

Fall 2021 MATH241 Discussion

This document can be found on [my website](#), named as “Discussion notes”. If needed, I will update further information in the same document.

Time & location

- Section BDH: TR 3-4pm, 441 Altgeld.
- Section BDI: TR 4-5pm, 441 Altgeld.

Contact

- Email: xinran4@illinois.edu. Please included MATH241 and your section number in your email subject. If you don't get a reply in two days, feel free to send me a reminder.
- **Any math question should be post on [Campuswire](#)** because it can be answered much quicker than email and we TAs can type math symbols on Campuswire.
- Office hour: Wed 4-6pm on Zoom.¹

Covid related

- **Face mask is required** all the time during discussion.
- Face shield is in general not acceptable unless one holds a [DRES accommodation letter](#).²
- **According to university policy, student who does not wear a proper face covering will be asked to put on one or to leave the class room.**
- **If the student refuse to leave, I'll have to dismiss the class and report this to the undergraduate office.**
- In the case one tested positive, status can be checked by your instructor. I'll have to verify it before giving any excuse of being absent due to covid.³

Worksheet

- I will print copies and bring it to the classroom.
- You'll find group number written at the up right corner.
- If you prefer to work on a electronic version, you can find the worksheets on [Moodle](#) under "Worksheets" folder.
- The worksheets are long and you might not be able to finish them in calss, so you don't have to write down every single step.
- **Solutions will be avaiable on Moodle at 5pm** after discussion sections are done.
- Ask for hints when you get stuck on a problem.

¹Office hour is run for all students in MATH241, regardless of section.

²See accommodation below for more information.

³Also see the grading section below.

Grading

- **Attendance is required** in order to get full grade for discussion.
- You shouldn't come to the class if you are sick.
- **The lowest 4 scores will be dropped** in order to remediate unforeseen illness, change of location or any possible reason for missing a class.
- If you were ill for more than 4 classes and want to see if you could be excused from that, you'll need to provide documents such as DRES letter to your instructor.
- Worksheets will be graded in a scale of 0-5. They are **not** graded for correctness.
- Most likely you'll get a full mark. In case you are interested, here is a sample grading scale:

- 5 Most likely you'll get a full mark
- 4 Being late or leave early for 15 min
- 3 Being late or leave early for 25 min
- 2 Being late or leave early for 35 min
- 1 Not doing anything at all during the class
- 0 Not showing up for any reason.

Accommodation

- Please contact the Disability Resources & Educational Services ([DRES](#)), if you need any sort of accommodation.
- You'll need to email **both your instructor and me** once you get the accommodation letter.

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Worksheet 1

Chain rule

If $h(x) = g \circ f(x)$, then

$$h'(x) = g'(f(x)) \cdot f'(x).$$

Arc length of parameterized curve

Given a parameterized curve $(x(t), y(t))$, then the arc length between $(x(a), y(a))$ and $(x(b), y(b))$ is

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

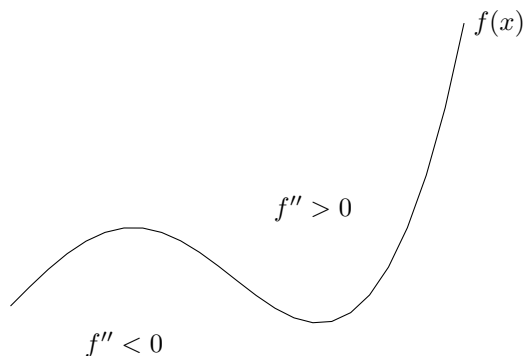
First and second derivative tests

We use the first and second derivative tests to determine local minimum and maximum.

First derivative tests. Compute $f'(x) = 0$ to find critical points.

Second derivative tests.

- If $f''(x) > 0$ for all x in the interval, then f is concave upward \implies local minimum.
- If $f''(x) < 0$ for all x in the interval, then f is concave downward \implies local maximum.



Taylor series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \\ &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \frac{f'''(a)}{3!} (x-a)^3 + \dots \end{aligned}$$

Substitution rule/Change of variable

Let $u = g(x)$, then

$$\int f(g(x)) \cdot g'(x) dx = \int f(u) du.$$

Open discussion: Sep 2nd

1. Find the angle between the planes $2x + y + 2z = 3$ and $4y + 3z = 1$.

Hint: Use normal vectors.

2. Determine if the following vectors lie in the same plane or not:

$$\mathbf{a} = (1, 4, -7), \mathbf{b} = (2, -1, 4) \text{ and } \mathbf{c} = (0, 9, -18).$$

Hint: Consider $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, draw a picture represent $\mathbf{b} \times \mathbf{c}$.

3. Find a counter example for the following:

$$\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c} \implies \mathbf{b} = \mathbf{c}.$$

Hint: Consider a triangle ABC . Let $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{CA} = \mathbf{b}$ and $\overrightarrow{CB} = \mathbf{c}$. What is $\mathbf{a} \times (\mathbf{b} - \mathbf{c})$?

4. Derive the following formula:

- Area of a parallelogram determined by the vectors \mathbf{a} and \mathbf{b} :

$$\text{Area} = |\mathbf{a} \times \mathbf{b}|.$$

Hint: Notes that $\mathbf{a} \times \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \sin \theta$.

- Volume of a parallelepiped determined by the vectors \mathbf{a} , \mathbf{b} and \mathbf{c} :

$$\text{Volume} = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|.$$

Hint: Try to find the height of this parallelepiped.

Open discussion: Sep 16th

1. Find the following limit or explain why it does not exist.

- $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$

- $\lim_{(x,y) \rightarrow (1,1)} \frac{2x^2 - xy - y^2}{x^2 - y^2}$

- $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + \sin y}{3x^2 + y}$

2. Determine whether the following functions are continuous at $(0,0)$ or not.

- $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

- $f(x, y) = \begin{cases} x \sin \frac{1}{y} + y \sin \frac{1}{x} & \text{if } xy \neq 0 \\ 0 & \text{if } xy = 0 \end{cases}$