

# Homework Assignments

MATH231

Spring 2022

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## Homework 0

**Due: Friday, Jan 21 (no need to turn in)**

1. Calculating Limits

- $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$
- $\lim_{h \rightarrow 0} \frac{(x + h)^2 - x^2}{h}$
- $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right)$

2. The Chain Rule

- $\frac{d}{dx} \ln(x + \sin x)$
- $\frac{d}{dx} \cos(x^2 e^x)$

3. Implicit Differentiation: Solve for  $\frac{dy}{dx}$  for the following implicit function.

- $x^2 + y^2 = r^2$ , where  $r$  is a constant
- $\frac{x + y}{x - y} = x$

4. Linear Approximations and Differentials: Find the Taylor polynomials of degree two approximating the given function centered at the given point.

- $f(x) = \sin(2x)$  at  $a = \frac{\pi}{2}$
- $f(x) = e^x$  at  $a = 1$

5. Mean Value Theorem: Determine if the Mean Value Theorem can be applied to the following function on the the given closed interval.

- $f(x) = 3 + \sqrt{x}, x \in [0, 4]$
- $f(x) = \frac{x}{1 + x}, x \in [1, 3]$

6. L'Hospital's Rule

- $\lim_{x \rightarrow 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$

- $\lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}}$

- $\lim_{x \rightarrow \infty} x \ln \left( 1 + \frac{3}{x} \right)$

7. The Fundamental Theorem of Calculus: Find the derivative of the following

- $\int_1^x \frac{1}{t^3 + 1} dt$

- $\int_1^{\sqrt{x}} \sin t dt$

- $\int_x^{2x} t^3 dt$

8. Substitution Rule

- $\int_{\frac{1}{2}}^0 \frac{x}{\sqrt{1 - 4x^2}} dx$

- $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\cos(\pi x)}{\sin^2(\pi x)} dx$

- $\int_0^1 x e^{4x^2+3} dx$

# Homework 1

Due: Friday, Feb 4, by the end of the class

## Instructions

- Write down the function  $u$  and  $v$  which you are using for substitution rule or integration by parts clearly in each problem.
- Note that for indefinite integrals, you need add constants to your final answers.

1. Integration by parts (You may also need to use substitution rule.)

- $\int \frac{\ln x}{x^2} dx$
- $\int x^2 \sin x dx$
- $\int (\ln x)^2 dx$
- $\int \arccos x dx$
- $\int e^{\sqrt{x}} dx$

2. Trigonometric integration: Evaluate the following integral of the form  $\int \sin^n x \cos^m x dx$ .

- $\int \sin^2 x \cos^3 x dx$
- $\int \cos^4 x dx$

3. Trigonometric substitution

- $\int \frac{x^2}{\sqrt{9-x^2}} dx$
- $\int \frac{1}{\sqrt{25+x^2}} dx$
- $\int \frac{1}{\sqrt{x^2+2x}} dx$
- $\int (x-2)^3 \sqrt{5+4x-x^2} dx$

## Homework 2

**Due: Friday, Feb 11, by the end of the class**

Note that for indefinite integrals, you need add constants to your final answers.

### 1. Partial Fractions

- $\int \frac{2x + 5}{x^2 + 4x + 8} \, dx$
- $\int \frac{2x^2 - x + 4}{(x^2 + 4)(x - 1)} \, dx$
- $\int \frac{x}{x^4 + 2x^2 + 2} \, dx$
- $\int \ln(x^2 + 1) \, dx$
- $\int \frac{1}{\sqrt{x} + x\sqrt{x}} \, dx$
- $\int \frac{1}{x + \sqrt[3]{x}} \, dx$

### 2. Approximate Integration

- Use the Midpoint Rule with  $n = 5$  to approximate  $\int_0^{10} x^2 \, dx$ .
- Use the Trapezoidal Rule with  $n = 6$  to approximate  $\int_0^\pi \sin^2 x \, dx$ .

### 3. Improper Integrals: compute the following integrals or show that it diverges.

- $\int_1^\infty \frac{1}{\sqrt{x}} \, dx$
- $\int_1^\infty \frac{1}{1 + x^2} \, dx$
- $\int_\pi^\infty \sin x \, dx$
- $\int_e^\infty \frac{1}{x \ln x} \, dx$
- $\int_{-\infty}^\infty x e^{-x^2} \, dx$

## Homework 3

Due: Friday, Feb 25, by the end of the class

### Instructions

Step 1. Write down what  $ds$  is before setting up an integral

Step 2. Substitute  $ds$  and simplify the integral as much as you can

Step 3. Compute the integral (substitution rule, integration by parts, etc.)

1. Arclength: for the following curves write down (do not evaluate) an integral w.r.t.  $x$  representing the length. Then write down an integral w.r.t.  $y$ .

- $y = x^3$  for  $x \in [1, 2]$ .

- $y = e^x$  for  $x \in [0, 2]$ .

2. Arclength: compute determine the arclength of the following curves

- $y = \frac{x^3}{6} + \frac{1}{2x}$  for  $x \in [1, 3]$ .

- $y = \cosh x$  for  $x \in [0, \ln 2]$ .

The hyperbolic cosine function  $\cosh x$  is given by  $\cosh x = \frac{e^x + e^{-x}}{2}$ .

- $y = \ln(\cos x)$  for  $x \in \left[0, \frac{\pi}{3}\right]$ .

3. Area of a Surface of Revolution: determine the area of the surface obtained by rotating the curve

- $y = \sqrt{9 - x^2}$  for  $x \in [-2, 2]$ , rotating about the  $x$ -axis.

- $y = x^2$  for  $x \in [1, 2]$ , rotating about the  $y$ -axis.

- $y = \frac{(x^2 + 2)^{3/2}}{3}$  for  $x \in [1, 2]$ , rotating about the  $y$ -axis.

Hint: For this question, it'd be easier if you treat  $ds$  as  $\sqrt{1 + (y')^2} dx$ .

## Homework 4

**Due: Friday, Mar 4, by the end of the class**

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

You might want to review Chapter 2 in the book for computing limits.

- $a_n = \frac{3 + 5n^2}{n + n^2}$

- $a_n = \frac{2n^4 - 11n + 5}{4n - 1}$

- $a_n = \frac{n^2 - 2n - 1}{n^3 + 3}$

- $a_n = \left(1 + \frac{2}{n}\right)^n$

2. Computing Series

- $\sum_{n=0}^{\infty} 9^{-\frac{n}{2}} 2^{1+n}$

- $\sum_{n=5}^{\infty} \frac{3}{n^2 - 7n + 12}$

Hint: recall  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ .

3. The Divergence Test: prove the following series diverges.

- $\sum_{n=2}^{\infty} \cos\left(\frac{n\pi}{2}\right)$

- $\sum_{n=1}^{\infty} \frac{1}{4 + e^{-n}}$

- $\sum_{n=0}^{\infty} \frac{e^n}{n^3 + n}$

4. The Integral Test: determine if the following series converges or diverges.

- $\sum_{n=1}^{\infty} \frac{n^4}{e^n}$

- $\sum_{n=1}^{\infty} \frac{n}{n^3 + 1}$

- $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$  for  $p > 1$  and for  $p \leq 1$  respectively.

## Homework 5

**Due: Friday, Apr 1, by the end of the class**

1. Comparison Test for Sequence

This question asks you to use the comparison test for a pedagogical reason. There are other ways to solve Q1, for example, using the limit comparison test.

$$\bullet \sum_{n=1}^{\infty} \frac{3n-2}{2n^3+5}$$

$$\bullet \sum_{n=1}^{\infty} \frac{1}{e^n + n^2}$$

2. The Limit Comparison Test

$$\bullet \sum_{n=1}^{\infty} \frac{n^2 - n + 5}{n^3 - 3n + 6}$$

$$\bullet \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$$

3. Alternating Series: determine if the following series converges absolutely, conditionally or diverges.

For this question, you may use whichever test you like. If a series converges, specify if it converges absolutely, conditionally, or both.

$$\bullet \sum_{n=1}^{\infty} \frac{(-3)^n n^2}{n!}$$

$$\bullet \sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$$

$$\bullet \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$$

$$\bullet \sum_{n=1}^{\infty} \frac{(-1)^n (\ln n)^2}{n}$$



## Homework 6

Due: Friday, Apr 8, by the end of the class

### Warning

1. When apply convergence tests such as alternating series test for the boundary cases, make sure you have check **all** the assumptions before apply the test.
2. Simplify your final answer until it is a **single** sum of the form  $\sum c_n(x-a)^n$ .
3. Don't forget the range of  $x$ .
4. Remember to compute the constant  $C$  for the term-by-term integration.

1. Power Series: determine the radius of convergence  $R$  and interval of convergence  $I$ .

- $\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{n \cdot 5^n}$
- $\sum_{n=1}^{\infty} \frac{(x+2)^n}{2^n \ln n}$
- $\sum_{n=1}^{\infty} \frac{n^2 x^{2n}}{(2n)!}$

2. Functions as Power Series: find the power series representations using substitution, term-by-term integration and differentiation.

- $\arctan x$       Hint:  $\arctan x = \int \frac{1}{1+x^2} dx$ .
- $\ln \left( \frac{1+x}{1-x} \right)$
- $\frac{7x-x}{3x^2+2x-1}$       Hint: Decompose the rational function first.

3. Taylor and Maclaurin Series: find the power series representation for by computing the  $n$ -th derivative  $f^{(n)}(0)$ .

- $f(x) = \sin(x)$  centered at  $\frac{\pi}{2}$ .
- $f(x) = (1+x)^k$  centered at 0.      Note that tshis proves the binomial theorem.

4. Applications:

- Using the Maclaurin series for  $f(x) = e^x$  and the alternating series estimation theorem in 11.5 to approximate  $\int_0^1 e^{-x^2} dx$  with error  $R < 0.04$ .

## Homework 7

Due: Friday, Apr 22, by the end of the class

- Unless otherwise stated,  $t \in (-\infty, \infty)$
- Q1 aims for helping you getting intuition to parameterizations. You may describe the curve by words if you find it difficult to draw.

1. Curves Defined by Parametric Equations: Draw or describe the following curves on the  $x, y$ -plane.

- $x = 2t + 3, y = 3t - 4$
- $x = t^2 - 3, y = 2t - 1$
- $x = 3 - \cos t, y = \sin t + 1$  for  $0 \leq t \leq 2\pi$
- $x = \sqrt{t+1}, y = \frac{1}{t+1}$

2. Derivatives: compute  $\frac{dy}{dx}$  for the following parameterizations.

- $x = 2t + 1, y = t^3 - 3t + 4$
- $x = 3 \cos t, y = 3 \sin t + 1$  for  $0 \leq t \leq 2\pi$
- $x = te^{-t}, y = 2t^2 + 1$

3. Second order derivatives: compute  $\frac{d^2y}{dx^2}$  for the following parameterizations.

- $x = t^2, y = t^3$
- $x = 3t^2, y = t^4 - 8t^2$
- $x = e^{-t}, y = t^3 + t + 1$

4. Arc Length with Parametric Curves: Compute the length of the curve

- $x = 2 \cos^2 t, y = 2 \cos t \sin t$  for  $0 \leq t \leq \pi$
- $x = t - \sin t, y = 1 - \cos t$  for  $0 \leq t \leq 2\pi$