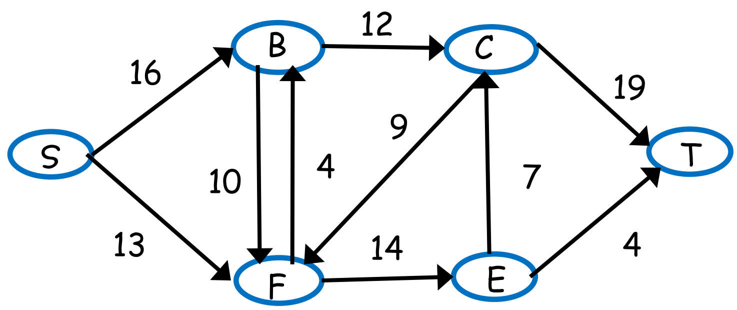
CSCI 570
Network Flow-1
Discussion Problems

1. Run the Ford-Fulkerson algorithm on the following network. How do you find a min-cut? Show a min-cut. Is a min-cut unique?



2(a). You have successfully computed a maximum s-t flow f for a network G = (V; E) with integer edge capacities. Your boss now gives you another network G' that is identical to G except that the capacity of exactly one edge is <u>decreased</u> by one. You are also explicitly given the edge whose capacity was changed. Describe how you can compute a maximum flow for G' in linear time.

2(b). Same as above but the capacity of exactly one edge is <u>increased</u> by one. How would you compute a maximum flow?

3. If we add the same positive number to the capacity of every directed edge, then the minimum cut (but not its value) remains unchanged. IF it is true, prove it, otherwise provide a counterexample.

4. In a daring burglary, someone attempted to steal all the candy bars from the CS department. Luckily, he was quickly detected, and now, the course staff and students will have to keep him from escaping from campus. In order to do so, they can be deployed to monitor strategic routes. More formally, we can think of the USC campus as a graph, in which the nodes are locations, and edges are pathways or corridors. One of the nodes (the instructor's office) is the burglar's starting point, and several nodes (the USC gates) are the escape points — if the burglar reaches any one of those, the candy bars will be gone forever. Students and staff can be placed to monitor the edges. As it is hard to hide that many candy bars, the burglar cannot pass by a monitored edge undetected. Give an algorithm to compute the minimum number of students/staff (and their placement) needed to ensure that the burglar cannot reach any escape points undetected.

5. We're asked to help the captain of the USC tennis team to arrange a series of matches against UCLA's team. Both teams have n players; the tennis rating of the i-th member of USC's team is a_i and the tennis rating for the k-th member of UCLA's team is b_k. We would like to set up a competition in which each person plays one match against a player from the opposite school. Our goal is to make as many matches as possible in which the USC player has a higher tennis rating than his or her opponent. Give an algorithm to decide which matches to arrange to achieve this objective

6. Professor Jones has determined that *x* priceless artifacts are located in a labyrinth. The labyrinth can be thought of as a graph, with each edge representing a path and each node an intersection of paths. All of the artifacts are in the same treasure room, which is located at one of the intersections. However, the artifacts are extremely burdensome, so Jones can only carry one artifact at a time. There is only one entrance to the labyrinth, which is also a node in the graph. The entrance serves as the only exit as well. All the paths are protected by human-eating vines, which will be woken up after someone passes the path, so Jones can only go through each path once. Give an algorithm that determines how many artifacts Jones can obtain and how he can do it.

7. We say that two paths are vertex-disjoint if they do not share any vertices (except s and t). Given a directed graph G = (V, E) with two distinguished nodes s, t. Design an algorithm to find the maximum number of vertex-disjoint s-t paths in G.

8. There are n students in a class. We want to choose a subset of k students as a committee. There has to be m1 number of freshmen, m2 number of sophomores, m3 number of juniors, and m4 number of seniors in the committee. Each student is from one of k departments, where k = m1 +m2 +m3 +m4. Exactly one student from each department has to be chosen for the committee. We are given a list of students, their home departments, and their class (freshman, sophomore, junior, senior). Describe an efficient algorithm based on network flow techniques to select who should be on the committee such that the above constraints are all satisfied.