78 }
    bool isSS_strict(P p1, P p2, P q1, P q2){
 81
         return crossOp(p1,p2,q1) * crossOp(p1,p2,q2) < 0 && crossOp(q1,q2,p1)
 82
                 * crossOp(q1,q2,p2) < 0;
 83 }
 84
    bool isMiddle(db a. db m. db b) {
         return sign(a - m) == 0 \mid \mid sign(b - m) == 0 \mid \mid (a < m != b < m);
 87 }
 89 bool isMiddle(Pa, Pm, Pb) {
         return isMiddle(a.x. m.x. b.x) && isMiddle(a.v. m.v. b.v):
 91 }
 92
    bool onSeg(P p1, P p2, P q){
         return crossOp(p1,p2,q) == 0 && isMiddle(p1, q, p2):
 95 }
 97 bool onSeg_strict(P p1, P p2, P q){
         return crossOp(p1,p2,q) == 0 && sign((q-p1).dot(p1-p2)) * sign((q-p2).
             dot(p1-p2)) < 0;
 99 }
101 P proj(P p1, P p2, P q) {
102
        P dir = p2 - p1;
         return p1 + dir * (dir.dot(q - p1) / dir.abs2());
104 }
106 P reflect(P p1, P p2, P q){
107
         return proj(p1,p2,q) * 2 - q;
108 }
109
110 db nearest(P p1,P p2,P q){
111
        P h = proj(p1, p2, q);
112
        if(isMiddle(p1,h,p2))
113
             return q.distTo(h);
114
         return min(p1.distTo(q),p2.distTo(q));
```

```
115 }
116
117 db disSS(P p1, P p2, P q1, P q2){
118
         if(isSS(p1,p2,q1,q2)) return 0;
119
         return min(min(nearest(p1,p2,q1),nearest(p1,p2,q2)), min(nearest(q1,q2,
             p1), nearest(q1,q2,p2)));
120 }
121
122 db rad(P p1,P p2){
         return atan21(p1.det(p2),p1.dot(p2));
123
124 }
125
126
     db incircle(P p1, P p2, P p3){
127
         db A = p1.distTo(p2);
         db B = p2.distTo(p3);
128
129
         db C = p3.distTo(p1);
         return sqrtl(A*B*C/(A+B+C));
130
131 }
132
133 //polygon
134
     db area(vector <P > ps){
135
         db ret = 0; rep(i,0,ps.size()) ret += ps[i].det(ps[(i+1)%ps.size()]);
136
137
         return ret/2;
138 }
139
     int contain(vector < P > ps, P p){ //2:inside,1:on seq,0:outside
140
141
         int n = ps.size(), ret = 0;
142
         rep(i,0,n){
             P u=ps[i],v=ps[(i+1)%n];
143
             if(onSeg(u,v,p)) return 1;
144
145
             if (cmp(u.y,v.y) \le 0) swap(u,v);
             if (cmp(p.y,u.y) > 0 \mid | cmp(p.y,v.y) \le 0) continue;
146
147
             ret ^= crossOp(p,u,v) > 0;
         }
148
149
         return ret*2;
150 }
151
152
     vector<P> convexHull(vector<P> ps) {
153
         int n = ps.size(); if(n <= 1) return ps;</pre>
         sort(ps.begin(), ps.end());
154
         vector < P > qs(n * 2); int k = 0;
155
         for (int i = 0; i < n; qs[k++] = ps[i++])
156
```

```
157
             while (k > 1 \&\& crossOp(qs[k - 2], qs[k - 1], ps[i]) \le 0) --k;
158
         for (int i = n - 2, t = k; i \ge 0; qs[k++] = ps[i--])
159
             while (k > t \&\& crossOp(qs[k - 2], qs[k - 1], ps[i]) \le 0) --k;
160
         qs.resize(k - 1);
161
         return qs;
162 }
163
     vector<P> convexHullNonStrict(vector<P> ps) {
165
         //caution: need to unique the Ps first
166
         int n = ps.size(); if(n <= 1) return ps;</pre>
167
         sort(ps.begin(), ps.end());
168
         vector\langle P \rangle qs(n * 2); int k = 0;
         for (int i = 0; i < n; qs[k++] = ps[i++])
169
170
             while (k > 1 \&\& crossOp(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;
171
         for (int i = n - 2, t = k; i \ge 0; qs[k++] = ps[i--])
172
             while (k > t \&\& cross0p(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;
173
         qs.resize(k - 1);
174
         return qs;
175 }
176
177
    db convexDiameter(vector < P > ps) {
178
         int n = ps.size(): if(n \le 1) return 0:
179
         int is = 0, js = 0; rep(k,1,n) is = ps[k] < ps[is]?k:is, js = ps[js] < ps[
             k]?k:js;
180
         int i = is, i = is:
         db ret = ps[i].distTo(ps[i]);
181
182
183
             if((ps[(i+1)\%n]-ps[i]).det(ps[(j+1)\%n]-ps[j]) >= 0)
184
                  (++j)%=n;
185
             else
186
                 (++i)%=n:
187
             ret = max(ret,ps[i].distTo(ps[j]));
188
         }while(i!=is || j!=js);
189
         return ret:
190 ጉ
191
     vector<P> convexCut(const vector<P>&ps, P q1, P q2) {
193
         vector <P> qs;
194
         int n = ps.size();
195
         rep(i,0,n){
196
             P p1 = ps[i], p2 = ps[(i+1)%n];
             int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
197
198
             if(d1 \ge 0) qs.pb(p1);
```

```
199
             if(d1 * d2 < 0) qs.pb(isLL(p1,p2,q1,q2));
         }
200
201
         return qs;
202 }
203
204
     //min dist
205
     db min dist(vector < P > & ps.int l.int r) {
206
         if(r-1<=5){
207
             db ret = 1e100:
208
209
             rep(i,l,r) rep(j,l,i) ret = min(ret,ps[i].distTo(ps[j]));
             return ret;
210
         }
211
212
         int m = (1+r) >> 1;
         db ret = min(min dist(ps.l.m).min dist(ps.m.r)):
213
         vector < P > qs; rep(i,l,r) if(abs(ps[i].x-ps[m].x) <= ret) qs.pb(ps[i]);
214
         sort(qs.begin(), qs.end(),[](Pa,Pb) -> bool {return a.y<b.y; });</pre>
215
         rep(i,1,qs.size()) for(int j=i-1;j>=0&&qs[j].y>=qs[i].y-ret;--j)
216
217
             ret = min(ret,qs[i].distTo(qs[j]));
         return ret:
218
219 }
220
    int type(P o1.db r1.P o2.db r2){
221
         db d = o1.distTo(o2);
222
223
         if(cmp(d,r1+r2) == 1) return 4:
         if(cmp(d,r1+r2) == 0) return 3;
224
         if(cmp(d,abs(r1-r2)) == 1) return 2;
225
         if(cmp(d,abs(r1-r2)) == 0) return 1;
226
227
         return 0:
228 }
229
     vector<P> isCL(P o,db r,P p1,P p2){
         db x = (p1-o) \cdot dot(p2-p1), y = (p2-p1) \cdot abs2(), d = x * x - y * ((p1-o))
231
             abs2() - r*r);
232
         if(sign(d) < 0) return {};</pre>
233
         d = max(d,0.0); P = p1 - (p2-p1)*(x/y), dr = (p2-p1)*(sqrt(d)/y);
         return {m-dr,m+dr}; //along dir: p1->p2
234
235 }
237 vector <P is CC (P o1, db r1, P o2, db r2) { //need to check whether two
         circles are the same
         db d = o1.distTo(o2):
238
         if (cmp(d, r1 + r2) == 1) return {};
239
```

```
240
         d = min(d, r1 + r2):
241
         db y = (r1 * r1 + d * d - r2 * r2) / (2 * d), x = sqrt(r1 * r1 - y * y);
242
         P dr = (o2 - o1).unit();
         P q1 = o1 + dr * y, q2 = dr.rot90() * x;
243
244
         return {q1-q2,q1+q2}; //along circle 1
245
246
    vector<P> tanCP(P o, db r, P p) {
248
         db x = (p - o).abs2(), d = x - r * r;
249
         if (sign(d) <= 0) return {}; // on circle => no tangent
250
         P q1 = o + (p - o) * (r * r / x);
251
         P q2 = (p - o).rot90() * (r * sqrt(d) / x);
252
         return {q1-q2,q1+q2}; //counter clock-wise
253 }
254
255
256
     vector<L> extanCC(P o1, db r1, P o2, db r2) {
257
         vector<L> ret:
258
         if (cmp(r1, r2) == 0) {
259
             P dr = (o2 - o1).unit().rot90() * r1:
260
             ret.pb(\{\{01 + dr, 02 + dr\}\}), ret.pb(\{\{01 - dr, 02 - dr\}\});
261
         } else {
262
             P p = (o2 * r1 - o1 * r2) / (r1 - r2):
263
             vector < P > ps = tanCP(o1, r1, p), qs = tanCP(o2, r2, p);
264
             rep(i.0.min(ps.size().qs.size())) ret.pb({{ps[i]. qs[i]}}): //c1
                 counter-clock wise
265
         }
         return ret:
267 }
268
    vector<L> intanCC(P o1, db r1, P o2, db r2) {
270
         vector<L> ret:
271
         P p = (o1 * r2 + o2 * r1) / (r1 + r2):
272
         vector\langle P \rangle ps = tanCP(o1,r1,p), qs = tanCP(o2,r2,p);
273
         rep(i,0,min(ps.size(),qs.size())) ret.pb({{ps[i], qs[i]}}); //c1 counter
             -clock wise
274
         return ret:
275 }
276
277 db areaCT(db r, P p1, P p2){
278
         vector\langle P \rangle is = isCL(P(0,0),r,p1,p2);
279
         if(is.empty()) return r*r*rad(p1,p2)/2;
280
         bool b1 = cmp(p1.abs2(),r*r) == 1, b2 = cmp(p2.abs2(),r*r) == 1;
```

```
281
         if(b1 && b2){
             if(sign((p1-is[0]).dot(p2-is[0])) <= 0 &&
282
                 sign((p1-is[0]).dot(p2-is[0])) \le 0)
283
284
             return r*r*(rad(p1,is[0]) + rad(is[1],p2))/2 + is[0].det(is[1])/2;
             else return r*r*rad(p1,p2)/2;
285
        }
286
         if(b1) return (r*r*rad(p1,is[0]) + is[0].det(p2))/2;
287
         if(b2) return (p1.det(is[1]) + r*r*rad(is[1],p2))/2:
288
289
         return p1.det(p2)/2;
290 }
291
     bool parallel(L 10, L 11) { return sign( 10.dir().det( 11.dir() ) ) == 0; }
293
    bool sameDir(L 10, L 11) { return parallel(10, 11) && sign(10.dir().dot(11.
         dir()) ) == 1; }
295
     bool cmp (Pa, Pb) {
296
297
         if (a.quad() != b.quad()) {
298
             return a.quad() < b.quad();</pre>
        } else {
299
300
             return sign(a.det(b)) > 0;
        }
301
302 }
303
     bool operator < (L 10, L 11) {
304
305
         if (sameDir(10, 11)) {
             return 11.include(10[0]);
306
        } else {
307
308
             return cmp( 10.dir(), 11.dir() );
        }
309
310 }
311
    bool check(L u. L v. L w) {
312
313
         return w.include(isLL(u,v));
314 }
315
     vector<P> halfPlaneIS(vector<L> &1) {
         sort(1.begin(), 1.end());
317
318
         deque<L> q;
         for (int i = 0; i < (int)1.size(); ++i) {</pre>
319
             if (i && sameDir(l[i], l[i - 1])) continue;
320
             while (q.size() > 1 \&\& !check(q[q.size() - 2], q[q.size() - 1], 1[i])
321
                 ])) q.pop back();
```

```
322
             while (q.size() > 1 && !check(q[1], q[0], l[i])) q.pop_front();
323
             q.push back(l[i]);
324
325
         while (q.size() > 2 \& \& !check(q[q.size() - 2], q[q.size() - 1], q[0]))
             .pop_back();
326
         while (q.size() > 2 && !check(q[1], q[0], q[q.size() - 1])) q.pop_front
             ();
327
         vector<P> ret:
328
         for (int i = 0; i < (int)q.size(); ++i) ret.push back(isLL(q[i], q[(i +
             1) % a.size()1)):
329
         return ret;
330 }
331
332
    P inCenter(P A, P B, P C) {
333
         double a = (B - C).abs(), b = (C - A).abs(), c = (A - B).abs();
334
         return (A * a + B * b + C * c) / (a + b + c);
335 }
336
337 P circumCenter(P a, P b, P c) {
338
        P bb = b - a, cc = c - a:
339
         double db = bb.abs2(), dc = cc.abs2(), d = 2 * bb.det(cc);
340
        return a - P(bb.v * dc - cc.v * db. cc.x * db - bb.x * dc) / d:
341 }
342
343 P othroCenter(P a, P b, P c) {
344
        P ba = b - a, ca = c - a, bc = b - c;
345
        double Y = ba.y * ca.y * bc.y,
346
        A = ca.x * ba.y - ba.x * ca.y
347
        x0 = (Y + ca.x * ba.y * b.x - ba.x * ca.y * c.x) / A,
        v0 = -ba.x * (x0 - c.x) / ba.v + ca.v:
349
         return {x0, y0};
350 }
```

4.5 圆面积并.cpp

```
db intergal(db x,db y,db r,db L,db R){
    return r*r*(R-L) + x*r*(sinl(R) - sinl(L)) + y*r*(-cosl(R) + cosl(L));
}

db calc_area_circle(P c,db r,db L,db R){
    return intergal(c.x,c.y,r,L,R) / 2;
}
```

```
9 db norm(db x){
        while(x < 0) x += 2 * PI;
10
        while(x > 2 * PI) x -= 2 * PI;
11
12
        return x;
13 }
14
15 P cs[N]; db rs[N];
17 void work(){
18
        vector<int> cand = {}:
19
        rep(i,0,m){
20
            bool ok = 1;
21
            rep(j,0,m) if(i!=j){
22
                if(rs[i] > rs[i] + EPS && rs[i] + cs[i].distTo(cs[i]) <= rs[i] +
                     EPS){
                    ok = 0; break;
24
                if(cs[i] == cs[j] \&\& cmp(rs[i],rs[j]) == 0 \&\& j < i){
                    ok = 0; break;
                }
27
            }
28
29
            if(ok) cand.pb(i);
        }
30
31
32
        rep(i,0,cand.size()) cs[i] = cs[cand[i]], rs[i] = rs[cand[i]];
        m = cand.size();
33
34
35
        db area = 0:
36
37
        //work
38
        rep(i,0,m){
39
            vector<pair<db,int>> ev = {{0,0},{2*PI,0}};
40
41
            int cur = 0;
42
43
            rep(j,0,m) if(j!=i){
                auto ret = isCC(cs[i],rs[i],cs[j],rs[j]);
                if(!ret.empty()){
                    db l = (ret[0] - cs[i]).alpha();
47
                    db r = (ret[1] - cs[i]).alpha();
                    1 = norm(1); r = norm(r);
48
                    ev.pb({1,1}); ev.pb({r,-1});
49
50
                    if(1 > r) ++cur;
```

```
51
52
            }
53
54
            sort(ev.begin(), ev.end());
            rep(j,0,ev.size() - 1){
56
                cur += ev[j].se;
                if(cur == 0){
                    area += calc_area_circle(cs[i],rs[i],ev[j].fi,ev[j+1].fi);
59
            }
61
       }
62 }
```

5 Graph

5.1 bellmanford.cpp

```
1 vector < PII > e[N];
2
3 template <typename T>
   void add(int u, int v, T w) {
       e[u].eb(v, w);
6 }
8 template <typename T>
   vector<T> bellmanford(vector<pair<int, T>> *g, int start) {
10
       // assert(0 <= start && start < q.n);
       // maybe use inf = numeric_limits<T>::max() / 4
11
12
       const T inf = numeric limits<T>::max() / 4:
       vector<T> dist(n. inf):
14
       dist[start] = 0;
15
       int cnt = 0:
16
       while (true) {
17
           bool upd = 0:
18
           cnt++;
19
           for (int i = 0; i < n; i++) {
                for (auto [to, cost] : e[i]) {
20
21
                    if (dist[to] > dist[i] + cost) {
22
                        upd = 1:
23
                        dist[to] = dist[i] + cost;
24
                   }
25
                }
```

```
26 }
27 if (!upd || cnt == n) {
28 break;
29 }
30 }
31 return dist;
32 // returns inf if there's no path
33 }
```

5.2 BlockCutTree.cpp

```
struct BlockCutTree {
       int n:
        std::vector<std::vector<int>> adj;
       std::vector<int> dfn, low, stk;
       int cnt, cur;
        std::vector<std::pair<int, int>> edges;
       BlockCutTree() {}
       BlockCutTree(int n) {
10
            init(n):
11
       }
12
13
       void init(int n) {
            this -> n = n;
14
            adj.assign(n, {});
15
            dfn.assign(n, -1);
16
17
            low.resize(n):
            stk.clear();
18
            cnt = cur = 0:
19
20
            edges.clear();
21
       }
22
       void addEdge(int u, int v) {
23
24
            adj[u].push_back(v);
            adj[v].push_back(u);
25
26
       }
27
       void dfs(int x) {
28
29
            stk.push_back(x);
            dfn[x] = low[x] = cur++;
30
31
32
            for (auto y : adj[x]) {
```

```
33
                if (dfn[y] == -1) {
34
                    dfs(y);
35
                    low[x] = std::min(low[x], low[y]);
36
                    if (low[v] == dfn[x]) {
37
                        int v:
                        do {
38
                            v = stk.back();
                            stk.pop_back();
41
                            edges.emplace_back(n + cnt, v);
42
                        } while (v != y);
43
                        edges.emplace_back(x, n + cnt);
44
                        cnt++;
                    }
45
                } else {
46
47
                    low[x] = std::min(low[x], dfn[y]);
48
            }
49
       }
50
51
52
        std::pair<int, std::vector<std::pair<int, int>>> work() {
53
            for (int i = 0; i < n; i++) {
54
                if (dfn[i] == -1) {
                    stk.clear();
                    dfs(i);
57
                }
            return {cnt, edges};
61 };
```

5.3 boruvka.cpp

```
1  /**
2  * while component > 1:
3  * for each component:
4  * find select[i]
5  * for each component:
6  * if select[i]!= i:
7  * merge(i, select[i])
8  * component--
9  */
10
11  ll ans = 0, cnt = n;
```

```
13
        fill(select + 1, select + n + 1, -1);
        vector<int> cand:
14
15
        for (int i = 1; i <= n; i++) {
            cand.push_back(col[i]);
16
       }
17
        ranges::sort(cand);
18
        cand.erase(unique(all(cand)), cand.end());
19
20
21
        for (auto id : cand) {
22
            for (auto x : S[id]) remove(x);
23
            for (auto x : S[id]) {
                auto [opt, w] = get(x);
24
25
                if (select[id] == -1 || w < mn[id]) {</pre>
                    select[id] = opt, mn[id] = w;
26
                }
27
            }
28
29
            for (auto x : S[id]) insert(x);
30
       }
31
        for (int i = 1; i <= n; i++) if (col[i] == i) {
32
            int i = col[select[i]]:
33
34
            if (i == i) continue:
35
            ans += mn[i];
36
            merge(i, j);
            cnt--;
       }
39 }
```

5.4 dijfast.cpp

12 while (cnt > 1) {

```
1 vector<PII> e[N];
2
3 template <typename T>
4 void add(int u, int v, T w) {
5    e[u].eb(v, w);
6 }
7
8 template <typename T>
9 vector<T> dijkstra(vector<pair<int, T>> *g, int start) {
10    // assert(0 <= start && start < g.n);
11    // maybe use inf = numeric_limits<T>::max() / 4
12 vector<T> dist(n, numeric_limits<T>::max());
```

```
13
        priority_queue<pair<T, int>, vector<pair<T, int>>, greater<pair<T, int</pre>
            >>> s;
        dist[start] = 0:
14
15
        s.emplace(dist[start], start);
        while (!s.empty()) {
16
17
            T expected = s.top().first;
18
            int i = s.top().second;
19
            s.pop();
20
            if (dist[i] != expected) {
21
                continue:
22
23
            for (auto [to, cost] : g[i]) {
                if (dist[i] + cost < dist[to]) {</pre>
24
                     dist[to] = dist[i] + cost;
26
                     s.emplace(dist[to], to);
27
                }
28
            }
        }
        return dist;
31
        // returns numeric limits <T>::max() if there's no path
32 }
```

5.5 dijkstra.cpp

```
1 vector < PII > e[N];
3 template <typename T>
   void add(int u, int v, T w) {
        e[u].eb(v, w);
6 }
   template <typename T>
9 vector<T> dijkstra(vector<pair<int, T>> *g, int start) {
       // assert(0 <= start && start < g.n);
11
       // maybe use inf = numeric limits <T>::max() / 4
        const T inf = numeric limits<T>::max();
       vector<T> dist(n, inf);
14
       vector<int> was(n. 0):
15
       dist[start] = 0;
16
       while (true) {
17
           int cur = -1:
18
           for (int i = 0; i < n; i++) {
19
                if (was[i] || dist[i] == inf) continue:
```

```
20
                if (cur == -1 || dist[i] < dist[cur]) {</pre>
21
                     cur = i;
22
                }
23
            }
            if (cur == -1 || dist[cur] == inf) {
24
25
                break:
26
            was[cur] = 1;
27
28
            for (auto [to, cost] : g[cur]) {
                dist[to] = min(dist[to], dist[cur] + cost);
29
30
            }
        }
31
        return dist:
32
33
        // returns inf if there's no path
34 }
```

5.6 dinic.cpp

```
1 template < typename T>
2 struct FlowGraph {
        static const int V = 1015;
       static const int E = 100015;
       int s, t, vtot;
       int head[V], etot;
       int dis[V], cur[V];
       struct edge {
           int v, nxt;
10
           T f:
11
       e[E * 2];
       void addedge(int u, int v, T f) {
12
            e[etot] = {v, head[u], f};
13
14
           head[u] = etot++;
           e[etot] = {u, head[v], 0};
15
           head[v] = etot++;
16
17
       }
18
       bool bfs() {
19
           for (int i = 1; i <= vtot; i++) {
                dis[i] = 0:
20
21
                cur[i] = head[i];
22
           }
23
            queue < int > q;
24
            q.push(s); dis[s] = 1;
25
            while (!q.empty()) {
```

```
26
                int u = q.front(); q.pop();
27
                for (int i = head[u]; i != -1; i = e[i].nxt) {
28
                    if (e[i].f && !dis[e[i].v]) {
29
                        int v = e[i].v;
30
                        dis[v] = dis[u] + 1:
                        if (v == t) return true;
31
32
                        q.push(v);
                    }
34
                }
            }
35
36
            return false;
37
       }
       T dfs(int u, T m) {
38
39
            if (u == t) return m;
40
            T flow = 0:
41
            for (int i = cur[u]; i != -1; cur[u] = i = e[i].nxt) {
42
                if (e[i].f && dis[e[i].v] == dis[u] + 1) {
                    T f = dfs(e[i].v, min(m, e[i].f));
44
                    e[i].f -= f;
                    e[i ^ 1].f += f;
45
46
                    m -= f;
47
                    flow += f:
48
                    if (!m) break:
49
               }
50
51
            if (!flow) dis[u] = -1;
            return flow:
53
       }
54
       T dinic() {
55
            T flow = 0:
56
            while (bfs()) flow += dfs(s, numeric limits<T>::max());
57
            return flow:
58
       }
59
        void init(int _s, int _t, int _vtot) {
60
            s = _s;
61
            t = _t;
62
            vtot = _vtot;
            etot = 0;
            for (int i = 1; i <= vtot; i++) head[i] = -1;
66 };
```

5.7 dinic-tourist.cpp

```
template <typename T>
    class flow_graph {
    public:
     static constexpr T eps = (T)1e-9;
     struct edge {
       int from;
       int to;
       T c;
10
       T f:
     };
11
12
13
     vector<vector<int>> g;
     vector<edge> edges;
14
     int n:
16
     int st;
     int fin:
17
18
     T flow;
19
     flow_graph(int _n, int _st, int _fin) : n(_n), st(_st), fin(_fin) {
20
21
       assert(0 <= st && st < n && 0 <= fin && fin < n && st != fin);
22
       g.resize(n);
       flow = 0;
24
     }
25
26
     void clear flow() {
27
       for (const edge &e : edges) {
28
         e.f = 0;
       }
       flow = 0;
31
     }
32
     int add(int from, int to, T forward_cap, T backward_cap) {
33
34
       assert(0 \leq from && from \leq n && 0 \leq to && to \leq n):
       int id = (int)edges.size();
35
       g[from].push_back(id);
36
       edges.push_back({from, to, forward_cap, 0});
37
       g[to].push_back(id + 1);
38
39
       edges.push_back({to, from, backward_cap, 0});
       return id;
41
42 };
```

```
43
44 template <typename T>
    class dinic {
    public:
47
     flow_graph<T> &g;
48
49
     vector<int> ptr;
     vector<int> d:
51
     vector<int> q;
52
     dinic(flow_graph<T> &_g) : g(_g) {
       ptr.resize(g.n);
55
       d.resize(g.n);
       q.resize(g.n);
57
     }
58
     bool expath() {
       fill(d.begin(), d.end(), -1);
61
       q[0] = g.fin;
       d[g.fin] = 0;
62
        int beg = 0, end = 1;
        while (beg < end) {
65
          int i = q[beg++];
66
         for (int id : g.g[i]) {
67
            const auto &e = g.edges[id];
68
            const auto &back = g.edges[id ^ 1];
            if (back.c - back.f > g.eps && d[e.to] == -1) {
70
              d[e.to] = d[i] + 1;
71
              if (e.to == g.st) {
72
                return true:
73
74
              q[end++] = e.to;
75
76
         }
77
       }
78
        return false;
79
80
81
     T dfs(int v, T w) {
       if (v == g.fin) {
83
          return w;
84
        int &j = ptr[v];
```

```
while (j \ge 0) {
           int id = g.g[v][j];
 87
           const auto &e = g.edges[id];
 88
 89
           if (e.c - e.f > g.eps && d[e.to] == d[v] - 1) {
             T t = dfs(e.to, min(e.c - e.f, w));
 90
             if (t > g.eps) {
 91
               g.edges[id].f += t;
 92
               g.edges[id ^ 1].f -= t;
 93
 94
               return t;
             }
 95
 96
          }
 97
           j--;
         }
 98
 99
         return 0;
      }
100
101
102
       T max flow() {
         while (expath()) {
103
           for (int i = 0; i < g.n; i++) {
104
             ptr[i] = (int)g.g[i].size() - 1;
105
106
           T big_add = 0;
107
           while (true) {
108
             T add = dfs(g.st, numeric_limits<T>::max());
109
110
             if (add <= g.eps) {
111
               break;
112
113
             big_add += add;
114
115
           if (big_add <= g.eps) {</pre>
116
             break;
117
           }
           g.flow += big_add;
118
119
         }
         return g.flow;
120
121
      }
122
       vector<bool> min cut() {
123
         max_flow();
124
125
         vector<bool> ret(g.n);
         for (int i = 0; i < g.n; i++) {
126
           ret[i] = (d[i] != -1);
127
128
         }
```

```
129 return ret;
130 }
131 };
```

5.8 eulerian-digraph.cpp

```
optional < vector < int >> eulerian_path (int n, const vector < PII > &E) {
     vector<int> res;
     if (E.empty()) return res;
     vector < VI > adj(n + 1);
     vector<int> in(n + 1);
     for (int i = 0; i < ssize(E); i++) {
      auto [u, v] = E[i];
       adj[u].push_back(i);
       in[v] += 1;
11
12
     int s = -1, hi = 0, lo = 0:
13
     for (int i = 1; i <= n; i++) {
14
       if (SZ(adj[i]) == in[i]) continue;
       if (abs(SZ(adj[i]) - in[i]) > 1) return {};
16
       if (SZ(adj[i]) > in[i]) {
         hi++, s = i;
17
18
       } else {
19
          lo++;
20
       }
21
22
     if (!(hi == 0 && lo == 0) && !(hi == 1 && lo == 1)) {
23
        return {};
24
     }
     for (int i = 1; s == -1 && i <= n; i++)
26
       if (!adj[i].empty()) s = i;
27
28
     auto Dfs = [&](auto &Dfs, int u) -> void {
29
        while (!adj[u].empty()) {
          auto id = adj[u].back();
30
31
          adj[u].pop_back();
          int v = E[id].second;
          Dfs(Dfs, v);
          res.push_back(v);
35
       }
36
     };
37
     Dfs(Dfs, s);
```

```
38    if (SZ(res) != SZ(E)) return {};
39     ranges::reverse(res);
40    return res;
41 }
```

5.9 eulerian-undigraph.cpp

```
optional < vector < int >> eulerian_path(int n, const vector < PII > &E) {
      vector<int> res:
     if (E.empty()) return res;
     vector < VI > adj(n + 1);
     for (int i = 0; i < ssize(E); i++) {
       auto [u, v] = E[i];
       adj[u].push_back(i);
       adj[v].push_back(i);
10
11
      int s = -1, odd = 0;
      for (int i = 1; i <= n; i++) {
13
       if (ssize(adj[i]) % 2 == 0) continue;
14
       if (++odd > 2) return {};
       s = i;
15
16
      for (int i = 1; s == -1 \&\& i <= n; i++)
17
18
       if (!adj[i].empty()) s = i;
19
      vector<int> vis(ssize(E));
20
21
      auto Dfs = [&](auto &Dfs, int u) -> void {
22
       while (!adj[u].empty()) {
         auto id = adj[u].back();
23
24
         adj[u].pop back();
         if (vis[id]) continue;
         vis[id] = 1:
         int v = u ^ E[id].fi ^ E[id].se;
         Dfs(Dfs, v);
28
         res.push back(v);
       }
30
31
     };
     Dfs(Dfs, s);
     if (SZ(res) != SZ(E)) return {};
33
     ranges::reverse(res);
34
     return res:
36 }
```

5.10 hungarian.cpp

```
1 vector<vector<int>> e(SZ(rloc));
2 vector<int> match(SZ(cloc), -1), vis(SZ(rloc));
3 for (auto [u, v] : E) {
     e[u].push back(v);
5 }
6 auto find = [\&] (auto&& find, int x) -> bool {
     vis[x] = 1;
    for (auto y : e[x]) {
       if (match[v] == -1 || (!vis[match[v]] && find(find, match[v]))) {
10
         match[y] = x;
11
         return true;
12
    }
13
14
     return false;
15 }:
16 auto DFSMatching = [&]() {
   int res = 0;
   rep(i, 0, SZ(rloc)) {
    fill(all(vis), 0);
    if (find(find, i)) res++;
21
   }
     return res;
23 }:
```

5.11 KM.cpp

```
#include <bits/stdc++.h>
using namespace std;
using l1 = long long;

// L <= R, 左边完全匹配
// 最小权完备匹配

// 最小权完备匹配

// 带权匹配: 使得该二分图的权值和最大(或最小)的匹配。
// 最大匹配: 使得该二分图边数最多的匹配。
// 完备匹配: 使得点数较小的点集中每个点都被匹配的匹配。
// 完善匹配: 使得点数较小的点集中每个点都被匹配的匹配。
// 完美匹配: 所有点都被匹配的匹配。
// 定理1: 最大匹配数 = 最小点覆盖数 (Konig 定理)
// 定理2: 最大匹配数 = 最大独立数
// 定理3: 最小路径覆盖数 = 顶点数 - 最大匹配数
// 定理3: 最小路径覆盖数 = 顶点数 - 最大匹配数
```

```
17 // 定义: 在二分图中, 求最少的点集, 使得每一条边至少都有端点在这个点集中。
18 // 二分图的最小点覆盖 = 二分图的最大匹配
20 // 二分图的最少边覆盖
21 // 定义: 在二分图中, 求最少的边, 使得他们覆盖所有的点, 并且每一个点只被一条
       边 覆 盖。
22 // 二分图的最少边覆盖 = 点数 - 二分图的最大匹配
24 // 二分图的最大独立集
25 // 定义: 在二分图中, 选最多的点, 使得任意两个点之间没有直接边连接。
26 // 二分图的最大独立集 = 点数 - 二分图的最大匹配
28 template < class T>
   pair<T, vector<int>> hungarian(const vector<vector<T>> &a) {
       if (a.empty()) return {0, {}};
30
31
       int n = a.size() + 1, m = a[0].size() + 1;
       vector<T> u(n), v(m); // 顶标
32
       vector<int> p(m). ans(n - 1):
34
       for (int i = 1; i < n; i++) {
35
          p[0] = i:
36
          int j0 = 0;
          vector<T> dist(m. numeric limits<T>::max()):
37
38
          vector<int> pre(m, -1);
39
          vector<bool> done(m + 1);
40
          do { // dijkstra
41
              done[j0] = true;
              int i0 = p[j0], j1;
             T delta = numeric limits<T>::max();
44
              for (int j = 1; j < m; j++) if (!done[j]) {
                 auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
                 if (cur < dist[j]) dist[j] = cur, pre[j] = j0;</pre>
46
                 if (dist[j] < delta) delta = dist[j], j1 = j;</pre>
             }
48
49
              for (int j = 0; j < m; j++) {
                 if (done[j]) u[p[j]] += delta, v[j] -= delta;
50
51
                  else dist[j] -= delta;
52
             }
              j0 = j1;
53
54
          } while (p[j0]);
          while (j0) { // update alternating path
55
              int j1 = pre[j0];
56
57
              p[j0] = p[j1], j0 = j1;
```

58

}

```
}
       for (int j = 1; j < m; j++) {
61
           if (p[j]) ans [p[j] - 1] = j - 1;
62
       }
       return {-v[0], ans}: // min cost
64 }
65
66 int L. R. m:
67 int main() {
       scanf("%d%d%d", &L, &R, &m);
       R = max(L, R);
    auto a = vector<vector<11>>(L, vector<11>(R, 0));
71
       for (int i = 0: i < m: i++) {
72
           int u, v, w;
73
           scanf("%d%d%d", &u, &v, &w);
74
           --u; --v;
           a[u][v] = -w;
76
77
       auto [val, ans] = hungarian(a);
78
       printf("%lld\n", -val);
       for (int i = 0; i < L; i++) {
           if (a[i][ans[i]] >= 0) ans[i] = -1:
           printf("d%c", ans[i] + 1, " | n | [i == L - 1] );
       }
83 }
```

5.12 kosaraju.cpp

```
1 vector<int> e[maxn], erev[maxn];
2 vector<int> c. out:
3 vector<vector<int>> scc:
4 int vis[maxn];
5 void dfs(int u) {
       vis[u] = 1;
       for (auto v : e[u]) if (!vis[v]) dfs(v):
       out.pb(u);
9 }
10 void dfs rev(int u) {
11 vis[u] = 1;
       for (auto v : erev[u]) if (!vis[v]) dfs rev(v):
13
       c.pb(u);
14 }
15 void solve() {
```

```
16
        cin >> n >> m;
        rep(i, 1, m) {
17
            int u, v;
18
19
            cin >> u >> v;
            e[u].pb(v);
20
            erev[v].pb(u);
21
22
        }
        rep(i, 1, n) if (!vis[i]) dfs(i);
23
        fill(vis + 1, vis + n + 1, 0);
24
        reverse(all(out)):
25
26
        for (auto v : out) if (!vis[v]) {
27
                c.clear();
                dfs rev(v):
28
29
                scc.pb(c);
30
            }
31 }
```

5.13 kruskal 重构树.cpp

1 /**

```
* 构建后是一颗二叉树,如果按最小生成树建立的话是大根堆。
   *性质:原图中两个点间所有路径上的边最大权值的最小值=最小生成树上两点简单路
       径的边最大权值
   * = kruskal 重构树上两点LCA的权值。
   * 重构树中代表原树中的点的节点全是叶子节点, 其余节点都代表了一条边的边权。
   * 利用这个性质可以找到点P的简单路径上边权最大值小于lim深度最小的节点。
   * 要求最小权值最大值,可以建最大生成树的重构树从而达到一样的效果。
   */
  vector<tuple<11, 11, 11>> E;
11 rep(i, 1, m) {
12
     int u, v, w;
13
     cin >> u >> v >> w:
     E.emplace_back(w, u, v);
15 }
16 ranges::sort(E);
17 for (auto [w, u, v] : E) {
     u = find(u), v = find(v):
18
19
     if (u == v) continue;
     int p = ++idx:
20
21
     lim[p] = w;
22
     fa[u] = p, fa[v] = p;
23
     e[p].push_back(u);
```

5.14 MCMF.cpp

```
1 template < typename T >
2 struct MinCostGraph {
        static const int V = 20100;
        static const int E = 201000;
        int s, t, vtot;
        int head[V], etot;
        T dis[V], flow, cost;
        int pre[V];
        bool vis[V]:
10
11
        struct edge {
12
            int v, nxt;
13
            T f, c;
14
        } e[E * 2]:
15
        void addedge(int u,int v, T f, T c, T f2 = 0){
16
            e[etot] = {v, head[u], f, c}; head[u] = etot++;
17
            e[etot] = \{u, head[v], f2, -c\}; head[v] = etot++;
18
       }
19
20
        bool spfa() {
21
            T inf = numeric limits<T>::max() / 2:
22
            for (int i = 1; i <= vtot; i++) {
23
                dis[i] = inf:
24
                vis[i] = false;
25
                pre[i] = -1;
            }
26
27
            dis[s] = 0;
28
            vis[s] = true:
            queue < int > q;
29
30
            q.push(s);
            while (!q.empty()) {
31
32
                int u = q.front();
33
                for (int i = head[u]: ~i: i = e[i].nxt) {
34
                    int v = e[i].v;
35
                    if (e[i].f && dis[v] > dis[u] + e[i].c) {
36
                         dis[v] = dis[u] + e[i].c:
```

```
37
                         pre[v] = i;
                         if (!vis[v]) {
38
                             vis[v] = 1:
39
40
                             q.push(v);
                        }
41
                    }
42
                }
43
44
                q.pop();
45
                vis[u] = false;
            }
46
47
            return dis[t] != inf;
       }
48
49
50
        void augment() {
51
            int u = t:
            T f = numeric_limits<T>::max();
52
53
            while (~pre[u]) {
                f = min(f, e[pre[u]].f);
54
                u = e[pre[u] ^ 1].v;
55
            }
56
57
            flow += f;
            cost += f * dis[t]:
58
59
            u = t:
60
            while (~pre[u]) {
61
                e[pre[u]].f -= f;
                e[pre[u] ^ 1].f += f;
62
                u = e[pre[u] ^ 1].v;
63
64
            }
65
       }
66
67
        pair<T, T> solve() {
            flow = 0;
69
            cost = 0;
70
            while (spfa()) augment();
71
            return {flow, cost};
72
73
        void init(int s_, int t_, int vtot_) {
74
            s = s_{;}
75
            t = t_;
76
            vtot = vtot_;
            etot = 0;
77
78
            for (int i = 1; i <= vtot; i++) head[i] = -1;
79
       }
```

```
80 };
```

5.15 MCMFfast.cpp

```
1 template <typename flow_t = int, typename cost_t = long long>
2 struct MCMF_SSPA {
3
        int N;
4
        vector<vector<int>> adj;
5
        struct edge_t {
6
            int dest;
7
           flow_t cap;
8
            cost_t cost;
9
       }:
10
        vector<edge_t> edges;
11
12
        vector<char> seen;
13
        vector<cost_t> pi;
14
       vector<int> prv;
15
16
        explicit MCMF SSPA(int N ): N(N ), adj(N), pi(N, O), prv(N) {}
17
18
        void addEdge(int from, int to, flow t cap, cost t cost) {
            assert(cap >= 0);
19
20
            int e = int(edges.size());
21
            edges.emplace back(edge t{to, cap, cost});
22
            edges.emplace_back(edge_t{from, 0, -cost});
23
            adj[from].push back(e);
24
            adj[to].push_back(e+1);
25
       }
26
27
        const cost_t INF_COST = numeric_limits<cost_t>::max() / 4;
28
        const flow t INF FLOW = numeric limits<flow t>::max() / 4;
29
        vector<cost_t> dist;
30
        __gnu_pbds::priority_queue<pair<cost_t, int>> q;
31
        vector<typename decltype(q)::point_iterator> its;
32
        void path(int s) {
33
            dist.assign(N, INF_COST);
            dist[s] = 0:
34
35
36
            its.assign(N, q.end());
37
            its[s] = q.push({0, s});
38
39
            while (!q.empty()) {
```

```
40
                int i = q.top().second; q.pop();
                cost t d = dist[i];
41
                for (int e : adj[i]) {
43
                    if (edges[e].cap) {
                        int j = edges[e].dest;
44
                        cost_t nd = d + edges[e].cost;
45
                        if (nd < dist[j]) {</pre>
46
                             dist[j] = nd;
47
48
                             prv[i] = e;
                             if (its[j] == q.end()) {
49
50
                                 its[j] = q.push({-(dist[j] - pi[j]), j});
51
                             } else {
                                 q.modify(its[j], {-(dist[j] - pi[j]), j});
52
53
                            }
54
                        }
                    }
55
56
                }
            }
57
58
            swap(pi, dist);
59
       }
60
61
62
        vector<pair<flow t. cost t>> maxflow(int s. int t) {
            assert(s != t);
63
64
            flow t totFlow = 0: cost t totCost = 0:
            vector<pair<flow t, cost t>> res;
65
            while (path(s), pi[t] < INF_COST) {</pre>
                flow t curFlow = numeric limits<flow t>::max();
67
68
                for (int cur = t; cur != s; ) {
                    int e = prv[cur];
69
                    int nxt = edges[e^1].dest;
70
71
                    curFlow = min(curFlow, edges[e].cap);
72
                    cur = nxt;
73
                }
74
                totFlow += curFlow;
75
                totCost += pi[t] * curFlow;
                for (int cur = t; cur != s; ) {
                    int e = prv[cur];
77
78
                    int nxt = edges[e^1].dest;
79
                    edges[e].cap -= curFlow;
                    edges[e^1].cap += curFlow;
80
81
                    cur = nxt:
82
                }
```

5.16 MCMFfull.cpp

```
1 template <typename T, typename C>
2 class MCMF {
3
    public:
4
      static constexpr T eps = (T) 1e-9;
5
6
      struct edge {
        int from:
        int to;
9
       T c:
10
       T f;
11
        C cost;
12
     }:
13
14
      int n:
15
      vector<vector<int>> g;
16
      vector<edge> edges;
17
      vector<C> d:
18
      vector<C> pot;
19
      __gnu_pbds::priority_queue<pair<C, int>> q;
20
      vector<typename decltype(q)::point_iterator> its;
21
      vector<int> pe:
22
      const C INF C = numeric limits<C>::max() / 2:
23
24
      explicit MCMF(int n_{-}) : n(n_{-}), g(n), d(n), pot(n, 0), its(n), pe(n) {}
25
26
      int add(int from, int to, T forward cap, T backward cap, C edge cost) {
27
        assert(0 \leq from && from \leq n && 0 \leq to && to \leq n):
28
        assert(forward_cap >= 0 && backward_cap >= 0);
29
        int id = static_cast<int>(edges.size());
30
        g[from].push back(id);
31
        edges.push back({from. to. forward cap. 0. edge cost}):
32
        g[to].push back(id + 1);
33
        edges.push_back({to, from, backward_cap, 0, -edge_cost});
34
        return id:
```

```
35
     }
36
      void expath(int st) {
37
       fill(d.begin(), d.end(), INF_C);
38
39
       q.clear();
       fill(its.begin(), its.end(), q.end());
40
       its[st] = q.push({pot[st], st});
41
       d[st] = 0:
       while (!q.empty()) {
43
         int i = q.top().second;
44
45
         q.pop();
         its[i] = q.end();
46
         for (int id : g[i]) {
47
            const edge &e = edges[id];
48
            int j = e.to;
49
            if (e.c - e.f > eps && d[i] + e.cost < d[j]) {</pre>
50
51
              d[i] = d[i] + e.cost;
              pe[j] = id;
52
             if (its[j] == q.end()) {
53
               its[j] = q.push({pot[j] - d[j], j});
54
55
              } else {
56
                q.modify(its[j], {pot[j] - d[j], j});
57
             }
58
            }
59
         }
       }
60
       swap(d, pot);
61
62
     }
63
64
      pair<T, C> calc(int st, int fin) { // max flow min cost
       T flow = 0;
65
       C cost = 0:
       bool ok = true;
67
68
       for (auto& e : edges) {
69
         if (e.c - e.f > eps && e.cost + pot[e.from] - pot[e.to] < 0) {
70
            ok = false;
71
            break;
         }
72
73
       }
74
       if (ok) {
75
          expath(st);
       } else {
76
77
          vector<int> deg(n, 0);
```

```
78
           for (int i = 0; i < n; i++) {
 79
             for (int eid : g[i]) {
               auto& e = edges[eid];
 80
 81
               if (e.c - e.f > eps) {
 82
                 deg[e.to] += 1;
 83
               }
 84
             }
           }
           vector<int> que;
           for (int i = 0; i < n; i++) {
 87
             if (deg[i] == 0) {
               que.push back(i);
 90
             }
           }
 91
 92
           for (int b = 0; b < (int) que.size(); b++) {</pre>
 93
             for (int eid : g[que[b]]) {
 94
               auto& e = edges[eid];
               if (e.c - e.f > eps) {
                 deg[e.to] -= 1;
 96
                 if (deg[e.to] == 0) {
 97
 98
                    que.push_back(e.to);
 99
                 }
100
               }
101
             }
102
           }
103
           fill(pot.begin(), pot.end(), INF_C);
104
           pot[st] = 0;
105
           if (static_cast<int>(que.size()) == n) {
106
             for (int v : que) {
107
               if (pot[v] < INF_C) {</pre>
108
                 for (int eid : g[v]) {
109
                    auto& e = edges[eid];
110
                   if (e.c - e.f > eps) {
111
                     if (pot[v] + e.cost < pot[e.to]) {</pre>
112
                        pot[e.to] = pot[v] + e.cost;
113
                        pe[e.to] = eid;
114
                     }
115
                   }
116
                 }
117
               }
118
             }
           } else {
119
120
             que.assign(1, st);
```

```
121
             vector < bool > in_queue(n, false);
             in queue[st] = true;
122
             for (int b = 0; b < (int) que.size(); b++) {</pre>
123
124
               int i = que[b];
               in_queue[i] = false;
125
               for (int id : g[i]) {
126
                 const edge &e = edges[id];
127
                 if (e.c - e.f > eps && pot[i] + e.cost < pot[e.to]) {
128
129
                   pot[e.to] = pot[i] + e.cost;
                   pe[e.to] = id;
130
                   if (!in_queue[e.to]) {
131
                     que.push back(e.to);
132
                     in_queue[e.to] = true;
133
134
                   }
                 }
135
136
               }
137
          }
138
139
         // debug(pot[fin]);
140
         while (pot[fin] < INF_C) \{ // < 0
141
          T push = numeric_limits<T>::max();
142
           int v = fin:
143
144
           while (v != st) {
145
             const edge &e = edges[pe[v]];
             push = min(push, e.c - e.f);
146
             v = e.from;
147
148
          }
149
           v = fin;
           while (v != st) {
150
             edge &e = edges[pe[v]];
151
152
             e.f += push;
             edge &back = edges[pe[v] ^ 1];
153
154
             back.f -= push;
             v = e.from;
155
156
          }
           flow += push;
157
           cost += push * pot[fin];
158
159
           expath(st);
160
         return {flow, cost};
161
162
```

163 };

5.17 prim.cpp

```
1 vector < PII > e[N];
   template <typename T>
    void add(int u, int v, T w) {
        e[u].eb(v, w);
6 }
   template <typename T>
   T prim(vector<pair<int, T>> *g, int start) {
        const T inf = numeric_limits<T>::max() / 4;
11
        T res = 0:
12
        vector<T> dist(n, inf);
13
        dist[start] = 0;
14
        priority_queue<pair<T, int>, vector<pair<T, int>>, greater<pair<T, int</pre>
15
        s.emplace(dist[start], start);
        vector<int> was(n, 0);
16
17
        while (!s.empty()) {
            T expected = s.top().first;
18
19
            int i = s.top().second;
20
            s.pop();
21
            if (dist[i] != expected || was[i]) {
22
                 continue:
23
            }
24
            was[i] = 1;
25
            res += expected;
26
            for (auto [to, cost] : g[i]) {
27
                if (cost < dist[to]) {</pre>
                    dist[to] = cost;
                    s.emplace(dist[to], to);
29
31
            }
32
        }
        return res;
34 }
```

5.18 PushRelabel.cpp

```
1 /**
2 * Author: Simon Lindholm
3 * Date: 2015-02-24
4 * License: CCO
```

```
* Source: Wikipedia, tinyKACTL
    * Description: Push-relabel using the highest label selection rule and the
         gap heuristic. Quite fast in practice.
    * To obtain the actual flow, look at positive values only.
    * Time: $0(V^2\sqrt E)$
    * Status: Tested on Kattis and SPOJ, and stress-tested
11 #pragma once
12
    struct PushRelabel {
13
     typedef vector<int> vi;
14
     struct Edge {
15
16
       int dest, back;
17
       11 f, c;
18
     }:
     vector<vector<Edge>> g;
19
     vector<11> ec;
21
     vector < Edge *> cur;
22
     vector<vi> hs; vi H;
     PushRelabel(int n): g(n), ec(n), cur(n), hs(2*n), H(n) {}
23
24
      void addEdge(int s, int t, ll cap, ll rcap=0) {
25
       if (s == t) return:
26
27
       g[s].push_back({t, SZ(g[t]), 0, cap});
28
       g[t].push_back({s, SZ(g[s])-1, 0, rcap});
29
     }
30
31
     void addFlow(Edge& e, ll f) {
32
       Edge &back = g[e.dest][e.back];
       if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
33
34
       e.f += f; e.c -= f; ec[e.dest] += f;
35
       back.f -= f; back.c += f; ec[back.dest] -= f;
     }
36
37
     11 calc(int s, int t) {
       int v = SZ(g); H[s] = v; ec[t] = 1;
38
39
       vi co(2*v); co[0] = v-1;
       rep(i,0,v-1) cur[i] = g[i].data();
       for (Edge& e : g[s]) addFlow(e, e.c);
41
42
       for (int hi = 0;;) {
43
          while (hs[hi].empty()) if (!hi--) return -ec[s];
44
         int u = hs[hi].back(); hs[hi].pop_back();
45
46
         while (ec[u] > 0) // discharge u
```

```
47
              if (cur[u] == g[u].data() + SZ(g[u])) {
48
                H[u] = 1e9;
                for (Edge& e : g[u]) if (e.c && H[u] > H[e.dest]+1)
49
50
                   H[u] = H[e.dest]+1, cur[u] = &e;
                if (++co[H[u]], !--co[hi] && hi < v)
51
52
                   rep(i,0,v-1) if (hi < H[i] && H[i] < v)
                     --co[H[i]], H[i] = v + 1;
54
                hi = H[u]:
55
             } else if (\operatorname{cur}[u] \rightarrow c \&\& H[u] == H[\operatorname{cur}[u] \rightarrow \operatorname{dest}] + 1)
56
                addFlow(*cur[u], min(ec[u], cur[u]->c));
57
              else ++cur[u];
         }
      }
      bool leftOfMinCut(int a) { return H[a] >= SZ(g); }
61 };
```

5.19 tarjan 割点.cpp

```
1 vector<int> g[maxn], ans;
2 stack<int> stk:
3 int dfn[maxn], cut[maxn], low[maxn], idx;
  void dfs(int x. int f) {
       low[x] = dfn[x] = ++idx;
       stk.push(x);
       int ch = 0;
       for (auto v : g[x]) {
10
           if (!dfn[y]) {
11
                ch++;
12
                dfs(v. x):
13
               low[x] = min(low[x], low[y]);
14
                if (low[y] >= dfn[x]) cut[x] = 1;
15
           } else {
                if (y != f) low[x] = min(low[x], dfn[y]);
17
           }
       if (x == 1 \&\& ch <= 1) cut[x] = 0;
       if (cut[x]) ans.pb(x);
21 }
```

5.20 tarjan 割边.cpp

```
1 vector < PII > g[maxn];
```

```
2 stack<int> stk;
3 int dfn[maxn], ins[maxn], low[maxn];
4 int idx, tot:
5 VI ans;
   void dfs(int x, int f) {
       low[x] = dfn[x] = ++idx;
       stk.push(x);
       ins[x] = 1:
10
       for (auto [y, id] : g[x]) {
11
           if (!dfn[y]) {
12
                dfs(y, id);
               low[x] = min(low[x], low[y]);
13
14
15
                if (ins[y] \&\& id != f) low[x] = min(low[x], dfn[y]);
           }
16
       }
17
18
       if (low[x] >= dfn[x]) {
19
            ++tot:
20
            while (true) {
                int cur = stk.top();
21
22
                stk.pop();
                ins[cur] = 0;
23
                if (cur == x) break;
24
25
           }
26
           if (f != 0) ans.pb(f);
       }
27
28 }
```

5.21 tarjan 强连通分量.cpp

```
1 vector<int> g[maxn];
2 stack<int> stk;
3 int dfn[maxn], ins[maxn], low[maxn], belong[maxn];
   int idx, tot;
   void dfs(int x) {
       low[x] = dfn[x] = ++idx;
       ins[x] = 1;
       stk.push(x);
       for (auto y : g[x]) {
10
11
           if (!dfn[y]) {
12
               dfs(y);
               low[x] = min(low[x], low[y]);
13
```

```
14
           } else {
                if (ins[y]) low[x] = min(low[x], dfn[y]);
           }
16
17
       }
       if (low[x] >= dfn[x]) {
18
            ++tot;
19
20
            while (true) {
21
                int cur = stk.top(); stk.pop();
22
                ins[cur] = 0;
                belong[cur] = tot;
23
                if (cur == x) break;
25
           }
       }
26
27 }
```

5.22 tarjan 点双.cpp

```
1 vector<int> g[maxn];
2 stack<int> stk;
3 int dfn[maxn], low[maxn], idx, tot, cut[maxn];
   vector < int > bcc[maxn];
5
   void dfs(int x, int f) {
       low[x] = dfn[x] = ++idx;
       stk.push(x);
       int ch = 0;
10
       for (auto y : g[x]) {
11
            if (!dfn[y]) {
12
                ch++;
13
                dfs(y, x);
14
                low[x] = min(low[x], low[y]);
15
                if (low[y] >= dfn[x]) {
16
                    cut[x] = 1;
17
                    ++tot;
18
                    bcc[tot].pb(x);
                    while (true) {
19
20
                        int cur = stk.top();
21
                        stk.pop();
22
                        bcc[tot].pb(cur);
                        if (cur == y) break;
24
                    }
25
                }
26
           } else {
```

```
void dfs(int x, int f) {
       low[x] = dfn[x] = ++idx:
       stk.push(x);
       for (auto [y, id] : g[x]) {
           if (!dfn[y]) {
11
                dfs(v, id);
               low[x] = min(low[x], low[v]):
13
           } else {
14
                if (id != f) low[x] = min(low[x], dfn[y]);
           }
15
16
       }
17
       if (low[x] >= dfn[x]) {
           ++tot;
            while (true) {
                int cur = stk.top();
                stk.pop();
                belong[cur] = tot;
                bcc[tot].pb(cur);
24
                if (cur == x) break;
           }
       }
27 }
```

5.24 twosat.cpp

```
1 class twosat {
2 public:
3    digraph<int> g;
4    int n;
5
```

```
twosat(int _n) : g(digraph<int>(2 * _n)), n(_n) {
7
       }
8
9
       // (v[x] == value x)
10
       inline void add(int x. int value x) {
            assert(0 \leq x && x \leq n):
11
12
            assert(0 <= value x && value x <= 1);
            g.add(2 * x + (value_x ^1), 2 * x + value_x);
       }
14
15
16
       // (v[x] == value x // v[y] == value y)
17
        inline void add(int x, int value x, int y, int value y) {
            assert(0 <= x && x < n && 0 <= v && v < n):
18
19
            assert(0 <= value x && value x <= 1 && 0 <= value y && value y <= 1)
            g.add(2 * x + (value_x ^ 1), 2 * y + value_y);
21
            g.add(2 * y + (value y \hat{} 1), 2 * x + value x);
22
23
24
        inline vector<int> solve() {
25
            int cnt;
26
            vector<int> c = find scc(g, cnt):
27
            vector<int> res(n):
            for (int i = 0; i < n; i++) {
29
                if (c[2 * i] == c[2 * i + 1]) {
                    return vector < int > ();
                res[i] = (c[2 * i] < c[2 * i + 1]);
34
            return res:
36 }:
```

5.25 差分约束系统.cpp

```
      1 /**

      2 Description:

      3 求解方程组 x_u - x_v <= w_i, 求出的x_i为满足条件的最大值</td>

      4 转化为x_u <= x_v + w_i</td>

      6 问题等价于求最短路 (bellmanford或Floyd)

      6 即加一条有向边add(u, v, w), dist[v] = min(dist[v], dist[u] + w)

      7 求最小值 (满足条件情况下尽量小)等价于求(-x_i)最大(或者转化为求最长路)

      8 求非负解只需要添加超级节点S、S向各个点连边(S + 0 <= x i), 再设dist[S]</td>
```

```
10 void solve() {
11
        cin >> n >> m;
        vector<int> dist(n, 0);
12
        vector<vector<PII>>> g(n);
13
14
        rep(i, 0, m - 1) {
15
            int u, v, w;
16
            cin >> u >> v >> w;
            u--, v--;
17
18
            g[u].eb(v, -w);
        }
19
        bool ok = 1;
20
21
        rep(i, 1, n) {
22
            bool upd = 0;
            rep(u, 0, n - 1) {
23
24
                for (auto [v, w] : g[u]) {
                    if (dist[v] < dist[u] + w) {</pre>
25
                         dist[v] = dist[u] + w;
                        upd = 1;
27
                    }
28
                }
29
            }
30
31
            if (!upd) break;
32
            // 仍然有约束未满足
33
            if (i == n && upd) ok = 0;
        }
34
        if (!ok) {
35
36
            return cout << -1 << '\n', void();
        }
37
38
        rep(i, 0, n - 1) {
39
            cout << dist[i] << "_\\n"[i == n - 1];</pre>
40
        }
41 }
```

= 0

6 Math

6.1 binom.cpp

```
vector<Mint> fact(1, 1);
vector<Mint> inv_fact(1, 1);
```

```
4 Mint C(int n, int k) {
       if (k < 0 | | k > n) {
6
           return 0:
7
       }
       while ((int)fact.size() < n + 1) {</pre>
           fact.push_back(fact.back() * (int)fact.size());
10
           inv fact.push back(1 / fact.back());
11
12
       return fact[n] * inv fact[k] * inv fact[n - k];
13 }
14
   const int mod = 1000000007;
16 const int T = 1000000:
17 ll fact[] = {};
18 11 powmod(11 a, 11 b) {
       ll ret = 1;
       for (; b; b >>= 1) {
           if (b & 1) ret = ret * a % mod:
           a = a * a \%mod;
23
       }
24
       return ret;
25 }
26 ll fac(int n) {
       ll v = fact[n / T];
       for (int i = n / T * T + 1; i <= n; i++)
           v = v * i \% mod;
       return v;
31 }
32 11 binom(int n, int m) {
       if (m < 0 || m > n) return 0;
       return fac(n) * powmod(fac(m) * fac(n - m) % mod, mod - 2) % mod;
35 }
```

6.2 bsgs.cpp

```
int bsgs(int a, int b, int m) { // a^x=b(mod m)
int res = m + 1;
int t = sqrt(m) + 2;

ll d = powmod(a, t, m);

ll cnt = 1;
   //map < int, int > p;
hs.init();

for (int i = 1; i <= t; i++) {</pre>
```

```
cnt = cnt * d % m:
           //if (!p.count(cnt)) p[cnt] = i;
10
           if (hs.query(cnt) == -1) hs.insert(cnt, i);
11
12
       }
       cnt = b:
13
       for (int i = 1: i <= t: i++) {
14
           cnt = cnt * a % m;
15
           //if (p.count(cnt)) res = min(res, p[cnt] * t - i);
17
           int tmp = hs.query(cnt);
           if (tmp != -1) res = min(res, tmp * t - i);
18
19
20
       if (res >= m) res = -1;
       return res:
21
22 }
```

6.3 cantor.cpp

1 ll fac[maxn], A[maxn], w[maxn];

```
2 void init(int n) {
                                    fac[0] = 1;
                                     rep(i, 1, n) fac[i] = fac[i - 1] * i % mod:
     6 ll cantor(int w[], int n) {
                                     ll ans = 1:
                                     for (int i = 1; i \le n; i++) { // can optimize by BIT
                                                        for (int j = i + 1; j \le n; j++) {
                                                                          if (w[i] > w[j]) A[i]++;
  11
                                                      }
  12
                                    }
                                     for (int i = 1: i < n: i++) {
                                                        ans += A[i] * fac[n - i]:
  14
  15
                                     }
 16
                                      return ans:
17 }
                   void decanter(ll x, int n) { // x - rank n - r
  20
                                     vector<int> rest(n. 0):
 21
                                     iota(rest.begin(), rest.end(), 1); // rest->1,2,3,4...
  22
                                     for (int i = 1: i <= n: i++) {
  23
                                                        A[i] = x / fac[n - i];
                                                        x %= fac[n - i];
  25
                                    }
```

6.4 EXCRT modequ exgcd.cpp

```
1 ll exgcd(ll a, ll b, ll &x, ll &y) {
      if (b == 0) {
        x = 1, v = 0:
          return a;
      11 d = exgcd(b, a \% b, y, x);
      y = (a / b) * x;
      return d:
9 }
12 11 modequ(11 a, 11 b, 11 m) {
    11 x, y;
   11 d = exgcd(a, m, x, y);
   if (b % d != 0) return -1;
      m /= d; a /= d; b /= d;
   x = x * b % m:
      if (x < 0) x += m;
       return x:
20 }
21
22 void merge(ll &a, ll &b, ll c, ll d) {
      if (a == -1 || b == -1) return;
24
      11 x, v;
      ll g = exgcd(b, d, x, y);
       if ((c - a) % g != 0) {
          a = -1, b = -1;
          return;
      }
      d /= g;
    11 t = ((c - a) / g) \% d * x \% d;
      if (t < 0) t += d:
      a = b * t + a;
       b = b * d:
35 }
```

6.5 factor.cpp

```
namespace Factor {
2
        const int N=1010000;
        ll C, fac[10010], n, mut, a[1001000];
        int T, cnt, i, l, prime[N], p[N], psize, _cnt;
        ll _e[100],_pr[100];
        vector<ll> d;
        inline 11 mul(11 a,11 b,11 p) {
            if (p<=1000000000) return a*b%p;
            else if (p \le 100000000000011) return (((a*(b>>20)%p) \le 20)+(a*(b))
                 &((1<<20)-1)))%p;
            else {
10
11
                11 d=(11)floor(a*(long double)b/p+0.5);
12
                11 ret=(a*b-d*p)%p;
                if (ret<0) ret+=p;
13
                return ret:
14
15
            }
        }
16
17
        void prime table(){
18
            int i,j,tot,t1;
            for (i=1;i<=psize;i++) p[i]=i;</pre>
19
20
            for (i=2,tot=0;i<=psize;i++){</pre>
                if (p[i]==i) prime[++tot]=i;
21
22
                for (j=1;j<=tot && (t1=prime[j]*i)<=psize;j++){</pre>
23
                    p[t1]=prime[j];
                     if (i%prime[j]==0) break;
24
                }
25
26
            }
27
        }
        void init(int ps) {
28
29
            psize=ps;
30
            prime table();
        }
31
32
        ll powl(ll a,ll n,ll p) {
33
            ll ans=1:
            for (;n;n>>=1) {
34
35
                if (n&1) ans=mul(ans,a,p);
36
                a=mul(a,a,p);
            }
37
38
            return ans:
39
        }
40
        bool witness(ll a,ll n) {
41
            int t=0:
```

```
42
            ll u=n-1:
43
            for (;~u&1;u>>=1) t++;
44
            11 x=pow1(a,u,n),_x=0;
45
            for (;t;t--) {
46
                 _x=mul(x,x,n);
47
                if (_x==1 && x!=1 && x!=n-1) return 1;
48
                 x = _x;
49
            }
50
            return x!=1;
51
        }
52
        bool miller(ll n) {
53
            if (n<2) return 0;
54
            if (n<=psize) return p[n]==n;</pre>
55
            if (~n&1) return 0;
56
            for (int j=0; j <=7; j++) if (witness(rng()%(n-1)+1,n)) return 0;
57
            return 1;
58
        }
59
        11 gcd(ll a,ll b) {
60
            ll ret=1;
            while (a!=0) {
61
62
                 if ((~a&1) && (~b&1)) ret <<=1,a>>=1,b>>=1;
                 else if (-a\&1) a>>=1: else if (-b\&1) b>>=1:
64
                 else {
65
                     if (a<b) swap(a,b);</pre>
66
                     a-=b:
67
            }
69
            return ret*b;
70
        }
71
        ll rho(ll n) {
72
            while (1) {
73
                 11 X=rng()%n,Y,Z,T=1,*1Y=a,*1X=1Y;
74
                 int tmp=20;
75
                 C=rng()%10+3;
                 X=mul(X,X,n)+C;*(1Y++)=X;1X++;
76
77
                 Y = mul(X, X, n) + C; *(1Y++) = Y;
78
                 for(;X!=Y;) {
79
                     11 t=X-Y+n;
80
                     Z=mul(T,t,n);
81
                     if(Z==0) return gcd(T,n);
82
                     tmp--;
                     if (tmp==0) {
83
84
                         tmp=20;
```

```
Z=\gcd(Z,n);
 85
                          if (Z!=1 && Z!=n) return Z;
 86
                      }
 87
 88
                      T=Z;
                      Y = *(1Y + +) = mul(Y, Y, n) + C;
 89
                      Y = *(1Y + +) = mul(Y, Y, n) + C:
 90
                      X = *(1X + +);
 91
                 }
 93
             }
         }
 94
 95
         void factor(ll n) {
             for (int i=0;i<cnt;i++) {</pre>
 96
                  if (n%fac[i]==0) n/=fac[i],fac[cnt++]=fac[i];}
 97
             if (n<=psize) {</pre>
 98
                  for (;n!=1;n/=p[n]) fac[cnt++]=p[n];
 99
100
                  return;
101
             if (miller(n)) fac[cnt++]=n:
102
             else {
103
                 11 x=rho(n):
104
105
                  _factor(x);_factor(n/x);
             }
106
         }
107
108
         void dfs(ll x,int dep) {
109
             if (dep==_cnt) d.pb(x);
             else {
110
                  dfs(x,dep+1);
111
                  for (int i=1; i \le e[dep]; i++) dfs(x*=pr[dep], dep+1);
112
113
             }
114
         }
         void norm() {
115
116
             sort(fac,fac+cnt);
             cnt=0;
117
             rep(i,0,cnt-1) if (i==0||fac[i]!=fac[i-1]) _pr[_cnt]=fac[i],_e[_cnt
118
                  ++]=1:
119
                  else e[ cnt-1]++;
         }
120
         vector<ll> getd() {
121
122
             d.clear();
123
             dfs(1,0);
             return d;
124
125
         }
126
         vector<ll> factor(ll n) {
```

```
127
             cnt=0:
128
             _factor(n);
129
             norm();
130
             return getd();
131
         }
132
         vector<PLL> factorG(ll n) {
133
             cnt=0;
134
             factor(n):
135
             norm();
136
             vector < PLL > d:
137
             rep(i,0,_cnt-1) d.pb(mp(_pr[i],_e[i]));
138
             return d;
        }
139
140
         bool is primitive(ll a,ll p) {
141
             assert(miller(p));
142
             vector<PLL> D=factorG(p-1);
143
             rep(i,0,SZ(D)-1) if (powl(a,(p-1)/D[i].fi,p)==1) return 0;
144
             return 1:
145
        }
146
        11 phi(11 n) {
147
             auto d=factorG(n);
148
             for (auto p:d) n=n/p.fi*(p.fi-1);
149
             return n:
150
        }
151 }
```

6.6 fft.cpp

```
namespace fft {
      typedef double dbl:
3
4
      struct num {
5
       dbl x, y;
       num() { x = y = 0; }
       num(dbl x, dbl y) : x(x), y(y) { }
8
     };
9
10
     inline num operator+(num a. num b) { return num(a.x + b.x. a.v + b.v): }
11
      inline num operator-(num a, num b) { return num(a.x - b.x, a.y - b.y); }
12
     inline num operator*(num a. num b) { return num(a.x * b.x - a.v * b.v. a.x
          * b.v + a.v * b.x); }
13
      inline num conj(num a) { return num(a.x, -a.y); }
14
```

```
int base = 1:
15
      vector < num > roots = \{\{0, 0\}, \{1, 0\}\};
16
      vector < int > rev = \{0, 1\}:
17
18
      const dbl PI = acosl(-1.0):
19
20
      void ensure base(int nbase) {
21
22
        if (nbase <= base) {
23
          return;
24
        }
25
        rev.resize(1 << nbase);
        for (int i = 0; i < (1 << nbase); i++) {
26
          rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1)):
27
28
        }
29
        roots.resize(1 << nbase):
30
        while (base < nbase) {</pre>
          dbl \ angle = 2 * PI / (1 << (base + 1));
            num z(cos(angle), sin(angle)):
          for (int i = 1 << (base - 1); i < (1 << base); i++) {
33
            roots[i << 1] = roots[i]:</pre>
34
35 //
            roots[(i << 1) + 1] = roots[i] * z;
36
            dbl angle i = angle * (2 * i + 1 - (1 << base)):
            roots[(i << 1) + 1] = num(cos(angle i), sin(angle i));
37
38
         }
39
          base++:
        }
41
     }
42
43
      void fft(vector<num> &a, int n = -1) {
44
        if (n == -1) {
45
         n = a.size();
46
        assert((n & (n - 1)) == 0):
47
48
        int zeros = __builtin_ctz(n);
        ensure_base(zeros);
49
50
        int shift = base - zeros;
51
        for (int i = 0; i < n; i++) {
          if (i < (rev[i] >> shift)) {
52
            swap(a[i], a[rev[i] >> shift]);
53
54
         }
        }
55
56 /*
         for (int k = 1: k < n: k <<= 1) {
57
          for (int \ i = 0; \ i < n; \ i += 2 * k) {
```

```
for (int j = 0; j < k; j++) {
               num z = a[i + j + k] * roots[j + k];
               a \lceil i + j + k \rceil = a \lceil i + j \rceil - z:
61
               a[i + j] = a[i + j] + z;
            7
          7
        7*/
         for (int len = 1: len < n: len <<= 1) {
           for (int i = 0; i < n; i += 2 * len) {
67
             for (int j = i, k = i + len; j < i + len; j++, k++) {
               num z = a[k] * roots[k - i];
               a[k] = a[i] - z;
              a[i] = a[i] + z:
70
71
            }
72.
          }
        }
74
75
76
      vector < num > fa, fb;
77
78
       vector<long long> multiply(vector<int> &a, vector<int> &b) {
79
        int need = a.size() + b.size() - 1:
        int nbase = 0:
81
        while ((1 << nbase) < need) nbase++;
82
        ensure base(nbase):
        int sz = 1 << nbase;</pre>
        if (sz > (int) fa.size()) {
           fa.resize(sz):
86
        }
87
        for (int i = 0: i < sz: i++) {
          int x = (i < (int) a.size() ? a[i] : 0);
          int y = (i < (int) b.size() ? b[i] : 0);</pre>
90
          fa[i] = num(x, y);
91
        }
92
        fft(fa, sz);
         num r(0, -0.25 / sz);
         for (int i = 0; i <= (sz >> 1); i++) {
94
          int j = (sz - i) & (sz - 1);
96
          num z = (fa[j] * fa[j] - conj(fa[i] * fa[i])) * r;
97
          if (i != j) {
           fa[i] = (fa[i] * fa[i] - conj(fa[i] * fa[i])) * r;
          }
100
           fa[i] = z;
```

```
101
        }
         fft(fa, sz);
102
         vector<long long> res(need):
103
104
         for (int i = 0; i < need; i++) {
          res[i] = fa[i].x + 0.5:
105
106
        }
107
         return res;
108
109
       vector<int> multiply mod(vector<int> &a. vector<int> &b. int m. int eg =
110
         int need = a.size() + b.size() - 1;
111
         int nbase = 0:
112
113
         while ((1 << nbase) < need) nbase++;
114
         ensure base(nbase):
         int sz = 1 \ll nbase;
115
         if (sz > (int) fa.size()) {
116
          fa.resize(sz):
117
118
         for (int i = 0: i < (int) a.size(): i++) {
119
          int x = (a[i] \% m + m) \% m;
120
          fa[i] = num(x & ((1 << 15) - 1), x >> 15):
121
        }
122
         fill(fa.begin() + a.size(), fa.begin() + sz, num {0, 0});
123
124
        fft(fa. sz):
         if (ea) {
125
           copy(fa.begin(), fa.begin() + sz, fb.begin());
126
        } else {
127
128
           if (sz > (int) fb.size()) {
129
             fb.resize(sz):
130
131
           for (int i = 0: i < (int) b.size(): i++) {
             int x = (b[i] \% m + m) \% m:
132
             fb[i] = num(x & ((1 << 15) - 1), x >> 15);
133
134
135
          fill(fb.begin() + b.size(), fb.begin() + sz, num {0, 0});
          fft(fb. sz):
136
        }
137
138
         dbl ratio = 0.25 / sz:
139
         num r2(0, -1);
         num r3(ratio, 0);
140
         num r4(0, -ratio);
141
142
         num r5(0, 1);
```

```
143
         for (int i = 0: i \le (sz >> 1): i++) {
144
           int j = (sz - i) & (sz - 1);
145
           num a1 = (fa[i] + coni(fa[i])):
146
           num a2 = (fa[i] - conj(fa[j])) * r2;
147
           num b1 = (fb[i] + coni(fb[i])) * r3:
148
           num b2 = (fb[i] - conj(fb[j])) * r4;
149
           if (i != j) {
150
            num c1 = (fa[j] + conj(fa[i]));
151
             num c2 = (fa[j] - conj(fa[i])) * r2;
152
            num d1 = (fb[i] + coni(fb[i])) * r3:
153
            num d2 = (fb[j] - conj(fb[i])) * r4;
154
            fa[i] = c1 * d1 + c2 * d2 * r5;
155
            fb[i] = c1 * d2 + c2 * d1:
156
          }
157
           fa[i] = a1 * b1 + a2 * b2 * r5:
158
          fb[j] = a1 * b2 + a2 * b1;
159
        }
160
        fft(fa. sz):
161
         fft(fb, sz);
162
        vector<int> res(need):
163
         for (int i = 0; i < need; i++) {
164
          long long aa = fa[i].x + 0.5:
165
          long long bb = fb[i].x + 0.5:
166
          long long cc = fa[i].v + 0.5;
167
          res[i] = (aa + ((bb \% m) << 15) + ((cc \% m) << 30)) \% m:
168
        }
169
        return res:
170
171
172
       vector<int> square mod(vector<int> &a. int m) {
173
        return multiply_mod(a, a, m, 1);
174
      }
175
      // fft::multiply uses dbl, outputs vector<long long> of rounded values
      // fft::multiply_mod might work for res.size() up to 2^21
176
177
      // typedef long double dbl;
                                                       up to 2^25 (but takes a lot
            of memory)
178 }:
```

6.7 fftfast.cpp

```
1 // FFT_MAXN = 2^k
2 // fft_init() to precalc FFT_MAXN-th roots
```

```
4 typedef long double db;
5 const int FFT MAXN = 262144;
6 const int N = 3.1e5:
7 const db pi = acosl(-1.);
   struct cp {
       db a. b:
9
       cp operator+(const cp &y) const { return (cp){a + y.a, b + y.b}; }
10
       cp operator-(const cp &y) const { return (cp){a - y.a, b - y.b}; }
11
       cp operator*(const cp &y) const { return (cp){a * y.a - b * y.b, a * y.b
12
             + b * v.a}: 
       cp operator!() const { return (cp){a, -b}; };
13
14 } nw[FFT MAXN + 1];
15 int bitrev[FFT MAXN]:
16 void dft(cp *a, int n, int flag = 1) {
17
       int d = 0:
       while ((1 \ll d) * n != FFT MAXN) d++;
18
19
       rep(i, 0, n - 1) if (i < (bitrev[i] >> d)) swap(a[i], a[bitrev[i] >> d])
20
       for (int 1 = 2; 1 <= n; 1 <<= 1) {
21
           int del = FFT MAXN / 1 * flag:
22
           for (int i = 0; i < n; i += 1) {
                cp * le = a + i, *ri = a + i + (l >> 1), *w = flag == 1 ? nw : nw
23
                     + FFT MAXN:
24
                rep(k, 0, 1 / 2 - 1) {
25
                    cp ne = *ri * *w:
26
                    *ri = *le - ne, *le = *le + ne;
                   le++, ri++, w += del;
28
               }
29
           }
30
31
       if (flag != 1) rep(i, 0, n - 1) a[i].a /= n, a[i].b /= n;
32 }
33 void fft init() {
34
       int L = 0;
35
       while ((1 << L) != FFT_MAXN) L++;
36
       bitrev[0] = 0:
       rep(i, 1, FFT_MAXN - 1) bitrev[i] = bitrev[i >> 1] >> 1 | ((i & 1) << (L
             - 1));
38
       nw[0] = nw[FFT_MAXN] = (cp){1, 0};
       rep(i, 0, FFT_MAXN)
39
       nw[i] = (cp){cos1(2 * pi / FFT MAXN * i), sin1(2 * pi / FFT MAXN * i)};
40
             // veru slow
41 }
```

```
42
43 void convo(db *a, int n, db *b, int m, db *c) {
        static cp f[FFT MAXN >> 1]. g[FFT MAXN >> 1]. t[FFT MAXN >> 1]:
45
       int N = 2:
46
       while (N \le n + m) N \le 1:
47
       rep(i, 0, N - 1) if (i & 1) {
            f[i >> 1].b = (i <= n) ? a[i] : 0.0;
            g[i >> 1].b = (i <= m) ? b[i] : 0.0:
50
       }
51
       else {
52
            f[i >> 1].a = (i <= n) ? a[i] : 0.0;
53
            g[i >> 1].a = (i <= m) ? b[i] : 0.0;
54
       }
       dft(f, N >> 1);
       dft(g, N >> 1);
56
       int del = FFT MAXN / (N >> 1);
        cp qua = (cp)\{0, 0.25\}, one = (cp)\{1, 0\}, four = (cp)\{4, 0\}, *w = nw;
       rep(i. 0. N / 2 - 1) {
            int j = i ? (N >> 1) - i : 0;
            t[i] = (four * !(f[j] * g[j]) - (!f[j] - f[i]) * (!g[j] - g[i]) * (
61
                one + *w)) * qua;
            w += del:
63
       }
       dft(t. N >> 1. -1):
       rep(i, 0, n + m) c[i] = (i & 1) ? t[i >> 1].a : t[i >> 1].b:
66 }
67
68 int mo:
69 void mul(int *a, int *b, int n) { // n \le N, 0 \le a[i], b[i] \le mo
70
        static cp f[N], g[N], t[N], r[N];
71
       int nn = 2:
72.
       while (nn \le n + n) nn \le 1;
       rep(i, 0, nn - 1) {
73
74
            f[i] = (i \le n) ? (cp){(db)(a[i] >> 15), (db)(a[i] & 32767)} : (cp)
            g[i] = (i \le n) ? (cp) \{ (db) (b[i] >> 15), (db) (b[i] & 32767) \} : (cp)
                {0. 0}:
76
       }
77
       swap(n, nn);
78
       dft(f, n, 1);
       dft(g, n, 1);
       rep(i, 0, n - 1) {
81
            int j = i ? n - i : 0;
```

```
t[i] = ((f[i] + !f[i]) * (!g[i] - g[i]) + (!f[i] - f[i]) * (g[i] + !
82
                                                                                                        g[i])) * (cp){0, 0.25};
83
                                                                           r[i] = (!f[j] - f[i]) * (!g[j] - g[i]) * (cp){-0.25, 0} + (cp){0,}
                                                                                                        0.25} * (f[i] + !f[j]) * (g[i] + !g[j]);
                                                 }
84
                                                  dft(t, n, -1);
 85
                                                  dft(r, n, -1);
                                                  rep(i, 0, n - 1)
 87
                                                  a[i] = ((11(t[i].a + 0.5) \% mo << 15) + 11(r[i].a + 0.5) + (11(r[i].b + 0.5)) + (11(r[i].b 
                                                                            0.5) % mo << 30)) % mo:
89 }
```

6.8 fftnew.cpp

```
namespace fft {
   typedef double dbl;
   struct num {
     dbl x, y;
     num() { x = y = 0; }
     num(dbl x_, dbl y_) : x(x_), y(y_) {}
9 }:
10
11 inline num operator+(num a, num b) { return num(a.x + b.x, a.y + b.y); }
12 inline num operator-(num a, num b) { return num(a.x - b.x, a.y - b.y); }
13 inline num operator*(num a, num b) { return num(a.x * b.x - a.y * b.y, a.x *
         b.v + a.v * b.x): }
14 inline num conj(num a) { return num(a.x, -a.y); }
16 int base = 1:
   vector<num> roots = \{\{0, 0\}, \{1, 0\}\};
   vector<int> rev = \{0, 1\};
19
   const dbl PI = static cast<dbl>(acosl(-1.0));
21
    void ensure base(int nbase) {
23
     if (nbase <= base) {
24
       return;
25
26
     rev.resize(1 << nbase);
27
     for (int i = 0; i < (1 << nbase); i++) {
       rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1)):
```

```
29
     roots.resize(1 << nbase);</pre>
31
      while (base < nbase) {</pre>
32
        dbl angle = 2 * PI / (1 << (base + 1));
33 //
            num z(cos(angle), sin(angle));
34
        for (int i = 1 << (base - 1); i < (1 << base); i++) {
35
          roots[i << 1] = roots[i];</pre>
36 //
              roots \lceil (i \ll 1) + 1 \rceil = roots \lceil i \rceil * z:
37
          dbl angle i = angle * (2 * i + 1 - (1 << base));
          roots[(i << 1) + 1] = num(cos(angle_i), sin(angle_i));</pre>
       }
        base++;
42 }
43
    void fft(vector<num>& a, int n = -1) {
     if (n == -1) {
        n = (int) a.size():
47
     assert((n & (n - 1)) == 0):
      int zeros = builtin ctz(n);
      ensure base(zeros):
      int shift = base - zeros:
      for (int i = 0; i < n; i++) {
       if (i < (rev[i] >> shift)) {
          swap(a[i], a[rev[i] >> shift]);
       }
56
57
      for (int k = 1; k < n; k <<= 1) {
58
        for (int i = 0: i < n: i += 2 * k) {
          for (int j = 0; j < k; j++) {
            num z = a[i + j + k] * roots[j + k];
61
            a[i + j + k] = a[i + j] - z;
            a[i + j] = a[i + j] + z;
          }
        }
66 }
   vector < num > fa, fb;
    vector<int64_t> square(const vector<int>& a) {
71
      if (a.empty()) {
```

```
72
         return {}:
 73
      int need = (int) a.size() + (int) a.size() - 1;
 74
 75
       int nbase = 1:
       while ((1 << nbase) < need) nbase++:
 76
 77
       ensure_base(nbase);
       int sz = 1 << nbase;</pre>
      if ((sz >> 1) > (int) fa.size()) {
 79
        fa.resize(sz >> 1);
 80
      }
 81
 82
      for (int i = 0; i < (sz >> 1); i++) {
        int x = (2 * i < (int) a.size() ? a[2 * i] : 0);
 83
        int v = (2 * i + 1 < (int) a.size() ? a[2 * i + 1] : 0):
 84
 85
        fa[i] = num(x, y);
      }
 86
      fft(fa, sz >> 1);
 87
      num r(1.0 / (sz >> 1), 0.0);
      for (int i = 0: i \le (sz >> 2): i++) {
 89
        int j = ((sz >> 1) - i) & ((sz >> 1) - 1);
 90
        num fe = (fa[i] + coni(fa[i])) * num(0.5, 0):
 91
 92
         num fo = (fa[i] - conj(fa[j])) * num(0, -0.5);
         num aux = fe * fe + fo * fo * roots(sz >> 1) + i * roots(sz >> 1) + i
 93
             1:
 94
         num tmp = fe * fo;
 95
        fa[i] = r * (coni(aux) + num(0, 2) * coni(tmp));
         fa[j] = r * (aux + num(0, 2) * tmp);
      }
      fft(fa, sz >> 1);
 98
 99
      vector<int64 t> res(need);
      for (int i = 0: i < need: i++) {
100
        res[i] = llround(i \% 2 == 0 ? fa[i >> 1].x : fa[i >> 1].y);
101
102
      }
      return res:
103
104 }
105
    vector<int64 t> multiply(const vector<int>& a, const vector<int>& b) {
      if (a.empty() || b.empty()) {
107
        return {};
108
109
      }
      if (a == b) {
110
      return square(a);
111
112
     int need = (int) a.size() + (int) b.size() - 1;
113
```

```
1114
       int nbase = 1:
115
       while ((1 << nbase) < need) nbase++;</pre>
       ensure base(nbase):
117
       int sz = 1 << nbase;</pre>
118
      if (sz > (int) fa.size()) {
119
        fa.resize(sz);
120
121
      for (int i = 0: i < sz: i++) {
122
        int x = (i < (int) a.size() ? a[i] : 0);</pre>
123
      int y = (i < (int) b.size() ? b[i] : 0);</pre>
124
        fa[i] = num(x, y);
125
      }
126
      fft(fa. sz):
127
       num r(0, -0.25 / (sz >> 1));
128
      for (int i = 0: i <= (sz >> 1): i++) {
129
      int j = (sz - i) & (sz - 1);
130
        num z = (fa[i] * fa[i] - coni(fa[i] * fa[i])) * r;
        fa[i] = (fa[i] * fa[i] - coni(fa[i] * fa[i])) * r:
132
        fa[i] = z;
133
     }
134
      for (int i = 0; i < (sz >> 1); i++) {
135
        num A0 = (fa[i] + fa[i + (sz >> 1)]) * num(0.5, 0):
136
        num A1 = (fa[i] - fa[i + (sz >> 1)]) * num(0.5, 0) * roots[(sz >> 1) + i
            ];
137
        fa[i] = A0 + A1 * num(0, 1):
138
139
      fft(fa, sz >> 1);
140
      vector<int64 t> res(need);
141
      for (int i = 0; i < need; i++) {
142
        res[i] = llround(i % 2 == 0 ? fa[i >> 1].x : fa[i >> 1].v):
143
144
      return res:
145 }
147 vector<int> multiply_mod(const vector<int>& a, const vector<int>& b, int m)
148
      if (a.empty() || b.empty()) {
149
        return {};
150
151
      int eq = (a.size() == b.size() && a == b);
      int need = (int) a.size() + (int) b.size() - 1;
153
      int nbase = 0:
       while ((1 << nbase) < need) nbase++;
```

```
ensure_base(nbase);
155
       int sz = 1 << nbase;</pre>
156
       if (sz > (int) fa.size()) {
157
         fa.resize(sz);
158
      }
159
       for (int i = 0; i < (int) a.size(); i++) {
160
161
         int x = (a[i] \% m + m) \% m;
         fa[i] = num(x & ((1 << 15) - 1), x >> 15);
162
      }
163
      fill(fa.begin() + a.size(), fa.begin() + sz, num {0, 0});
164
       fft(fa, sz);
165
       if (sz > (int) fb.size()) {
166
         fb.resize(sz):
167
168
      }
      if (eq) {
169
         copy(fa.begin(), fa.begin() + sz, fb.begin());
170
171
172
         for (int i = 0: i < (int) b.size(): i++) {
173
           int x = (b[i] \% m + m) \% m;
           fb[i] = num(x & ((1 << 15) - 1), x >> 15);
174
175
         fill(fb.begin() + b.size(), fb.begin() + sz, num {0, 0});
176
         fft(fb, sz):
177
      }
178
       dbl ratio = 0.25 / sz:
179
       num r2(0, -1);
180
       num r3(ratio, 0);
181
       num r4(0, -ratio);
182
183
       num r5(0, 1);
       for (int i = 0: i \le (sz >> 1): i++) {
184
         int j = (sz - i) & (sz - 1);
185
186
         num a1 = (fa[i] + conj(fa[j]));
         num a2 = (fa[i] - conj(fa[j])) * r2;
187
188
         num b1 = (fb[i] + conj(fb[j])) * r3;
         num b2 = (fb[i] - conj(fb[j])) * r4;
189
         if (i != j) {
190
           num c1 = (fa[j] + conj(fa[i]));
191
           num c2 = (fa[j] - conj(fa[i])) * r2;
192
193
           num d1 = (fb[j] + conj(fb[i])) * r3;
           num d2 = (fb[j] - conj(fb[i])) * r4;
194
           fa[i] = c1 * d1 + c2 * d2 * r5;
195
           fb[i] = c1 * d2 + c2 * d1:
196
         }
197
```

```
198
        fa[i] = a1 * b1 + a2 * b2 * r5;
199
        fb[j] = a1 * b2 + a2 * b1;
200
      }
201
       fft(fa, sz);
202
      fft(fb, sz):
203
       vector<int> res(need);
204
      for (int i = 0; i < need; i++) {
205
        int64 t aa = llround(fa[i].x):
206
        int64 t bb = llround(fb[i].x);
207
        int64_t cc = llround(fa[i].y);
208
        res[i] = static cast<int>((aa + ((bb \% m) << 15) + ((cc \% m) << 30)) \% m
             );
      }
209
210
       return res;
211 }
212
213 } // namespace fft
```

6.9 FST.cpp

```
1 void fst(VI &a.bool inv) {
        for (int n=SZ(a),step=1;step<n;step*=2) {</pre>
3
            for (int i=0; i< n; i+=2*step) rep(j,i,i+step-1) {
                int &u=a[j],&v=a[j+step];
                tie(u,v)=
                inv?PII(v-u,u):PII(v,u+v); // AND
                inv?PII(v,u-v):PII(u+v,u); // OR
                PII(u+v.u-v): // XOR
           }
       }
10
        if (inv) for (auto &x : a) x/=SZ(a); // XOR only
11
12 }
13 VI conv(VI a, VI b) {
14
       fst(a,0),fst(b,0);
       rep(i,0,SZ(a)-1) a[i]=a[i]*b[i];
        fst(a,1); return a;
17 }
```

6.10 FWT.cpp

```
1    11 f[maxn], g[maxn], h[maxn];
2    int main() {
3       for (int i = 0; i < n; i++) {</pre>
```

```
4
            for (int j = 0; j < bit(n); j++) {
                if ((j & bit(i)) == 0) {
                    f[j] += f[j + bit(i)];
                    g[j] += g[j + bit(i)];
               }
            }
9
10
11
        for (int i = 0; i < bit(n); i++) {
12
            f[i] %= mod;
13
            g[i] %= mod;
14
            h[i] = f[i] * g[i] % mod;
15
16
       for (int i = 0; i < n; i++) {
17
            for (int j = 0; j < bit(n); j++) {
18
                if ((j & bit(i)) == 0)
19
                    h[j] = h[j + bit(i)];
20
            }
       }
21
22
        for (int i = 0; i < bit(n); i++) {
            h[i] %= mod;
23
            if (h[i] < 0) h[i] += mod;
24
25
       }
26
27
        11 \text{ ans} = 0;
28
        rep(i, 0, bit(n) - 1) ans ^= h[i];
29
        cout << ans << '\n';
30 }
```

6.11 gauss(合数).cpp

```
1 void gauss(int n) {
       int ans = 1;
       //rep(i,1,n) rep(j,1,n) p[i][j]%=mod;
       for (int i = 1; i <= n; i++) {
            for (int j = i + 1; j \le n; j++) {
                int x = i, y = j;
                while (p[x][i]) {
                   int t = p[y][i] / p[x][i];
                    for (int k = i; k \le n; k++)
10
                        p[y][k] = (p[y][k] - p[x][k] * t) % mod;
11
                    swap(x, y);
12
13
                if (x == i) {
```

6.12 gauss.cpp

```
1 ll f[N][N];
2 11 v[N], a[N]:
3 void gauss() {
       for (int i = 1; i <= n; i++) {
            for (int j = i; j \le n; j++) {
                if (f[j][i] > f[i][i]) {
                    swap(v[i], v[j]);
                    for (int k = 1; k \le n; k++)
9
                        swap(f[j][k], f[i][k]);
10
               }
11
           }
12
            for (int j = i + 1; j \le n; j++) {
13
                if (f[j][i]) {
14
                    int delta = f[j][i] * fpow(f[i][i], mod - 2) % mod;
15
                    for (int k = i; k <= n; k++) {
16
                        f[j][k] -= f[i][k] * delta % mod;
17
                        if (f[j][k] < 0)
18
                            f[j][k] += mod;
19
                   }
20
                    v[j] -= v[i] * delta % mod;
21
                    if (v[j] < mod)
22
                        v[i] += mod;
23
               }
           }
24
25
       for (int j = n; j > 0; j--) {
27
            for (int k = j + 1; k \le n; k++) {
28
                v[j] -= f[j][k] * a[k] % mod;
29
                if (v[j] < 0)
30
                    v[j] += mod;
31
32
            a[j] = v[j] * fpow(f[j][j], mod - 2) % mod;
33
       }
34 }
```

6.13 linearbasis.cpp

```
struct linear_base {
       ll w[64];
       11 zero = 0;
       11 \text{ tot} = -1;
       void clear() {
           rep(i, 0, 63) w[i] = 0;
           zero = 0;
           tot = -1;
10
       void insert(ll x) {
           for (int i = 62; i >= 0; i--) {
11
12
               if (x & bit(i))
                   if (!w[i]) {w[i] = x; return;}
13
                    else x ^= w[i];
14
15
16
           zero++;
       }
17
18
       void build() {
19
           rep(i, 0, 63) rep(j, 0, i - 1) {
                if (w[i]&bit(j)) w[i] ^= w[j];
20
21
           }
           for (int i = 0; i <= 62; i++) {
22
                if (w[i] != 0) w[++tot] = w[i];
           }
24
       }
25
26
       11 qmax() {
           ll res = 0:
27
28
           for (int i = 62; i >= 0; i--) {
                res = max(res, res ^ w[i]);
30
           }
31
           return res;
32
       }
33
       bool check(ll x) {
34
           for (int i = 62; i >= 0; i--) {
35
               if (x & bit(i))
                   if (!w[i]) return false;
37
                    else x ^= w[i];
38
           }
39
           return true:
40
       }
41
       11 query(11 k) {
42
           ll res = 0:
```

```
43
           // if (zero) k=1;
           // if (k \ge bit(tot)) return -1;
45
           for (int i = tot; i >= 0; i--) {
                if (k & bit(i)) {
47
                   res = max(res, res ^ w[i]);
               } else {
                   res = min(res, res ^ w[i]);
50
51
           }
           return res;
       }
54 };
```

6.14 lucas.cpp

```
1 11 fac[maxn], fnv[maxn];
3 ll binom(ll a, ll b) {
       if (b > a \mid | b < 0) return 0;
       return fac[a] * fnv[a - b] % p * fnv[b] % p;
6 }
8 ll lucas(ll a, ll b, ll p) {
       11 \text{ ans} = 1;
       while (a > 0 | | b > 0) {
           ans = (ans * binom(a % p, b % p)) % p;
           a /= p, b /= p;
13
       }
14
       return ans;
15 }
17 int main() {
    cin >> p >> T;
   fac[0] = 1;
    rep(i, 1, p - 1) fac[i] = fac[i - 1] * i % p;
       fnv[p - 1] = powmod(fac[p - 1], p - 2, p);
       per(i, p - 2, 0) fnv[i] = fnv[i + 1] * (i + 1) % p;
       assert(fnv[0] == 1);
24 }
```

6.15 mathdiv.cpp

```
1 ll floor_div(ll x, ll y) {
```

```
assert(y != 0);
        if (y < 0) {
           y = -y;
            x = -x;
        if (x \ge 0) return x / y;
        return (x + 1) / y - 1;
10 ll ceil div(ll x, ll y) {
11
        assert(y != 0);
12
        if (y < 0) {
13
           y = -y;
14
            x = -x:
15
       }
16
       if (x \le 0) return x / y;
        return (x - 1) / y + 1;
17
18 }
```

6.16 matrix.cpp

```
1 template <typename T>
2 vector<vector<T>> operator*(const vector<vector<T>>& a, const vector<vector<
        T>>& b) {
     if (a.empty() || b.empty()) {
        return {{}};
     }
     vector<vector<T>> c(a.size(), vector<T>(b[0].size()));
      for (int i = 0; i < static_cast<int>(c.size()); i++) {
       for (int j = 0; j < static_cast < int > (c[0].size()); <math>j++) {
         c[i][i] = 0:
10
         for (int k = 0; k < static_cast<int>(b.size()); k++) {
            c[i][i] += a[i][k] * b[k][i];
11
12
         }
       }
14
     return c;
15
16 }
17
   template <typename T>
    vector<vector<T>>& operator *= (vector<vector<T>>& a. const vector<vector<T>>&
         b) {
     return a = a * b;
21 }
```

```
22
   template <typename T, typename U>
   vector<vector<T>> power(const vector<vector<T>>& a, const U& b) {
      assert(b >= 0);
26
     vector<U> binary:
27
     U bb = b:
     while (bb > 0) {
      binary.push_back(bb & 1);
30
       bb >>= 1;
31
     }
32
     vector<vector<T>> res(a.size(), vector<T>(a.size()));
     for (int i = 0; i < static cast<int>(a.size()); i++) {
       res[i][i] = 1:
34
35
     7
     for (int j = (int)binary.size() - 1; j >= 0; j--) {
       res *= res;
       if (binary[j] == 1) {
         res *= a:
40
       }
41
42
     return res;
43 }
```

6.17 matrixfast.cpp

```
1 Description: Basic operations on square matrices.
2 Usage: Matrix<int, 3> A;
3 \quad A.d = \{\{\{1, 2, 3\}\}, \{\{4, 5, 6\}\}, \{\{7, 8, 9\}\}\}\};
   vector < int > vec = \{1, 2, 3\};
   vec = (A^N) * vec:
   template < class T, int N> struct Matrix {
        typedef Matrix M;
        array<array<T, N>, N> d{};
10
        M operator*(const M& m) const {
11
            Ma;
12
            rep(i, 0, N) rep(j, 0, N)
            rep(k, 0, N) a.d[i][j] += d[i][k] * m.d[k][j];
13
14
            return a;
15
       }
16
        vector<T> operator*(const vector<T>& vec) const {
17
            vector<T> ret(N);
18
            rep(i, 0, N) rep(j, 0, N) ret[i] += d[i][j] * vec[j];
```

```
19
            return ret;
        }
20
21
        M operator^(ll p) const {
22
            assert(p >= 0);
            M a. b(*this):
23
            rep(i, 0, N) a.d[i][i] = 1;
24
            while (p) {
                if (p \& 1) a = a * b;
27
                b = b * b;
                p >>= 1;
29
30
            return a;
31
32 };
```

6.18 MillerRabbin pollard modmul.cpp

```
1 /*ModMulLL.h
2 Description: Calculate a b mod c (or a
3 b mod c) for 0 a, b c 7.2 • 10^18
4 Time: O (1) for modmul, O (log b) for modpow*/
5 /*ull modmul(ull a, ull b, ull M) {
        ll \ ret = a * b - M * ull(1.L / M * a * b):
        return ret + M * (ret < 0) - M * (ret >= (ll)M);
8 7
   ull modpow(ull b, ull e, ull mod) {
10
       ull \ ans = 1;
        for (: e: b = modmul(b, b, mod), e \neq 2)
11
12
            if (e \& 1) ans = modmul(ans, b, mod);
13
        return ans:
14 }*/
15 ll modmul(ll a, ll b, ll m) {
       a %= m, b %= m;
16
17
       11 d = ((1db)a * b / m);
       d = a * b - d * m:
18
19
       if (d \ge m) d = m;
       if (d < 0) d += m;
21
       return d:
22 }
23 ll modpow(ll a, ll b, ll p) {
24
       ll ans = 1:
25
       while (b) {
26
           if (b & 1) ans = modmul(ans, a, p):
```

```
27
           a = modmul(a, a, p); b >>= 1;
       } return ans;
29 }
30 /*MillerRabin.h
31 Description: Deterministic Miller-Rabin primality test. Guaranteed to
32 work for numbers up to 7 · 1018; for larger numbers, use Python and extend A
         randomly.
33 Time: 7 times the complexity of a^b mod c.*/
34 bool isPrime(ll n) {
       if (n < 2 | | n % 6 % 4 != 1) return (n | 1) == 3:
       11 A[] = \{2, 325, 9375, 28178, 450775, 9780504, 1795265022\},
                  s = __builtin_ctzll(n - 1), d = n >> s;
       for (11 a : A) { // ^ count trailing zeroes
           ll p = modpow(a \% n, d, n), i = s;
40
           while (p != 1 && p != n - 1 && a % n && i--)
                p = modmul(p, p, n);
41
           if (p != n - 1 && i != s) return 0;
42
43
       }
       return 1;
45 }
46 /*Factor.h
47 Description: Pollard-rho randomized factorization algorithm. Returns
   prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
49 Time: O(n^{1/4}), less for numbers with small factors.*/
50 11 pollard(11 n) {
51
       auto f = [n](11 x) \{ return modmul(x, x, n) + 1; \};
       11 x = 0, y = 0, t = 30, prd = 2, i = 1, q;
       while (t++ \% 40 || _-gcd(prd, n) == 1) {
54
           if (x == y) x = ++i, y = f(x);
           if ((q = modmul(prd, max(x, y) - min(x, y), n))) prd = q;
           x = f(x), y = f(f(y));
57
       }
       return __gcd(prd, n);
59 }
60 vector<11> factor(11 n) {
       if (n == 1) return {};
       if (isPrime(n)) return {n};
       ll x = pollard(n);
       auto 1 = factor(x), r = factor(n / x);
       1.insert(1.end(), all(r));
       return 1:
67 }
```

6.19 ntt(polynomial).cpp

```
#include<bits/stdc++.h>
    using namespace std;
   const int mod = 998244353;
6 inline void add(int &x, int y) {
     x += y;
     if (x \ge mod) {
       x -= mod;
11 }
12
13 inline void sub(int &x, int y) {
    x -= v;
14
    if (x < 0) {
      x += mod;
17
18 }
   inline int mul(int x, int y) {
      return (long long) x * y % mod;
22 }
24 inline int power(int x, int y) {
     int res = 1:
25
    for (; y; y >>= 1, x = mul(x, x)) {
     if (y & 1) {
27
         res = mul(res, x);
       }
30
31
     return res;
32 }
33
34 inline int inv(int a) {
     a %= mod;
     if (a < 0) {
       a += mod:
37
     int b = mod. u = 0. v = 1:
39
     while (a) {
41
      int t = b / a;
       b -= t * a:
```

```
swap(a, b);
       u -= t * v;
       swap(u, v);
46
47
    if (u < 0) {
     u += mod;
     return u:
51 }
52
   namespace ntt {
   int base = 1, root = -1, max base = -1;
55 vector<int> rev = \{0, 1\}, roots = \{0, 1\};
56
57 void init() {
    int temp = mod - 1;
    \max base = 0;
    while (temp % 2 == 0) {
61
    temp >>= 1;
      ++max_base;
    }
    root = 2:
     while (true) {
       if (power(root, 1 << max_base) == 1 && power(root, 1 << (max_base - 1))</pre>
           != 1) {
         break;
       }
       ++root;
70
71 }
72
73 void ensure_base(int nbase) {
    if (\max base == -1) {
    init();
76
     if (nbase <= base) {
78
       return;
79
    assert(nbase <= max_base);
    rev.resize(1 << nbase);
     for (int i = 0; i < 1 << nbase; ++i) {
      rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (nbase - 1));
84
    }
```

```
roots.resize(1 << nbase);</pre>
 85
       while (base < nbase) {
 86
         int z = power(root, 1 << (max base - 1 - base)):
 87
 88
         for (int i = 1 << (base - 1); i < 1 << base; ++i) {
           roots[i << 1] = roots[i]:</pre>
 89
           roots[i << 1 | 1] = mul(roots[i], z);
 90
        }
 92
         ++base:
 93
 94 }
 95
     void dft(vector<int> &a) {
       int n = a.size(), zeros = __builtin_ctz(n);
 97
 98
       ensure base(zeros);
       int shift = base - zeros:
 99
       for (int i = 0; i < n; ++i) {
100
         if (i < rev[i] >> shift) {
101
102
           swap(a[i], a[rev[i] >> shift]);
103
        }
      }
104
105
       for (int i = 1; i < n; i <<= 1) {
         for (int i = 0: i < n: i += i << 1) {
106
           for (int k = 0: k < i: ++k) {
107
             int x = a[j + k], y = mul(a[j + k + i], roots[i + k]);
108
109
             a[i + k] = (x + v) \% mod:
             a[j + k + i] = (x + mod - y) \% mod;
110
          }
111
112
         }
113
      }
114 }
115
116
     vector<int> multiply(vector<int> a, vector<int> b) {
       int need = a.size() + b.size() - 1. nbase = 0:
117
118
       while (1 << nbase < need) {
         ++nbase;
119
120
      }
       ensure_base(nbase);
121
       int sz = 1 << nbase;</pre>
122
123
       a.resize(sz);
124
       b.resize(sz);
       bool equal = a == b;
125
       dft(a);
126
       if (equal) {
127
```

```
128
         b = a:
129
      } else {
130
         dft(b):
131
132
      int inv sz = inv(sz):
133
       for (int i = 0; i < sz; ++i) {
134
         a[i] = mul(mul(a[i], b[i]), inv sz);
135
136
      reverse(a.begin() + 1, a.end());
137
      dft(a):
138
       a.resize(need);
139
       return a;
140 }
141
|142 vector<int> inverse_new(const vector<int> &a) {
143
       assert(!a.empty());
144
       int n = (int) a.size();
       vector<int> b = {inv(a[0])}:
146
       while ((int) b.size() < n) {</pre>
147
         vector<int> x(a.begin(), a.begin() + min(a.size(), b.size() << 1));</pre>
148
         x.resize(b.size() << 1);</pre>
149
         b.resize(b.size() << 1):</pre>
150
         vector<int> c = b:
151
         // NTT<T>::fft(c);
152
         // NTT < T > :: fft(x):
153
         dft(c);
154
         dft(x):
         // Modular<T> inv = 1 / static cast<Modular<T>>((int) x.size());
156
         int inv sz = inv((int)x.size());
157
         for (int i = 0; i < (int) x.size(); i++) {
           // x[i] *= c[i] * inv;
159
           x[i] = mul(x[i], mul(c[i], inv_sz));
160
         }
161
         reverse(x.begin() + 1, x.end());
162
         // NTT < T > :: fft(x);
163
         dft(x):
164
         rotate(x.begin(), x.begin() + (x.size() >> 1), x.end());
         fill(x.begin() + (x.size() >> 1), x.end(), 0);
166
         // NTT < T > :: fft(x);
167
         dft(x):
         for (int i = 0; i < (int) x.size(); i++) {
           // x \lceil i \rceil *= c \lceil i \rceil * inv:
169
170
           x[i] = mul(x[i], mul(c[i], inv sz));
```

```
171
         }
172
         reverse(x.begin() + 1, x.end());
         // NTT < T > :: fft(x):
173
174
         dft(x);
         for (int i = 0: i < ((int) x.size() >> 1): i++) {
175
          // b [i + ((int) x.size() >> 1)] = -x[i]:
176
          int t = 0; sub(t, x[i]);
177
          b[i + ((int) x.size() >> 1)] = t:
178
179
        }
      }
180
       b.resize(n);
181
       return b;
182
183 }
184
     vector<int> inverse(vector<int> a) {
185
       int n = a.size(), m = (n + 1) >> 1;
186
       if (n == 1) {
187
         return vector<int>(1, inv(a[0]));
188
189
      } else {
         vector<int> b = inverse(vector<int>(a.begin(), a.begin() + m));
190
191
         int need = n << 1, nbase = 0;
         while (1 << nbase < need) {
192
          ++nbase:
193
194
         }
195
         ensure base(nbase):
         int sz = 1 \ll nbase;
196
         a.resize(sz):
197
         b.resize(sz);
198
199
         dft(a);
         dft(b):
200
         int inv sz = inv(sz);
201
202
         for (int i = 0: i < sz: ++i) {
           a[i] = mul(mul(mod + 2 - mul(a[i], b[i]), b[i]), inv sz);
203
204
         }
         reverse(a.begin() + 1, a.end());
205
         dft(a);
206
         a.resize(n):
207
         return a;
208
209
210 }
211 }
213 using ntt::multiply;
```

```
214 using ntt::inverse;
215
     vector<int>& operator += (vector<int> &a. const vector<int> &b) {
       if (a.size() < b.size()) {
217
        a.resize(b.size()):
218
219
220
      for (int i = 0; i < b.size(); ++i) {
221
        add(a[i], b[i]):
222
223
      return a:
224 }
225
226 vector<int> operator + (const vector<int> &a. const vector<int> &b) {
227
       vector<int> c = a;
228
      return c += b:
229 }
230
231 vector<int>& operator -= (vector<int> &a. const vector<int> &b) {
      if (a.size() < b.size()) {
233
        a.resize(b.size()):
234
235
      for (int i = 0: i < b.size(): ++i) {
236
        sub(a[i], b[i]);
237
      }
238
      return a:
239 }
240
    vector<int> operator - (const vector<int> &a, const vector<int> &b) {
242
      vector<int> c = a;
243
      return c -= b:
244 }
245
     vector<int>& operator *= (vector<int> &a, const vector<int> &b) {
247
      if (min(a.size(), b.size()) < 128) {
248
        vector<int> c = a:
249
        a.assign(a.size() + b.size() - 1, 0);
250
        for (int i = 0; i < c.size(); ++i) {
251
          for (int j = 0; j < b.size(); ++j) {
252
             add(a[i + j], mul(c[i], b[j]));
253
          }
254
        }
255
      } else {
256
        a = multiply(a, b);
```

```
}
257
258
      return a;
259 }
260
     vector<int> operator * (const vector<int> &a. const vector<int> &b) {
261
       vector<int> c = a:
262
263
      return c *= b;
264 }
265
     vector<int>& operator /= (vector<int> &a. const vector<int> &b) {
266
       int n = a.size(), m = b.size();
267
       if (n < m) {
268
        a.clear():
269
270
      } else {
        vector<int> c = b:
271
272
        reverse(a.begin(), a.end());
        reverse(c.begin(), c.end());
273
        c.resize(n - m + 1):
274
        a *= inverse(c);
275
        a.erase(a.begin() + n - m + 1, a.end()):
276
277
        reverse(a.begin(), a.end());
278
279
      return a:
280 }
281
     vector<int> operator / (const vector<int> &a, const vector<int> &b) {
282
       vector<int> c = a:
283
      return c /= b;
284
285 }
286
     vector<int>& operator %= (vector<int> &a, const vector<int> &b) {
288
       int n = a.size(). m = b.size():
       if (n >= m) {
289
290
        vector<int> c = (a / b) * b;
        a.resize(m - 1);
291
292
        for (int i = 0; i < m - 1; ++i) {
          sub(a[i], c[i]);
        }
294
295
      }
       return a;
296
297 }
298
299 vector<int> operator % (const vector<int> &a, const vector<int> &b) {
```

```
300
       vector<int> c = a:
301
       return c %= b;
302 }
303
304
     vector<int> derivative(const vector<int> &a) {
305
      int n = a.size():
306
      vector<int> b(n - 1);
      for (int i = 1: i < n: ++i) {
308
        b[i - 1] = mul(a[i], i);
309
      }
310
      return b;
311 }
312
313 vector<int> primitive(const vector<int> &a) {
314
      int n = a.size():
315
      vector<int> b(n + 1), invs(n + 1);
316
      for (int i = 1; i <= n; ++i) {
317
        invs[i] = i == 1 ? 1 : mul(mod - mod / i, invs[mod % i]):
        b[i] = mul(a[i - 1], invs[i]);
319
     }
320
      return b;
321 }
322
    vector<int> logarithm(const vector<int> &a) {
324
      vector<int> b = primitive(derivative(a) * inverse(a)):
325
      b.resize(a.size());
326
      return b:
327 }
328
329
     vector<int> exponent(const vector<int> &a) {
330
       vector<int> b(1, 1);
331
       while (b.size() < a.size()) {</pre>
332
        vector<int> c(a.begin(), a.begin() + min(a.size(), b.size() << 1));</pre>
333
        add(c[0], 1);
334
        vector<int> old_b = b;
335
        b.resize(b.size() << 1);</pre>
         c -= logarithm(b);
337
        c *= old b;
         for (int i = b.size() >> 1; i < b.size(); ++i) {</pre>
339
          b[i] = c[i];
340
        }
341
342
       b.resize(a.size());
```

```
return b:
343
344 }
345
     vector<int> power(vector<int> a, int m) {
346
       int n = a.size(), p = -1;
347
       vector<int> b(n);
348
       for (int i = 0; i < n; ++i) {
349
         if (a[i]) {
350
           p = i;
351
352
           break:
         }
353
       }
354
       if (p == -1) {
355
356
         b[0] = !m;
357
         return b:
358
       if ((long long) m * p >= n) {
359
         return b:
360
      }
361
       int mu = power(a[p], m), di = inv(a[p]);
362
363
       vector<int> c(n - m * p);
       for (int i = 0: i < n - m * p: ++i) {
364
         c[i] = mul(a[i + p], di);
365
366
      }
       c = logarithm(c):
367
       for (int i = 0; i < n - m * p; ++i) {
368
         c[i] = mul(c[i], m);
369
370
      }
371
       c = exponent(c);
       for (int i = 0; i < n - m * p; ++i) {
372
         b[i + m * p] = mul(c[i], mu);
373
374
      }
       return b;
375
376 }
377
     vector<int> sqrt(const vector<int> &a) {
378
       vector<int> b(1, 1):
379
       while (b.size() < a.size()) {</pre>
380
381
         vector<int> c(a.begin(), a.begin() + min(a.size(), b.size() << 1));</pre>
         vector<int> old_b = b;
382
         b.resize(b.size() << 1);</pre>
383
         c *= inverse(b):
384
         for (int i = b.size() >> 1; i < b.size(); ++i) {</pre>
385
```

```
b[i] = mul(c[i], (mod + 1) >> 1);
387
        }
388
      }
389
      b.resize(a.size());
390
      return b:
391 }
392
     vector<int> multiply_all(int 1, int r, vector<vector<int>> &all) {
394
      if (1 > r) {
395
        return vector<int>();
396
      } else if (1 == r) {
397
        return all[1];
398
     } else {
399
      int y = (1 + r) >> 1;
400
        return multiply_all(1, y, all) * multiply_all(y + 1, r, all);
401
     }
402 }
403
     vector<int> evaluate(const vector<int> &f, const vector<int> &x) {
      int n = x.size();
405
406
      if (!n) {
407
        return vector<int>():
408
409
      vector<vector<int>> up(n * 2);
410
      for (int i = 0; i < n; ++i) {
411
        up[i + n] = vector < int > \{(mod - x[i]) \% mod, 1\};
412
413
      for (int i = n - 1; i; --i) {
414
        up[i] = up[i << 1] * up[i << 1 | 1];
415
416
      vector<vector<int>> down(n * 2);
417
      down[1] = f % up[1];
418
      for (int i = 2; i < n * 2; ++i) {
419
        down[i] = down[i >> 1] % up[i];
420
421
      vector<int> v(n);
422
      for (int i = 0; i < n; ++i) {
423
        v[i] = down[i + n][0];
424
425
      return y;
426 }
427
   vector<int> interpolate(const vector<int> &x, const vector<int> &y) {
```

```
int n = x.size();
429
       vector<vector<int>> up(n * 2);
430
       for (int i = 0: i < n: ++i) {
431
432
         up[i + n] = vector < int > \{(mod - x[i]) \% mod, 1\};
      }
433
       for (int i = n - 1; i; --i) {
434
         up[i] = up[i << 1] * up[i << 1 | 1];
435
436
       vector<int> a = evaluate(derivative(up[1]), x);
437
       for (int i = 0: i < n: ++i) {
438
         a[i] = mul(y[i], inv(a[i]));
      }
440
       vector<vector<int>> down(n * 2):
441
442
       for (int i = 0; i < n; ++i) {
         down[i + n] = vector < int > (1, a[i]);
443
444
      for (int i = n - 1; i; --i) {
445
         down[i] = down[i << 1] * up[i << 1 | 1] + down[i << 1 | 1] * up[i << 1];
446
447
       return down[1]:
448
449 }
450
    int main() {
452
453 }
```

6.20 PrimitiveRoot.cpp

```
#include <bits/stdc++.h>
   using namespace std;
   typedef long long 11;
    int pf[1010];
   11 powmod(ll a. ll b. ll mod) {
     ll res = 1;
     a %= mod;
     for (: b: b >>= 1) {
11
       if (b & 1) res = res * a % mod;
       a = a * a \% mod:
13
     }
14
      return res;
15 }
```

```
16
17 int main() {
     int T:
19
     scanf("%d", &T);
     for (int tc = 0; tc < T; tc++) {
21
       int p;
       scanf("%d", &p);
       int m = p - 1, t = 0;
24
       for (int i = 2; i * i <= m; i++) {
25
          if (m % i == 0) {
           pf[t++] = i;
            while (m \% i == 0) m /= i;
         }
29
       }
       if (m != 1) pf[t++] = m;
31
        for (int g = 1; g < p; g++) {
32
          bool valid = true;
          for (int i = 0: i < t: i++)
34
            if (powmod(g, (p - 1) / pf[i], p) == 1) {
35
              valid = false:
              break;
           }
          if (valid) {
            printf("%d\n", g);
40
            break:
         }
41
       }
```

6.21 simplex.cpp

```
1 /**
2 * Author: Stanford
3 * Source: Stanford Notebook
4 * License: MIT
5 * Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$.
6 * Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise.
7 * The input vector is set to an optimal $x$ (or in the unbounded case, an arbitrary solution fulfilling the constraints).
8 * Numerical stability is not quaranteed. For better performance, define
```

```
variables such that $x = 0$ is viable.
    * Usage:
    * vvd A = \{\{1, -1\}, \{-1, 1\}, \{-1, -2\}\}\}:
   * vd b = \{1, 1, -4\}, c = \{-1, -1\}, x;
   * T val = LPSolver(A, b, c).solve(x);
   * Time: O(NM * \#pivots), where a pivot may be e.q. an edge relaxation. O
         (2^n) in the general case.
     * Status: seems to work?
15
16
    typedef long double T; // long double, Rational, double + mod <P>...
    typedef vector <T> vd;
    typedef vector<vd> vvd;
20
21 const T eps = 1e-8, inf = 1/.0;
   #define MP make_pair
    #define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j
   struct LPSolver {
26
       int m. n:
27
        vector<int> N, B;
28
        vvd D:
29
30
        LPSolver(const vvd& A, const vd& b, const vd& c) :
31
            m(b.size()), n(c.size()), N(n+1), B(m), D(m+2, vd(n+2)) {
                for(int i = 0; i < m; i++){
32
                    for(int j = 0; j < n; j++){
34
                        D[i][i] = A[i][i];
35
                    }
                }
37
                for(int i = 0; i < m; i++){
                    B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];
39
40
                for(int j = 0; j < n; j++){
41
                    N[j] = j; D[m][j] = -c[j];
                }
                N[n] = -1; D[m+1][n] = 1;
44
            }
45
46
        void pivot(int r, int s) {
            T *a = D[r].data(), inv = 1 / a[s];
47
48
            for(int i = 0; i < m+2; i++){
49
                if (i != r && abs(D[i][s]) > eps) {
```

```
50
                    T *b = D[i].data(), inv2 = b[s] * inv;
51
                    for(int j = 0; j < n+2; j++){
52
                        b[i] -= a[i] * inv2:
53
                    }
54
                    b[s] = a[s] * inv2:
55
                }
57
            for(int j = 0; j < n+2; j++){
58
                if (j != s) D[r][j] *= inv;
59
           }
            for(int i = 0; i < m+2; i++){
61
                if (i != r) D[i][s] *= -inv;
62
           }
           D[r][s] = inv;
64
            swap(B[r], N[s]);
       }
65
66
67
        bool simplex(int phase) {
            int x = m + phase - 1;
            for (::) {
69
70
                int s = -1;
71
                for(int j = 0; j < n+1; j++){
                    if (N[j] != -phase) ltj(D[x]);
73
74
                if (D[x][s] >= -eps) return true;
                int r = -1;
                for(int i = 0; i < m; i++){
77
                    if (D[i][s] <= eps) continue;</pre>
78
                    if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
79
                            < MP(D[r][n+1] / D[r][s], B[r])) r = i:
                if (r == -1) return false:
                pivot(r, s);
84
       }
85
        T solve(vd &x) {
            int r = 0;
            for(int i = 1; i < m; i++){
                if (D[i][n+1] < D[r][n+1]) r = i;
           }
            if (D[r][n+1] < -eps) {
91
                pivot(r, n);
```

```
if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
 93
                 for(int i = 0; i < m; i++) if (B[i] == -1) {
 94
 95
                     int s = 0:
 96
                     for(int j = 1; j < n+1; j++){
 97
                         ltj(D[i]);
                     }
 98
                     pivot(i, s);
                 }
100
101
             }
102
             bool ok = simplex(1); x = vd(n);
             for(int i = 0; i < m; i++){
103
                 if (B[i] < n) \times [B[i]] = D[i][n+1];
104
105
             }
106
             return ok ? D[m][n+1] : inf;
         }
107
108 };
```

6.22 区间互质.cpp

```
1 int p[N / 5], num;
2 void prime(int n) {
        num = 0;
        for (int i = 2; i * i <= n; i++) {
            if ((n % i) == 0) {
                p[++num] = i;
                while ((n \% i) == 0) n /= i;
            }
        if (n > 1) p[++num] = n;
11 }
12 11 solve(11 r, int k) {
13
        prime(k);
14
        11 \text{ res} = 0:
15
        for (int i = 1; i < (1 << num); i++) {
16
            int k = 0:
17
            ll div = 1;
18
            for (int j = 1; j <= num; j++) {
19
                if (i & (1 << (j - 1))) {
20
                    k++;
21
                    div *= p[j];
                }
23
            }
            if (k % 2)
24
```

6.23 幂转下降幂 (求幂和).cpp

```
1 ll comb[N][N];
2 ll s[maxn], inv[maxn], p;
3 // 1^k+2^k+...+n^k
4 void solve() {
        cin >> k >> n >> p;
       rep(i, 0, k + 1) {
            comb[i][0] = comb[i][i] = 1;
            rep(j, 1, i - 1) {
                comb[i][j] = (comb[i - 1][j - 1] + comb[i - 1][j]) % p;
10
            }
       }
11
12
        inv[1] = 1;
13
        rep(i, 2, k + 1) inv[i] = (p - p / i) * inv[p % i] % p;
14
        assert(inv[k] * k % p == 1);
15
16
        11 pw = 1;
17
        // (k+1)*S[k] = (n+1)^{(k+1)} - [0-k-1](k+1,j)*S[j]-1
18
        rep(i, 0, k) {
19
            pw = pw * (n + 1) \% p;
20
            s[i] = (pw - 1 + p) \% p;
21
            rep(j, 0, i - 1) {
                s[i] = (s[i] - comb[i + 1][j] * s[j] % p + p) % p;
23
24
            s[i] = s[i] * inv[i + 1] % p;
        cout << s[k] << '\n';
27 }
```

6.24 扩展欧拉定理.cpp

```
1 // mod {min(b, b % phi + phi)}
```

6.25 拉格朗日插值.cpp

```
1 // k阶多项式(需要k+1个点)
2 // 求在点n 上的值
3 // O(k)
4 ll lagrange(ll n, int k) {
        vector<11> x(k + 5), y(k + 5);
        rep(i, 1, k + 1) {
           x[i] = i;
            // y[i] = (y[i-1] + powmod(i,k-1,mod)) \% mod;
9
10
        if (n <= k + 1) return y[n];
11
12
        vector<ll> fac(k + 5);
       fac[0] = 1:
13
14
       ll coe = 1;
        rep(i, 1, k + 4) fac[i] = fac[i - 1] * i % mod;
15
        rep(i, 1, k + 1) coe = coe * (n - i + mod) % mod;
16
       11 \text{ ans} = 0;
17
       rep(i, 1, k + 1) {
18
            11 \text{ sgn} = (((k + 1 - i) \% 2) ? -1 : 1);
19
            11 f1 = powmod(fac[i - 1] * fac[k + 1 - i] % mod, mod - 2, mod);
21
            11 f2 = powmod(n - i, mod - 2, mod);
            ans += sgn * coe * f1 % mod * f2 % mod * v[i] % mod;
            ans = (ans + mod) % mod:
23
24
       }
25
        return ans;
26 }
```

6.26 整除分块.cpp

```
1 void solve() {
2     u64 ans = 0;
3     cin >> n;
4     for (ll l = 1; l <= n; l++) {
5         ll d = n / l, r = n / d;
6         ans += (l + r) * (r - l + 1) / 2 * d;
7         l = r;
8     }
9 }</pre>
```

6.27 枚举子集.cpp

```
1 void solve() {
       f[0] = 1;
       for (int i = 1; i < (111 << n); i++) {
           int t = i;
5
         11 res = 0;
           while (true) {
               if (t == 0) break;
               t = (t - 1)&i;
               res = (res + f[t]) % mod:
           }
10
11
           f[i] = res * i;
12
       }
13 }
```

6.28 枚举超集.cpp

```
void solve() {
    for (int i = 1; i < (111 << n); i++) {
        int t = i;
        while (true) {
            t = (t + 1) | i;
            if (t == bit(n) - 1) break;
        }
    }
}</pre>
```

6.29 狄利克雷卷积.cpp

```
1 const int N = 1010000;
```

```
2 int p[N], pr[N / 5], n, tot;
   unsigned int A, B, C, mu[N], f[N], g[N];
5 inline unsigned int rng61() {
        A ^= A << 16:
        A ^= A >> 5;
        A ^= A << 1;
        unsigned int t = A;
        A = B;
        B = C:
11
12
        C = t A;
13
        return C;
14 }
15
   int main() {
16
17
        scanf("%d%u%u%u", &n, &A, &B, &C);
18
        for (int i = 1; i <= n; i++)
19
            f[i] = rng61();
20
        p[1] = 1; mu[1] = 1;
21
22
        for (int i = 2; i <= n; i++) {
            if (!p[i]) p[i] = i, mu[i] = (uint)-1, pr[++tot] = i;
23
            for (int j = 1; j \le tot && pr[j] * i \le n; j++) {
24
25
                p[i * pr[j]] = pr[j];
26
                if (p[i] == pr[j]) {
                    mu[i * pr[j]] = 0;
27
                    break;
28
               } else {
29
30
                    mu[i * pr[j]] = (uint)-mu[i];
                }
31
32
            }
33
        7
34
        for (int d1 = 1; d1 \le n; d1++)
            for (int d2 = 1; d1 * d2 <= n; d2++)
35
                g[d1 * d2] += f[d1] * mu[d2];
36
37
        uint ans = 0;
        for (int i = 1; i <= n; i++) ans ^= g[i];
38
39
        printf("%u\n", ans);
40 }
```

6.30 线性筛常见积性函数.cpp

```
1 const int N = 20010000;
```

```
2 int p[N], pr[N / 5], n, pe[N], tot;
3 uint f[N], a, b, ans;
5 void compute(int n, function < void(int) > calcpe) {
       ans = 0:
       f[1] = 1;
       for (int i = 2; i <= n; i++) {
           if (i == pe[i])
10
                calcpe(i);
11
           else
12
                f[i] = f[pe[i]] * f[i / pe[i]];
13
       }
14
       for (uint i = 1: i <= n: i++) {
15
           ans \hat{} = (a * i * f[i] + b);
16
       }
17
       printf("%u\n", ans);
18 }
19
20 int main() {
21
       scanf("%d%u%u", &n, &a, &b);
22
       p[1] = 1;
23
       for (int i = 2: i <= n: i++) {
24
           if (!p[i]) p[i] = i, pe[i] = i, pr[++tot] = i;
25
           for (int j = 1; j <= tot && pr[j] * i <= n; j++) {
26
                p[i * pr[j]] = pr[j];
27
                if (p[i] == pr[j]) {
                    pe[i * pr[j]] = pe[i] * pr[j];
29
                   break;
30
               } else {
31
                    pe[i * pr[j]] = pr[j];
               }
32
33
           }
34
       }
       // 因子个数,因子和,欧拉函数,莫比乌斯函数
35
36
       compute(n, [&](int x) {
37
           f[x] = f[x / p[x]] + 1;
38
       });
39
40
       compute(n, [&](int x) {
41
           f[x] = f[x / p[x]] + x;
42
       });
43
44
        compute(n, [&](int x) {
```

6.31 莫比乌斯反演 gcd 常见结论.cpp

```
1 // u * 1 = e, phi * 1 = id, phi = id * u
2 const int N = 10100000, M = 10000000;
3 int p[N], pr[N / 5], n, tot;
   int mu[N], smu[N];
6 int main() {
       p[1] = 1; mu[1] = 1;
       for (int i = 2; i <= M; i++) {
            if (!p[i]) p[i] = i, mu[i] = -1, pr[++tot] = i;
            for (int j = 1; j <= tot && pr[j] * i <= M; j++) {
10
11
                p[i * pr[j]] = pr[j];
                if (p[i] == pr[j]) {
12
                    mu[i * pr[j]] = 0;
13
                    break;
14
               } else {
                    mu[i * pr[j]] = -mu[i];
16
17
               }
            }
18
19
       for (int i = 1: i \le M: i++)
20
21
            smu[i] = smu[i - 1] + mu[i]:
22
       int T;
23
       scanf("%d", &T);
24
       for (int tc = 0; tc < T; tc++) {
25
            int n, m;
            scanf("%d%d", &n, &m);
            if (n > m) swap(n, m);
28
            11 \text{ ans} = 0:
29
            for (int l = 1; l <= n; l++) {
                int n1 = n / 1, m1 = m / 1:
30
                int r = min(n / n1, m / m1);
31
32
                //l \dots r
33
                ans += 111 * (smu[r] - smu[1 - 1]) * n1 * m1:
```

7 String

7.1 ACAM.cpp

```
1 const int AC SIGMA = 26, AC V = 26, AC N = 810000;
2 struct AC_automaton {
        struct node {
            node *go[AC V], *fail, *f;
   // declare extra variables:
        } pool[AC_N], *cur, *root, *q[AC_N];
        node* newnode() {
            node *p = cur++;
    // init extra variables:
            return p;
11
        }
12 // CALL init() and CHECK all const variables:
        void init() { cur = pool; root = newnode(); }
        node* append(node *p, int w) {
14
            if (!p->go[w]) p->go[w] = newnode(), <math>p->go[w]->f = p;
16
            return p->go[w];
17
        }
18
        void build() {
19
            int t = 0:
20
            q[t++] = root;
21
            root->fail = root;
22
            rep(i, 0, AC_SIGMA - 1) if (root->go[i]) {
23
                 q[t++] = root->go[i];
24
                 root->go[i]->fail = root;
25
            } else {
26
                 root->go[i] = root;
            }
27
28
            rep(i, 1, t - 1) {
                 node *u = q[i];
30
                 rep(j, 0, AC_SIGMA - 1) if (u->go[j]) {
31
                     u \rightarrow go[j] \rightarrow fail = u \rightarrow fail \rightarrow go[j];
32
                     q[t++] = u->go[i];
```

```
33
                  } else {
34
                        u \rightarrow go[j] = u \rightarrow fail \rightarrow go[j];
                  }
36
              }
         }
38 } ac:
    typedef AC automaton::node ACnode;
    const int M = 2, N = 2.1e5;
42 struct node {
         node *son[M], *go[M], *fail;
43
         int cnt, vis, ins;
45 } pool[N], *cur = pool, *q[N], *root;
46
47 node *newnode() { return cur++: }
48 int t, n;
    void build() {
         t = 0:
51
         a[t++] = root:
52
         for (int i = 0; i < t; i++) {
53
54
              node *u = a[i]:
55
              for (int j = 0; j < M; j++) {
56
                   if (u->son[j]) {
57
                       u \rightarrow go[j] = u \rightarrow son[j];
                        if (u != root)
                            u \rightarrow go[j] \rightarrow fail = u \rightarrow fail \rightarrow go[j];
59
60
                        else
61
                            u->go[j]->fail = root;
                       q[t++] = u->son[j];
62
63
                  } else {
64
                        if (u != root)
                            u \rightarrow go[j] = u \rightarrow fail \rightarrow go[j];
65
66
                        else
                            u \rightarrow go[j] = root;
67
                  }
70
71 }
72
    void insert(string &s) {
74
         node *cur = root:
75
         for (auto c : s) {
```

```
76          int w = c - '0';
77          if (!cur->son[w]) {
78                cur->son[w] = newnode();
79          }
80          cur = cur->son[w];
81     }
82     cur->cnt = 1;
83 }
```

7.2 hash61.cpp

```
1 struct hash61 {
     static const uint64_t md = (1LL << 61) - 1;
     static uint64_t step;
     static vector <uint64 t> pw;
4
5
     uint64 t addmod(uint64 t a, uint64 t b) const {
       a += b:
       if (a \ge md) a -= md;
       return a;
10
     }
11
12
     uint64_t submod(uint64_t a, uint64_t b) const {
13
       a += md - b;
    if (a >= md) a -= md;
14
       return a:
16
    }
17
18
     uint64 t mulmod(uint64 t a, uint64 t b) const {
19
       uint64 t 11 = (uint32 t) a, h1 = a >> 32, 12 = (uint32 t) b, h2 = b >>
20
       uint64 t 1 = 11 * 12, m = 11 * h2 + 12 * h1, h = h1 * h2;
21
       uint64 t ret = (1 \& md) + (1 >> 61) + (h << 3) + (m >> 29) + (m << 35 >>
             3) + 1;
       ret = (ret & md) + (ret >> 61):
       ret = (ret & md) + (ret >> 61);
24
       return ret - 1;
25
26
     void ensure_pw(int sz) {
       int cur = (int) pw.size();
29
       if (cur < sz) {
30
         pw.resize(sz);
```

```
31
          for (int i = cur; i < sz; i++) {
           pw[i] = mulmod(pw[i - 1], step);
32
33
         }
34
       }
     }
35
36
37
     vector<uint64 t> pref;
38
     int n:
39
     template < typename T>
40
41
     hash61(const T& s) {
       n = (int) s.size();
42
       ensure_pw(n + 1);
43
44
       pref.resize(n + 1);
       pref[0] = 1;
45
       for (int i = 0; i < n; i++) {
          pref[i + 1] = addmod(mulmod(pref[i], step), s[i]);
48
49
     }
50
51
      inline uint64 t operator()(const int from, const int to) const {
       assert(0 <= from && from <= to && to <= n - 1):
52
       return submod(pref[to + 1], mulmod(pref[from], pw[to - from + 1]));
53
54
     }
55 }:
   uint64_t hash61::step = (md >> 2) + rng() % (md >> 1);
   vector<uint64_t> hash61::pw = vector<uint64_t>(1, 1);
```

7.3 kmp.cpp

```
1 template <typename T>
2 vector<int> kmp_table(int n, const T &s) {
3    vector<int> p(n, 0);
4    int k = 0;
5    for (int i = 1; i < n; i++) {
6       while (k > 0 && !(s[i] == s[k])) {
7         k = p[k - 1];
8       }
9       if (s[i] == s[k]) {
10         k++;
11       }
12       p[i] = k;
```

```
13
14
     return p;
15 }
16
17 template <typename T>
18 vector<int> kmp_table(const T &s) {
      return kmp table((int) s.size(), s);
20 }
21
22 template <typename T>
23 vector < int > kmp_search(int n, const T &s, int m, const T &w, const vector <
     assert(n >= 1 && (int) p.size() == n);
     vector<int> res;
26
     int k = 0:
     for (int i = 0; i < m; i++) {
       while (k > 0 \&\& (k == n \mid | !(w[i] == s[k])))  {
29
         k = p[k - 1];
30
       }
       if (w[i] == s[k]) {
         k++;
       }
34
       if (k == n) {
35
          res.push_back(i - n + 1);
36
       }
     }
37
38
     return res;
      // returns 0-indexed positions of occurrences of s in w
40 }
41
42 template <typename T>
   vector<int> kmp_search(const T &s, const T &w, const vector<int> &p) {
      return kmp_search((int) s.size(), s, (int) w.size(), w, p);
45 }
```

7.4 manacherfast.cpp

```
1 template <typename T>
2 vector<int> manacher(int n, const T &s) {
3    if (n == 0) {
4      return vector<int>();
5    }
6    vector<int> res(2 * n - 1, 0);
```

```
int 1 = -1, r = -1:
      for (int z = 0; z < 2 * n - 1; z++) {
        int i = (z + 1) >> 1:
10
        int j = z \gg 1;
        int p = (i \ge r ? 0 : min(r - i, res[2 * (1 + r) - z])):
11
        while (j + p + 1 < n \&\& i - p - 1 >= 0) {
12
          if (!(s[j + p + 1] == s[i - p - 1])) {
14
15
         }
16
          p++;
17
        if (j + p > r) {
         1 = i - p:
          r = j + p;
21
        res[z] = p;
24
     return res:
      // res[2 * i] = odd radius in position i
      // res[2 * i + 1] = even \ radius \ between \ positions \ i \ and \ i + 1
      // s = "abaa" \rightarrow res = \{0, 0, 1, 0, 0, 1, 0\}
     // in other words, for every z from 0 to 2 * n - 2:
     // calculate i = (z + 1) \gg 1 and i = z \gg 1
     // now there is a palindrome from i - res[z] to j + res[z]
31
     // (watch out for i > j and res[z] = 0)
32 }
   template <typename T>
35 vector<int> manacher(const T &s) {
      return manacher((int) s.size(), s);
37 }
```

7.5 MinRotation.cpp

```
1 Description: Finds the lexicographically smallest rotation of a string.
2 Usage: rotate(v.begin(), v.begin() + minRotation(v), v.end());
3 Time: 0 (N)
4
5 int minRotation(string s) {
    int a = 0, N = sz(s); s += s;
    rep(b, 0, N) rep(k, 0, N) {
        if (a + k == b || s[a + k] < s[b + k]) {b += max(0, k - 1); break;}
        if (s[a + k] > s[b + k]) { a = b; break; }
```

```
10 }
11 return a;
12 }
```

7.6 PAM.cpp

```
struct PAM {
        struct T {
            array<int, 10> tr;
            int fail, len, tag;
           T(): fail(0), len(0), tag(0) {
                tr.fill(0);
           }
       };
       vector<T> t;
       vector<int> stk:
11
        int newnode(int len) {
12
            t.emplace_back();
13
            t.back().len = len;
            return (int)t.size() - 1;
14
15
       }
16
       PAM() : t(2) {
17
           t[0].fail = 1, t[0].len = 0;
18
            t[1].fail = 0, t[1].len = -1;
19
            stk.push back(-1);
20
       }
21
        int getfail(int v) {
22
            while (stk.end()[-2 - t[v].len] != stk.back()) {
23
                v = t[v].fail;
24
           }
25
            return v:
26
27
       int insert(int lst. int c. int td) {
28
            stk.emplace_back(c);
29
            int x = getfail(lst):
            if (!t[x].tr[c]) {
30
31
                int u = newnode(t[x].len + 2);
                t[u].fail = t[getfail(t[x].fail)].tr[c];
                t[x].tr[c] = u;
33
34
            t[t[x].tr[c]].tag += td;
35
36
            return t[x].tr[c];
37
       }
```

```
int build(int n) {
38
            int ans = 0;
39
40
            for (int i = (int)t.size() - 1; i > 1; i--) {
                t[t[i].fail].tag += t[i].tag;
41
                if (t[i].len > n) {
                    continue;
                }
                ans = (ans + 111 * t[i].tag * t[i].tag % M * t[i].len) % M;
            }
47
            return ans;
48
       }
49 };
```

7.7 rollingHash.cpp

```
typedef pair<int,int> hashv;
    const ll mod1=1000000007;
    const 11 mod2=1000000009;
   // prefixSum trick for high dimensions
   hashv operator + (hashv a, hashv b) {
       int c1=a.fi+b.fi,c2=a.se+b.se;
       if (c1>=mod1) c1-=mod1;
       if (c2 \ge mod2) c2 = mod2:
10
11
       return mp(c1,c2);
12 }
13
   hashv operator - (hashv a, hashv b) {
       int c1=a.fi-b.fi.c2=a.se-b.se:
15
16
       if (c1<0) c1+=mod1;
       if (c2<0) c2+=mod2;
17
       return mp(c1,c2);
18
19 }
20
   hashv operator * (hashv a,hashv b) {
        return mp(111*a.fi*b.fi%mod1,111*a.se*b.se%mod2);
23 }
```

7.8 SA.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
```

```
3
4 const int N = 101000;
5 char s[N]:
6 int sa[N], rk[N], ht[N], n;
7 // 0-based sa 表示第i大的为哪个, rk 表示第i个后缀第几大
8 // ht表示 lcp(sa[i], sa[i-1])
   void buildSA(char *s, int *sa, int *rk, int *ht, int n, int m = 128) {
        static int x[N], y[N], c[N];
11
       s[n] = 0;
       for (int i = 0; i < m; i++) c[i] = 0;
13
       for (int i = 0; i < n; i++) c[x[i] = s[i]]++;
14
       for (int i = 1; i < m; i++) c[i] += c[i - 1];
15
       for (int i = n - 1: i \ge 0: i--) sa[--c[x[i]]] = i:
16
       for (int k = 1; k < n; k <<= 1) {
17
           int p=0;
18
           for (int i = n - 1; i \ge n - k; i--) y[p++] = i;
19
           for (int i = 0; i < n; i++) if (sa[i] >= k) v[p++] = sa[i] - k;
           for (int i = 0; i < m; i++) c[i] = 0;
21
           for (int i = 0; i < n; i++) c[x[y[i]]]++;
22
           for (int i = 1; i < m; i++) c[i] += c[i - 1];
23
           for (int i = n - 1; i \ge 0; i--) sa[--c[x[y[i]]]] = y[i];
24
           swap(x, v):
25
           p = 1; x[sa[0]] = 0; y[n] = -1;
26
           for (int i = 1; i < n; i++) {
27
               if (y[sa[i-1]] == y[sa[i]] && y[sa[i-1] + k] == y[sa[i] + k]
                    x[sa[i]] = p - 1;
                else
30
                    x[sa[i]] = p++;
31
32
           if (p == n) break;
33
34
       }
       for (int i = 0; i < n; i++) rk[sa[i]] = i;
       int k = 0;
37
       for (int i = 0; i < n; i++) {
           k = max(k - 1, 0);
           if (rk[i] == 0) continue;
40
           int j = sa[rk[i] - 1];
41
           while (s[i + k] == s[j + k]) k++;
42
           ht[rk[i]] = k;
43
       }
44 }
```

```
45
   int LCP(int u, int v) {
        if (u == v) return n - u:
47
48
        if (rk[u] > rk[v]) swap(u, v);
        // RMO(ht. rk[u] + 1. rk[v])
49
50 }
51
52 int main() {
        scanf("%s", s);
53
        n = strlen(s):
54
55
        buildSA(s, sa, rk, ht, n);
        for (int i = 0; i < n; i++) printf("d_{\perp}", sa[i] + 1); puts("");
56
        for (int i = 1; i < n; i++) printf("%d<sub>||</sub>", ht[i]); puts("");
57
58 }
```

7.9 SAfast.cpp

```
template <typename T>
   vector<int> suffix_array(int n, const T &s, int char_bound) {
      vector<int> a(n);
     if (n == 0) {
        return a;
     }
      if (char bound != -1) {
        vector<int> aux(char_bound, 0);
        for (int i = 0: i < n: i++) {
10
          aux[s[i]]++;
11
       }
12
        int sum = 0;
        for (int i = 0: i < char bound: i++) {
13
14
          int add = aux[i]:
15
         aux[i] = sum;
         sum += add:
16
17
       for (int i = 0: i < n: i++) {
18
19
         a[aux[s[i]]++] = i;
20
       }
21
     } else {
22
        iota(a.begin(), a.end(), 0);
        sort(a.begin(), a.end(), [&s](int i, int i) { return s[i] < s[i]: }):</pre>
23
24
25
      vector<int> sorted_by_second(n);
     vector<int> ptr_group(n);
```

```
vector<int> new_group(n);
     vector<int> group(n);
     group[a[0]] = 0;
30
     for (int i = 1; i < n; i++) {
        group[a[i]] = group[a[i - 1]] + (!(s[a[i]] == s[a[i - 1]]));
31
32
     }
33
     int cnt = group [a[n - 1]] + 1;
     int step = 1:
35
     while (cnt < n) {
       int at = 0:
36
37
       for (int i = n - step; i < n; i++) {
38
          sorted by second[at++] = i;
39
40
        for (int i = 0; i < n; i++) {
41
         if (a[i] - step >= 0) {
42
            sorted_by_second[at++] = a[i] - step;
43
         }
       }
44
        for (int i = n - 1; i \ge 0; i--) {
          ptr_group[group[a[i]]] = i;
46
47
48
       for (int i = 0: i < n: i++) {
         int x = sorted by second[i]:
50
          a[ptr_group[group[x]]++] = x;
51
       }
52
        new_group[a[0]] = 0;
        for (int i = 1; i < n; i++) {
          if (group[a[i]] != group[a[i - 1]]) {
54
55
            new_group[a[i]] = new_group[a[i - 1]] + 1;
56
         } else {
            int pre = (a[i - 1] + step >= n ? -1 : group[a[i - 1] + step]);
57
            int cur = (a[i] + step >= n ? -1 : group[a[i] + step]);
            new_group[a[i]] = new_group[a[i - 1]] + (pre != cur);
60
         }
61
       }
        swap(group, new_group);
        cnt = group[a[n - 1]] + 1;
        step <<= 1;
64
65
66
     return a;
67 }
   template <typename T>
```

```
vector<int> suffix_array(const T &s, int char_bound) {
       return suffix array((int) s.size(), s, char bound);
 72 }
 73
    template <typename T>
     vector<int> build_lcp(int n, const T &s, const vector<int> &sa) {
       assert((int) sa.size() == n);
      vector<int> pos(n);
 77
 78
      for (int i = 0; i < n; i++) {
        pos[sa[i]] = i;
 79
 80
      vector<int> lcp(max(n - 1, 0));
 81
      int k = 0:
      for (int i = 0; i < n; i++) {
 83
 84
        k = max(k - 1, 0):
        if (pos[i] == n - 1) {
          k = 0;
        } else {
          int j = sa[pos[i] + 1];
          while (i + k < n &  j + k < n &  s[i + k] == s[j + k]) {
 90
            k++;
          }
 91
          lcp[pos[i]] = k;
 93
        }
 94
 95
      return lcp;
 97
    template <typename T>
     vector<int> build_lcp(const T &s, const vector<int> &sa) {
      return build_lcp((int) s.size(), s, sa);
100
101 }
    7.10 SAM.cpp
```

```
struct SAM {
static constexpr int ALPHABET_SIZE = 26;
struct Node {
    int len;
    int link;
std::array<int, ALPHABET_SIZE> next;
Node() : len{}, link{}, next{} {}
};
```

```
std::vector<Node> t;
10
        SAM() {
11
            init():
12
       }
13
       void init() {
14
            t.assign(2, Node());
15
            t[0].next.fill(1);
16
            t[0].len = -1:
17
       }
       int newNode() {
18
19
            t.emplace_back();
20
            return t.size() - 1;
21
       }
       int extend(int p, int c) {
23
            if (t[p].next[c]) {
24
                int q = t[p].next[c];
25
                if (t[q].len == t[p].len + 1) {
26
                    return q;
27
                int r = newNode():
28
29
                t[r].len = t[p].len + 1;
                t[r].link = t[a].link:
31
                t[r].next = t[q].next;
32
                t[q].link = r;
                while (t[p].next[c] == q) {
                    t[p].next[c] = r;
34
                    p = t[p].link;
36
37
                return r;
38
            }
39
            int cur = newNode();
40
            t[cur].len = t[p].len + 1;
            while (!t[p].next[c]) {
41
42
                t[p].next[c] = cur;
43
                p = t[p].link;
44
            t[cur].link = extend(p, c);
            return cur;
47
48 };
```

7.11 SA-IS.cpp

```
2 * Time Complexity: Suffix Array: O(N + Character Set Size) time and space
                     //
           128 --- ASCII
                                                       LCP: O(N) time and space
           * Usage:
                              1. Suffix Array (returns s.size() elements, NOT considering
           O-length/empty suffix)
                                           auto sa = suffix array(s); // s is the input string with
                     ASCII
           characters
                                            auto sa wide char = suffix array(s, LIM); // LIM = max(s[i])
                     + 2.
           s is the string with arbitary big characters.
                              2. LCP:
                                          auto lcp = LCP(s, suffix array(s)); // returns s.size()
                     elements.
           where lcp[i]=LCP(sa[i], sa[i+1])
         * Status: Tested (DMOJ: ccc03s4, SPOJ: SARRAY (100pts), Yosupo's: Suffix
          & Number of Substrings, CodeForces EDU
17
          */
18 // Based on: Rickypon, https://judge.yosupo.jp/submission/10105
         void induced_sort(const std::vector<int>& vec, int val_range,
20
                                                   std::vector<int>& SA, const std::vector<bool>& sl,
                                                   const std::vector<int>& lms idx) {
21
22
                  std::vector<int> l(val range, 0), r(val range, 0);
23
                  for (int c : vec) {
                           if (c + 1 < val_range) ++1[c + 1];
24
                           ++r[c]:
25
26
                 }
27
                  std::partial sum(1.begin(), 1.end(), 1.begin());
28
                  std::partial_sum(r.begin(), r.end(), r.begin());
                  std::fill(SA.begin(), SA.end(), -1);
29
30
                  for (int i = (int)lms idx.size() - 1; i >= 0; --i)
                           SA[--r[vec[lms idx[i]]]] = lms idx[i]:
31
32
                  for (int i : SA)
                           if (i >= 1 && sl[i - 1]) SA[l[vec[i - 1]]++] = i - 1:
33
34
                  std::fill(r.begin(), r.end(), 0);
35
                  for (int c : vec) ++r[c];
                  std::partial sum(r.begin(), r.end(), r.begin());
37
                  for (int k = (int)SA.size() - 1, i = SA[k]; k \ge 1; k \ge 1;
38
                           if (i >= 1 && !sl[i - 1]) {
```

```
39
                SA[--r[vec[i-1]]] = i-1:
40
           }
41 }
42
   std::vector<int> SA IS(const std::vector<int>& vec. int val range) {
44
        const int n = vec.size():
        std::vector<int> SA(n), lms idx;
       std::vector<bool> sl(n):
47
       sl[n - 1] = false;
       for (int i = n - 2; i \ge 0; --i) {
48
49
            sl[i] = (vec[i] > vec[i + 1] || (vec[i] == vec[i + 1] && sl[i + 1]))
           if (sl[i] && !sl[i + 1]) lms idx.push back(i + 1):
50
51
       }
52
       std::reverse(lms_idx.begin(), lms_idx.end());
       induced sort(vec, val range, SA, sl, lms idx);
54
        std::vector<int> new lms idx(lms idx.size()), lms vec(lms idx.size());
       for (int i = 0, k = 0; i < n; ++i)
56
            if (!sl[SA[i]] \&\& SA[i] >= 1 \&\& sl[SA[i] - 1]) {
57
                new lms idx[k++] = SA[i]:
           }
59
       int cur = 0:
       SA[n - 1] = cur:
61
       for (size t k = 1; k < new lms idx.size(); ++k) {</pre>
62
           int i = new lms idx[k - 1], i = new lms idx[k]:
63
           if (vec[i] != vec[j]) {
                SA[i] = ++cur:
                continue;
67
           bool flag = false:
           for (int a = i + 1, b = j + 1; ++a, ++b) {
                if (vec[a] != vec[b]) {
70
                    flag = true;
71
                    break:
72
73
                if ((!sl[a] && sl[a - 1]) || (!sl[b] && sl[b - 1])) {
74
                    flag = !((!sl[a] \&\& sl[a - 1]) \&\& (!sl[b] \&\& sl[b - 1]));
75
                    break;
76
                }
77
           }
78
           SA[j] = (flag ? ++cur : cur);
79
       }
80
       for (size t i = 0; i < lms idx.size(); ++i) lms vec[i] = SA[lms idx[i]];
```

```
if (cur + 1 < (int)lms_idx.size()) {</pre>
 81
             auto lms SA = SA IS(lms vec, cur + 1);
 82
             for (size_t i = 0; i < lms_idx.size(); ++i) {</pre>
 83
 84
                 new lms idx[i] = lms idx[lms SA[i]];
             }
 85
         }
 86
 87
         induced sort(vec, val range, SA, sl, new lms idx);
         return SA:
 88
 89 }
 90
     std::vector<int> suffix array(const std::string& s, const char first = 'a',
                               const char last = 'z') {
 92
         std::vector<int> vec(s.size() + 1):
 93
 94
         std::copy(std::begin(s), std::end(s), std::begin(vec));
         for (auto& x : vec) x -= (int)first - 1:
 95
         vec.back() = 0;
         auto ret = SA IS(vec, (int)last - (int)first + 2);
 97
 98
         ret.erase(ret.begin());
 99
         return ret;
100 }
101 // Author: https://codeforces.com/blog/entry/12796?#comment-175287
102 // Uses kasai's algorithm linear in time and space
     std::vector<int> LCP(const std::string& s. const std::vector<int>& sa) {
103
104
         int n = s.size(), k = 0;
105
         std::vector<int> lcp(n), rank(n):
         for (int i = 0; i < n; i++) rank[sa[i]] = i;
106
         for (int i = 0; i < n; i++, k ? k-- : 0) {
107
             if (rank[i] == n - 1) {
108
109
                 k = 0:
110
                 continue:
111
             }
112
             int j = sa[rank[i] + 1];
             while (i + k < n \&\& j + k < n \&\& s[i + k] == s[j + k]) k++;
113
114
             lcp[rank[i]] = k;
         }
115
116
         lcp[n-1] = 0;
         return lcp;
117
118 }
119
120 template <typename T, class F = function<T(const T&, const T&)>>
     class SparseTable {
121
      public:
122
      int n;
123
```

```
124
       vector<vector<T>> mat:
125
       F func;
126
127
       SparseTable(const vector<T>& a, const F& f) : func(f) {
128
         n = static cast<int>(a.size()):
129
        int max_log = 32 - __builtin_clz(n);
130
        mat.resize(max log);
131
         mat[0] = a:
132
        for (int j = 1; j < max log; <math>j++) {
133
           mat[i].resize(n - (1 << i) + 1):
134
           for (int i = 0; i \le n - (1 \le j); i++) {
135
             mat[j][i] = func(mat[j-1][i], mat[j-1][i+(1 << (j-1))]);
136
          }
137
        }
138
      }
139
140
      T get(int from, int to) const {
141
         assert(0 <= from && from <= to && to <= n - 1):
142
        int lg = 32 - __builtin_clz(to - from + 1) - 1;
143
        return func(mat[lg][from], mat[lg][to - (1 << lg) + 1]):
144
     }
145 }:
```

7.12 Z.cpp

```
1 template <typename T>
2 vector<int> z function(int n, const T &s) {
     vector<int> z(n, n):
     int 1 = 0, r = 0;
    for (int i = 1: i < n: i++) {
       z[i] = (i > r ? 0 : min(r - i + 1, z[i - 1])):
6
7
       while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]]) {
         z[i]++:
8
9
       }
       if (i + z[i] - 1 > r) {
11
         1 = i:
         r = i + z[i] - 1;
12
13
14
     }
     return z:
16 }
17
18 template <typename T>
```

```
19 vector<int> z_function(const T &s) {
20 return z_function((int) s.size(), s);
21 }
```

8 Basic

8.1 AST.py

```
class Solution:
        def calculate(self, s: str) -> int:
            sign = ['+', '-', '*', '/', '(', ')']
            v = []
            n11m = ''
            for c in s:
                if c in sign:
                    if num:
                        v.append(num)
10
                        num = ''
                    if c == '-' and (not v or v[-1] == '('):
11
12
                        v.append('0')
13
                    v.append(c)
                elif c.isnumeric():
14
15
                    num += c
16
            if num:
17
                v.append(num)
18
            stk0 = []
19
20
            stk1 = []
            for e in v:
21
22
                if e.isnumeric():
23
                    stk0.append(e)
                elif e in ['+', '-']:
24
25
                    while stk1 and stk1[-1] in ['*', '/', '+', '-']:
                         stk0.append(stk1.pop())
26
27
                    stk1.append(e)
28
                elif e in ['*', '/', '(']:
29
                    stk1.append(e)
30
                else:
31
                    while stk1 and stk1[-1] != '(':
32
                        stk0.append(stk1.pop())
33
                    stk1.pop()
34
            while stk1:
```

```
35
                stk0.append(stk1.pop())
36
37
            res = []
38
            for e in stk0:
39
                if e.isnumeric():
40
                     res.append(int(e))
41
42
                     v = res.pop()
43
                    u = res.pop()
44
                     if e == '+':
45
                         res.append(u + v)
46
                    if e == '-':
47
                         res.append(u - v)
48
                    if e == '*':
49
                         res.append(u * v)
50
                    if e == '/':
51
                         res.append(u //v)
            return res[0]
```

8.2 bitset.cpp

```
1 template <int len = 1>
2 void solve(int n) {
3
        if (n > len) {
4
            solve < std::min(len*2, MAXLEN)>(n);
5
            return:
6
        // solution using bitset < len >
8 }
9
   struct Bitset {
11
       vector<ull> b;
12
       int n:
       Bitset(int x = 0) {
13
14
            n = x;
15
            b.resize((n + 63) / 64, 0);
16
       }
17
18
        int get(int x) {
19
            return (b[x >> 6] >> (x & 63)) & 1:
20
       }
21
22
        void set(int x, int y) {
```

```
23
           b[x >> 6] = 1ULL << (x & 63);
24
           if (!y) b[x >> 6] ^= 1ULL << (x & 63);
       }
25
26
27
       Bitset &operator&=(const Bitset &another) {
28
           rep(i, 0, min(SZ(b), SZ(another.b)) - 1) {
29
                b[i] &= another.b[i];
31
           return (*this);
       }
32
33
34
       Bitset operator&(const Bitset &another)const {
            return (Bitset(*this) &= another):
35
36
       }
37
38
       Bitset &operator|=(const Bitset &another) {
39
            rep(i, 0, min(SZ(b), SZ(another.b)) - 1) {
                b[i] |= another.b[i]:
41
           }
           return (*this);
42
43
       }
44
45
       Bitset operator | (const Bitset & another) const {
            return (Bitset(*this) |= another);
46
47
       }
48
49
       Bitset &operator^=(const Bitset &another) {
           rep(i, 0, min(SZ(b), SZ(another.b)) - 1) {
50
51
               b[i] ^= another.b[i];
52
           }
53
           return (*this);
54
       }
55
56
       Bitset operator^(const Bitset &another)const {
57
            return (Bitset(*this) ^= another);
58
       }
59
       Bitset &operator>>=(int x) {
60
61
           if (x & 63) {
62
                rep(i, 0, SZ(b) - 2) {
                   b[i] >>= (x & 63);
                   b[i] = (b[i + 1] << (64 - (x & 63)));
65
               }
```

```
b.back() >>= (x & 63):
67
             }
 68
69
             x >>= 6;
             rep(i, 0, SZ(b) - 1) {
 70
 71
                 if (i + x < SZ(b)) b[i] = b[i + x];
                 else b[i] = 0;
 74
             return (*this);
 75
         }
 76
 77
         Bitset operator>>(int x)const {
             return (Bitset(*this) >>= x);
 78
 79
        }
 80
         Bitset &operator<<=(int x) {</pre>
             if (x & 63) {
                 for (int i = SZ(b) - 1: i >= 1: i--) {
 84
                     b[i] <<= (x \& 63);
                     b[i] = b[i - 1] >> (64 - (x & 63));
                 b[0] <<= x & 63:
             }
             x >>= 6;
91
             for (int i = SZ(b) - 1; i \ge 0; i--) {
                 if (i - x \ge 0) b[i] = b[i - x]:
                 else b[i] = 0;
 94
95
             return (*this):
        }
96
97
         Bitset operator << (int x) const {</pre>
             return (Bitset(*this) <<= x);</pre>
100
        }
101 };
```

8.3 fastIO.cpp

```
1 static struct FastInput {
2   static constexpr int BUF_SIZE = 1 << 20;
3   char buf[BUF_SIZE];
4   size_t chars_read = 0;</pre>
```

```
size_t buf_pos = 0;
     FILE *in = stdin;
      char cur = 0:
    inline char get_char() {
       if (buf_pos >= chars_read) {
10
11
          chars read = fread(buf, 1, BUF SIZE, in);
12
         buf_pos = 0;
         buf[0] = (chars read == 0 ? -1 : buf[0]);
13
       }
14
15
        return cur = buf[buf_pos++];
     }
16
17
18
     template <typename T>
19
     inline void tie(T) {}
20
      inline explicit operator bool() {
21
       return cur != -1:
22
23
     }
24
25
      inline static bool is blank(char c) {
26
        return c <= '..':
27
     }
28
29
     inline bool skip_blanks() {
30
        while (is blank(cur) && cur != -1) {
         get_char();
31
32
       }
33
       return cur != -1;
34
     }
35
36
      inline FastInput& operator>>(char& c) {
37
        skip_blanks();
38
       c = cur;
        get_char();
39
40
        return *this;
41
     }
42
      inline FastInput& operator>>(string& s) {
43
44
       if (skip_blanks()) {
45
         s.clear();
          do {
46
47
            s += cur;
```

```
48
         } while (!is_blank(get_char()));
49
       }
50
       return *this:
51
52
      template <typename T>
53
      inline FastInput& read integer(T& n) {
       // unsafe, doesn't check that characters are actually digits
56
       n = 0:
57
       if (skip_blanks()) {
         int sign = +1;
         if (cur == '-') {
          sign = -1:
61
           get char();
62
         }
          do {
           n += n + (n << 3) + cur - '0';
         } while (!is_blank(get_char()));
         n *= sign;
       }
67
       return *this;
70
71
      template <typename T>
72
     inline typename enable_if < is_integral < T > :: value, FastInput & > :: type
          operator>>(T& n) {
       return read_integer(n);
74
75
76
     #if !defined( WIN32) || defined( WIN64)
     inline FastInput& operator>>( int128& n) {
77
78
       return read_integer(n);
79
     }
     #endif
81
     template <typename T>
     inline typename enable_if < is_floating_point < T > :: value, FastInput & > :: type
          operator>>(T& n) {
       // not sure if really fast, for compatibility only
84
       n = 0:
       if (skip_blanks()) {
          string s;
          (*this) >> s;
```

```
sscanf(s.c_str(), "%lf", &n);
         }
 90
         return *this:
 91
 92
     } fast_input;
 94
     #define cin fast input
 97
     static struct FastOutput {
       static constexpr int BUF_SIZE = 1 << 20;</pre>
 98
 99
       char buf[BUF SIZE];
       size t buf pos = 0;
100
       static constexpr int TMP_SIZE = 1 << 20;</pre>
101
102
       char tmp[TMP SIZE];
       FILE *out = stdout;
103
104
105
       inline void put char(char c) {
         buf[buf_pos++] = c;
106
         if (buf_pos == BUF_SIZE) {
107
           fwrite(buf, 1, buf_pos, out);
108
109
           buf pos = 0;
         }
110
       }
111
112
113
       ~FastOutput() {
         fwrite(buf, 1, buf_pos, out);
114
      }
115
116
117
       inline FastOutput& operator<<(char c) {</pre>
118
         put_char(c);
119
         return *this;
120
      }
121
122
       inline FastOutput& operator<<(const char* s) {</pre>
123
         while (*s) {
124
           put_char(*s++);
         }
125
         return *this;
126
127
      }
128
       inline FastOutput& operator<<(const string& s) {</pre>
129
         for (int i = 0; i < (int) s.size(); i++) {
130
131
           put_char(s[i]);
```

```
132
         }
133
         return *this;
134
      }
135
136
       template <typename T>
137
       inline char* integer_to_string(T n) {
138
         // beware of TMP SIZE
139
         char* p = tmp + TMP_SIZE - 1;
140
         if (n == 0) {
141
           *--p = '0';
142
         } else {
143
           bool is negative = false;
144
           if (n < 0) {
145
             is negative = true;
146
             n = -n:
147
           }
148
           while (n > 0) {
149
             *--p = (char) ('0' + n % 10);
150
             n /= 10;
151
           }
152
           if (is negative) {
             *--p = '-':
154
           }
155
         }
156
         return p;
157
158
159
       template <typename T>
160
       inline typename enable_if<is_integral<T>::value, char*>::type stringify(T
           n) {
161
         return integer_to_string(n);
162
      }
163
164
       #if !defined(_WIN32) || defined(_WIN64)
165
       inline char* stringify(__int128 n) {
166
         return integer_to_string(n);
167
       }
168
       #endif
169
170
       template <typename T>
171
       inline typename enable_if < is_floating_point < T > :: value, char * > :: type
           stringify(T n) {
172
         sprintf(tmp, "%.17f", n);
```

```
173
         return tmp;
       }
174
175
176
       template <typename T>
       inline FastOutput& operator << (const T& n) {</pre>
177
         auto p = stringify(n);
178
         for (; *p != 0; p++) {
179
180
           put_char(*p);
181
         return *this:
182
183
     } fast output;
184
185
    #define cout fast output
```

8.4 FastMod.cpp

```
Description: Compute a % b about 5 times faster than usual, where b is
constant but not known at compile time. Returns a value congruent to a
(mod b) in the range [0, 2b).

typedef unsigned long long ull;
struct FastMod {
ull b, m;
FastMod(ull b) : b(b), m(-1ULL / b) {}
ull reduce(ull a) { // a % b + (0 or b)}
return a - (ull)((__uint128_t(m) * a) >> 64) * b;
}

}
```

8.5 intervalContainer.cpp

```
Description: Add and remove intervals from a set of disjoint intervals.

Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).

Time: O (log N)

setpi>compare the added interval (setsetpii>& is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
        R = max(R, it->second);
        before = it = is.erase(it):
}
```

```
12
       }
13
       if (it != is.begin() && (--it)->second >= L) {
           L = min(L, it->first):
15
           R = max(R, it->second);
           is.erase(it):
16
17
18
       return is.insert(before, {L, R});
19 }
   void removeInterval(set<pii>& is, int L, int R) {
21
       if (L == R) return:
       auto it = addInterval(is, L, R);
       auto r2 = it->second;
       if (it->first == L) is.erase(it):
       else (int&)it->second = L;
       if (R != r2) is.emplace(R, r2);
27 }
```

8.6 lineContainer.cpp

```
1 /**
     * Author: Simon Lindholm
    * Date: 2017-04-20
     * License: CCO
    * Source: own work
    * Description: Container where you can add lines of the form kx+m, and
         query maximum values at points x.
    * Useful for dynamic programming (``convex hull trick'').
    * Time: O(\loa N)
     * Status: stress-tested
   #pragma once
12
    struct Line {
13
14
        mutable 11 k, m, p;
15
        bool operator<(const Line& o) const { return k < o.k: }</pre>
16
        bool operator<(ll x) const { return p < x; }</pre>
17 };
18
    struct LineContainer : multiset<Line, less<>>> {
       // (for doubles, use inf = 1/.0, div(a,b) = a/b)
21
        static const ll inf = LLONG MAX;
22
       ll div(ll a, ll b) { // floored division
23
            return a / b - ((a ^ b) < 0 && a % b): }
```

```
24
        bool isect(iterator x. iterator v) {
25
            if (y == end()) return x \rightarrow p = inf, 0;
26
            if (x->k == y->k) x->p = x->m > y->m ? inf : -inf:
27
            else x->p = div(y->m - x->m, x->k - y->k);
28
            return x->p >= v->p:
29
       }
30
        void add(ll k, ll m) {
31
            auto z = insert(\{k, m, 0\}), y = z++, x = y;
            while (isect(y, z)) z = erase(z);
32
            if (x != begin() \&\& isect(--x, y)) isect(x, y = erase(y));
33
34
            while ((y = x) != begin() && (--x)->p >= y->p)
35
                isect(x, erase(y));
       }
36
37
        11 query(11 x) {
38
            assert(!emptv()):
39
            auto 1 = *lower bound(x);
40
            return 1.k * x + 1.m;
       }
42 };
```

8.7 mint.cpp

```
1 template<int MOD. int RT> struct mint {
        static const int mod = MOD;
        static constexpr mint rt() { return RT; } // primitive root for FFT
       int v; explicit operator int() const { return v; } // explicit -> don't
            silently convert to int
       mint():v(0) {}
        mint(11 \ v) \{ v = int((-MOD < v \&\& v < MOD) ? v : v % MOD); \}
            if (v < 0) v += MOD: 
       bool operator == (const mint& o) const {
            return v == o.v: }
       friend bool operator!=(const mint& a. const mint& b) {
10
11
            return !(a == b): }
12
       friend bool operator < (const mint& a. const mint& b) {
            return a.v < b.v; }
13
14
15
       mint& operator+=(const mint& o) {
           if ((v += o.v) >= MOD) v -= MOD;
16
17
           return *this: }
       mint& operator -= (const mint& o) {
18
19
            if ((v -= o.v) < 0) v += MOD:
20
            return *this: }
```

```
21
        mint& operator *= (const mint& o) {
22
            v = int((11)v*o.v%MOD); return *this; }
23
        mint& operator/=(const mint& o) { return (*this) *= inv(o): }
24
        friend mint pow(mint a, ll p) {
25
            mint ans = 1: assert(p >= 0):
26
            for (; p; p /= 2, a *= a) if (p&1) ans *= a;
27
            return ans; }
28
        friend mint inv(const mint& a) { assert(a.v != 0):
            return pow(a,MOD-2); }
29
30
31
        mint operator-() const { return mint(-v); }
32
        mint& operator++() { return *this += 1; }
        mint& operator -- () { return *this -= 1: }
33
34
        friend mint operator+(mint a, const mint& b) { return a += b; }
35
       friend mint operator-(mint a. const mint& b) { return a -= b: }
        friend mint operator*(mint a, const mint& b) { return a *= b; }
37
        friend mint operator/(mint a, const mint& b) { return a /= b; }
38 };
39
   const int MOD=998244353:
    using mi = mint<MOD,5>; // 5 is primitive root for both common mods
42
43 namespace simp {
44
        vector < mi > fac.ifac.invn:
45
        void check(int x) {
46
            if (fac.empty()) {
47
                fac={mi(1).mi(1)}:
                ifac={mi(1).mi(1)}:
48
49
                invn={mi(0),mi(1)};
50
            }
            while (SZ(fac)<=x) {
51
52
                int n=SZ(fac).m=SZ(fac)*2:
53
                fac.resize(m):
54
                ifac.resize(m):
55
                invn.resize(m):
                for (int i=n;i<m;i++) {</pre>
57
                    fac[i]=fac[i-1]*mi(i);
                    invn[i]=mi(MOD-MOD/i)*invn[MOD%i];
                    ifac[i]=ifac[i-1]*invn[i];
59
60
               }
61
            }
62
63
        mi gfac(int x) {
```

```
check(x); return fac[x];
64
       }
65
        mi ginv(int x) {
66
            check(x); return invn[x];
67
       }
68
        mi gifac(int x) {
69
70
            check(x); return ifac[x];
71
72
        mi binom(int n,int m) {
            if (m < 0 \mid | m > n) return mi(0):
73
74
            return gfac(n)*gifac(m)*gifac(n - m);
       }
75
76 }
```

8.8 pbds.cpp

```
#include <bits/extc++.h>
   using namespace __gnu_cxx;
   using namespace __gnu_pbds;
5 #include < ext/pb_ds/assoc_container.hpp>
  #include<ext/pb ds/tree policy.hpp>
   #include<ext/pb_ds/hash_policy.hpp>
   #include<ext/pb ds/trie policy.hpp>
   #include<ext/pb_ds/priority_queue.hpp>
10
   pairing_heap_tag: 配对堆
12 thin_heap_tag: 斐波那契堆
   binomial_heap_tag: 二项堆
   binary_heap_tag: 二叉堆
15
   __gnu_pbds::priority_queue<PII, greater<PII>, pairing_heap_tag> q;
   __gnu_pbds::priority_queue<PII, greater<PII>, pairing_heap_tag>::
       point_iterator its[N];
   its[v] = q.push({dis[v], v});
   q.modify(its[v], {dis[v], v});
21
   可以将两个优先队列中的元素合并 (无任何约束)
  使用方法为a.join(b)
   此时优先队列b内所有元素就被合并进优先队列a中,且优先队列b被清空
26 cc_hash_table < string, int > mp1拉链法
```

```
27 gp_hash_table<string, int> mp2查探法
```

8.9 simu.cpp

```
db rnd(db 1, db r) {
      static uniform_real_distribution < db > u(0, 1);
      static default_random_engine e(rng());
      return 1 + (r - 1) * u(e); // u(rng);
5 }
   db eval(pair<db, db> x) { ... }
    void simulate_anneal() {
10
      pair < db, db > cur(rnd(0, 10000), rnd(0, 10000));
     for (double k = 10000; k > 1e-5; k *= 0.99) {
11
       // [start, end, step]
13
       pair<db, db> nxt(cur.fi + rnd(-k, k), cur.se + rnd(-k, k));
       db delta = eval(nxt) - eval(cur):
14
15
       if (exp(-delta / k) > rnd(0, 1)) {
          cur = nxt;
17
       }
18
19 }
20
21
    * https://codeforces.com/gym/104813/submission/234982955
    * The 9th CCPC (Harbin) 2023
24
     * Author: QwertyPi
25
    */
26 LD Prob() {
      static uniform_real_distribution<> dist(0.0, 1.0);
28
      return dist(rng);
29 }
30
   LD Sigma(LD x) { return 1 / (1 + exp(-x)); }
31
   LD overall max score = 0;
33 for (int main_loop = 0; main_loop < 5; main_loop++) {
     vector<LD> e(n, (LD)1 / n);
35
     for (int tr = 0; tr < 1000; tr++) {
       vector<LD> ne(n):
37
       for (int i = 0; i < n; i++) {
38
          ne[i] = Prob();
39
       }
```

```
40
        LD s = accumulate(all(ne), 0.0L);
       for (int i = 0; i < n; i++) {
41
         ne[i] /= s:
43
       }
       if (eval(ne) > eval(e)) e = ne;
44
     }
45
     LD t = (LD) 0.0002;
     LD max score = 0:
47
48
      const LD depr = 0.999995;
      const int tries = 2E6:
49
      const int loop = 1E5;
50
51
     LD score old = eval(e):
52
53
     for (int tr = 0; tr < tries; tr++) {
   #ifdef LOCAL
54
55
        if (tr % loop == loop - 1) {
          cout << fixed << setprecision(10) << "current_|score_|=| " << max score
57
               << ", utu=u" << t << '\n';
58
       }
   #endif
59
60
        int x = rng() \% n, y = rng() \% n;
       if (e[x] < t \mid | x == v) {
61
        t *= depr:
62
63
         continue;
64
       }
        e[x] = t;
        e[y] += t;
        LD score new = eval(e);
67
        if (score_new > score_old) { // ok
69
70
       } else { // revert
71
         e[x] += t:
72
         e[v] -= t;
73
74
        score_old = score_new;
75
        max_score = max(max_score, score_new);
        t *= depr;
76
77
78
     overall_max_score = max(overall_max_score, max_score);
79 #ifdef LOCAL
      cout << "Loop, #" << main loop << ":, " << max score << '\n';
81 #endif
```

82 }

8.10 sort.cpp

```
1 void merge sort(int q[], int l, int r) {
       if (1 >= r) return;
       int mid = 1 + r >> 1;
       merge_sort(q, 1, mid);
       merge_sort(q, mid + 1, r);
6
7
       int k = 0, i = 1, j = mid + 1;
       while (i <= mid && j <= r)
           if (q[i] <= q[i])
10
                tmp[k++] = q[i++];
11
           else
12
                tmp[k++] = q[j++];
13
14
       while (i <= mid)
15
           tmp[k++] = q[i++];
16
       while (j <= r)
17
           tmp[k++] = q[j++];
18
19
       for (i = 1, j = 0; i \le r; i++, j++) q[i] = tmp[j];
20 }
21
22 void quick_sort(int q[], int 1, int r) {
       if (1 >= r) return;
       int i = 1 - 1, j = r + 1, x = q[1 + r >> 1];
       while (i < i) {
26
           do i ++; while (q[i] < x);
27
           do j --; while (q[j] > x);
28
           if (i < j) swap(q[i], q[j]);</pre>
29
       }
       quick_sort(q, 1, j), quick_sort(q, j + 1, r);
31 }
32
33 template < class T>
34 void radixsort(T *a, 11 n) {
       int base = 0;
       rep(i, 1, n) sa[i] = i;
37
       rep(k, 1, 5) {
           rep(i, 0, 255) c[i] = 0;
           rep(i, 1, n) c[(a[i] >> base) & 255]++:
40
           rep(i, 1, 255) c[i] += c[i - 1];
41
           per(i, n, 1) {
42
                rk[sa[i]] = c[(a[sa[i]] >> base) & 255]--:
```

```
43 }
44 rep(i, 1, n) sa[rk[i]] = i;
45 base += 7;
46 }
47 }
```

8.11 高精度.cpp

```
1 vector<int> add(vector<int> &A, vector<int> &B) {
        if (A.size() < B.size()) return add(B, A);</pre>
       vector<int> C:
       int t = 0;
       for (int i = 0; i < A.size(); i ++ ) {
            t += A[i]:
            if (i < B.size()) t += B[i];</pre>
            C.push_back(t % 10);
            t /= 10;
10
       }
11
       if (t) C.push_back(t);
12
        return C;
13 }
14
15 vector<int> sub(vector<int> &A, vector<int> &B) {
16
        vector<int> C;
       for (int i = 0, t = 0; i < A.size(); i ++ ) {
17
            t = A[i] - t;
18
19
            if (i < B.size()) t -= B[i];</pre>
            C.push_back((t + 10) % 10);
20
21
            if (t < 0) t = 1;
22
            else t = 0;
```

```
23
       }
       while (C.size() > 1 && C.back() == 0) C.pop back();
       return C:
26 }
27
28 vector<int> mul(vector<int> &A, int b) {
       vector<int> C;
       int t = 0:
       for (int i = 0; i < A.size() || t; i ++ ) {
31
           if (i < A.size()) t += A[i] * b;</pre>
32
33
           C.push_back(t % 10);
34
           t /= 10;
       }
35
       while (C.size() > 1 && C.back() == 0) C.pop back();
37
       return C:
38 }
39
40 vector<int> div(vector<int> &A. int b. int &r) {
41
       vector<int> C;
       r = 0:
42
43
       for (int i = A.size() - 1; i >= 0; i -- ) {
           r = r * 10 + A[i]:
45
           C.push_back(r / b);
           r %= b;
46
47
       }
       reverse(C.begin(), C.end());
       while (C.size() > 1 && C.back() == 0) C.pop_back();
       return C;
51 }
```