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CSCI -570 Spring 2018 - HW2

1.) Graded Problems :

1.

$$f_1(n) = n^{2.5} = n^2 \cdot n^{1/2}$$
$$f_2(n) = \sqrt{2} \times n^{0.5}$$
$$f_3(n) = n+10$$
$$f_4(n) = 10^n$$
$$f_5(n) = 100^n$$
$$f_6(n) = n^2 \log n$$

log	polynomials	Exponents
	$f_2 < f_3 < f_6 < f_1 <$	$< f_4 < f_5$

for any $c > 2$, since $\log_2 2 = 1$

2.

$$g_1 = 2^{(\log n)^{1/2}}$$
$$g_2 = 2^n$$
$$g_3 = n(\log n)^3$$
$$g_4 = n^{4/3} = n \cdot n^{1/3}$$
$$g_5 = n^{\log n}$$
$$g_6 = 2^{2^n}$$
$$g_7 = 2^{n^2}$$

$$g_1 = 2^{(\log n)^{1/2}}$$

$$\log g_1 = (\log n)^{1/2} \cdot \log 2 = (\log n)^{1/2} = (z)^{1/2}$$
$$\log g_5 = \log n \cdot \log n = (\log n)^2 = (z)^2$$

$$\Rightarrow \boxed{g_1 < g_5}$$

$$\log g_3 = \log(n(\log n)^3) = \log n + 3\log(\log n)$$
$$= z + 3\log z$$

$$\Rightarrow \boxed{g_3 < g_5}$$

$\therefore \quad g_3 < g_4 \; \Big| \; < g_1 < g_5 < g_2 < g_7 < g_6$

polynomials | exponents

3. $f(n) = O(g(n))$

$\leq c\, g(n)$

a.) $\log_2 f(n) \leq c \log_2 (g(n))$

This is false when,

$g(n) = 1$, for all n

$f(n) = 2$, for all n

$\log_2 (g(n)) = 0$

This is true when,

$g(n) \geq 2$, for all $n \geq n_0$

b) $2^{f(n)} \leq c \cdot 2^{g(n)}$

$\log \underbrace{f(n)}_{2n} \leq \log \underbrace{g(n)}_{n}$

$4^n \leq c \cdot 2^n$

$\therefore$ False

c.) $f(n) \leq c \cdot g(n)$, for all $n \geq n_0$

$(f(n))^2 \leq c^2 \cdot (g(n))^2$, for all $n \geq n_0$

4. (a). $f \leq c\, a(h)$

$g \geq c\, b(h)$

$f + g \neq O(h) \neq \Omega(h)$

$\therefore$ False

b.) It costs less, if both the solutions are run in the same environment (processor, memory, etc.).

c.) $F(n) = 4n + \sqrt{3} \cdot n^{1/2}$ => exponent function

$\leq / \geq (n^{1/2})$

∴ False, since F(n) is not defined

d.) False

If T is both a depth-first search tree and a BFS tree rooted at u, then G cannot contain any edges that don't belong to T.

e.) True

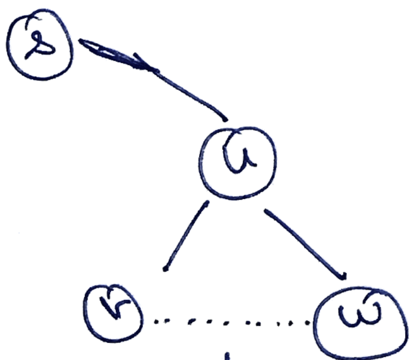
5.) Run BFS tree recursively

if (G = = T)

Contains no cycles

else

some edge e = (v, w) that belongs to G and not T



cycle found