MAT 207

TUTORIAL SERIES IV
Limits and Continuity

1 Using the definition sfow that:

(i) $\lim_{x\to 0} \sin x = 0$ (ii) $\lim_{x\to 0} \cos x = 0$ (iii) $\lim_{x\to 2} x^2 = 4$

2 Consider the function $f(x) = 2x^2 - 8x + 2$.

(i)Find the y-intercept of f (ii) Complete the square of $f(x)$.

(iii) Sketch the graph of f.

3 Evaluate the following limits:

(i) $\lim_{x\to 0}(x^2-3)(2x-5)$ (ii) $\lim_{x\to 2} \dfrac{x^2-1}{\sqrt{1+x^2}}$

(iii) $\lim_{x\to 1^-} \ln\left(\frac{1-x}{1+x}\right)$ (vi) $\lim_{x\to -\infty} e^{x^3-x^2+1}$

(v) $\lim_{x\to 0} \sin\sqrt{|x|} + x^3 \cos\dfrac{1}{x^2}$ (vi) $\lim_{x\to 1} \dfrac{x^m-1}{x-1}$, $m \in \mathbb{N}^*$

4 Evaluate the following limits:

(i) $\lim_{x\to 0} \sin x^3 + \tan x$ (ii) $\lim_{x\to 0} \dfrac{\sin x}{x}$

(iii) $\lim_{x\to 0} \dfrac{\tan x}{x}$ (vi) $\lim_{x\to 0} x \sin\dfrac{1}{x}$

(v) $\lim_{x\to 1} \cos\dfrac{\pi}{2}x\left(\sqrt{4x^5+3x^3+2x^2+1}\right)$ (vi) $\lim_{x\to +\infty} e^{-x}\cos\dfrac{3x^2+2}{\sqrt{x^2+1}}$

5 Evaluate the following limits:

(i) $\lim_{x\to -1}\left(\dfrac{1}{(x+1)^2}+x^2+2\right)$ (ii) $\lim_{x\to 1^-} \dfrac{1}{x-1}$

(iii) $\lim_{x\to 0^+} \dfrac{x^7-1}{x^4}$ (iv) $\lim_{x\to +\infty} x^2 - x$

(v) $\lim_{x\to -\infty} x^6 + x$ (vi) $\lim_{x\to +\infty}\left(\sqrt{x^3-x^2}-\sqrt{x^3+x^2+1}\right)$

(vii) $\lim_{x\to +\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$ (viii) $\lim_{x\to +\infty} x \sin\dfrac{\pi}{x}$.

6 A crop is planted in a soil where the nitrogen level is x., then the crop yield f can be modeled by the function $f(x) = a(x+1)\ln(1+\frac{1}{x+1}) + b$, $x \geq 0$ where a and b are constants. If the level of Nitrogen is 2, what is the crop yield? What happens to the crop yield as the nitrogen level is increased indefinitely? Explain the significance of your results.

7 Sketch the function $f(x) = \begin{cases} x-2 & x \leq 2 \\ 2x+1 & x > 2 \end{cases}$ and determine whether or not f is continuous at $x = 2$.

8 Determine the limit of the function $f(x)$ as x tends to $x_0 = 0$ in the following cases and determine whether or not one can define a prolongation of f by continuity.

(i) $f(x) = \dfrac{|\sin x|}{x}$

(ii) $f(x) = \begin{cases} x^4 cox\frac{1}{x} & \text{if} \quad x > 0 \\ \sin\sqrt{-x} & \text{if} \quad x < 0 \end{cases}$

9 Consider the function $f :]-1,1[\to \mathbb{R}$ defined by $f(x) = \dfrac{x}{1-|x|}$.

(i) Determine whether the function f is continuous at $x = 0$.

(ii) Consider the restriction g of the function on $[0,1[$. Determine the range of g. Show that g is incerasing and then bijective from its domain to its range and determine its inverse g^{-1}.

(iii) Consider the restriction h of the function on $]-1,0]$. Determine the range of h. Show that h is decerasing and then bijective from its domain to its range and determine its inverse h^{-1}.

iv Find the inverse f^{-1} of f and determine whether f^{-1} is continuous or not at $x = 0$.

10 Find the constants a and c such that the function $f(x) = \begin{cases} x^3 + c & \text{for} \quad x < 0 \\ ax + c^2 & \text{for} \quad 0 < x < 1 \\ \arctan x & \text{for} \quad x \geq 1 \end{cases}$ is continuous for all x.