Decomposing peer-effects by directionality to model the diffusion of behaviors across social networks

ONLINE SUPPORTING INFORMATION

1 Additional Simulation Results

In this online supporting information, we include Figure S1 with the general results across values of $\alpha = (\alpha_1, \alpha_2, \alpha_3)^T$ referred to in Section 3.2 of the main text. The plots show the bias, mean-squared-error (MSE), and coverage of the estimated peer-effect (α_0) using the influence matrix constructed from the undirected adjacency matrix compared to each of the inbound (α_1), outbound (α_2), and mutual (α_3) peer-effects. The simulated network size is 100 with a density of 0.2.

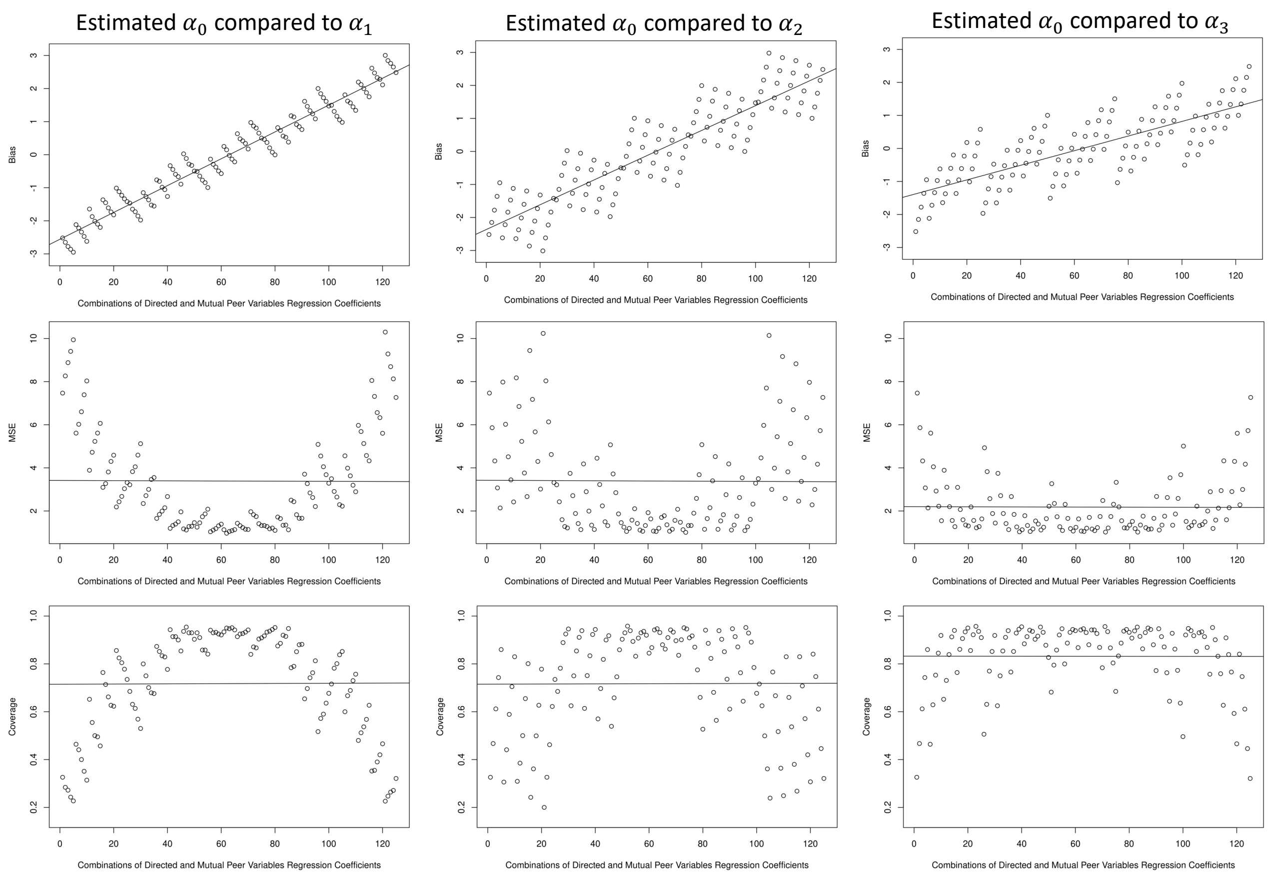

Figure S1: Bias, mean-squared-error (MSE), and coverage of estimated directed peer-effects $(\alpha_1, \alpha_2, \alpha_3)^T$ when the model in Equation (3) of the main text generates the data but the model with a single peer-effect (denoted $alpha_0$) for an undirected weight matrix is used for estimation. The x-axis is ordered by the combinations of $(\alpha_1, \alpha_2, \alpha_3)^T$ whereby α_1 varies fastest followed by α_2 and then by α_3 (leading to the patterns reflecting the repeated mini-trends in several of the subplots).

2 Additional Information on Model Fit and Statistics

In this section, we first present the generalized logit transformation that we performed on the outcome variable in the main text to account for its skewed distribution, along with statistics of its distribution. We then present statistics indicating the model fit of the models that are included in the main text.

Table S1: Statistics and distribution of prescribing index I_{all} in 2015 (the outcome variable).

Range	Min	Max								
	0	1								
Quantiles	5th	25th	50th	75th	95th					
	0.750	0.947	1.000	1.000	1.000					
Ten most common values	1	0.818	0.778	0.750	0.800	0.600	0.833	0.867	0.846	0.667
Proportion of occurrence	0.7086	0.0071	0.0070	0.0067	0.0067	0.0058	0.0058	0.0056	0.0054	0.0053
Total number of unique values	595									

To account for the skewness of $I_{all}^{(it)}$, which in our empirical sample took values over the interval $[0, 1]$ (although mathematically values as small as -1 could occur), instead of the log transformation, we used the following generalized logit transformation to retain the observations with $I_{all}^{(it)} = 0$ since taking the log transformation leads to infinity. Suppose I denotes a generic prescribing index with support $[a, b]$and i denotes the ith subject on which I is evaluated. Denoting I_{new} as the new transformed index, the generalized logit transformation can be specified as,

$$I_{new} = log(\frac{I - a + \epsilon}{b - I + \epsilon}), \tag{1}$$

where $a \leq I \leq b$, $\epsilon = 0.5 \times min(\{i \in I \mid i \neq 0\})$, and $\epsilon > 0$. In our case $a = 0$ and $b = 1$ and set $\epsilon = 0.056$.

We repeated the endogenous and exogenous peer-effect analyses described in the main text using this generalized logit transformation of I_{all}. Figures S2 and S3 are the plots of residuals vs. fitted values of both the endogenous and exogenous peer-effect models in the main text (those of model 1 in Tables 3 and 4 in the main text). One possible explanation for the lines on the residual plots is because I_{all} takes some commonly occurring values. As the corresponding fitted value gets bigger, the residual forms a negatively sloping trend. The most common of the commonly occurring values is 1, which occurs when the physician is never associated with transitioning a patient to a less risky state. This gets transformed to the value 2.944. The corresponding fitted values range from 1.75 to 3.05. As a result, the residual slopes downwards from about 1.2 to -0.05 forming a line at the top of the plot. Such behavior is purely a function of the semi-discrete nature of I_{all}. Here we present a list of the 10 most commonly occurring values of I_{all} and the proportion of times that these values occur (Table S1). We then compute the total number of unique values that I_{all} takes. We also report the mean of the residuals to confirm that as a whole the residuals are centered at 0 (see Tables S2 and S3 for more details).

Table S2: Statistics of the model fit of the endogenous peer-effect model in the main text (see model 1 in Table 3 in the main text).

Mean of Resd.	Correlation of resd. and fit. (p-value)	AIC, BIC
-1.7558e-13	0.0029 (0.7806)	21333.28, 21461.99

Table S3: Statistics of the model fit of the exogenous peer-effect model in the main text (see model 1 in Table 4 in the main text).

Mean of Resd.	Correlation of resd. and fit. (p-value)	AIC, BIC
-1.7524e-13	0.0061 (0.5566)	21658.88, 21787.59

3 Additional Peer-effect Linear Model Results

In this section, we present results of both the endogenous and exogenous peer-effect linear models (Tables S4 and S5) without transforming the outcome variable, i.e., the modeling the value of the prescribing index I_{all} itself.

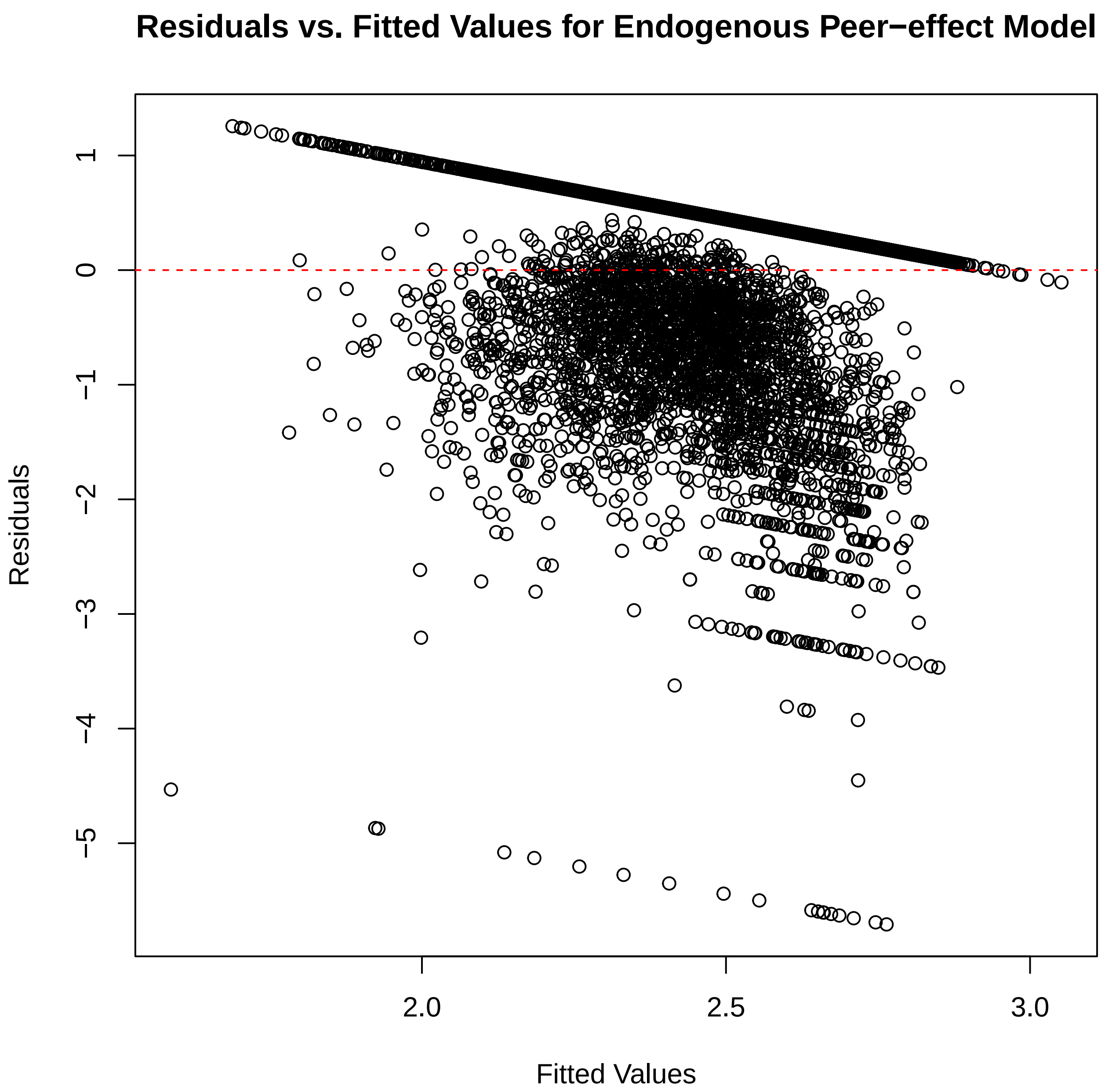

Figure S2: The plot of residuals vs. fitted values of the endogenous peer-effect model in the main text using the generalized transformation of I_{all}, without accounting for the modification by the ego physician's network strength.

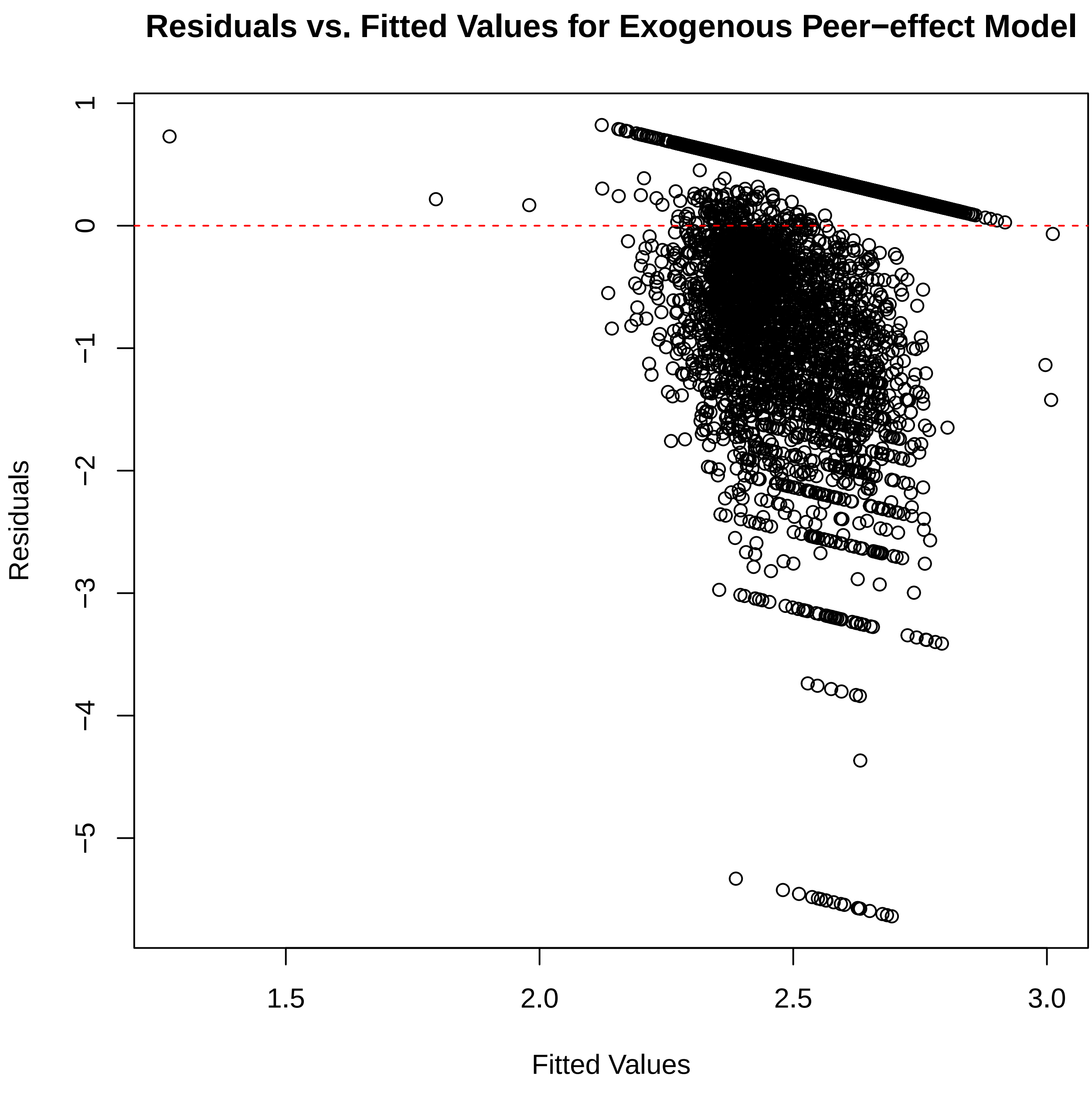

Figure S3: The plot of residuals vs. fitted values of the exogenous peer-effect model in the main text using the generalized transformation of I_{all}, without accounting for the modification by the ego physician's network strength.

Table S4: Estimated peer-effect linear models of the prescribing index (I_{all}) allowing directional patient-sharing, with and without effect modification by ego's network strength.

	Model 1			**Model 2**		
Term	**Est.**	**p-value**	**95% CI**	**Est.**	**p-value**	**95% CI**
(Intercept)	0.8176	0.000	$[0.7614, 0.8738]$	0.8315	0.000	$[0.7721, 0.8909]$
Lag ego I_{all}	0.0640	0.000	$[0.0555, 0.0725]$	0.0624	0.000	$[0.0539, 0.0709]$
Lag peer I_{all} (in)	0.0016	0.931	$[-0.0344, 0.0375]$	-0.0064	0.738	$[-0.0437, 0.0310]$
Lag peer I_{all} (out)	0.0285	0.143	$[-0.0096, 0.0665]$	0.0200	0.320	$[-0.0194, 0.0594]$
Lag peer I_{all} (mut)	0.0567	0.001	$[0.0217, 0.0917]$	0.0443	0.018	$[0.0075, 0.0811]$
ego patients (100s)	0.0010	0.508	$[-0.0019, 0.0039]$	0.0069	0.028	$[0.0007, 0.0131]$
peer patients (in) (100s)	0.0019	0.651	$[-0.0063, 0.0101]$	0.0017	0.683	$[-0.0065, 0.0099]$
peer patients (out) (100s)	-0.0010	0.818	$[-0.0092, 0.0073]$	-0.0006	0.890	$[-0.0088, 0.0077]$
peer patients (mut) (100s)	0.0038	0.201	$[-0.0020, 0.0096]$	0.0053	0.079	$[-0.0006, 0.0111]$
ego specialty (egoSpec) (Ref = PCP)	0.0477	0.002	$[0.0177, 0.0777]$	0.0343	0.029	$[0.0036, 0.0649]$
peerSpecIn	0.0040	0.704	$[-0.0168, 0.0249]$	0.0019	0.858	$[-0.0190, 0.0227]$
peerSpecOut	-0.0209	0.036	$[-0.0404, -0.0013]$	-0.0206	0.040	$[-0.0402, -0.0009]$
peerSpecMut	-0.0005	0.961	$[-0.0200, 0.0191]$	0.0043	0.667	$[-0.0153, 0.0240]$
egoSpec * peerSpecIn	-0.0488	0.006	$[-0.0837, -0.0139]$	-0.0373	0.038	$[-0.0726, -0.0021]$
egoSpec * peerSpecOut	0.0079	0.633	$[-0.0244, 0.0402]$	0.0202	0.225	$[-0.0125, 0.0529]$
egoSpec * peerSpecMut	-0.0369	0.013	$[-0.0660, -0.0078]$	-0.0390	0.009	$[-0.0681, -0.0099]$
in-strength (iStr)				0.0048	0.803	$[-0.0330, 0.0427]$
out-strength (oStr)				0.0324	0.113	$[-0.0077, 0.0726]$
mut-strength (mStr)				0.0033	0.727	$[-0.0152, 0.0218]$
Lag peer I_{all} (in) * iStr				-0.0193	0.285	$[-0.0548, 0.0161]$
Lag peer I_{all} (out) * oStr				-0.0175	0.376	$[-0.0563, 0.0213]$
Lag peer I_{all} (mut) * mStr				-0.0111	0.261	$[-0.0306, 0.0083]$

Note: The directionality of relationships is denoted by abbreviations in the parentheses of the lagged peer-effect terms. Abbreviations: iStr = in-strength (the total number of patient visits that the ego physician received from peer physicians in dyads with only an inbound tie to the ego physician); oStr = out-strength (the total number of patient visits involving dyads with only an outbound tie originating from the ego physician); mStr = mutual-strength (the total number of shared-patient visits to the ego and their peers across dyads involving the ego in which patient-sharing is non-null in both directions); peerSpecIn, peerSpecOut, and peerSpecMut = the weighted proportions of a focal physician's peers who are specialists based on inbound, outbound, and mutual patient-sharing relationships. Model 2 includes peer-effect modification by the ego physician's network strength for inbound, outbound, and mutual dyads.

Table S5: Estimated linear models of weighted average hip fracture experienced by patients of peer physicians on the ego physician's subsequent prescribing index (I_{all}).

	Model 1			**Model 2**		
Term	**Est.**	**p-value**	**95% CI**	**Est.**	**p-value**	**95% CI**
(Intercept)	0.9516	0.000	$[0.9244, 0.9789]$	0.9294	0.000	$[0.9010, 0.9578]$
Lag ego event	-0.0039	0.116	$[-0.0088, 0.0010]$	-0.0025	0.312	$[-0.0074, 0.0024]$
Lag peer event (in)	0.0038	0.775	$[-0.0220, 0.0295]$	0.0068	0.605	$[-0.0189, 0.0325]$
Lag peer event (out)	-0.0087	0.550	$[-0.0373, 0.0199]$	-0.0070	0.650	$[-0.0370, 0.0231]$
Lag peer event (mut)	0.0030	0.768	$[-0.0170, 0.0231]$	0.0090	0.381	$[-0.0111, 0.0292]$
ego patients (100s)	0.0012	0.420	$[-0.0017, 0.0041]$	0.0100	0.002	$[0.0036, 0.0164]$
peer patients (in)	0.0024	0.568	$[-0.0059, 0.0108]$	0.0022	0.599	$[-0.0061, 0.0105]$
peer patients (out)	-0.0003	0.949	$[-0.0087, 0.0081]$	0.0006	0.884	$[-0.0078, 0.0090]$
peer patients (mut)	0.0033	0.281	$[-0.0027, 0.0092]$	0.0053	0.082	$[-0.0007, 0.0112]$
ego specialty (egoSpec) (Ref = PCP)	0.0531	0.001	$[0.0227, 0.0836]$	0.0348	0.028	$[0.0038, 0.0659]$
peerSpecIn	0.0078	0.486	$[-0.0142, 0.0298]$	0.0064	0.565	$[-0.0155, 0.0284]$
peerSpecOut	-0.0296	0.006	$[-0.0507, -0.0085]$	-0.0275	0.011	$[-0.0488, -0.0063]$
peerSpecMut	-0.0026	0.800	$[-0.0226, 0.0175]$	0.0038	0.713	$[-0.0164, 0.0239]$
egoSpec * peerSpecIn	-0.0599	0.001	$[-0.0953, -0.0246]$	-0.0441	0.016	$[-0.0798, -0.0083]$
egoSpec * peerSpecOut	0.0075	0.656	$[-0.0253, 0.0402]$	0.0237	0.161	$[-0.0095, 0.0569]$
egoSpec * peerSpecMut	-0.0356	0.018	$[-0.0651, -0.0061]$	-0.0362	0.016	$[-0.0657, -0.0067]$
in-strength (iStr)				-0.0104	0.334	$[-0.0315, 0.0107]$
out-strength (oStr)				0.0141	0.192	$[-0.0071, 0.0354]$
mut-strength (mStr)				-0.0083	0.050	$[-0.0166, -0.00001]$
Lag peer event (in) * iStr				0.0083	0.325	$[-0.0082, 0.0247]$
Lag peer event (out) * oStr				-0.0019	0.896	$[-0.0307, 0.0269]$
Lag peer event (mut) * mStr				0.0045	0.201	$[-0.0024, 0.0113]$

Note: The expressions in the parentheses of the lag peer event terms denote the level of occurrence of hip fracture among peer-physicians' patients for inbound, outbound, and mutual patient-sharing. The directions in the parentheses of peer patient predictors denote the patient volume of peer physicians with the given type of relationships to the ego physician. Abbreviations: peerSpecIn, peerSpecOut, and peerSpecMut = the weighted proportions of a focal physician's peers who are specialists based on inbound, outbound, and mutual patient-sharing relationships.

4 Peer-effect of Alternative Risky Prescribing Measures

Recall Equation (8) in the main text that defines an 8×8 matrix $\mathbf{M}^{(it)}$ that summarizes physician i's contribution to their patients' prescription state transitions in year $t \in \{2014, 2015\}$, alternative prescribing indexes can be derived that capture various perspectives of a physician's prescribing behavior. For example, here we define another prescribing index I_{OBS} that quantifies a physician's contribution to bringing patients to the prescription state that involves an opioid, a benzodiazepine, and a non-benzodiazepine sedative-hypnotics, concurrently (the riskiest prescription state). Mathematically, I_{OBS} is defined as,

$$I_{OBS}^{(it)} = \frac{\sum_{u<8} C_{u,8}^{(it)}}{\sum_v \sum_{u<v} C_{u,v}^{(it)} + \sum_u \sum_{u>v} C_{u,v}^{(it)}} \tag{2}$$

We implemented the same generalized logit transformation outlined in Equation 1 on prescribing index $I_{OBS}^{(it)}$ and then performed endogenous peer-effect analysis in regards to $I_{OBS}^{(it)}$. As shown in Table S6, we didn't find significant evidence of peer-association from inbound, outbound, and mutual peers in regards to prescribing index $I_{OBS}^{(it)}$. Notably, in terms of peer-effect modification by physician network strength, we found that physicians with a higher level of network out-strength are less impacted ($Est. = -2.3284, p = 0.036$) by the risky prescribing of their outbound peers in regards to $I_{OBS}^{(it)}$.

Table S6: Estimated peer-effect models of the prescribing index (I_{OBS}) allowing directional patient-sharing, with and without effect modification by ego's network strength.

	Model 1			Model 2		
Term	**Est.**	**p-value**	**95% CI**	**Est.**	**p-value**	**95% CI**
(Intercept)	−5.6349	0.000	$[-5.8869, -5.3830]$	−5.3699	0.000	$[-5.6357, -5.1042]$
Lag ego I_{OBS}	3.8290	0.000	$[3.4158, 4.2422]$	3.7611	0.000	$[3.3484, 4.1738]$
Lag peer I_{OBS} (in)	−0.2538	0.784	$[-2.0654, 1.5578]$	−0.2144	0.825	$[-2.1116, 1.6827]$
Lag peer I_{OBS} (out)	1.9278	0.055	$[-0.0441, 3.8997]$	1.3156	0.209	$[-0.7360, 3.3673]$
Lag peer I_{OBS} (mut)	1.3757	0.127	$[-0.3892, 3.1405]$	1.0405	0.248	$[-0.7231, 2.8042]$
ego patients (100s)	0.0349	0.020	$[0.0056, 0.0642]$	−0.1058	0.001	$[-0.1690, -0.0427]$
peer patients (in) (100s)	−0.1006	0.018	$[-0.1838, -0.0174]$	−0.0985	0.020	$[-0.1816, -0.0154]$
peer patients (out) (100s)	0.0204	0.632	$[-0.0633, 0.1042]$	0.0060	0.889	$[-0.0777, 0.0897]$
peer patients (mut) (100s)	−0.0076	0.800	$[-0.0666, 0.0514]$	−0.0232	0.444	$[-0.0827, 0.0362]$
ego specialty (egoSpec) (Ref = PCP)	−0.7917	0.000	$[-1.0955, -0.4878]$	−0.6236	0.000	$[-0.9338, -0.3133]$
peerSpecIn	−0.7046	0.000	$[-0.9156, -0.4937]$	−0.6653	0.000	$[-0.8762, -0.4544]$
peerSpecOut	−0.0651	0.514	$[-0.2605, 0.1303]$	−0.0529	0.597	$[-0.2486, 0.1429]$
peerSpecMut	0.4548	0.000	$[0.2574, 0.6523]$	0.3860	0.000	$[0.1879, 0.5841]$
egoSpec * peerSpecIn	0.9013	0.000	$[0.5479, 1.2547]$	0.7467	0.000	$[0.3897, 1.1037]$
egoSpec * peerSpecOut	0.3852	0.021	$[0.0575, 0.7128]$	0.2071	0.221	$[-0.1244, 0.5385]$
egoSpec * peerSpecMut	−0.1400	0.353	$[-0.4354, 0.1554]$	−0.1105	0.463	$[-0.4055, 0.1845]$
in-strength (iStr)				0.1449	0.178	$[-0.0660, 0.3557]$
out-strength (oStr)				0.0091	0.933	$[-0.2032, 0.2214]$
mut-strength (mStr)				0.0004	0.992	$[-0.0818, 0.0827]$
Lag peer I_{OBS} (in) * iStr				0.1667	0.870	$[-1.8227, 2.1561]$
Lag peer I_{OBS} (out) * oStr				−2.3284	0.036	$[-4.5043, -0.1524]$
Lag peer I_{OBS} (mut) * mStr				−0.2226	0.551	$[-0.9550, 0.5098]$

5 Peer-effect Modification by Physician Network Closeness Centrality

In addition to using network strength as an effect modifier, we also explored using closeness centrality to capture a physician's prominence in a network based on the efficiency of information sharing. We specifically employed undirected closeness centrality (one that ignores the direction of edges when considering the distance from one node to another in a network) and repeated the endogenous peer-effect model with this alternative effect modifier. As shown in Table S7, there is insufficient evidence to conclude that physicians with different levels of prominence measured by undirected closeness centrality in the network are impacted differentially by their inbound, outbound, or mutual peers in terms of risky-prescribing.

Table S7: Estimated peer-effect models of the prescribing index (I_{all}) allowing directional patient-sharing, with and without effect modification by ego's network strength.

Term	**Est.**	**p-value**	**95% CI**
(Intercept)	1.4468	0.000	$[1.0496, 1.8440]$
Lag ego I_{all}	0.5260	0.000	$[0.4671, 0.5848]$
Lag peer I_{all} (in)	0.0600	0.639	$[-0.1906, 0.3106]$
Lag peer I_{all} (out)	0.2464	0.070	$[-0.0204, 0.5132]$
Lag peer I_{all} (mut)	0.5820	0.000	$[0.3379, 0.8262]$
ego patients (100s)	−0.0160	0.120	$[-0.0361, 0.0042]$
peer patients (in) (100s)	0.0232	0.429	$[-0.0343, 0.0807]$
peer patients (out) (100s)	−0.0147	0.619	$[-0.0725, 0.0431]$
peer patients (mut) (100s)	0.0191	0.358	$[-0.0217, 0.0598]$
ego specialty (egoSpec) (Ref = PCP)	0.3605	0.001	$[0.1515, 0.5696]$
peerSpecIn	0.0957	0.196	$[-0.0495, 0.2409]$
peerSpecOut	−0.2595	0.000	$[-0.3955, -0.1235]$
peerSpecMut	−0.1944	0.005	$[-0.3306, -0.0582]$
egoSpec * peerSpecIn	−0.4283	0.001	$[-0.6716, -0.1850]$
egoSpec * peerSpecOut	0.0764	0.507	$[-0.1492, 0.3020]$
egoSpec * peerSpecMut	−0.1752	0.090	$[-0.3780, 0.0275]$
ClsCent	0.4075	0.318	$[-0.3923, 1.2074]$
Lag peer I_{all} (in) * ClsCent	−0.0151	0.933	$[-0.3691, 0.3389]$
Lag peer I_{all} (out) * ClsCent	−0.3637	0.249	$[-0.9817, 0.2542]$
Lag peer I_{all} (mut) * ClsCent	−0.0486	0.520	$[-0.1969, 0.0996]$

Abbreviation: ClsCent = the undirected closeness centrality of a focal physician in the shared-patient network.