COT 305

A TABONG PHOTOCOPY

UNIVERSITY OF BUEA

COLLEGE OF TECHNOLOGY

END OF SEMESTER EXAMINATION

DEPARTMENT : Electrical Electronics Engineering	COURSE INSTRUCTOR: P. Ndambomve
MONTH : February	COURSE CODE & NUMBER: COT305
YEAR : 2019	COURSE TITLE: Mathematics 3
DATE & TIME : 28-02-2019	CREDIT VALUE: Six Credits
TIME ALLOWED : Two Hours	PERIOD:08:00 10:00

Instructions : Answer **Question 1** and any other **TWO Questions**. All necessary work must be shown and must be neatly and orderly presented.

Question 1 (10 marks)
What do you understand by the following expressions with respect to ordinary differential equations: (i) first order linear equation, (ii) first order Nonlinear equation, (iii) homogeneous equation, (iv) non-homogeneous equation, (v) a general solution, (vi) a particular solution, (vii) Laplace transform of a function, (viii) Fourier series expansion of a function.

Question 2 (4, 3, 6, 4, 13 marks)
Consider a circuit containing an electromotive force E (supplied by a battery or generator), a resistor R, an inductor L, and a capacitor C, in series. If the charge on the capacitor at time t is $Q = Q(t)$, then, the current is the rate of change of Q with respect to t, i.e., $I = \frac{dQ}{dt}$. It is known from physics that the voltage drops accross the resistor, inductor, and capacitor are RI, $L\frac{dI}{dt}$, $\frac{Q}{C}$, respectively.

1) State *Kirchhoff's voltage law*.
2) Using the above law, obtain a first order differential equation for the current.
3) Solve the equation in 2) for $E = 0$.
4) Derive a second order linear differential equation with constant coefficients for the charge. (Hint: Use the fact that the current is the rate of change of the charge with respect to time).
5) Using the equation obtained in 4) above, find the charge and the current at time t in the circuit if $R = 40\,\Omega$, $L = 1\,H$, $C = 16 \times 10^{-4}$, $E(t) = 100\cos(10t)$, and the initial charge and current are both 0.

Question 3 (15, 4, 6, 5 marks)

1) Solve each of the following systems of ordinary differential equations.

$$\begin{cases} x' = \frac{1}{3}(-x + 2y) \\ y' = \frac{1}{3}(4x + y). \end{cases} \qquad (S_4)$$

$$\begin{cases} x' = x - y \\ y' = 2x - 2y. \end{cases} \qquad (S_5)$$

2) The objective of this question is the transformation of a 2^{nd} order linear differential equation into a system of 2 ordinary differential equations.
Consider the following linear differential equation

$$x'' + 4x' + 3x = 0, \qquad (E)$$

and let us set:

$$u = x \text{ and } v = x'.$$

1) What is the system of differential equations (S) satisfied by (u, v) when x satisfies (E) (and vice-versa)?

2) Solve (S) and deduce the general solution of (E).

3) Does the above results for (E) agree with the formulas of the general solutions of a 2^{nd} order linear homogeneous differential equation with constant coefficients ? .

Question 4 (10, 5, 5, 5, 5 marks).

1) Find the solution of the following Initial Value Problem using Laplace transform.

$$\begin{cases} y'' - 2y' + 2y = e^{-t} \\ y(0) = 0, \quad y'(0) = 1, \end{cases} \qquad (1)$$

2) Consider the function $f : [-1, 1] \to \mathbb{R}$ defined by $f(x) = x^2$.

i) Show that the Fourier series expansion of f is given by:

$$f(x) = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(n\pi x).$$

ii) Show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

iii) Compute the value of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$.

iv) Deduce the values of $\displaystyle\sum_{n\ odd} \frac{1}{n^2}$ and $\displaystyle\sum_{n\ even} \frac{1}{n^2}$.

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