University of Buea   College of Technology

Department of Electrical and Electronic Engineering
EEC 209: Mathematics I
Test, 05 February 2018
15.00-17.00

Exercise 1 (15 marks)

Consider the sequence $\{x_n\}$ defined for all $n \in \mathbb{N}$ by

$$x_0 = 1 \quad \text{and} \quad x_n = \frac{1}{4}x_{n-1} + 1, \ n \geq 1.$$

1. Evaluate x_1, x_2 and x_3. (1+1+1 marks).

2. We consider now the sequence $\{y_n\}$ defined for all $n \in \mathbb{N}^*$ by

$$y_n = x_n - x_{n-1}, \ n \geq 1.$$

   (a) Evaluate y_1, y_2 and y_3. (1+1+1 marks).

   (b) Prove that $\{y_n\}$ is a geometric progression and find its ratio. (2 marks).

   (c) Deduce, according to n, the general term y_n. (2 marks).

3. Consider the sum $S_n = y_1 + y_2 + \cdots + y_n$.

   (a) Prove that $S_n = \frac{1}{3}\left(1 - \left(\frac{1}{4}\right)^n\right)$ and find $\lim_{n\to\infty} S_n$. (2+1 marks).

   (b) Prove that $S_n = x_n - x_0$ and deduce $\lim_{n\to\infty} x_n$. (1+1 marks).

Exercise 2 (30 marks)

1. Find the domain of definition of the following functions and indicate which of them are even, odd, periodic (precise the period where applicable) or neither even, odd nor periodic: a) $f(x) = |x|$; b) $f(x) = \frac{1}{x}$; c) $f(x) = \cos 2x$; d) $f(x) = \frac{x+1}{x^2+x-2}$. (3×4 marks).

2. Find the domain of definition and the domain of study of the following: a) $f(x) = x^2$; b) $f(x) = \sin 2x$. (2+2 marks).

3. Find the indicated limits if they exists:

   a) $\lim_{x\to 1}(1 - 2x^2)$; b) $\lim_{x\to 3}\frac{x^2-2x-3}{x-3}$; c) $\lim_{x\to+\infty}\sqrt{x^2+4} - x$; d) $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$. (7 marks).

4. Consider the function $f(x) = \begin{cases} x^2 - 1 & \text{if} \quad x \leq 1 \\ \ln x & \text{if} \quad x > 1. \end{cases}$

   (a) Determine the domain of definition of f. (1 mark).

   (b) Evaluate $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$. Can we say that $\lim_{x\to 1} f(x)$ exists? Justify your answer. (4 marks).

   (c) Is f continuous at $x_0 = 1$? Explain. (2 marks).