Spring 2021 MATH231 Section CDQ Discussion

WF 9-10am

This document can be found here or on my website. I will continue update it until the end of semester.

Contact

- TA for section CDQ: Xinran Yu
- Email: xinran4@illinois.edu. Please included MATH231 in your email subject.
- Office hour: Wed 10-11am[1]

Zoom

- Please use your cameras and microphones in breakout rooms.
- Interrupt me/using the "Raise Hand" feature on Zoom to ask questions.
- You can call me into your breakout room/return to the main room to ask for help.
- It is also possible for me to join your breakout rooms randomly to check if you have any questions.

Worksheet

- Worksheet can be found on Moodle under Groupwork folder.
- Ask for hints when you get stuck on a problem.

Submission

- Submit on Moodle under Groupwork folder.
- **1 submission per group**. Once a file is uploaded, everyone in the same group will be able to see/edit the file. [2]
- Group remains the same until each midterm.
- 1st worksheet of the week is due on **Thursday** at **8AM** CST. [3]
- 2nd worksheet of the week is due on **Saturday** at **8AM** CST.
- Worksheet solutions available at 12:30PM CST on the due date.

Grading

Worksheets are graded with 2, 1 or 0.

2 - the worksheet uploaded is satisfactory

1 - the worksheet uploaded is unsatisfactory and needs improvement. Your TA will comment on what should be improved for next time.

0 - the worksheet was not uploaded

[1] Office hour is run for all students in MATH231
[2] Groups are assigned randomly by Moodle
[3] Central Standard Time

Contents

Worksheet 1

Recall

Theorem 1.1 (Fundamental Theorem of Calculus)**.** *Ref p.26*

*Part 1 If $f(x)$ is **continuous** over an interval $[a, b]$, and the function $F(x)$ is defined by*

$$F(x) = \int_a^x f(t)\,\mathrm{d}t, \quad x \in [a, b]$$

then $F'(x) = f(x)$ over $[a, b]$.

*Part 2 If $f(x)$ is **continuous** over an interval $[a, b]$, and $F(x)$ is any antiderivative of $f(x)$ i.e. $F'(x) = f(x)$, then*

$$\int_a^b f(x)\,\mathrm{d}x = F(a) - F(b).$$

Example 1.2. Let

$$g(x) = \int_a^{b(x)} f(t)\,\mathrm{d}t$$

Apply chain rule and FTC

$$g'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\int_c^{b(x)} f(t)\,\mathrm{d}t = b'(x) \cdot f(b(x)).$$

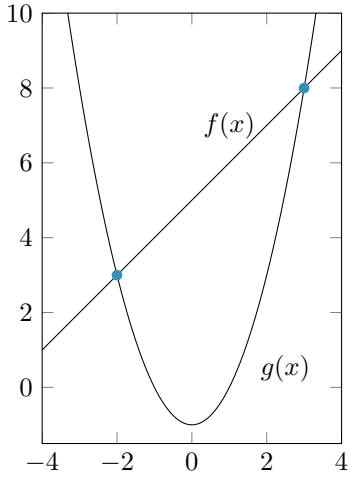
Worksheet 2

Recall

- **Substitution rule/Change of variable**: let $u = g(x)$, then

$$\int f(g(x)) \cdot g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u. \tag{Q1-3}$$

- **Compute area between curves.** (Q4-7)
  - Draw the graph.
  - Find intersection points by solving $f(x) = g(x)$, say they are $x = a$ and $x = b$.
  - Area $= \int_a^b f(x) - g(x)\,\mathrm{d}x$.



Worksheet 3

Volume of a Solid of Revolution: Slices of volume are circles. Ref

$$\text{Vol} = \int_a^b \pi f(x)^2 \, \mathrm{d}x.$$

Q3. Slices are squares/triangles.

$$\text{Vol} = \int_a^b \text{ Area of slices } \mathrm{d}x.$$

E.g.

$$\text{Vol} = \int_a^b f(x)^2 \, \mathrm{d}x.$$

Worksheet 4

Volume by Cylindrical Shells:

$$\text{Vol} = \int_a^b 2\pi r \cdot f(x)\,\mathrm{d}x.$$

Rotation about y-axis: $r = x$.
Rotation about the vertical line $x = a$: $r = |a - x|$.

Worksheet 5

Recall

- Since $\sin x$ is oscillating between -1 and 1, $\lim_{x\to\infty} \sin x$ does not exists.
- we can use L'Hopital's Rule to compute indeterminate forms "$\frac{0}{0}$" and "$\frac{\infty}{\infty}$".

Theorem 5.1 (L'Hopital's Rule)**.**
Assumptions:

$$\begin{aligned} &\boldsymbol{f(x) \to 0} \quad \text{as } x \to a, \\ &\boldsymbol{g(x) \to 0}, \\ &\boldsymbol{g'(x) \neq 0}. \end{aligned}$$

Conclusion:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

Warning: check the assumptions before applying L'Hopital's Rule.

Worksheet 6

Integration by Parts:

$$(uv)' = u'v + uv' \implies \int \boldsymbol{u}\,\mathrm{d}\boldsymbol{v} = \boldsymbol{uv} - \int \boldsymbol{v}\,\mathrm{d}\boldsymbol{u}.$$

Choose u based on which of these comes first, (search "integration by parts what to choose as u"):

(1) **L**ogarithmic functions: $\ln x$

(2) **I**nverse trigonometric functions: $\arcsin x$

(3) **A**lgebraic functions: x

(4) **T**rigonometric functions: $\sin x$

(5) **E**xponential functions: e^x

Worksheet 7

Worksheet 8

Recall: $1 + \tan^2 x = \sec^2 x$.

Worksheet 9

Substitution rule/Change of variable: let $u = g(x)$, then

$$\int f(g(x)) \cdot g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u.$$

Integration by Parts:

$$\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u.$$

Worksheet 10

Partial Fractions decomposition: Find A, B such that

$$\frac{1}{(x-a)(x-b)} = \frac{A}{x-a} - \frac{B}{x-b}.$$

The following table is from this website

Factor in denominator	Term in partial fraction decomposition
$ax+b$	$\dfrac{A}{ax+b}$
$(ax+b)^k$	$\dfrac{A_1}{ax+b} + \dfrac{A_2}{(ax+b)^2} + \cdots + \dfrac{A_k}{(ax+b)^k}, k = 1, 2, 3, \ldots$
ax^2+bx+c	$\dfrac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^k$	$\dfrac{A_1x+B_1}{ax^2+bx+c} + \dfrac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \dfrac{A_kx+B_k}{(ax^2+bx+c)^k}, k = 1, 2, 3, \ldots$

Typo in solution

Q3.

$$\frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2} + \frac{E}{x-1} + \frac{F}{(x-1)^2} + \frac{G}{(x-1)^3}.$$

Worksheet 11

Improper integrals: There are two types of improper integrals $\int_a^b f(x)\,\mathrm{d}x$:

(1) a or b (or both) infinite, e.g $\int_1^\infty \frac{1}{x}\,\mathrm{d}x$.

(2) The function $f(x)$ blows up in the interval $[a,b]$, e.g $\int_0^1 \ln x\,\mathrm{d}x$.

To compute improper integrals, e.g.:

$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{b\to\infty}\int_1^b \frac{1}{x}\,\mathrm{d}x$$