

Homework Assignments

MATH231

Spring 2022

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Homework 0

Due: Friday, Jan 21 (no need to turn in)

1. Calculating Limits

- $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$
- $\lim_{h \rightarrow 0} \frac{(x + h)^2 - x^2}{h}$
- $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$

2. The Chain Rule

- $\frac{d}{dx} \ln(x + \sin x)$
- $\frac{d}{dx} \cos(x^2 e^x)$

3. Implicit Differentiation: Solve for $\frac{dy}{dx}$ for the following implicit function.

- $x^2 + y^2 = r^2$, where r is a constant
- $\frac{x + y}{x - y} = x$

4. Linear Approximations and Differentials: Find the Taylor polynomials of degree two approximating the given function centered at the given point.

- $f(x) = \sin(2x)$ at $a = \frac{\pi}{2}$
- $f(x) = e^x$ at $a = 1$

5. Mean Value Theorem: Determine if the Mean Value Theorem can be applied to the following function on the the given closed interval.

- $f(x) = 3 + \sqrt{x}, x \in [0, 4]$
- $f(x) = \frac{x}{1 + x}, x \in [1, 3]$

6. L'Hospital's Rule

- $\lim_{x \rightarrow 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$

- $\lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}}$

- $\lim_{x \rightarrow \infty} x \ln \left(1 + \frac{3}{x} \right)$

7. The Fundamental Theorem of Calculus: Find the derivative of the following

- $\int_1^x \frac{1}{t^3 + 1} dt$

- $\int_1^{\sqrt{x}} \sin t dt$

- $\int_x^{2x} t^3 dt$

8. Substitution Rule

- $\int_{\frac{1}{2}}^0 \frac{x}{\sqrt{1 - 4x^2}} dx$

- $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\cos(\pi x)}{\sin^2(\pi x)} dx$

- $\int_0^1 x e^{4x^2+3} dx$

Homework 1

Due: Friday, Feb 4, by the end of the class

Instructions

- Write down the function u and v which you are using for substitution rule or integration by parts clearly in each problem.
- Note that for indefinite integrals, you need add constants to your final answers.

1. Integration by parts (You may also need to use substitution rule.)

- $\int \frac{\ln x}{x^2} dx$
- $\int x^2 \sin x dx$
- $\int (\ln x)^2 dx$
- $\int \arccos x dx$
- $\int e^{\sqrt{x}} dx$

2. Trigonometric integration: Evaluate the following integral of the form $\int \sin^n x \cos^m x dx$.

- $\int \sin^2 x \cos^3 x dx$
- $\int \cos^4 x dx$

3. Trigonometric substitution

- $\int \frac{x^2}{\sqrt{9-x^2}} dx$
- $\int \frac{1}{\sqrt{25+x^2}} dx$
- $\int \frac{1}{\sqrt{x^2+2x}} dx$
- $\int (x-2)^3 \sqrt{5+4x-x^2} dx$

Homework 2

Due: Friday, Feb 11, by the end of the class

Note that for indefinite integrals, you need add constants to your final answers.

1. Partial Fractions

- $\int \frac{2x + 5}{x^2 + 4x + 8} \, dx$
- $\int \frac{2x^2 - x + 4}{(x^2 + 4)(x - 1)} \, dx$
- $\int \frac{x}{x^4 + 2x^2 + 2} \, dx$
- $\int \ln(x^2 + 1) \, dx$
- $\int \frac{1}{\sqrt{x} + x\sqrt{x}} \, dx$
- $\int \frac{1}{x + \sqrt[3]{x}} \, dx$

2. Approximate Integration

- Use the Midpoint Rule with $n = 5$ to approximate $\int_0^{10} x^2 \, dx$.
- Use the Trapezoidal Rule with $n = 6$ to approximate $\int_0^\pi \sin^2 x \, dx$.

3. Improper Integrals: compute the following integrals or show that it diverges.

- $\int_1^\infty \frac{1}{\sqrt{x}} \, dx$
- $\int_1^\infty \frac{1}{1 + x^2} \, dx$
- $\int_\pi^\infty \sin x \, dx$
- $\int_e^\infty \frac{1}{x \ln x} \, dx$
- $\int_{-\infty}^\infty x e^{-x^2} \, dx$

Homework 3

Due: Friday, Feb 25, by the end of the class

Instructions

Step 1. Write down what ds is before setting up an integral

Step 2. Substitute ds and simplify the integral as much as you can

Step 3. Compute the integral (substitution rule, integration by parts, etc.)

1. Arclength: for the following curves write down (do not evaluate) an integral w.r.t. x representing the length. Then write down an integral w.r.t. y .

- $y = x^3$ for $x \in [1, 2]$.

- $y = e^x$ for $x \in [0, 2]$.

2. Arclength: compute determine the arclength of the following curves

- $y = \frac{x^3}{6} + \frac{1}{2x}$ for $x \in [1, 3]$.

- $y = \cosh x$ for $x \in [0, \ln 2]$.

The hyperbolic cosine function $\cosh x$ is given by $\cosh x = \frac{e^x + e^{-x}}{2}$.

- $y = \ln(\cos x)$ for $x \in \left[0, \frac{\pi}{3}\right]$.

3. Area of a Surface of Revolution: determine the area of the surface obtained by rotating the curve

- $y = \sqrt{9 - x^2}$ for $x \in [-2, 2]$, rotating about the x -axis.

- $y = x^2$ for $x \in [1, 2]$, rotating about the y -axis.

- $y = \frac{(x^2 + 2)^{3/2}}{3}$ for $x \in [1, 2]$, rotating about the y -axis.

Hint: For this question, it'd be easier if you treat ds as $\sqrt{1 + (y')^2} dx$.

Homework 4

Due: Friday, Mar 4, by the end of the class

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

You might want to review Chapter 2 in the book for computing limits.

- $a_n = \frac{3 + 5n^2}{n + n^2}$

- $a_n = \frac{2n^4 - 11n + 5}{4n - 1}$

- $a_n = \frac{n^2 - 2n - 1}{n^3 + 3}$

- $a_n = \left(1 + \frac{2}{n}\right)^n$

2. Computing Series

- $\sum_{n=0}^{\infty} 9^{-\frac{n}{2}} 2^{1+n}$

- $\sum_{n=5}^{\infty} \frac{3}{n^2 - 7n + 12}$

Hint: recall $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

3. The Divergence Test: prove the following series diverges.

- $\sum_{n=2}^{\infty} \cos\left(\frac{n\pi}{2}\right)$

- $\sum_{n=1}^{\infty} \frac{1}{4 + e^{-n}}$

- $\sum_{n=0}^{\infty} \frac{e^n}{n^3 + n}$

4. The Integral Test: determine if the following series converges or diverges.

- $\sum_{n=1}^{\infty} \frac{n^4}{e^n}$

- $\sum_{n=1}^{\infty} \frac{n}{n^3 + 1}$

- $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ for $p > 1$ and for $p \leq 1$ respectively.