

Ruye Wang, Nick Richardson

 $Harvey\ Mudd\ College$ $Department\ of\ Engineering,\ Department\ of\ Mathematics$

 $\textit{E-mail:} \verb| rwang@hmc.edu|, \verb| nrichardson@g.hmc.edu|$

Contents

Ι	Fundamentals	2
1	Probabilities and Inference	2
2	Linear Algebra	2
3	Signal Processing	2
4	Optimization	2
II	Representation	3
5	Smoothing, Compression, and Information Loss	3
6	Basic Basis Function Expansion	4
7	Adaptive Basis Function Methods	5
8	Latent Processes	5
9	Automatic Structure Discovery	5
10	Metric Learning	5
11	Representation examples	5
II	I Inference	6
12	Linear methods	6
13	Adaptive Basis Function Methods	6
IV	Application 4	7
14	Step Zero	7
15	Exploratory Data Analysis	7
16	Model Selection	7
17	Hyperparameter Selection	-

8 Datasets and Distribution Shift	
19 Application: Computer Vision	7
20 Application: Linguistics & Genomics	7
21 Application: Robotics & Control	7
V Frontiers	8
22 Model Composition	8
23 Returning to Control: Machine Learning in Society	8
24 Brain Machine Interfacing	8
25 Machine Intelligence: What will it take?	8
VI Appendices	8

Contents (Extendend)

Ι	Fu	ndamentals	2	
1	Pro	babilities and Inference		
	1.1	Probability	2	
	1.2	Collections of Probabilities	2	
		1.2.1 Joint Probability	2	
		1.2.2 Marginal Probability	2	
		1.2.3 Conditional Probability	2	
	1.3	Independence	2	
	1.4	Product Rule & Sum Rule	2	
	1.5	Bayes' Theorem	2	
	1.6	Expectation	2	
	1.7	Variance & Covariance	2	
	1.8	Information & Entropy	2	
	1.9	Bounds on Signal Compression	2	
		Kullback-Leibler Divergence	2	
		Inductive Bias	2	
	1.12	The Language of Inference	2	
		1.12.1 Prior	2	
		1.12.2 Likelihood	2	
		1.12.3 Evidence/Marginal Likelihood	2	
	1.13	Point Estimation	2	
		1.13.1 Maximum Likelihood	2	
		1.13.2 Maximum a Posteriori	2	
		Laplace's Method	2	
		Variational Inference	2	
	1.16	16 Markov Chain Monte Carlo		
		1.16.1 Importance Sampling	2	
		1.16.2 Rejection Sampling	2	
		1.16.3 Hamiltonian MCMC	2	
2	Line	ear Algebra	-	
		Vectors	6	
		2.1.1 Vector addition and scaling	6	
		2.1.2 Inner Products	6	
		2.1.3 Vector Norms	6	
		2.1.4 Angle	6	
		2.1.5 Vector Mean and Variance	6	
		2.1.6 Linearity and Approximations from Calculus	6	

		2.1.7	Linear Independence and Basis
		2.1.8	Orthogonality
		2.1.9	Vector Mean and Variance
		2.1.10	Vectorized data
		2.1.11	k-means clustering: A Taste of Automatic Structure Discovery
	2.2	Matric	es
		2.2.1	Matrix addition and transpose
		2.2.2	Matrix-vector and matrix-matrix multiplication
		2.2.3	Geometric matrices
		2.2.4	Convolution matrices
		2.2.5	Finite Differences
		2.2.6	Matrix representations of graphs
		2.2.7	Affine functions
		2.2.8	Systems of Linear Equations & Compositions of Linear Functions
		2.2.9	Matrix Inverses & Psuedo-inverses
		2.2.10	Matrix Powers & Eigensystems
		2.2.11	Common matrix factorizations
		2.2.12	Singular Value Decomposition
		2.2.13	Least Squares
n	G:		
3	_	al Pro	ocessing
3	Sig r. 3.1	al Pro Basic l	ocessing Processing
3	_	Basic I	Processing Rescaling and centering
3	_	Basic 1 3.1.1 3.1.2	Processing Rescaling and centering Histograms
3	_	Basic 1 3.1.1 3.1.2 3.1.3	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality
3	_	Basic I 3.1.1 3.1.2 3.1.3 3.1.4	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals
3	_	Basic I 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling
3	_	Basic I 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5 3.1.6	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling Linear Filters
3	_	Basic I 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5 3.1.6 3.1.7	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling Linear Filters Nonlinear Filters
3	_	Basic I 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5 3.1.6 3.1.7 3.1.8	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling Linear Filters Nonlinear Filters Fourier Transform
3	_	Basic 1 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5 3.1.6 3.1.7 3.1.8 3.1.9	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling Linear Filters Nonlinear Filters Fourier Transform Wavelet Transforms
3	3.1	Basic I 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5 3.1.6 3.1.7 3.1.8 3.1.9 3.1.10	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling Linear Filters Nonlinear Filters Fourier Transform Wavelet Transforms Shapelets
3	_	Basic 1 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5 3.1.6 3.1.7 3.1.8 3.1.9 3.1.10 Signal	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling Linear Filters Nonlinear Filters Fourier Transform Wavelet Transforms Shapelets Inference
3	3.1	Basic I 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5 3.1.6 3.1.7 3.1.8 3.1.9 3.1.10 Signal 3.2.1	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling Linear Filters Nonlinear Filters Fourier Transform Wavelet Transforms Shapelets Inference Kalman Filter
3	3.1	Basic 1 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5 3.1.6 3.1.7 3.1.8 3.1.9 3.1.10 Signal	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling Linear Filters Nonlinear Filters Fourier Transform Wavelet Transforms Shapelets Inference
	3.1	Basic I 3.1.1 3.1.2 3.1.3 3.1.4 3.1.5 3.1.6 3.1.7 3.1.8 3.1.9 3.1.10 Signal 3.2.1	Processing Rescaling and centering Histograms Spectral Decay and effective dimensionality Moving Functionals Interpolation and Resampling Linear Filters Nonlinear Filters Fourier Transform Wavelet Transforms Shapelets Inference Kalman Filter Hidden Markov model

3

Representation

 \mathbf{II}

6	Bas	sic Basis Function Expansion	4
	6.1	Polynomial Basis	4
	6.2	Periodic Basis	4
	6.3	Piecewise Linear	4
	6.4	Radial Basis Functions	4
	6.5	Sigmoid Functions	4
7	\mathbf{Ad}	aptive Basis Function Methods	5
	7.1	Kernels	5
		7.1.1 The Gram matrix	5
		7.1.2 The Kernel Trick	5
		7.1.3 Learning the Kernel	5
	7.2	Gaussian Processes	5
	7.3	Neural Networks	5
	7.4	Kernel Machines	5
8	Lat	ent Processes	5
	8.1	Discrete Latent Processes	5
	8.2	Continuous Latent Processes	5
	8.3	Latent Processes as Probabilistic Graphical Models	5
9	Au	tomatic Structure Discovery	5
	9.1	Clustering and Graphical Community Identification	5
	9.2	Hierarchical clustering	5
	9.3	Mean and mediod based clustering	5
	9.4	Spectral clustering	5
	9.5	Principal components analysis	5
	9.6	Linear discriminant analysis	5
	9.7	T-distributed stochastic neighbor embedding	5
	9.8	Laplacian Eigenmaps and UMAP	5
10) Me	tric Learning	5
11	$\mathbf{Re}_{\mathbf{I}}$	presentation examples	5
	11.1	Image and video	5
	11.2	2 Speech and sound	5
	11.3	3 Text and genetic sequence	5
77	т.	T. C	6
П	III Inference		

3

3

3

3

5 Smoothing, Compression, and Information Loss

Geometric Compression

5.0.2

5.0.1 Basis sparsity and the Fourier Transform

5.0.3 Introduction to Dimensionality Reduction

12 Linear methods		
12.1 Linear Regression	6	
12.1.1 Least Squares and Linear Regression	6	
12.1.2 Basis Function Expansion	6	
12.1.3 Solution Penalties: A Return to Vector Norms and Geometry	6	
12.1.4 Bayesian Linear Regression	6	
12.2 Logistic Regression	6	
12.3 Generalized Linear Models	6	
12.3.1 The Exponential Family	6	
12.3.2 Conjugacy and Priors	6	
13 Adaptive Basis Function Methods		
13.1 Inference with Neural Networks	6	
13.1.1 Single Neurons	6	
13.1.2 Feedforward Neural Networks	6	
13.1.3 Structured Neural Networks	6	
13.1.4 Weight Intialization and Random Matrices	6	
13.1.5 Optimization in Neural Networks	6	
13.1.6 Probabilistic Neural Networks	6	
13.2 Inference with Kernel Machines	6	
13.3 Inference with Gaussian Processes	6	
IV Application	7	
14 Step Zero	7	
14.1 Understanding the Problem You Care About	7	
14.2 Specifying the Inductive Bias	7	
15 Exploratory Data Analysis	7	
15.1 Dataset Versioning	7	
15.2 Visualization	7	
15.3 Guaging Problem Difficulty	7	
15.4 Starting Simply	7	
16 Model Selection	7	
16.1 Standard Model Selection	7	
16.2 Bayesian Model Selection	7	
17 Hyperparameter Selection	7	
17.1 Heuristics	7	
17.2 Bayesian Optimization	7	

18 Data	18 Datasets and Distribution Shift	
18.1	Training on the Test Set	7
18.2	Internal Covariate Shift	7
18.3	Distribution Shift	7
19 App	olication: Computer Vision	7
20 App	olication: Linguistics & Genomics	7
21 App	olication: Robotics & Control	7
V Fr	contiers	8
22 Mod	del Composition	8
22.1	Probabilistic Graphics Models and Adaptive Basis Methods	8
23 Retu	urning to Control: Machine Learning in Society	8
23.1	Autonomous Vehicles	8
	Autonomy in Weapon Systems	8
23.3	Robustness in Machine Learning	8
	in Machine Interfacing	8
	Studying the Brain	8
24.2	Imitating the Brain	8
24.3	Machine intelligence as analogies for neurocomputation	8
	chine Intelligence: What will it take?	8
	The AI effect	8
	Hype and Reality	8
25.3	Constructing Cockroaches	8
VI A	Appendices	8
	Notation	9
	Calculus	9
20.0	25.5.1 Derivatives	9
	25.5.2 Chain Rule & Product Rule	5
	25.5.3 Integrals	8
25.6	Complex numbers as dynamic objects	8
	ı v	_

OPTIMIZATION 2

Fundamentals

1 Probabilities and Inference

- 1.1 Probability
- 1.2 Collections of Probabilities
- 1.2.1 Joint Probability
- 1.2.2 Marginal Probability
- 1.2.3 Conditional Probability
- 1.3 Independence
- 1.4 Product Rule & Sum Rule
- 1.5 Bayes' Theorem
- 1.6 Expectation
- 1.7 Variance & Covariance
- 1.8 Information & Entropy
- 1.9 Bounds on Signal Compression
- 1.10 Kullback-Leibler Divergence
- 1.11 Inductive Bias
- 1.12 The Language of Inference
- 1.12.1 Prior
- 1.12.2 Likelihood
- 1.12.3 Evidence/Marginal Likelihood
- 1.13 Point Estimation
- 1.13.1 Maximum Likelihood
- 1.13.2 Maximum a Posteriori
- 1.14 Laplace's Method
- 1.15 Variational Inference
- 1.16 Markov Chain Monte Carlo
- 1.16.1 Importance Sampling
- 1.16.2 Rejection Sampling
- 1.16.3 Hamiltonian MCMC

2 Linear Algebra

- 2.1 Vectors
- 2.1.1 Vector addition and scaling
- 2.1.2 Inner Products
- 2.1.3 Vector Norms
- 2.1.4 Angle
- 2.1.5 Vector Mean and Variance
- 2.1.6 Linearity and Approximations from Calculus

PART

PART

II

Representation

5 Smoothing, Compression, and Information Loss

- 5.0.1 Basis sparsity and the Fourier Transform
- 5.0.2 Geometric Compression
- 5.0.3 Introduction to Dimensionality Reduction

6 Basic Basis Function Expansion

- 6.1 Polynomial Basis
- 6.2 Periodic Basis
- 6.3 Piecewise Linear
- 6.4 Radial Basis Functions
- 6.5 Sigmoid Functions

REPRESENTATION EXAMPLES

5

7 Adaptive Basis Function Methods

- 7.1 Kernels
- 7.1.1 The Gram matrix
- 7.1.2 The Kernel Trick
- 7.1.3 Learning the Kernel
- 7.2 Gaussian Processes
- 7.3 Neural Networks
- 7.4 Kernel Machines
- 8 Latent Processes
- 8.1 Discrete Latent Processes
- 8.2 Continuous Latent Processes
- 8.3 Latent Processes as Probabilistic Graphical Models
- 9 Automatic Structure Discovery
- 9.1 Clustering and Graphical Community Identification
- 9.2 Hierarchical clustering
- 9.3 Mean and mediod based clustering
- 9.4 Spectral clustering
- 9.5 Principal components analysis
- 9.6 Linear discriminant analysis
- 9.7 T-distributed stochastic neighbor embedding
- 9.8 Laplacian Eigenmaps and UMAP
- 10 Metric Learning
- 11 Representation examples
- 11.1 Image and video
- 11.2 Speech and sound
- 11.3 Text and genetic sequence

PART



Inference

12 Linear methods

- 12.1 Linear Regression
- 12.1.1 Least Squares and Linear Regression
- 12.1.2 Basis Function Expansion
- 12.1.3 Solution Penalties: A Return to Vector Norms and Geometry
- 12.1.4 Bayesian Linear Regression
- 12.2 Logistic Regression
- 12.3 Generalized Linear Models
- 12.3.1 The Exponential Family
- 12.3.2 Conjugacy and Priors

13 Adaptive Basis Function Methods

- 13.1 Inference with Neural Networks
- 13.1.1 Single Neurons
- 13.1.2 Feedforward Neural Networks
- 13.1.3 Structured Neural Networks
- 13.1.4 Weight Intialization and Random Matrices
- 13.1.5 Optimization in Neural Networks
- 13.1.6 Probabilistic Neural Networks
- 13.2 Inference with Kernel Machines
- 13.3 Inference with Gaussian Processes

PART

IV

Application

- 14 Step Zero
- 14.1 Understanding the Problem You Care About
- 14.2 Specifying the Inductive Bias
- 15 Exploratory Data Analysis
- 15.1 Dataset Versioning
- 15.2 Visualization
- 15.3 Guaging Problem Difficulty
- 15.4 Starting Simply
- 16 Model Selection
- 16.1 Standard Model Selection
- 16.2 Bayesian Model Selection
- 17 Hyperparameter Selection
- 17.1 Heuristics
- 17.2 Bayesian Optimization
- 18 Datasets and Distribution Shift
- 18.1 Training on the Test Set
- 18.2 Internal Covariate Shift
- 18.3 Distribution Shift
- 19 Application: Computer Vision
- 20 Application: Linguistics & Genomics
- 21 Application: Robotics & Control

Frontiers

V

PART

- 22 Model Composition
- 22.1 Probabilistic Graphics Models and Adaptive Basis Methods
- 23 Returning to Control: Machine Learning in Society
- 23.1 Autonomous Vehicles
- 23.2 Autonomy in Weapon Systems
- 23.3 Robustness in Machine Learning
- 24 Brain Machine Interfacing
- 24.1 Studying the Brain
- 24.2 Imitating the Brain
- 24.3 Machine intelligence as analogies for neurocomputation
- 25 Machine Intelligence: What will it take?
- 25.1 The AI effect
- 25.2 Hype and Reality
- 25.3 Constructing Cockroaches

Appendices

- 25.4 Notation
- 25.5 Calculus
- 25.5.1 Derivatives
- 25.5.2 Chain Rule & Product Rule
- 25.5.3 Integrals
- 25.6 Complex numbers as dynamic objects

