Machine Learning

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Discussion Set 6.2 University of Southern California

Kernel Methods

Problem 1

Which of the following is true about kernel function?

a) If k_1 and k_2 are kernel, then $c_1 k_1 + c_2 k_2$ is a kernel too for any $c_1, c_2 \in R$.
b) Kernel function must be symmetric, that is, $k(x, x') = k(x', x)$.
c) If k is a kernel, then -k is a kernel too.
d) If k is a kernel, then ln k is a kernel too.

Problem 2

Prove that

$$K(x, z) = \varphi(x)^{\mathrm{T}} \varphi(z) = \|x - z\|^2$$

is not a valid kernel by the Mercer theorem (K is a kernel if and only if the Gram matrix is positive semidefinite)

Problem 3

What is the corresponding $\phi(x)$ for the polynomial kernel of 2-dimensional vectors:

$$K(x, z) = \left(x^T z + c\right)^d$$

where c=1 and d=2.

Problem 4

Consider a polynomial kernel defined as

$$K(x, z) = \left(x^T z\right)^{50}$$

where x and z are 4-dimensional vectors.

What is a dimension of a correspondent $\phi(x)$ vector?

Problem 5

What is the corresponding $\phi(x)$ for the Gaussian kernel of 2-dimensional vectors:

$$k(x, z) = e^{-\|x - z\|^2}$$

Problem 6

Kernels in Logistic Regression

(1) Extend it to mapping ϕ.

(2) Kernelize the algorithm by kernel k.