CSCI-567: Machine Learning (Fall 2019)

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U of Southern California

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Outline

1. Administration
2. Decision tree
3. Naive Bayes

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2. Decision tree
3. Naive Bayes

Administrative stuff

- PA1 is released
- Programming Office Hours are within the assignment writeup
- Theory assignment will be released by Friday
- Theory Office Hours are are the syllabus
- Use Piazza for short questions

Outline

1 Administration

2 Decision tree
- The model
- Learning a decision tree

3 Naive Bayes

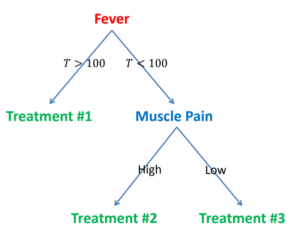
Decision tree

Decision tree is another ML model for classification:

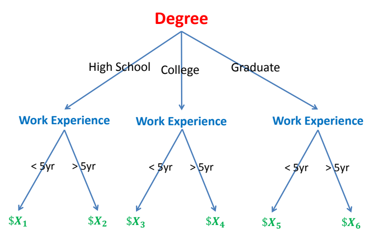
- the learned function is represented by a decision tree.
- also can be represented as sets of if-then rules.
- used to be very popular, successfully applied to a broad range of tasks.

Example

Medical treatment

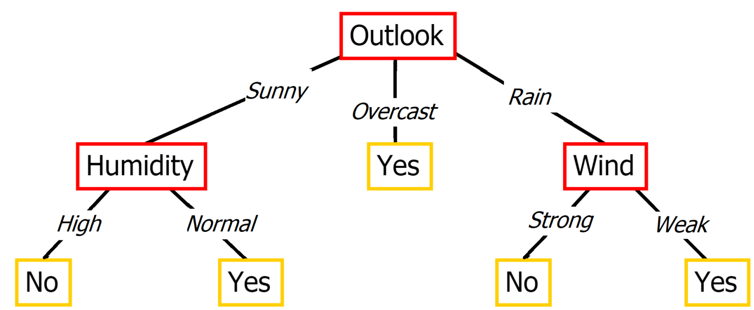


Salary in a company



- Each node in the tree specifies a test of some attribute
- Each branch from a node corresponds to one of the possible values for this attribute

Example: playing tennis



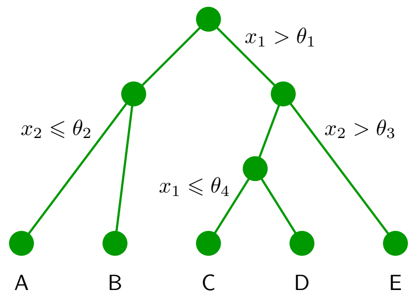
Decision Trees represent disjunctions of conjunctions.

A more abstract example of decision trees with five classes

Input: $\boldsymbol{x} = (x_1, x_2)$, assume just two features

Output: $f(\boldsymbol{x})$ determined naturally by traversing the tree

- start from the root
- test at each node to decide which child to visit next
- finally the leaf gives the prediction $f(\boldsymbol{x})$

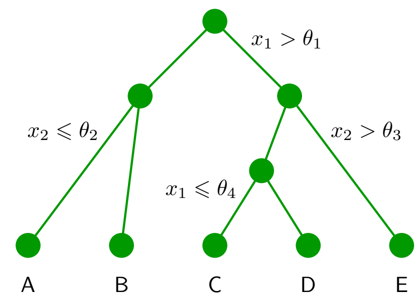


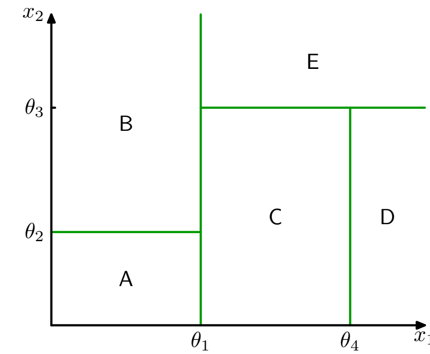
For example, $f((\theta_1 - 1, \theta_2 + 1)) = \text{B}$

Complex to formally write it down, but easy to represent pictorially or to code.

The decision boundary

Corresponds to a classifier with boundaries:

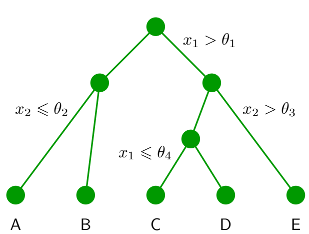




Parameters

Parameters to learn for a decision tree:

- the structure of the tree, such as the depth, #branches, #nodes, etc
  - the structure of a tree is *not fixed in advance, but learned from data*
  - some of them are sometimes considered as hyperparameters
- the test at each internal node
  - which feature(s) to test on?
  - categorical vs. continuous attributes.
  - if the feature is continuous, what threshold $(\theta_1, \theta_2, \ldots)$?



- the value/prediction of the leaves (A, B, . . .)

Learning the parameters

Given a set of examples (training set), both positive and negative, the task is to construct a decision tree.

Using the resulting decision tree, we want to classify new instances of examples (either as *yes* or *no*).

The typical approach is to find the parameters that minimize some loss.

This is unfortunately *not feasible for trees*

- suppose there are Z nodes, there are roughly #features^Z different ways to decide
- enumerating all these configurations to find the one that minimizes some loss is too computationally expensive.

Instead, we turn to some **greedy top-down approach**.

A running example

[Russell & Norvig, AIMA]

- 12 examples
- predict whether a customer will wait for a table at a restaurant
- 10 features (all discrete)

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*T*	*F*	*F*	*T*	*Some*	*$$$*	*F*	*T*	*French*	*0–10*	*T*
X_2	*T*	*F*	*F*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*30–60*	*F*
X_3	*F*	*T*	*F*	*F*	*Some*	*$*	*F*	*F*	*Burger*	*0–10*	*T*
X_4	*T*	*F*	*T*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*10–30*	*T*
X_5	*T*	*F*	*T*	*F*	*Full*	*$$$*	*F*	*T*	*French*	*>60*	*F*
X_6	*F*	*T*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Italian*	*0–10*	*T*
X_7	*F*	*T*	*F*	*F*	*None*	*$*	*T*	*F*	*Burger*	*0–10*	*F*
X_8	*F*	*F*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Thai*	*0–10*	*T*
X_9	*F*	*T*	*T*	*F*	*Full*	*$*	*T*	*F*	*Burger*	*>60*	*F*
X_{10}	*T*	*T*	*T*	*T*	*Full*	*$$$*	*F*	*T*	*Italian*	*10–30*	*F*
X_{11}	*F*	*F*	*F*	*F*	*None*	*$*	*F*	*F*	*Thai*	*0–10*	*F*
X_{12}	*T*	*T*	*T*	*T*	*Full*	*$*	*F*	*F*	*Burger*	*30–60*	*T*

List of attributes

- Alternate: whether there is a suitable alternative restaurant nearby.
- Bar. whether the restaurant has a comfortable bar area to wait in.
- Fri/Sat: true on Fridays and Saturdays.
- Hungry: whether we are hungry.
- Patrons: how many people are in the restaurant (values are None, Some, and Full).
- Price: the restaurant's price range ($, $$, $$$).
- Raining: whether it is raining outside.
- Reservation: whether we made a reservation.
- Type: the kind of restaurant (French, Italian, Thai, or Burger).
- WaitEstimate: the wait estimated by the host (0-10 minutes, and so).

First step: how to build the root?

I.e., which feature should we test at the root? Examples:

Which split is better?

- intuitively "patrons" is a better feature since it leads to "more pure" or "more certain" children
- How to quantitatively measure which one is better?

Choosing the Best Attribute

Use Shannon's information theory to choose the attribute that gives the smallest *entropy* that is defined by:

$$H(P) = -\sum_{k=1}^{\mathsf{C}} P(Y=k) \log P(Y=k)$$

- Entropy is always non-negative
- Entropy is a measure of impurity (disorder).
- maximized if P is uniform ($H(P) = \log \mathsf{C}$): most uncertain case

$$H(P) = -\sum_{k=1}^{\mathsf{C}} \frac{1}{\mathsf{C}} \log \frac{1}{\mathsf{C}} = \mathsf{C} \times \frac{1}{\mathsf{C}} \log \mathsf{C} = \log \mathsf{C}$$

- minimized if P focuses on one class ($H(P) = 0$): most certain case
  - $0 \log 0$ is defined naturally as $\lim_{z \to 0+} z \log z = 0$

Examples of entropy

H = 2bits *H = 1.0bits* *H = 0.3bits* *H = 0.0bits*

$$H(P) = -\frac{1}{16}\log\frac{1}{16} - \frac{15}{16}\log\frac{15}{16} = 0.34$$

Our greed (in the greedy approach) is to minimize entropy.

Restaurant example

Entropy of each child if root tests on "patrons"

For "None" branch

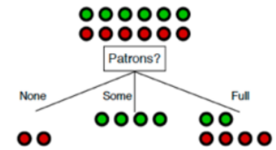
$$-\left(\frac{0}{0+2}\log\frac{0}{0+2} + \frac{2}{0+2}\log\frac{2}{0+2}\right) = 0$$

For "Some" branch

$$-\left(\frac{4}{4+0}\log\frac{4}{4+0} + \frac{0}{4+0}\log\frac{0}{4+0}\right) = 0$$

For "Full" branch

$$-\left(\frac{2}{2+4}\log\frac{2}{2+4} + \frac{4}{2+4}\log\frac{4}{2+4}\right) \approx 0.9$$



So how good is choosing "patrons" overall?

Very naturally, we take the weighted average of entropy:

$$\frac{2}{12}\times 0 + \frac{4}{12}\times 0 + \frac{6}{12}\times 0.9 = 0.45$$

Measure of uncertainty of a split

Suppose we split based on a discrete feature A, the uncertainty can be measured by the **conditional entropy** $H(Y \mid A = v)$.

The entropy of Y among only those records in which A has value v:

$$H(Y \mid A) = \sum_v P(A = v) H(Y \mid A = v)$$

A	Y
Some	*T*
Full	*F*
Some	*T*
Full	*T*
Full	*F*
Some	*T*
None	*F*
Some	*T*
Full	*F*
Full	*F*
None	*F*
Full	*T*

- Y = will wait.
- $H(Y \mid A = None) = 0$
- $H(Y \mid A = Some) = 0$
- $H(Y \mid A = Full) = 0.9$
- $H(Y \mid A) = \frac{2}{12}\times 0 + \frac{4}{12}\times 0 + \frac{6}{12}\times 0.9 = 0.45$

Restaurant example

Entropy of each child if root tests on "type"

For "French" branch

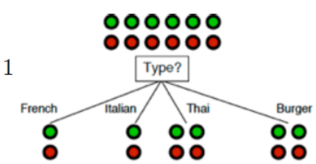
$$-\left(\frac{1}{1+1}\log\frac{1}{1+1} + \frac{1}{1+1}\log\frac{1}{1+1}\right) = 1$$

For "Italian" branch

$$-\left(\frac{1}{1+1}\log\frac{1}{1+1} + \frac{1}{1+1}\log\frac{1}{1+1}\right) = 1$$

For "Thai" and "Burger" branches

$$-\left(\frac{2}{2+2}\log\frac{2}{2+2} + \frac{2}{2+2}\log\frac{2}{2+2}\right) = 1$$



The conditional entropy is $\frac{2}{12}\times 1 + \frac{2}{12}\times 1 + \frac{4}{12}\times 1 + \frac{4}{12}\times 1 = 1 > 0.45$

Pick the feature that leads to the smallest conditional entropy.

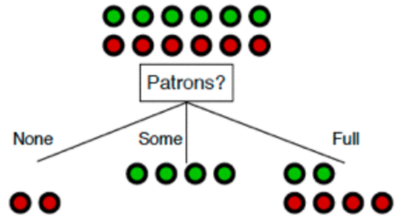
So splitting with "patrons" is better than splitting with "type".

We are now done with building the root (this is also called a **stump**).

Repeat recursively

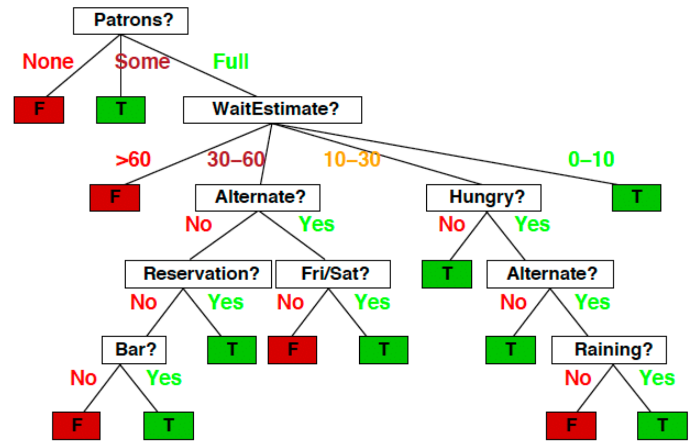
Split each child in the same way.

- but no need to split children "none" and "some": they are pure already and become leaves
- for "full", repeat, focusing on those 6 examples:



	Alt	Bar	Fri	Hun	Pat	Price	Rain	Res	Type	Est	WillWait
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

Greedily we build the tree and get this



Again, very easy to interpret.

Putting it together: the ID3 algorithm

DecisionTreeLearning(Examples, Features)

- if Examples have the same class, return a leaf with this class
- else if Features is empty, return a leaf with the majority class
- else if Examples is empty, return a leaf with majority class of parent
- else

  find the best feature A to split (e.g. based on conditional entropy)

  Tree $\leftarrow$ a root with test on A

  For each value a of A:

  Child $\leftarrow$ **DecisionTreeLearning**(Examples with $A = a$, Features$-\{A\}$)
  add **Child** to **Tree** as a new branch
- return Tree

Information Gain

Entropy as a measure of the purity.
Now we define a measure of the **effectiveness** of an attribute

Information Gain of a node n with children Values(A) is

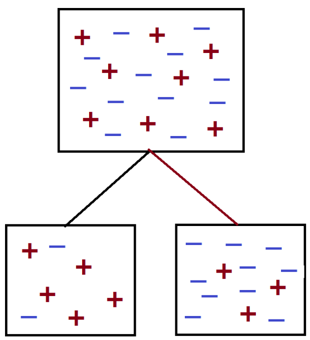
$$\text{Gain}(n, A) = \text{Entropy}(S_n) - \sum_{m \in \text{Values}(A)} \frac{|S_m|}{|S_n|} \text{Entropy}(S_m)$$

where S_n and S_m are the subsets of training examples that belong to the node n and one of its child node m respectively.

Information Gain = entropy(parent) – [average entropy(children)]

Gain(n, A) is the expected reduction in entropy caused by partitioning on the values of attribute A.

Calculating Information Gain



- Parent Entropy

$$-\frac{8}{20}\log\frac{8}{20} - \frac{12}{20}\log\frac{12}{20} = 0.97$$

- Left Child Entropy

$$-\frac{5}{7}\log\frac{5}{7} - \frac{2}{7}\log\frac{2}{7} = 0.86$$

- Right Child Entropy

$$-\frac{3}{13}\log\frac{3}{13} - \frac{10}{13}\log\frac{10}{13} = 0.78$$

Information Gain

$$0.97 - \frac{7}{20}\times 0.86 - \frac{13}{20}\times 0.78 = 0.97 - 0.81 = 0.16$$

Variants

Popular decision tree algorithms (e.g. C4.5, CART, etc) are all based on this framework.

Variants:

- replace entropy by **Gini impurity**:

$$G(P) = \sum_{k=1}^{C} P(Y=k)(1 - P(Y=k))$$

is used to provide an indication of how "pure" the leaf nodes are.

Overfitting

Some reasons for overfitting:

- Large number of attributes
- Too little training data
- Many kinds of "noise"(same feature but different classes),values of attributes are incorrect , classes are incorrect)

How can we avoid overfitting?

- Stop growing when you reach some depth or number of nodes
- Stop growing when data split is not statistically significant
- Acquire more training data
- Remove irrelevant attributes
- Grow a full tree, then **prune** it

Reduced-Error Pruning

Pruning is done by replacing a whole subtree by a leaf node and assigning the most common class to that node.

Split data into training and validation sets.

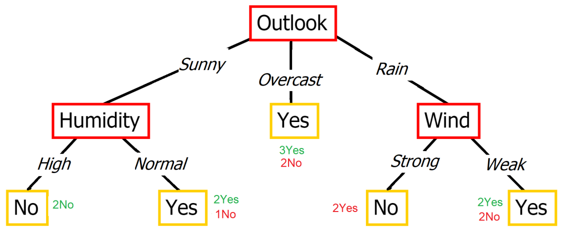
Grow a full tree based on training set.

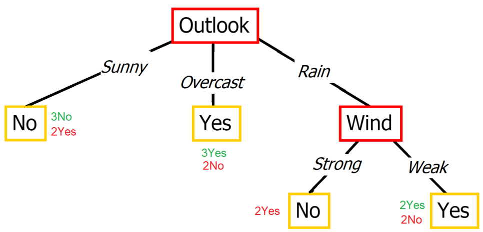
Do pruning until it is harmful:

- Evaluate impact on validation set of pruning each possible node.
- Greedily remove the node that most improves validation set accuracy.

Accuracy is the number of correct predictions made divided by the total number of predictions made.

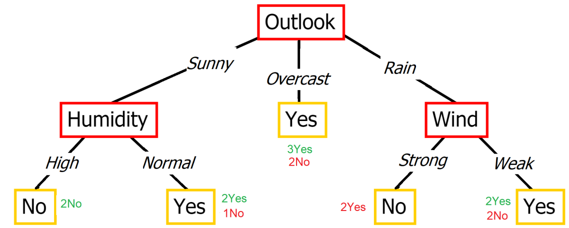
Reduced-Error Pruning: Example

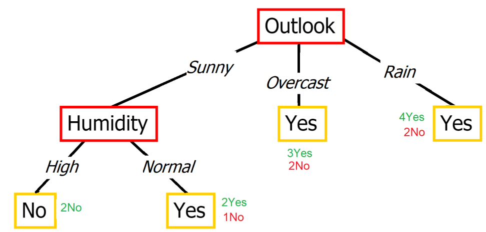




- Accuracy $= \frac{9}{16}$
- Let us remove Humidity.
- We will assign No, to this node, since there are three No examples.
- New accuracy $= \frac{8}{16}$
- Do not prune it!

Reduced-Error Pruning: Example





- Accuracy $= \frac{9}{16}$
- Let us remove Wind.
- We will assign Yes, to this node, since there are four Yes examples.
- New accuracy $= \frac{11}{16}$
- Prune it!

Outline

1. Administration
2. Decision tree
3. Naive Bayes
   - Motivating example
   - Naive Bayes: informal definition
   - Parameter estimation

Bayes optimal classifier

Suppose the data $(\boldsymbol{x}_n, y_n)$ is drawn from a joint distribution p, the **Bayes optimal classifier** is

$$f^*(\boldsymbol{x}) = \operatorname*{argmax}_{c \in [\mathsf{C}]} P(c \mid \boldsymbol{x})$$

i.e. predict the class with the largest conditional probability.

How hard is it to learn the optimal classifier?

Exponential, 2^D.

Bayes rule

Recall the **Bayes rule** is

$$P(Y \mid X) = \frac{P(X \mid Y)P(Y)}{P(X)}$$

How does it help us?

We can use a conditional independency.

A "naive" assumption

Naive Bayes assumption: **the x_d are conditionally independent given y**, which means

$$P(\boldsymbol{x} \mid y = c) = \prod_{d=1}^{\mathsf{D}} P(x_d \mid y = c)$$

This reduces complexity to linear.

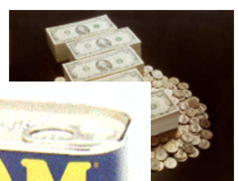
Is this a reasonable assumption? Sometimes yes.

More often this assumption is *unrealistic and "naive"*, but still Naive Bayes can work very well even if the assumption is wrong.

A daily battle

Great news: I will be rich!

FROM THE DESK OF MR. AMINU SALEH
DIRECTOR, FOREIGN OPERATIONS DEPARTMENT
AFRI BANK PLC
Afribank Plaza,
14th Floormoney344.jpg
51/55 Broad Street,
P.M.B 12021 Lagos-Nigeria



Attention: Honorable Beneficiary,

IMMEDIATE PAYMENT NOTIFICATION

It is my modest obligation to write you th ... owed payment through our most respected financial institution (AFRI BANK PLC). I a ... tions Department, AFRI Bank Plc, NIGERIA. The British Government, in conjunction w ... NITED NATIONS ORGANIZATION on foreign payment matters, has empowered ... tion, to handle all foreign payments and release them to their appropriate benefici ... leral Reserve Bank.

To facilitate the process of this transactior ... ıtion below:

1) Your full Name and Address:
2) Phones, Fax and Mobile No. :
3) Profession, Age and Marital Status:
4) Copy of any valid form of your Identification:

How to tell spam from ham?

FROM THE DESK OF MR. AMINU SALEH
DIRECTOR, FOREIGN OPERATIONS DEPARTMENT
AFRI BANK PLC
Afribank Plaza,
14th Floormoney344.jpg
51/55 Broad Street,
P.M.B 12021 Lagos-Nigeria

Attention: Honorable Beneficiary,

IMMEDIATE PAYMENT NOTIFICATION VALUED AT **US$10 MILLION**

Dear Dr.Sha,

I just would like to remind you of your scheduled presentation for CS597, Monday October 13, 12pm at OHE122.

If there is anything that you would need, please do not hesitate to contact me.

sincerely,

Christian Siagian

Intuition

How human solves the problem?

Spam emails

concentrated use of a lot of words like "money", "free", "bank account", "viagara"

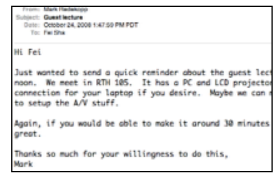
Ham emails

word usage pattern is more spread out

Simple strategy: count the words

Bag-of-word representation of documents (and textual data)

$$\begin{pmatrix} \text{free} & 100 \\ \text{money} & 2 \\ \vdots & \vdots \\ \text{account} & 2 \\ \vdots & \vdots \end{pmatrix}$$


From: Mark Redekopp
Subject: Guest lecture
Date: October 24, 2008 1:47:50 PM PDT
To: Fei Sha

Hi Fei

Just wanted to send a quick reminder about the guest lect
noon. We meet in RTH 105. It has a PC and LCD projector
connection for your laptop if you desire. Maybe we can
to setup the A/V stuff.

Again, if you would be able to make it around 30 minutes
great.

Thanks so much for your willingness to do this,
Mark

$$\begin{pmatrix} \text{free} & 1 \\ \text{money} & 1 \\ \vdots & \vdots \\ \text{account} & 2 \\ \vdots & \vdots \end{pmatrix}$$

Weighted sum of those telltale words

$$\begin{pmatrix} \text{free} & 100 \\ \text{money} & 2 \\ \vdots & \vdots \\ \text{account} & 2 \\ \vdots & \vdots \end{pmatrix}$$

$$\begin{pmatrix} 100 \times 0.2 \\ 2 \times 0.3 \\ \vdots \\ 2 \times 0.3 \\ \vdots \end{pmatrix} = 3.2$$

$$\begin{pmatrix} 100 \times 0.01 \\ 2 \times 0.02 \\ \vdots \\ 2 \times 0.01 \\ \vdots \end{pmatrix} = 1.03$$

different weights for spam and ham: representing how compatible the word usage pattern is to different category

Our intuitive model of classification

Assign weight to each word

Compute compatibility score to "spam"

of "free" x a_{free} + # of "account" x $a_{account}$ + # of "money" x a_{money}

Compute compatibility score to "ham":

of "free" x b_{free} + # of "account" x $b_{account}$ + # of "money" x b_{money}

Make a decision:

if spam score > ham score then spam

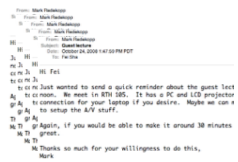
else ham

How we get the weights?

Learning from experience

get a lot of spams

get a lot of hams



But what to optimize?

Naive Bayes model for identifying spams

Class label: binary

y = {spam, ham}

Features: word counts in the document (Bag-of-word)

Ex: x = {('free', 100), ('lottery', 10), ('money', 10), , ('identification', 1)...}

Each pair is in the format of (w_i, $\#w_i$), namely, a unique word in the dictionary, and the number of times it shows up

Naive Bayes classification rule

For any document x, we need to compute

$$p(\text{spam}|x) \quad \text{and} \quad p(\text{ham}|x)$$

Using Bayes rule, this gives rise to

$$p(\text{spam}|x) = \frac{p(x|\text{spam})p(\text{spam})}{p(x)}, \quad p(\text{ham}|x) = \frac{p(x|\text{ham})p(\text{ham})}{p(x)}$$

It is convenient to compute the logarithms, so we need only to compare

$$\log[p(x|\text{spam})p(\text{spam})] \quad \text{versus} \quad \log[p(x|\text{ham})p(\text{ham})]$$

as the denominators are the same

Classifier in the linear form of compatibility scores

$$\log[p(x|\text{spam})p(\text{spam})] = \log\left[\prod_i p(w_i|\text{spam})^{\#w_i} p(\text{spam})\right]$$
$$= \sum_i \#w_i \log p(w_i|\text{spam}) + \log p(\text{spam})$$

Similarly, we have

$$\log[p(x|\text{ham})p(\text{ham})] = \sum_i \#w_i \log p(w_i|\text{ham}) + \log p(\text{ham})$$

Namely, we are back to the idea of comparing weighted sum of # of word occurrences!

$\log p(\text{spam})$ *and* $\log p(\text{ham})$ *are called "priors" or "bias" (they are not in our intuition but they are crucially needed)*

Formal definition of Naive Bayes

General case

Given random variables $X_1, \ldots, X_D$ and a variable $Y \in [\mathsf{C}]$, the Naive Bayes model defines the joint distribution

$$
\begin{aligned}
&P(X_1 = x_1, \ldots, X_D = x_D, Y = c) = \\
&P(Y = c) P(X_1 = x_1, \ldots, X_D = x_D \mid Y = c) = \\
&P(Y = c) \prod_{d=1}^{\mathsf{D}} P(X_d = x_d \mid X_1 = x_1, \ldots, X_{d-1} = x_{d-1}, Y = c) = \\
&P(Y = c) \prod_{d=1}^{\mathsf{D}} P(X_d = x_d \mid Y = c)
\end{aligned}
$$

The first equality is by the definition of a joint probability.

The second is by the chain rule (see the next slide).

The last one follows by a conditional independence (Naive Bayes assumption)

Chain rule example

Prove the following

$$P(x_1, x_2 \mid y) = P(x_1 \mid y) P(x_2 \mid x_1, y)$$

Proof. Convert each term to a joint probability

$$\frac{P(x_1, x_2, y)}{P(y)} = \frac{P(x_1, y)}{P(y)} \frac{P(x_2, x_1, y)}{P(x_1, y)}$$

Naive Bayes assumption

$$P(x_1, x_2 \mid y) = P(x_1 \mid y) P(x_2 \mid y)$$

How to predict?

The **prediction** for a new example x is

$$
\begin{aligned}
\operatorname*{argmax}_{c \in [\mathsf{C}]} P(y = c \mid \boldsymbol{x}) &= \operatorname*{argmax}_{c \in [\mathsf{C}]} \frac{P(\boldsymbol{x} \mid y = c) P(y = c)}{P(\boldsymbol{x})} \\
&= \operatorname*{argmax}_{c \in [\mathsf{C}]} \left(P(y = c) \prod_{d=1}^{\mathsf{D}} P(x_d \mid y = c) \right) \\
&= \operatorname*{argmax}_{c \in [\mathsf{C}]} \left(\ln P(y = c) + \sum_{d=1}^{\mathsf{D}} \ln P(x_d \mid y = c) \right)
\end{aligned}
$$

These two probability terms can be estimated from data.

For discrete features.

For a label $c \in [\mathsf{C}]$,

$$P(y = c) = \frac{|\{n : y_n = c\}|}{N} = \frac{\text{\#of data points labeled as c}}{\text{N}}$$

For each possible value k of a discrete feature d,

$$P(x_d = k \mid y = c) = \frac{|\{n : x_{nd} = k, y_n = c\}|}{|\{n : y_n = c\}|}$$

They can be estimated separately.

Translating back to our problem of detecting spam emails

- Collect a lot of ham and spam emails as training examples
- Estimate the "bias"

$$p(\text{ham}) = \frac{\#\text{of ham emails}}{\#\text{of emails}}, \quad p(\text{spam}) = \frac{\#\text{of spam emails}}{\#\text{of emails}}$$

- Estimate the weights (i.e., $p(\text{dollar}|\text{ham})$ etc)

$$p(\text{funny\_word}|\text{ham}) = \frac{\#\text{of funny\_word in ham emails}}{\#\text{of words in ham emails}}$$
$$p(\text{funny\_word}|\text{spam}) = \frac{\#\text{of funny\_word in spam emails}}{\#\text{of words in spam emails}}$$

- Compute argmax (slide 40).

Continuous features

If the feature is continuous, we can do parametric estimation, via a Gaussian

$$P(x_d = x \mid y = c) = \frac{1}{\sqrt{2\pi}\sigma_{cd}} \exp\left(-\frac{(x-\mu_{cd})^2}{2\sigma_{cd}^2}\right)$$

where μ_{cd} and σ_{cd}^2 are the empirical mean and variance of feature d among all examples with label c.

We will see more on this model later in the course.

In particular, Naive Bayes model with missing labels.

Generative classifier

Naïve Bayes $P(X, Y = c)$ is a generative model.

We can generate a sample of the data

$$P(X) = \sum_c P(X \mid Y = c)$$

We estimate parameters $P(X \mid Y = c), P(Y = c)$ directly from training data.

Later we will consider a descriminative classifier - Logistic Regression, $P(Y = c \mid X)$. In that model we cannot obtain a sample of the data, because $P(X)$ is not available.