Part 1:

Direct proof:

x is even: x = 2k

x\*y = 2k\*y=2(k\*y)

Any number which can be divided by 2 is even, so 2(k\*y) is even

And x\*y is even.

Part2:

Contra positive:

If h is not an injection, then f or g is not injection

If h is not an injection there are too cases:

Case 1: there must be an x that x!=y, h(x)=h(y)

Because of h: f o g (composition)

f or g must have this x. Which make the one who have it become not injection.

Then f or g are not injections

Case 2:there must be an x that x = y, h(x)!=h(y)

Because of h: f o g (composition)

f or g must have this x. Which make the one who have it become not injection.

Then f or g are not injections

So, If h is not an injection, then f or g are is injection

So, If f and g are injections, then h is an injection.

Part3:

Proof by contradiction:

Consider that (x, y) is an element of (AXB)U(AXC)

Iff (x,y) is in (AXB)U(AXC)

Iff (x,y ) is in AXB or (x,y) is in AXC

Iff x is in A and y is in B or x is in A and y is in C

iff x is an element of A and y is an element of B or C

So, (x, y) is an element of AX(BUC)

So, AX(BUC)= (AXB)U(AXC)

Part4：

Counter example:

i =3 is odd

i^2 =9 is odd

The proposition is not true in all cases.

Part5:

Proof by induction:

Step1:Base case: n = 0

I^2+I+1 = 1 is odd.

Step2: Ok at k: Assume k^2+k+1 is odd

Step3: Is it ok at (k+1):

(K+1)^2+(k+1)+1=k^2+2k+1+k+1+1 = k^2+3k+3= (k^2+k+1)+2k+2

Because, 2k+2 = 2(k+1) is even

So (k^2+k+1)+2k+2 = odd + even = odd.