Homework Assignments

MATH231

Spring 2022

Homework 0

Due: Friday, Jan 21 (no need to turn in)

1. Calculating Limits
   - $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$
   - $\lim_{h \to 0} \dfrac{(x + h)^2 - x^2}{h}$
   - $\lim_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{x^2 + x} \right)$
2. The Chain Rule
   - $\dfrac{\mathrm{d}}{\mathrm{d}x} \ln(x + \sin x)$
   - $\dfrac{\mathrm{d}}{\mathrm{d}x} \cos(x^2 e^x)$
3. Implicit Differentiation: Solve for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for the following implicit function.
   - $x^2 + y^2 = r^2$, where r is a constant
   - $\dfrac{x + y}{x - y} = x$
4. Linear Approximations and Differentials: Find the Taylor polynomials of degree two approximating the given function centered at the given point.
   - $f(x) = \sin(2x)$ at $a = \frac{\pi}{2}$
   - $f(x) = e^x$ at $a = 1$
5. Mean Value Theorem: Determine if the Mean Value Theorem can be applied to the following function on the the given closed interval.
   - $f(x) = 3 + \sqrt{x}, x \in [0, 4]$
   - $f(x) = \dfrac{x}{1 + x}, x \in [1, 3]$

6. L'Hospital's Rule
   - $\lim_{x \to 2} \dfrac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$
   - $\lim_{x \to \infty} (e^x + x)^{\frac{1}{x}}$
   - $\lim_{x \to \infty} x \ln\left(1 + \dfrac{3}{x}\right)$

7. The Fundamental Theorem of Calculus: Find the derivative of the following
   - $\displaystyle\int_1^x \frac{1}{t^3 + 1} \, \mathrm{d}t$
   - $\displaystyle\int_1^{\sqrt{x}} \sin t \, \mathrm{d}t$
   - $\displaystyle\int_x^{2x} t^3 \, \mathrm{d}t$

8. Substitution Rule
   - $\displaystyle\int_{\frac{1}{2}}^0 \frac{x}{\sqrt{1 - 4x^2}} \, \mathrm{d}x$
   - $\displaystyle\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\cos(\pi x)}{\sin^2(\pi x)} \, \mathrm{d}x$
   - $\displaystyle\int_0^1 x e^{4x^2+3} \, \mathrm{d}x$

Homework 1

Due: Friday, Feb 4, by the end of the class

Instructions

- Write down the function u and v which you are using for substitution rule or integration by parts clearly in each problem.
- Note that for indefinite integrals, you need add constants to your final answers.

1. Integration by parts (You may also need to use substitution rule.)
   - $\displaystyle\int \frac{\ln x}{x^2}\,\mathrm{d}x$
   - $\displaystyle\int x^2 \sin x\,\mathrm{d}x$
   - $\displaystyle\int (\ln x)^2\,\mathrm{d}x$
   - $\displaystyle\int \arccos x\,\mathrm{d}x$
   - $\displaystyle\int e^{\sqrt{x}}\,\mathrm{d}x$

2. Trigonometric integration: Evaluate the following integral of the form $\displaystyle\int \sin^n x \cos^m x\,\mathrm{d}x$.
   - $\displaystyle\int \sin^2 x \cos^3 x\,\mathrm{d}x$
   - $\displaystyle\int \cos^4 x\,\mathrm{d}x$

3. Trigonometric substitution
   - $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,\mathrm{d}x$
   - $\displaystyle\int \frac{1}{\sqrt{25+x^2}}\,\mathrm{d}x$
   - $\displaystyle\int \frac{1}{\sqrt{x^2+2x}}\,\mathrm{d}x$
   - $\displaystyle\int (x-2)^3\sqrt{5+4x-x^2}\,\mathrm{d}x$

Homework 2

Due: Friday, Feb 11, by the end of the class

Note that for indefinite integrals, you need add constants to your final answers.

1. Partial Fractions
   - $\displaystyle\int \frac{2x+5}{x^2+4x+8}\,\mathrm{d}x$
   - $\displaystyle\int \frac{2x^2-x+4}{(x^2+4)(x-1)}\,\mathrm{d}x$
   - $\displaystyle\int \frac{x}{x^4+2x^2+2}\,\mathrm{d}x$
   - $\displaystyle\int \ln(x^2+1)\,\mathrm{d}x$
   - $\displaystyle\int \frac{1}{\sqrt{x}+x\sqrt{x}}\,\mathrm{d}x$
   - $\displaystyle\int \frac{1}{x+\sqrt[3]{x}}\,\mathrm{d}x$

2. Approximate Integration
   - Use the Midpoint Rule with $n=5$ to approximate $\displaystyle\int_0^{10} x^2\,\mathrm{d}x$.
   - Use the Trapezoidal Rule with $n=6$ to approximate $\displaystyle\int_0^{\pi} \sin^2 x\,\mathrm{d}x$.

3. Improper Integrals: compute the following integrals or show that it diverges.
   - $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}}\,\mathrm{d}x$
   - $\displaystyle\int_1^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$
   - $\displaystyle\int_{\pi}^{\infty} \sin x\,\mathrm{d}x$
   - $\displaystyle\int_e^{\infty} \frac{1}{x\ln x}\,\mathrm{d}x$
   - $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\,\mathrm{d}x$

Homework 3

Due: Friday, Feb 25, by the end of the class

Instructions

Step 1. Write down what $\mathrm{d}s$ is before setting up an integral

Step 2. Substitute $\mathrm{d}s$ and simplify the integral as much as you can

Step 3. Compute the integral (substitution rule, integration by parts, etc.)

1. Arclength: for the following curves write down (do not evaluate) an integral w.r.t. x representing the length. Then write down an integral w.r.t. y.
   - $y = x^3$ for $x \in [1, 2]$.
   - $y = e^x$ for $x \in [0, 2]$.
2. Arclength: compute determine the arclength of the following curves
   - $y = \dfrac{x^3}{6} + \dfrac{1}{2x} =$ for $x \in [1, 3]$.
   - $y = \cosh x$ for $x \in [0, \ln 2]$.

     The hyperbolic cosine function $\cosh x$ is given by $\cosh x = \dfrac{e^x + e^{-x}}{2}$.
   - $y = \ln(\cos x)$ for $x \in \left[0, \dfrac{\pi}{3}\right]$.
3. Area of a Surface of Revolution: determine the area of the surface obtained by rotating the curve
   - $y = \sqrt{9 - x^2}$ for $x \in [-2, 2]$, rotating about the x-axis.
   - $y = x^2$ for $x \in [1, 2]$, rotating about the y-axis.
   - $y = \dfrac{(x^2 + 2)^{3/2}}{3}$ for $x \in [1, 2]$, rotating about the y-axis.

     Hint: For this question, it'd be easier if you treat $\mathrm{d}s$ as $\sqrt{1 + (y')^2}\,\mathrm{d}x$.

Homework 4

Due: Friday, Mar 4, by the end of the class

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

   You might want to review Chapter 2 in the book for computing limits.

   - $a_n = \dfrac{3+5n^2}{n+n^2}$
   - $a_n = \dfrac{2n^4 - 11n + 5}{4n-1}$
   - $a_n = \dfrac{n^2-2n-1}{n^3+3}$
   - $a_n = \left(1 + \dfrac{2}{n}\right)^n$

2. Computing Series

   - $\displaystyle\sum_{n=0}^{\infty} 9^{-\frac{n}{2}} 2^{1+n}$
   - $\displaystyle\sum_{n=5}^{\infty} \frac{3}{n^2-7n+12}$ Hint: recall $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

3. The Divergence Test: prove the following series diverges.

   - $\displaystyle\sum_{n=2}^{\infty} \cos\left(\frac{n\pi}{2}\right)$
   - $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4+e^{-n}}$
   - $\displaystyle\sum_{n=0}^{\infty} \frac{e^n}{n^3+n}$

4. The Integral Test: determine if the following series converges or diverges.

   - $\displaystyle\sum_{n=1}^{\infty} \frac{n^4}{e^n}$
   - $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+1}$
   - $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ for $p > 1$ and for $p \leq 1$ respectively.

Homework 5

Due: Friday, Apr 1, by the end of the class

1. Comparison Test for Sequence

   This question asks you to use the comparison test for a pedagogical reason. There are other ways to solve Q1, for example, using the limit comparison test.

   - $\displaystyle\sum_{n=1}^{\infty} \frac{3n-2}{2n^3+5}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^n+n^2}$

2. The Limit Comparison Test

   - $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-n+5}{n^3-3n+6}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$

3. Alternating Series: determine if the following series converges absolutely, conditionally or diverges.

   For this question, you may use whichever test you like. If a series converges, specify if it converges absolutely, conditionally, or both.

   - $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n n^2}{n!}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$

   - $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(\ln n)^2}{n}$

Homework 6

Due: Friday, Apr 8, by the end of the class

Warning

1. When apply convergence tests such as alternating series test for the boundary cases, make sure you have check **all** the assumptions before apply the test.
2. Simplify your final answer until it is a **single** sum of the form $\sum c_n(x-a)^n$.
3. Don't forget the range of x.
4. Remember to compute the constant C for the term-by-term integration.

1. Power Series: determine the radius of convergence R and interval of convergence I.
   - $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{(x-3)^n}{n\cdot 5^n}$
   - $\displaystyle\sum_{n=1}^{\infty}\frac{(x+2)^n}{2^n\ln n}$
   - $\displaystyle\sum_{n=1}^{\infty}\frac{n^2x^{2n}}{(2n)!}$

2. Functions as Power Series: find the power series representations using substitution, term-by-term integration and differentiation.
   - $\arctan x$ Hint: $\arctan x = \displaystyle\int \frac{1}{1+x^2}\,\mathrm{d}x$.
   - $\ln\left(\dfrac{1+x}{1-x}\right)$
   - $\dfrac{7x-x}{3x^2+2x-1}$ Hint: Decompose the rational function first.

3. Taylor and Maclaurin Series: find the power series representation for by computing the n-th derivative $f^{(n)}(0)$.
   - $f(x)=\sin(x)$ centered at $\dfrac{\pi}{2}$.
   - $f(x)=(1+x)^k$ centered at 0. Note that tshis proves the binomial theorem.

4. Applications:
   - Using the Maclaurin series for $f(x)=e^x$ and the alternating series estimation theorem in 11.5 to approximate $\displaystyle\int_0^1 e^{-x^2}\,\mathrm{d}x$ with error $R<0.04$.