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CSCI 570 - SPRING 2018 - HW4

1. Graded Problems:

1.) $d_1 < d_2 \ldots\ldots d_{n-1} < d_n$

for all n
  if $(d_n - d_{n-1})$ < remaining fuel tank
    Fill gas at d_n
  else
    Fill gas at d_{n-1}
endfor

Car filled gas in $G: g_1 \ldots\ldots g_k$

Say G is of size k.

Say there is an optimal solution O

$O: h_1 \ldots h_m$

$h_1 \le g_1$ - since it's greedy the car fills at 1st step

By exchange argument, new solution is

$\{g_1, h_2 \ldots h_m\}$ - optimal solution

Assume,

$\{g_1, g_2 \ldots g_{l-1}, h_l \ldots h_k\}$

is an optimal solution - Induction hypothesis.

$h_l \le g_{l+1}$ - there is fuel left to be filled at the next station

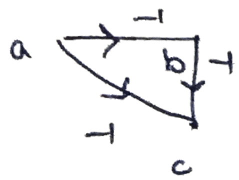
By induction,

$\{g_1, g_2, g_3 \ldots g_k\}$ is an optimal solution

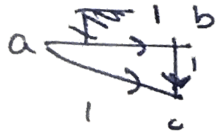
for { if .... else } $\rightarrow O(n)$

2:) Dijkstra Algo :

$$\text{if } (d(u) \geq (d(v) + l_e))$$



$a \to c \Rightarrow (a,b), (b,c) = -2$



$a \to c \Rightarrow (a,c) = 1$.

Dijkstra algorithm while comparision of -ve values, it reverses the result.

3:) Say our greedy algorithm fits boxes B into the first k trucks $\{b_1, b_2 \dots b_j\}$

Say there is an optimal solution O : $\{B_1, B_2 \dots B_i\}$ for the first k trucks

$i \leq j$

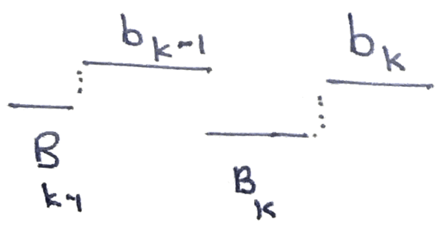
$k = 1$;

$B_1 < b_1$ - greedy also fits more boxes

$k = k-1$;

$B_{k-1} \leq b_{k-1}$ - induction hypothesis

For k ;

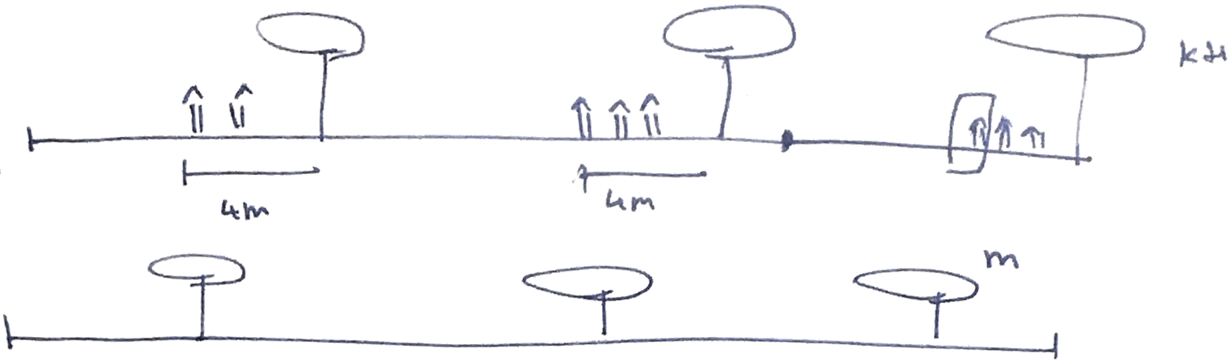


$b_k > B_k$, since it can fit more boxes into the k^{th} truck.

4.)

Greedy Solution:

Sort →



Optimal: Solution

Greedy (k+1) base station is to the right of Optimal (k+1) base station

$i=1$ → true , otherwise 1^{st} house is not Covered

~~k~~m of optimal will not cover the rightmost house

⇒ $k \leq m$ - by induction

That one particular house will not get coverage from $(m-1)$, so m has to be placed in optimal solution.

Sort: $O(n \log n)$

Placement: $O(n)$