RAHUL ETHIRAJ
ethirgj@usc.edu

CSCI-570-SPRNG 2018 - HW5

1) Graded Problems :

1. $T(n) = 7\,T\left(\frac{n}{2}\right) + n^2$

$T'(n) = a\,T'\left(\frac{n}{4}\right) + n^2 \log n$

T(n):

$f(n) = n^2$

$n^{\log_2 7} \approx n^{2.8}$

$\Rightarrow T(n) = \Theta(n^{2.8})$

T'(n):

$f(n) = n^2 \log n$

$n^{\log_4 a}$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$\frac{\log_2 a}{2} < \log_2 7$$

$$\log a < 5.6$$

$$a < \log^{-1}(5.6) < 48.5$$

$\Rightarrow \boxed{a = 48}$

$T(n) = \Theta(n^{2.75})$

2. a) $T(n) = 4\,T\left(\frac{n}{2}\right) + n^2 \log n$

$f(n) = n^2 \log n$

$n^{\log_2 4} = n^2$

Case (ii) : $\boxed{T(n) = \Theta(n^2 \log^2 n)}$

b) $T(n) = 8\,T\left(\frac{n}{6}\right) + n \log n$

$f(n) = n \log n$

$n^{\log_6 8} = \frac{\log_2 8}{\log_2 6} = 1.16$

$\Rightarrow \boxed{n^{1.16}}$

Case (iii) $\Rightarrow$ $\boxed{T(n) = n \log n}$

$$8 \times \frac{n}{6} \times \log_6 n \leq c \times n \log n$$

$$\log_6 n \leq c \cdot \log n$$

$$\log n - \log 6 \leq c \cdot \log n$$

$$\boxed{\log n \leq c \cdot \log n + 2.5}$$ ✓

c.) $T(n) = \sqrt{6006}\, T\left(\frac{n}{2}\right) + n^{\sqrt{6006}}$

$$f(n) = n^{\sqrt{6006}} = n^{77.5}$$

$$n^{\log_b a} = n^{\log_2 \sqrt{6006}} = n^{6.28}$$

Case (iii) $\Rightarrow$ $\boxed{T(n) = n^{\sqrt{6006}}}$

$$\sqrt{6006} \times \left(\frac{n}{2}\right)^{\sqrt{6006}} \leq c \cdot n^{\sqrt{6006}}$$

$$\frac{77.5}{2^{\sqrt{6006}}} \leq c$$

$$77.5 \leq c \times 2^{77.5}$$

$$\Rightarrow \boxed{c < 1}$$

d.) $T(n) = 10\, T\left(\frac{n}{2}\right) + 2^n$

$$n^{\log_2 10} = n^{3.32}$$

Case (i) $\Rightarrow$ $\boxed{T(n) = n^{\log_2 10}}$

e.) $T(n) = 2\, T(\sqrt{n}) + \log_2 n$

$$f(n) = \log n$$

Let $n = 2^m \Rightarrow \log n = m \log 2$

$$T(2^m) = 2\, T(2^{m/2}) + \log n$$

$$\underbrace{T(\exp(2^m))}_{\text{Composition of functions}} = S(m) = 2S\left(\frac{m}{2}\right) + m$$

$f(m) = m$

$m^{\log_2 2} = m$

Case (ii) $\rightarrow$ $S(m) = \Theta(m \cdot \log m)$

$\Rightarrow$ $\boxed{T(n) = \Theta(\log n \cdot \log(\log n))}$

f.) $T^2(n) = T\left(\frac{n}{2}\right) \cdot T(2n) - T(n) \cdot T\left(\frac{n}{2}\right)$

$T(n) = T(n)^{-1} \cdot T\left(\frac{n}{2}\right) \cdot T(2n) - T\left(\frac{n}{2}\right)$

$\Rightarrow T(n) = T\left(\frac{n}{2}\right)\left(\left(T(n)^{-1} \cdot T(2n)\right) - 1\right)$

doubt

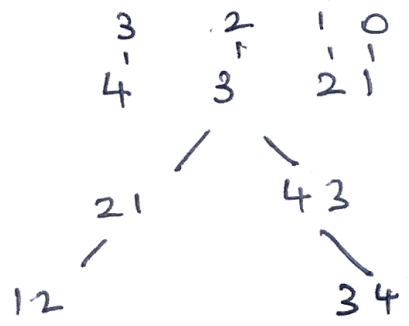
g.) $T(n) = 2T\left(\frac{n}{2}\right) - \sqrt{n}$

$f(n) = -(n)^{1/2}$

$n^{\log_2 2} = n$

Case (i) $\Rightarrow$ $\boxed{T(n) = \Theta(n)}$

3.) List A, n



$T(n) = 2T\left(\frac{n}{2}\right) + n$

$f(n) = n$

$n^{\log_2 2} = n$

Case (ii) $\Rightarrow$ $\boxed{T(n) = \Theta(n \log n)}$

4.)

1 2 3 (2) — (3.5) — 4 5 6 (5)

4 5 6 (5) — (3.5) — 1 2 3 (2)

```
median (n, A, B)
{
    if (n == 1)
        avg (A[n], B[n]) // base case
    else
    {
        if (median(A) < median(B))
            median ( ⌈n/2⌉, A[⌊n/2⌋, n], B[0, ⌊n/2⌋] )
        else
            median ( ⌈n/2⌉, A[0, ⌊n/2⌋], B[⌈n/2⌉, n] )
    }
}
```

$a = 1$, $b = 2$

$T(n) = 1 \times T\left(\frac{n}{2}\right) + c$

$f(n) = O(1)$

$n^{\log_2 1} = n^0 = O(1)$

Case (ii) $\Rightarrow$ $\boxed{T(n) = \Theta(\log n)}$