

1.) Graded Problems :

1.)  $k$  types :  $1 \leq k \leq n$   
 $0 \leq w \leq W$

$$OPT(k, w) = \max \{ OPT(k-1, w), OPT(w - w_k) + v_k \times i \}$$

↓  
do not include  
item of type  $k$

↓  
include item of type  $k$

Base cases

$$OPT(0, 0) = 0$$

$$OPT(k, w) = OPT(k-1, w), \text{ if } w_k > w$$

$$\Rightarrow \text{Complexity} = \Theta(nw)$$

2.)

$s_i, s_{i+1}, \dots, s_k$  be the sub-string in  $s$ .

$$0 \leq k \leq n$$

$$OPT(k) = \max_{0 \leq i < k \wedge s_{i+1:k} \text{ is in dict.}} (0, OPT(i))$$

Base cases:  ~~$OPT(0) = -1$~~   $OPT(0) = -1$

$$\Rightarrow \text{Complexity} = \Theta(nm)$$

↙  
To traverse  
the given string

↘  
To find all  
words in dict.

3.)

let balloons be  $l$  to  $r$

$l \dots k \dots r$

$$OPT(l, r) = \max_{l \leq k \leq r} \{ OPT(l, k-1) + OPT(k+1, r) + \text{num}[l] \times \text{num}[k] \times \text{num}[r] \}$$

$$0 \leq k \leq n$$

Base cases :

$$\text{OPT}(0,0) = 0, \text{ if } s_2 \leq l$$

$$\Rightarrow \text{Complexity} : \underset{\substack{\downarrow \\ \text{Array \& sub-array}}}{O(n^2)} \times \underset{\substack{\downarrow \\ \text{max}}}{O(n)} = \Theta(n^3)$$

4.)

$\text{Opt}(i) \rightarrow$  score of 1<sup>st</sup>  $i$  characters

$$j \leq i$$

$$1 \leq i \leq n$$

$$\text{Opt}(i) = \min_{j \leq i} \{ \text{opt}(j-1) + \text{Quality}(j \dots n) \}$$

Base case :  $\text{OPT}(0) = 0$

$$\Rightarrow \text{Complexity} = \Theta(n^2)$$

String      Sub-string