UNIVERSITY OF BUEA
COLLEGE OF TECHNOLOGY
ENTRANCE EXAMINATION INTO FIRST YEAR OF BTECH PROGRAM

PAPER TITLE: MATHEMATICS
DATE: 25/09/19
TIME ALLOWED 3 HOURS
TIME: 08.00 – 11.00
COEFFICIENT: 4

SECTION 1, 1hr 30min

INSTRUCTIONS: Answer all questions in this section. Write down only the letter that corresponds to the best answer to each question. Each question carries two marks.

1) The function f is defined by
$\begin{cases} \cos(x+3\theta), -\frac{\pi}{2} \leq x \leq 0 \\ x^2, 0 \leq x \leq \frac{\pi}{2} \end{cases}$ *where θ is a positive constant and continous at x = 0*. The value of θ is:

(A) π (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) 0

2) The sum, S_n of the first terms of a sequence is given by $S_n = 3^n - 1$. The 4th term of the sequence is

(A) 80 (B) 26 (C) 18 (D) 54

3) $Lim_{x\to 1} \frac{x^n - x}{x^m - x} =$

(A) $\frac{m}{n}$ (B) $\frac{n}{m}$ (C) mn (D) $\frac{n-1}{m-1}$

4) $\int_0^1 \frac{2x^2}{(x^2+1)2} dx =$

(A) 1+In2 (B) $\frac{1}{2}(-1+In2)$ (C) (3+In2) (D) 1+In4

5) The plane whose vector parametric equation is $\underset{r}{\rightarrow} 2i - 3j + pk + \alpha(2i+j) + \mu(-1+j+2k)$ *contains the point* $\underset{ro}{\rightarrow} = 4i + j$. The value of the real constant p is:

(A) 2 (B) -8 (C) -4 (D) 6

6) The minimum value of $y = \frac{x}{x^2+1}$ *for which x is real*

(A) $\frac{-1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{-1}{4}$ (D) $\frac{1}{2}$

7) The x coordinate of the point where one of the asymptotes of the curve $f(x) = \frac{x(x-3)}{x^2-5x+4}$ *cuts it is*

(A) 1 (B) 2 (C) 4 (D) 3

8) $\frac{d}{dx}(log_{10}x^2) =$

(A) $\frac{2x}{In10}$ (B) $\frac{xIn2}{10}$ (C) $\frac{2}{xIn10}$ (D) $\frac{In10}{x(In2)}$

9) Given that Z^2-4Z+5=0, where Z is a complex number. The value of $\left|\frac{1}{z}\ is\right|$

(A) $\sqrt{5}$ (B) $\frac{1}{\sqrt{5}}$ (C) $\frac{1}{2\sqrt{5}}$ (D) $\frac{1}{20}$

10) The line symmetrical of the curve $f(x) = x^2 - 4x + 3$ *is*

(A) x=1 (B) y=3 (C) x=2 (D) x=-1

11) The value of x for which $((1+8x)^4 \equiv \left(\frac{3\sqrt{3}}{5}\right)^8$ *is:*

(A) $\frac{2}{25}$ (B) $\frac{1}{100}$ (C) $\frac{27}{25}$ (D) $\frac{1}{50}$

12) $Cos^4\theta - Sin^4\theta \equiv$

(A) $(\sin\theta - \cos\theta)(\cos\theta - \sin\theta)$ (B) $\cos2\theta$ (C) $\cos4\theta - \sin4\theta)$ (D) $\cos^2\theta + \sin^2\theta$

13) $\int_0^1 \frac{tan^{-1}x}{1+x^2}dx =$

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{32}$ (C) $\frac{\pi^2}{64}$ (D) $\frac{\pi^2}{16}$

14) Given that $(x-1)\frac{dy}{dx} = 1 - y$ *and that* $y = -2$ *when* $x = 2$. The general solution of this differential is

(A) $y = \frac{4-x}{3-x}$ (B) $y = 4 - 3x$ (C) $y = (3 - 3x)(x - 1)$ (D) $y = \frac{x-4}{x-1}$

15) Given that $\cot\beta = -\sqrt{2}$ and that $\frac{3}{2}\pi < \beta < 2\pi$, the value of $\cos\beta$ is:

(A) $\frac{-1}{\sqrt{2}}$ (B) $\frac{\sqrt{2}}{\sqrt{3}}$ (C) $\frac{-1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{2}}$

16) The equation of the normal to the curve $y = x + \frac{1}{x}$ at the point where x = 1 is;

(A) Y = 2 (B) x = 2 (C) y = 0 (D) x = 0

17) The set of values of x for which $log_x 3 - log_3 x = 0$ *is*:

(A) $\{3\}$ (B) $\left\{3, \frac{1}{3}\right\}$ (C) $\{1,3\}$ (D) $\left\{\frac{1}{3}\right\}$

18) Given that sinx = cosx. The value of $tan\frac{x}{2}$ *is*:

(A) $-1 + \sqrt{2}$ (B) $-2 - \sqrt{2}$ (C) $1 - 2\sqrt{2}$ (D) $\frac{1}{2}(1 - \sqrt{2})$

19) The Cartesian equation of a cycle C is defined by $X^2 + Y^2 - 2X - 4Y = 3$ value of the constant k for which x + y = k is a tangent to C is:

(A) 1 (B) 8 (C) $\sqrt{2}$ (D) 7

20) Given that $g(x) = e^{-x}\text{In}x$, then g'(x) is :

(A) $\frac{e^x}{x} + g(x)$ (B) $xe^{-x} - g(x)$ (C) $xe^{-x} - g(x)$ (D) $\frac{e^{-x}}{x} - g(x)$

21) Given that k∈ R, $\int \frac{x+2}{x+1}dx$ *is* :

(A) X + In(x + 1) + k (B) In(x + 2) + k (C) Inx(x + 1) + k (D) x + Ink(x + 1)

22) The value of $\tan^{-1}(3) + \tan^{-1}(2)$ is:

(A) $\frac{\pi}{4}$ (B) $\frac{5}{6}\pi$ (C) $\frac{3}{4}\pi$ (D) $\frac{5}{4}\pi$

23) The perpendicular distance from the point Q(1,2,3) to the plane 2x + y + 2z =1 is;

(A) $\frac{11}{9}$ (B) $\frac{11}{3}$ (C) 1 (D) 3

24) The value of y for which $e^y - 2e^{-y} - 1=0$ is;

(A) e^{-1} (B) In2 (C) e^2 (D) 2

25) the roots of the equation $3x^2 - 5x + k = 0$ is:

(A) $\frac{-11}{36}$ (B) $\frac{-13}{12}$ (C) $\frac{-11}{2}$ (D) $\frac{-11}{36}$

26) If $\alpha - 2, \beta - 2$ are roots of $3x^2 - 5x = 4$. Find the value of $\alpha + \beta$

(A) $^5/_3$ (B) $^4/_3$ (C) $^{17}/_3$ (D) $^{16}/_3$

27) The greatest value of $5\sin\theta + 12\cos\theta$ is

(A) -13 (B) 13 (C) 17 (D) -17

28) If logx + log(x – 30) = 1. The value of x is;

(A) -2 (B) -2.5 (C) 5 (D) 2.5

29) A translation of y=f(x) through 2 units parallel to the x – axis is represented by

(A) f(x) + 2 (B) f(x) – 2 (C) f(x) + 2 (D) f(x) – 2

30) if the sum of the first 2n terms of the series 2 + 5 + 8 + ……. Is 29n + 10 the value of n is?

(A) 8 (B) 5 (C) 10 (D) 15

31) The parametric equations of a curve are: $x = 2/t$, $y = 3t$. The Cartesian equation of the tangent to the curve at the point A, where $t=1$.
(A) $2x - 3y = -5$ (B) $3x + 2y = 12$ (C) $2x + 3y = -5$ (D) $3x - 2y = 8$

32) The exact value of $sin\frac{2}{3}\pi + sin\frac{7}{3}\pi$ is
(A) $\frac{3^{1/2}}{4}$ (B) 0 (C) $\frac{1}{\sqrt{6}}$ (D) $3^{1/2}$

33) The value of the real constant k for which $(x + 2)$ divides the expression $2x^3 - 5x^2 + kx + 18$ exactly is
(A) -27 (B) -18 (C) -9 (D) 7

34) $\int \frac{x+3}{x+2} dx = ?$
(A) $x + In|x + 2| + k$ (B) $x + In|x + 3| + k$ (C) $In|x + 3| + k$ (D) $In|x + 2| + k$

35) $\frac{x+3}{(x-2)^2} = ?$
(A) $\frac{4}{(x-2)} + \frac{3}{(x-2)^2}$ (B) $\frac{1}{(x-2)} + \frac{5}{(x-2)^2}$ (C) $x + \frac{1}{(x-2)^2}$ (D) $\frac{2}{(x-2)} + \frac{5}{(x-2)^2}$

36) The interval in which the equation $x^4 - x^2 - 1$ has a root is
(A) $0 \le x \le 1$ (B) $2 \le x \le 3$ (C) $0 \le x$ ½ (D) $1 \le x \le 1$

37) The period of $y = \sin 4x$ is :
(A) 2π (B) $\pi/4$ (C) 4π (D) $\pi/2$

38) The sum of the first n terns of a sequence is given by $S_n = 3^n - 1$. The fourth term is
(A) 80 (B) 26 (C) 18 (D) 54

39) The x coordinate of the point of inflexion of the curve $y = \frac{x^3}{6} - \frac{x^2}{2} + x$ are
(A) 2/3 (B) 0 (C) -5/3 (D) 1

40) Given that $g(x) = e^{-x}Inx$, then $g'(x)$ is:
(A) $\frac{e^x}{x+g(x)}$ (B) $xe^{-x} - g(x)$ (C) $xe^{-x} - g(x)$ (D) $\frac{e^{-x}}{x-g(x)}$

41) The value of y for which $e^y + 2e^y - 1 = 0$ is
(A) e^{-1} (B) In2 (C) e^2 (D) 2

42) The volume generated when the area bounded by the curve $y = 2 - x^2$ and the line $y = 1$ is rotated completely about the line $y = 1$ is:
(A) $\frac{56\pi}{6}$ (B) $\frac{2\pi}{3}$ (C) $\frac{62\pi}{5}$ (D) $\frac{16\pi}{15}$

43) Given that the complex number $Z = p - qi$. The length of z is;
(A) $p^2 + q^2$ (B) $(p^2 + q^2)^{\frac{1}{2}}$ (C) $(p^2 - q^2)^{\frac{1}{2}}$ (D) $(p^2 - q^2)$

44) the greatest value of the constant k if the expression $k + 8x - 2x^2$ is never positive is;
(A) -2 (B) -8 (C) 4 (D) 0

45) The circles $C1: x^2 + y^2 = 4$ and $C2: x^2 + y^2 - 2x = 0$ touch externally. This equation of the common tangent at the point of contact is:
(A) $y = 2$ (B) $x = 2$ (C) $x = 4$ (D) $y = 0$

SECTION 2: (1hr 30min):

Answer any three (03) questions of your choice in this section

1. a) Find the centre and radius of the circle $x^2 + y^2 - 4x - 6y - 12 = 0$
   b) Find the points of intersection of the line y = 2x + 4 and the given circle and prove that the length of the chord cut off is $4\sqrt{5}$
   c) If b is a constant, evaluate $\int (b^2 - t^2)^3 \sqrt{t} dt$
   d) Evaluate $\lim_{x\to 0}(\sqrt{x^2+9} - \frac{x^2+3x)}{x})$
2. a) If $z = \frac{(1+i)}{(2-i)'}$ find the real and imaginary parts of $\frac{z^2-1}{z}$
   b) If $z = x + iy$ *and* $z^5 = 1$, *show that* $4x(y^4 - x^4) = 1$
   c) A first year student in COT Buea has five blue and four white balls in his left pocket and four blue and five white balls in his right pocket. If he transfers one ball at random from his left to his right pocket, what is the probability of his drawing a blue ball from his right pocket?

3. a) Simplify the complex number $z = i^{45} - i^7 + 5i^{20} + 3i^{30}$.
   b) Let $z = \frac{9-3i}{2+i}$. Express z in the form $a + bi$ and hence or otherwise find modulus and argument of z.
4. a) The fourth term of an arithmetic progression is 22 and the seventh term of the progression is 34. Find its twenty third term.
   b) Prove, using Mathematical induction, that $\sum_{k=1}^{n} 3k = \frac{n(3n-1)+4}{2}$.

5. i) Find $\frac{dy}{dx}$ given that

   a) $y = \frac{(2x+1)^7}{(x^2+1)^5}$ b) $y^2 + 2x^3y + 5x = 7$

   ii) Evaluate the following integrals.

   a) $\int \frac{1}{3x} - \frac{3}{x^2} + \sqrt{x}\, dx$ b) $\int \frac{3x-3}{(x^2-2x+6)^5}\, dx$ C) $\int x \ln 2x \ dx$