# Documenting Historical Mathematical Models

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## 1. Project Goals

- Examine a selection of the University of Illinois' 400+ historical mathematical models.
- Document and trace the origins of previously undocumented models.
- Develop visual materials for a display in Altgeld Hall and the Library's Friends event on November 15, 2024.
- Provide mathematical descriptions of the models for the University Library's Digital Library Collection.

## 2. History

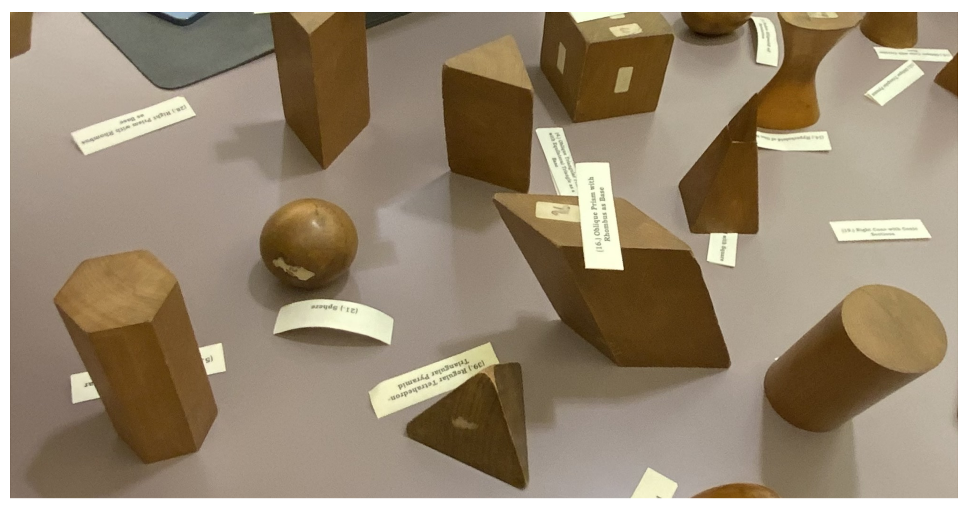
The University of Illinois owns one of the largest collections of historical mathematical models worldwide, comprising approximately 400 pieces from the late 19th and early 20th centuries. These include models purchased from Germany and those created by the faculty and students at the Department of Mathematics.

Edgar Townsend, the second chair of the department (1905-1928) and dean of the College of Science (1905-1913), played a pivotal role in acquiring many of these models, which feature works by renowned German mathematicians Alexander Brill, Martin Schilling, and Felix Klein. Arnold Emch, hired by Townsend, later enriched the collection with unique contributions [3]. After the ongoing renovations, the models will be re-displayed in Altgeld Hall, preserving their historical and educational value. A selection of these models has been digitized and made available online [2].

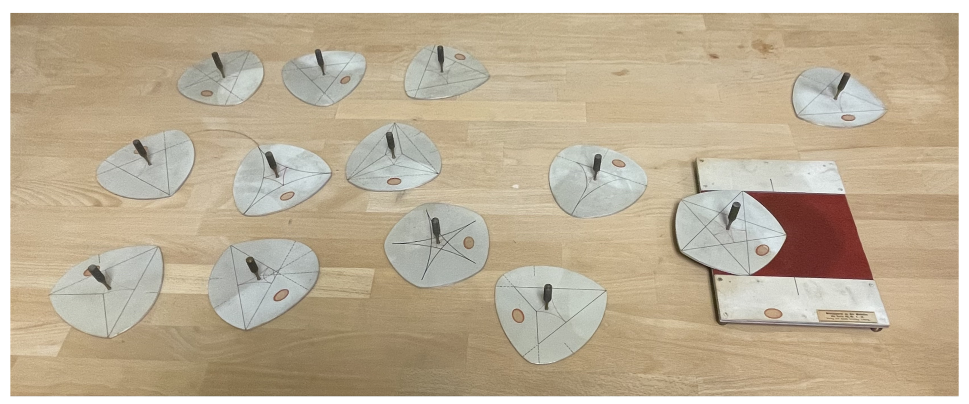
## 3. Overview of the Project

This project highlights two historic collections previously displayed in Altgeld Hall

- **Wood Models (Quadric Surfaces and Polyhedral Surfaces)**: A set of 39 wooden models illustrating geometric surfaces such as ellipsoids, hyperboloids, and conic sections.



- **Schilling Series 40 (Curves of Constant Width):** A collection of 19 models, comprising 16 curves of constant width, 3 surfaces of constant width, and measurement tools that demonstrate the intriguing properties of constant-width geometries.



## 4. Curve of Constant Width of an Isosceles Triangle (Model 14)

### Mathematical Description

This **curve of constant width** (the blue curves) is based on an isosceles triangle (the black solid lines) with equal legs longer than the bent base, creating a symmetrical form.

The outer blue curve consists of 7 smooth circular arcs formed by sectors of varying radii originating from the triangle's vertices, either inside or outside the triangle.

The **middle width curve** (the red curves) comprises 3 intersecting arcs: 2 congruent lower arcs and a wider upper arc, all originating from the triangle's vertices. On this curve lie all the midpoints of the the diameters of this shape.

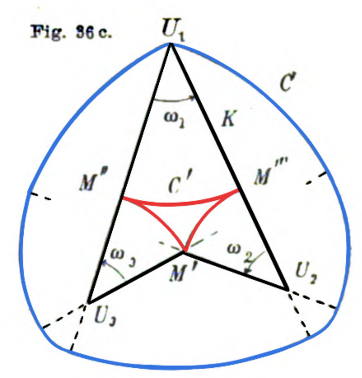


Figure 3: Curves of constant width of an isosceles triangle [5].

The radii are chosen to ensure the constant-width property, where the distance between parallel tangents is uniform. The result is a closed, symmetric, smooth shape derived geometrically from the triangle.

## 5. Curve of Constant Width of a Special Equiangular Star (Model 18)

### Description of the Figure

- The **basis** (black solid lines) for this curve of constant width consists of a regular star with two of its intersecting edges extended to form the legs of an isosceles triangle, and with the triangle closed by another 12 cm edge.
- To construct the curve of constant width we first extend the solid legs of the regular star to 12 cm (dashed lines.)
- The **curve of constant width** is then made up of eight circular arcs, with centers at the vertices of the black lines and radii chosen appropriately.
- The resulting curve has reflectional symmetry and constant width.

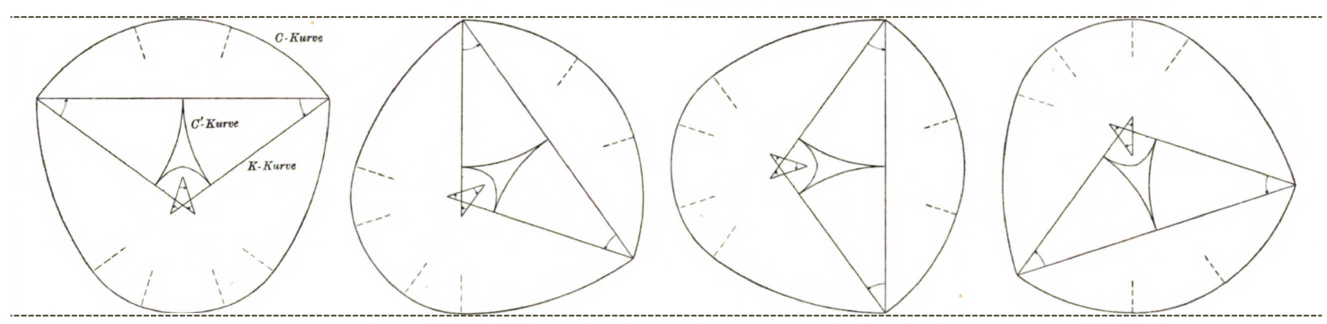


Figure 5: Curve shown at various rotations [5]

### Properties of Curves of Constant Width

An important property of curves of constant width is that the distance between two parallel support lines must be constant no matter how you rotate the curve. Shown above are two such support lines enclosing this model at various rotations.

## 6. Ellipsoid (Wood Model 9)

### Mathematical description

The mathematical representation of the prolate ellipsoid is given by the following equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (a > 0, b > 0, c > 0).$$

The parametric form of the prolate ellipsoid is as follows:

$$x = a \sin\theta \cos\phi, \quad y = b \sin\theta \sin\phi, \quad z = c \cos\theta.$$

### Geometric properties

- The prolate ellipsoid is characterized by its elongated shape.
- It has two equal shorter axes and one longer axis, creating a symmetry around the long axis.
- The vertical and horizontal cross sections of the ellipsoid are ellipses.

Figure 6: Wooden Model 9 kept in the library

## 7. Right Cone with Conic Section (Wood Model 19)

**Conic sections** are curves formed by the intersection of the surface of a right cone and a plane. Typically, they can be expressed by quadratic equations of the form:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

In the figure on the right:

- $\alpha$ represents the acute angle between the normal vector and the axis of the cone.
- $\beta$ represents the angle of the cone.

By cutting a right cone with a plane, we get:

- A circle, if $\alpha = 0°$.
- An ellipse, if $\alpha < \beta$.
- A parabola, if $\alpha = \beta$.
- A hyperbola, if $\alpha > \beta$.

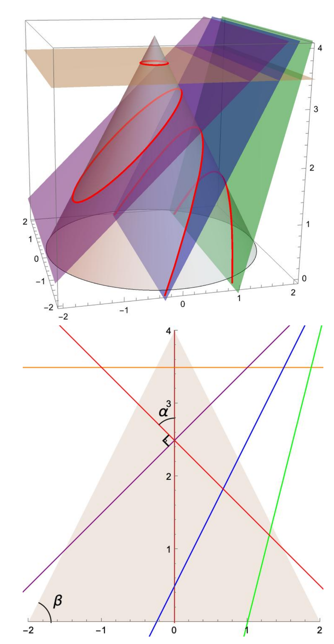


Figure 7: Conic sections, created with Mathematica

### References

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[2] U. of Illinois Urbana-Champaign. The Altgeld math models. https://mathmodels.illinois.edu/cgi-bin/cview?SITEID=4&ID=342, n.d. Accessed: 2024-11-20.

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[4] H. Rademacher and O. Toeplitz. Curves of constant breadth. In *Enjoyment of Mathematics: Selections from Mathematics for the Amateur*, chapter 29. Princeton University Press, 1994.

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[6] M. Schilling. *Verlag von Modellen für den Höheren Mathematischen Unterricht*. 1913.

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