

etc. and ... series

(I) Prove each of the following

- (i) $f: x \mapsto x^5 + 3x^4 + x + 1$ has a real zero (root).
- (ii) $f: x \mapsto 2x^4 - x^3 + x^2 - 1$ has a zero in $[0, 1]$.
- (iii) The equation $\cos x = x$ has a real solution.
- (iv) The equation $x^3 + x + 3 = 0$ has a unique solution in $[-2, -1]$.

(II) Evaluate the following indefinite integrals

- (i) $\int (2+x^2)^3 dx$ (ii) $\int x(3-x^3)^2 dx$ (iii) $\int \left(\frac{1+x}{x}\right)^3 dx$
- (iv) $\int \frac{x^2+x+1}{\sqrt{x}} dx$ (v) $\int \frac{x^3}{1+x^2} dx$ (vi) $\int \frac{x^2}{1-x^2} dx$
- (vii) $\int \frac{e^{3x}+1}{e^x+1} dx$ (viii) $\int (e^{-x} + e^{2x} + e^{-3x}) dx$
- (ix) $\int \sin^7 x \cos x dx$ (x) $\int \frac{1}{x \ln x} dx$ (xi) $\int \tan x dx$

(III) Use the change of variable $x = a \sin t$ or $x = a \cos t$ to perform the following

- (i) $\int_0^1 \sqrt{1-x^2} dx$ (ii) $\int_{-1}^0 \sqrt{1-x^2} dx$ (iii) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$

(IV) Evaluate the following

- (i) $\int \frac{x}{(x+1)(x+2)(x-5)} dx$ (ii) $\int \frac{2x^4-1}{(x-1)(x-2)} dx$
- (iii) $\int \frac{x^5+x^4-8}{x^3-4x} dx$ (iv) $\int \frac{x^4}{(x^2-1)(x+2)} dx$

$$(V) \int \frac{1}{1-\sqrt{x}} dx$$

$$(VI) \int \frac{1}{\sin x} dx$$

$$(VII) \int \frac{1}{5-3\cos x} dx$$

$$(VIII) \int \frac{\cos x + 2}{2\sin x - \cos x + 5} dx$$

$$(IX) \int \frac{\sin x}{1 + \sin x} dx$$

Exercises

Proove that the equation $x^3 + 10x - 4 = 0$ has a real root in $[0, 1]$

2) Evaluate the following indefinite integrals

a) $\int x^2(1-x^2) dx$

b) $\int (x+4)^{\frac{1}{5}} dx$

c) $\int e^x \ln x dx$

d) $\int \frac{x}{\sqrt{x-1}} dx$

e) $\int x^2 \cos 2x dx$

f) $\int x^3 e^x dx$

3) Find the values of the following definite integrals

a) $\int_0^1 3\sqrt{5x+1} dx$

b) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$

c) $\int_{-1}^1 x(2+5x)^7 dx$

d) $\int_0^{\frac{1}{2}} \frac{dx}{2+x^2}$

e) $\int_0^1 x^2 \sqrt{1+x} dx$

f) $\int_1^{\pi} e^{2x} \cos x dx$

g) $\int_0^3 \frac{1}{x^2+x+1} dx$