

Spring 2021 MATH231 Section CDQ Discussion

WF 9-10am

This document can be found [here](#) or on [my website](#). I will continue update it until the end of semester.

Contact

- TA for section CDQ: Xinran Yu
- Email: xinran4@illinois.edu. Please included MATH231 in your email subject.
- Office hour: Wed 10-11am¹

Zoom

- Please use your cameras and microphones in breakout rooms.
- Interrupt me/using the “Raise Hand” feature on Zoom to ask questions.
- You can call me into your breakout room/return to the main room to ask for help.
- It is also possible for me to join your breakout rooms randomly to check if you have any questions.

Worksheet

- Worksheet can be found on [Moodle](#) under Groupwork folder.
- Ask for hints when you get stuck on a problem.

Submission

- Submit on [Moodle](#) under Groupwork folder.
- **1 submission per group.** Once a file is uploaded, everyone in the same group will be able to see/edit the file. ²
- Group remains the same until each midterm.
- 1st worksheet of the week is due on **Thursday** at **8AM** CST. ³
- 2nd worksheet of the week is due on **Saturday** at **8AM** CST.
- Worksheet solutions available at 12:30PM CST on the due date.

Grading

Worksheets are graded with 2, 1 or 0.

2 - the worksheet uploaded is satisfactory

1 - the worksheet uploaded is unsatisfactory and needs improvement. Your TA will comment on what should be improved for next time.

0 - the worksheet was not uploaded

¹Office hour is run for all students in MATH231

²Groups are assigned randomly by Moodle

³Central Standard Time

Contents

1	Worksheet 1	3
2	Worksheet 2	4
3	Worksheet 3	5
4	Worksheet 4	6
5	Worksheet 5	7
6	Worksheet 6	8
7	Worksheet 7	9
8	Worksheet 8	9
9	Worksheet 9	10
10	Worksheet 10	11

Worksheet 1

Recall

Theorem 1.1 (Fundamental Theorem of Calculus). [Ref p.26](#)

Part 1 If $f(x)$ is **continuous** over an interval $[a, b]$, and the function $F(x)$ is defined by

$$F(x) = \int_a^x f(t) \, dt, \quad x \in [a, b]$$

then $F'(x) = f(x)$ over $[a, b]$.

Part 2 If $f(x)$ is **continuous** over an interval $[a, b]$, and $F(x)$ is any antiderivative of $f(x)$ i.e. $F'(x) = f(x)$, then

$$\int_a^b f(x) \, dx = F(a) - F(b).$$

Example 1.2. Let

$$g(x) = \int_a^{b(x)} f(t) \, dt$$

Apply chain rule and FTC

$$g'(x) = \frac{d}{dx} \int_c^{b(x)} f(t) \, dt = b'(x) \cdot f(b(x)).$$

Worksheet 2

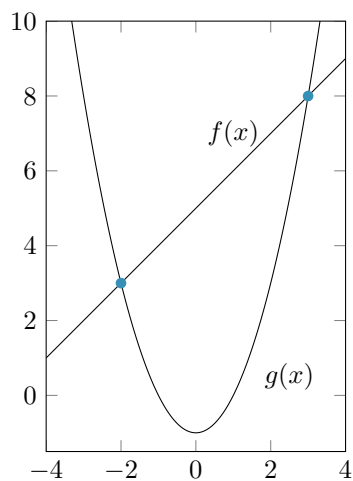
Recall

- **Substitution rule/Change of variable:** let $u = g(x)$, then

$$\int f(g(x)) \cdot g'(x) \, dx = \int f(u) \, du. \quad (\text{Q1-3})$$

- **Compute area between curves.** (Q4-7)

- Draw the graph.
- Find intersection points by solving $f(x) = g(x)$, say they are $x = a$ and $x = b$.
- Area = $\int_a^b f(x) - g(x) \, dx$.



Worksheet 3

Volume of a Solid of Revolution: Slices of volume are circles. [Ref](#)

$$\text{Vol} = \int_a^b \pi f(x)^2 \, dx.$$

Q3. Slices are squares/triangles.

$$\text{Vol} = \int_a^b \text{Area of slices} \, dx.$$

E.g.

$$\text{Vol} = \int_a^b f(x)^2 \, dx.$$

Worksheet 4

Volume by Cylindrical Shells:

$$\text{Vol} = \int_a^b 2\pi r \cdot f(x) \, dx.$$

Rotation about y -axis: $r = x$.

Rotation about the vertical line $x = a$: $r = |a - x|$.

Worksheet 5

Recall

- Since $\sin x$ is oscillating between -1 and 1, $\lim_{x \rightarrow \infty} \sin x$ does not exist.
- we can use L'Hopital's Rule to compute indeterminate forms " $\frac{0}{0}$ " and " $\frac{\infty}{\infty}$ ".

Theorem 5.1 (L'Hopital's Rule).

Assumptions:

$$\begin{aligned}f(x) &\rightarrow 0 \quad \text{as } x \rightarrow a, \\g(x) &\rightarrow 0, \\g'(x) &\neq 0.\end{aligned}$$

Conclusion:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$$

Warning: check the assumptions before applying L'Hopital's Rule.

Worksheet 6

Integration by Parts:

$$(uv)' = u'v + uv' \implies \int u \, dv = uv - \int v \, du.$$

Choose u based on which of these comes first, (search “[integration by parts what to choose as \$u\$](#) ”):

- (1) **L**ogarithmic functions: $\ln x$
- (2) **I**nverse trigonometric functions: $\arcsin x$
- (3) **A**lgebraic functions: x
- (4) **T**rigonometric functions: $\sin x$
- (5) **E**xponential functions: e^x

Worksheet 7

Worksheet 8

Recall: $1 + \tan^2 x = \sec^2 x$.

Worksheet 9

Substitution rule/Change of variable: let $u = g(x)$, then

$$\int f(g(x)) \cdot g'(x) \, dx = \int f(u) \, du.$$

Integration by Parts:

$$\int u \, dv = uv - \int v \, du.$$

Worksheet 10

Partial Fractions decomposition: Find A, B such that

$$\frac{1}{(x-a)(x-b)} = \frac{A}{x-a} - \frac{B}{x-b}.$$

The following table is from this [website](#)

Factor in denominator	Term in partial fraction decomposition
$ax + b$	$\frac{A}{ax + b}$
$(ax + b)^k$	$\frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \cdots + \frac{A_k}{(ax + b)^k}, k = 1, 2, 3, \dots$
$ax^2 + bx + c$	$\frac{Ax + B}{ax^2 + bx + c}$
$(ax^2 + bx + c)^k$	$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}, k = 1, 2, 3, \dots$