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CSCI - 570 - HW 10

Graded Problems :

1.)

$$e \in E$$

$$\text{if } e \text{ is satisfied} \Rightarrow P[e] = \frac{2}{3}$$

$$E[\text{no. of satisfied edges}] = E \left[ \sum_{e \in E} X_e \right]$$

$$= \sum_{e \in E} E[X_e]$$

$$= \frac{2}{3} |E|$$

$$C \leq |E|$$

optimal no. of edges

$$\geq \frac{2}{3} C$$

2.)

2 passengers :

$$\begin{array}{ccc} \frac{1}{2} & \frac{2}{2} & \\ \uparrow & \uparrow & \checkmark \\ \hline \uparrow & \uparrow & \times \end{array} \quad \frac{1}{2}$$

3 passengers :

$$\begin{array}{ccc} \frac{1}{3} & \frac{2}{3} & \frac{3}{3} \\ \uparrow & \uparrow & \uparrow \quad \checkmark \\ \hline \uparrow & \uparrow & \checkmark \\ \hline & \uparrow & \checkmark \\ \hline \uparrow & \uparrow & \times \\ \hline & \uparrow & \times \\ \hline \uparrow & \uparrow & \times \end{array} \quad \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow h^{\text{th}} \text{ passenger} = P(e) = \frac{1}{2}$$

4.)

Las Vegas: keep picking until 1  
 Monte Carlo: " " " " or till k times

Las Vegas :

Expected no. of trials = 2  $\Rightarrow O(1)$

Monte Carlo :

$$P(1) = \left[ 1 - \left( \frac{1}{2} \right)^k \right] \Rightarrow O(k)$$

2.)

1000 songs

Artist

Songs

1	→	10
⋮		⋮
100	→	10
		<hr/>
		1000

$$P[E] = \frac{10}{1000} \times \frac{9}{1000} \times \frac{8}{1000}$$

$$C \leq |E|$$

$$\Rightarrow \geq P[E] \times C$$