Spring 2021 MATH231 Section CDQ Discussion

WF 9-10am

This document can be found on here. I will continue update it until the end of semester.

Contact

- TA for section CDQ: Xinran Yu
- Email: xinran4@illinois.edu. Please included MATH231 in your email subject.
- Office hour: Wed 10-11am[1]

Zoom

- Please use your cameras and microphones in breakout rooms.
- Interrupt me/using the "Raise Hand" feature on Zoom to ask questions.
- You can call me into your breakout room/return to the main room to ask for help.
- It is also possible for me to join your breakout rooms randomly to check if you have any questions.

Worksheet

- Worksheet can be found on Moodle under Groupwork folder.
- Ask for hints when you get stuck on a problem.

Submission

- Submit on Moodle under Groupwork folder.
- **1 submission per group**. Once a file is uploaded, everyone in the same group will be able to see/edit the file. [2]
- Group remains the same until each midterm.
- 1st worksheet of the week is due on **Thursday** at **8AM** CST. [3]
- 2nd worksheet of the week is due on **Saturday** at **8AM** CST.
- Worksheet solutions available at 12:30PM CST on the due date.

Grading

Worksheets are graded with 2, 1 or 0.

2 - the worksheet uploaded is satisfactory

1 - the worksheet uploaded is unsatisfactory and needs improvement. Your TA will comment on what should be improved for next time.

0 - the worksheet was not uploaded

[1] Office hour is run for all students in MATH231
[2] Groups are assigned randomly by Moodle
[3] Central Standard Time

Contents

Worksheet 1

Recall

Theorem 1.1 (Fundamental Theorem of Calculus)**.** *Ref p.26*

Part 1 *If $f(x)$ is **continuous** over an interval $[a, b]$, and the function $F(x)$ is defined by*

$$F(x) = \int_a^x f(t)\,\mathrm{d}t, \quad x \in [a, b]$$

then $F'(x) = f(x)$ over $[a, b]$.

Part 2 *If $f(x)$ is **continuous** over an interval $[a, b]$, and $F(x)$ is any antiderivative of $f(x)$ i.e. $F'(x) = f(x)$, then*

$$\int_a^b f(x)\,\mathrm{d}x = F(a) - F(b).$$

Example 1.2. Let

$$g(x) = \int_a^{b(x)} f(t)\,\mathrm{d}t$$

Apply chain rule and FTC

$$g'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\int_c^{b(x)} f(t)\,\mathrm{d}t = b'(x) \cdot f(b(x)).$$

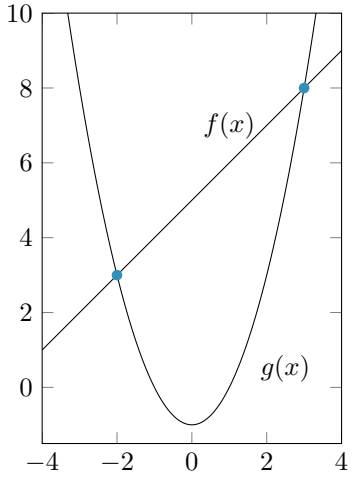
Worksheet 2

Recall

- **Substitution rule/Change of variable**: let $u = g(x)$, then

$$\int f(g(x)) \cdot g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u. \tag{Q1-3}$$

- **Compute area between curves.** (Q4-7)
  - Draw the graph.
  - Find intersection points by solving $f(x) = g(x)$, say they are $x = a$ and $x = b$.
  - Area $= \int_a^b f(x) - g(x)\,\mathrm{d}x$.



Worksheet 3

Volume of a Solid of Revolution: Slices of volume are circles. Ref

$$\text{Vol} = \int_a^b \pi f(x)^2 \, \mathrm{d}x.$$

Q3. Slices are squares/triangles.

$$\text{Vol} = \int_a^b \text{ Area of slices } \mathrm{d}x.$$

E.g.

$$\text{Vol} = \int_a^b f(x)^2 \, \mathrm{d}x.$$

Worksheet 4

Volume by Cylindrical Shells:

$$\text{Vol} = \int_a^b 2\pi r \cdot f(x)\,\mathrm{d}x.$$

Rotation about y-axis: $r = x$.
Rotation about the vertical line $x = a$: $r = |a - x|$.

Worksheet 5

Recall

- Since $\sin x$ is oscillating between -1 and 1, $\lim_{x\to\infty} \sin x$ does not exists.
- we can use L'Hopital's Rule to compute indeterminate forms "$\frac{0}{0}$" and "$\frac{\infty}{\infty}$".

Theorem 5.1 (L'Hopital's Rule)**.**
Assumptions:

$$\begin{aligned} &\boldsymbol{f(x) \to 0} \quad \text{as } x \to a, \\ &\boldsymbol{g(x) \to 0}, \\ &\boldsymbol{g'(x) \neq 0}. \end{aligned}$$

Conclusion:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

Warning: check the assumptions before applying L'Hopital's Rule.