

CSCI 570
NP-completeness
Discussion Problems

1. Show that vertex cover remains NP-Complete even if the instances are restricted to graphs with only even degree vertices. Let us call this problem VC-even. Prove: $VC \leq_p VC\text{-even}$.
2. You are given an undirected graph $G = (V, E)$ and for each vertex v , you are given a number $p(v)$ that denotes the number of pebbles placed on v . We will now play a game where the following move is the only move allowed. You can pick a vertex u that contains at least two pebbles, and remove two pebbles from u and add one pebble to an adjacent vertex. The objective of the game is to perform a sequence of moves such that we are left with exactly one pebble in the whole graph. Show that the problem of deciding if we can reach the objective is NP-complete.

3. Given SAT in Conjunctive Normal Form (CNF)

$$(X_1 \vee \neg X_3) \wedge (X_1 \vee \neg X_2 \vee X_4 \vee X_5) \wedge \dots$$

with any number of clauses and any number of literals in each clause. Prove that SAT is polynomial time reducible to 3SAT.

4. Assuming that finding a Hamiltonian Cycle (HC) in a graph is NP-complete, prove that finding a Hamiltonian Path is also NP-complete. HP is a path that visits each vertex exactly once and isn't required to return to its starting point.