Homework Assignments

MATH231

Spring 2022

Homework 0

Due: Friday, Jan 21 (no need to turn in)

1. Calculating Limits

   - $\displaystyle \lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$

   - $\displaystyle \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h}$

   - $\displaystyle \lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$

2. The Chain Rule

   - $\dfrac{\mathrm{d}}{\mathrm{d}x} \ln(x + \sin x)$

   - $\dfrac{\mathrm{d}}{\mathrm{d}x} \cos(x^2 e^x)$

3. Implicit Differentiation: Solve for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for the following implicit function.

   - $x^2 + y^2 = r^2$, where r is a constant

   - $\dfrac{x + y}{x - y} = x$

4. Linear Approximations and Differentials: Find the Taylor polynomials of degree two approximating the given function centered at the given point.

   - $f(x) = \sin(2x)$ at $a = \frac{\pi}{2}$

   - $f(x) = e^x$ at $a = 1$

5. Mean Value Theorem: Determine if the Mean Value Theorem can be applied to the following function on the the given closed interval.

   - $f(x) = 3 + \sqrt{x}, x \in [0, 4]$

   - $f(x) = \dfrac{x}{1 + x}, x \in [1, 3]$

6. L'Hospital's Rule

- $\displaystyle \lim_{x \to 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$

- $\displaystyle \lim_{x \to \infty} (e^x + x)^{\frac{1}{x}}$

- $\displaystyle \lim_{x \to \infty} x \ln \left(1 + \frac{3}{x}\right)$

7. The Fundamental Theorem of Calculus: Find the derivative of the following

- $\displaystyle \int_1^x \frac{1}{t^3 + 1} \, dt$

- $\displaystyle \int_1^{\sqrt{x}} \sin t \, dt$

- $\displaystyle \int_x^{2x} t^3 \, dt$

8. Substitution Rule

- $\displaystyle \int_{\frac{1}{2}}^0 \frac{x}{\sqrt{1 - 4x^2}} \, dx$

- $\displaystyle \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\cos(\pi x)}{\sin^2(\pi x)} \, dx$

- $\displaystyle \int_0^1 x e^{4x^2 + 3} \, dx$

Homework 1

Due: Friday, Feb 4, by the end of the class

1. Integration by parts (You may also need to use substitution rule.)

   - $$\int \frac{\ln x}{x^2} \, \mathrm{d}x$$

   - $$\int x^2 \sin x \, \mathrm{d}x$$

   - $$\int (\ln x)^2 \, \mathrm{d}x$$

   - $$\int \arccos x \, \mathrm{d}x$$

   - $$\int e^{\sqrt{x}} \, \mathrm{d}x$$

2. Trigonometric integration: Evaluate the following integral of the form $\int \sin^n x \cos^m x \, \mathrm{d}x$.

   - $$\int \sin^2 x \cos^3 x \, \mathrm{d}x$$

   - $$\int \cos^4 x \, \mathrm{d}x$$

3. Trigonometric substitution

   - $$\int \frac{x^2}{\sqrt{9 - x^2}} \, \mathrm{d}x$$

   - $$\int \frac{1}{\sqrt{25 + x^2}} \, \mathrm{d}x$$

   - $$\int \frac{1}{\sqrt{x^2 + 2x}} \, \mathrm{d}x$$

   - $$\int (x - 2)^3 \sqrt{5 + 4x - x^2} \, \mathrm{d}x$$

Homework 2

Due: Friday, Feb 11, by the end of the class

Note that for indefinite integrals, you need add constants to your final answers.

1. Partial Fractions

   - $\displaystyle\int \frac{2x+5}{x^2+4x+8}\,\mathrm{d}x$

   - $\displaystyle\int \frac{2x^2-x+4}{(x^2+4)(x-1)}\,\mathrm{d}x$

   - $\displaystyle\int \frac{x}{x^4+2x^2+2}\,\mathrm{d}x$

   - $\displaystyle\int \ln(x^2+1)\,\mathrm{d}x$

   - $\displaystyle\int \frac{1}{\sqrt{x}+x\sqrt{x}}\,\mathrm{d}x$

   - $\displaystyle\int \frac{1}{x+\sqrt[3]{x}}\,\mathrm{d}x$

2. Approximate Integration

   - Use the Midpoint Rule with $n=5$ to approximate $\displaystyle\int_0^{10} x^2\,\mathrm{d}x$.

   - Use the Trapezoidal Rule with $n=6$ to approximate $\displaystyle\int_0^{\pi} \sin^2 x\,\mathrm{d}x$.

3. Improper Integrals: compute the following integrals or show that it diverges.

   - $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}}\,\mathrm{d}x$

   - $\displaystyle\int_1^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$

   - $\displaystyle\int_{\pi}^{\infty} \sin x\,\mathrm{d}x$

   - $\displaystyle\int_e^{\infty} \frac{1}{x\ln x}\,\mathrm{d}x$

   - $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\,\mathrm{d}x$

Homework 3

Due: Friday, Feb 25, by the end of the class

> **Instructions**
> Step 1. Write down what $\mathrm{d}s$ is before setting up an integral
> Step 2. Substitute $\mathrm{d}s$ and simplify the integral as much as you can
> Step 3. Compute the integral (substitution rule, integration by parts, etc.)

1. Arclength: for the following curves write down (do not evaluate) an integral w.r.t. x representing the length. Then write down an integral w.r.t. y.

   - $y = x^3$ for $x \in [1, 2]$.

   - $y = e^x$ for $x \in [0, 2]$.

2. Arclength: compute determine the arclength of the following curves

   - $y = \dfrac{x^3}{6} + \dfrac{1}{2x} = $ for $x \in [1, 3]$.

   - $y = \cosh x$ for $x \in [0, \ln 2]$.

     The hyperbolic cosine function $\cosh x$ is given by $\cosh x = \dfrac{e^x + e^{-x}}{2}$.

   - $y = \ln(\cos x)$ for $x \in \left[0, \dfrac{\pi}{3}\right]$.

3. Area of a Surface of Revolution: determine the area of the surface obtained by rotating the curve

   - $y = \sqrt{9 - x^2}$ for $x \in [-2, 2]$, rotating about the x-axis.

   - $y = x^2$ for $x \in [1, 2]$, rotating about the y-axis.

   - $y = \dfrac{(x^2 + 2)^{3/2}}{3}$ for $x \in [1, 2]$, rotating about the y-axis.

     Hint: For this question, it'd be easier if you treat $\mathrm{d}s$ as $\sqrt{1 + (y')^2}\, \mathrm{d}x$.

Homework 4

Due: Friday, Mar 4, by the end of the class

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

   You might want to review Chapter 2 in the book for computing limits.

   - $a_n = \dfrac{3 + 5n^2}{n + n^2}$

   - $a_n = \dfrac{2n^4 - 11n + 5}{4n - 1}$

   - $a_n = \dfrac{n^2 - 2n - 1}{n^3 + 3}$

   - $a_n = \left(1 + \dfrac{2}{n}\right)^n$

2. Computing Series

   - $\displaystyle\sum_{n=0}^{\infty} 9^{-\frac{n}{2}} 2^{1+n}$

   - $\displaystyle\sum_{n=5}^{\infty} \dfrac{3}{n^2 - 7n + 12}$ $\qquad$ Hint: recall $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{n(n+1)}$.

3. The Divergence Test: prove the following series diverges.

   - $\displaystyle\sum_{n=2}^{\infty} \cos\left(\dfrac{n\pi}{2}\right)$

   - $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{4 + e^{-n}}$

   - $\displaystyle\sum_{n=0}^{\infty} \dfrac{e^n}{n^3 + n}$

4. The Integral Test: determine if the following series converges or diverges.

   - $\displaystyle\sum_{n=1}^{\infty} \dfrac{n^4}{e^n}$

   - $\displaystyle\sum_{n=1}^{\infty} \dfrac{n}{n^3 + 1}$

   - $\displaystyle\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^p}$ for $p > 1$ and for $p \le 1$ respectively.

Homework 5

Due: Friday, Apr 1, by the end of the class

1. Comparison Test for Sequence

   This question asks you to use the comparison test for a pedagogical reason. There are other ways to solve Q1, for example, using the limit comparison test.

   - $\displaystyle\sum_{n=1}^{\infty} \frac{3n-2}{2n^3+5}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^n+n^2}$

2. The Limit Comparison Test

   - $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-n+5}{n^3-3n+6}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$

3. Alternating Series: determine if the following series converges absolutely, conditionally or diverges.

   For this question, you may use whichever test you like. If a series converges, specify if it converges absolutely, conditionally, or both.

   - $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n n^2}{n!}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (\ln n)^2}{n}$

Homework 6

Due: Friday, Apr 8, by the end of the class

> **Warning**
> 1. You might need some convergence tests such as alternating series test for the boundary cases. Make sure you have check **all** the assumptions before apply the test.
> 2. Simplify your final answer until it is a **single** sum of the form $\sum c_n (x - a)^n$.
> 3. Don't forget the range of x.
> 4. Remember to compute the constant C for the term-by-term integration.

1. Power Series: determine the radius of convergence R and interval of convergence I.

   - $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{n \cdot 5^n}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{2^n \ln n}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^{2n}}{(2n)!}$

2. Functions as Power Series: find the power series representations using substitution, term-by-term integration and differentiation.

   - $\arctan x$ Hint: $\arctan x = \displaystyle\int \frac{1}{1+x^2} \, dx$.

   - $\ln\left(\dfrac{1+x}{1-x}\right)$

   - $\dfrac{7x - x}{3x^2 + 2x - 1}$ Hint: Decompose the rational function first.

3. Taylor and Maclaurin Series: find the power series representation for by computing the n-th derivative $f^{(n)}(0)$.

   - $f(x) = \sin(x)$ centered at $\dfrac{\pi}{2}$.

   - $f(x) = (1 + x)^k$ centered at 0.   Note that tshis proves the binomial theorem.

4. Applications:

   - Using the Maclaurin series for $f(x) = e^x$ and the alternating series estimation theorem in 11.5 to approximate $\displaystyle\int_0^1 e^{-x^2} \, dx$ with error $R < 0.04$.