Last time Calc I Review
This time Integration by parts

Motivation

using Calc I knowledge (FTC) we know the antiderivative of x is $\frac{1}{2}x^2+C$
$\cos x$ $\sin x$

It is natural to ask what is the antiderivative of $\ln x$?
$\tan x$?

To solve this problem we need a new tool.

Recall product rule

$$(uv)' = u'v + uv'$$

if we integrate on both sides w.r.t. x.

$$\Rightarrow \int uv \, dx = \int uv' dx + \int u'v \, dx$$
$$= \int u \, dv + \int v \, du$$

rearrange

$$\Rightarrow \boxed{\int u \, dv = uv - \int v \, du}$$

Example 1 compute $\int \ln x \, dx$

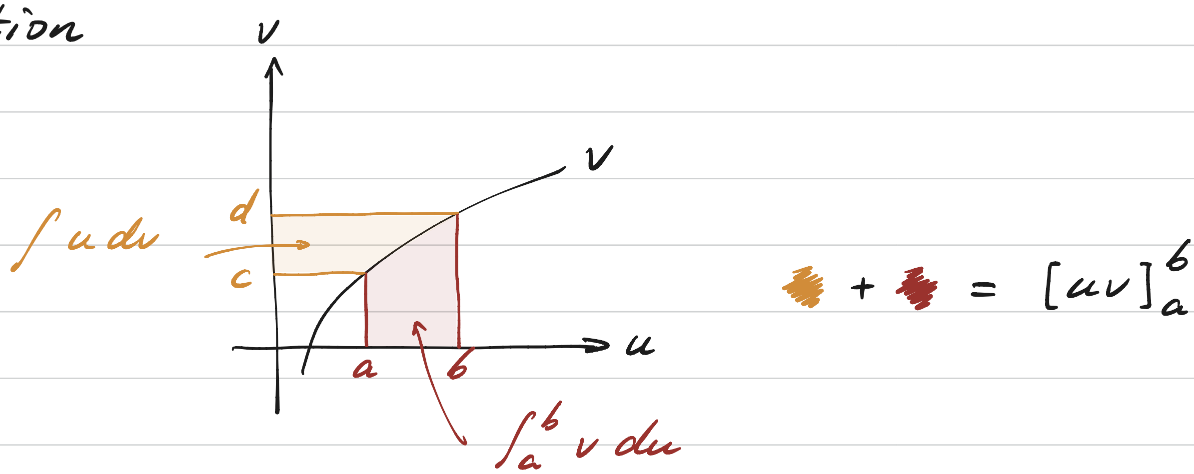
take $u = \ln x \qquad v = x$

$$\int \ln x \, dx = x \ln x - \int x \, d\ln x$$

$$= x \ln x - \int \underbrace{x \tfrac{1}{x}}_{=1} \, dx$$

$$= x \ln x - x + C$$

Intuition



NB. There's a list of choices called LIATE rule tells which function to choose as u first. However in general there's no easy way to tell immediately which function to take as u or v.

L	ln log
I	arcsin arccos arctan ...
A	algebraic x. $1+x^2$
T	sin cos ...
E	e^x

Example 2 $\int \underbrace{\arctan x}_{u}\, d\underbrace{x}_{v}$

$$= x \arctan x - \int x \, d \arctan x$$

$$= x \arctan x - \underbrace{\int \frac{x}{1+x^2} \, dx}_{\text{use substitution law}}$$

$$= \frac{1}{2} \int \frac{d(x^2+1)}{x^2+1} = \frac{1}{2} \ln(1+x^2) + C$$

$$= x \arctan x - \frac{1}{2} \ln(1+x^2) + C$$

Example 3 $\int x e^x \, dx = \int \underbrace{x}_{u}\, d\underbrace{e^x}_{v}$

$$= x e^x - \int e^x \, dx$$

$$= x e^x - e^x + C$$

Step

1. Choose functon u.
2. The choice of v depends on u.

   That is, in order to match the integration by parts formula, say we are computing $\int f(x)\, dx$

   Then we claim $dv = \frac{f(x)}{u(x)} dx$, so that

$$\int f \, dx = \int u \cdot \frac{f}{u} \, dx = \int u \, dv$$