CE 810 Homework 3

Due by the start of class on Wed. 9/24

1. (50pt) Write a Python program to obtain an incremental-iterative solution for the system in Figure 1 using an incremental load vector of $< \Delta U_e, \Delta W_e >=< 70N, -7N >$ until the force W_e is -259 *N*. Make two plots: one for (U_e vs. u) and another for ($-W_e$ vs. -w). Use the following dimensions and properties:
   $EA = 5 \times 10^7 N$, $z = 25mm$, $l = 2500mm$, $K_s = 2.35N/mm$

For convergence test, use a tolerance of 1e-4 for the norm of the unbalanced force vector. To submit your codes, please "Fork" the GitHub repo xinlong-du/CE810Stability, clone it to your local computer, work in the folder with your name, and use "Pull request" to submit your work before the due time.

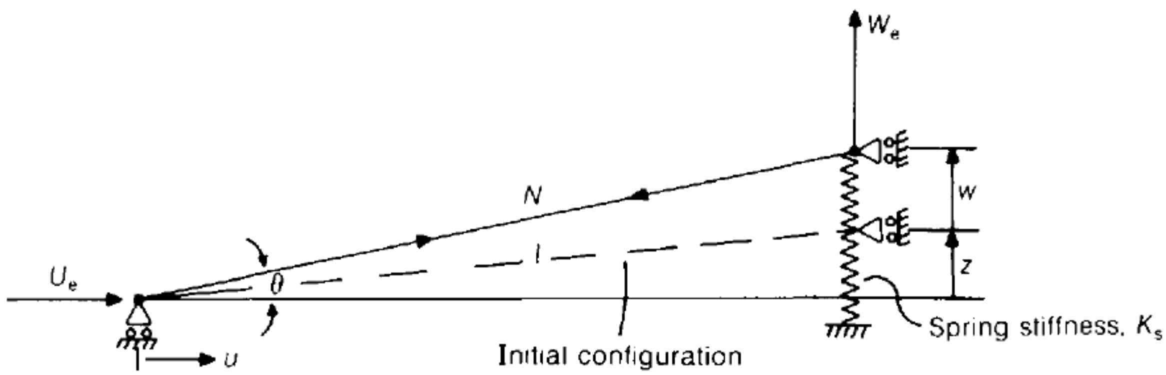


Figure 1. A 2-DOF system (cross-sectional area=A, Young's modulus=E).

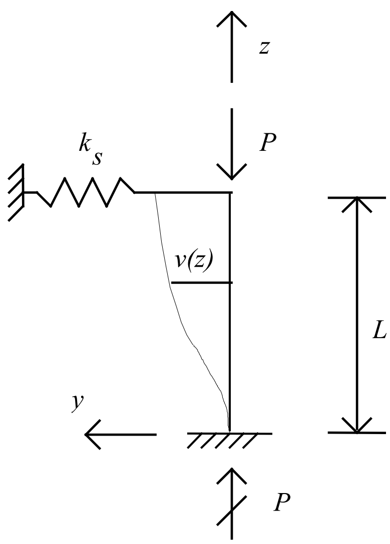
2. (50pt) The fourth order ordinary differential equation for flexural buckling of a column is:

$$v^{iv}(z) + k^2 v''(z) = 0$$
$$k^2 = \frac{P}{EI}$$

The general solution to this fourth order equation takes the form:

$$v(z) = A + Bz + C\cos(kz) + D\sin(kz)$$

The column below is fixed at its bottom and is supported at its top by a translational spring (which resists force in the y direction only; the column at the top is free to move in the z direction and to rotate about the x axis). The spring has stiffness k_s. The column is free to sway to the side at its top, as shown. Use the coordinate system that is shown!



Cross-sectional area: A_g

In-plane moment of inertia: I

Modulus of Elasticity: E

Deflection: $v(z)$

Use the coordinate system shown!

The applied load is P, area is A_g, the modulus is E, the moment of inertia in the plane of buckling is I, and the deflection in the plane of buckling is $v(z)$. The spring stiffness is $k_s = gEI/L^3$, where g is a real number that will vary between zero and infinity depending on the relative stiffness of the spring compared to the stiffness of the column.

a) Determine the characteristic equation for this column in terms of g, EI, L, and k. Your derivation should be clean, neat, concise, and *must* highlight the following items along the way:
   - Draw this column, its coordinate system, and **the buckled shape**.
   - The governing fourth order differential equation for this buckling problem (you do not need to re-derive this equation – it is given above).

- The general solution for the buckled deflection, *v(z)* (you do not need to rederive this equation – it is given above).
- The boundary conditions of the problem.
- The characteristic equation of the homogenous solution to the problem.

b) Determine mathematically the critical buckling load for this column *when the translational stiffness of the spring approaches infinity*. **Draw the buckled shape clearly and indicate the dimension *KL*.**

c) When $k_s \to \infty$, what is the value of the effective length factor for this column (determine this by inspection directly from your critical load equation)?

d) Hint: One of the four boundary conditions that you will need to solve this problem will relate the shear in the column to the stiffness of the spring times the lateral deflection of the spring. Note that the *positive* shear force in the column at incipient buckling equals $-EIv'''(z) - Pv'(z)$.