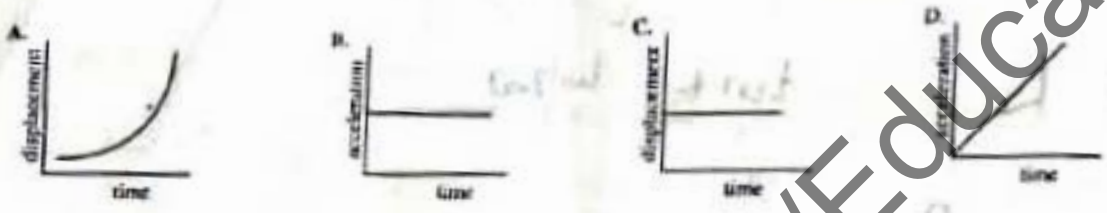
UNIVERSITY OF BUEA
COLLEGE OF TECHNOLOGY
ENTRANCE EXAMINATION INTO FIRST YEAR OF BTECH PROGRAM

PAPER TITLE: PHYSICS
DATE: 18/10/18 **TIME:** 12.00 – 15.00
TIME ALLOWED: 3 HOURS **COEFFICIENT:** 4

SECTION 1, 1hr 30min

NSTRUCTIONS: Answer all questions in this section. Write down only the letter that corresponds to he best answer to each question. Each question carries two marks.

. Which of the following graphs best represents a particle with constant velocity?



. A car accelerates at a constant rate from 0 to 25 ms^{-1} over a distance of 25 m. Approximately how long do ke the car to reach the velocity of 25 ms^{-1}?

A. 1s B. 2 s C. 4 s D. 8 s

. A driver moving at a constant speed of 20 ms^{-1} sees an accident up ahead and hits the brakes. If the ecelerates at a constant rate of -5 ms^{-2}, how far does the car go before it comes to a stop?

A. 10m B. 20m C. 40m D. 100m

Two balls are dropped from a tall tower. The balls are the same size. But ball X has greater mass than bal hen both balls have reached terminal velocity, which of the following is true?

A. The force of air resistance on either ball is zero.
B. Ball X has greater velocity.
C. The ball X has greater acceleration.
D. The acceleration of both balls is 9.8 ms^{-2}

If F is the force of air resistance on an object with mass m moving at a constant velocity, which of the follov st describes the acceleration of the object when the force of air resistance is reduced by a factor of 4?

A. $\frac{F}{m}$ B. $\frac{3F}{4m}$ C. $\frac{F}{4m}$ D. $\frac{3F}{4m}$

Which of the following is true of the magnitudes of velocity and acceleration, as the ball rolls down the sl shown? Note: Please ignore any centripetal acceleration.

The velocity and the acceleration increase.
The velocity and the acceleration decrease.
The velocity increases and the acceleration decreases.
The velocity decreases and the acceleration Increases.

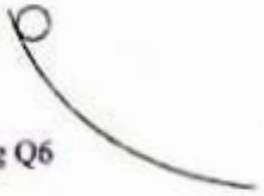


Fig Q6

7. If the radius of the orbit of a satellite orbiting the earth is reduced by a factor of 2, the gravitation the earth will:

A. decrease by a factor of 2.
B. remain the same.
C. increase by a factor of 2.
D. increase by a factor of 4.

8. If the rear wheels of the truck pictured below drive the truck forward, then the frictional force on the rear tires due to the road is:

A. kinetic and in the direction of A.
B. kinetic and in the direction of B.
C. static and in the direction of A.
D. static and in the direction of B.

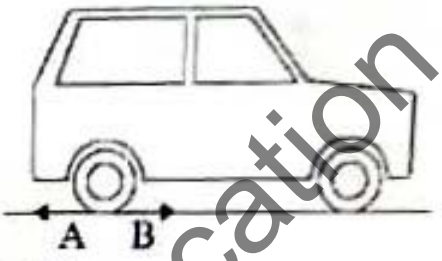


Fig Q8

9. If a rope capable of withstanding 900N of tension is attached to a wall as shown, what is the maxim that can be applied in the direction of *F* before the rope will break?

A. 300 N B. 450 N
C. 900 N D. 1800 N

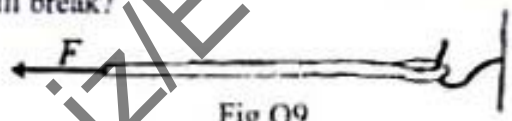


Fig Q9

10. The diagram below shows two different masses hung from identical Hooke's law springs. The Hooke's law constant *k* for the springs is equal to:

A. 2 *N/cm* B. 5 *N/cm* C. 10 *N/cm* D. 20 *N/cm*

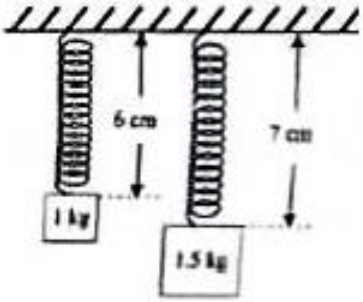


Fig Q10

11. The pulley shown below is old and rusted. When the 50 kg mass is allowed to drop, friction in the pulley creates a constant 200 N force upward. What is the tension in the

A. 0N B. 200 N C. 400 N D. 600 N

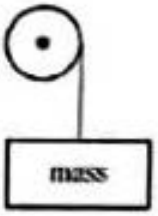
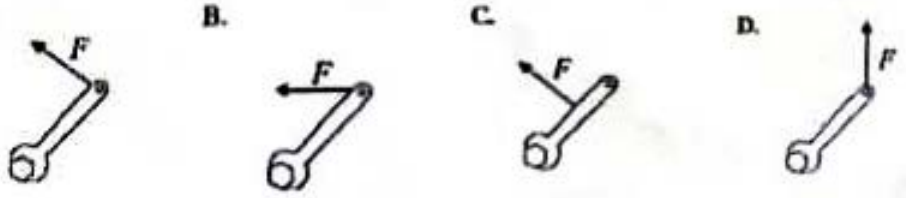


Fig Q11

12. Which of the following describes a situation requiring no net force?
A. A car starts from rest and reaches a speed of 80 km/hr after 15 seconds.
B. A bucket is lowered from a rooftop at a constant speed of 2 *m/s*.
C. A skater glides along the ice, gradually slowing from 10 *m/s* to 5 *m/s*.
D. The pendulum of a clock moves back and forth at a constant frequency of 0.5 hertz.

13. If all of the forces below have equal magnitude, which one creates the most torque?

A. B. C. D.



4. A 100 N force is applied as shown to a 10 kg object for 2 seconds. If the object is initially at rest, what is its nal velocity? (Ignore friction)

A. 8.7 ms^{-1} B. 1 ms^{-1} C. 17.4 ms^{-1} D. 34.8 ms^{-1}

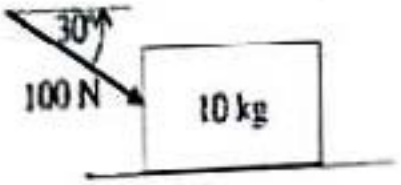


Fig Q14

15. A winch is used to lift heavy objects to the top of building under construction. A winch with a power of 50 kW was replaced with a new winch with a power of 100 kW. Which of the following statements about the new winch is NOT true?

A. The new winch can do twice as much work in the same time as the old winch.
B. The new winch takes twice as much time to do the same work as the old winch.
C. The new winch can raise objects with twice as much mass at the same speed as the old winch.
D. The new winch can raise objects with the same mass at twice the speed of the old winch.

16. A 3kg cat sitting on a 1.5 kg piece of cardboard on a frozen lake wants to jump to shore without touching the ice. If there is no friction between the cardboard and the ice, when the cat jumps, the cardboard will move in the opposite direction with a velocity:

A. half as great as the cat's velocity. B. equal to the cat's velocity.
C. twice as great as the cat's velocity. D. four times as great as the cat's velocity.

17. Ball A moving at 12 ms^{-1} collides elastically with ball B as shown. If both balls have the same mass, what is the final velocity of ball A?

A. 3 ms^{-1} B. 6 ms^{-1} C. 9 ms^{-1} D. 12 ms^{-1}

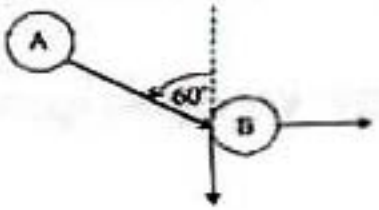


Fig Q17

18. An inventor designs a machine that he claims will lift a 30kg object with the application of only a 25N force. If the inventor is correct, what is the shortest possible distance through which the force must be applied for each meter that the object is raised?

A. 5m B. 8m C. 12 m D. 15m

19. The pulley system shown below operates as a modified lever. Pulley A and pulley B turn together so when a person pulls on rope A the mass attached to rope B will be lifted. Which of the following changes to the system will reduce the force needed to lift the mass?

A. Increase the diameter of pulley A.
B. Increase the length of rope B.
C. Increase the length of rope A.
D. Increase the diameter of pulley B.

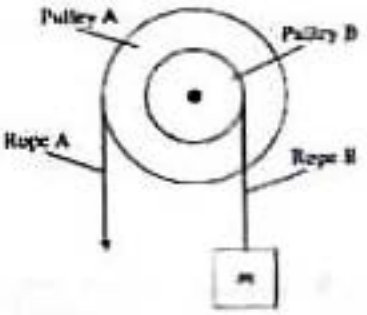


Fig Q19

20. The half-life of substance X is 45 years, and it decomposes to substance Y. A sample from a meteorite was taken which contained 1.5% of X and 13.5% of Y by mass. If substance Y is not normally found on a meteorite, what is the approximate age of the meteorite?

A. 45 years B. 100 years C. 140 years D. 270 years

21. In nuclear fission, a uranium nucleus combines with a neutron, becomes unstable, and splits into Ce plus two neutrons. The change in the mass of the interacting parts is 0.211 amu. How much energy is relea this reaction? (Note: $e^2 = 931.5$ MeV/amu)

A. 98 MeV B. 130 MeV C. 157 MeV D. 197 MeV

22. Mercury has specific gravity of 13.6. The column of mercury in the barometer below has a height $h = 76$ cm. If a similar barometer were made with water, what would be the approximate height h of the column of water?

Fig Q22

A. 5.6 cm B. 76 cm C. 154 cm D. 1034 cm

23. An ideal fluid with pressure P flows through a horizontal pipe with radius r. If the radius of the p increased by a factor of 2, which of the following most likely gives the new pressure?

A. P B. $4P$ C. $16P$ D. The new pressure cannot be determined

24. All of the Following would increase the volume flow rate of a fluid being pumped through a pipe EXC

A. increasing the pressure difference between the ends of the pipe.
B. decreasing the fluid viscosity.
C. increasing the length of the pipe.
D. increasing the pipe radius.

25. Water in moist soil rises through capillary action. The intermolecular forces between water molecules

A. weaker than the intermolecular forces between water and soil molecules.
B. equal to the intermolecular forces between water and soil molecules.
C. stronger than the intermolecular forces between water and soil molecules.
D. The comparative strength between the intermolecular forces cannot be determined with the information

26. The Young's modulus for bone is 9×10^9 N/m^2. What is the percent change in length of a tibia with a sectional area of 6 cm^2, if it experiences a compressive force of 5.4×10^3 N?

A. 0.001% B. 0.1% C. 1% D. 10%

27. The bulk modulus for a substance would be most important to a researcher who is testing material that be:

A. used in high tension cables. B. submerged deep in the ocean.
C. subjected to high temperatures. D. transported at great speeds.

28. If an ocean wave hits a particular beach once every 4 seconds, and the wave peaks are 12 meters apart, what velocity are the waves coming into shore?

A. 3 ms^{-1} B. 4 ms^{-1} C. 12 ms^{-1} D. 48 ms^{-1}

29. Waves generally travel faster in solids than in gases because:

A. The density of solids is generally greater than the density of gases.
B. The density of gases is generally greater than the density of solids.
C. Solids are less compressible than gases.
D. Gases are less compressible than solids.

30. If the intensity of a sound is doubled, the decibel level will increase by:
A. less than 10dB. B. exactly 10 dB. C. more than 10 dB. D. exactly 20 dB.

31. How many wavelengths are shown between the dotted lines in the wave form below?

A. 1 B. 2 C. 3 D. 4

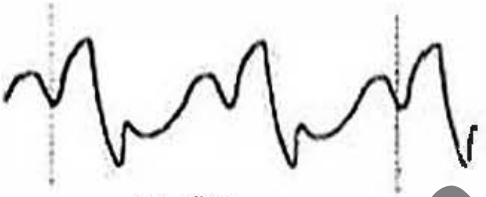


Fig Q31

32. If a guitar string is 0.5 m long, what is the wavelength of its third harmonic?
A. 0.25 m B. 1m C. 0.5 m D. 1m 0.33 m

33. Two violinists are playing together, slightly out of tune. If one violinist produces a frequency of 883 Hz and the other produces a frequency of 879 Hz, beats would be heard with a frequency of:
A. 2Hz. B. 4Hz. C. 881 Hz. D. 1762 Hz.

34. In order for two sound waves to have an audible beat frequency, the two waves must be:
A. in phase. B. out of phase. C. close in frequency. D. of the same wavelength.

35. All of the following are examples of harmonic motion EXCEPT:
A. a pendulum moving back and forth
B. a skydiver falling through the atmosphere.
C. a car moving around a circular track.
D. a string vibrating on a musical instrument.

36. Two charged metal plates are placed one meter apart creating a constant electric field between them. A 0 coulomb charged particle is placed in the space between them. The particle experiences a force of 100N due the electric field. What is the potential difference between the plates?
A. 1V B. 10V C. 1000V D. 100V

37. The electric field for two point charges A and B is shown belo
Which of the following is true?
A. Both charges are positive.
B. Both charges are negative.
C. The charges have opposite charges.
D. The charges can not be determined.

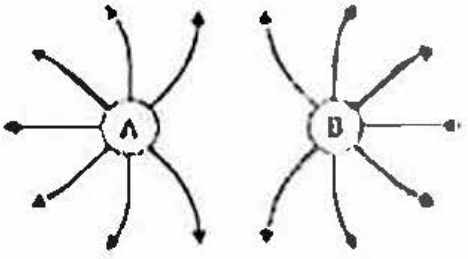


Fig Q37

38. Each resistor in the circuit below has a resistance of 2 Ω. The battery is a 12 volt battery. What is the current across resistor B?

A. 1 A B. 2A
C. 3A D. 4A

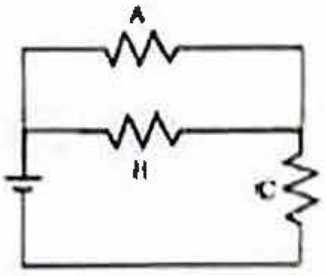


Fig Q38

39. What is the energy required to operate a 60 W light bulb for 1 minute?
A. 1 J B. 60J C. 360J D. 3600 J

40. The magnetic field created by a long straight current carrying wire:
A. decreases in strength proportionally with the distance from the wire.
B. decreases in strength with the square of the distance from the wire.
C. increases in strength proportionally with the distance from the wire.
D. increases in strength with the square of the distance from the wire.

41. A charged particle moves horizontally through a magnetic field which points directly upward. The force o the particle due to the magnetic field is:
A. perpendicular to the magnetic field and parallel to the velocity of the particle.
B. parallel to the magnetic field and perpendicular to the velocity of the particle.
C. perpendicular to the magnetic field and perpendicular to the velocity of the particle.
D. parallel to the magnetic field and parallel to the velocity of the particle.

42. A particle of mass m is fired into a magnetic field of strength B at a speed v. The particle travels in a circula path inside the field with a radius r. Which of the following expressions gives the magnitude of the charge on the particle?

A. $\frac{vB}{mr}$ B. $\frac{mv}{Br}$ C. $\frac{mr}{v^2B}$ D. $\frac{mv^2}{Br}$

43. If a light on a dimmer switch is gradually turned down, it will generally show a red glow at the moment before it is turned off. This is because red light:
A. has less energy than light of any other color.
B. moves more quickly through air than light of any other color.
C. bas more energy than light of any other color.
D. moves more slowly through air than light of any other color.

44. When an object is 10 cm from a certain converging lens, the image is magnified by a factor of 1.5. What is the distance of the image?
A. 3.3 cm B. 6.6 cm C. 10 cm D. 15 cm

45. An inverted image is created 5 ill in front of a mirror. Which of the following could be true about the mirror and the object?
A. The mirror is convex with less than a 5 ill focal distance.
B. The mirror is concave with less than a 5 ill focal distance.
C. The mirror is convex with more than a 5 m focal distance.
D. The mirror is concave with more than a 5 ill focal distance.

SECTION 2: (1hr 30min):

Answer any three (03) questions of your choice in this section

Q1a. A shot putter releases the shot some distance above the level ground with a velocity of 12.0 m/s, 51.0° above the h orizontal. The shot hits the ground 2.08s later. You can ignore air resistance.

(i) What are the components of the shot's acceleration while in flight? **(2 marks)**

(ii) What are the components of the shot's velocity at the beginning and at the end of its trajectory? **(5 marks)**

(iii) How far did she throw the shot horizontally? **(2 marks)**

(iv) How high was the shot above the ground when she released it? **(2 marks)**

Q1b. Two crates, one with mass 4.0kg and the other with mass 6.0kg, sit on the frictionless surface of a frozen pond, connected by a light rope (Fig. Q1c). A woman wearing golf shoes (so she can get traction on the ice) pulls horizontally on the 6.0kg crate with a force F that gives the crate an acceleration of 2.5 ms^{-2}.

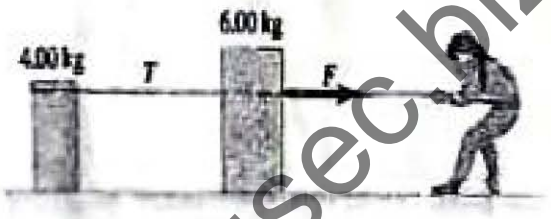


Figure Q1c.

(i) What is the acceleration of the 4.0kg crate? **(2 marks)**

(ii) Draw a free-body diagram for the 4.0kg crate. Use that diagram and Newton's second law to find the tension T in the rope that connects the two crates. **(3 marks)**

(iii) Draw a free-body diagram for the 6.0kg crate. What is the direction of the net force on the 6.0kg crate? Which is larger in magnitude, force T or force F? **(3 marks)**

(iv) Use part (iii) and Newton's second law to calculate the magnitude of the force F. **(2 marks)**

1c. A light rope is attached to a block with mass 4.0 kg that rests on a frictionless, horizontal surface. The horizontal rope passes over a frictionless, massless pulley, and a block with mass m is suspended from the other end. When the blocks are released, the tension in the rope is 10.0 N.

(i) Draw two free-body diagrams, one for the 4.0kg block and one for the block with mass m. **(4 marks)**

(ii) What is the acceleration of either block? **(2 marks)**

(iii) Find the mass m ofthe hanging block. **(2 marks)**

(iv) How does the tension compare to the weight of the hanging block? **(2 marks)**

Q2a. A luggage handler pulls a 20.0kg suitcase op a ramp inclined at 25.0° above the horizontal by a force F of magnitude 140 N that acts parallel to the ramp. The coefficient of kinetic friction between the ramp and the incline is $\mu_k = 0.30$. If the suitcase travels 3.80 m along the ramp, calculate

(i) the work done on the suitcase by the force F; **(3 marks)**

(ii) the work done on the suitcase by the gravitational force; **(3 marks)**

(iii) the work done on the suitcase by the normal force; **(2 marks)**

(iv) the work done on the suitcase by the friction force; **(3 marks)**

(v) the total work done on the suitcase. **(3 marks)**

(vi) If the speed of the suitcase is zero at the bottom of the ramp, what is its speed after it has traveled 3.80 m along the ramp? **(3 marks)**

Q2b. A uniform ladder 5.0 m long rests against a frictionless, vertical wall with its lower end 3.0 m from the wall. The ladder weighs 160 N. The coefficient of static friction between the foot of the ladder and the ground is 0.40. A man weighing 740 N climbs slowly up the ladder. Start by drawing a free-body diagram of the ladder.

(i) What is the maximum frictional force that the ground can exert on the ladder at its lower end? **(3 marks)**

(ii) What is the actual frictional force when the man has climbed 1.0 m along the ladder? **(3 marks)**

(iii) How far along the ladder can the man climb before the ladder starts to slip? **(3 marks)**

Q2c. A uniform, 255N rod that is 2.0 m long carries a 225N weight at its right end and an unknown weight W toward the left end (Fig. Q2c). When W is placed 50.0 cm from the left end of the rod, the system just balances horizontally when the fulcrum is located 75.0 cm from the right end.

Figure Q2c.

(i) Find W. **(2 marks)**

(ii) If W is now moved 25.0 cm to the right, how far and in what direction must the fulcrum be moved to restore balance? **(3 marks)**

Q3a. Transverse waves on a string have wave speed 8.0m/s, amplitude 0.07m, and wavelength 0.32 m. The waves travel in the $-x$ direction, and at $t = 0$ the $x = 0$ end of the string has its maximum upward displacement.

(i) Find the frequency, period, and wave number of these waves. **(6 marks)**

(ii) Write a wave function describing the wave. **(3 marks)**

(iii) Find the transverse displacement of a particle at $x = 0.36$m at time $t = 0.15$s. **(2 marks)**

(iv) How much time must elapse from the instant in part (iii) until the particle at $x = 0.36$ m next has maximum upward displacement? **(3 marks)**

3b. In a constant-volume process, 209 J of energy is transferredby heat to 1.00 mol of an ideal monatomic g initially at 300 K.

i) Clearly distinguish among *temperature, heat,* and *internal energy.* **(3 marks)**

Find

(ii) the increase in internal energy of the gas, **(2 marks)**

(iii) the work done on it, and **(2 marks)**

(iv) its final temperature. **(3 marks)**

Q3c. (i) In our model of the kinetic theory of gases, molecules were viewed as hard spheres colliding elastical with the walls of the container. Is this model realistic? **(2 marks)**

In an insulated vessel, 250 g of ice at 0°C is added to 600 g of water at 18.0°C.

(ii) What is the final temperature of the system? **(2 marks)**

(iii) How much ice remains when the system reaches equilibrium? **(3 marks)**

Q4a. (i) Two protons in a molecule are separated by 3.80×10^{-10} m. Find the electric force exerted by one proto onthe other. **(3 marks)**

(ii) How does the magnitude of this force compareto the magnitude of the gravitational force betweenthe tw protons? **(3 marks)**

(iii)What must be the charge-to-mass ratio of a particle if the magnitude of the gravitationalforce between two o these particles equals thernagnitude of electric force between them? **(3 marks)**

Q4b. Three capacitors are connected to a battery as shown inFigure Q4b. Their capacitances are $C_1 = 3C$ $C_2 = C$, and $C_3 = 5C$.

(i) What is the equivalent capacitance of this set of capacitors? **(2 marks)**

(ii) State the ranking of the capacitors according to the charge they store, from largest tosmallest. **(2 marks)**

(iii) Rank the capacitors according to the potential differences across them, from largest to smallest. **(2 marks)**

(iv) If C_1 is increased, what happens to the charge stored by each of the capacitors? **(3 marks)**

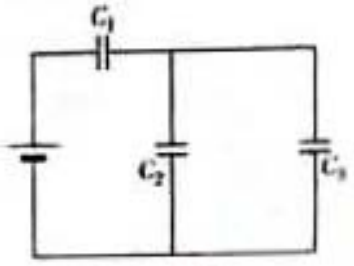


Figure Q4b

1/2

20

3/20

Q4c. (i) Find the equivalent resistance between points *a* and *b* in Figure Q4c. **(3 marks)**

(ii) A potential difference of 34.0 V is applied between points *a* and *b*. Calculate the current in each resistor. **(6 marks)**

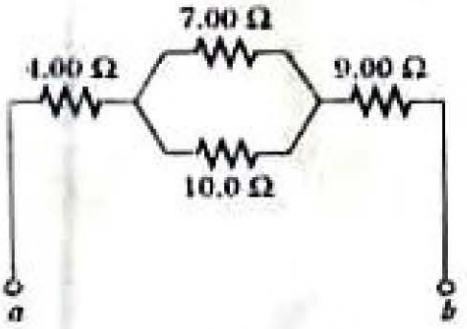


Figure Q4c

Q4d. A series *RL* circuit with $L = 3.0$H and a series *RC* circuitwith $C = 3.0\mu$F have equal time constants. If the twocircuits contain the same resistance *R*,

(i) what is the value of *R* and **(2 marks)**

(ii) what is the time constant? **(2 marks)**

Q5a. Light of wavelength 700 nm is incident on the face of afused quartz prism at an angle of 75.0° (with respect to thenormal to the surface). The apex angle of the prism is 60.0°. Use the value of *n* as 1.458 and calculate the angle

(i) of refraction at this first surface, **(2 marks)**

(ii) of incidence at the second surface, **(3 marks)**

(iii) of refraction at the second surface, and **(2 marks)**

Q5b. (i) A converging lens has a focal length of 20.0 cm. Locate the image for object distances of 40.0 cm, and 10.0 cm. For each case, state whether the image is real or virtual and upright or inverted. Find the magnification in each case. **(6, 6 marks)**

(ii) The wavelength of red helium–neon laser light in air is 632.8 nm. What is its frequency, wavelength in glass that has an index of refraction of 1.50? **(2, 2 marks)**

(iii) A person sees clearly when he wears eyeglasses that have apower of −4.0 diopters and sit 2.0 cm in front of hiseyes. If the person wants to switch to contact lenses, which are placed directly on the eyes, what lens power should beprescribed? **(2 marks)**

Q5c. A laser used to weld detached retinas emits light with a wavelength of 652 nm in pulses that are 20.0 ms in duration. The average power during each pulse is 0.600 W.

(i) How much energy is in each pulse in joules? In electron volts? **(2 marks)**

(ii) What is the energy of one photon in joules? In electron volts? **(2 marks)**

(iii) How many photons are in each pulse? **(2 marks)**

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PAPER TITLE: MATHEMATICS
DATE: 18/10/18
TIME ALLOWED 3 HOURS

TIME: 08.00 – 11.00
COEFFICIENT: 4

SECTION 1. 1hr 30min

INSTRUCTIONS: Answer all questions in this section. Write down only the letter that corresponds to the best answer to each question. Each question carries two marks.

1. The quadratic equation $ax^2 + bx + c = 0$ has real and distinct roots if
   A) $b^2 - 4ac \geq 0$ B) $b^2 - 4ac = 0$ C) $b^2 - 4ac < 0$ D) $b^2 - 4ac > 0$
2. The solution of the inequality $\frac{(x+1)(x+4)}{x-2} \leq 0$ is
   A) $\{x : x \leq -1 \text{ or } 2 < x \leq 4\}$ B) $\{x : x \leq -1 \text{ or } 2 \leq x \leq 4\}$
   C) $\{x : -1 \leq x < 2 \text{ or } x \geq 4\}$ D) $\{x : -1 \leq x \leq \text{ or } x > 4\}$
3. $\log_3 27 + \frac{1}{2}\log_2 16 - 3\log_5 125$ simplifies to
   A) 1 B) -4 C) 2 D) -2.
4. The solution of the equation $2^{2x} - 3(2^x) - 4 = 0$ is
   A) $x = 2$ B) $x = 4$ C) $x = -1$ or $x = 4$ D) $x = -1$ or $x = 2$
5. Let A={1, 2, 3, 4} and R a relation defined on A, where R={(1,1), (1,2), (2, 1), (2, 3), (3,3), (3,2)}. The
   A) reflexive B) symmetric C) anti-symmetric D) transitive.
6. A function $f: \mathbb{R} \to \mathbb{R}$ is given by $f(x) = \frac{x}{\sqrt{x-1}}$. The domain of f is
   A) $\{x : x < 1\}$ B) $\{x : x \leq 1\}$ C) $\{x : x > 1\}$ D) $\{x : x \geq 1\}$.
7. The function $f: \mathbb{R} \to \mathbb{R}$, where $f(x) = x^2 + 5$ is
   A) Odd B) periodic C) has a maximum value D) even.
8. The value of a for which $x + 2$ is a factor of the polynomial $f(x) = x^3 - x^2 + ax + 2$ is
   A) 3 B) -2 C) -5 D) 4.
9. The partial fraction decomposition of the function $f(x) = \frac{x^2-3x+1}{(x+3)(x+1)^2}$ takes the form A) $\frac{P}{x+3} + \frac{Q}{(x+1)^2}$ B) $\frac{P}{x+3} + \frac{Qx+R}{(x+1)^2}$ C) $\frac{P}{x+3} + \frac{Q}{x+1}$ D) $\frac{P}{x+3} + \frac{Q}{x+1} + \frac{R}{(x+1)^2}$
10. The trigonometric expression $\frac{1-\sin^2 x}{\cot^2 x}$ is identical to
    A) $\cot x \sin x$ B) $\sec^2 x \tan x$ C) $\sec^2 x$ D) $\sin^2 x$.
11. If $\sec x = \frac{13}{5}$, then $\sin x$ is
    A) $\frac{5}{13}$ B) $\frac{12}{13}$ C) $\frac{5}{12}$ D) $\frac{12}{5}$
12. The cosine of the angle between the vectors $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 4\mathbf{j} - 3\mathbf{k}$ is
    A) $-\frac{2}{15}$ B) $\frac{2}{15}$ C) $\frac{2}{45}$ D) $-\frac{2}{45}$.
13. The value of α for which the vectors $\mathbf{a} = 5\mathbf{i} + \alpha\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ are perpendicular is
    A) 2 B) -2 C) $-\frac{1}{2}$ D) $\frac{1}{2}$.
14. The complex number $z = \frac{3-2i}{1+5i}$ in standard form is
    A) $-\frac{7}{11} + \frac{1}{2}i$ B) $\frac{7}{6} + \frac{1}{2}i$ C) $\frac{17}{6} + \frac{1}{2}i$ D) $\frac{13}{6} - \frac{1}{2}i$.

15. The complex $z = -1 - \sqrt{3}\,i$ in Euler form is

A) $2e^{-\frac{\pi}{3}i}$ B) $4e^{\frac{\pi}{3}i}$ C) $4e^{-\frac{\pi}{6}i}$ D) $2e^{\frac{\pi}{3}i}$

16. If $z = \cos 14\pi + i \sin 14\pi$, then $z^{\frac{1}{2}}$ is

A) 1 B) -1 C) $1+i$ D) $-1-i$.

17. The center c and radius r of the circle $x^2 + y^2 - 2x + 3y - 3 = 0$ is

A) $c = \left(1, -\frac{3}{2}\right)$, $r = 4$ B) $c = \left(-1, \frac{3}{2}\right)$ $r = 16$.

C) $c = (-2, 3)$, $r = 4$ D) $c = (2, -3)$, $r = 16$

18. The number of 3 digit numbers that can be formed using the digits 0, 1, 2, 3 is

A) 48 B) 64 C) 24 D) 18

19. The number of ways in which a committee of 5 persons can be selected from 3 men and 6 women given that there must be at least a man in the committee is

A) 45 B) 105 C) 70 D) 115

20. If $y = x^2 + \frac{1}{x}$ then $\frac{dy}{dx}$ is

A) $2x + \ln x$ B) $2x + x^{-2}$ C) $2x - x^{-2}$ D) $\frac{x^3}{3} + \ln x$.

21. Given that $f(x) = e^{3x} \sin 2x$ then $f'(x)$ is

A) $3e^{3x} \sin 2x - 2e^{3x} \cos 2x$ B) $6e^{3x} \cos 2x$

C) $3e^{3x} \sin 2x + 2e^{3x} \cos 2x$ D) $6e^{3x} \sin 2x + e^{3x} \cos 2x$.

22. If $y = \frac{\ln x}{x}$ then $\frac{dy}{dx}$ is equal to

A) $\frac{1}{x^2}$ B) $\frac{1+\ln x}{x^2}$ C) $\frac{1+\ln x}{x}$ D) $\frac{1}{x}$

23. The equation of a curve is given parametrically by $y = \cos 2t$, $x = \frac{1}{t}$. $\frac{dy}{dx}$ is

A) $2t^2 \sin 2t$ B) $-2t^2 \sin 2t$ C) $\frac{2\sin 2t}{t^2}$ D) $-\frac{2\sin 2t}{t^2}$.

24. The maximum value of the function $f: \mathbb{R} \to \mathbb{R}$ where $f(x) = 2 - 3x - x^2$ occurs at point where x is A) -1.5 B) 1.5 C) 0 D) 1

25. The gradient of the curve $y = xe^{2x}$ is at $x = 0$ is

A) 0 B) 1 C) 2 D) $\frac{1}{2}$.

26. The position of a particle at any time t is given by $2t^3 - 5t + 7$. The acceleration of the particle

A) $6t^2 - 5$ B) $2t^2 - 5$ C) $12t$ D) $12t - 5$.

27. $\int x^2 + 2x + 5\,dx$ is

A) $2x + 2$ B) $x^3 + x^2$ C) $\frac{x^3}{3} + x^2 + K$ D) $3x^3 + x^2 + K$.

28. $\int \frac{x}{x^2+5}\,dx$ is

A) $\frac{1}{2}\ln(x^2+5) + K$ B) $\ln(x^2+5) + K$ C) $2\ln(x^2+5) + K$ D) $\frac{x^3}{3} + 5x + K$.

29. $\int \sin 3x \cos x\,dx$ is

A) $\frac{1}{4}\cos 4x + \frac{1}{2}\cos 2x + K$ B) $-\left[\frac{1}{4}\cos 4x + \frac{1}{2}\cos 2x\right] + K$

C) $\frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x + K$ D) $-\left[\frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x\right] + K$

30. $\int x^2(x^3+2)^7\,dx$ is

A) $\frac{(x^3+2)^8}{8} + K$ B) $\frac{3(x^3+2)^8}{8} + K$ C) $\frac{8(x^3+2)^8}{3} + K$ D) $\frac{(x^3+2)^8}{24} + K$.

31. The solution of the initial value problem $\frac{dy}{dx} = xy$ given that $y = 1$ when $x = 0$ is

A) $\ln y = x^2 + 1$ B) $\ln y = x^2$ C) $\ln y = \frac{x^2}{2} + 1$ D) $\ln y = \frac{x^2}{2}$.

32. The value(s) of x for which the matrix $\begin{vmatrix} 2 & 0 & -1 \\ -2 & x & 5 \\ 0 & 1 & x \end{vmatrix}$ is singular is

A) 2 and -2 B) 2 C) -2 D) 4 and -4.

33. A bag contains 3 white balls and 4 red balls. Two balls are selected from the bag without replacement. The probability that both balls are of different colours is

A) $\frac{4}{7}$ B) $\frac{2}{7}$ C) $\frac{1}{7}$ D) $\frac{3}{7}$.

34. The sum of the first n terms of a sequence is given by $S_n = 3n^2 + 7$. The 10^{th} term of the sequence is

A) 307 B) 27 C) 278 D) 63.

35. $\lim_{x \to -2}(1 - x^2)$ is

A) 2 B) 5 C) -3 D) 3

36. Simplify $\sqrt{75}$

A) $5\sqrt{3}$ B) $3\sqrt{5}$ C) $5\sqrt{5}$ D) $3\sqrt{3}$

37. Find the value of $\tan 30^0$

A) 0 B) $\frac{1}{\sqrt{3}}$ C) 1 D) $\sqrt{3}$

38. Differentiate $\sin 3x$

A) $\cos 3x$ B) $3 \sin 3x$ C) $3\cos 3x$ D) $\cos x$

39. Integrate $x^{3/2}$

A) $\frac{3}{5}x^{3/2} + c$ B) $\frac{2}{5}x^{5/2} + c$ C) $x^{3/2} + c$ D) $\frac{2}{5}x^{3/2} + c$

40. Solve $A = \int_{-2}^{1.2} x\,dy$

A) $32\frac{2}{3}$ B) $33\frac{2}{3}$ C) $34\frac{2}{3}$ D) $33\frac{1}{3}$

41. Find the volume of the solid $V = \int_1^6 \pi x^2 dy$

A) $\frac{\pi}{5}$ B) $\frac{5\pi}{93}$ C) $\frac{5\pi}{93}$ D) $\frac{93\pi}{5}$

42. If 3, a, 7 are three consecutive terms of an arithmetic progression, find the value of a.

A) 5 B) -5 C) -10 D) 10

43. The limit as x tends to 0 of the function g defined as $g(x) = \frac{3}{x+1}$ is

A) 3 B) 1 C) 0 D) ∞

44. If $A = \frac{e^{1+\ln 2}}{e^{1+\ln 5}}$, the simplified form of A is

A) $\frac{1+\ln 3}{1+\ln 5}$ B) $\frac{\ln 3}{\ln 5}$ C) $\frac{2}{5}$ D) $-\frac{2}{5}$

45. The first term of a geometric progression is 2 and the common ratio is 3. Find the fourth term.

A) -54 B) -53 C) 54 D) -54

SECTION 2: (1hr 30min):

Answer any three (03) questions of your choice in this section

1. a) The roots of the quadratic equation $x^2 + 3x - 7 = 0$ are α and β. Find the quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$.

   b) Solve the inequality $\frac{x^2+3}{x+1} < 3$.

   c) Solve for x, the equation $3(7^{2x}) - 7^x - 2 = 0$.

2. a) Express $\sin x - \sqrt{3}\cos x$ in the form $r\sin(x - \alpha)$, where α is an acute angle. If $f(x) = \sin x - \sqrt{3}\cos x + 5$, find the maximum value of $\frac{1}{f(x)}$.

   b) Prove that $\frac{1+\sin x}{\cos x} - \frac{\cos x}{1-\sin x} = 0$.

3. Find the inverse of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 1 & 1 & -3 \\ 3 & 2 & 1 \end{pmatrix}$.

   Hence or otherwise, solve the system of equations

   $$2x + 3y = 4$$
   $$x + y - 3z = -1$$
   $$3x + 2y + z = 6$$

4. a) Find $\int x^2 \ln x \, dx$.

   b) Evaluate $\int_0^{\frac{\pi}{2}} \sin 3x \sin x \, dx$.

   c) Solve the differential equation $\frac{dy}{dx} = y^2 \cos x$ given that $y = -1$ when $x = 0$.

5. a) Find $\frac{dy}{dx}$, given that $x^2 + y^3 e^x + \ln y = 5$.

   b) If $y = \cos 2t$ and $x = t^3$ find $\frac{d^2y}{dx^2}$.

   c) Find the equation of the tangent to the curve $y = 4x^2 + \frac{1}{x}$ at the point $x = 1$.

6. In order to monitor buses of Musango express agency, the garage manager decides to the number of breakdown of the buses using the sequence (U_n) *defined by* $U_{n+1} = 1.05U_n + 4$. Given that $U_0 = 40$, is the number of breakdown by the buses of this agency from 1^{st} *of January* 2000, and that for every $n \in N$, U_n is the number of breakdown of the buses as from 1^{st} of January of the year $(2000 + n)$

   a) Calculate U_1 *and* U_2.

   b) Given that for all natural numbers n, $V_n = U_n + 80$.

   i) Express V_{n+1} *in terms of* V_n, hence deduce the nature of (V_n).

   ii) Express V_n *in terms of* n *and deduce that:* $U_n = 120 \times (1.05)^n - 80$.

   c) Find the number of breakdown that will be registered by 1^{st} of January 2020.