Homework Assignments

MATH231

Spring 2022

Homework 0

Due: Friday, Jan 21 (no need to turn in)

1. Calculating Limits

   - $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

   - $\lim\limits_{h \to 0} \dfrac{(x + h)^2 - x^2}{h}$

   - $\lim\limits_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{x^2 + x} \right)$

2. The Chain Rule

   - $\dfrac{\mathrm{d}}{\mathrm{d}x} \ln(x + \sin x)$

   - $\dfrac{\mathrm{d}}{\mathrm{d}x} \cos(x^2 e^x)$

3. Implicit Differentiation: Solve for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for the following implicit function.

   - $x^2 + y^2 = r^2$, where r is a constant

   - $\dfrac{x + y}{x - y} = x$

4. Linear Approximations and Differentials: Find the Taylor polynomials of degree two approximating the given function centered at the given point.

   - $f(x) = \sin(2x)$ at $a = \frac{\pi}{2}$

   - $f(x) = e^x$ at $a = 1$

5. Mean Value Theorem: Determine if the Mean Value Theorem can be applied to the following function on the the given closed interval.

   - $f(x) = 3 + \sqrt{x}, x \in [0, 4]$

   - $f(x) = \dfrac{x}{1 + x}, x \in [1, 3]$

6. L'Hospital's Rule

- $\displaystyle\lim_{x \to 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$

- $\displaystyle\lim_{x \to \infty} (e^x + x)^{\frac{1}{x}}$

- $\displaystyle\lim_{x \to \infty} x \ln\left(1 + \frac{3}{x}\right)$

7. The Fundamental Theorem of Calculus: Find the derivative of the following

- $\displaystyle\int_1^x \frac{1}{t^3 + 1}\, dt$

- $\displaystyle\int_1^{\sqrt{x}} \sin t\, dt$

- $\displaystyle\int_x^{2x} t^3\, dt$

8. Substitution Rule

- $\displaystyle\int_{\frac{1}{2}}^0 \frac{x}{\sqrt{1 - 4x^2}}\, dx$

- $\displaystyle\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\cos(\pi x)}{\sin^2(\pi x)}\, dx$

- $\displaystyle\int_0^1 xe^{4x^2 + 3}\, dx$

Homework 1

Due: Friday, Feb 4, by the end of the class

Instructions

- Write down the function u and v which you are using for substitution rule or integration by parts clearly in each problem.

- Note that for indefinite integrals, you need add constants to your final answers.

1. Integration by parts (You may also need to use substitution rule.)

   - $\int \dfrac{\ln x}{x^2} \, \mathrm{d}x$

   - $\int x^2 \sin x \, \mathrm{d}x$

   - $\int (\ln x)^2 \, \mathrm{d}x$

   - $\int \arccos x \, \mathrm{d}x$

   - $\int e^{\sqrt{x}} \, \mathrm{d}x$

2. Trigonometric integration: Evaluate the following integral of the form $\int \sin^n x \cos^m x \, \mathrm{d}x$.

   - $\int \sin^2 x \cos^3 x \, \mathrm{d}x$

   - $\int \cos^4 x \, \mathrm{d}x$

3. Trigonometric substitution

   - $\int \dfrac{x^2}{\sqrt{9 - x^2}} \, \mathrm{d}x$

   - $\int \dfrac{1}{\sqrt{25 + x^2}} \, \mathrm{d}x$

   - $\int \dfrac{1}{\sqrt{x^2 + 2x}} \, \mathrm{d}x$

   - $\int (x - 2)^3 \sqrt{5 + 4x - x^2} \, \mathrm{d}x$

Homework 2

Due: Friday, Feb 11, by the end of the class

Note that for indefinite integrals, you need add constants to your final answers.

1. Partial Fractions

   - $\displaystyle \int \frac{2x+5}{x^2+4x+8}\,\mathrm{d}x$

   - $\displaystyle \int \frac{2x^2-x+4}{(x^2+4)(x-1)}\,\mathrm{d}x$

   - $\displaystyle \int \frac{x}{x^4+2x^2+2}\,\mathrm{d}x$

   - $\displaystyle \int \ln(x^2+1)\,\mathrm{d}x$

   - $\displaystyle \int \frac{1}{\sqrt{x}+x\sqrt{x}}\,\mathrm{d}x$

   - $\displaystyle \int \frac{1}{x+\sqrt[3]{x}}\,\mathrm{d}x$

2. Approximate Integration

   - Use the Midpoint Rule with $n=5$ to approximate $\displaystyle \int_0^{10} x^2\,\mathrm{d}x$.

   - Use the Trapezoidal Rule with $n=6$ to approximate $\displaystyle \int_0^{\pi} \sin^2 x\,\mathrm{d}x$.

3. Improper Integrals: compute the following integrals or show that it diverges.

   - $\displaystyle \int_1^{\infty} \frac{1}{\sqrt{x}}\,\mathrm{d}x$

   - $\displaystyle \int_1^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$

   - $\displaystyle \int_{\pi}^{\infty} \sin x\,\mathrm{d}x$

   - $\displaystyle \int_e^{\infty} \frac{1}{x\ln x}\,\mathrm{d}x$

   - $\displaystyle \int_{-\infty}^{\infty} xe^{-x^2}\,\mathrm{d}x$

Homework 3

Due: Friday, Feb 25, by the end of the class

> **Instructions**
>
>   Step 1. Write down what $\mathrm{d}s$ is before setting up an integral
>
>   Step 2. Substitute $\mathrm{d}s$ and simplify the integral as much as you can
>
>   Step 3. Compute the integral (substitution rule, integration by parts, etc.)

1. Arclength: for the following curves write down (do not evaluate) an integral w.r.t. x representing the length. Then write down an integral w.r.t. y.

   - $y = x^3$ for $x \in [1, 2]$.

   - $y = e^x$ for $x \in [0, 2]$.

2. Arclength: compute determine the arclength of the following curves

   - $y = \dfrac{x^3}{6} + \dfrac{1}{2x} =$ for $x \in [1, 3]$.

   - $y = \cosh x$ for $x \in [0, \ln 2]$.

     The hyperbolic cosine function $\cosh x$ is given by $\cosh x = \dfrac{e^x + e^{-x}}{2}$.

   - $y = \ln(\cos x)$ for $x \in \left[0, \dfrac{\pi}{3}\right]$.

3. Area of a Surface of Revolution: determine the area of the surface obtained by rotating the curve

   - $y = \sqrt{9 - x^2}$ for $x \in [-2, 2]$, rotating about the x-axis.

   - $y = x^2$ for $x \in [1, 2]$, rotating about the y-axis.

   - $y = \dfrac{(x^2 + 2)^{3/2}}{3}$ for $x \in [1, 2]$, rotating about the y-axis.

     Hint: For this question, it'd be easier if you treat $\mathrm{d}s$ as $\sqrt{1 + (y')^2}\, \mathrm{d}x$.