Fall 2021 MATH241 Discussion

This document can be found on my website, named as "Discussion notes". If needed, I will update further information in the same document.

Time & location

- Section BDH: TR 3-4pm, 441 Altgeld.
- Section BDI: TR 4-5pm, 441 Altgeld.

Contact

- Email: xinran4@illinois.edu. Please included MATH241 and your section number in your email subject. If you don't get a reply in two days, feel free to send me a reminder.
- **Any math question should be post on Campuswire** because it can be answered much quicker than email and we TAs can type math symbols on Campuswire.
- Office hour: Wed 4-6pm on Zoom.[1]

Covid related

- **Face mask is required** all the time during discussion.
- Face shield is in general not acceptable unless one holds a DRES accommodation letter.[2]
- **According to university policy, student who does not wear a proper face covering will be asked to put on one or to leave the class room.**
- **If the student refuse to leave, I'll have to dismiss the class and report this to the undergraduate office.**
- In the case one tested positive, status can be checked by your instructor. I'll have to verify it before giving any excuse of being absent due to covid.[3]

Worksheet

- I will print copies and bring it to the classroom.
- You'll find group number written at the up right corner.
- If you prefer to work on a electronic version, you can find the worksheets on Moodle under "Worksheets" folder.
- The worksheets are long and you might not be able to finish them in calss, so you don't have to write down every single step.
- **Solutions will be avaliable on Moodle at 5pm** after discussion sections are done.
- Ask for hints when you get stuck on a problem.

[1] Office hour is run for all students in MATH241, regardless of section.
[2] See accommodation below for more information.
[3] Also see the grading section below.

Grading

- **Attendance is required** in order to get full grade for discussion.
- You shouldn't come to the class if you are sick.
- **The lowest 4 scores will be dropped** in order to remediate unforeseen illness, change of location or any possible reason for missing a class.
- If you were ill for more than 4 classes and want to see if you could be excused from that, you'll need to provide documents such as DRES letter to your instructor.
- Worksheets will be graded in a scale of 0-5. They are **not** graded for correctness.
- Most likely you'll get a full mark. In case you are interested, here is a sample grading scale:

  5 Most likely you'll get a full mark

  4 Being late or leave early for 15 min

  3 Being late or leave early for 25 min

  2 Being late or leave early for 35 min

  1 Not doing anything at all during the class

  0 Not showing up for any reason.

Accommodation

- Please contact the Disability Resources & Educational Services (DRES), if you need any sort of accommodation.
- You'll need to email **both your instructor and me** once you get the accommodation letter.

Contents

Worksheet 1

Chain rule

If $h(x) = g \circ f(x)$, then
$$h'(x) = g'(f(x)) \cdot f'(x).$$

Arc length of parameterized curve

Given a parameterized curve $(x(t), y(t))$, then the arc length between $(x(a), y(a))$ and $(x(b), y(b))$ is
$$s = \int_a^b \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t.$$

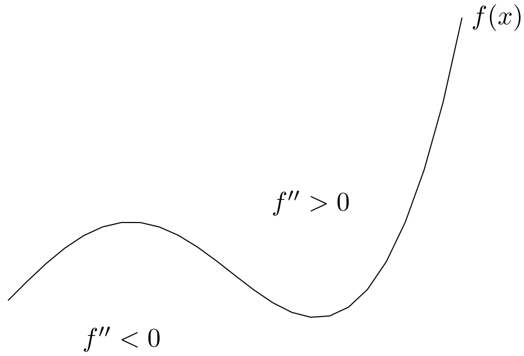
First and second derivative tests

We use the first and second derivative tests to determine local minimum and maximum.

First derivative tests. Compute $f'(x) = 0$ to find critical points.

Second derivative tests.

- If $f''(x) > 0$ for all x in the interval, then f is concave upward $\implies$ local minimum.
- If $f''(x) < 0$ for all x in the interval, then f is concave downward $\implies$ local maximum.



Taylor series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \\ &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots \end{aligned}$$

Substitution rule/Change of variable

Let $u = g(x)$, then
$$\int f(g(x)) \cdot g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u.$$

Open discussion: Sep 2nd

1. Find the angle between the planes $2x + y + 2z = 3$ and $4y + 3z = 1$.

   Hint: Use normal vectors.

2. Determine if the following vectors lie in the same plane or not:

   $$\mathbf{a} = \langle 1, 4, -7 \rangle, \ \mathbf{b} = \langle 2, -1, 4 \rangle \ \text{ and } \ \mathbf{c} = \langle 0, 9, -18 \rangle.$$

   Hint: Consider $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, draw a picture represent $\mathbf{b} \times \mathbf{c}$.

3. Find a counter example for the following:

   $$\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c} \implies \mathbf{b} = \mathbf{c}.$$

   Hint: Consider a triangle ABC. Let $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{CA} = \mathbf{b}$ and $\overrightarrow{CB} = \mathbf{c}$. What is $\mathbf{a} \times (\mathbf{b} - \mathbf{c})$?

4. Derive the following formula:

   - Area of a parallelogram determined by the vectors $\mathbf{a}$ and $\mathbf{b}$:

     $$\text{Area} = |\mathbf{a} \times \mathbf{b}|.$$

     Hint: Notes that $\mathbf{a} \times \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \sin\theta$.

   - Volume of a parallelepiped determined by the vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$:

     $$\text{Volume} = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|.$$

     Hint: Try to find the height of this parallelepiped.

Open discussion: Sep 16th & Exam 1 checklist

1. Find the following limit or explain why it does not exsit.

   - $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$

     DNE. Hint: Approach $(0,0)$ along $x=0$ and $y=0$ respectively.

   - $\lim_{(x,y)\to(1,1)} = \dfrac{2x^2-xy-y^2}{x^2-y^2}$

     $$\lim_{(x,y)\to(1,1)} \frac{2x^2-xy-y^2}{x^2-y^2} = \lim_{(x,y)\to(1,1)} \frac{(2x+y)(x-y)}{(x-y)(x+y)} = \lim_{(x,y)\to(1,1)} \frac{2x+y}{x+y} = \frac{3}{2}.$$

   - $\lim_{(x,y)\to(0,0)} \dfrac{x^2+\sin y}{3x^2+y}$

     DNE. Hint: Approach $(0,0)$ along $x=0$ we have

     $$\lim_{(x,y)\to(0,0)} \frac{x^2+\sin y}{3x^2+y} = \lim_{y\to 0} \frac{\sin y}{y} = 1.$$

     Approach $(0,0)$ along $y=0$ we have

     $$\lim_{(x,y)\to(0,0)} \frac{x^2+\sin y}{3x^2+y} = \lim_{x\to 0} \frac{x^2+1}{3x^2} = \frac{1}{3}.$$

2. Determine whether the following functions are continuous at $(0,0)$ or not.

   - $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y)\neq(0,0) \\ 0 & \text{if } (x,y)=(0,0) \end{cases}$

     The function is not continuous at $(0,0)$. Hint: use $y=ax$.

   - $f(x,y) = \begin{cases} x\sin\dfrac{1}{y} + y\sin\dfrac{1}{x} & \text{if } xy\neq 0 \\ 0 & \text{if } xy=0 \end{cases}$

     The function is continuous at $(0,0)$:

     $$\left| x\sin\frac{1}{y} + y\sin\frac{1}{x} \right| \leq |x|+|y| \xrightarrow{(x,y)\to(0,0)} 0.$$

Exam 1 Check list.

magnitute (scalar) → distance between points

dot product (scalar) → $\cos\theta$ → projection → orthogonal

$\cos\theta$ → angle between 2 planes/lines

cross product (vector) → magnitute = area of

triple product (scalar) → volume of

triple product (scalar) → determine if $\vec{a}, \vec{b}, \vec{c}$ or A, B, C, D lie on the same plane

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line ⟷ directional vector $\vec{v}$ + a point on the line P

plane ⟷ normal vector $\vec{n}$ + a point on the plane. P

→ e.g. given a line l
find the plane // to l and contain a certain pt

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match level sets & figure

- check certain pts.
- symmetry (odd / even function, switch x and y)
- $f(x,y) \to \infty$ at some pt / $x, y \to \infty$.

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know the equations of qudraic surfaces

computing limits.

Open discussion: Sep 21st

1. Let $f(x,y) = e^{xy}$ and $g(x,y) = f\left(\sin(x^2+y), x^3+2y+1\right)$. Compute g_x.

2. Let $f(x,y) = x^2y$. Find the derivative of f in the direction of $\mathbf{v} = \langle 3, 4\rangle$ at the point $P = (2, \frac{1}{4})$.

3. Find the direction for which the directional derivative of $f(x,y) = 3x^2 - 4xy + 2y^2$ at $P = (\frac{1}{2}, 1)$. What is the maximum value?

4. Open-ended question:
   - What is an intuitive explanation of the 1-dimensional chain rule?
     Could it be generalized to the chain rule in higher dimension?
   - What is a geometric intuition for directional derivatives? Say $f : \mathbb{R}^2 \to \mathbb{R}$.