

Team Reference Document

tourist

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1 data

1.1 bstnode.cpp

```
1 class node {
2     public:
3         int id;
4         node* l;
5         node* r;
6         node* p;
7         bool rev;
8         int sz;
9         // declare extra variables:
10
11
12     node(int _id) {
13         id = _id;
14         l = r = p = nullptr;
15         rev = false;
16         sz = 1;
17         // init extra variables:
18
```

```
19     }
20
21     // push everything else:
22     void push_stuff() {
23
24     }
25
26     void unsafe_reverse() {
27         push_stuff(); // !! edu 112
28         rev ^= 1;
29         swap(l, r);
30         pull();
31     }
32
33     // apply changes:
34     void unsafe_apply() {
35
36     }
37
38     void push() {
39         if (rev) {
40             if (l != nullptr) {
41                 l->unsafe_reverse();
42             }
43             if (r != nullptr) {
44                 r->unsafe_reverse();
45             }
46             rev = 0;
47         }
48         push_stuff();
49     }
50
51     void pull() {
52         sz = 1;
53         // now init from self:
54
55         if (l != nullptr) {
56             l->p = this;
57             sz += l->sz;
58             // now pull from l:
59
60         }
61         if (r != nullptr) {
```

```

62     r->p = this;
63     sz += r->sz;
64     // now pull from r:
65
66 }
67 }
68 };
69
70 void debug_node(node* v, string pref = "") {
71     #ifdef LOCAL
72         if (v != nullptr) {
73             debug_node(v->r, pref + "└");
74             cerr << pref << "-" << "└" << v->id << '\n';
75             debug_node(v->l, pref + "└");
76         } else {
77             cerr << pref << "-" << "└" << "nullptr" << '\n';
78         }
79     #endif
80 }

```

1.2 disjointsparsetable.cpp

```

1  template <typename T, typename F>
2  class DisjointSparseTable {
3  public:
4      int n;
5      vector<vector<T>> mat;
6      F func;
7
8      DisjointSparseTable(const vector<T>& a, const F& f) : n(int(a.size())),
9          func(f) {
10         mat.push_back(a);
11         for (int p = 1; (1 << p) < n; p++) {
12             mat.emplace_back(n);
13             for (int mid = 1 << p; mid < n; mid += 1 << (p + 1)) {
14                 mat[p][mid - 1] = a[mid - 1];
15                 for (int j = mid - 2; j >= mid - (1 << p); j--) {
16                     mat[p][j] = func(a[j], mat[p][j + 1]);
17                 }
18                 mat[p][mid] = a[mid];
19                 for (int j = mid + 1; j < min(n, mid + (1 << p)); j++) {
20                     mat[p][j] = func(mat[p][j - 1], a[j]);

```

```

21     }
22 }
23 }
24
25 T Query(int l, int r) const {
26     assert(0 <= l && l < r && r <= n);
27     if (r - l == 1) {
28         return mat[0][l];
29     }
30     int p = bit_width(unsigned(l ^ (r - 1))) - 1;
31     return func(mat[p][l], mat[p][r - 1]);
32 }
33 };

```

1.3 dsu.cpp

```

1  class dsu {
2  public:
3      vector<int> p;
4      int n;
5
6      dsu(int _n) : n(_n) {
7          p.resize(n);
8          iota(p.begin(), p.end(), 0);
9      }
10
11     inline int get(int x) {
12         return (x == p[x] ? x : (p[x] = get(p[x])));
13     }
14
15     inline bool unite(int x, int y) {
16         x = get(x);
17         y = get(y);
18         if (x != y) {
19             p[x] = y;
20             return true;
21         }
22         return false;
23     }
24 };

```

1.4 fenwick.cpp

```

1  template <typename T>
2  class FenwickTree {
3  public:
4      vector<T> fenw;
5      int n;
6      int pw;
7
8      FenwickTree() : n(0) {}
9      FenwickTree(int n_) : n(n_) {
10         fenw.resize(n);
11         pw = bit_floor(unsigned(n));
12     }
13
14     void Modify(int x, T v) {
15         assert(0 <= x && x < n);
16         while (x < n) {
17             fenw[x] += v;
18             x |= x + 1;
19         }
20     }
21
22     T Query(int x) {
23         assert(0 <= x && x <= n);
24         T v{};
25         while (x > 0) {
26             v += fenw[x - 1];
27             x &= x - 1;
28         }
29         return v;
30     }
31
32     // Returns the length of the longest prefix with sum <= c
33     int MaxPrefix(T c) {
34         T v{};
35         int at = 0;
36         for (int len = pw; len > 0; len >>= 1) {
37             if (at + len <= n) {
38                 auto nv = v;
39                 nv += fenw[at + len - 1];
40                 if (!(c < nv)) {
41                     v = nv;
42                     at += len;
43                 }

```

```

44         }
45     }
46     assert(0 <= at && at <= n);
47     return at;
48 }
49 };

```

1.5 fenwick2d.cpp

```

1  template <typename T>
2  class FenwickTree2D {
3  public:
4      vector<vector<T>> fenw;
5      int n, m;
6
7      FenwickTree() : n(0), m(0) {}
8      FenwickTree2D(int n_, int m_) : n(n_), m(m_) {
9         fenw.resize(n);
10         for (int i = 0; i < n; i++) {
11             fenw[i].resize(m);
12         }
13     }
14
15     void Modify(int i, int j, T v) {
16         assert(0 <= i && i < n && 0 <= j && j < m);
17         int x = i;
18         while (x < n) {
19             int y = j;
20             while (y < m) {
21                 fenw[x][y] += v;
22                 y |= y + 1;
23             }
24             x |= x + 1;
25         }
26     }
27
28     T Query(int i, int j) {
29         assert(0 <= i && i <= n && 0 <= j && j <= m);
30         T v{};
31         int x = i;
32         while (x > 0) {
33             int y = j;
34             while (y > 0) {

```

```

35         v += fenw[x - 1][y - 1];
36         y &= y - 1;
37     }
38     x &= x - 1;
39 }
40 return v;
41 }
42 };

```

1.6 fenwicknode.cpp

```

1 struct FenwickTreeNode {
2     ${0}... a = ...;
3
4     inline void operator += (FenwickTreeNode &other) {
5         a = ...(a, other.a);
6     }
7
8     inline bool operator < (FenwickTreeNode &other) {
9         return a < other.a;
10    }
11 };

```

1.7 hashmap.cpp

```

1 // #include<bits/extc++.h>
2 #include <ext/pb_ds/assoc_container.hpp>
3
4 struct splitmix64_hash {
5     static uint64_t splitmix64(uint64_t x) {
6         // http://xorshift.di.unimi.it/splitmix64.c
7         x += 0x9e3779b97f4a7c15;
8         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
9         x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
10        return x ^ (x >> 31);
11    }
12
13    size_t operator()(uint64_t x) const {
14        static const uint64_t FIXED_RANDOM = std::chrono::
15            steady_clock::now().time_since_epoch().count();
16        return splitmix64(x + FIXED_RANDOM);
17    }
18 };

```

```

18
19 template <typename K, typename V, typename Hash = splitmix64_hash>
20 using HashMap = __gnu_pbds::gp_hash_table<K, V, Hash>;
21
22 template <typename K, typename Hash = splitmix64_hash>
23 using HashSet = HashMap<K, __gnu_pbds::null_type, Hash>;

```

1.8 linkcut.cpp

```

1 template <bool rooted>
2 class link_cut_tree {
3 public:
4     int n;
5     vector<node*> nodes;
6
7     link_cut_tree(int _n) : n(_n) {
8         nodes.resize(n);
9         for (int i = 0; i < n; i++) {
10             nodes[i] = new node(i);
11         }
12     }
13
14     int add_node() {
15         int id = (int) nodes.size();
16         nodes.push_back(new node(id));
17         return id;
18     }
19
20     void expose(node* v) {
21         node* r = nullptr;
22         node* u = v;
23         while (u != nullptr) {
24             splay(u);
25             u->push();
26             u->r = r;
27             u->pull();
28             r = u;
29             u = u->p;
30         }
31         splay(v);
32         assert(v->p == nullptr);
33     }
34

```

```

35 int get_root(int i) {
36     node* v = nodes[i];
37     expose(v);
38     return get_leftmost(v)->id;
39 }
40
41 bool link(int i, int j) { // for rooted: (x, parent[x])
42     if (i == j) {
43         return false;
44     }
45     node* v = nodes[i];
46     node* u = nodes[j];
47     if (rooted) {
48         splay(v);
49         if (v->p != nullptr || v->l != nullptr) {
50             return false; // not a root
51         }
52     } else {
53         make_root(i);
54     }
55     expose(u);
56     if (v->p != nullptr) {
57         return false;
58     }
59     v->p = u;
60     return true;
61 }
62
63 bool cut(int i, int j) { // for rooted: (x, parent[x])
64     if (i == j) {
65         return false;
66     }
67     node* v = nodes[i];
68     node* u = nodes[j];
69     expose(u);
70     splay(v);
71     if (v->p != u) {
72         if (rooted) {
73             return false;
74         }
75         swap(u, v);
76         expose(u);
77         splay(v);

```

```

78         if (v->p != u) {
79             return false;
80         }
81     }
82     v->p = nullptr;
83     return true;
84 }
85
86 bool cut(int i) { // only for rooted
87     assert(rooted);
88     node* v = nodes[i];
89     expose(v);
90     v->push();
91     if (v->l == nullptr) {
92         return false; // already a root
93     }
94     v->l->p = nullptr;
95     v->l = nullptr;
96     v->pull();
97     return true;
98 }
99
100 bool connected(int i, int j) {
101     if (i == j) {
102         return true;
103     }
104     node* v = nodes[i];
105     node* u = nodes[j];
106     expose(v);
107     expose(u);
108     return v->p != nullptr;
109 }
110
111 int lca(int i, int j) {
112     if (i == j) {
113         return i;
114     }
115     node* v = nodes[i];
116     node* u = nodes[j];
117     expose(v);
118     expose(u);
119     if (v->p == nullptr) {
120         return -1;

```

```

121     }
122     splay(v);
123     if (v->p == nullptr) {
124         return v->id;
125     }
126     return v->p->id;
127 }
128
129 bool is_ancestor(int i, int j) {
130     if (i == j) {
131         return true;
132     }
133     node* v = nodes[i];
134     node* u = nodes[j];
135     expose(u);
136     splay(v);
137     return v->p == nullptr && u->p != nullptr;
138 }
139
140 void make_root(int i) {
141     assert(!rooted);
142     node* v = nodes[i];
143     expose(v);
144     reverse(v);
145 }
146
147 node* get_path_from_root(int i) {
148     node* v = nodes[i];
149     expose(v);
150     return v;
151 }
152
153 template <typename... T>
154 void apply(int i, T... args) {
155     node* v = nodes[i];
156     splay_tree::apply(v, args...);
157 }
158 };

```

1.9 pbds.cpp

```

1 #include <ext/pb_ds/assoc_container.hpp>
2

```

```

3 template <typename K, typename V, typename Comp = std::less<K>>
4 using ordered_map = __gnu_pbds::tree<
5     K, V, Comp,
6     __gnu_pbds::rb_tree_tag,
7     __gnu_pbds::tree_order_statistics_node_update
8 >;
9
10 template <typename K, typename Comp = std::less<K>>
11 using ordered_set = ordered_map<K, __gnu_pbds::null_type, Comp>;
12
13 // Supports
14 // auto iterator = ordered_set().find_by_order(idx); // (0-indexed)
15 // int num_strictly_smaller = ordered_set().order_of_key(key);

```

1.10 queue.cpp

```

1 template <typename T, typename F>
2 class Queue {
3 public:
4     vector<T> pref;
5     vector<pair<T, T>> suf;
6     F func;
7
8     Queue(const F& f) : func(f) {}
9
10    bool Empty() { return pref.empty() && suf.empty(); }
11    int Size() { return int(pref.size()) + int(suf.size()); }
12    void Clear() { pref.clear(); suf.clear(); }
13
14    void Push(T t) {
15        if (suf.empty()) {
16            suf.emplace_back(t, t);
17        } else {
18            suf.emplace_back(t, func(suf.back().second, t));
19        }
20    }
21
22    void Pop() {
23        if (!pref.empty()) {
24            pref.pop_back();
25            return;
26        }
27        assert(!suf.empty());

```

```

28     if (suf.size() > 1) {
29         pref.resize(suf.size() - 1);
30         pref[0] = suf.back().first;
31         for (int i = 1; i < int(pref.size()); i++) {
32             pref[i] = func(suf[int(suf.size()) - 1 - i].first, pref[i - 1]);
33         }
34     }
35     suf.clear();
36 }
37
38 T Get() {
39     assert(!Empty());
40     if (pref.empty()) {
41         return suf.back().second;
42     }
43     if (suf.empty()) {
44         return pref.back();
45     }
46     return func(pref.back(), suf.back().second);
47 }
48 };
49
50 template <typename T, typename F>
51 Queue<T, F> MakeQueue(const F& f) {
52     return Queue<T, F>(f);
53 }

```

1.11 segtree.cpp

```

1 class segtree {
2 public:
3     struct node {
4         // don't forget to set default value (used for leaves)
5         // not necessarily neutral element!
6         ... a = ...;
7
8         void apply(int l, int r, ... v) {
9             ...
10        }
11    };
12
13    node unite(const node &a, const node &b) const {
14        node res;

```

```

15        ...
16        return res;
17    }
18
19    inline void push(int x, int l, int r) {
20        int y = (l + r) >> 1;
21        int z = x + ((y - l + 1) << 1);
22        // push from x into (x + 1) and z
23        ...
24        /*
25         if (tree[x].add != 0) {
26             tree[x + 1].apply(l, y, tree[x].add);
27             tree[z].apply(y + 1, r, tree[x].add);
28             tree[x].add = 0;
29         }
30        */
31    }
32
33    inline void pull(int x, int z) {
34        tree[x] = unite(tree[x + 1], tree[z]);
35    }
36
37    int n;
38    vector<node> tree;
39
40    void build(int x, int l, int r) {
41        if (l == r) {
42            return;
43        }
44        int y = (l + r) >> 1;
45        int z = x + ((y - l + 1) << 1);
46        build(x + 1, l, y);
47        build(z, y + 1, r);
48        pull(x, z);
49    }
50
51    template <typename M>
52    void build(int x, int l, int r, const vector<M> &v) {
53        if (l == r) {
54            tree[x].apply(l, r, v[l]);
55            return;
56        }
57        int y = (l + r) >> 1;

```



```

58     int z = x + ((y - 1 + 1) << 1);
59     build(x + 1, 1, y, v);
60     build(z, y + 1, r, v);
61     pull(x, z);
62 }
63
64 node get(int x, int l, int r, int ll, int rr) {
65     if (ll <= 1 && r <= rr) {
66         return tree[x];
67     }
68     int y = (l + r) >> 1;
69     int z = x + ((y - 1 + 1) << 1);
70     push(x, 1, r);
71     node res{};
72     if (rr <= y) {
73         res = get(x + 1, 1, y, ll, rr);
74     } else {
75         if (ll > y) {
76             res = get(z, y + 1, r, ll, rr);
77         } else {
78             res = unite(get(x + 1, 1, y, ll, rr), get(z, y + 1, r, ll, rr));
79         }
80     }
81     pull(x, z);
82     return res;
83 }
84
85 template <typename... M>
86 void modify(int x, int l, int r, int ll, int rr, const M&... v) {
87     if (ll <= 1 && r <= rr) {
88         tree[x].apply(1, r, v...);
89         return;
90     }
91     int y = (l + r) >> 1;
92     int z = x + ((y - 1 + 1) << 1);
93     push(x, 1, r);
94     if (ll <= y) {
95         modify(x + 1, 1, y, ll, rr, v...);
96     }
97     if (rr > y) {
98         modify(z, y + 1, r, ll, rr, v...);
99     }
100    pull(x, z);

```

```

101 }
102
103 int find_first_knowingly(int x, int l, int r, const function<bool(const
104     node&)> &f) {
105     if (l == r) {
106         return l;
107     }
108     push(x, 1, r);
109     int y = (l + r) >> 1;
110     int z = x + ((y - 1 + 1) << 1);
111     int res;
112     if (f(tree[x + 1])) {
113         res = find_first_knowingly(x + 1, 1, y, f);
114     } else {
115         res = find_first_knowingly(z, y + 1, r, f);
116     }
117     pull(x, z);
118     return res;
119 }
120
121 int find_first(int x, int l, int r, int ll, int rr, const function<bool(
122     const node&)> &f) {
123     if (ll <= 1 && r <= rr) {
124         if (!f(tree[x])) {
125             return -1;
126         }
127         return find_first_knowingly(x, 1, r, f);
128     }
129     push(x, 1, r);
130     int y = (l + r) >> 1;
131     int z = x + ((y - 1 + 1) << 1);
132     int res = -1;
133     if (ll <= y) {
134         res = find_first(x + 1, 1, y, ll, rr, f);
135     }
136     if (rr > y && res == -1) {
137         res = find_first(z, y + 1, r, ll, rr, f);
138     }
139     pull(x, z);
140     return res;
141 }
142
143 int find_last_knowingly(int x, int l, int r, const function<bool(const

```

```

        node&>> &f) {
142     if (l == r) {
143         return l;
144     }
145     push(x, l, r);
146     int y = (l + r) >> 1;
147     int z = x + ((y - l + 1) << 1);
148     int res;
149     if (f(tree[z])) {
150         res = find_last_knowingly(z, y + 1, r, f);
151     } else {
152         res = find_last_knowingly(x + 1, l, y, f);
153     }
154     pull(x, z);
155     return res;
156 }
157
158 int find_last(int x, int l, int r, int ll, int rr, const function<bool(
        const node&>> &f) {
159     if (ll <= l && r <= rr) {
160         if (!f(tree[x])) {
161             return -1;
162         }
163         return find_last_knowingly(x, l, r, f);
164     }
165     push(x, l, r);
166     int y = (l + r) >> 1;
167     int z = x + ((y - l + 1) << 1);
168     int res = -1;
169     if (rr > y) {
170         res = find_last(z, y + 1, r, ll, rr, f);
171     }
172     if (ll <= y && res == -1) {
173         res = find_last(x + 1, l, y, ll, rr, f);
174     }
175     pull(x, z);
176     return res;
177 }
178
179 segtree(int _n) : n(_n) {
180     assert(n > 0);
181     tree.resize(2 * n - 1);
182     build(0, 0, n - 1);

```

```

183 }
184
185 template <typename M>
186 segtree(const vector<M> &v) {
187     n = v.size();
188     assert(n > 0);
189     tree.resize(2 * n - 1);
190     build(0, 0, n - 1, v);
191 }
192
193 node get(int ll, int rr) {
194     assert(0 <= ll && ll <= rr && rr <= n - 1);
195     return get(0, 0, n - 1, ll, rr);
196 }
197
198 node get(int p) {
199     assert(0 <= p && p <= n - 1);
200     return get(0, 0, n - 1, p, p);
201 }
202
203 template <typename... M>
204 void modify(int ll, int rr, const M&... v) {
205     assert(0 <= ll && ll <= rr && rr <= n - 1);
206     modify(0, 0, n - 1, ll, rr, v...);
207 }
208
209 // find_first and find_last call all FALSE elements
210 // to the left (right) of the sought position exactly once
211
212 int find_first(int ll, int rr, const function<bool(const node&>> &f) {
213     assert(0 <= ll && ll <= rr && rr <= n - 1);
214     return find_first(0, 0, n - 1, ll, rr, f);
215 }
216
217 int find_last(int ll, int rr, const function<bool(const node&>> &f) {
218     assert(0 <= ll && ll <= rr && rr <= n - 1);
219     return find_last(0, 0, n - 1, ll, rr, f);
220 }
221 };

```

1.12 sparsetable.cpp

```

1 template <typename T, typename F>

```

```

2 class SparseTable {
3 public:
4     int n;
5     vector<vector<T>> mat;
6     F func;
7
8     SparseTable(const vector<T>& a, const F& f) : func(f) {
9         n = static_cast<int>(a.size());
10        int max_log = 32 - __builtin_clz(n);
11        mat.resize(max_log);
12        mat[0] = a;
13        for (int j = 1; j < max_log; j++) {
14            mat[j].resize(n - (1 << j) + 1);
15            for (int i = 0; i <= n - (1 << j); i++) {
16                mat[j][i] = func(mat[j - 1][i], mat[j - 1][i + (1 << (j - 1))]);
17            }
18        }
19    }
20
21    T get(int from, int to) const {
22        assert(0 <= from && from <= to && to <= n - 1);
23        int lg = 32 - __builtin_clz(to - from + 1) - 1;
24        return func(mat[lg][from], mat[lg][to - (1 << lg) + 1]);
25    }
26 };

```

1.13 splay.cpp

```

1 namespace splay_tree {
2
3 bool is_bst_root(node* v) {
4     if (v == nullptr) {
5         return false;
6     }
7     return (v->p == nullptr || (v->p->l != v && v->p->r != v));
8 }
9
10 void rotate(node* v) {
11     node* u = v->p;
12     assert(u != nullptr);
13     u->push();
14     v->push();
15     v->p = u->p;

```

```

16     if (v->p != nullptr) {
17         if (v->p->l == u) {
18             v->p->l = v;
19         }
20         if (v->p->r == u) {
21             v->p->r = v;
22         }
23     }
24     if (v == u->l) {
25         u->l = v->r;
26         v->r = u;
27     } else {
28         u->r = v->l;
29         v->l = u;
30     }
31     u->pull();
32     v->pull();
33 }
34
35 void splay(node* v) {
36     if (v == nullptr) {
37         return;
38     }
39     while (!is_bst_root(v)) {
40         node* u = v->p;
41         if (!is_bst_root(u)) {
42             if ((u->l == v) ^ (u->p->l == u)) {
43                 rotate(v);
44             } else {
45                 rotate(u);
46             }
47         }
48         rotate(v);
49     }
50 }
51
52 pair<node*, int> find(node* v, const function<int(node*)> &go_to) {
53     // go_to returns: 0 -- found; -1 -- go left; 1 -- go right
54     // find returns the last vertex on the descent and its go_to
55     if (v == nullptr) {
56         return {nullptr, 0};
57     }
58     splay(v);

```

```

59     int dir;
60     while (true) {
61         v->push();
62         dir = go_to(v);
63         if (dir == 0) {
64             break;
65         }
66         node* u = (dir == -1 ? v->l : v->r);
67         if (u == nullptr) {
68             break;
69         }
70         v = u;
71     }
72     splay(v);
73     return {v, dir};
74 }
75
76 node* get_leftmost(node* v) {
77     return find(v, [&](node*) { return -1; }).first;
78 }
79
80 node* get_rightmost(node* v) {
81     return find(v, [&](node*) { return 1; }).first;
82 }
83
84 node* get_kth(node* v, int k) { // 0-indexed
85     pair<node*, int> p = find(v, [&](node* u) {
86         if (u->l != nullptr) {
87             if (u->l->sz > k) {
88                 return -1;
89             }
90             k -= u->l->sz;
91         }
92         if (k == 0) {
93             return 0;
94         }
95         k--;
96         return 1;
97     });
98     return (p.second == 0 ? p.first : nullptr);
99 }
100
101 int get_position(node* v) { // 0-indexed

```

```

102     splay(v);
103     return (v->l != nullptr ? v->l->sz : 0);
104 }
105
106 node* get_bst_root(node* v) {
107     splay(v);
108     return v;
109 }
110
111 pair<node*, node*> split(node* v, const function<bool(node*)> &is_right) {
112     if (v == nullptr) {
113         return {nullptr, nullptr};
114     }
115     pair<node*, int> p = find(v, [&](node* u) { return is_right(u) ? -1 : 1;
116         });
117     v = p.first;
118     v->push();
119     if (p.second == -1) {
120         node* u = v->l;
121         if (u == nullptr) {
122             return {nullptr, v};
123         }
124         v->l = nullptr;
125         u->p = v->p;
126         u = get_rightmost(u);
127         v->p = u;
128         v->pull();
129         return {u, v};
130     } else {
131         node* u = v->r;
132         if (u == nullptr) {
133             return {v, nullptr};
134         }
135         v->r = nullptr;
136         u->p = v->p;
137         u = get_leftmost(u);
138         v->p = u;
139         v->pull();
140         return {v, u};
141     }
142 }
143
144 pair<node*, node*> split_leftmost_k(node* v, int k) {
145     return split(v, [&](node* u) {
146         int left_and_me = (u->l != nullptr ? u->l->sz : 0) + 1;
147         if (k >= left_and_me) {

```

```

144     k -= left_and_me;
145     return false;
146 }
147 return true;
148 });
149 }
150
151 node* merge(node* v, node* u) {
152     if (v == nullptr) {
153         return u;
154     }
155     if (u == nullptr) {
156         return v;
157     }
158     v = get_rightmost(v);
159     assert(v->r == nullptr);
160     splay(u);
161     v->push();
162     v->r = u;
163     v->pull();
164     return v;
165 }
166
167 int count_left(node* v, const function<bool(node*)> &is_right) {
168     if (v == nullptr) {
169         return 0;
170     }
171     pair<node*, int> p = find(v, [&](node* u) { return is_right(u) ? -1 : 1;
172         });
173     node* u = p.first;
174     return (u->l != nullptr ? u->l->sz : 0) + (p.second == 1);
175 }
176
177 node* add(node* r, node* v, const function<bool(node*)> &go_left) {
178     pair<node*, node*> p = split(r, go_left);
179     return merge(p.first, merge(v, p.second));
180 }
181
182 node* remove(node* v) { // returns the new root
183     splay(v);
184     v->push();
185     node* x = v->l;
186     node* y = v->r;

```

```

186     v->l = v->r = nullptr;
187     node* z = merge(x, y);
188     if (z != nullptr) {
189         z->p = v->p;
190     }
191     v->p = nullptr;
192     v->push();
193     v->pull(); // now v might be reusable...
194     return z;
195 }
196
197 node* next(node* v) {
198     splay(v);
199     v->push();
200     if (v->r == nullptr) {
201         return nullptr;
202     }
203     v = v->r;
204     while (v->l != nullptr) {
205         v->push();
206         v = v->l;
207     }
208     splay(v);
209     return v;
210 }
211
212 node* prev(node* v) {
213     splay(v);
214     v->push();
215     if (v->l == nullptr) {
216         return nullptr;
217     }
218     v = v->l;
219     while (v->r != nullptr) {
220         v->push();
221         v = v->r;
222     }
223     splay(v);
224     return v;
225 }
226
227 int get_size(node* v) {
228     splay(v);

```

```

229     return (v != nullptr ? v->sz : 0);
230 }
231
232 template<typename... T>
233 void do_apply(node* v, T... args) {
234     splay(v);
235     v->unsafe_apply(args...);
236 }
237
238 void reverse(node* v) {
239     splay(v);
240     v->unsafe_reverse();
241 }
242
243 } // namespace splay_tree
244
245 using namespace splay_tree;

```

1.14 treap.cpp

```

1 namespace treap {
2
3 pair<node*, int> find(node* v, const function<int(node*)>& go_to) {
4     // go_to returns: 0 -- found; -1 -- go left; 1 -- go right
5     // find returns the last vertex on the descent and its go_to
6     if (v == nullptr) {
7         return {nullptr, 0};
8     }
9     int dir;
10    while (true) {
11        v->push();
12        dir = go_to(v);
13        if (dir == 0) {
14            break;
15        }
16        node* u = (dir == -1 ? v->l : v->r);
17        if (u == nullptr) {
18            break;
19        }
20        v = u;
21    }
22    return {v, dir};
23 }

```

```

24
25 node* get_leftmost(node* v) {
26     return find(v, [&](node*) { return -1; }).first;
27 }
28
29 node* get_rightmost(node* v) {
30     return find(v, [&](node*) { return 1; }).first;
31 }
32
33 node* get_kth(node* v, int k) { // 0-indexed
34     pair<node*, int> p = find(v, [&](node* u) {
35         if (u->l != nullptr) {
36             if (u->l->sz > k) {
37                 return -1;
38             }
39             k -= u->l->sz;
40         }
41         if (k == 0) {
42             return 0;
43         }
44         k--;
45         return 1;
46     });
47     return (p.second == 0 ? p.first : nullptr);
48 }
49
50 int get_position(node* v) { // 0-indexed
51     int k = (v->l != nullptr ? v->l->sz : 0);
52     while (v->p != nullptr) {
53         if (v == v->p->r) {
54             k++;
55             if (v->p->l != nullptr) {
56                 k += v->p->l->sz;
57             }
58         }
59         v = v->p;
60     }
61     return k;
62 }
63
64 node* get_bst_root(node* v) {
65     while (v->p != nullptr) {
66         v = v->p;

```

```

67     }
68     return v;
69 }
70
71 pair<node*, node*> split(node* v, const function<bool(node*)> &is_right) {
72     if (v == nullptr) {
73         return {nullptr, nullptr};
74     }
75     v->push();
76     if (is_right(v)) {
77         pair<node*, node*> p = split(v->l, is_right);
78         if (p.first != nullptr) {
79             p.first->p = nullptr;
80         }
81         v->l = p.second;
82         v->pull();
83         return {p.first, v};
84     } else {
85         pair<node*, node*> p = split(v->r, is_right);
86         v->r = p.first;
87         if (p.second != nullptr) {
88             p.second->p = nullptr;
89         }
90         v->pull();
91         return {v, p.second};
92     }
93 }
94
95 pair<node*, node*> split_leftmost_k(node* v, int k) {
96     return split(v, [&](node* u) {
97         int left_and_me = (u->l != nullptr ? u->l->sz : 0) + 1;
98         if (k >= left_and_me) {
99             k -= left_and_me;
100             return false;
101         }
102         return true;
103     });
104 }
105
106 node* merge(node* v, node* u) {
107     if (v == nullptr) {
108         return u;
109     }

```

```

110     if (u == nullptr) {
111         return v;
112     }
113     if (v->P > u->P) {
114         // if (rng() % (v->sz + u->sz) < (unsigned int) v->sz) {
115             v->push();
116             v->r = merge(v->r, u);
117             v->pull();
118             return v;
119         } else {
120             u->push();
121             u->l = merge(v, u->l);
122             u->pull();
123             return u;
124         }
125     }
126
127 int count_left(node* v, const function<bool(node*)> &is_right) {
128     if (v == nullptr) {
129         return 0;
130     }
131     v->push();
132     if (is_right(v)) {
133         return count_left(v->l, is_right);
134     }
135     return (v->l != nullptr ? v->l->sz : 0) + 1 + count_left(v->r, is_right);
136 }
137
138 node* add(node* r, node* v, const function<bool(node*)> &go_left) {
139     pair<node*, node*> p = split(r, go_left);
140     return merge(p.first, merge(v, p.second));
141 }
142
143 node* remove(node* v) { // returns the new root
144     v->push();
145     node* x = v->l;
146     node* y = v->r;
147     node* p = v->p;
148     v->l = v->r = v->p = nullptr;
149     v->push();
150     v->pull(); // now v might be reusable...
151     node* z = merge(x, y);
152     if (p == nullptr) {

```

```

153     if (z != nullptr) {
154         z->p = nullptr;
155     }
156     return z;
157 }
158 if (p->l == v) {
159     p->l = z;
160 }
161 if (p->r == v) {
162     p->r = z;
163 }
164 while (true) {
165     p->push();
166     p->pull();
167     if (p->p == nullptr) {
168         break;
169     }
170     p = p->p;
171 }
172 return p;
173 }
174
175 node* next(node* v) {
176     if (v->r == nullptr) {
177         while (v->p != nullptr && v->p->r == v) {
178             v = v->p;
179         }
180         return v->p;
181     }
182     v->push();
183     v = v->r;
184     while (v->l != nullptr) {
185         v->push();
186         v = v->l;
187     }
188     return v;
189 }
190
191 node* prev(node* v) {
192     if (v->l == nullptr) {
193         while (v->p != nullptr && v->p->l == v) {
194             v = v->p;
195         }

```

```

196         return v->p;
197     }
198     v->push();
199     v = v->l;
200     while (v->r != nullptr) {
201         v->push();
202         v = v->r;
203     }
204     return v;
205 }
206
207 int get_size(node* v) {
208     return (v != nullptr ? v->sz : 0);
209 }
210
211 template<typename... T>
212 void apply(node* v, T... args) {
213     v->unsafe_apply(args...);
214 }
215
216 void reverse(node* v) {
217     v->unsafe_reverse();
218 }
219
220 } // namespace treap
221
222 using namespace treap;

```

2 flows

2.1 blossom.cpp

```

1  template <typename T>
2  vector<int> find_max_unweighted_matching(const undigraph<T>& g) {
3      vector<int> mate(g.n, -1);
4      vector<int> label(g.n);
5      vector<int> parent(g.n);
6      vector<int> orig(g.n);
7      queue<int> q;
8      vector<int> aux(g.n, -1);
9      int aux_time = -1;
10     auto lca = [&](int x, int y) {

```



```

11     aux_time++;
12     while (true) {
13         if (x != -1) {
14             if (aux[x] == aux_time) {
15                 return x;
16             }
17             aux[x] = aux_time;
18             if (mate[x] == -1) {
19                 x = -1;
20             } else {
21                 x = orig[parent[mate[x]]];
22             }
23         }
24         swap(x, y);
25     }
26 };
27 auto blossom = [&](int v, int w, int a) {
28     while (orig[v] != a) {
29         parent[v] = w;
30         w = mate[v];
31         if (label[w] == 1) {
32             label[w] = 0;
33             q.push(w);
34         }
35         orig[v] = orig[w] = a;
36         v = parent[w];
37     }
38 };
39 auto augment = [&](int v) {
40     while (v != -1) {
41         int pv = parent[v];
42         int nv = mate[pv];
43         mate[v] = pv;
44         mate[pv] = v;
45         v = nv;
46     }
47 };
48 auto bfs = [&](int root) {
49     fill(label.begin(), label.end(), -1);
50     iota(orig.begin(), orig.end(), 0);
51     while (!q.empty()) {
52         q.pop();
53     }

```

```

54     q.push(root);
55     label[root] = 0;
56     while (!q.empty()) {
57         int v = q.front();
58         q.pop();
59         for (int id : g.g[v]) {
60             auto &e = g.edges[id];
61             int x = e.from ^ e.to ^ v;
62             if (label[x] == -1) {
63                 label[x] = 1;
64                 parent[x] = v;
65                 if (mate[x] == -1) {
66                     augment(x);
67                     return true;
68                 }
69                 label[mate[x]] = 0;
70                 q.push(mate[x]);
71                 continue;
72             }
73             if (label[x] == 0 && orig[v] != orig[x]) {
74                 int a = lca(orig[v], orig[x]);
75                 blossom(x, v, a);
76                 blossom(v, x, a);
77             }
78         }
79     }
80     return false;
81 };
82 auto greedy = [&]() {
83     vector<int> order(g.n);
84     iota(order.begin(), order.end(), 0);
85     shuffle(order.begin(), order.end(), mt19937(787788));
86     for (int i : order) {
87         if (mate[i] == -1) {
88             for (int id : g.g[i]) {
89                 auto &e = g.edges[id];
90                 int to = e.from ^ e.to ^ i;
91                 if (i != to && mate[to] == -1) {
92                     mate[i] = to;
93                     mate[to] = i;
94                     break;
95                 }
96             }

```

```

97     }
98 }
99 };
100 greedy();
101 for (int i = 0; i < g.n; i++) {
102     if (mate[i] == -1) {
103         bfs(i);
104     }
105 }
106 return mate;
107 }

```

2.2 dinic-edge-ids.cpp

```

1  template <typename T>
2  class flow_graph {
3  public:
4      static constexpr T eps = (T) 1e-9;
5
6      struct edge {
7          int from;
8          int to;
9          T c;
10         T f;
11     };
12
13     vector<vector<int>> g;
14     vector<edge> edges;
15     int n;
16     int st, fin;
17     T flow;
18
19     vector<int> ptr;
20     vector<int> d;
21     vector<int> q;
22
23     flow_graph(int _n, int _st, int _fin) : n(_n), st(_st), fin(_fin) {
24         assert(0 <= st && st < n && 0 <= fin && fin < n && st != fin);
25         g.resize(n);
26         ptr.resize(n);
27         d.resize(n);
28         q.resize(n);
29         flow = 0;

```

```

30     }
31
32     void clear_flow() {
33         for (const edge &e : edges) {
34             e.f = 0;
35         }
36         flow = 0;
37     }
38
39     void add(int from, int to, T forward_cap, T backward_cap) {
40         assert(0 <= from && from < n && 0 <= to && to < n);
41         g[from].push_back((int) edges.size());
42         edges.push_back({from, to, forward_cap, 0});
43         g[to].push_back((int) edges.size());
44         edges.push_back({to, from, backward_cap, 0});
45     }
46
47     bool expath() {
48         fill(d.begin(), d.end(), -1);
49         q[0] = fin;
50         d[fin] = 0;
51         int beg = 0, end = 1;
52         while (beg < end) {
53             int i = q[beg++];
54             for (int id : g[i]) {
55                 const edge &e = edges[id];
56                 const edge &back = edges[id ^ 1];
57                 if (back.c - back.f > eps && d[e.to] == -1) {
58                     d[e.to] = d[i] + 1;
59                     if (e.to == st) {
60                         return true;
61                     }
62                     q[end++] = e.to;
63                 }
64             }
65         }
66         return false;
67     }
68
69     T dfs(int v, T w) {
70         if (v == fin) {
71             return w;
72         }

```

```

73     int &j = ptr[v];
74     while (j >= 0) {
75         int id = g[v][j];
76         const edge &e = edges[id];
77         if (e.c - e.f > eps && d[e.to] == d[v] - 1) {
78             T t = dfs(e.to, min(e.c - e.f, w));
79             if (t > eps) {
80                 edges[id].f += t;
81                 edges[id ^ 1].f -= t;
82                 return t;
83             }
84         }
85         j--;
86     }
87     return 0;
88 }
89
90 T max_flow() {
91     while (expath()) {
92         for (int i = 0; i < n; i++) {
93             ptr[i] = (int) g[i].size() - 1;
94         }
95         T big_add = 0;
96         while (true) {
97             T add = dfs(st, numeric_limits<T>::max());
98             if (add <= eps) {
99                 break;
100             }
101             big_add += add;
102         }
103         if (big_add <= eps) {
104             break;
105         }
106         flow += big_add;
107     }
108     return flow;
109 }
110
111 vector<bool> min_cut() {
112     max_flow();
113     vector<bool> ret(n);
114     for (int i = 0; i < n; i++) {
115         ret[i] = (d[i] != -1);

```

```

116     }
117     return ret;
118 }
119
120 // Maximum flow / minimum cut, Dinic's algorithm
121 // Usage:
122 // 1) flow_graph<T> g(n, start, finish); [T == int / long long / double]
123 // 2) g.add(from, to, forward_cap, backward_cap);
124 // 3) cout << g.max_flow() << endl;
125 // 4) vector<bool> cut = g.min_cut();
126 //     for (auto &e : g.edges)
127 //         if (cut[e.from] != cut[e.to]) ; // edge e = (e.from -> e.to) is
128 //         cut
129 };

```

2.3 dinic-old.cpp

```

1  template <typename T>
2  class flow_graph {
3  public:
4      static constexpr T eps = (T) 1e-9;
5
6      struct edge {
7          int to;
8          T c;
9          T f;
10         int rev;
11     };
12
13     vector<vector<edge>> g;
14     vector<int> ptr;
15     vector<int> d;
16     vector<int> q;
17     int n;
18     int st, fin;
19     T flow;
20
21     flow_graph(int _n, int _st, int _fin) : n(_n), st(_st), fin(_fin) {
22         assert(0 <= st && st < n && 0 <= fin && fin < n && st != fin);
23         g.resize(n);
24         ptr.resize(n);
25         d.resize(n);
26         q.resize(n);

```

```

27     flow = 0;
28 }
29
30 void clear_flow() {
31     for (int i = 0; i < n; i++) {
32         for (edge &e : g[i]) {
33             e.f = 0;
34         }
35     }
36     flow = 0;
37 }
38
39 void add(int from, int to, T forward_cap, T backward_cap) {
40     assert(0 <= from && from < n && 0 <= to && to < n);
41     int from_size = g[from].size();
42     int to_size = g[to].size();
43     g[from].push_back({to, forward_cap, 0, to_size});
44     g[to].push_back({from, backward_cap, 0, from_size});
45 }
46
47 bool expath() {
48     fill(d.begin(), d.end(), -1);
49     q[0] = fin;
50     d[fin] = 0;
51     int beg = 0, end = 1;
52     while (beg < end) {
53         int i = q[beg++];
54         for (const edge &e : g[i]) {
55             const edge &back = g[e.to][e.rev];
56             if (back.c - back.f > eps && d[e.to] == -1) {
57                 d[e.to] = d[i] + 1;
58                 if (e.to == st) {
59                     return true;
60                 }
61                 q[end++] = e.to;
62             }
63         }
64     }
65     return false;
66 }
67
68 T dfs(int v, T w) {
69     if (v == fin) {

```

```

70         return w;
71     }
72     int &j = ptr[v];
73     while (j >= 0) {
74         const edge &e = g[v][j];
75         if (e.c - e.f > eps && d[e.to] == d[v] - 1) {
76             T t = dfs(e.to, min(e.c - e.f, w));
77             if (t > eps) {
78                 g[v][j].f += t;
79                 g[e.to][e.rev].f -= t;
80                 return t;
81             }
82         }
83         j--;
84     }
85     return 0;
86 }
87
88 T max_flow() {
89     while (expath()) {
90         for (int i = 0; i < n; i++) {
91             ptr[i] = (int) g[i].size() - 1;
92         }
93         T big_add = 0;
94         while (true) {
95             T add = dfs(st, numeric_limits<T>::max());
96             if (add <= eps) {
97                 break;
98             }
99             big_add += add;
100         }
101         if (big_add <= eps) {
102             break;
103         }
104         flow += big_add;
105     }
106     return flow;
107 }
108
109 vector<bool> min_cut() {
110     max_flow();
111     vector<bool> ret(n);
112     for (int i = 0; i < n; i++) {

```

```

113     ret[i] = (d[i] != -1);
114 }
115 return ret;
116 }
117 };

```

2.4 dinic.cpp

```

1  template <typename T>
2  class dinic {
3  public:
4      flow_graph<T> &g;
5
6      vector<int> ptr;
7      vector<int> d;
8      vector<int> q;
9
10     dinic(flow_graph<T> &_g) : g(_g) {
11         ptr.resize(g.n);
12         d.resize(g.n);
13         q.resize(g.n);
14     }
15
16     bool expath() {
17         fill(d.begin(), d.end(), -1);
18         q[0] = g.fin;
19         d[g.fin] = 0;
20         int beg = 0, end = 1;
21         while (beg < end) {
22             int i = q[beg++];
23             for (int id : g.g[i]) {
24                 const auto &e = g.edges[id];
25                 const auto &back = g.edges[id ^ 1];
26                 if (back.c - back.f > g.eps && d[e.to] == -1) {
27                     d[e.to] = d[i] + 1;
28                     if (e.to == g.st) {
29                         return true;
30                     }
31                     q[end++] = e.to;
32                 }
33             }
34         }
35         return false;

```

```

36     }
37
38     T dfs(int v, T w) {
39         if (v == g.fin) {
40             return w;
41         }
42         int &j = ptr[v];
43         while (j >= 0) {
44             int id = g.g[v][j];
45             const auto &e = g.edges[id];
46             if (e.c - e.f > g.eps && d[e.to] == d[v] - 1) {
47                 T t = dfs(e.to, min(e.c - e.f, w));
48                 if (t > g.eps) {
49                     g.edges[id].f += t;
50                     g.edges[id ^ 1].f -= t;
51                     return t;
52                 }
53             }
54             j--;
55         }
56         return 0;
57     }
58
59     T max_flow() {
60         while (expath()) {
61             for (int i = 0; i < g.n; i++) {
62                 ptr[i] = (int) g.g[i].size() - 1;
63             }
64             T big_add = 0;
65             while (true) {
66                 T add = dfs(g.st, numeric_limits<T>::max());
67                 if (add <= g.eps) {
68                     break;
69                 }
70                 big_add += add;
71             }
72             if (big_add <= g.eps) {
73                 break;
74             }
75             g.flow += big_add;
76         }
77         return g.flow;
78     }

```

```

79
80     vector<bool> min_cut() {
81         max_flow();
82         vector<bool> ret(g.n);
83         for (int i = 0; i < g.n; i++) {
84             ret[i] = (d[i] != -1);
85         }
86         return ret;
87     }
88 };

```

2.5 fastflow-other.cpp

```

1  // https://pastebin.com/exQM152L
2
3  // Doesn't walk through the whole path during augment at the cost of bigger
   constant
4  // Not recommended to use with double
5
6  template <typename T>
7  class flow_graph {
8  public:
9      static constexpr T eps = (T) 1e-9;
10
11     struct edge {
12         int to;
13         T c;
14         T f;
15         int rev;
16     };
17
18     vector<vector<edge>> g;
19     vector<int> ptr;
20     vector<int> d;
21     vector<int> q;
22     vector<int> cnt_on_layer;
23     vector<int> prev_edge;
24     vector<T> to_push;
25     vector<T> pushed;
26     vector<int> smallest;
27     bool can_reach_sink;
28
29     int n;

```

```

30     int st, fin;
31     T flow;
32
33     flow_graph(int _n, int _st, int _fin) : n(_n), st(_st), fin(_fin) {
34         assert(0 <= st && st < n && 0 <= fin && fin < n && st != fin);
35         g.resize(n);
36         ptr.resize(n);
37         d.resize(n);
38         q.resize(n);
39         cnt_on_layer.resize(n + 1);
40         prev_edge.resize(n);
41         to_push.resize(n);
42         pushed.resize(n);
43         smallest.resize(n);
44         flow = 0;
45     }
46
47     void clear_flow() {
48         for (int i = 0; i < n; i++) {
49             for (edge &e : g[i]) {
50                 e.f = 0;
51             }
52         }
53         flow = 0;
54     }
55
56     void add(int from, int to, T forward_cap, T backward_cap) {
57         assert(0 <= from && from < n && 0 <= to && to < n);
58         int from_size = g[from].size();
59         int to_size = g[to].size();
60         g[from].push_back({to, forward_cap, 0, to_size});
61         g[to].push_back({from, backward_cap, 0, from_size});
62     }
63
64     bool expath() {
65         fill(d.begin(), d.end(), n);
66         q[0] = fin;
67         d[fin] = 0;
68         fill(cnt_on_layer.begin(), cnt_on_layer.end(), 0);
69         cnt_on_layer[n] = n - 1;
70         cnt_on_layer[0] = 1;
71         int beg = 0, end = 1;
72         while (beg < end) {

```

```

73     int i = q[beg++];
74     for (const edge &e : g[i]) {
75         const edge &back = g[e.to][e.rev];
76         if (back.c - back.f > eps && d[e.to] == n) {
77             cnt_on_layer[d[e.to]]--;
78             d[e.to] = d[i] + 1;
79             cnt_on_layer[d[e.to]]++;
80             q[end++] = e.to;
81         }
82     }
83 }
84 return (d[st] != n);
85 }
86
87 void rollback(int &v) {
88     edge &e = g[v][prev_edge[v]];
89     if (pushed[v]) {
90         edge &back = g[e.to][e.rev];
91         back.f += pushed[v];
92         e.f -= pushed[v];
93         pushed[e.to] += pushed[v];
94         to_push[e.to] -= pushed[v];
95         pushed[v] = 0;
96     }
97     v = e.to;
98 }
99
100 void augment(int &v) {
101     pushed[v] += to_push[v];
102     to_push[v] = 0;
103     int new_v = smallest[v];
104     while (v != new_v) {
105         rollback(v);
106     }
107 }
108
109 void retreat(int &v) {
110     int new_dist = n - 1;
111     for (const edge &e : g[v]) {
112         if (e.c - e.f > eps && d[e.to] < new_dist) {
113             new_dist = d[e.to];
114         }
115     }

```

```

116     cnt_on_layer[d[v]]--;
117     if (cnt_on_layer[d[v]] == 0) {
118         if (new_dist + 1 > d[v]) {
119             can_reach_sink = false;
120         }
121     }
122     d[v] = new_dist + 1;
123     cnt_on_layer[d[v]]++;
124     if (v != st) {
125         rollback(v);
126     }
127 }
128
129 T max_flow() {
130     can_reach_sink = true;
131     for (int i = 0; i < n; i++) {
132         ptr[i] = (int) g[i].size() - 1;
133     }
134     if (expath()) {
135         int v = st;
136         to_push[v] = numeric_limits<T>::max();
137         smallest[v] = v;
138         while (d[st] < n) {
139             while (ptr[v] >= 0) {
140                 const edge &e = g[v][ptr[v]];
141                 if (e.c - e.f > eps && d[e.to] == d[v] - 1) {
142                     prev_edge[e.to] = e.rev;
143                     to_push[e.to] = to_push[v];
144                     smallest[e.to] = smallest[v];
145                     if (e.c - e.f < to_push[e.to]) {
146                         to_push[e.to] = e.c - e.f;
147                         smallest[e.to] = v;
148                     }
149                     v = e.to;
150                     if (v == fin) {
151                         augment(v);
152                     }
153                     break;
154                 }
155                 ptr[v]--;
156             }
157             if (ptr[v] < 0) {
158                 ptr[v] = (int) g[v].size() - 1;

```

```

159         retreat(v);
160         if (!can_reach_sink) {
161             break;
162         }
163     }
164 }
165 while (v != st) {
166     rollback(v);
167 }
168 flow += pushed[st];
169 pushed[st] = 0;
170 }
171 return flow;
172 }
173
174 vector<bool> min_cut() {
175     max_flow();
176     assert(!expath());
177     vector<bool> ret(n);
178     for (int i = 0; i < n; i++) {
179         ret[i] = (d[i] != n);
180     }
181     return ret;
182 }
183 };

```

2.6 fastflow.cpp

```

1 // https://pastebin.com/exQM152L
2
3 template <typename T>
4 class flow_graph {
5 public:
6     static constexpr T eps = (T) 1e-9;
7
8     struct edge {
9         int to;
10        T c;
11        T f;
12        int rev;
13    };
14
15     vector<vector<edge>> g;

```

```

16     vector<int> ptr;
17     vector<int> d;
18     vector<int> q;
19     vector<int> cnt_on_layer;
20     vector<int> prev_edge;
21     bool can_reach_sink;
22
23     int n;
24     int st, fin;
25     T flow;
26
27     flow_graph(int _n, int _st, int _fin) : n(_n), st(_st), fin(_fin) {
28         assert(0 <= st && st < n && 0 <= fin && fin < n && st != fin);
29         g.resize(n);
30         ptr.resize(n);
31         d.resize(n);
32         q.resize(n);
33         cnt_on_layer.resize(n + 1);
34         prev_edge.resize(n);
35         flow = 0;
36     }
37
38     void clear_flow() {
39         for (int i = 0; i < n; i++) {
40             for (edge &e : g[i]) {
41                 e.f = 0;
42             }
43         }
44         flow = 0;
45     }
46
47     void add(int from, int to, T forward_cap, T backward_cap) {
48         assert(0 <= from && from < n && 0 <= to && to < n);
49         int from_size = g[from].size();
50         int to_size = g[to].size();
51         g[from].push_back({to, forward_cap, 0, to_size});
52         g[to].push_back({from, backward_cap, 0, from_size});
53     }
54
55     bool expath() {
56         fill(d.begin(), d.end(), n);
57         q[0] = fin;
58         d[fin] = 0;

```



```

59     fill(cnt_on_layer.begin(), cnt_on_layer.end(), 0);
60     cnt_on_layer[n] = n - 1;
61     cnt_on_layer[0] = 1;
62     int beg = 0, end = 1;
63     while (beg < end) {
64         int i = q[beg++];
65         for (const edge &e : g[i]) {
66             const edge &back = g[e.to][e.rev];
67             if (back.c - back.f > eps && d[e.to] == n) {
68                 cnt_on_layer[d[e.to]]--;
69                 d[e.to] = d[i] + 1;
70                 cnt_on_layer[d[e.to]]++;
71                 q[end++] = e.to;
72             }
73         }
74     }
75     return (d[st] != n);
76 }
77
78 T augment(int &v) {
79     T cur = numeric_limits<T>::max();
80     int i = fin;
81     while (i != st) {
82         const edge &e = g[i][prev_edge[i]];
83         const edge &back = g[e.to][e.rev];
84         cur = min(cur, back.c - back.f);
85         i = e.to;
86     }
87     i = fin;
88     while (i != st) {
89         edge &e = g[i][prev_edge[i]];
90         edge &back = g[e.to][e.rev];
91         back.f += cur;
92         e.f -= cur;
93         i = e.to;
94         if (back.c - back.f <= eps) {
95             v = i;
96         }
97     }
98     return cur;
99 }
100
101 int retreat(int v) {

```

```

102     int new_dist = n - 1;
103     for (const edge &e : g[v]) {
104         if (e.c - e.f > eps && d[e.to] < new_dist) {
105             new_dist = d[e.to];
106         }
107     }
108     cnt_on_layer[d[v]]--;
109     if (cnt_on_layer[d[v]] == 0) {
110         if (new_dist + 1 > d[v]) {
111             can_reach_sink = false;
112         }
113     }
114     d[v] = new_dist + 1;
115     cnt_on_layer[d[v]]++;
116     if (v != st) {
117         v = g[v][prev_edge[v]].to;
118     }
119     return v;
120 }
121
122 T max_flow() {
123     can_reach_sink = true;
124     for (int i = 0; i < n; i++) {
125         ptr[i] = (int) g[i].size() - 1;
126     }
127     if (expath()) {
128         int v = st;
129         while (d[st] < n) {
130             while (ptr[v] >= 0) {
131                 const edge &e = g[v][ptr[v]];
132                 if (e.c - e.f > eps && d[e.to] == d[v] - 1) {
133                     prev_edge[e.to] = e.rev;
134                     v = e.to;
135                     if (v == fin) {
136                         flow += augment(v);
137                     }
138                     break;
139                 }
140                 ptr[v]--;
141             }
142             if (ptr[v] < 0) {
143                 ptr[v] = (int) g[v].size() - 1;
144                 v = retreat(v);

```

```

145         if (!can_reach_sink) {
146             break;
147         }
148     }
149 }
150 }
151 return flow;
152 }
153
154 vector<bool> min_cut() {
155     max_flow();
156     assert(!expath());
157     vector<bool> ret(n);
158     for (int i = 0; i < n; i++) {
159         ret[i] = (d[i] != n);
160     }
161     return ret;
162 }
163 };

```

2.7 flow-decomposition.cpp

```

1 template <typename T>
2 class flow_decomposition {
3 public:
4     const flow_graph<T> &g;
5
6     vector<vector<int>>> paths;
7     vector<T> path_flows;
8     vector<vector<int>>> cycles;
9     vector<T> cycle_flows;
10
11     flow_decomposition(const flow_graph<T> &_g) : g(_g) {
12     }
13
14     void decompose() {
15         vector<T> fs(g.edges.size());
16         for (int i = 0; i < (int) g.edges.size(); i++) {
17             fs[i] = g.edges[i].f;
18         }
19         paths.clear();
20         path_flows.clear();
21         cycles.clear();

```

```

22     cycle_flows.clear();
23     vector<int> ptr(g.n);
24     for (int i = 0; i < g.n; i++) {
25         ptr[i] = (int) g.g[i].size() - 1;
26     }
27     vector<int> was(g.n, -1);
28     int start = g.st;
29     for (int iter = 0; ; iter++) {
30         bool found_start = false;
31         while (true) {
32             if (ptr[start] >= 0) {
33                 int id = g.g[start][ptr[start]];
34                 if (fs[id] > g.eps) {
35                     found_start = true;
36                     break;
37                 }
38                 ptr[start]--;
39                 continue;
40             }
41             start = (start + 1) % g.n;
42             if (start == g.st) {
43                 break;
44             }
45         }
46         if (!found_start) {
47             break;
48         }
49         vector<int> path;
50         bool is_cycle = false;
51         int v = start;
52         while (true) {
53             if (v == g.fin) {
54                 break;
55             }
56             if (was[v] == iter) {
57                 bool found = false;
58                 for (int i = 0; i < (int) path.size(); i++) {
59                     int id = path[i];
60                     auto &e = g.edges[id];
61                     if (e.from == v) {
62                         path.erase(path.begin(), path.begin() + i);
63                         found = true;
64                         break;

```

```

65     }
66 }
67 assert(found);
68 is_cycle = true;
69 break;
70 }
71 was[v] = iter;
72 bool found = false;
73 while (ptr[v] >= 0) {
74     int id = g.g[v][ptr[v]];
75     if (fs[id] > g.eps) {
76         path.push_back(id);
77         v = g.edges[id].to;
78         found = true;
79         break;
80     }
81     ptr[v]--;
82 }
83 assert(found);
84 }
85 T path_flow = numeric_limits<T>::max();
86 for (int id : path) {
87     path_flow = min(path_flow, fs[id]);
88 }
89 for (int id : path) {
90     fs[id] -= path_flow;
91     fs[id ^ 1] += path_flow;
92 }
93 if (is_cycle) {
94     cycles.push_back(path);
95     cycle_flows.push_back(path_flow);
96 } else {
97     paths.push_back(path);
98     path_flows.push_back(path_flow);
99 }
100 }
101 for (const T& f : fs) {
102     assert(-g.eps <= f && f <= g.eps);
103 }
104 }
105 };

```

2.8 flow-graph.cpp

```

1  template <typename T>
2  class flow_graph {
3  public:
4      static constexpr T eps = (T) 1e-9;
5
6      struct edge {
7          int from;
8          int to;
9          T c;
10         T f;
11     };
12
13     vector<vector<int>> g;
14     vector<edge> edges;
15     int n;
16     int st;
17     int fin;
18     T flow;
19
20     flow_graph(int _n, int _st, int _fin) : n(_n), st(_st), fin(_fin) {
21         assert(0 <= st && st < n && 0 <= fin && fin < n && st != fin);
22         g.resize(n);
23         flow = 0;
24     }
25
26     void clear_flow() {
27         for (const edge &e : edges) {
28             e.f = 0;
29         }
30         flow = 0;
31     }
32
33     int add(int from, int to, T forward_cap, T backward_cap) {
34         assert(0 <= from && from < n && 0 <= to && to < n);
35         int id = (int) edges.size();
36         g[from].push_back(id);
37         edges.push_back({from, to, forward_cap, 0});
38         g[to].push_back(id + 1);
39         edges.push_back({to, from, backward_cap, 0});
40         return id;
41     }
42 };

```

2.9 gomory-hu-old.cpp

```
1  template <typename T>
2  forest<T> gomory_hu(const undigraph<T> &g) {
3      int n = g.n;
4      if (n == 1) {
5          return forest<T>(n);
6      }
7      flow_graph<T> fg(n, 0, 1);
8      for (auto &e : g.edges) {
9          fg.add(e.from, e.to, e.cost, e.cost);
10     }
11     vector<vector<int>> dist(n, vector<int>(n, numeric_limits<T>::max()));
12     function<void(vector<int>>> dfs = [&g, &n, &fg, &dist, &dfs](vector<int>
        group) {
13         int sz = group.size();
14         if (sz == 1) {
15             return;
16         }
17         fg.clear_flow();
18         fg.st = group[0];
19         fg.fin = group[1];
20         T flow = fg.max_flow();
21         vector<bool> cut = fg.min_cut();
22         for (int i = 0; i < n; i++) {
23             for (int j = i + 1; j < n; j++) {
24                 if (cut[i] != cut[j]) {
25                     dist[i][j] = min(dist[i][j], flow);
26                 }
27             }
28         }
29         vector<int> new_groups[2];
30         for (int v : group) {
31             new_groups[(int) cut[v]].push_back(v);
32         }
33         for (int id = 0; id < 2; id++) {
34             dfs(new_groups[id]);
35         }
36     };
37     vector<int> group(n);
38     iota(group.begin(), group.end(), 0);
39     dfs(group);
40     undigraph<T> mg(n);
41     for (int i = 0; i < n; i++) {
```

```
42         for (int j = i + 1; j < n; j++) {
43             mg.add(i, j, -dist[i][j]);
44         }
45     }
46     T foo;
47     vector<int> ids = mst(mg, foo);
48     forest<T> ret(n);
49     for (int id : ids) {
50         auto &e = mg.edges[id];
51         ret.add(e.from, e.to, -e.cost);
52     }
53     return ret;
54     // don't be lazy next time!
55     // implement a proper gomory-hu tree
56 }
```

2.10 gomory-hu.cpp

```
1  template <typename T>
2  forest<T> gomory_hu(const undigraph<T>& g) {
3      int n = g.n;
4      flow_graph<T> fg(n, 0, 1);
5      for (auto& e : g.edges) {
6          fg.add(e.from, e.to, e.cost, e.cost);
7      }
8      forest<T> ret(n);
9      vector<int> pr(n, 0);
10     for (int i = 1; i < n; i++) {
11         fg.clear_flow();
12         fg.st = i;
13         fg.fin = pr[i];
14         T flow = fg.max_flow();
15         vector<bool> cut = fg.min_cut();
16         for (int j = i + 1; j < n; j++) {
17             if (cut[j] == cut[i] && pr[j] == pr[i]) {
18                 pr[j] = i;
19             }
20         }
21         ret.add(i, pr[i], flow);
22     }
23     return ret;
24     // can be optimized by compressing components
25 }
```

2.11 hungarian-arrays.cpp

```
1 template <typename T>
2 class hungarian {
3 public:
4     static const int MAX_N = ... + 1;
5
6     int n;
7     int m;
8     T a[MAX_N][MAX_N];
9     T u[MAX_N];
10    T v[MAX_N];
11    int pa[MAX_N];
12    int pb[MAX_N];
13    int way[MAX_N];
14    T minv[MAX_N];
15    bool used[MAX_N];
16    T inf;
17
18    hungarian(int _n, int _m) : n(_n), m(_m) {
19        assert(n <= m);
20        T zero = T{};
21        fill(u, u + n + 1, zero);
22        fill(v, v + m + 1, zero);
23        fill(pa, pa + n + 1, -1);
24        fill(pb, pb + m + 1, -1);
25        inf = numeric_limits<T>::max();
26    }
27
28    inline void add_row(int i) {
29        fill(minv, minv + m + 1, inf);
30        fill(used, used + m + 1, false);
31        pb[m] = i;
32        pa[i] = m;
33        int j0 = m;
34        do {
35            used[j0] = true;
36            int i0 = pb[j0];
37            T delta = inf;
38            int j1 = -1;
39            for (int j = 0; j < m; j++) {
40                if (!used[j]) {
41                    T cur = a[i0][j] - u[i0] - v[j];
42                    if (cur < minv[j]) {
```

```
43                        minv[j] = cur;
44                        way[j] = j0;
45                    }
46                    if (minv[j] < delta) {
47                        delta = minv[j];
48                        j1 = j;
49                    }
50                }
51            }
52            for (int j = 0; j <= m; j++) {
53                if (used[j]) {
54                    u[pb[j]] += delta;
55                    v[j] -= delta;
56                } else {
57                    minv[j] -= delta;
58                }
59            }
60            j0 = j1;
61        } while (pb[j0] != -1);
62        do {
63            int j1 = way[j0];
64            pb[j0] = pb[j1];
65            pa[pb[j0]] = j0;
66            j0 = j1;
67        } while (j0 != m);
68    }
69
70    inline T current_score() {
71        return -v[m];
72    }
73
74    inline T solve() {
75        for (int i = 0; i < n; i++) {
76            add_row(i);
77        }
78        return current_score();
79    }
80 };
```

2.12 hungarian.cpp

```
1 template <typename T>
2 class hungarian {
```

```

3 public:
4     int n;
5     int m;
6     vector<vector<T>> a;
7     vector<T> u;
8     vector<T> v;
9     vector<int> pa;
10    vector<int> pb;
11    vector<int> way;
12    vector<T> minv;
13    vector<bool> used;
14    T inf;
15
16    hungarian(int _n, int _m) : n(_n), m(_m) {
17        assert(n <= m);
18        a = vector<vector<T>>(n, vector<T>(m));
19        u = vector<T>(n + 1);
20        v = vector<T>(m + 1);
21        pa = vector<int>(n + 1, -1);
22        pb = vector<int>(m + 1, -1);
23        way = vector<int>(m, -1);
24        minv = vector<T>(m);
25        used = vector<bool>(m + 1);
26        inf = numeric_limits<T>::max();
27    }
28
29    inline void add_row(int i) {
30        fill(minv.begin(), minv.end(), inf);
31        fill(used.begin(), used.end(), false);
32        pb[m] = i;
33        pa[i] = m;
34        int j0 = m;
35        do {
36            used[j0] = true;
37            int i0 = pb[j0];
38            T delta = inf;
39            int j1 = -1;
40            for (int j = 0; j < m; j++) {
41                if (!used[j]) {
42                    T cur = a[i0][j] - u[i0] - v[j];
43                    if (cur < minv[j]) {
44                        minv[j] = cur;
45                        way[j] = j0;

```

```

46                }
47                if (minv[j] < delta) {
48                    delta = minv[j];
49                    j1 = j;
50                }
51            }
52        }
53        for (int j = 0; j <= m; j++) {
54            if (used[j]) {
55                u[pb[j]] += delta;
56                v[j] -= delta;
57            } else {
58                minv[j] -= delta;
59            }
60        }
61        j0 = j1;
62    } while (pb[j0] != -1);
63    do {
64        int j1 = way[j0];
65        pb[j0] = pb[j1];
66        pa[pb[j0]] = j0;
67        j0 = j1;
68    } while (j0 != m);
69    }
70
71    inline T current_score() {
72        return -v[m];
73    }
74
75    inline T solve() {
76        for (int i = 0; i < n; i++) {
77            add_row(i);
78        }
79        return current_score();
80    }
81 };

```

2.13 matching.cpp

```

1 class matching {
2 public:
3     vector<vector<int>> g;
4     vector<int> pa;

```

```

5  vector<int> pb;
6  vector<int> was;
7  int n, m;
8  int res;
9  int iter;
10
11 matching(int _n, int _m) : n(_n), m(_m) {
12     assert(0 <= n && 0 <= m);
13     pa = vector<int>(n, -1);
14     pb = vector<int>(m, -1);
15     was = vector<int>(n, 0);
16     g.resize(n);
17     res = 0;
18     iter = 0;
19 }
20
21 void add(int from, int to) {
22     assert(0 <= from && from < n && 0 <= to && to < m);
23     g[from].push_back(to);
24 }
25
26 bool dfs(int v) {
27     was[v] = iter;
28     for (int u : g[v]) {
29         if (pb[u] == -1) {
30             pa[v] = u;
31             pb[u] = v;
32             return true;
33         }
34     }
35     for (int u : g[v]) {
36         if (was[pb[u]] != iter && dfs(pb[u])) {
37             pa[v] = u;
38             pb[u] = v;
39             return true;
40         }
41     }
42     return false;
43 }
44
45 int solve() {
46     while (true) {
47         iter++;

```

```

48         int add = 0;
49         for (int i = 0; i < n; i++) {
50             if (pa[i] == -1 && dfs(i)) {
51                 add++;
52             }
53         }
54         if (add == 0) {
55             break;
56         }
57         res += add;
58     }
59     return res;
60 }
61
62 int run_one(int v) {
63     if (pa[v] != -1) {
64         return 0;
65     }
66     iter++;
67     return (int) dfs(v);
68 }
69 };

```

2.14 mcmf-slow.cpp

```

1  template <typename T, typename C>
2  class mcmf {
3  public:
4      static constexpr T eps = (T) 1e-9;
5
6      struct edge {
7          int from;
8          int to;
9          T c;
10         T f;
11         C cost;
12     };
13
14     vector<vector<int>>> g;
15     vector<edge> edges;
16     vector<C> d;
17     vector<int> q;
18     vector<bool> in_queue;

```

```

19     vector<int> pe;
20     int n;
21     int st, fin;
22     T flow;
23     C cost;
24
25     mcmf(int _n, int _st, int _fin) : n(_n), st(_st), fin(_fin) {
26         assert(0 <= st && st < n && 0 <= fin && fin < n && st != fin);
27         g.resize(n);
28         d.resize(n);
29         in_queue.resize(n);
30         pe.resize(n);
31         flow = 0;
32         cost = 0;
33     }
34
35     void clear_flow() {
36         for (const edge &e : edges) {
37             e.f = 0;
38         }
39         flow = 0;
40     }
41
42     void add(int from, int to, T forward_cap, T backward_cap, C cost) {
43         assert(0 <= from && from < n && 0 <= to && to < n);
44         g[from].push_back((int) edges.size());
45         edges.push_back({from, to, forward_cap, 0, cost});
46         g[to].push_back((int) edges.size());
47         edges.push_back({to, from, backward_cap, 0, -cost});
48     }
49
50     bool expath() {
51         fill(d.begin(), d.end(), numeric_limits<C>::max());
52         q.clear();
53         q.push_back(st);
54         d[st] = 0;
55         in_queue[st] = true;
56         int beg = 0;
57         bool found = false;
58         while (beg < (int) q.size()) {
59             int i = q[beg++];
60             if (i == fin) {
61                 found = true;

```

```

62         }
63         in_queue[i] = false;
64         for (int id : g[i]) {
65             const edge &e = edges[id];
66             if (e.c - e.f > eps && d[i] + e.cost < d[e.to]) {
67                 d[e.to] = d[i] + e.cost;
68                 pe[e.to] = id;
69                 if (!in_queue[e.to]) {
70                     q.push_back(e.to);
71                     in_queue[e.to] = true;
72                 }
73             }
74         }
75     }
76     if (found) {
77         T push = numeric_limits<T>::max();
78         int v = fin;
79         while (v != st) {
80             const edge &e = edges[pe[v]];
81             push = min(push, e.c - e.f);
82             v = e.from;
83         }
84         v = fin;
85         while (v != st) {
86             edge &e = edges[pe[v]];
87             e.f += push;
88             edge &back = edges[pe[v] ^ 1];
89             back.f -= push;
90             v = e.from;
91         }
92         flow += push;
93         cost += push * d[fin];
94     }
95     return found;
96 }
97
98 pair<T, C> max_flow_min_cost() {
99     while (expath()) {}
100     return {flow, cost};
101 }
102 };

```


2.15 mcmf.cpp

```
1 #include <bits/extc++.h>
2
3 template <typename T, typename C>
4 class MCMF {
5 public:
6     static constexpr T eps = (T) 1e-9;
7
8     struct edge {
9         int from;
10        int to;
11        T c;
12        T f;
13        C cost;
14    };
15
16    int n;
17    vector<vector<int>>> g;
18    vector<edge> edges;
19    vector<C> d;
20    vector<C> pot;
21    __gnu_pbds::priority_queue<pair<C, int>> q;
22    vector<typename decltype(q)::point_iterator> its;
23    vector<int> pe;
24    const C INF_C = numeric_limits<C>::max() / 2;
25
26    explicit MCMF(int n_) : n(n_), g(n), d(n), pot(n, 0), its(n), pe(n) {}
27
28    int add(int from, int to, T forward_cap, T backward_cap, C edge_cost) {
29        assert(0 <= from && from < n && 0 <= to && to < n);
30        assert(forward_cap >= 0 && backward_cap >= 0);
31        int id = static_cast<int>(edges.size());
32        g[from].push_back(id);
33        edges.push_back({from, to, forward_cap, 0, edge_cost});
34        g[to].push_back(id + 1);
35        edges.push_back({to, from, backward_cap, 0, -edge_cost});
36        return id;
37    }
38
39    void expath(int st) {
40        fill(d.begin(), d.end(), INF_C);
41        q.clear();
42        fill(its.begin(), its.end(), q.end());
```

```
43        its[st] = q.push({pot[st], st});
44        d[st] = 0;
45        while (!q.empty()) {
46            int i = q.top().second;
47            q.pop();
48            its[i] = q.end();
49            for (int id : g[i]) {
50                const edge &e = edges[id];
51                int j = e.to;
52                if (e.c - e.f > eps && d[i] + e.cost < d[j]) {
53                    d[j] = d[i] + e.cost;
54                    pe[j] = id;
55                    if (its[j] == q.end()) {
56                        its[j] = q.push({pot[j] - d[j], j});
57                    } else {
58                        q.modify(its[j], {pot[j] - d[j], j});
59                    }
60                }
61            }
62        }
63        swap(d, pot);
64    }
65
66    pair<T, C> max_flow_min_cost(int st, int fin) {
67        T flow = 0;
68        C cost = 0;
69        bool ok = true;
70        for (auto& e : edges) {
71            if (e.c - e.f > eps && e.cost + pot[e.from] - pot[e.to] < 0) {
72                ok = false;
73                break;
74            }
75        }
76        if (ok) {
77            expath(st);
78        } else {
79            vector<int> deg(n, 0);
80            for (int i = 0; i < n; i++) {
81                for (int eid : g[i]) {
82                    auto& e = edges[eid];
83                    if (e.c - e.f > eps) {
84                        deg[e.to] += 1;
85                    }
86                }
87            }
88        }
89    }
90}
```

```

86     }
87 }
88 vector<int> que;
89 for (int i = 0; i < n; i++) {
90     if (deg[i] == 0) {
91         que.push_back(i);
92     }
93 }
94 for (int b = 0; b < (int) que.size(); b++) {
95     for (int eid : g[que[b]]) {
96         auto& e = edges[eid];
97         if (e.c - e.f > eps) {
98             deg[e.to] -= 1;
99             if (deg[e.to] == 0) {
100                 que.push_back(e.to);
101             }
102         }
103     }
104 }
105 fill(pot.begin(), pot.end(), INF_C);
106 pot[st] = 0;
107 if (static_cast<int>(que.size()) == n) {
108     for (int v : que) {
109         if (pot[v] < INF_C) {
110             for (int eid : g[v]) {
111                 auto& e = edges[eid];
112                 if (e.c - e.f > eps) {
113                     if (pot[v] + e.cost < pot[e.to]) {
114                         pot[e.to] = pot[v] + e.cost;
115                         pe[e.to] = eid;
116                     }
117                 }
118             }
119         }
120     }
121 } else {
122     que.assign(1, st);
123     vector<bool> in_queue(n, false);
124     in_queue[st] = true;
125     for (int b = 0; b < (int) que.size(); b++) {
126         int i = que[b];
127         in_queue[i] = false;
128         for (int id : g[i]) {

```

```

129             const edge &e = edges[id];
130             if (e.c - e.f > eps && pot[i] + e.cost < pot[e.to]) {
131                 pot[e.to] = pot[i] + e.cost;
132                 pe[e.to] = id;
133                 if (!in_queue[e.to]) {
134                     que.push_back(e.to);
135                     in_queue[e.to] = true;
136                 }
137             }
138         }
139     }
140 }
141 }
142 while (pot[fin] < INF_C) {
143     T push = numeric_limits<T>::max();
144     int v = fin;
145     while (v != st) {
146         const edge &e = edges[pe[v]];
147         push = min(push, e.c - e.f);
148         v = e.from;
149     }
150     v = fin;
151     while (v != st) {
152         edge &e = edges[pe[v]];
153         e.f += push;
154         edge &back = edges[pe[v] ^ 1];
155         back.f -= push;
156         v = e.from;
157     }
158     flow += push;
159     cost += push * pot[fin];
160     expath(st);
161 }
162 return {flow, cost};
163 }
164 };

```

2.16 mincut.cpp

```

1 template <typename T>
2 pair<T, vector<bool>> MinCut(vector<vector<T>> g) {
3     int n = static_cast<int>(g.size());
4     for (int i = 0; i < n; i++) {

```

```

5     assert(static_cast<int>(g[i].size()) == n);
6 }
7 for (int i = 0; i < n; i++) {
8     for (int j = i + 1; j < n; j++) {
9         assert(g[i][j] == g[j][i]);
10    }
11 }
12 vector<vector<bool>> v(n, vector<bool>(n));
13 for (int i = 0; i < n; i++) {
14     v[i][i] = true;
15 }
16 vector<T> w(n);
17 vector<bool> exists(n, true);
18 vector<bool> in_a(n);
19 T best_cost = numeric_limits<T>::max();
20 vector<bool> best_cut;
21 for (int ph = 0; ph < n - 1; ph++) {
22     fill(in_a.begin(), in_a.end(), false);
23     fill(w.begin(), w.end(), T(0));
24     int prev = -1;
25     for (int it = 0; it < n - ph; it++) {
26         int sel = -1;
27         for (int i = 0; i < n; i++) {
28             if (exists[i] && !in_a[i] && (sel == -1 || w[i] > w[sel])) {
29                 sel = i;
30             }
31         }
32         if (it == n - ph - 1) {
33             if (w[sel] < best_cost) {
34                 best_cost = w[sel];
35                 best_cut = v[sel];
36             }
37             for (int i = 0; i < n; i++) {
38                 v[prev][i] = v[prev][i] | v[sel][i];
39                 g[prev][i] += g[sel][i];
40                 g[i][prev] += g[i][sel];
41             }
42             exists[sel] = false;
43             break;
44         }
45         in_a[sel] = true;
46         for (int i = 0; i < n; i++) {
47             w[i] += g[sel][i];

```

```

48         }
49         prev = sel;
50     }
51 }
52 return make_pair(best_cost, best_cut);
53 }

```

3 geometry

3.1 Point.cpp

```

1 template <typename T>
2 struct TPoint {
3     T x;
4     T y;
5     int id;
6
7     TPoint() : x(0), y(0), id(-1) {}
8     TPoint(const T& x_, const T& y_) : x(x_), y(y_), id(-1) {}
9     TPoint(const T& x_, const T& y_, int id_) : x(x_), y(y_), id(id_) {}
10
11     static constexpr T eps = static_cast<T>(1e-9);
12
13     inline TPoint operator+(const TPoint& rhs) const { return TPoint(x + rhs.x
14         , y + rhs.y); }
15     inline TPoint operator-(const TPoint& rhs) const { return TPoint(x - rhs.x
16         , y - rhs.y); }
17     inline TPoint operator*(const T& rhs) const { return TPoint(x * rhs, y *
18         rhs); }
19     inline TPoint operator/(const T& rhs) const { return TPoint(x / rhs, y /
20         rhs); }
21
22     friend T smul(const TPoint& a, const TPoint& b) {
23         return a.x * b.x + a.y * b.y;
24     }
25
26     friend T vmul(const TPoint& a, const TPoint& b) {
27         return a.x * b.y - a.y * b.x;
28     }
29
30     inline T abs2() const {
31         return x * x + y * y;
32     }

```

```

28     }
29
30     inline bool operator<(const TPoint& rhs) const {
31         return (y < rhs.y || (y == rhs.y && x < rhs.x));
32     }
33
34     inline bool is_upper() const {
35         return (y > eps || (abs(y) <= eps && x > eps));
36     }
37
38     inline int cmp_polar(const TPoint& rhs) const {
39         assert(abs(x) > eps || abs(y) > eps);
40         assert(abs(rhs.x) > eps || abs(rhs.y) > eps);
41         bool a = is_upper();
42         bool b = rhs.is_upper();
43         if (a != b) {
44             return (a ? -1 : 1);
45         }
46         long long v = x * rhs.y - y * rhs.x;
47         return (v > eps ? -1 : (v < -eps ? 1 : 0));
48     }
49 };
50
51 using Point = TPoint<long long>;
52 //using Point = TPoint<long double>;
53
54 template <typename T>
55 string to_string(const TPoint<T>& p) {
56     return "(" + to_string(p.x) + ", " + to_string(p.y) + ")";
57 }

```

4 graph

4.1 bicone.cpp

```

1 template <typename T>
2 vector<int> find_bicone(dfs_undigraph<T> &g, int &cnt) {
3     g.dfs_all();
4     vector<int> vertex_comp(g.n);
5     cnt = 0;
6     for (int i : g.order) {
7         if (g.pv[i] == -1 || g.min_depth[i] == g.depth[i]) {

```

```

8         vertex_comp[i] = cnt++;
9     } else {
10         vertex_comp[i] = vertex_comp[g.pv[i]];
11     }
12 }
13 return vertex_comp;
14 }

```

4.2 biconv.cpp

```

1 template <typename T>
2 vector<int> find_biconv(dfs_undigraph<T> &g, int &cnt) {
3     g.dfs_all();
4     vector<int> vertex_comp(g.n);
5     cnt = 0;
6     for (int i : g.order) {
7         if (g.pv[i] == -1) {
8             vertex_comp[i] = -1;
9             continue;
10        }
11        if (g.min_depth[i] >= g.depth[g.pv[i]]) {
12            vertex_comp[i] = cnt++;
13        } else {
14            vertex_comp[i] = vertex_comp[g.pv[i]];
15        }
16    }
17    vector<int> edge_comp(g.edges.size(), -1);
18    for (int id = 0; id < (int) g.edges.size(); id++) {
19        int x = g.edges[id].from;
20        int y = g.edges[id].to;
21        int z = (g.depth[x] > g.depth[y] ? x : y);
22        edge_comp[id] = vertex_comp[z];
23    }
24    return edge_comp;
25 }

```

4.3 bridges.cpp

```

1 template <typename T>
2 vector<bool> find_bridges(dfs_undigraph<T> &g) {
3     g.dfs_all();
4     vector<bool> bridge(g.edges.size(), false);
5     for (int i = 0; i < g.n; i++) {

```

```

6     if (g.pv[i] != -1 && g.min_depth[i] == g.depth[i]) {
7         bridge[g.pe[i]] = true;
8     }
9 }
10 return bridge;
11 }

```

4.4 cutpoints.cpp

```

1 template <typename T>
2 vector<bool> find_cutpoints(dfs_undigraph<T> &g) {
3     g.dfs_all();
4     vector<bool> cutpoint(g.n, false);
5     for (int i = 0; i < g.n; i++) {
6         if (g.pv[i] != -1 && g.min_depth[i] >= g.depth[g.pv[i]]) {
7             cutpoint[g.pv[i]] = true;
8         }
9     }
10    vector<int> children(g.n, 0);
11    for (int i = 0; i < g.n; i++) {
12        if (g.pv[i] != -1) {
13            children[g.pv[i]]++;
14        }
15    }
16    for (int i = 0; i < g.n; i++) {
17        if (g.pv[i] == -1 && children[i] < 2) {
18            cutpoint[i] = false;
19        }
20    }
21    return cutpoint;
22 }

```

4.5 cycles.cpp

```

1 template <typename T>
2 vector<vector<int>> find_cycles(const graph<T> &g, int bound_cnt = 1 << 30,
3     int bound_size = 1 << 30) {
4     vector<int> was(g.n, -1);
5     vector<int> st;
6     vector<vector<int>> cycles;
7     int total_size = 0;
8     function<void(int, int)> dfs = [&](int v, int pe) {
9         if ((int) cycles.size() >= bound_cnt || total_size >= bound_size) {

```

```

9         return;
10    }
11    was[v] = (int) st.size();
12    for (int id : g.g[v]) {
13        if (id == pe) {
14            continue;
15        }
16        auto &e = g.edges[id];
17        int to = e.from ^ e.to ^ v;
18        if (was[to] >= 0) {
19            vector<int> cycle(1, id);
20            for (int j = was[to]; j < (int) st.size(); j++) {
21                cycle.push_back(st[j]);
22            }
23            cycles.push_back(cycle);
24            total_size += (int) cycle.size();
25            if ((int) cycles.size() >= bound_cnt || total_size >= bound_size) {
26                was[v] = -2;
27                return;
28            }
29            continue;
30        }
31        if (was[to] == -1) {
32            st.push_back(id);
33            dfs(to, id);
34            st.pop_back();
35        }
36    }
37    was[v] = -2;
38 };
39 for (int i = 0; i < g.n; i++) {
40     if (was[i] == -1) {
41         dfs(i, -1);
42     }
43 }
44 return cycles;
45 // cycles are given by edge ids, all cycles are simple
46 // breaks after getting bound_cnt cycles or total_size >= bound_size
47 // digraph: finds at least one cycle in every connected component (if not
48 broken)
49 // undigraph: finds cycle basis
50 }

```

```

51 template <typename T>
52 vector<int> edges_to_vertices(const graph<T> &g, const vector<int> &
    edge_cycle) {
53     int sz = (int) edge_cycle.size();
54     vector<int> vertex_cycle;
55     if (sz <= 2) {
56         vertex_cycle.push_back(g.edges[edge_cycle[0]].from);
57         if (sz == 2) {
58             vertex_cycle.push_back(g.edges[edge_cycle[0]].to);
59         }
60     } else {
61         for (int i = 0; i < sz; i++) {
62             int j = (i + 1) % sz;
63             auto &e = g.edges[edge_cycle[i]];
64             auto &other = g.edges[edge_cycle[j]];
65             if (other.from == e.from || other.to == e.from) {
66                 vertex_cycle.push_back(e.to);
67             } else {
68                 vertex_cycle.push_back(e.from);
69             }
70         }
71     }
72     return vertex_cycle;
73     // only for simple cycles!
74 }

```

4.6 dfs-digraph-useless.cpp

```

1 template <typename T>
2 class dfs_digraph : public digraph<T> {
3 public:
4     using digraph<T>::edges;
5     using digraph<T>::g;
6     using digraph<T>::n;
7
8     vector<int> pv;
9     vector<int> pe;
10    vector<int> order;
11    vector<int> pos;
12    vector<int> end;
13    vector<int> sz;
14    vector<int> root;
15    vector<int> depth;

```

```

16    vector<T> dist;
17
18    dfs_digraph(int _n) : digraph<T>(_n) {
19    }
20
21    void clear() {
22        pv.clear();
23        pe.clear();
24        order.clear();
25        pos.clear();
26        end.clear();
27        sz.clear();
28        root.clear();
29        depth.clear();
30        dist.clear();
31    }
32
33    void init() {
34        pv = vector<int>(n, -1);
35        pe = vector<int>(n, -1);
36        order.clear();
37        pos = vector<int>(n, -1);
38        end = vector<int>(n, -1);
39        sz = vector<int>(n, 0);
40        root = vector<int>(n, -1);
41        depth = vector<int>(n, -1);
42        dist = vector<T>(n);
43    }
44
45 private:
46    void do_dfs(int v) {
47        pos[v] = (int) order.size();
48        order.push_back(v);
49        sz[v] = 1;
50        for (int id : g[v]) {
51            if (id == pe[v]) {
52                continue;
53            }
54            auto &e = edges[id];
55            int to = e.from ^ e.to ^ v;
56            // well, this is controversial...
57            if (depth[to] != -1) {
58                continue;

```

```

59     }
60     depth[to] = depth[v] + 1;
61     dist[to] = dist[v] + e.cost;
62     pv[to] = v;
63     pe[to] = id;
64     root[to] = (root[v] != -1 ? root[v] : to);
65     do_dfs(to);
66     sz[v] += sz[to];
67 }
68 end[v] = (int) order.size() - 1;
69 }
70
71 void do_dfs_from(int v) {
72     depth[v] = 0;
73     dist[v] = T{};
74     root[v] = v;
75     pv[v] = pe[v] = -1;
76     do_dfs(v);
77 }
78
79 public:
80     int dfs_one_unsafe(int v) {
81         // run init() before this
82         // then run this with the required v's
83         do_dfs_from(v);
84         return v;
85     }
86
87     int dfs(int v) {
88         init();
89         do_dfs_from(v);
90         // assert((int) order.size() == n);
91         return v;
92     }
93
94     void dfs_many(const vector<int> &roots) {
95         init();
96         for (int v : roots) {
97             if (depth[v] == -1) {
98                 do_dfs_from(v);
99             }
100         }
101         // assert((int) order.size() == n);

```

```

102     }
103
104     vector<int> dfs_all() {
105         init();
106         vector<int> roots;
107         for (int v = 0; v < n; v++) {
108             if (depth[v] == -1) {
109                 roots.push_back(v);
110                 do_dfs_from(v);
111             }
112         }
113         assert((int) order.size() == n);
114         return roots;
115     }
116 };

```

4.7 dfs-forest.cpp

```

1  template <typename T>
2  class dfs_forest : public forest<T> {
3  public:
4      using forest<T>::edges;
5      using forest<T>::g;
6      using forest<T>::n;
7
8      vector<int> pv;
9      vector<int> pe;
10     vector<int> order;
11     vector<int> pos;
12     vector<int> end;
13     vector<int> sz;
14     vector<int> root;
15     vector<int> depth;
16     vector<T> dist;
17
18     dfs_forest(int _n) : forest<T>(_n) {
19     }
20
21     void init() {
22         pv = vector<int>(n, -1);
23         pe = vector<int>(n, -1);
24         order.clear();
25         pos = vector<int>(n, -1);

```

```

26     end = vector<int>(n, -1);
27     sz = vector<int>(n, 0);
28     root = vector<int>(n, -1);
29     depth = vector<int>(n, -1);
30     dist = vector<T>(n);
31 }
32
33 void clear() {
34     pv.clear();
35     pe.clear();
36     order.clear();
37     pos.clear();
38     end.clear();
39     sz.clear();
40     root.clear();
41     depth.clear();
42     dist.clear();
43 }
44
45 private:
46 void do_dfs(int v) {
47     pos[v] = (int) order.size();
48     order.push_back(v);
49     sz[v] = 1;
50     for (int id : g[v]) {
51         if (id == pe[v]) {
52             continue;
53         }
54         auto &e = edges[id];
55         int to = e.from ^ e.to ^ v;
56         depth[to] = depth[v] + 1;
57         dist[to] = dist[v] + e.cost;
58         pv[to] = v;
59         pe[to] = id;
60         root[to] = (root[v] != -1 ? root[v] : to);
61         do_dfs(to);
62         sz[v] += sz[to];
63     }
64     end[v] = (int) order.size() - 1;
65 }
66
67 void do_dfs_from(int v) {
68     depth[v] = 0;

```

```

69     dist[v] = T{};
70     root[v] = v;
71     pv[v] = pe[v] = -1;
72     do_dfs(v);
73 }
74
75 public:
76 void dfs(int v, bool clear_order = true) {
77     if (pv.empty()) {
78         init();
79     } else {
80         if (clear_order) {
81             order.clear();
82         }
83     }
84     do_dfs_from(v);
85 }
86
87 void dfs_all() {
88     init();
89     for (int v = 0; v < n; v++) {
90         if (depth[v] == -1) {
91             do_dfs_from(v);
92         }
93     }
94     assert((int) order.size() == n);
95 }
96 };

```

4.8 dfs-undigraph.cpp

```

1  template <typename T>
2  class dfs_undigraph : public undigraph<T> {
3  public:
4      using undigraph<T>::edges;
5      using undigraph<T>::g;
6      using undigraph<T>::n;
7
8      vector<int> pv;
9      vector<int> pe;
10     vector<int> order;
11     vector<int> pos;
12     vector<int> end;

```



```

13     vector<int> sz;
14     vector<int> root;
15     vector<int> depth;
16     vector<int> min_depth;
17     vector<T> dist;
18     vector<int> was;
19     int attempt;
20
21     dfs_undigraph(int _n) : undigraph<T>(_n) {
22     }
23
24     void init() {
25         pv = vector<int>(n, -1);
26         pe = vector<int>(n, -1);
27         order.clear();
28         pos = vector<int>(n, -1);
29         end = vector<int>(n, -1);
30         sz = vector<int>(n, 0);
31         root = vector<int>(n, -1);
32         depth = vector<int>(n, -1);
33         min_depth = vector<int>(n, -1);
34         dist = vector<T>(n);
35         was = vector<int>(n, -1);
36         attempt = 0;
37     }
38
39     void clear() {
40         pv.clear();
41         pe.clear();
42         order.clear();
43         pos.clear();
44         end.clear();
45         sz.clear();
46         root.clear();
47         depth.clear();
48         min_depth.clear();
49         dist.clear();
50         was.clear();
51     }
52
53     private:
54     void do_dfs(int v) {
55         was[v] = attempt;

```

```

56         pos[v] = (int) order.size();
57         order.push_back(v);
58         sz[v] = 1;
59         min_depth[v] = depth[v];
60         for (int id : g[v]) {
61             if (id == pe[v]) {
62                 continue;
63             }
64             auto &e = edges[id];
65             int to = e.from ^ e.to ^ v;
66             if (was[to] == attempt) {
67                 min_depth[v] = min(min_depth[v], depth[to]);
68                 continue;
69             }
70             depth[to] = depth[v] + 1;
71             dist[to] = dist[v] + e.cost;
72             pv[to] = v;
73             pe[to] = id;
74             root[to] = (root[v] != -1 ? root[v] : to);
75             do_dfs(to);
76             sz[v] += sz[to];
77             min_depth[v] = min(min_depth[v], min_depth[to]);
78         }
79         end[v] = (int) order.size() - 1;
80     }
81
82     void do_dfs_from(int v) {
83         ++attempt;
84         depth[v] = 0;
85         dist[v] = T{};
86         root[v] = v;
87         pv[v] = pe[v] = -1;
88         do_dfs(v);
89     }
90
91     public:
92     void dfs(int v, bool clear_order = true) {
93         if (pv.empty()) {
94             init();
95         } else {
96             if (clear_order) {
97                 order.clear();
98             }

```

```

99     }
100     do_dfs_from(v);
101 }
102
103 void dfs_all() {
104     init();
105     for (int v = 0; v < n; v++) {
106         if (depth[v] == -1) {
107             do_dfs_from(v);
108         }
109     }
110     assert((int) order.size() == n);
111 }
112 };

```

4.9 digraph.cpp

```

1  template <typename T>
2  class digraph : public graph<T> {
3  public:
4      using graph<T>::edges;
5      using graph<T>::g;
6      using graph<T>::n;
7
8      digraph(int _n) : graph<T>(_n) {
9      }
10
11     int add(int from, int to, T cost = 1) {
12         assert(0 <= from && from < n && 0 <= to && to < n);
13         int id = (int) edges.size();
14         g[from].push_back(id);
15         edges.push_back({from, to, cost});
16         return id;
17     }
18
19     digraph<T> reverse() const {
20         digraph<T> rev(n);
21         for (auto &e : edges) {
22             rev.add(e.to, e.from, e.cost);
23         }
24         return rev;
25     }
26 };

```

4.10 dijkstra-set.cpp

```

1  template <typename T>
2  vector<T> dijkstra(const graph<T> &g, int start) {
3      assert(0 <= start && start < g.n);
4      vector<T> dist(g.n, numeric_limits<T>::max());
5      dist[start] = 0;
6      set<pair<T, int>> s;
7      s.emplace(dist[start], start);
8      while (!s.empty()) {
9          int i = s.begin()->second;
10         s.erase(s.begin());
11         for (int id : g.g[i]) {
12             auto &e = g.edges[id];
13             int to = e.from ^ e.to ^ i;
14             if (dist[i] + e.cost < dist[to]) {
15                 s.erase({dist[to], to});
16                 dist[to] = dist[i] + e.cost;
17                 s.emplace(dist[to], to);
18             }
19         }
20     }
21     return dist;
22     // returns numeric_limits<T>::max() if there's no path
23 }

```

4.11 dijkstra.cpp

```

1  template <typename T>
2  vector<T> dijkstra(const graph<T> &g, int start) {
3      assert(0 <= start && start < g.n);
4      vector<T> dist(g.n, numeric_limits<T>::max());
5      priority_queue<pair<T, int>, vector<pair<T, int>>, greater<pair<T, int>>>
6          s;
7      dist[start] = 0;
8      s.emplace(dist[start], start);
9      while (!s.empty()) {
10         T expected = s.top().first;
11         int i = s.top().second;
12         s.pop();
13         if (dist[i] != expected) {
14             continue;
15         }
16         for (int id : g.g[i]) {

```

```

16     auto &e = g.edges[id];
17     int to = e.from ^ e.to ^ i;
18     if (dist[i] + e.cost < dist[to]) {
19         dist[to] = dist[i] + e.cost;
20         s.emplace(dist[to], to);
21     }
22 }
23 }
24 return dist;
25 // returns numeric_limits<T>::max() if there's no path
26 }

```

4.12 dominators.cpp

```

1 template <typename T>
2 vector<int> find_dominators(const digraph<T> &g, int root) {
3     int n = g.n;
4     vector<int> pos(n, -1);
5     vector<int> order;
6     vector<int> parent(n, -1);
7     function<void(int)> dfs = [&g, &pos, &order, &parent, &dfs](int v) {
8         pos[v] = (int) order.size();
9         order.push_back(v);
10        for (int id : g.g[v]) {
11            auto &e = g.edges[id];
12            int u = e.to;
13            if (pos[u] == -1) {
14                parent[u] = v;
15                dfs(u);
16            }
17        }
18    };
19    dfs(root);
20    vector<int> p(n), best(n);
21    iota(p.begin(), p.end(), 0);
22    iota(best.begin(), best.end(), 0);
23    vector<int> sdom = pos;
24    function<int(int)> find_best = [&p, &best, &sdom, &find_best](int x) {
25        if (p[x] != x) {
26            int u = find_best(p[x]);
27            if (sdom[u] < sdom[best[x]]) {
28                best[x] = u;
29            }

```

```

30        p[x] = p[p[x]];
31    }
32    if (sdom[best[p[x]]] < sdom[best[x]]) {
33        best[x] = best[p[x]];
34    }
35    return best[x];
36 };
37 digraph<int> g_rev = g.reverse();
38 vector<int> idom(n, -1);
39 vector<int> link(n, 0);
40 vector<vector<int>> bucket(n);
41 for (int it = (int) order.size() - 1; it >= 0; it--) {
42     int w = order[it];
43     for (int id : g_rev.g[w]) {
44         auto &e = g_rev.edges[id];
45         int u = e.to;
46         if (pos[u] != -1) {
47             sdom[w] = min(sdom[w], sdom[find_best(u)]);
48         }
49     }
50     idom[w] = order[sdom[w]];
51     for (int u : bucket[w]) {
52         link[u] = find_best(u);
53     }
54     for (int id : g.g[w]) {
55         auto &e = g.edges[id];
56         int u = e.to;
57         if (parent[u] == w) {
58             p[u] = w;
59         }
60     }
61     bucket[order[sdom[w]]].push_back(w);
62 }
63 for (int it = 1; it < (int) order.size(); it++) {
64     int w = order[it];
65     idom[w] = idom[link[w]];
66 }
67 return idom;
68 // idom[i] -- immediate dominator for vertex i
69 }

```

4.13 eulerian.cpp

```

1  template <typename T>
2  vector<int> find_eulerian_path(const graph<T> &g, int &root) {
3      // in_deg and out_deg are fake for undigraph!
4      vector<int> in_deg(g.n, 0);
5      vector<int> out_deg(g.n, 0);
6      int cnt_edges = 0;
7      for (int id = 0; id < (int) g.edges.size(); id++) {
8          cnt_edges++;
9          auto &e = g.edges[id];
10         out_deg[e.from]++;
11         in_deg[e.to]++;
12     }
13     root = -1;
14     int odd = 0;
15     for (int i = 0; i < g.n; i++) {
16         if ((in_deg[i] + out_deg[i]) % 2 == 1) {
17             odd++;
18             if (root == -1 || out_deg[i] - in_deg[i] > out_deg[root] - in_deg[root]
19                 ]) {
20                 root = i;
21             }
22         }
23     }
24     if (odd > 2) {
25         root = -1;
26         return vector<int>();
27     }
28     if (root == -1) {
29         root = 0;
30         while (root < g.n && in_deg[root] + out_deg[root] == 0) {
31             root++;
32         }
33         if (root == g.n) {
34             // an empty path
35             root = 0;
36             return vector<int>();
37         }
38     }
39     vector<bool> used(g.edges.size(), false);
40     vector<int> ptr(g.n, 0);
41     vector<int> balance(g.n, 0);
42     vector<int> res(cnt_edges);
43     int stack_ptr = 0;

```

```

43     int write_ptr = cnt_edges;
44     int v = root;
45     while (true) {
46         bool found = false;
47         while (ptr[v] < (int) g.g[v].size()) {
48             int id = g.g[v][ptr[v]++];
49             if (used[id]) {
50                 continue;
51             }
52             used[id] = true;
53             res[stack_ptr++] = id;
54             auto &e = g.edges[id];
55             balance[v]++;
56             v ^= e.from ^ e.to;
57             balance[v]--;
58             found = true;
59             break;
60         }
61         if (!found) {
62             if (stack_ptr == 0) {
63                 break;
64             }
65             int id = res[--stack_ptr];
66             res[--write_ptr] = id;
67             auto &e = g.edges[id];
68             v ^= e.from ^ e.to;
69         }
70     }
71     int disbalance = 0;
72     for (int i = 0; i < g.n; i++) {
73         disbalance += abs(balance[i]);
74     }
75     if (write_ptr != 0 || disbalance > 2) {
76         root = -1;
77         return vector<int>();
78     }
79     return res;
80     // returns edge ids in the path (or the cycle if it exists)
81     // root == -1 if there is no path
82     // (or res.empty(), but this is also true when there are no edges)
83 }

```

4.14 forest.cpp

```
1 template <typename T>
2 class forest : public graph<T> {
3 public:
4     using graph<T>::edges;
5     using graph<T>::g;
6     using graph<T>::n;
7
8     forest(int _n) : graph<T>(_n) {
9     }
10
11     int add(int from, int to, T cost = 1) {
12         assert(0 <= from && from < n && 0 <= to && to < n);
13         int id = (int) edges.size();
14         assert(id < n - 1);
15         g[from].push_back(id);
16         g[to].push_back(id);
17         edges.push_back({from, to, cost});
18         return id;
19     }
20 };
```

4.15 graph.cpp

```
1 template <typename T>
2 class graph {
3 public:
4     struct edge {
5         int from;
6         int to;
7         T cost;
8     };
9
10    vector<edge> edges;
11    vector<vector<int>> g;
12    int n;
13
14    graph(int _n) : n(_n) {
15        g.resize(n);
16    }
17
18    virtual int add(int from, int to, T cost) = 0;
19 };
```

4.16 hld-forest.cpp

```
1 template <typename T>
2 class hld_forest : public dfs_forest<T> {
3 public:
4     using dfs_forest<T>::edges;
5     using dfs_forest<T>::g;
6     using dfs_forest<T>::n;
7     using dfs_forest<T>::pv;
8     using dfs_forest<T>::sz;
9     using dfs_forest<T>::root;
10    using dfs_forest<T>::pos;
11    using dfs_forest<T>::end;
12    using dfs_forest<T>::order;
13    using dfs_forest<T>::depth;
14    using dfs_forest<T>::dfs;
15    using dfs_forest<T>::dfs_all;
16
17    vector<int> head;
18    vector<int> visited;
19
20    hld_forest(int _n) : dfs_forest<T>(_n) {
21        visited.resize(n);
22    }
23
24    void build_hld(const vector<int> &vs) {
25        for (int tries = 0; tries < 2; tries++) {
26            if (vs.empty()) {
27                dfs_all();
28            } else {
29                order.clear();
30                for (int v : vs) {
31                    dfs(v, false);
32                }
33                assert((int) order.size() == n);
34            }
35            if (tries == 1) {
36                break;
37            }
38            for (int i = 0; i < n; i++) {
39                if (g[i].empty()) {
40                    continue;
41                }
42                int best = -1, bid = 0;
```

```

43     for (int j = 0; j < (int) g[i].size(); j++) {
44         int id = g[i][j];
45         int v = edges[id].from ^ edges[id].to ^ i;
46         if (pv[v] != i) {
47             continue;
48         }
49         if (sz[v] > best) {
50             best = sz[v];
51             bid = j;
52         }
53     }
54     swap(g[i][0], g[i][bid]);
55 }
56 }
57 head.resize(n);
58 for (int i = 0; i < n; i++) {
59     head[i] = i;
60 }
61 for (int i = 0; i < n - 1; i++) {
62     int x = order[i];
63     int y = order[i + 1];
64     if (pv[y] == x) {
65         head[y] = head[x];
66     }
67 }
68 }
69
70 void build_hld(int v) {
71     build_hld(vector<int>(1, v));
72 }
73
74 void build_hld_all() {
75     build_hld(vector<int>());
76 }
77
78 bool apply_on_path(int x, int y, bool with_lca, function<void(int,int,bool
79     )> f) {
80     // f(x, y, up): up -- whether this part of the path goes up
81     assert(!head.empty());
82     int z = lca(x, y);
83     if (z == -1) {
84         return false;
85     }

```

```

85     {
86         int v = x;
87         while (v != z) {
88             if (depth[head[v]] <= depth[z]) {
89                 f(pos[z] + 1, pos[v], true);
90                 break;
91             }
92             f(pos[head[v]], pos[v], true);
93             v = pv[head[v]];
94         }
95     }
96     if (with_lca) {
97         f(pos[z], pos[z], false);
98     }
99     {
100         int v = y;
101         int cnt_visited = 0;
102         while (v != z) {
103             if (depth[head[v]] <= depth[z]) {
104                 f(pos[z] + 1, pos[v], false);
105                 break;
106             }
107             visited[cnt_visited++] = v;
108             v = pv[head[v]];
109         }
110         for (int at = cnt_visited - 1; at >= 0; at--) {
111             v = visited[at];
112             f(pos[head[v]], pos[v], false);
113         }
114     }
115     return true;
116 }
117
118 inline bool anc(int x, int y) {
119     return (pos[x] <= pos[y] && end[y] <= end[x]);
120 }
121
122 inline int go_up(int x, int up) {
123     int target = depth[x] - up;
124     if (target < 0) {
125         return -1;
126     }
127     while (depth[head[x]] > target) {

```

```

128     x = pv[head[x]];
129 }
130 return order[pos[x] - depth[x] + target];
131 }
132
133 inline int lca(int x, int y) {
134     if (root[x] != root[y]) {
135         return -1;
136     }
137     while (head[x] != head[y]) {
138         if (depth[head[x]] > depth[head[y]]) {
139             x = pv[head[x]];
140         } else {
141             y = pv[head[y]];
142         }
143     }
144     return depth[x] < depth[y] ? x : y;
145 }
146 };

```

4.17 hld-forest-old.cpp

```

1 template <typename T>
2 class hld_forest_old : public lca_forest<T> {
3 public:
4     using lca_forest<T>::edges;
5     using lca_forest<T>::g;
6     using lca_forest<T>::n;
7     using lca_forest<T>::pv;
8     using lca_forest<T>::sz;
9     using lca_forest<T>::pos;
10    using lca_forest<T>::order;
11    using lca_forest<T>::depth;
12    using lca_forest<T>::dfs;
13    using lca_forest<T>::dfs_all;
14    using lca_forest<T>::lca;
15    using lca_forest<T>::build_lca;
16
17    vector<int> head;
18    vector<int> visited;
19
20    hld_forest_old(int _n) : lca_forest<T>(_n) {
21        visited.resize(n);

```

```

22    }
23
24    void build_hld(const vector<int> &vs) {
25        for (int tries = 0; tries < 2; tries++) {
26            if (vs.empty()) {
27                dfs_all();
28            } else {
29                order.clear();
30                for (int v : vs) {
31                    dfs(v, false);
32                }
33                assert((int) order.size() == n);
34            }
35            if (tries == 1) {
36                break;
37            }
38            for (int i = 0; i < n; i++) {
39                if (g[i].empty()) {
40                    continue;
41                }
42                int best = -1, bid = 0;
43                for (int j = 0; j < (int) g[i].size(); j++) {
44                    int id = g[i][j];
45                    int v = edges[id].from ^ edges[id].to ^ i;
46                    if (pv[v] != i) {
47                        continue;
48                    }
49                    if (sz[v] > best) {
50                        best = sz[v];
51                        bid = j;
52                    }
53                }
54                swap(g[i][0], g[i][bid]);
55            }
56        }
57        build_lca();
58        head.resize(n);
59        for (int i = 0; i < n; i++) {
60            head[i] = i;
61        }
62        for (int i = 0; i < n - 1; i++) {
63            int x = order[i];
64            int y = order[i + 1];

```

```

65     if (pv[y] == x) {
66         head[y] = head[x];
67     }
68 }
69 }
70
71 void build_hld(int v) {
72     build_hld(vector<int>(1, v));
73 }
74
75 void build_hld_all() {
76     build_hld(vector<int>());
77 }
78
79 bool apply_on_path(int x, int y, bool with_lca, function<void(int,int,bool
    >> f) {
80     // f(x, y, up): up -- whether this part of the path goes up
81     assert(!head.empty());
82     int z = lca(x, y);
83     if (z == -1) {
84         return false;
85     }
86     {
87         int v = x;
88         while (v != z) {
89             if (depth[head[v]] <= depth[z]) {
90                 f(pos[z] + 1, pos[v], true);
91                 break;
92             }
93             f(pos[head[v]], pos[v], true);
94             v = pv[head[v]];
95         }
96     }
97     if (with_lca) {
98         f(pos[z], pos[z], false);
99     }
100     {
101         int v = y;
102         int cnt_visited = 0;
103         while (v != z) {
104             if (depth[head[v]] <= depth[z]) {
105                 f(pos[z] + 1, pos[v], false);
106                 break;

```

```

107     }
108     visited[cnt_visited++] = v;
109     v = pv[head[v]];
110 }
111 for (int at = cnt_visited - 1; at >= 0; at--) {
112     v = visited[at];
113     f(pos[head[v]], pos[v], false);
114 }
115 }
116 return true;
117 }
118 };

```

4.18 lca-forest.cpp

```

1  template <typename T>
2  class lca_forest : public dfs_forest<T> {
3  public:
4      using dfs_forest<T>::edges;
5      using dfs_forest<T>::g;
6      using dfs_forest<T>::n;
7      using dfs_forest<T>::pv;
8      using dfs_forest<T>::pos;
9      using dfs_forest<T>::end;
10     using dfs_forest<T>::depth;
11
12     int h;
13     vector<vector<int>>> pr;
14
15     lca_forest(int _n) : dfs_forest<T>(_n) {
16     }
17
18     inline void build_lca() {
19         assert(!pv.empty());
20         int max_depth = 0;
21         for (int i = 0; i < n; i++) {
22             max_depth = max(max_depth, depth[i]);
23         }
24         h = 1;
25         while ((1 << h) <= max_depth) {
26             h++;
27         }
28         pr.resize(n);

```



```

29     for (int i = 0; i < n; i++) {
30         pr[i].resize(h);
31         pr[i][0] = pv[i];
32     }
33     for (int j = 1; j < h; j++) {
34         for (int i = 0; i < n; i++) {
35             pr[i][j] = (pr[i][j - 1] == -1 ? -1 : pr[pr[i][j - 1]][j - 1]);
36         }
37     }
38 }
39
40 inline bool anc(int x, int y) {
41     return (pos[x] <= pos[y] && end[y] <= end[x]);
42 }
43
44 inline int go_up(int x, int up) {
45     assert(!pr.empty());
46     up = min(up, (1 << h) - 1);
47     for (int j = h - 1; j >= 0; j--) {
48         if (up & (1 << j)) {
49             x = pr[x][j];
50             if (x == -1) {
51                 break;
52             }
53         }
54     }
55     return x;
56 }
57
58 inline int lca(int x, int y) {
59     assert(!pr.empty());
60     if (anc(x, y)) {
61         return x;
62     }
63     if (anc(y, x)) {
64         return y;
65     }
66     for (int j = h - 1; j >= 0; j--) {
67         if (pr[x][j] != -1 && !anc(pr[x][j], y)) {
68             x = pr[x][j];
69         }
70     }
71     return pr[x][0];

```

```

72     }
73 };

```

4.19 mst.cpp

```

1  template <typename T>
2  vector<int> find_mst(const undigraph<T> &g, T &ans) {
3      vector<int> order(g.edges.size());
4      iota(order.begin(), order.end(), 0);
5      sort(order.begin(), order.end(), [&g](int a, int b) {
6          return g.edges[a].cost < g.edges[b].cost;
7      });
8      dsu d(g.n);
9      vector<int> ans_list;
10     ans = 0;
11     for (int id : order) {
12         auto &e = g.edges[id];
13         if (d.get(e.from) != d.get(e.to)) {
14             d.unite(e.from, e.to);
15             ans_list.push_back(id);
16             ans += e.cost;
17         }
18     }
19     return ans_list;
20     // returns edge ids of minimum "spanning" forest
21 }

```

4.20 scc.cpp

```

1  template <typename T>
2  vector<int> find_scc(const digraph<T> &g, int &cnt) {
3      digraph<T> g_rev = g.reverse();
4      vector<int> order;
5      vector<bool> was(g.n, false);
6      function<void(int)> dfs1 = [&](int v) {
7          was[v] = true;
8          for (int id : g.g[v]) {
9              auto &e = g.edges[id];
10             int to = e.to;
11             if (!was[to]) {
12                 dfs1(to);
13             }
14         }

```

```

15     order.push_back(v);
16 };
17 for (int i = 0; i < g.n; i++) {
18     if (!was[i]) {
19         dfs1(i);
20     }
21 }
22 vector<int> c(g.n, -1);
23 function<void(int)> dfs2 = [&](int v) {
24     for (int id : g_rev.g[v]) {
25         auto &e = g_rev.edges[id];
26         int to = e.to;
27         if (c[to] == -1) {
28             c[to] = c[v];
29             dfs2(to);
30         }
31     }
32 };
33 cnt = 0;
34 for (int id = g.n - 1; id >= 0; id--) {
35     int i = order[id];
36     if (c[i] != -1) {
37         continue;
38     }
39     c[i] = cnt++;
40     dfs2(i);
41 }
42 return c;
43 // c[i] <= c[j] for every edge i -> j
44 }

```

4.21 topsort.cpp

```

1 template <typename T>
2 vector<int> find_topsort(const digraph<T> &g) {
3     vector<int> deg(g.n, 0);
4     for (int id = 0; id < (int) g.edges.size(); id++) {
5         deg[g.edges[id].to]++;
6     }
7     vector<int> x;
8     for (int i = 0; i < g.n; i++) {
9         if (deg[i] == 0) {
10             x.push_back(i);

```

```

11     }
12 }
13 for (int ptr = 0; ptr < (int) x.size(); ptr++) {
14     int i = x[ptr];
15     for (int id : g.g[i]) {
16         auto &e = g.edges[id];
17         int to = e.to;
18         if (--deg[to] == 0) {
19             x.push_back(to);
20         }
21     }
22 }
23 if ((int) x.size() != g.n) {
24     return vector<int>();
25 }
26 return x;
27 }

```

4.22 tree-dp.cpp

```

1 struct Data {
2     ${0}... a = ...;
3 };
4 auto Unite = [&](const Data& a, const Data& b) -> Data {
5     return ...;
6 };
7 auto AddVertex = [&](const Data& a, int v) -> Data {
8     return ...;
9 };
10 auto MoveUp = [&](const Data& a, int v, int eid) -> Data {
11     auto& e = g.edges[eid];
12     return ...;
13 };
14 g.dfs(0);
15 vector<Data> down(g.n);
16 vector<Data> up(g.n);
17 vector<Data> dp(g.n);
18 {
19     for (int it = g.n - 1; it >= 0; it--) {
20         int i = g.order[it];
21         for (int eid : g.g[i]) {
22             auto& e = g.edges[eid];
23             int to = e.from ^ e.to ^ i;

```

```

24     if (to == g.pv[i]) {
25         continue;
26     }
27     down[i] = Unite(down[i], MoveUp(down[to], i, eid));
28 }
29 down[i] = AddVertex(down[i], i);
30 }
31 for (int it = 0; it < g.n; it++) {
32     int i = g.order[it];
33     vector<int> children;
34     vector<Data> vals;
35     for (int eid : g.g[i]) {
36         auto& e = g.edges[eid];
37         int to = e.from ^ e.to ^ i;
38         if (to == g.pv[i]) {
39             continue;
40         }
41         children.push_back(to);
42         vals.push_back(MoveUp(down[to], i, eid));
43     }
44     vector<Data> suf(vals.size() + 1);
45     for (int j = int(vals.size()) - 1; j >= 0; j--) {
46         suf[j] = Unite(vals[j], suf[j + 1]);
47     }
48     Data pref;
49     if (g.pv[i] != -1) {
50         pref = MoveUp(up[i], i, g.pe[i]);
51     }
52     for (int j = 0; j < int(vals.size()); j++) {
53         up[children[j]] = AddVertex(Unite(pref, suf[j + 1]), i);
54         pref = Unite(pref, vals[j]);
55     }
56     dp[i] = AddVertex(pref, i);
57 }
58 }

```

4.23 twosat.cpp

```

1 class twosat {
2 public:
3     digraph<int> g;
4     int n;
5

```

```

6     twosat(int _n) : g(digraph<int>(2 * _n)), n(_n) {
7     }
8
9     // (v[x] == value_x)
10    inline void add(int x, int value_x) {
11        assert(0 <= x && x < n);
12        assert(0 <= value_x && value_x <= 1);
13        g.add(2 * x + (value_x ^ 1), 2 * x + value_x);
14    }
15
16    // (v[x] == value_x || v[y] == value_y)
17    inline void add(int x, int value_x, int y, int value_y) {
18        assert(0 <= x && x < n && 0 <= y && y < n);
19        assert(0 <= value_x && value_x <= 1 && 0 <= value_y && value_y <= 1);
20        g.add(2 * x + (value_x ^ 1), 2 * y + value_y);
21        g.add(2 * y + (value_y ^ 1), 2 * x + value_x);
22    }
23
24    inline vector<int> solve() {
25        int cnt;
26        vector<int> c = find_scc(g, cnt);
27        vector<int> res(n);
28        for (int i = 0; i < n; i++) {
29            if (c[2 * i] == c[2 * i + 1]) {
30                return vector<int>();
31            }
32            res[i] = (c[2 * i] < c[2 * i + 1]);
33        }
34        return res;
35    }
36 };

```

4.24 undigraph.cpp

```

1 template <typename T>
2 class undigraph : public graph<T> {
3 public:
4     using graph<T>::edges;
5     using graph<T>::g;
6     using graph<T>::n;
7
8     undigraph(int _n) : graph<T>(_n) {
9     }

```

```

10
11     int add(int from, int to, T cost = 1) {
12         assert(0 <= from && from < n && 0 <= to && to < n);
13         int id = (int) edges.size();
14         g[from].push_back(id);
15         g[to].push_back(id);
16         edges.push_back({from, to, cost});
17         return id;
18     }
19 };

```

5 misc

5.1 debug.cpp

```

1  template <typename A, typename B>
2  string to_string(pair<A, B> p);
3
4  template <typename A, typename B, typename C>
5  string to_string(tuple<A, B, C> p);
6
7  template <typename A, typename B, typename C, typename D>
8  string to_string(tuple<A, B, C, D> p);
9
10 string to_string(const string& s) {
11     return '"' + s + '"';
12 }
13
14 string to_string(const char* s) {
15     return to_string((string) s);
16 }
17
18 string to_string(bool b) {
19     return (b ? "true" : "false");
20 }
21
22 string to_string(vector<bool> v) {
23     bool first = true;
24     string res = "{";
25     for (int i = 0; i < static_cast<int>(v.size()); i++) {
26         if (!first) {
27             res += ", ";

```

```

28         }
29         first = false;
30         res += to_string(v[i]);
31     }
32     res += "}";
33     return res;
34 }
35
36 template <size_t N>
37 string to_string(bitset<N> v) {
38     string res = "";
39     for (size_t i = 0; i < N; i++) {
40         res += static_cast<char>('0' + v[i]);
41     }
42     return res;
43 }
44
45 template <typename A>
46 string to_string(A v) {
47     bool first = true;
48     string res = "{";
49     for (const auto &x : v) {
50         if (!first) {
51             res += ", ";
52         }
53         first = false;
54         res += to_string(x);
55     }
56     res += "}";
57     return res;
58 }
59
60 template <typename A, typename B>
61 string to_string(pair<A, B> p) {
62     return "(" + to_string(p.first) + ", " + to_string(p.second) + ")";
63 }
64
65 template <typename A, typename B, typename C>
66 string to_string(tuple<A, B, C> p) {
67     return "(" + to_string(get<0>(p)) + ", " + to_string(get<1>(p)) + ", " +
68         to_string(get<2>(p)) + ")";
69 }

```

```

70 template <typename A, typename B, typename C, typename D>
71 string to_string(tuple<A, B, C, D> p) {
72     return "(" + to_string(get<0>(p)) + "," + to_string(get<1>(p)) + "," +
73         to_string(get<2>(p)) + "," + to_string(get<3>(p)) + ")";
74 }
75 void debug_out() { cerr << endl; }
76
77 template <typename Head, typename... Tail>
78 void debug_out(Head H, Tail... T) {
79     cerr << " " << to_string(H);
80     debug_out(T...);
81 }
82
83 #ifdef LOCAL
84 #define debug(...) cerr << "[" << __VA_ARGS__ << "]:", debug_out(
85     __VA_ARGS__)
86 #else
87 #define debug(...) 42
88 #endif

```

5.2 fastinput.cpp

```

1 static struct FastInput {
2     static constexpr int BUF_SIZE = 1 << 20;
3     char buf[BUF_SIZE];
4     size_t chars_read = 0;
5     size_t buf_pos = 0;
6     FILE *in = stdin;
7     char cur = 0;
8
9     inline char get_char() {
10         if (buf_pos >= chars_read) {
11             chars_read = fread(buf, 1, BUF_SIZE, in);
12             buf_pos = 0;
13             buf[0] = (chars_read == 0 ? -1 : buf[0]);
14         }
15         return cur = buf[buf_pos++];
16     }
17
18     inline void tie(int) {}
19
20     inline explicit operator bool() {

```

```

21         return cur != -1;
22     }
23
24     inline static bool is_blank(char c) {
25         return c <= ' ';
26     }
27
28     inline bool skip_blanks() {
29         while (is_blank(cur) && cur != -1) {
30             get_char();
31         }
32         return cur != -1;
33     }
34
35     inline FastInput& operator>>(char& c) {
36         skip_blanks();
37         c = cur;
38         return *this;
39     }
40
41     inline FastInput& operator>>(string& s) {
42         if (skip_blanks()) {
43             s.clear();
44             do {
45                 s += cur;
46             } while (!is_blank(get_char()));
47         }
48         return *this;
49     }
50
51     template <typename T>
52     inline FastInput& read_integer(T& n) {
53         // unsafe, doesn't check that characters are actually digits
54         n = 0;
55         if (skip_blanks()) {
56             int sign = +1;
57             if (cur == '-') {
58                 sign = -1;
59                 get_char();
60             }
61             do {
62                 n += n * 10 + cur - '0';
63             } while (!is_blank(get_char()));

```

```

64     n *= sign;
65 }
66 return *this;
67 }
68
69 template <typename T>
70 inline typename enable_if<is_integral<T>::value, FastInput&>::type
71     operator>>(T& n) {
72     return read_integer(n);
73 }
74
75 #if !defined(_WIN32) || defined(_WIN64)
76 inline FastInput& operator>>(__int128& n) {
77     return read_integer(n);
78 }
79 #endif
80
81 template <typename T>
82 inline typename enable_if<is_floating_point<T>::value, FastInput&>::type
83     operator>>(T& n) {
84     // not sure if really fast, for compatibility only
85     n = 0;
86     if (skip_blanks()) {
87         string s;
88         (*this) >> s;
89         sscanf(s.c_str(), "%lf", &n);
90     }
91     return *this;
92 }
93 #define cin fast_input

```

5.3 fastoutput.cpp

```

1 static struct FastOutput {
2     static constexpr int BUF_SIZE = 1 << 20;
3     char buf[BUF_SIZE];
4     size_t buf_pos = 0;
5     static constexpr int TMP_SIZE = 1 << 20;
6     char tmp[TMP_SIZE];
7     FILE *out = stdout;
8

```

```

9     inline void put_char(char c) {
10         buf[buf_pos++] = c;
11         if (buf_pos == BUF_SIZE) {
12             fwrite(buf, 1, buf_pos, out);
13             buf_pos = 0;
14         }
15     }
16
17     ~FastOutput() {
18         fwrite(buf, 1, buf_pos, out);
19     }
20
21     inline FastOutput& operator<<(char c) {
22         put_char(c);
23         return *this;
24     }
25
26     inline FastOutput& operator<<(const char* s) {
27         while (*s) {
28             put_char(*s++);
29         }
30         return *this;
31     }
32
33     inline FastOutput& operator<<(const string& s) {
34         for (int i = 0; i < (int) s.size(); i++) {
35             put_char(s[i]);
36         }
37         return *this;
38     }
39
40     template <typename T>
41     inline char* integer_to_string(T n) {
42         // beware of TMP_SIZE
43         char* p = tmp + TMP_SIZE - 1;
44         if (n == 0) {
45             *--p = '0';
46         } else {
47             bool is_negative = false;
48             if (n < 0) {
49                 is_negative = true;
50                 n = -n;
51             }

```

```

52     while (n > 0) {
53         *--p = (char) ('0' + n % 10);
54         n /= 10;
55     }
56     if (is_negative) {
57         *--p = '-';
58     }
59 }
60 return p;
61 }
62
63 template <typename T>
64 inline typename enable_if<is_integral<T>::value, char*>::type stringify(T
        n) {
65     return integer_to_string(n);
66 }
67
68 #if !defined(_WIN32) || defined(_WIN64)
69 inline char* stringify(_int128 n) {
70     return integer_to_string(n);
71 }
72 #endif
73
74 template <typename T>
75 inline typename enable_if<is_floating_point<T>::value, char*>::type
        stringify(T n) {
76     sprintf(tmp, "%.17f", n);
77     return tmp;
78 }
79
80 template <typename T>
81 inline FastOutput& operator<<(const T& n) {
82     auto p = stringify(n);
83     for (; *p != 0; p++) {
84         put_char(*p);
85     }
86     return *this;
87 }
88 } fast_output;
89
90 #define cout fast_output

```

5.4 lis.cpp

```

1  template<typename T>
2  int lis(const vector<T>& a) {
3      vector<T> u;
4      for (const T& x : a) {
5          auto it = lower_bound(u.begin(), u.end(), x);
6          if (it == u.end()) {
7              u.push_back(x);
8          } else {
9              *it = x;
10         }
11     }
12     return (int) u.size();
13 }

```

5.5 pragma.cpp

```

1  #pragma GCC optimize("O3,unroll-loops")
2  // #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")

```

5.6 radix.cpp

```

1  namespace radix {
2
3  vector<int> p(65537);
4
5  template<typename T>
6  void SortShift(vector<T>& a, vector<T>& new_a, int shift) {
7      assert(a.size() == new_a.size());
8      int n = static_cast<int>(a.size());
9      fill(p.begin(), p.end(), 0);
10     for (int i = 0; i < n; i++) p[1 + ((a[i] >> shift) & 0xffff)]++;
11     for (int i = 1; i <= 65536; i++) p[i] += p[i - 1];
12     for (int i = 0; i < n; i++) new_a[p[(a[i] >> shift) & 0xffff]++] = a[i];
13 }
14
15 void Sort(vector<int32_t>& a) {
16     constexpr int32_t flip = static_cast<int32_t>(1) << 31;
17     for (auto& aa : a) aa ^= flip;
18     vector<int32_t> b(a.size());
19     SortShift(a, b, 0);
20     SortShift(b, a, 16);

```

```

21     for (auto& aa : a) aa ^= flip;
22 }
23
24 void Sort(vector<uint32_t>& a) {
25     vector<uint32_t> b(a.size());
26     SortShift(a, b, 0);
27     SortShift(b, a, 16);
28 }
29
30 void Sort(vector<int64_t>& a) {
31     constexpr int64_t flip = static_cast<int64_t>(1) << 63;
32     for (auto& aa : a) aa ^= flip;
33     vector<int64_t> b(a.size());
34     SortShift(a, b, 0);
35     SortShift(b, a, 16);
36     SortShift(a, b, 32);
37     SortShift(b, a, 48);
38     for (auto& aa : a) aa ^= flip;
39 }
40
41 void Sort(vector<uint64_t>& a) {
42     vector<uint64_t> b(a.size());
43     SortShift(a, b, 0);
44     SortShift(b, a, 16);
45     SortShift(a, b, 32);
46     SortShift(b, a, 48);
47 }
48
49 } // namespace radix

```

5.7 rng.cpp

```

1 mt19937_64 rng((unsigned int) chrono::steady_clock::now().time_since_epoch()
    .count());

```

6 numeric

6.1 bm.cpp

```

1 template <typename T>
2 vector<T> BM(vector<T> a) {
3     vector<T> p = {1};

```

```

4     vector<T> q = {1};
5     int l = 0;
6     for (int r = 1; r <= (int) a.size(); r++) {
7         T delta = 0;
8         for (int j = 0; j <= l; j++) {
9             delta += a[r - 1 - j] * p[j];
10        }
11        q.insert(q.begin(), 0);
12        if (delta != 0) {
13            vector<T> t = p;
14            if (q.size() > t.size()) {
15                t.resize(q.size());
16            }
17            for (int i = 0; i < (int) q.size(); i++) {
18                t[i] -= delta * q[i];
19            }
20            if (2 * l <= r - 1) {
21                q = p;
22                T od = 1 / delta;
23                for (T& x : q) {
24                    x *= od;
25                }
26                l = r - 1;
27            }
28            swap(p, t);
29        }
30    }
31    assert((int) p.size() == l + 1);
32    // assert(l * 2 + 30 < (int) a.size());
33    reverse(p.begin(), p.end());
34    return p;
35 }

```

6.2 extgcd.cpp

```

1 template<typename T>
2 T extgcd(T a, T b, T &x, T &y) {
3     if (a == 0) {
4         x = 0;
5         y = 1;
6         return b;
7     }
8     T p = b / a;

```



```

9   T g = extgcd(b - p * a, a, y, x);
10  x -= p * y;
11  return g;
12 }
13
14 template<typename T>
15 bool diophantine(T a, T b, T c, T &x, T &y, T &g) {
16     if (a == 0 && b == 0) {
17         if (c == 0) {
18             x = y = g = 0;
19             return true;
20         }
21         return false;
22     }
23     if (a == 0) {
24         if (c % b == 0) {
25             x = 0;
26             y = c / b;
27             g = abs(b);
28             return true;
29         }
30         return false;
31     }
32     if (b == 0) {
33         if (c % a == 0) {
34             x = c / a;
35             y = 0;
36             g = abs(a);
37             return true;
38         }
39         return false;
40     }
41     g = extgcd(a, b, x, y);
42     if (c % g != 0) {
43         return false;
44     }
45     T dx = c / a;
46     c -= dx * a;
47     T dy = c / b;
48     c -= dy * b;
49     x = dx + (T) ((__int128) x * (c / g) % b);
50     y = dy + (T) ((__int128) y * (c / g) % a);
51     g = abs(g);

```

```

52     return true;
53     // |x|, |y| <= max(|a|, |b|, |c|) [tested]
54 }
55
56 bool crt(long long k1, long long m1, long long k2, long long m2, long long &
57         k, long long &m) {
58     k1 %= m1;
59     if (k1 < 0) k1 += m1;
60     k2 %= m2;
61     if (k2 < 0) k2 += m2;
62     long long x, y, g;
63     if (!diophantine(m1, -m2, k2 - k1, x, y, g)) {
64         return false;
65     }
66     long long dx = m2 / g;
67     long long delta = x / dx - (x % dx < 0);
68     k = m1 * (x - dx * delta) + k1;
69     m = m1 / g * m2;
70     assert(0 <= k && k < m);
71     return true;
72 }
73 // for distinct prime modulus
74 template <typename T>
75 void crt_garner(const vector<int>& p, const vector<int>& a, T& res) {
76     assert(p.size() == a.size());
77     auto inverse = [&](int q, int m) {
78         q %= m;
79         if (q < 0) q += m;
80         int b = m, u = 0, v = 1;
81         while (q) {
82             int t = b / q;
83             b -= t * q; swap(q, b);
84             u -= t * v; swap(u, v);
85         }
86         assert(b == 1);
87         if (u < 0) u += m;
88         return u;
89     };
90     vector<int> x(p.size());
91     for (int i = 0; i < (int) p.size(); i++) {
92         assert(0 <= a[i] && a[i] < p[i]);
93         x[i] = a[i];

```

```

94     for (int j = 0; j < i; j++) {
95         x[i] = (int) ((long long) (x[i] - x[j]) * inverse(p[j], p[i]) % p[i]);
96         if (x[i] < 0) x[i] += p[i];
97     }
98 }
99 res = 0;
100 for (int i = (int) p.size() - 1; i >= 0; i--) {
101     res = res * p[i] + x[i];
102 }
103 }

```

6.3 factorizer.cpp

```

1 namespace factorizer {
2
3 template <typename T>
4 struct FactorizerVarMod { static T value; };
5 template <typename T>
6 T FactorizerVarMod<T>::value;
7
8 template <typename T>
9 bool IsPrime(T n, const vector<T>& bases) {
10     if (n < 2) {
11         return false;
12     }
13     vector<T> small_primes = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29};
14     for (const T& x : small_primes) {
15         if (n % x == 0) {
16             return n == x;
17         }
18     }
19     if (n < 31 * 31) {
20         return true;
21     }
22     int s = 0;
23     T d = n - 1;
24     while ((d & 1) == 0) {
25         d >>= 1;
26         s++;
27     }
28     FactorizerVarMod<T>::value = n;
29     for (const T& a : bases) {
30         if (a % n == 0) {

```

```

31         continue;
32     }
33     Modular<FactorizerVarMod<T>> cur = a;
34     cur = power(cur, d);
35     if (cur == 1) {
36         continue;
37     }
38     bool witness = true;
39     for (int r = 0; r < s; r++) {
40         if (cur == n - 1) {
41             witness = false;
42             break;
43         }
44         cur *= cur;
45     }
46     if (witness) {
47         return false;
48     }
49 }
50 return true;
51 }
52
53 bool IsPrime(int64_t n) {
54     return IsPrime(n, {2, 325, 9375, 28178, 450775, 9780504, 1795265022});
55 }
56
57 bool IsPrime(int32_t n) {
58     return IsPrime(n, {2, 7, 61});
59 }
60
61 // but if you really need uint64_t version...
62 /*
63 bool IsPrime(uint64_t n) {
64     if (n < 2) {
65         return false;
66     }
67     vector<uint32_t> small_primes = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29};
68     for (uint32_t x : small_primes) {
69         if (n == x) {
70             return true;
71         }
72         if (n % x == 0) {
73             return false;

```

```

74     }
75 }
76 if (n < 31 * 31) {
77     return true;
78 }
79 uint32_t s = __builtin_ctzll(n - 1);
80 uint64_t d = (n - 1) >> s;
81 function<bool(uint64_t)> witness = [&n, &s, &d](uint64_t a) {
82     uint64_t cur = 1, p = d;
83     while (p > 0) {
84         if (p & 1) {
85             cur = (__uint128_t) cur * a % n;
86         }
87         a = (__uint128_t) a * a % n;
88         p >>= 1;
89     }
90     if (cur == 1) {
91         return false;
92     }
93     for (uint32_t r = 0; r < s; r++) {
94         if (cur == n - 1) {
95             return false;
96         }
97         cur = (__uint128_t) cur * cur % n;
98     }
99     return true;
100 };
101 vector<uint64_t> bases_64bit = {2, 325, 9375, 28178, 450775, 9780504,
102     1795265022};
103 for (uint64_t a : bases_64bit) {
104     if (a % n == 0) {
105         return true;
106     }
107     if (witness(a)) {
108         return false;
109     }
110 }
111 return true;
112 */
113
114 vector<int> least = {0, 1};
115 vector<int> primes;

```

```

116 int precalculated = 1;
117
118 void RunLinearSieve(int n) {
119     n = max(n, 1);
120     least.assign(n + 1, 0);
121     primes.clear();
122     for (int i = 2; i <= n; i++) {
123         if (least[i] == 0) {
124             least[i] = i;
125             primes.push_back(i);
126         }
127         for (int x : primes) {
128             if (x > least[i] || i * x > n) {
129                 break;
130             }
131             least[i * x] = x;
132         }
133     }
134     precalculated = n;
135 }
136
137 void RunSlowSieve(int n) {
138     n = max(n, 1);
139     least.assign(n + 1, 0);
140     for (int i = 2; i * i <= n; i++) {
141         if (least[i] == 0) {
142             for (int j = i * i; j <= n; j += i) {
143                 if (least[j] == 0) {
144                     least[j] = i;
145                 }
146             }
147         }
148     }
149     primes.clear();
150     for (int i = 2; i <= n; i++) {
151         if (least[i] == 0) {
152             least[i] = i;
153             primes.push_back(i);
154         }
155     }
156     precalculated = n;
157 }
158

```

```

159 void RunSieve(int n) {
160     RunLinearSieve(n);
161 }
162
163 template <typename T>
164 vector<pair<T, int>> MergeFactors(const vector<pair<T, int>>& a, const
    vector<pair<T, int>>& b) {
165     vector<pair<T, int>> c;
166     int i = 0;
167     int j = 0;
168     while (i < (int) a.size() || j < (int) b.size()) {
169         if (i < (int) a.size() && j < (int) b.size() && a[i].first == b[j].first
            ) {
170             c.emplace_back(a[i].first, a[i].second + b[j].second);
171             ++i;
172             ++j;
173             continue;
174         }
175         if (j == (int) b.size() || (i < (int) a.size() && a[i].first < b[j].
            first)) {
176             c.push_back(a[i++]);
177         } else {
178             c.push_back(b[j++]);
179         }
180     }
181     return c;
182 }
183
184 template <typename T>
185 vector<pair<T, int>> RhoC(const T& n, const T& c) {
186     if (n <= 1) {
187         return {};
188     }
189     if ((n & 1) == 0) {
190         return MergeFactors({{2, 1}}, RhoC(n / 2, c));
191     }
192     if (IsPrime(n)) {
193         return {{n, 1}};
194     }
195     FactorizerVarMod<T>::value = n;
196     Modular<FactorizerVarMod<T>> x = 2;
197     Modular<FactorizerVarMod<T>> saved = 2;
198     T power = 1;

```

```

199     T lam = 1;
200     while (true) {
201         x = x * x + c;
202         T g = __gcd((x - saved)(), n);
203         if (g != 1) {
204             return MergeFactors(RhoC(g, c + 1), RhoC(n / g, c + 1));
205         }
206         if (power == lam) {
207             saved = x;
208             power <= 1;
209             lam = 0;
210         }
211         lam++;
212     }
213     return {};
214 }
215
216 template <typename T>
217 vector<pair<T, int>> Rho(const T& n) {
218     return RhoC(n, static_cast<T>(1));
219 }
220
221 template <typename T>
222 vector<pair<T, int>> Factorize(T x) {
223     if (x <= 1) {
224         return {};
225     }
226     if (x <= precalculated) {
227         vector<pair<T, int>> ret;
228         while (x > 1) {
229             if (!ret.empty() && ret.back().first == least[x]) {
230                 ret.back().second++;
231             } else {
232                 ret.emplace_back(least[x], 1);
233             }
234             x /= least[x];
235         }
236         return ret;
237     }
238     if (x <= static_cast<int64_t>(precalculated) * precalculated) {
239         vector<pair<T, int>> ret;
240         if (!IsPrime(x)) {
241             for (T i : primes) {

```

```

242     T t = x / i;
243     if (i > t) {
244         break;
245     }
246     if (x == t * i) {
247         int cnt = 0;
248         while (x % i == 0) {
249             x /= i;
250             cnt++;
251         }
252         ret.emplace_back(i, cnt);
253         if (IsPrime(x)) {
254             break;
255         }
256     }
257 }
258 }
259 if (x > 1) {
260     ret.emplace_back(x, 1);
261 }
262 return ret;
263 }
264 return Rho(x);
265 }
266
267 template <typename T>
268 vector<T> BuildDivisorsFromFactors(const vector<pair<T, int>>& factors) {
269     vector<T> divisors = {1};
270     for (auto& p : factors) {
271         int sz = (int) divisors.size();
272         for (int i = 0; i < sz; i++) {
273             T cur = divisors[i];
274             for (int j = 0; j < p.second; j++) {
275                 cur *= p.first;
276                 divisors.push_back(cur);
277             }
278         }
279     }
280     sort(divisors.begin(), divisors.end());
281     return divisors;
282 }
283
284 } // namespace factorizer

```

6.4 fft.cpp

```

1  // make it understandable one day...
2  namespace fft {
3
4  typedef double dbl;
5
6  struct num {
7      dbl x, y;
8      num() { x = y = 0; }
9      num(dbl x_, dbl y_) : x(x_), y(y_) {}
10 };
11
12 inline num operator+(num a, num b) { return num(a.x + b.x, a.y + b.y); }
13 inline num operator-(num a, num b) { return num(a.x - b.x, a.y - b.y); }
14 inline num operator*(num a, num b) { return num(a.x * b.x - a.y * b.y, a.x *
    b.y + a.y * b.x); }
15 inline num conj(num a) { return num(a.x, -a.y); }
16
17 int base = 1;
18 vector<num> roots = {{0, 0}, {1, 0}};
19 vector<int> rev = {0, 1};
20
21 const dbl PI = static_cast<dbl>(acosl(-1.0));
22
23 void ensure_base(int nbase) {
24     if (nbase <= base) {
25         return;
26     }
27     rev.resize(1 << nbase);
28     for (int i = 0; i < (1 << nbase); i++) {
29         rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));
30     }
31     roots.resize(1 << nbase);
32     while (base < nbase) {
33         dbl angle = 2 * PI / (1 << (base + 1));
34         // num z(cos(angle), sin(angle));
35         for (int i = 1 << (base - 1); i < (1 << base); i++) {
36             roots[i << 1] = roots[i];
37             // roots[(i << 1) + 1] = roots[i] * z;
38             dbl angle_i = angle * (2 * i + 1 - (1 << base));
39             roots[(i << 1) + 1] = num(cos(angle_i), sin(angle_i));
40         }
41         base++;

```

```

42     }
43 }
44
45 void fft(vector<num>& a, int n = -1) {
46     if (n == -1) {
47         n = (int) a.size();
48     }
49     assert((n & (n - 1)) == 0);
50     int zeros = __builtin_ctz(n);
51     ensure_base(zeros);
52     int shift = base - zeros;
53     for (int i = 0; i < n; i++) {
54         if (i < (rev[i] >> shift)) {
55             swap(a[i], a[rev[i] >> shift]);
56         }
57     }
58     for (int k = 1; k < n; k <= 1) {
59         for (int i = 0; i < n; i += 2 * k) {
60             for (int j = 0; j < k; j++) {
61                 num z = a[i + j + k] * roots[j + k];
62                 a[i + j + k] = a[i + j] - z;
63                 a[i + j] = a[i + j] + z;
64             }
65         }
66     }
67 }
68
69 vector<num> fa, fb;
70
71 vector<int64_t> square(const vector<int>& a) {
72     if (a.empty()) {
73         return {};
74     }
75     int need = (int) a.size() + (int) a.size() - 1;
76     int nbase = 1;
77     while ((1 << nbase) < need) nbase++;
78     ensure_base(nbase);
79     int sz = 1 << nbase;
80     if ((sz >> 1) > (int) fa.size()) {
81         fa.resize(sz >> 1);
82     }
83     for (int i = 0; i < (sz >> 1); i++) {
84         int x = (2 * i < (int) a.size() ? a[2 * i] : 0);

```

```

85         int y = (2 * i + 1 < (int) a.size() ? a[2 * i + 1] : 0);
86         fa[i] = num(x, y);
87     }
88     fft(fa, sz >> 1);
89     num r(1.0 / (sz >> 1), 0.0);
90     for (int i = 0; i <= (sz >> 2); i++) {
91         int j = ((sz >> 1) - i) & ((sz >> 1) - 1);
92         num fe = (fa[i] + conj(fa[j])) * num(0.5, 0);
93         num fo = (fa[i] - conj(fa[j])) * num(0, -0.5);
94         num aux = fe * fe + fo * fo * roots[(sz >> 1) + i] * roots[(sz >> 1) + i];
95         num tmp = fe * fo;
96         fa[i] = r * (conj(aux) + num(0, 2) * conj(tmp));
97         fa[j] = r * (aux + num(0, 2) * tmp);
98     }
99     fft(fa, sz >> 1);
100     vector<int64_t> res(need);
101     for (int i = 0; i < need; i++) {
102         res[i] = llround(i % 2 == 0 ? fa[i >> 1].x : fa[i >> 1].y);
103     }
104     return res;
105 }
106
107 vector<int64_t> multiply(const vector<int>& a, const vector<int>& b) {
108     if (a.empty() || b.empty()) {
109         return {};
110     }
111     if (a == b) {
112         return square(a);
113     }
114     int need = (int) a.size() + (int) b.size() - 1;
115     int nbase = 1;
116     while ((1 << nbase) < need) nbase++;
117     ensure_base(nbase);
118     int sz = 1 << nbase;
119     if (sz > (int) fa.size()) {
120         fa.resize(sz);
121     }
122     for (int i = 0; i < sz; i++) {
123         int x = (i < (int) a.size() ? a[i] : 0);
124         int y = (i < (int) b.size() ? b[i] : 0);
125         fa[i] = num(x, y);
126     }

```

```

127     fft(fa, sz);
128     num r(0, -0.25 / (sz >> 1));
129     for (int i = 0; i <= (sz >> 1); i++) {
130         int j = (sz - i) & (sz - 1);
131         num z = (fa[j] * fa[j] - conj(fa[i] * fa[i])) * r;
132         fa[j] = (fa[i] * fa[i] - conj(fa[j] * fa[j])) * r;
133         fa[i] = z;
134     }
135     for (int i = 0; i < (sz >> 1); i++) {
136         num A0 = (fa[i] + fa[i + (sz >> 1)]) * num(0.5, 0);
137         num A1 = (fa[i] - fa[i + (sz >> 1)]) * num(0.5, 0) * roots[(sz >> 1) + i
            ];
138         fa[i] = A0 + A1 * num(0, 1);
139     }
140     fft(fa, sz >> 1);
141     vector<int64_t> res(need);
142     for (int i = 0; i < need; i++) {
143         res[i] = llround(i % 2 == 0 ? fa[i >> 1].x : fa[i >> 1].y);
144     }
145     return res;
146 }
147
148 vector<int> multiply_mod(const vector<int>& a, const vector<int>& b, int m)
149 {
150     if (a.empty() || b.empty()) {
151         return {};
152     }
153     int eq = (a.size() == b.size() && a == b);
154     int need = (int) a.size() + (int) b.size() - 1;
155     int nbase = 0;
156     while ((1 << nbase) < need) nbase++;
157     ensure_base(nbase);
158     int sz = 1 << nbase;
159     if (sz > (int) fa.size()) {
160         fa.resize(sz);
161     }
162     for (int i = 0; i < (int) a.size(); i++) {
163         int x = (a[i] % m + m) % m;
164         fa[i] = num(x & ((1 << 15) - 1), x >> 15);
165     }
166     fill(fa.begin() + a.size(), fa.begin() + sz, num {0, 0});
167     fft(fa, sz);
168     if (sz > (int) fb.size()) {

```

```

168         fb.resize(sz);
169     }
170     if (eq) {
171         copy(fa.begin(), fa.begin() + sz, fb.begin());
172     } else {
173         for (int i = 0; i < (int) b.size(); i++) {
174             int x = (b[i] % m + m) % m;
175             fb[i] = num(x & ((1 << 15) - 1), x >> 15);
176         }
177         fill(fb.begin() + b.size(), fb.begin() + sz, num {0, 0});
178         fft(fb, sz);
179     }
180     dbl ratio = 0.25 / sz;
181     num r2(0, -1);
182     num r3(ratio, 0);
183     num r4(0, -ratio);
184     num r5(0, 1);
185     for (int i = 0; i <= (sz >> 1); i++) {
186         int j = (sz - i) & (sz - 1);
187         num a1 = (fa[i] + conj(fa[j]));
188         num a2 = (fa[i] - conj(fa[j])) * r2;
189         num b1 = (fb[i] + conj(fb[j])) * r3;
190         num b2 = (fb[i] - conj(fb[j])) * r4;
191         if (i != j) {
192             num c1 = (fa[j] + conj(fa[i]));
193             num c2 = (fa[j] - conj(fa[i])) * r2;
194             num d1 = (fb[j] + conj(fb[i])) * r3;
195             num d2 = (fb[j] - conj(fb[i])) * r4;
196             fa[i] = c1 * d1 + c2 * d2 * r5;
197             fb[i] = c1 * d2 + c2 * d1;
198         }
199         fa[j] = a1 * b1 + a2 * b2 * r5;
200         fb[j] = a1 * b2 + a2 * b1;
201     }
202     fft(fa, sz);
203     fft(fb, sz);
204     vector<int> res(need);
205     for (int i = 0; i < need; i++) {
206         int64_t aa = llround(fa[i].x);
207         int64_t bb = llround(fb[i].x);
208         int64_t cc = llround(fa[i].y);
209         res[i] = static_cast<int>((aa + ((bb % m) << 15) + ((cc % m) << 30)) % m
            );

```

```

210     }
211     return res;
212 }
213
214 } // namespace fft
215
216 template <typename T>
217 typename enable_if<is_same<typename Modular<T>::Type, int>::value, vector<
    Modular<T>>>::type operator*(
218     const vector<Modular<T>>& a,
219     const vector<Modular<T>>& b) {
220     if (a.empty() || b.empty()) {
221         return {};
222     }
223     if (min(a.size(), b.size()) < 150) {
224         vector<Modular<T>> c(a.size() + b.size() - 1, 0);
225         for (int i = 0; i < (int) a.size(); i++) {
226             for (int j = 0; j < (int) b.size(); j++) {
227                 c[i + j] += a[i] * b[j];
228             }
229         }
230         return c;
231     }
232     vector<int> a_mul(a.size());
233     for (int i = 0; i < (int) a.size(); i++) {
234         a_mul[i] = static_cast<int>(a[i]);
235     }
236     vector<int> b_mul(b.size());
237     for (int i = 0; i < (int) b.size(); i++) {
238         b_mul[i] = static_cast<int>(b[i]);
239     }
240     vector<int> c_mul = fft::multiply_mod(a_mul, b_mul, T::value);
241     vector<Modular<T>> c(c_mul.size());
242     for (int i = 0; i < (int) c.size(); i++) {
243         c[i] = c_mul[i];
244     }
245     return c;
246 }
247
248 template <typename T>
249 typename enable_if<is_same<typename Modular<T>::Type, int>::value, vector<
    Modular<T>>>::type& operator*=(
250     vector<Modular<T>>& a,

```

```

251     const vector<Modular<T>>& b) {
252     return a = a * b;
253 }

```

6.5 fwht.cpp

```

1 namespace fwht {
2
3 template<typename T>
4 void hadamard(vector<T> &a) {
5     int n = a.size();
6     for (int k = 1; k < n; k <= 1) {
7         for (int i = 0; i < n; i += 2 * k) {
8             for (int j = 0; j < k; j++) {
9                 T x = a[i + j];
10                T y = a[i + j + k];
11                a[i + j] = x + y;
12                a[i + j + k] = x - y;
13            }
14        }
15    }
16 }
17
18 template<typename T>
19 vector<T> multiply(vector<T> a, vector<T> b) {
20     int eq = (a == b);
21     int n = 1;
22     while (n < (int) max(a.size(), b.size())) {
23         n <= 1;
24     }
25     a.resize(n);
26     b.resize(n);
27     hadamard(a);
28     if (eq) b = a; else hadamard(b);
29     for (int i = 0; i < n; i++) {
30         a[i] *= b[i];
31     }
32     hadamard(a);
33     T q = 1 / static_cast<T>(n);
34     for (int i = 0; i < n; i++) {
35         a[i] *= q;
36     }
37     return a;

```



```

38 }
39
40 } // namespace fwht

```

6.6 gauss.cpp

```

1  const double eps = 1e-9;
2
3  bool IsZero(double v) {
4      return abs(v) < 1e-9;
5  }
6
7  enum GAUSS_MODE {
8      DEGREE, ABS
9  };
10
11 template <typename T>
12 void GaussianElimination(vector<vector<T>>& a, int limit, GAUSS_MODE mode =
    DEGREE) {
13     if (a.empty() || a[0].empty()) {
14         return;
15     }
16     int h = static_cast<int>(a.size());
17     int w = static_cast<int>(a[0].size());
18     for (int i = 0; i < h; i++) {
19         assert(w == static_cast<int>(a[i].size()));
20     }
21     assert(limit <= w);
22     vector<int> deg(h);
23     for (int i = 0; i < h; i++) {
24         for (int j = 0; j < w; j++) {
25             deg[i] += !IsZero(a[i][j]);
26         }
27     }
28     int r = 0;
29     for (int c = 0; c < limit; c++) {
30         int id = -1;
31         for (int i = r; i < h; i++) {
32             if (!IsZero(a[i][c]) && (id == -1 || (mode == DEGREE && deg[i] < deg[
                id]) || (mode == ABS && abs(a[id][c]) < abs(a[i][c])))) {
33                 id = i;
34             }
35         }

```

```

36         if (id == -1) {
37             continue;
38         }
39         if (id > r) {
40             swap(a[r], a[id]);
41             swap(deg[r], deg[id]);
42             for (int j = c; j < w; j++) {
43                 a[id][j] = -a[id][j];
44             }
45         }
46         vector<int> nonzero;
47         for (int j = c; j < w; j++) {
48             if (!IsZero(a[r][j])) {
49                 nonzero.push_back(j);
50             }
51         }
52         T inv_a = 1 / a[r][c];
53         for (int i = r + 1; i < h; i++) {
54             if (IsZero(a[i][c])) {
55                 continue;
56             }
57             T coeff = -a[i][c] * inv_a;
58             for (int j : nonzero) {
59                 if (!IsZero(a[i][j])) deg[i]--;
60                 a[i][j] += coeff * a[r][j];
61                 if (!IsZero(a[i][j])) deg[i]++;
62             }
63         }
64         ++r;
65     }
66     for (r = h - 1; r >= 0; r--) {
67         for (int c = 0; c < limit; c++) {
68             if (!IsZero(a[r][c])) {
69                 T inv_a = 1 / a[r][c];
70                 for (int i = r - 1; i >= 0; i--) {
71                     if (IsZero(a[i][c])) {
72                         continue;
73                     }
74                     T coeff = -a[i][c] * inv_a;
75                     for (int j = c; j < w; j++) {
76                         a[i][j] += coeff * a[r][j];
77                     }
78                 }

```

```

79         break;
80     }
81 }
82 }
83 }
84
85 template <typename T>
86 T Determinant(vector<vector<T>> /*&a*/ a) {
87     if (a.empty()) {
88         return T{1};
89     }
90     assert(a.size() == a[0].size());
91     GaussianElimination(a, static_cast<int>(a[0].size()));
92     T d{1};
93     for (int i = 0; i < a.h; i++) {
94         d *= a[i][i];
95     }
96     return d;
97 }
98
99 template <typename T>
100 int Rank(vector<vector<T>> /*&a*/ a) {
101     if (a.empty()) {
102         return 0;
103     }
104     GaussianElimination(a, static_cast<int>(a[0].size()));
105     int rank = 0;
106     for (int i = 0; i < static_cast<int>(a.size()); i++) {
107         for (int j = 0; j < static_cast<int>(a[i].size()); j++) {
108             if (!IsZero(a[i][j])) {
109                 ++rank;
110                 break;
111             }
112         }
113     }
114     return rank;
115 }
116
117 template <typename T>
118 vector<T> SolveLinearSystem(vector<vector<T>> /*&a*/ a, const vector<T>& b,
119                             int w) {
120     int h = static_cast<int>(a.size());
121     assert(h == static_cast<int>(b.size()));

```

```

121     if (h > 0) {
122         assert(w == static_cast<int>(a[0].size()));
123     }
124     for (int i = 0; i < h; i++) {
125         a[i].push_back(b[i]);
126     }
127     GaussianElimination(a, w);
128     vector<T> x(w, 0);
129     for (int i = 0; i < h; i++) {
130         for (int j = 0; j < w; j++) {
131             if (!IsZero(a[i][j])) {
132                 x[j] = a[i][w] / a[i][j];
133                 break;
134             }
135         }
136     }
137     return x;
138 }
139
140 template <typename T>
141 vector<vector<T>> Inverse(vector<vector<T>> /*&a*/ a) {
142     if (a.empty()) {
143         return a;
144     }
145     int h = static_cast<int>(a.size());
146     for (int i = 0; i < h; i++) {
147         assert(h == static_cast<int>(a[i].size()));
148     }
149     for (int i = 0; i < h; i++) {
150         a[i].resize(2 * h);
151         a[i][i + h] = 1;
152     }
153     GaussianElimination(a, h);
154     for (int i = 0; i < h; i++) {
155         if (IsZero(a[i][i])) {
156             return {{}};
157         }
158     }
159     vector<vector<T>> b(h);
160     for (int i = 0; i < h; i++) {
161         b[i] = vector<T>(a[i].begin() + h, a[i].end());
162         T coeff = 1 / a[i][i];
163         for (int j = 0; j < h; j++) {

```

```

164     b[i][j] *= coeff;
165 }
166 }
167 return b;
168 }

```

6.7 matrix.cpp

```

1 template <typename T, size_t N, size_t M, size_t K>
2 array<array<T, K>, N> operator*(const array<array<T, M>, N>& a, const array<
    array<T, K>, M>& b) {
3     array<array<T, K>, N> c;
4     for (size_t i = 0; i < N; i++) {
5         for (size_t j = 0; j < K; j++) {
6             c[i][j] = 0;
7             for (size_t k = 0; k < M; k++) {
8                 c[i][j] += a[i][k] * b[k][j];
9             }
10        }
11    }
12    return c;
13 }
14
15 template <typename T>
16 vector<vector<T>> operator*(const vector<vector<T>>& a, const vector<vector<
    T>>& b) {
17     if (a.empty() || b.empty()) {
18         return {{}};
19     }
20     vector<vector<T>> c(a.size(), vector<T>(b[0].size()));
21     for (int i = 0; i < static_cast<int>(c.size()); i++) {
22         for (int j = 0; j < static_cast<int>(c[0].size()); j++) {
23             c[i][j] = 0;
24             for (int k = 0; k < static_cast<int>(b.size()); k++) {
25                 c[i][j] += a[i][k] * b[k][j];
26             }
27         }
28     }
29     return c;
30 }
31
32 template <typename T>
33 vector<vector<T>>& operator*=(vector<vector<T>>& a, const vector<vector<T>>&

```

```

    b) {
34     return a = a * b;
35 }
36
37 template <typename T, typename U>
38 vector<vector<T>> power(const vector<vector<T>>& a, const U& b) {
39     assert(b >= 0);
40     vector<U> binary;
41     U bb = b;
42     while (bb > 0) {
43         binary.push_back(bb & 1);
44         bb >>= 1;
45     }
46     vector<vector<T>> res(a.size(), vector<T>(a.size()));
47     for (int i = 0; i < static_cast<int>(a.size()); i++) {
48         res[i][i] = 1;
49     }
50     for (int j = (int) binary.size() - 1; j >= 0; j--) {
51         res *= res;
52         if (binary[j] == 1) {
53             res *= a;
54         }
55     }
56     return res;
57 }

```

6.8 mint.cpp

```

1 template <typename T>
2 T inverse(T a, T m) {
3     T u = 0, v = 1;
4     while (a != 0) {
5         T t = m / a;
6         m -= t * a; swap(a, m);
7         u -= t * v; swap(u, v);
8     }
9     assert(m == 1);
10    return u;
11 }
12
13 template <typename T>
14 class Modular {
15 public:

```

```

16 using Type = typename decay<decltype(T::value)>::type;
17
18 constexpr Modular() : value() {}
19 template <typename U>
20 Modular(const U& x) {
21     value = normalize(x);
22 }
23
24 template <typename U>
25 static Type normalize(const U& x) {
26     Type v;
27     if (-mod() <= x && x < mod()) v = static_cast<Type>(x);
28     else v = static_cast<Type>(x % mod());
29     if (v < 0) v += mod();
30     return v;
31 }
32
33 const Type& operator()() const { return value; }
34 template <typename U>
35 explicit operator U() const { return static_cast<U>(value); }
36 constexpr static Type mod() { return T::value; }
37
38 Modular& operator+=(const Modular& other) { if ((value += other.value) >=
    mod()) value -= mod(); return *this; }
39 Modular& operator-=(const Modular& other) { if ((value -= other.value) <
    0) value += mod(); return *this; }
40 template <typename U> Modular& operator+=(const U& other) { return *this
    += Modular(other); }
41 template <typename U> Modular& operator-=(const U& other) { return *this
    -= Modular(other); }
42 Modular& operator++() { return *this += 1; }
43 Modular& operator--() { return *this -= 1; }
44 Modular operator++(int) { Modular result(*this); *this += 1; return result
    ; }
45 Modular operator--(int) { Modular result(*this); *this -= 1; return result
    ; }
46 Modular operator-() const { return Modular(-value); }
47
48 template <typename U = T>
49 typename enable_if<is_same<typename Modular<U>::Type, int>::value, Modular
    >::type& operator*=(const Modular& rhs) {
50     value = normalize(static_cast<int64_t>(value) * static_cast<int64_t>(rhs
        .value));

```

```

51     return *this;
52 }
53 template <typename U = T>
54 typename enable_if<is_same<typename Modular<U>::Type, long long>::value,
    Modular>::type& operator*=(const Modular& rhs) {
55     long long q = static_cast<long long>(static_cast<long double>(value) *
        rhs.value / mod());
56     value = normalize(value * rhs.value - q * mod());
57     return *this;
58 }
59 template <typename U = T>
60 typename enable_if<!is_integral<typename Modular<U>::Type>::value, Modular
    >::type& operator*=(const Modular& rhs) {
61     value = normalize(value * rhs.value);
62     return *this;
63 }
64
65 Modular& operator/=(const Modular& other) { return *this *= Modular(
    inverse(other.value, mod())); }
66
67 friend const Type& abs(const Modular& x) { return x.value; }
68
69 template <typename U>
70 friend bool operator==(const Modular<U>& lhs, const Modular<U>& rhs);
71
72 template <typename U>
73 friend bool operator<(const Modular<U>& lhs, const Modular<U>& rhs);
74
75 template <typename V, typename U>
76 friend V& operator>>(V& stream, Modular<U>& number);
77
78 private:
79     Type value;
80 };
81
82 template <typename T> bool operator==(const Modular<T>& lhs, const Modular<T
    >& rhs) { return lhs.value == rhs.value; }
83 template <typename T, typename U> bool operator==(const Modular<T>& lhs, U
    rhs) { return lhs == Modular<T>(rhs); }
84 template <typename T, typename U> bool operator==(U lhs, const Modular<T>&
    rhs) { return Modular<T>(lhs) == rhs; }
85
86 template <typename T> bool operator!=(const Modular<T>& lhs, const Modular<T

```

```

    >& rhs) { return !(lhs == rhs); }
87 template <typename T, typename U> bool operator!=(const Modular<T>& lhs, U
    rhs) { return !(lhs == rhs); }
88 template <typename T, typename U> bool operator!=(U lhs, const Modular<T>&
    rhs) { return !(lhs == rhs); }
89
90 template <typename T> bool operator<(const Modular<T>& lhs, const Modular<T
    >& rhs) { return lhs.value < rhs.value; }
91
92 template <typename T> Modular<T> operator+(const Modular<T>& lhs, const
    Modular<T>& rhs) { return Modular<T>(lhs) += rhs; }
93 template <typename T, typename U> Modular<T> operator+(const Modular<T>& lhs
    , U rhs) { return Modular<T>(lhs) += rhs; }
94 template <typename T, typename U> Modular<T> operator+(U lhs, const Modular<
    T>& rhs) { return Modular<T>(lhs) += rhs; }
95
96 template <typename T> Modular<T> operator-(const Modular<T>& lhs, const
    Modular<T>& rhs) { return Modular<T>(lhs) -= rhs; }
97 template <typename T, typename U> Modular<T> operator-(const Modular<T>& lhs
    , U rhs) { return Modular<T>(lhs) -= rhs; }
98 template <typename T, typename U> Modular<T> operator-(U lhs, const Modular<
    T>& rhs) { return Modular<T>(lhs) -= rhs; }
99
100 template <typename T> Modular<T> operator*(const Modular<T>& lhs, const
    Modular<T>& rhs) { return Modular<T>(lhs) *= rhs; }
101 template <typename T, typename U> Modular<T> operator*(const Modular<T>& lhs
    , U rhs) { return Modular<T>(lhs) *= rhs; }
102 template <typename T, typename U> Modular<T> operator*(U lhs, const Modular<
    T>& rhs) { return Modular<T>(lhs) *= rhs; }
103
104 template <typename T> Modular<T> operator/(const Modular<T>& lhs, const
    Modular<T>& rhs) { return Modular<T>(lhs) /= rhs; }
105 template <typename T, typename U> Modular<T> operator/(const Modular<T>& lhs
    , U rhs) { return Modular<T>(lhs) /= rhs; }
106 template <typename T, typename U> Modular<T> operator/(U lhs, const Modular<
    T>& rhs) { return Modular<T>(lhs) /= rhs; }
107
108 template<typename T, typename U>
109 Modular<T> power(const Modular<T>& a, const U& b) {
110     assert(b >= 0);
111     Modular<T> x = a, res = 1;
112     U p = b;
113     while (p > 0) {

```

```

114         if (p & 1) res *= x;
115         x *= x;
116         p >>= 1;
117     }
118     return res;
119 }
120
121 template <typename T>
122 bool IsZero(const Modular<T>& number) {
123     return number() == 0;
124 }
125
126 template <typename T>
127 string to_string(const Modular<T>& number) {
128     return to_string(number());
129 }
130
131 // U == std::ostream? but done this way because of fastoutput
132 template <typename U, typename T>
133 U& operator<<(U& stream, const Modular<T>& number) {
134     return stream << number();
135 }
136
137 // U == std::istream? but done this way because of fastinput
138 template <typename U, typename T>
139 U& operator>>(U& stream, Modular<T>& number) {
140     typename common_type<typename Modular<T>::Type, long long>::type x;
141     stream >> x;
142     number.value = Modular<T>::normalize(x);
143     return stream;
144 }
145
146 // using ModType = int;
147
148 // struct VarMod { static ModType value; };
149 // ModType VarMod::value;
150 // ModType& md = VarMod::value;
151 // using Mint = Modular<VarMod>;
152
153 constexpr int md = ${0};
154 using Mint = Modular<std::integral_constant<decay<decltype(md)>::type, md>>;
155
156 // vector<Mint> fact(1, 1);

```

```

157 // vector<Mint> inv_fact(1, 1);
158
159 // Mint C(int n, int k) {
160 //     if (k < 0 || k > n) {
161 //         return 0;
162 //     }
163 //     while ((int) fact.size() < n + 1) {
164 //         fact.push_back(fact.back() * (int) fact.size());
165 //         inv_fact.push_back(1 / fact.back());
166 //     }
167 //     return fact[n] * inv_fact[k] * inv_fact[n - k];
168 // }

```

6.9 ntt.cpp

```

1  template <typename T>
2  class NTT {
3  public:
4      using Type = typename decay<decltype(T::value)>::type;
5
6      static Type md;
7      static Modular<T> root;
8      static int base;
9      static int max_base;
10     static vector<Modular<T>> roots;
11     static vector<int> rev;
12
13     static void clear() {
14         root = 0;
15         base = 0;
16         max_base = 0;
17         roots.clear();
18         rev.clear();
19     }
20
21     static void init() {
22         md = T::value;
23         assert(md >= 3 && md % 2 == 1);
24         auto tmp = md - 1;
25         max_base = 0;
26         while (tmp % 2 == 0) {
27             tmp /= 2;
28             max_base++;

```

```

29     }
30     root = 2;
31     while (power(root, (md - 1) >> 1) == 1) {
32         root++;
33     }
34     assert(power(root, md - 1) == 1);
35     root = power(root, (md - 1) >> max_base);
36     base = 1;
37     rev = {0, 1};
38     roots = {0, 1};
39 }
40
41 static void ensure_base(int nbase) {
42     if (md != T::value) {
43         clear();
44     }
45     if (roots.empty()) {
46         init();
47     }
48     if (nbase <= base) {
49         return;
50     }
51     assert(nbase <= max_base);
52     rev.resize(1 << nbase);
53     for (int i = 0; i < (1 << nbase); i++) {
54         rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));
55     }
56     roots.resize(1 << nbase);
57     while (base < nbase) {
58         Modular<T> z = power(root, 1 << (max_base - 1 - base));
59         for (int i = 1 << (base - 1); i < (1 << base); i++) {
60             roots[i << 1] = roots[i];
61             roots[(i << 1) + 1] = roots[i] * z;
62         }
63         base++;
64     }
65 }
66
67 static void fft(vector<Modular<T>> &a) {
68     int n = (int) a.size();
69     assert((n & (n - 1)) == 0);
70     int zeros = __builtin_ctz(n);
71     ensure_base(zeros);

```

```

72     int shift = base - zeros;
73     for (int i = 0; i < n; i++) {
74         if (i < (rev[i] >> shift)) {
75             swap(a[i], a[rev[i] >> shift]);
76         }
77     }
78     for (int k = 1; k < n; k <= 1) {
79         for (int i = 0; i < n; i += 2 * k) {
80             for (int j = 0; j < k; j++) {
81                 Modular<T> x = a[i + j];
82                 Modular<T> y = a[i + j + k] * roots[j + k];
83                 a[i + j] = x + y;
84                 a[i + j + k] = x - y;
85             }
86         }
87     }
88 }
89
90 static vector<Modular<T>> multiply(vector<Modular<T>> a, vector<Modular<T
91     >> b) {
92     if (a.empty() || b.empty()) {
93         return {};
94     }
95     int eq = (a == b);
96     int need = (int) a.size() + (int) b.size() - 1;
97     int nbase = 0;
98     while ((1 << nbase) < need) nbase++;
99     ensure_base(nbase);
100     int sz = 1 << nbase;
101     a.resize(sz);
102     b.resize(sz);
103     fft(a);
104     if (eq) b = a; else fft(b);
105     Modular<T> inv_sz = 1 / static_cast<Modular<T>>(sz);
106     for (int i = 0; i < sz; i++) {
107         a[i] *= b[i] * inv_sz;
108     }
109     reverse(a.begin() + 1, a.end());
110     fft(a);
111     a.resize(need);
112     return a;
113 }

```

```

114
115 template <typename T> typename NTT<T>::Type NTT<T>::md;
116 template <typename T> Modular<T> NTT<T>::root;
117 template <typename T> int NTT<T>::base;
118 template <typename T> int NTT<T>::max_base;
119 template <typename T> vector<Modular<T>> NTT<T>::roots;
120 template <typename T> vector<int> NTT<T>::rev;
121
122 template <typename T>
123 vector<Modular<T>> inverse(const vector<Modular<T>>& a) {
124     assert(!a.empty());
125     int n = (int) a.size();
126     vector<Modular<T>> b = {1 / a[0]};
127     while ((int) b.size() < n) {
128         vector<Modular<T>> x(a.begin(), a.begin() + min(a.size(), b.size() << 1)
129             );
130         x.resize(b.size() << 1);
131         b.resize(b.size() << 1);
132         vector<Modular<T>> c = b;
133         NTT<T>::fft(c);
134         NTT<T>::fft(x);
135         Modular<T> inv = 1 / static_cast<Modular<T>>((int) x.size());
136         for (int i = 0; i < (int) x.size(); i++) {
137             x[i] *= c[i] * inv;
138         }
139         reverse(x.begin() + 1, x.end());
140         NTT<T>::fft(x);
141         rotate(x.begin(), x.begin() + (x.size() >> 1), x.end());
142         fill(x.begin() + (x.size() >> 1), x.end(), 0);
143         NTT<T>::fft(x);
144         for (int i = 0; i < (int) x.size(); i++) {
145             x[i] *= c[i] * inv;
146         }
147         reverse(x.begin() + 1, x.end());
148         NTT<T>::fft(x);
149         for (int i = 0; i < ((int) x.size() >> 1); i++) {
150             b[i + ((int) x.size() >> 1)] = -x[i];
151         }
152     }
153     b.resize(n);
154     return b;
155 }

```

```

156 template <typename T>
157 vector<Modular<T>> inverse_old(vector<Modular<T>> a) {
158     assert(!a.empty());
159     int n = (int) a.size();
160     if (n == 1) {
161         return {1 / a[0]};
162     }
163     int m = (n + 1) >> 1;
164     vector<Modular<T>> b = inverse(vector<Modular<T>>(a.begin(), a.begin() + m
        ));
165     int need = n << 1;
166     int nbase = 0;
167     while ((1 << nbase) < need) {
168         ++nbase;
169     }
170     NTT<T>::ensure_base(nbase);
171     int size = 1 << nbase;
172     a.resize(size);
173     b.resize(size);
174     NTT<T>::fft(a);
175     NTT<T>::fft(b);
176     Modular<T> inv = 1 / static_cast<Modular<T>>(size);
177     for (int i = 0; i < size; ++i) {
178         a[i] = (2 - a[i] * b[i]) * b[i] * inv;
179     }
180     reverse(a.begin() + 1, a.end());
181     NTT<T>::fft(a);
182     a.resize(n);
183     return a;
184 }
185
186 template <typename T>
187 vector<Modular<T>> operator*(const vector<Modular<T>>& a, const vector<
    Modular<T>>& b) {
188     if (a.empty() || b.empty()) {
189         return {};
190     }
191     if (min(a.size(), b.size()) < 150) {
192         vector<Modular<T>> c(a.size() + b.size() - 1, 0);
193         for (int i = 0; i < (int) a.size(); i++) {
194             for (int j = 0; j < (int) b.size(); j++) {
195                 c[i + j] += a[i] * b[j];
196             }

```

```

197     }
198     return c;
199 }
200 return NTT<T>::multiply(a, b);
201 }
202
203 template <typename T>
204 vector<Modular<T>>& operator*=(vector<Modular<T>>& a, const vector<Modular<T
    >>& b) {
205     return a = a * b;
206 }

```

6.10 poly.cpp

```

1 template <typename T>
2 vector<T>& operator+=(vector<T>& a, const vector<T>& b) {
3     if (a.size() < b.size()) {
4         a.resize(b.size());
5     }
6     for (int i = 0; i < (int) b.size(); i++) {
7         a[i] += b[i];
8     }
9     return a;
10 }
11
12 template <typename T>
13 vector<T> operator+(const vector<T>& a, const vector<T>& b) {
14     vector<T> c = a;
15     return c += b;
16 }
17
18 template <typename T>
19 vector<T>& operator--=(vector<T>& a, const vector<T>& b) {
20     if (a.size() < b.size()) {
21         a.resize(b.size());
22     }
23     for (int i = 0; i < (int) b.size(); i++) {
24         a[i] -= b[i];
25     }
26     return a;
27 }
28
29 template <typename T>

```



```

30 vector<T> operator-(const vector<T>& a, const vector<T>& b) {
31     vector<T> c = a;
32     return c -= b;
33 }
34
35 template <typename T>
36 vector<T> operator-(const vector<T>& a) {
37     vector<T> c = a;
38     for (int i = 0; i < (int) c.size(); i++) {
39         c[i] = -c[i];
40     }
41     return c;
42 }
43
44 template <typename T>
45 vector<T> operator*(const vector<T>& a, const vector<T>& b) {
46     if (a.empty() || b.empty()) {
47         return {};
48     }
49     vector<T> c(a.size() + b.size() - 1, 0);
50     for (int i = 0; i < (int) a.size(); i++) {
51         for (int j = 0; j < (int) b.size(); j++) {
52             c[i + j] += a[i] * b[j];
53         }
54     }
55     return c;
56 }
57
58 template <typename T>
59 vector<T>& operator*=(vector<T>& a, const vector<T>& b) {
60     return a = a * b;
61 }
62
63 template <typename T>
64 vector<T> inverse(const vector<T>& a) {
65     assert(!a.empty());
66     int n = (int) a.size();
67     vector<T> b = {1 / a[0]};
68     while ((int) b.size() < n) {
69         vector<T> a_cut(a.begin(), a.begin() + min(a.size(), b.size() << 1));
70         vector<T> x = b * b * a_cut;
71         b.resize(b.size() << 1);
72         for (int i = (int) b.size() >> 1; i < (int) min(x.size(), b.size()); i

```

```

        ++) {
73         b[i] = -x[i];
74     }
75 }
76 b.resize(n);
77 return b;
78 }
79
80 template <typename T>
81 vector<T>& operator/=(vector<T>& a, const vector<T>& b) {
82     int n = (int) a.size();
83     int m = (int) b.size();
84     if (n < m) {
85         a.clear();
86     } else {
87         vector<T> d = b;
88         reverse(a.begin(), a.end());
89         reverse(d.begin(), d.end());
90         d.resize(n - m + 1);
91         a *= inverse(d);
92         a.erase(a.begin() + n - m + 1, a.end());
93         reverse(a.begin(), a.end());
94     }
95     return a;
96 }
97
98 template <typename T>
99 vector<T> operator/(const vector<T>& a, const vector<T>& b) {
100     vector<T> c = a;
101     return c /= b;
102 }
103
104 template <typename T>
105 vector<T>& operator%=(vector<T>& a, const vector<T>& b) {
106     int n = (int) a.size();
107     int m = (int) b.size();
108     if (n >= m) {
109         vector<T> c = (a / b) * b;
110         a.resize(m - 1);
111         for (int i = 0; i < m - 1; i++) {
112             a[i] -= c[i];
113         }
114     }

```

```

115     return a;
116 }
117
118 template <typename T>
119 vector<T> operator%(const vector<T>& a, const vector<T>& b) {
120     vector<T> c = a;
121     return c %= b;
122 }
123
124 template <typename T, typename U>
125 vector<T> power(const vector<T>& a, const U& b, const vector<T>& c) {
126     assert(b >= 0);
127     vector<U> binary;
128     U bb = b;
129     while (bb > 0) {
130         binary.push_back(bb & 1);
131         bb >>= 1;
132     }
133     vector<T> res = vector<T>{1} % c;
134     for (int j = (int) binary.size() - 1; j >= 0; j--) {
135         res = res * res % c;
136         if (binary[j] == 1) {
137             res = res * a % c;
138         }
139     }
140     return res;
141 }
142
143 template <typename T>
144 vector<T> derivative(const vector<T>& a) {
145     vector<T> c = a;
146     for (int i = 0; i < (int) c.size(); i++) {
147         c[i] *= i;
148     }
149     if (!c.empty()) {
150         c.erase(c.begin());
151     }
152     return c;
153 }
154
155 template <typename T>
156 vector<T> primitive(const vector<T>& a) {
157     vector<T> c = a;

```

```

158     c.insert(c.begin(), 0);
159     for (int i = 1; i < (int) c.size(); i++) {
160         c[i] /= i;
161     }
162     return c;
163 }
164
165 template <typename T>
166 vector<T> logarithm(const vector<T>& a) {
167     assert(!a.empty() && a[0] == 1);
168     vector<T> u = primitive(derivative(a) * inverse(a));
169     u.resize(a.size());
170     return u;
171 }
172
173 template <typename T>
174 vector<T> exponent(const vector<T>& a) {
175     assert(!a.empty() && a[0] == 0);
176     int n = (int) a.size();
177     vector<T> b = {1};
178     while ((int) b.size() < n) {
179         vector<T> x(a.begin(), a.begin() + min(a.size(), b.size() << 1));
180         x[0] += 1;
181         vector<T> old_b = b;
182         b.resize(b.size() << 1);
183         x -= logarithm(b);
184         x *= old_b;
185         for (int i = (int) b.size() >> 1; i < (int) min(x.size(), b.size()); i
            ++) {
186             b[i] = x[i];
187         }
188     }
189     b.resize(n);
190     return b;
191 }
192
193 template <typename T>
194 vector<T> sqrt(const vector<T>& a) {
195     assert(!a.empty() && a[0] == 1);
196     int n = (int) a.size();
197     vector<T> b = {1};
198     while ((int) b.size() < n) {
199         vector<T> x(a.begin(), a.begin() + min(a.size(), b.size() << 1));

```

```

200     b.resize(b.size() << 1);
201     x *= inverse(b);
202     T inv2 = 1 / static_cast<T>(2);
203     for (int i = (int) b.size() >> 1; i < (int) min(x.size(), b.size()); i
        ++){
204         b[i] = x[i] * inv2;
205     }
206 }
207 b.resize(n);
208 return b;
209 }
210
211 template <typename T>
212 vector<T> multiply(const vector<vector<T>>& a) {
213     if (a.empty()) {
214         return {0};
215     }
216     function<vector<T>(int, int)> mult = [&](int l, int r) {
217         if (l == r) {
218             return a[l];
219         }
220         int y = (l + r) >> 1;
221         return mult(l, y) * mult(y + 1, r);
222     };
223     return mult(0, (int) a.size() - 1);
224 }
225
226 template <typename T>
227 T evaluate(const vector<T>& a, const T& x) {
228     T res = 0;
229     for (int i = (int) a.size() - 1; i >= 0; i--) {
230         res = res * x + a[i];
231     }
232     return res;
233 }
234
235 template <typename T>
236 vector<T> evaluate(const vector<T>& a, const vector<T>& x) {
237     if (x.empty()) {
238         return {};
239     }
240     if (a.empty()) {
241         return vector<T>(x.size(), 0);

```

```

242     }
243     int n = (int) x.size();
244     vector<vector<T>> st((n << 1) - 1);
245     function<void(int, int, int)> build = [&](int v, int l, int r) {
246         if (l == r) {
247             st[v] = vector<T>{-x[l], 1};
248         } else {
249             int y = (l + r) >> 1;
250             int z = v + ((y - l + 1) << 1);
251             build(v + 1, l, y);
252             build(z, y + 1, r);
253             st[v] = st[v + 1] * st[z];
254         }
255     };
256     build(0, 0, n - 1);
257     vector<T> res(n);
258     function<void(int, int, int, vector<T>)> eval = [&](int v, int l, int r,
        vector<T> f) {
259         f %= st[v];
260         if ((int) f.size() < 150) {
261             for (int i = l; i <= r; i++) {
262                 res[i] = evaluate(f, x[i]);
263             }
264             return;
265         }
266         if (l == r) {
267             res[l] = f[0];
268         } else {
269             int y = (l + r) >> 1;
270             int z = v + ((y - l + 1) << 1);
271             eval(v + 1, l, y, f);
272             eval(z, y + 1, r, f);
273         }
274     };
275     eval(0, 0, n - 1, a);
276     return res;
277 }
278
279 template <typename T>
280 vector<T> interpolate(const vector<T>& x, const vector<T>& y) {
281     if (x.empty()) {
282         return {};
283     }

```

```

284     assert(x.size() == y.size());
285     int n = (int) x.size();
286     vector<vector<T>> st((n << 1) - 1);
287     function<void(int, int, int)> build = [&](int v, int l, int r) {
288         if (l == r) {
289             st[v] = vector<T>{-x[l], 1};
290         } else {
291             int w = (l + r) >> 1;
292             int z = v + ((w - l + 1) << 1);
293             build(v + 1, l, w);
294             build(z, w + 1, r);
295             st[v] = st[v + 1] * st[z];
296         }
297     };
298     build(0, 0, n - 1);
299     vector<T> m = st[0];
300     vector<T> dm = derivative(m);
301     vector<T> val(n);
302     function<void(int, int, int, vector<T>>> eval = [&](int v, int l, int r,
303         vector<T> f) {
304         f %= st[v];
305         if ((int) f.size() < 150) {
306             for (int i = l; i <= r; i++) {
307                 val[i] = evaluate(f, x[i]);
308             }
309             return;
310         }
311         if (l == r) {
312             val[l] = f[0];
313         } else {
314             int w = (l + r) >> 1;
315             int z = v + ((w - l + 1) << 1);
316             eval(v + 1, l, w, f);
317             eval(z, w + 1, r, f);
318         }
319     };
320     eval(0, 0, n - 1, dm);
321     for (int i = 0; i < n; i++) {
322         val[i] = y[i] / val[i];
323     }
324     function<vector<T>(int, int, int)> calc = [&](int v, int l, int r) {
325         if (l == r) {
326             return vector<T>{val[l]};

```

```

327         }
328         int w = (l + r) >> 1;
329         int z = v + ((w - l + 1) << 1);
330         return calc(v + 1, l, w) * st[z] + calc(z, w + 1, r) * st[v + 1];
331     };
332     return calc(0, 0, n - 1);
333 }
334 //  $f[i] = 1^i + 2^i + \dots + up^i$ 
335 template <typename T>
336 vector<T> faulhaber(const T& up, int n) {
337     vector<T> ex(n + 1);
338     T e = 1;
339     for (int i = 0; i <= n; i++) {
340         ex[i] = e;
341         e /= i + 1;
342     }
343     vector<T> den = ex;
344     den.erase(den.begin());
345     for (auto& d : den) {
346         d = -d;
347     }
348     vector<T> num(n);
349     T p = 1;
350     for (int i = 0; i < n; i++) {
351         p *= up + 1;
352         num[i] = ex[i + 1] * (1 - p);
353     }
354     vector<T> res = num * inverse(den);
355     res.resize(n);
356     T f = 1;
357     for (int i = 0; i < n; i++) {
358         res[i] *= f;
359         f *= i + 1;
360     }
361     return res;
362 }
363
364 //  $(x + 1) * (x + 2) * \dots * (x + n)$ 
365 // (can be optimized with precomputed inverses)
366 template <typename T>
367 vector<T> sequence(int n) {
368     if (n == 0) {

```

```

369     return {1};
370 }
371 if (n % 2 == 1) {
372     return sequence<T>(n - 1) * vector<T>{n, 1};
373 }
374 vector<T> c = sequence<T>(n / 2);
375 vector<T> a = c;
376 reverse(a.begin(), a.end());
377 T f = 1;
378 for (int i = n / 2 - 1; i >= 0; i--) {
379     f *= n / 2 - i;
380     a[i] *= f;
381 }
382 vector<T> b(n / 2 + 1);
383 b[0] = 1;
384 for (int i = 1; i <= n / 2; i++) {
385     b[i] = b[i - 1] * (n / 2) / i;
386 }
387 vector<T> h = a * b;
388 h.resize(n / 2 + 1);
389 reverse(h.begin(), h.end());
390 f = 1;
391 for (int i = 1; i <= n / 2; i++) {
392     f /= i;
393     h[i] *= f;
394 }
395 vector<T> res = c * h;
396 return res;
397 }
398
399 template <typename T>
400 class OnlineProduct {
401 public:
402     const vector<T> a;
403     vector<T> b;
404     vector<T> c;
405
406     OnlineProduct(const vector<T>& a_) : a(a_) {}
407
408     T add(const T& val) {
409         int i = (int) b.size();
410         b.push_back(val);
411         if ((int) c.size() <= i) {

```

```

412             c.resize(i + 1);
413         }
414         c[i] += a[0] * b[i];
415         int z = 1;
416         while ((i & (z - 1)) == z - 1 && (int) a.size() > z) {
417             vector<T> a_mul(a.begin() + z, a.begin() + min(z << 1, (int) a.size())
418                 );
419             vector<T> b_mul(b.end() - z, b.end());
420             vector<T> c_mul = a_mul * b_mul;
421             if ((int) c.size() <= i + (int) c_mul.size()) {
422                 c.resize(i + c_mul.size() + 1);
423             }
424             for (int j = 0; j < (int) c_mul.size(); j++) {
425                 c[i + 1 + j] += c_mul[j];
426             }
427             z <<= 1;
428         }
429         return c[i];
430     };

```

6.11 primitive.cpp

```

1  template <typename T>
2  struct PrimitiveVarMod { static T value; };
3  template <typename T>
4  T PrimitiveVarMod<T>::value;
5
6  template <typename T, class F>
7  T GetPrimitiveRoot(const T& modulo, const F& factorize) {
8      if (modulo <= 0) {
9          return -1;
10     }
11     if (modulo == 1 || modulo == 2 || modulo == 4) {
12         return modulo - 1;
13     }
14     vector<pair<T, int>> modulo_factors = factorize(modulo);
15     if (modulo_factors[0].first == 2 && (modulo_factors[0].second != 1 ||
16         modulo_factors.size() != 2)) {
17         return -1;
18     }
19     if (modulo_factors[0].first != 2 && modulo_factors.size() != 1) {
20         return -1;

```

```

20 }
21 set<T> phi_factors;
22 T phi = modulo;
23 for (auto& d : modulo_factors) {
24     phi = phi / d.first * (d.first - 1);
25     if (d.second > 1) {
26         phi_factors.insert(d.first);
27     }
28     for (auto& e : factorize(d.first - 1)) {
29         phi_factors.insert(e.first);
30     }
31 }
32 PrimitiveVarMod<T>::value = modulo;
33 Modular<PrimitiveVarMod<T>> gen = 2;
34 while (gen != 0) {
35     if (power(gen, phi) != 1) {
36         continue;
37     }
38     bool ok = true;
39     for (auto& p : phi_factors) {
40         if (power(gen, phi / p) == 1) {
41             ok = false;
42             break;
43         }
44     }
45     if (ok) {
46         return gen();
47     }
48     gen++;
49 }
50 assert(false);
51 return -1;
52 }
53
54 template <typename T>
55 T GetPrimitiveRoot(const T& modulo) {
56     return GetPrimitiveRoot(modulo, factorizer::Factorize<T>);
57 }

```

6.12 simplex.cpp

```

1 typedef long double ld;
2

```

```

3 const ld eps = 1e-8;
4
5 vector<ld> simplex(vector<vector<ld>> a) {
6     int n = (int) a.size() - 1;
7     int m = (int) a[0].size() - 1;
8     vector<int> left(n + 1);
9     vector<int> up(m + 1);
10    iota(left.begin(), left.end(), m);
11    iota(up.begin(), up.end(), 0);
12    auto pivot = [&](int x, int y) {
13        swap(left[x], up[y]);
14        ld k = a[x][y];
15        a[x][y] = 1;
16        vector<int> pos;
17        for (int j = 0; j <= m; j++) {
18            a[x][j] /= k;
19            if (fabs(a[x][j]) > eps) {
20                pos.push_back(j);
21            }
22        }
23        for (int i = 0; i <= n; i++) {
24            if (fabs(a[i][y]) < eps || i == x) {
25                continue;
26            }
27            k = a[i][y];
28            a[i][y] = 0;
29            for (int j : pos) {
30                a[i][j] -= k * a[x][j];
31            }
32        }
33    };
34    while (1) {
35        int x = -1;
36        for (int i = 1; i <= n; i++) {
37            if (a[i][0] < -eps && (x == -1 || a[i][0] < a[x][0])) {
38                x = i;
39            }
40        }
41        if (x == -1) {
42            break;
43        }
44        int y = -1;
45        for (int j = 1; j <= m; j++) {

```

```

46     if (a[x][j] < -eps && (y == -1 || a[x][j] < a[x][y])) {
47         y = j;
48     }
49 }
50 if (y == -1) {
51     return vector<ld>(); // infeasible
52 }
53 pivot(x, y);
54 }
55 while (1) {
56     int y = -1;
57     for (int j = 1; j <= m; j++) {
58         if (a[0][j] > eps && (y == -1 || a[0][j] > a[0][y])) {
59             y = j;
60         }
61     }
62     if (y == -1) {
63         break;
64     }
65     int x = -1;
66     for (int i = 1; i <= n; i++) {
67         if (a[i][y] > eps && (x == -1 || a[i][0] / a[i][y] < a[x][0] / a[x][y]
68             )) {
69             x = i;
70         }
71     }
72     if (x == -1) {
73         return vector<ld>(); // unbounded
74     }
75     pivot(x, y);
76 }
77 vector<ld> ans(m + 1);
78 for (int i = 1; i <= n; i++) {
79     if (left[i] <= m) {
80         ans[left[i]] = a[i][0];
81     }
82 }
83 ans[0] = -a[0][0];
84 return ans;
85 }

```

6.13 sparsematrix.cpp

```

1  const double eps = 1e-9;
2
3  bool IsZero(double v) {
4      return abs(v) < 1e-9;
5  }
6
7  template <typename T>
8  class SparseMatrix {
9      public:
10         int h;
11         int w;
12         vector<map<int, T>> rows;
13         vector<map<int, T>> cols;
14
15         SparseMatrix(int h_, int w_) : h(h_), w(w_) {
16             rows.resize(h);
17             cols.resize(w);
18         }
19
20         void set(int i, int j, const T& value) {
21             if (IsZero(value)) {
22                 rows[i].erase(j);
23                 cols[j].erase(i);
24             } else {
25                 rows[i][j] = value;
26                 cols[j][i] = value;
27             }
28         }
29
30         void modify(int i, int j, const T& value) {
31             if (IsZero(value)) {
32                 return;
33             }
34             auto it = rows[i].find(j);
35             if (it == rows[i].end()) {
36                 rows[i][j] = value;
37                 cols[j][i] = value;
38             } else {
39                 it->second += value;
40                 if (IsZero(it->second)) {
41                     rows[i].erase(it);
42                     cols[j].erase(i);
43                 } else {

```

```

44     cols[j][i] = it->second;
45 }
46 }
47 }
48
49 T get(int i, int j) {
50     auto it = rows[i].find(j);
51     if (it == rows[i].end()) {
52         return T{};
53     }
54     return it->second;
55 }
56
57 void transpose() {
58     swap(h, w);
59     swap(rows, cols);
60 }
61 };
62
63 template <typename T>
64 void GaussianElimination(SparseMatrix<T>& a, int limit) {
65     assert(limit <= a.w);
66     int r = 0;
67     for (int c = 0; c < limit; c++) {
68         int mn = a.w + 1;
69         int id = -1;
70         for (auto& p : a.cols[c]) {
71             int i = p.first;
72             if (i >= r) {
73                 int sz = static_cast<int>(a.rows[i].size());
74                 if (sz < mn) {
75                     mn = sz;
76                     id = i;
77                 }
78             }
79         }
80         if (id == -1) {
81             continue;
82         }
83         if (id > r) {
84             set<int> s;
85             for (auto& p : a.rows[r]) {
86                 s.insert(p.first);

```

```

87     }
88     for (auto& p : a.rows[id]) {
89         s.insert(p.first);
90     }
91     for (int j : s) {
92         T tmp = a.get(r, j);
93         a.set(r, j, a.get(id, j));
94         a.set(id, j, -tmp);
95     }
96 }
97 T inv_a = 1 / a.get(r, c);
98 vector<int> touched_rows;
99 for (auto& p : a.cols[c]) {
100     int i = p.first;
101     if (i > r) {
102         touched_rows.push_back(i);
103         T coeff = -p.second * inv_a;
104         for (auto& q : a.rows[r]) {
105             if (q.first != c) {
106                 a.modify(i, q.first, coeff * q.second);
107             }
108         }
109     }
110 }
111 for (int i : touched_rows) {
112     a.set(i, c, 0);
113 }
114 ++r;
115 }
116 }
117
118 template <typename T>
119 T Determinant(SparseMatrix<T>*& a) {
120     assert(a.h == a.w);
121     GaussianElimination(a, a.w);
122     T d{1};
123     for (int i = 0; i < a.h; i++) {
124         d *= a.get(i, i);
125     }
126     return d;
127 }
128
129 template <typename T>

```



```

130 int Rank(SparseMatrix<T> /*&a*/ a) {
131     GaussianElimination(a, a.w);
132     int rank = 0;
133     for (int i = 0; i < a.h; i++) {
134         if (!a.rows[i].empty()) {
135             ++rank;
136         }
137     }
138     return rank;
139 }
140
141 template <typename T>
142 vector<T> SolveLinearSystem(SparseMatrix<T> /*&a*/ a, const vector<T>& b) {
143     assert(a.h == static_cast<int>(b.size()));
144     ++a.w;
145     a.cols.emplace_back();
146     for (int i = 0; i < a.h; i++) {
147         a.set(i, a.w - 1, b[i]);
148     }
149     GaussianElimination(a, a.w - 1);
150     vector<T> x(a.h, 0);
151     for (int r = a.h - 1; r >= 0; r--) {
152         int c = a.rows[r].begin()->first;
153         if (c == a.w - 1) {
154             return {};
155         }
156         x[c] = a.get(r, a.w - 1) / a.get(r, c);
157         vector<int> touched_rows;
158         for (auto& q : a.cols[c]) {
159             int i = q.first;
160             if (i < r) {
161                 touched_rows.push_back(i);
162                 a.modify(i, a.w - 1, -x[c] * q.second);
163             }
164         }
165         for (int i : touched_rows) {
166             a.set(i, c, 0);
167         }
168     }
169     return x;
170 }

```

7 segtree

7.1 dynamic-fenwick.cpp

```

1  template <typename T>
2  class DynamicFenwickTree {
3  public:
4      HashMap<int, T> fenw;
5      int n;
6      int pw;
7
8      DynamicFenwickTree() : n(0) {}
9      DynamicFenwickTree(int n_) : n(n_) {
10         pw = bit_floor(unsigned(n));
11     }
12
13     void Modify(int x, T v) {
14         assert(0 <= x && x < n);
15         while (x < n) {
16             fenw[x] += v;
17             x |= x + 1;
18         }
19     }
20
21     T Query(int x) {
22         assert(0 <= x && x <= n);
23         T v{};
24         while (x > 0) {
25             auto it = fenw.find(x - 1);
26             if (it != fenw.end()) {
27                 v += it->second;
28             }
29             x &= x - 1;
30         }
31         return v;
32     }
33
34     // Returns the length of the longest prefix with sum <= c
35     int MaxPrefix(T c) {
36         T v{};
37         int at = 0;
38         for (int len = pw; len > 0; len >>= 1) {
39             if (at + len <= n) {

```

```

40     auto nv = v;
41     auto it = fenw.find(at + len - 1);
42     if (it != fenw.end()) {
43         nv += it->second;
44     }
45     if (!(c < nv)) {
46         v = nv;
47         at += len;
48     }
49 }
50 }
51 assert(0 <= at && at <= n);
52 return at;
53 }
54 };

```

7.2 dynamic-lazy.cpp

```

1  template <typename Info, typename Tag, typename Index = int, bool Persistent
   = false, bool Commutative = false>
2  class DynamicLazySegmentTree {
3  public:
4      struct Node {
5          array<int, 2> c;
6          Info d;
7          Tag t;
8      };
9
10     Index n;
11     vector<Node> nodes;
12
13     DynamicLazySegmentTree(Index n_ = 0, int q = -1) : n(n_) {
14         if (q >= 0) {
15             nodes.reserve(2 + q * (Commutative ? 2 : 4) * bit_width(
16                 make_unsigned_t<Index>(2 * n - 1)));
17         }
18         nodes.resize(2);
19         nodes[1] = {{-1, -1}, Info::GetDefault(0, n), Tag()};
20     }
21
22     DynamicLazySegmentTree(const vector<Info>& init, int q = -1) : n(Index(
23         init.size())) {
24         if (q >= 0) {

```

```

23         nodes.reserve(2 * n + q * (Commutative ? 2 : 4) * bit_width(
24             make_unsigned_t<Index>(2 * n - 1)));
25     }
26     nodes.resize(2 * n);
27     auto Build = [&](auto&& self, int id, Index nl, Index nr) -> void {
28         if (nr - nl == 1) {
29             nodes[id] = {{-1, -1}, init[nl], Tag()};
30             return;
31         }
32         Index mid = (nl + nr) >> 1;
33         array<int, 2> c = {id + 1, id + 2 * (mid - nl)};
34         self(self, c[0], nl, mid);
35         self(self, c[1], mid, nr);
36         nodes[id] = {c, nodes[c[0]].d.Unite(nodes[c[1]].d), Tag()};
37     };
38     Build(Build, 1, 0, n);
39
40     void RefreshNode(int& id, Index nl, Index nr) {
41         if (id == -1) {
42             nodes.push_back({{-1, -1}, Info::GetDefault(nl, nr), Tag()});
43             id = int(nodes.size()) - 1;
44         } else {
45             if (Persistent) {
46                 nodes.push_back(nodes[id]);
47                 id = int(nodes.size()) - 1;
48             }
49         }
50     }
51
52     int ModifyImpl(Index l, Index r, const Tag& t, int id, Index nl, Index nr,
53         Tag above) {
54         RefreshNode(id, nl, nr);
55         if (!above.Empty()) {
56             above.ApplyTo(nodes[id].d, nl, nr);
57             above.ApplyTo(nodes[id].t);
58         }
59         if (nr <= l || nl >= r) {
60             return id;
61         }
62         if (l <= nl && nr <= r) {
63             t.ApplyTo(nodes[id].d, nl, nr);
64             t.ApplyTo(nodes[id].t);

```

```

64     return id;
65 }
66 Index mid = (nl + nr) >> 1;
67 if ((!Commutative && !nodes[id].t.Empty()) || l < mid) {
68     int got = ModifyImpl(l, r, t, nodes[id].c[0], nl, mid, Commutative ?
69         Tag() : nodes[id].t);
70     nodes[id].c[0] = got;
71 }
72 if ((!Commutative && !nodes[id].t.Empty()) || r > mid) {
73     int got = ModifyImpl(l, r, t, nodes[id].c[1], mid, nr, Commutative ?
74         Tag() : nodes[id].t);
75     nodes[id].c[1] = got;
76 }
77 auto lft = nodes[id].c[0] == -1 ? Info::GetDefault(nl, mid) : nodes[
78     nodes[id].c[0]].d;
79 auto rgt = nodes[id].c[1] == -1 ? Info::GetDefault(mid, nr) : nodes[
80     nodes[id].c[1]].d;
81 nodes[id].d = lft.Unite(rgt);
82 if (Commutative) {
83     nodes[id].t.ApplyTo(nodes[id].d, nl, nr);
84 } else {
85     nodes[id].t = Tag();
86 }
87 return id;
88 }
89
90 int Modify(Index l, Index r, const Tag& t, int root = Persistent ? -1 : 1)
91 {
92     assert(0 <= l && l <= r && r <= n && root >= 1);
93     return l == r ? root : ModifyImpl(l, r, t, root, 0, n, Tag());
94 }
95
96 int SetImpl(Index p, const Info& v, int id, Index nl, Index nr, Tag above)
97 {
98     RefreshNode(id, nl, nr);
99     if (!above.Empty()) {
100         above.ApplyTo(nodes[id].d, nl, nr);
101         above.ApplyTo(nodes[id].t);
102     }
103     if (p < nl || p >= nr) {
104         return id;
105     }
106     if (nr - nl == 1) {

```

```

101     nodes[id].d = v;
102 } else {
103     Index mid = (nl + nr) >> 1;
104     int got = SetImpl(p, v, nodes[id].c[0], nl, mid, nodes[id].t);
105     nodes[id].c[0] = got;
106     got = SetImpl(p, v, nodes[id].c[1], mid, nr, nodes[id].t);
107     nodes[id].c[1] = got;
108     auto lft = nodes[nodes[id].c[0]].d;
109     auto rgt = nodes[nodes[id].c[1]].d;
110     nodes[id].d = lft.Unite(rgt);
111     nodes[id].t = Tag();
112 }
113 return id;
114 }
115
116 int Set(Index p, const Info& v, int root = Persistent ? -1 : 1) {
117     assert(0 <= p && p < n && root >= 1);
118     return SetImpl(p, v, root, 0, n, Tag());
119 }
120
121 Info QueryImpl(Index l, Index r, int id, Index nl, Index nr, Tag t) {
122     if (id == -1) {
123         nl = max(l, nl);
124         nr = min(r, nr);
125         Info res = Info::GetDefault(nl, nr);
126         if (!t.Empty()) {
127             t.ApplyTo(res, nl, nr);
128         }
129         return res;
130     }
131     if (l <= nl && nr <= r) {
132         Info res = nodes[id].d;
133         if (!t.Empty()) {
134             t.ApplyTo(res, nl, nr);
135         }
136         return res;
137     }
138     auto nt = nodes[id].t;
139     if (!t.Empty()) {
140         t.ApplyTo(nt);
141     }
142     Index mid = (nl + nr) >> 1;
143     auto lft = l < mid ? QueryImpl(l, r, nodes[id].c[0], nl, mid, nt) : Info

```

```

    ();
144     auto rgt = r > mid ? QueryImpl(l, r, nodes[id].c[1], mid, nr, nt) : Info
        ();
145     return lft.Unite(rgt);
146 }
147
148 Info Query(Index l, Index r, int root = Persistent ? -1 : 1) {
149     assert(0 <= l && l <= r && r <= n && root >= 1);
150     return l == r ? Info() : QueryImpl(l, r, root, 0, n, Tag());
151 }
152
153 Info Get(Index p, int root = Persistent ? -1 : 1) {
154     assert(0 <= p && p < n && root >= 1);
155     int id = root;
156     Index nl = 0;
157     Index nr = n;
158     Tag t;
159     while (nr - nl > 1 && id != -1) {
160         auto nt = nodes[id].t;
161         if (!t.Empty()) {
162             t.ApplyTo(nt);
163         }
164         t = nt;
165         Index mid = (nl + nr) >> 1;
166         if (p < mid) {
167             id = nodes[id].c[0];
168             nr = mid;
169         } else {
170             id = nodes[id].c[1];
171             nl = mid;
172         }
173     }
174     Info res = id == -1 ? Info::GetDefault(p, p + 1) : nodes[id].d;
175     if (!t.Empty()) {
176         t.ApplyTo(res, p, p + 1);
177     }
178     return res;
179 }
180
181 template<int N, typename F>
182 Index MaxRight(array<int, N> roots, Index l, F f) {
183     assert(0 <= l && l <= n);
184     if (l == n) {

```

```

185         return n;
186     }
187     Index res = n;
188     array<Info, N> sums;
189     for (int i = 0; i < N; i++) {
190         sums[i] = Info();
191     }
192     array<Info, N> new_sums;
193     auto Dfs = [&](auto&& self, array<int, N> v, Index nl, Index nr, array<
        Tag, N> tags) -> void {
194         if (res != n) {
195             return;
196         }
197         array<int, N> to;
198         if (nl < 1) {
199             Index mid = (nl + nr) >> 1;
200             for (int i = 0; i < N; i++) {
201                 auto nt = v[i] == -1 ? Tag() : nodes[v[i]].t;
202                 tags[i].ApplyTo(nt);
203                 tags[i] = nt;
204             }
205             if (l < mid) {
206                 for (int i = 0; i < N; i++) {
207                     to[i] = v[i] == -1 ? -1 : nodes[v[i]].c[0];
208                 }
209                 self(self, to, nl, mid, tags);
210                 if (res != n) {
211                     return;
212                 }
213             }
214             for (int i = 0; i < N; i++) {
215                 to[i] = v[i] == -1 ? -1 : nodes[v[i]].c[1];
216             }
217             self(self, to, mid, nr, tags);
218             return;
219         }
220         for (int i = 0; i < N; i++) {
221             auto d = v[i] == -1 ? Info::GetDefault(nl, nr) : nodes[v[i]].d;
222             tags[i].ApplyTo(d, nl, nr);
223             new_sums[i] = sums[i].Unite(d);
224         }
225         if (f(new_sums)) {
226             sums = new_sums;

```

```

227     return;
228 }
229 while (nr - nl > 1) {
230     Index mid = (nl + nr) >> 1;
231     for (int i = 0; i < N; i++) {
232         auto nt = v[i] == -1 ? Tag() : nodes[v[i]].t;
233         tags[i].ApplyTo(nt);
234         tags[i] = nt;
235     }
236     for (int i = 0; i < N; i++) {
237         auto d = v[i] == -1 || nodes[v[i]].c[0] == -1 ? Info::GetDefault(
238             nl, mid) : nodes[nodes[v[i]].c[0]].d;
239         tags[i].ApplyTo(d, nl, mid);
240         new_sums[i] = sums[i].Unite(d);
241     }
242     if (f(new_sums)) {
243         sums = new_sums;
244         nl = mid;
245         for (int i = 0; i < N; i++) {
246             v[i] = v[i] == -1 ? -1 : nodes[v[i]].c[1];
247         }
248     } else {
249         nr = mid;
250         for (int i = 0; i < N; i++) {
251             v[i] = v[i] == -1 ? -1 : nodes[v[i]].c[0];
252         }
253     }
254     res = nl;
255 };
256 array<Tag, N> tags;
257 for (int i = 0; i < N; i++) {
258     tags[i] = Tag();
259 }
260 Dfs(Dfs, roots, 0, n, tags);
261 return res;
262 }
263
264 template<int N, typename F>
265 Index MinLeft(array<int, N> roots, Index r, F f) {
266     assert(0 <= r && r <= n);
267     if (r == 0) {
268         return 0;

```

```

269     }
270     Index res = 0;
271     array<Info, N> sums;
272     for (int i = 0; i < N; i++) {
273         sums[i] = Info();
274     }
275     array<Info, N> new_sums;
276     auto Dfs = [&](auto&& self, array<int, N> v, Index nl, Index nr, array<
277         Tag, N> tags) -> void {
278         if (res != 0) {
279             return;
280         }
281         array<int, N> to;
282         if (nr > r) {
283             Index mid = (nl + nr) >> 1;
284             for (int i = 0; i < N; i++) {
285                 auto nt = v[i] == -1 ? Tag() : nodes[v[i]].t;
286                 tags[i].ApplyTo(nt);
287                 tags[i] = nt;
288             }
289             if (r > mid) {
290                 for (int i = 0; i < N; i++) {
291                     to[i] = v[i] == -1 ? -1 : nodes[v[i]].c[1];
292                 }
293                 self(self, to, mid, nr, tags);
294                 if (res != 0) {
295                     return;
296                 }
297             }
298             for (int i = 0; i < N; i++) {
299                 to[i] = v[i] == -1 ? -1 : nodes[v[i]].c[0];
300             }
301             self(self, to, nl, mid, tags);
302             return;
303         }
304         for (int i = 0; i < N; i++) {
305             auto d = v[i] == -1 ? Info::GetDefault(nl, nr) : nodes[v[i]].d;
306             tags[i].ApplyTo(d, nl, nr);
307             new_sums[i] = d.Unite(sums[i]);
308         }
309         if (f(new_sums)) {
310             sums = new_sums;
311             return;

```

```

311     }
312     while (nr - nl > 1) {
313         Index mid = (nl + nr) >> 1;
314         for (int i = 0; i < N; i++) {
315             auto nt = v[i] == -1 ? Tag() : nodes[v[i]].t;
316             tags[i].ApplyTo(nt);
317             tags[i] = nt;
318         }
319         for (int i = 0; i < N; i++) {
320             auto d = v[i] == -1 || nodes[v[i]].c[1] == -1 ? Info::GetDefault(
321                 mid, nr) : nodes[nodes[v[i]].c[1]].d;
322             tags[i].ApplyTo(d, mid, nr);
323             new_sums[i] = d.Unite(sums[i]);
324         }
325         if (f(new_sums)) {
326             sums = new_sums;
327             nr = mid;
328             for (int i = 0; i < N; i++) {
329                 v[i] = v[i] == -1 ? -1 : nodes[v[i]].c[0];
330             }
331         } else {
332             nl = mid;
333             for (int i = 0; i < N; i++) {
334                 v[i] = v[i] == -1 ? -1 : nodes[v[i]].c[1];
335             }
336         }
337         res = nr;
338     };
339     array<Tag, N> tags;
340     for (int i = 0; i < N; i++) {
341         tags[i] = Tag();
342     }
343     Dfs(Dfs, roots, 0, n, tags);
344     return res;
345 }
346 };

```

7.3 dynamic-simple.cpp

```

1 template <typename Info, typename Index = int, bool Persistent = false>
2 class DynamicSimpleSegmentTree {
3 public:

```

```

4     struct Node {
5         array<int, 2> c;
6         Info d;
7     };
8
9     Index n;
10    vector<Node> nodes;
11
12    DynamicSimpleSegmentTree() : DynamicSimpleSegmentTree(0, -1) {}
13    DynamicSimpleSegmentTree(Index n_) : DynamicSimpleSegmentTree(n_, -1) {}
14    DynamicSimpleSegmentTree(const vector<Info>& a) : DynamicSimpleSegmentTree(
15        a, -1) {}
16
17    DynamicSimpleSegmentTree(Index n_, int q) : n(n_) {
18        if (q >= 0) {
19            nodes.reserve(2 + q * bit_width(make_unsigned_t<Index>(2 * n - 1)));
20        }
21        nodes.resize(2);
22        nodes[1] = {{-1, -1}, Info::GetDefault(0, n)};
23    }
24
25    DynamicSimpleSegmentTree(const vector<Info>& a, int q) : n(int(a.size()))
26    {
27        if (q >= 0) {
28            nodes.reserve(2 * n + q * bit_width(make_unsigned_t<Index>(2 * n - 1)));
29        }
30        nodes.resize(2 * n);
31        auto Build = [&](auto&& self, int id, int nl, int nr) -> void {
32            if (nr - nl == 1) {
33                nodes[id] = {{-1, -1}, a[nl]};
34                return;
35            }
36            int mid = (nl + nr) >> 1;
37            array<int, 2> c = {id + 1, id + 2 * (mid - nl)};
38            self(self, c[0], nl, mid);
39            self(self, c[1], mid, nr);
40            nodes[id] = {c, nodes[c[0]].d.Unite(nodes[c[1]].d)};
41        };
42        Build(Build, 1, 0, n);
43    }
44
45    int SetImpl(int root, Index p, const Info& v, Index nl, Index nr) {

```

```

44     int me;
45     if (root == -1) {
46         me = int(nodes.size());
47         nodes.push_back({{-1, -1}, Info::GetDefault(nl, nr)});
48     } else {
49         if (Persistent) {
50             me = int(nodes.size());
51             nodes.push_back(nodes[root]);
52         } else {
53             me = root;
54         }
55     }
56     if (nr - nl == 1) {
57         nodes[me].d = v;
58     } else {
59         Index mid = (nl + nr) >> 1;
60         if (p < mid) {
61             int got = SetImpl(nodes[me].c[0], p, v, nl, mid);
62             nodes[me].c[0] = got;
63         } else {
64             int got = SetImpl(nodes[me].c[1], p, v, mid, nr);
65             nodes[me].c[1] = got;
66         }
67         auto lft = nodes[me].c[0] == -1 ? Info::GetDefault(nl, mid) : nodes[
68             nodes[me].c[0]].d;
69         auto rgt = nodes[me].c[1] == -1 ? Info::GetDefault(mid, nr) : nodes[
70             nodes[me].c[1]].d;
71         nodes[me].d = lft.Unite(rgt);
72     }
73     return me;
74 }
75
76 int Set(Index p, const Info& v, int root = Persistent ? -1 : 1) {
77     assert(0 <= p && p < n && root >= 1);
78     return SetImpl(root, p, v, 0, n);
79 }
80
81 Info QueryImpl(int root, Index l, Index r, Index nl, Index nr) {
82     if (root == -1) {
83         return Info::GetDefault(max(l, nl), min(r, nr));
84     }
85     if (l <= nl && nr <= r) {
86         return nodes[root].d;
87     }

```

```

85     }
86     Index mid = (nl + nr) >> 1;
87     auto lft = l < mid ? QueryImpl(nodes[root].c[0], l, r, nl, mid) : Info()
88         ;
89     auto rgt = r > mid ? QueryImpl(nodes[root].c[1], l, r, mid, nr) : Info()
90         ;
91     return lft.Unite(rgt);
92 }
93
94 Info Query(Index l, Index r, int root = Persistent ? -1 : 1) {
95     assert(0 <= l && l <= r && r <= n && root >= 1);
96     if (l == r) {
97         return Info();
98     }
99     return QueryImpl(root, l, r, 0, n);
100 }
101
102 Info Get(Index p, int root = Persistent ? -1 : 1) {
103     assert(0 <= p && p < n && root >= 1);
104     Index nl = 0;
105     Index nr = n;
106     while (nr - nl > 1 && root != -1) {
107         Index mid = (nl + nr) >> 1;
108         if (p < mid) {
109             root = nodes[root].c[0];
110             nr = mid;
111         } else {
112             root = nodes[root].c[1];
113             nl = mid;
114         }
115     }
116     return root == -1 ? Info::GetDefault(p, p + 1) : nodes[root].d;
117 }
118
119 template<int N, typename F>
120 Index MaxRight(array<int, N> roots, Index l, F f) {
121     assert(0 <= l && l <= n);
122     if (l == n) {
123         return n;
124     }
125     Index res = n;
126     array<Info, N> sums;
127     for (int i = 0; i < N; i++) {

```

```

126     sums[i] = Info();
127 }
128 array<Info, N> new_sums;
129 auto Dfs = [&](auto&& self, array<int, N> v, Index nl, Index nr) -> void
130 {
131     if (res != n) {
132         return;
133     }
134     array<int, N> to;
135     if (nl < 1) {
136         Index mid = (nl + nr) >> 1;
137         if (1 < mid) {
138             for (int i = 0; i < N; i++) {
139                 to[i] = v[i] == -1 ? -1 : nodes[v[i]].c[0];
140             }
141             self(self, to, nl, mid);
142             if (res != n) {
143                 return;
144             }
145             for (int i = 0; i < N; i++) {
146                 to[i] = v[i] == -1 ? -1 : nodes[v[i]].c[1];
147             }
148             self(self, to, mid, nr);
149             return;
150         }
151         for (int i = 0; i < N; i++) {
152             new_sums[i] = sums[i].Unite(v[i] == -1 ? Info::GetDefault(nl, nr) :
153                 nodes[v[i]].d);
154         }
155         if (f(new_sums)) {
156             sums = new_sums;
157             return;
158         }
159         while (nr - nl > 1) {
160             Index mid = (nl + nr) >> 1;
161             for (int i = 0; i < N; i++) {
162                 new_sums[i] = sums[i].Unite(v[i] == -1 || nodes[v[i]].c[0] == -1 ?
163                     Info::GetDefault(nl, mid) : nodes[nodes[v[i]].c[0]].d);
164             }
165             if (f(new_sums)) {
166                 sums = new_sums;
167                 nl = mid;

```

```

166         for (int i = 0; i < N; i++) {
167             v[i] = v[i] == -1 ? -1 : nodes[v[i]].c[i];
168         }
169     } else {
170         nr = mid;
171         for (int i = 0; i < N; i++) {
172             v[i] = v[i] == -1 ? -1 : nodes[v[i]].c[0];
173         }
174     }
175 }
176 res = nl;
177 };
178 Dfs(Dfs, roots, 0, n);
179 return res;
180 }
181
182 template<int N, typename F>
183 Index MinLeft(array<int, N> roots, Index r, F f) {
184     assert(0 <= r && r <= n);
185     if (r == 0) {
186         return 0;
187     }
188     Index res = 0;
189     array<Info, N> sums;
190     for (int i = 0; i < N; i++) {
191         sums[i] = Info();
192     }
193     array<Info, N> new_sums;
194     auto Dfs = [&](auto&& self, array<int, N> v, Index nl, Index nr) -> void
195     {
196         if (res != 0) {
197             return;
198         }
199         array<int, N> to;
200         if (nr > r) {
201             Index mid = (nl + nr) >> 1;
202             if (r > mid) {
203                 for (int i = 0; i < N; i++) {
204                     to[i] = v[i] == -1 ? -1 : nodes[v[i]].c[1];
205                 }
206                 self(self, to, mid, nr);
207                 if (res != 0) {
208                     return;

```



```

208     }
209 }
210 for (int i = 0; i < N; i++) {
211     to[i] = v[i] == -1 ? -1 : nodes[v[i]].c[0];
212 }
213 self(self, to, nl, mid);
214 return;
215 }
216 for (int i = 0; i < N; i++) {
217     new_sums[i] = (v[i] == -1 ? Info::GetDefault(nl, nr) : nodes[v[i]].d
        ).Unite(sums[i]);
218 }
219 if (f(new_sums)) {
220     sums = new_sums;
221     return;
222 }
223 while (nr - nl > 1) {
224     Index mid = (nl + nr) >> 1;
225     for (int i = 0; i < N; i++) {
226         new_sums[i] = (v[i] == -1 || nodes[v[i]].c[1] == -1 ? Info::
            GetDefault(mid, nr) : nodes[nodes[v[i]].c[1]].d).Unite(sums[i
                ]);
227     }
228     if (f(new_sums)) {
229         sums = new_sums;
230         nr = mid;
231         for (int i = 0; i < N; i++) {
232             v[i] = v[i] == -1 ? -1 : nodes[v[i]].c[0];
233         }
234     } else {
235         nl = mid;
236         for (int i = 0; i < N; i++) {
237             v[i] = v[i] == -1 ? -1 : nodes[v[i]].c[1];
238         }
239     }
240 }
241 res = nr;
242 };
243 Dfs(Dfs, roots, 0, n);
244 return res;
245 }
246 };

```

7.4 info.cpp

```

1 struct Info {
2     ${0}... a = ...;
3
4     Info Unite(const Info& b) const {
5         Info res;
6         ...
7         return res;
8     }
9
10    static Info GetDefault([[maybe_unused]] int l, [[maybe_unused]] int r) {
11        return Info();
12    }
13 };

```

7.5 layout.cpp

```

1 namespace seg_tree {
2
3     // Floor of log2(a); index of highest 1-bit
4     inline int floor_log_2(int a) {
5         return a ? bit_width(unsigned(a)) - 1 : -1;
6     }
7
8     struct point {
9         int a;
10        point() : a(0) {}
11        explicit point(int a_) : a(a_) { assert(a >= -1); }
12
13        explicit operator bool () { return bool(a); }
14
15        // This is useful so you can directly do array indices
16        /* implicit */ operator int() const { return a; }
17
18        point c(bool z) const {
19            return point((a << 1) | z);
20        }
21
22        point operator [] (bool z) const {
23            return c(z);
24        }
25
26        point p() const {

```

```

27     return point(a >> 1);
28 }
29
30 friend std::ostream& operator << (std::ostream& o, const point& p) {
31     return o << int(p); }
32
33 template <typename F> void for_each(F f) const {
34     for (int v = a; v > 0; v >= 1) {
35         f(point(v));
36     }
37 }
38
39 template <typename F> void for_parents_down(F f) const {
40     // strictly greater than 0
41     for (int L = floor_log_2(a); L > 0; L--) {
42         f(point(a >> L));
43     }
44 }
45
46 template <typename F> void for_parents_up(F f) const {
47     for (int v = a >> 1; v > 0; v >= 1) {
48         f(point(v));
49     }
50 }
51
52 point& operator ++ () { ++a; return *this; }
53 point operator ++ (int) { return point(a++); }
54 point& operator -- () { --a; return *this; }
55 point operator -- (int) { return point(a--); }
56 };
57
58 struct range {
59     int a, b;
60     range() : a(1), b(1) {}
61     range(int a_, int b_) : a(a_), b(b_) {
62         assert(1 <= a && a <= b && b <= 2 * a);
63     }
64     explicit range(std::array<int, 2> r) : range(r[0], r[1]) {}
65
66     explicit operator std::array<int, 2>() const {
67         return {a, b};
68     }

```

```

69     const int& operator[] (bool z) const {
70         return z ? b : a;
71     }
72
73     friend std::ostream& operator << (std::ostream& o, const range& r) {
74         return o << "[" << r.a << ".." << r.b << ")"; }
75
76     // Iterate over the range from outside-in.
77     // Calls f(point a)
78     template <typename F> void for_each(F f) const {
79         for (int x = a, y = b; x < y; x >= 1, y >= 1) {
80             if (x & 1) f(point(x++));
81             if (y & 1) f(point(--y));
82         }
83
84         // Iterate over the range from outside-in.
85         // Calls f(point a, bool is_right)
86         template <typename F> void for_each_with_side(F f) const {
87             for (int x = a, y = b; x < y; x >= 1, y >= 1) {
88                 if (x & 1) f(point(x++), false);
89                 if (y & 1) f(point(--y), true);
90             }
91         }
92
93         // Iterate over the range from left to right.
94         // Calls f(point)
95         template <typename F> void for_each_l_to_r(F f) const {
96             int anc_depth = floor_log_2((a - 1) ^ b);
97             int anc_msk = (1 << anc_depth) - 1;
98             for (int v = (-a) & anc_msk; v; v &= v - 1) {
99                 int i = countr_zero(unsigned(v));
100                 f(point(((a - 1) >> i) + 1));
101             }
102             for (int v = b & anc_msk; v; ) {
103                 int i = floor_log_2(v);
104                 f(point((b >> i) - 1));
105                 v ^= (1 << i);
106             }
107         }
108
109         // Iterate over the range from right to left.
110         // Calls f(point)

```

```

111 template <typename F> void for_each_r_to_l(F f) const {
112     int anc_depth = floor_log_2((a - 1) ^ b);
113     int anc_msk = (1 << anc_depth) - 1;
114     for (int v = b & anc_msk; v; v &= v - 1) {
115         int i = countr_zero(unsigned(v));
116         f(point((b >> i) - 1));
117     }
118     for (int v = (-a) & anc_msk; v; ) {
119         int i = floor_log_2(v);
120         f(point(((a - 1) >> i) + 1));
121         v ^= (1 << i);
122     }
123 }
124
125 template <typename F> void for_parents_down(F f) const {
126     int x = a, y = b;
127     if ((x ^ y) > x) { x <= 1, std::swap(x, y); }
128     int dx = countr_zero(unsigned(x));
129     int dy = countr_zero(unsigned(y));
130     int anc_depth = floor_log_2((x - 1) ^ y);
131     for (int i = floor_log_2(x); i > dx; i--) {
132         f(point(x >> i));
133     }
134     for (int i = anc_depth; i > dy; i--) {
135         f(point(y >> i));
136     }
137 }
138
139 template <typename F> void for_parents_up(F f) const {
140     int x = a, y = b;
141     if ((x ^ y) > x) { x <= 1, std::swap(x, y); }
142     int dx = countr_zero(unsigned(x));
143     int dy = countr_zero(unsigned(y));
144     int anc_depth = floor_log_2((x - 1) ^ y);
145     for (int i = dx + 1; i <= anc_depth; i++) {
146         f(point(x >> i));
147     }
148     for (int v = y >> (dy + 1); v; v >= 1) {
149         f(point(v));
150     }
151 }
152 };
153

```

```

154 struct in_order_layout {
155     // Alias them in for convenience
156     using point = seg_tree::point;
157     using range = seg_tree::range;
158
159     int n, s;
160     in_order_layout() : n(0), s(0) {}
161     in_order_layout(int n_) : n(n_), s(n ? bit_ceil(unsigned(n)) : 0) {}
162
163     point get_point(int a) const {
164         assert(0 <= a && a < n);
165         a += s;
166         return point(a >= 2 * n ? a - n : a);
167     }
168
169     range get_range(int a, int b) const {
170         assert(0 <= a && a <= b && b <= n);
171         if (n == 0) return range();
172         a += s, b += s;
173         return range((a >= 2 * n ? 2 * (a - n) : a), (b >= 2 * n ? 2 * (b - n) :
174             b));
175     }
176
177     range get_range(std::array<int, 2> p) const {
178         return get_range(p[0], p[1]);
179     }
180
181     int get_leaf_index(point pt) const {
182         int a = int(pt);
183         assert(n <= a && a < 2 * n);
184         return (a < s ? a + n : a) - s;
185     }
186
187     std::array<int, 2> get_node_bounds(point pt) const {
188         int a = int(pt);
189         assert(1 <= a && a < 2 * n);
190         int l = countl_zero(unsigned(a)) - countl_zero(unsigned(2 * n - 1));
191         int x = a << 1, y = (a + 1) << 1;
192         assert(s <= x && x < y && y <= 2 * s);
193         return {(x >= 2 * n ? (x >> 1) + n : x) - s, (y >= 2 * n ? (y >> 1) + n
194             : y) - s};
195     }
196 }
197

```

```

195     int get_node_split(point pt) const {
196         int a = int(pt);
197         assert(1 <= a && a < n);
198         int l = countl_zero(unsigned(2 * a + 1)) - countl_zero(unsigned(2 * n -
199             1));
200         int x = (2 * a + 1) << 1;
201         assert(s <= x && x < 2 * s);
202         return (x >= 2 * n ? (x >> 1) + n : x) - s;
203     }
204     int get_node_size(point pt) const {
205         auto bounds = get_node_bounds(pt);
206         return bounds[1] - bounds[0];
207     }
208 };
209
210 struct circular_layout {
211     // Alias them in for convenience
212     using point = seg_tree::point;
213     using range = seg_tree::range;
214
215     int n;
216     circular_layout() : n(0) {}
217     circular_layout(int n_) : n(n_) {}
218
219     point get_point(int a) const {
220         assert(0 <= a && a < n);
221         return point(n + a);
222     }
223
224     range get_range(int a, int b) const {
225         assert(0 <= a && a <= b && b <= n);
226         if (n == 0) return range();
227         return range(n + a, n + b);
228     }
229
230     range get_range(std::array<int, 2> p) const {
231         return get_range(p[0], p[1]);
232     }
233
234     int get_leaf_index(point pt) const {
235         int a = int(pt);
236         assert(n <= a && a < 2 * n);

```

```

237         return a - n;
238     }
239
240     // Returns {x,y} so that 0 <= x < n and 1 <= y <= n
241     // If the point is non-wrapping, then 0 <= x < y <= n
242     std::array<int, 2> get_node_bounds(point pt) const {
243         int a = int(pt);
244         assert(1 <= a && a < 2 * n);
245         int l = countl_zero(unsigned(a)) - countl_zero(unsigned(2 * n - 1));
246         int s = bit_ceil(unsigned(n));
247         int x = a << 1, y = (a + 1) << 1;
248         assert(s <= x && x < y && y <= 2 * s);
249         return {(x >= 2 * n ? x >> 1 : x) - n, (y > 2 * n ? y >> 1 : y) - n};
250     }
251
252     // Returns the split point of the node, such that 1 <= s <= n.
253     int get_node_split(point pt) const {
254         int a = int(pt);
255         assert(1 <= a && a < n);
256         return get_node_bounds(pt.c(0))[1];
257     }
258
259     int get_node_size(point pt) const {
260         auto bounds = get_node_bounds(pt);
261         int r = bounds[1] - bounds[0];
262         return r > 0 ? r : r + n;
263     }
264 };
265
266 } // namespace seg_tree

```

7.6 lazy.cpp

```

1  template <typename Info, typename Tag>
2  class LazySegmentTree {
3  public:
4      int n;
5      vector<Info> infos;
6      vector<Tag> tags;
7      seg_tree::in_order_layout layout;
8
9      void Apply(seg_tree::point a, const Tag& t) {
10         auto [l, r] = layout.get_node_bounds(a);

```

```

11     if (!t.ApplyTo(infos[a], l, r)) {
12         assert(a < n);
13         DowndateNode(a);
14         Apply(a.c(0), t);
15         Apply(a.c(1), t);
16         UpdateNode(a);
17         return;
18     }
19     if (a < n) {
20         t.ApplyTo(tags[a]);
21     }
22 }
23
24 void DowndateNode(seg_tree::point a) {
25     if (!tags[a].Empty()) {
26         Apply(a.c(0), tags[a]);
27         Apply(a.c(1), tags[a]);
28         tags[a] = Tag();
29     }
30 }
31
32 void UpdateNode(seg_tree::point a) {
33     infos[a] = infos[a.c(0)].Unite(infos[a.c(1)]);
34 }
35
36 LazySegmentTree() : LazySegmentTree(0) {}
37 LazySegmentTree(int n_) : LazySegmentTree(vector<Info>(n_)) {}
38 LazySegmentTree(const vector<Info>& a) : n(int(a.size())) {
39     infos.resize(2 * n);
40     tags.resize(n);
41     layout = seg_tree::in_order_layout(n);
42     for (int i = 0; i < n; i++) {
43         infos[layout.get_point(i)] = a[i];
44     }
45     for (int i = n - 1; i >= 1; i--) {
46         UpdateNode(seg_tree::point(i));
47     }
48 }
49
50 void Modify(int l, int r, const Tag& t) {
51     auto rng = layout.get_range(l, r);
52     rng.for_parents_down([&](seg_tree::point a) {
53         DowndateNode(a);

```

```

54     });
55     rng.for_each([&](seg_tree::point a) {
56         Apply(a, t);
57     });
58     rng.for_parents_up([&](seg_tree::point a) {
59         UpdateNode(a);
60     });
61 }
62
63 void Set(int p, const Info& v) {
64     auto pt = layout.get_point(p);
65     pt.for_parents_down([&](seg_tree::point a) {
66         DowndateNode(a);
67     });
68     infos[pt] = v;
69     pt.for_parents_up([&](seg_tree::point a) {
70         UpdateNode(a);
71     });
72 }
73
74 Info Query(int l, int r) {
75     auto rng = layout.get_range(l, r);
76     rng.for_parents_down([&](seg_tree::point a) {
77         DowndateNode(a);
78     });
79     Info res;
80     rng.for_each_l_to_r([&](seg_tree::point a) {
81         res = res.Unite(infos[a]);
82     });
83     return res;
84 }
85
86 Info Get(int p) {
87     auto pt = layout.get_point(p);
88     pt.for_parents_down([&](seg_tree::point a) {
89         DowndateNode(a);
90     });
91     return infos[pt];
92 }
93
94 template<typename F>
95 int MaxRight(int l, F f) {
96     auto rng = layout.get_range(l, n);

```

```

97     rng.for_parents_down([&](seg_tree::point a) {
98         DowndateNode(a);
99     });
100     int res = n;
101     Info sum;
102     rng.for_each_l_to_r([&](seg_tree::point a) {
103         if (res != n) {
104             return;
105         }
106         auto new_sum = sum.Unite(infos[a]);
107         if (f(new_sum)) {
108             sum = new_sum;
109             return;
110         }
111         while (a < n) {
112             DowndateNode(a);
113             new_sum = sum.Unite(infos[a.c(0)]);
114             if (f(new_sum)) {
115                 sum = new_sum;
116                 a = a.c(1);
117             } else {
118                 a = a.c(0);
119             }
120         }
121         res = layout.get_node_bounds(a)[0];
122     });
123     return res;
124 }

125
126 template<typename F>
127 int MinLeft(int r, F f) {
128     auto rng = layout.get_range(0, r);
129     rng.for_parents_down([&](seg_tree::point a) {
130         DowndateNode(a);
131     });
132     int res = 0;
133     Info sum;
134     rng.for_each_r_to_l([&](seg_tree::point a) {
135         if (res != 0) {
136             return;
137         }
138         auto new_sum = infos[a].Unite(sum);
139         if (f(new_sum)) {

```

```

140         sum = new_sum;
141         return;
142     }
143     while (a < n) {
144         DowndateNode(a);
145         new_sum = infos[a.c(1)].Unite(sum);
146         if (f(new_sum)) {
147             sum = new_sum;
148             a = a.c(0);
149         } else {
150             a = a.c(1);
151         }
152     }
153     res = layout.get_node_bounds(a)[1];
154 });
155     return res;
156 }
157 };

```

7.7 simple.cpp

```

1  template <typename Info>
2  class SimpleSegmentTree {
3  public:
4      int n;
5      vector<Info> infos;
6      seg_tree::in_order_layout layout;
7
8      void UpdateNode(seg_tree::point a) {
9          infos[a] = infos[a.c(0)].Unite(infos[a.c(1)]);
10     }
11
12     SimpleSegmentTree(int n_) : SimpleSegmentTree(vector<Info>(n_)) {}
13
14     SimpleSegmentTree(const vector<Info>& a) : n(int(a.size())) {
15         assert(n > 0);
16         infos.resize(2 * n);
17         layout = seg_tree::in_order_layout(n);
18         for (int i = 0; i < n; i++) {
19             infos[layout.get_point(i)] = a[i];
20         }
21         for (int i = n - 1; i >= 1; i--) {
22             infos[i] = infos[2 * i].Unite(infos[2 * i + 1]);

```

```

23     }
24 }
25
26 void Set(int p, const Info& v) {
27     auto pt = layout.get_point(p);
28     infos[pt] = v;
29     pt.for_parents_up([&](seg_tree::point a) {
30         UpdateNode(a);
31     });
32 }
33
34 Info Query(int l, int r) {
35     auto rng = layout.get_range(l, r);
36     Info res;
37     rng.for_each_l_to_r([&](seg_tree::point a) {
38         res = res.Unite(infos[a]);
39     });
40     return res;
41 }
42
43 Info Get(int p) {
44     auto pt = layout.get_point(p);
45     return infos[pt];
46 }
47
48 template<typename F>
49 int MaxRight(int l, F f) {
50     auto rng = layout.get_range(l, n);
51     int res = n;
52     Info sum;
53     rng.for_each_l_to_r([&](seg_tree::point a) {
54         if (res != n) {
55             return;
56         }
57         auto new_sum = sum.Unite(infos[a]);
58         if (f(new_sum)) {
59             sum = new_sum;
60             return;
61         }
62         while (a < n) {
63             new_sum = sum.Unite(infos[a.c(0)]);
64             if (f(new_sum)) {
65                 sum = new_sum;

```

```

66             a = a.c(1);
67         } else {
68             a = a.c(0);
69         }
70     }
71     res = layout.get_node_bounds(a)[0];
72 });
73     return res;
74 }
75
76 template<typename F>
77 int MinLeft(int r, F f) {
78     auto rng = layout.get_range(0, r);
79     int res = 0;
80     Info sum;
81     rng.for_each_r_to_l([&](seg_tree::point a) {
82         if (res != 0) {
83             return;
84         }
85         auto new_sum = infos[a].Unite(sum);
86         if (f(new_sum)) {
87             sum = new_sum;
88             return;
89         }
90         while (a < n) {
91             new_sum = infos[a.c(1)].Unite(sum);
92             if (f(new_sum)) {
93                 sum = new_sum;
94                 a = a.c(0);
95             } else {
96                 a = a.c(1);
97             }
98         }
99         res = layout.get_node_bounds(a)[1];
100     });
101     return res;
102 }
103 };

```

7.8 tag.cpp

```

1 struct Tag {
2     ${0}... add = ...;

```

```

3
4  bool ApplyTo(Info& a, [[maybe_unused]] int l, [[maybe_unused]] int r)
        const {
5      ...
6      return true;
7  }
8
9  void ApplyTo(Tag& t) const {
10     ...
11     t.add += add;
12 }
13
14 bool Empty() const {
15     ...
16     return add == 0;
17 }
18 };

```

8 string

8.1 duval.cpp

```

1  template <typename T>
2  int duval(int n, const T &s) {
3      assert(n >= 1);
4      int i = 0, ans = 0;
5      while (i < n) {
6          ans = i;
7          int j = i + 1, k = i;
8          while (j < n + n && !(s[j % n] < s[k % n])) {
9              if (s[k % n] < s[j % n]) {
10                 k = i;
11             } else {
12                 k++;
13             }
14             j++;
15         }
16         while (i <= k) {
17             i += j - k;
18         }
19     }
20     return ans;

```

```

21     // returns 0-indexed position of the least cyclic shift
22 }
23
24 template <typename T>
25 int duval(const T &s) {
26     return duval((int) s.size(), s);
27 }

```

8.2 duval-prefixes.cpp

```

1  template <typename T>
2  vector<int> duval_prefixes(int n, const T &s) {
3      vector<int> z = z_function(n, s);
4      vector<int> ans(n, 0);
5      int i = 0, pos = 0;
6      while (i < n) {
7          int j = i, k = i;
8          while (j < n) {
9              j++;
10             if (j > pos) {
11                 if (z[k] <= pos - k && s[z[k]] < s[k + z[k]]) {
12                     int shift = (pos - i) / (j - k) * (j - k);
13                     ans[pos] = ans[pos - shift] + shift;
14                 } else {
15                     ans[pos] = i;
16                 }
17                 pos++;
18             }
19             if (s[k] < s[j]) k = i; else
20             if (!(s[j] < s[k])) k++; else
21             else break;
22         }
23         while (i <= k) {
24             i += j - k;
25         }
26     }
27     return ans;
28     // returns 0-indexed positions of the least cyclic shifts of all prefixes
29 }
30
31 template <typename T>
32 vector<int> duval_prefixes(const T &s) {
33     return duval_prefixes((int) s.size(), s);

```



```
34 }
```

8.3 hash61.cpp

```
1 struct hash61 {
2     static const uint64_t md = (1LL << 61) - 1;
3     static uint64_t step;
4     static vector<uint64_t> pw;
5
6     uint64_t addmod(uint64_t a, uint64_t b) const {
7         a += b;
8         if (a >= md) a -= md;
9         return a;
10    }
11
12    uint64_t submod(uint64_t a, uint64_t b) const {
13        a += md - b;
14        if (a >= md) a -= md;
15        return a;
16    }
17
18    uint64_t mulmod(uint64_t a, uint64_t b) const {
19        uint64_t l1 = (uint32_t) a, h1 = a >> 32, l2 = (uint32_t) b, h2 = b >>
20            32;
21        uint64_t l = l1 * l2, m = l1 * h2 + l2 * h1, h = h1 * h2;
22        uint64_t ret = (l & md) + (l >> 61) + (h << 3) + (m >> 29) + (m << 35 >>
23            3) + 1;
24        ret = (ret & md) + (ret >> 61);
25        ret = (ret & md) + (ret >> 61);
26        return ret - 1;
27    }
28
29    void ensure_pw(int sz) {
30        int cur = (int) pw.size();
31        if (cur < sz) {
32            pw.resize(sz);
33            for (int i = cur; i < sz; i++) {
34                pw[i] = mulmod(pw[i - 1], step);
35            }
36        }
37    }
38
39    vector<uint64_t> pref;
```

```
38     int n;
39
40     template<typename T>
41     hash61(const T& s) {
42         n = (int) s.size();
43         ensure_pw(n + 1);
44         pref.resize(n + 1);
45         pref[0] = 1;
46         for (int i = 0; i < n; i++) {
47             pref[i + 1] = addmod(mulmod(pref[i], step), s[i]);
48         }
49     }
50
51     inline uint64_t operator()(const int from, const int to) const {
52         assert(0 <= from && from <= to && to <= n - 1);
53         return submod(pref[to + 1], mulmod(pref[from], pw[to - from + 1]));
54     }
55 };
56
57 uint64_t hash61::step = (md >> 2) + rng() % (md >> 1);
58 vector<uint64_t> hash61::pw = vector<uint64_t>(1, 1);
```

8.4 kmp.cpp

```
1 template <typename T>
2 vector<int> kmp_table(int n, const T &s) {
3     vector<int> p(n, 0);
4     int k = 0;
5     for (int i = 1; i < n; i++) {
6         while (k > 0 && !(s[i] == s[k])) {
7             k = p[k - 1];
8         }
9         if (s[i] == s[k]) {
10             k++;
11         }
12         p[i] = k;
13     }
14     return p;
15 }
16
17 template <typename T>
18 vector<int> kmp_table(const T &s) {
19     return kmp_table((int) s.size(), s);
```

```

20 }
21
22 template <typename T>
23 vector<int> kmp_search(int n, const T &s, int m, const T &w, const vector<
    int> &p) {
24     assert(n >= 1 && (int) p.size() == n);
25     vector<int> res;
26     int k = 0;
27     for (int i = 0; i < m; i++) {
28         while (k > 0 && (k == n || !(w[i] == s[k]))) {
29             k = p[k - 1];
30         }
31         if (w[i] == s[k]) {
32             k++;
33         }
34         if (k == n) {
35             res.push_back(i - n + 1);
36         }
37     }
38     return res;
39     // returns 0-indexed positions of occurrences of s in w
40 }
41
42 template <typename T>
43 vector<int> kmp_search(const T &s, const T &w, const vector<int> &p) {
44     return kmp_search((int) s.size(), s, (int) w.size(), w, p);
45 }

```

8.5 manacher.cpp

```

1 template <typename T>
2 vector<int> manacher(int n, const T &s) {
3     if (n == 0) {
4         return vector<int>();
5     }
6     vector<int> res(2 * n - 1, 0);
7     int l = -1, r = -1;
8     for (int z = 0; z < 2 * n - 1; z++) {
9         int i = (z + 1) >> 1;
10        int j = z >> 1;
11        int p = (i >= r ? 0 : min(r - i, res[2 * (l + r) - z]));
12        while (j + p + 1 < n && i - p - 1 >= 0) {
13            if (!(s[j + p + 1] == s[i - p - 1])) {

```

```

14                break;
15            }
16            p++;
17        }
18        if (j + p > r) {
19            l = i - p;
20            r = j + p;
21        }
22        res[z] = p;
23    }
24    return res;
25    // res[2 * i] = odd radius in position i
26    // res[2 * i + 1] = even radius between positions i and i + 1
27    // s = "abaa" -> res = {0, 0, 1, 0, 0, 1, 0}
28    // in other words, for every z from 0 to 2 * n - 2:
29    // calculate i = (z + 1) >> 1 and j = z >> 1
30    // now there is a palindrome from i - res[z] to j + res[z]
31    // (watch out for i > j and res[z] = 0)
32 }
33
34 template <typename T>
35 vector<int> manacher(const T &s) {
36     return manacher((int) s.size(), s);
37 }

```

8.6 suffix-array.cpp

```

1 template <typename T>
2 vector<int> suffix_array(int n, const T &s, int char_bound) {
3     vector<int> a(n);
4     if (n == 0) {
5         return a;
6     }
7     if (char_bound != -1) {
8         vector<int> aux(char_bound, 0);
9         for (int i = 0; i < n; i++) {
10            aux[s[i]]++;
11        }
12        int sum = 0;
13        for (int i = 0; i < char_bound; i++) {
14            int add = aux[i];
15            aux[i] = sum;
16            sum += add;

```

```

17     }
18     for (int i = 0; i < n; i++) {
19         a[aux[s[i]]++] = i;
20     }
21 } else {
22     iota(a.begin(), a.end(), 0);
23     sort(a.begin(), a.end(), [&s](int i, int j) { return s[i] < s[j]; });
24 }
25 vector<int> sorted_by_second(n);
26 vector<int> ptr_group(n);
27 vector<int> new_group(n);
28 vector<int> group(n);
29 group[a[0]] = 0;
30 for (int i = 1; i < n; i++) {
31     group[a[i]] = group[a[i - 1]] + (!(s[a[i]] == s[a[i - 1]]));
32 }
33 int cnt = group[a[n - 1]] + 1;
34 int step = 1;
35 while (cnt < n) {
36     int at = 0;
37     for (int i = n - step; i < n; i++) {
38         sorted_by_second[at++] = i;
39     }
40     for (int i = 0; i < n; i++) {
41         if (a[i] - step >= 0) {
42             sorted_by_second[at++] = a[i] - step;
43         }
44     }
45     for (int i = n - 1; i >= 0; i--) {
46         ptr_group[group[a[i]]] = i;
47     }
48     for (int i = 0; i < n; i++) {
49         int x = sorted_by_second[i];
50         a[ptr_group[group[x]]++] = x;
51     }
52     new_group[a[0]] = 0;
53     for (int i = 1; i < n; i++) {
54         if (group[a[i]] != group[a[i - 1]]) {
55             new_group[a[i]] = new_group[a[i - 1]] + 1;
56         } else {
57             int pre = (a[i - 1] + step >= n ? -1 : group[a[i - 1] + step]);
58             int cur = (a[i] + step >= n ? -1 : group[a[i] + step]);
59             new_group[a[i]] = new_group[a[i - 1]] + (pre != cur);

```

```

60     }
61 }
62 swap(group, new_group);
63 cnt = group[a[n - 1]] + 1;
64 step <= 1;
65 }
66 return a;
67 }
68
69 template <typename T>
70 vector<int> suffix_array(const T &s, int char_bound) {
71     return suffix_array((int) s.size(), s, char_bound);
72 }
73
74 template <typename T>
75 vector<int> build_lcp(int n, const T &s, const vector<int> &sa) {
76     assert((int) sa.size() == n);
77     vector<int> pos(n);
78     for (int i = 0; i < n; i++) {
79         pos[sa[i]] = i;
80     }
81     vector<int> lcp(max(n - 1, 0));
82     int k = 0;
83     for (int i = 0; i < n; i++) {
84         k = max(k - 1, 0);
85         if (pos[i] == n - 1) {
86             k = 0;
87         } else {
88             int j = sa[pos[i] + 1];
89             while (i + k < n && j + k < n && s[i + k] == s[j + k]) {
90                 k++;
91             }
92             lcp[pos[i]] = k;
93         }
94     }
95     return lcp;
96 }
97
98 template <typename T>
99 vector<int> build_lcp(const T &s, const vector<int> &sa) {
100     return build_lcp((int) s.size(), s, sa);
101 }

```

8.7 z.cpp

```
1  template <typename T>
2  vector<int> z_function(int n, const T &s) {
3      vector<int> z(n, n);
4      int l = 0, r = 0;
5      for (int i = 1; i < n; i++) {
6          z[i] = (i > r ? 0 : min(r - i + 1, z[i - l]));
7          while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
8              z[i]++;
9          }
10         if (i + z[i] - 1 > r) {
11             l = i;
12             r = i + z[i] - 1;
13         }
14     }
15     return z;
16 }
17
18 template <typename T>
19 vector<int> z_function(const T &s) {
20     return z_function((int) s.size(), s);
21 }
```

9 template

9.1 hc.cpp

```
1  /**
2   *   author:   tourist
3   *   created:  $CURRENT_DATE.$CURRENT_MONTH.$CURRENT_YEAR $CURRENT_HOUR:
4               $CURRENT_MINUTE:$CURRENT_SECOND
5   */
6  #include <bits/stdc++.h>
7
8  using namespace std;
9
10 #ifdef LOCAL
11 #include "algo/debug.h"
12 #else
13 #define debug(...) 42
14 #endif
```

```
15 int main() {
16     ios::sync_with_stdio(false);
17     cin.tie(nullptr);
18     int tt;
19     cin >> tt;
20     for (int qq = 1; qq <= tt; qq++) {
21         cout << "Case_" << qq << ":_";
22         ${0}
23     }
24     return 0;
25 }
```

9.2 multithreaded.cpp

```
1  /**
2   *   author:   tourist
3   *   created:  $CURRENT_DATE.$CURRENT_MONTH.$CURRENT_YEAR $CURRENT_HOUR:
4               $CURRENT_MINUTE:$CURRENT_SECOND
5   */
6  #include <bits/stdc++.h>
7
8  using namespace std;
9
10 class Solution {
11 public:
12     int k;
13     string s, w;
14
15     void readData() {
16     }
17
18     void solve(stringstream& out) {
19     }
20 };
21
22 const int maxThreads = 8;
23 const int numTests = 1000;
24
25 stringstream out[numTests];
26 mutex mu;
27 int cur, tt;
```

```

29  thread threads[maxThreads];
30
31  void solutionRunner() {
32      while (true) {
33          Solution s;
34          int id;
35          mu.lock();
36          if (cur >= tt) {
37              mu.unlock();
38              return;
39          }
40          id = cur;
41          cur++;
42          s.readData();
43          mu.unlock();
44          s.solve(out[id]);
45      }
46  }
47
48  using namespace std::chrono;
49
50  long long now() {
51      milliseconds ms = duration_cast<milliseconds>(system_clock::now().
52          time_since_epoch());
53      return ms.count();
54  }
55
56  int main() {
57      ios::sync_with_stdio(false);
58      cin.tie(0);
59      long long start = now();
60      cin >> tt;
61      cur = 0;
62      for (int i = 0; i < maxThreads; i++) {
63          threads[i] = thread(solutionRunner);
64      }
65      for (int i = 0; i < maxThreads; i++) {
66          threads[i].join();
67      }
68      for (int i = 0; i < tt; i++) {
69          cout << "Case_" << i + 1 << ":_" << '\n';
70          cout << out[i].str();
71      }

```

```

71      cerr << "time_" << now() - start << "_ms" << endl;
72      return 0;
73  }

```

9.3 multithreaded2.cpp

```

1  /**
2   *   author:   tourist
3   *   created:  $CURRENT_DATE.$CURRENT_MONTH.$CURRENT_YEAR $CURRENT_HOUR:
4               $CURRENT_MINUTE:$CURRENT_SECOND
5   */
6
7  #ifndef _GLIBCXX_DEBUG
8
9  #include <bits/stdc++.h>
10
11  using namespace std;
12
13  #ifdef LOCAL
14  #include "algo/debug.h"
15  #else
16  #define debug(...) 42
17  #endif
18
19  mutex mut;
20  int qq = 0;
21  int tt;
22
23  class Solution {
24  public:
25      int test_id;
26
27      explicit Solution(int test_id_) : test_id(test_id_) {}
28
29      ${0}
30
31      void ReadData() {
32
33      }
34
35      void Solve(stringstream& out) {
36
37          mut.lock();
38          debug(++qq, tt, test_id, clock());

```

```

37     mut.unlock();
38 }
39 };
40
41 int main() {
42     ios::sync_with_stdio(false);
43     cin.tie(0);
44     cin >> tt;
45     vector<Solution> solutions;
46     solutions.reserve(tt);
47     for (int test_id = 0; test_id < tt; test_id++) {
48         solutions.emplace_back(test_id);
49         solutions.back().ReadData();
50     }
51     debug("input_read, kicking off");
52     vector<stringstream> outs(tt);
53     vector<thread> threads;
54     threads.reserve(tt);
55     for (int test_id = 0; test_id < tt; test_id++) {
56         threads.emplace_back(&Solution::Solve, &solutions[test_id], ref(outs[
57             test_id]));
58     }
59     for (int test_id = 0; test_id < tt; test_id++) {
60         threads[test_id].join();
61     }
62     for (int i = 0; i < tt; i++) {
63         cout << "Case_" << i + 1 << ":_" << outs[i].str();
64     }
65     return 0;
66 }

```

9.4 q1.cpp

```

1 /**
2  *   author:   tourist
3  *   created:  $CURRENT_DATE.$CURRENT_MONTH.$CURRENT_YEAR $CURRENT_HOUR:
4             $CURRENT_MINUTE:$CURRENT_SECOND
5  */
6 #include <bits/stdc++.h>
7 using namespace std;

```

```

8
9 #ifdef LOCAL
10 #include "algo/debug.h"
11 #else
12 #define debug(...) 42
13 #endif
14
15 int main() {
16     ios::sync_with_stdio(false);
17     cin.tie(nullptr);
18     ${0}
19     return 0;
20 }

```

9.5 qt.cpp

```

1 /**
2  *   author:   tourist
3  *   created:  $CURRENT_DATE.$CURRENT_MONTH.$CURRENT_YEAR $CURRENT_HOUR:
4             $CURRENT_MINUTE:$CURRENT_SECOND
5  */
6 #include <bits/stdc++.h>
7 using namespace std;
8
9 #ifdef LOCAL
10 #include "algo/debug.h"
11 #else
12 #define debug(...) 42
13 #endif
14
15 int main() {
16     ios::sync_with_stdio(false);
17     cin.tie(nullptr);
18     int tt;
19     cin >> tt;
20     while (tt--) {
21         ${0}
22     }
23     return 0;
24 }

```