CS273B: Deep learning for Genomics and Biomedicine

Lecture 3: Functional genomics, DenseNets, Backprop, Convnets & genomics applications 09/25/2017

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Outline

1

Functional

Genomics

Dense Neural

Networks

3

Convolutional architectures

- Multi-modal learning
- Multi-task learning

4

Training a DNN (SGD + Backprop)

Functional genomics

TGCCAAGCAGCAAAGTTTTGCTGCTGTTTATTTTTGTAGCTCTTACTATATTCT ACTTTTACCATTGAAAATATTGAGGAAGTTATTTATATTTCTATTTTTATATAT TATATATTTTATGTATTTTAATATTACTATTACACATAATTATTTTTTATATATATGA AGTACCAATGACTTCCTTTTCCAGAGCAATAATGAAATTTCACAGTATGAAA ATGGAAGAATCAATAAAATTATACGTGACCTGTGGCGAAGTACCTATCGTG GACAAGGTGAGTACCATGGTGTATCACAAATGCTCTTTCCAAAGCCCTCTCC GCAGCTCTTCCCCTTATGACCTCTCATCATGCCAGCATTACCTCCCTGGACCC CTTTCTAAGCATGTCTTTGAGATTTTCTAAGAATTCTTATCTTGGCAACATCTT GTAGCAAGAAATGTAAAGTTTTCTGTTCCAGAGCCTAACAGGACTTACATA TTTGACTGCAGTAGGCATTATATTTAGCTGATGACATAATAGGTTCTGTCATA GTGTAGATAGGGATAAGCCAAAATGCAATAAGAAAAACCATCCAGAGGAA ACTCTTTTTTTTTTTTTTTTTTTTTTTTCCAGATGGAGTCTCGCACTTC TCTGTCACCCGGGCTGGAGCGCAGTGGTGCAATCTTGGCTCACTGCAACCT CCACCTCCTGGGTTCAGGTGATTCTCCCACCTCAGCCTCCCGAGTAGTAGCT GGAATTACAGGTGCGCGCTCCCACACCTGGCTAATTTTTTTGTATTCTTAGTA GAGATGGGGTTTCACCATGTTGGCCAGGCTGGTCTCAAACTCCTGCCCTCA GGTGATCTGCCCACCTTGGCCTCCCAGTGTTGGGTTTACAGGCGTGAGCCA AGGCTGAGGAACTGGGCATCTGGGTTGCTTCTGGCCAGACCACCAGGCT CTTGAATCCTCCCAGCCAGAGAAAGAGTTTCCACACCAGCCATTGTTTTCCT CTGGTAATGTCAGCCTCATCTGTTGTTCCTAGGCTTACTTGATATGTTTGTAA ATGACAAAAGGCTACAGAGCATAGGTTCCTCTAAAATATTCTTCTTCCTGTGT CAGATATTGAATACATAGAAATACGGTCTGATGCCGATGAAAATGTATCAGCT TCTGATAAAAGGCGGAATTATAACTACCGAGTGGTGATGCTGAAGGGAGAC ACAGCCTTGGATATGCGAGGACGATGCAGTGCTGGACAAAAGGCAGGTAT CTCAAAAGCCTGGGGAGCCAACTCACCCAAGTAACTGAAAGAGAGAAACA AACATCAGTGCAGTGGAAGCACCCAAGGCTACACCTGAATGGTGGGAAGC TCTTTGCTGCTATATAAAATGAATCAGGCTCAGCTACTATTATT

Functional genomics: Decoding genome function

~ 3 billion nucleotides

TGCCAAGCAGCAAAGTTTTGCTGCTGTTTATTTTTGTAGCTCTTACTATATTCT ACTTTTACCATTGAAAATATTGAGGAAGTTATTTATATTTCTATTTTTATATAT TATATATTTTATGTATTTTAATATTACTATTACACATAATTATTTTTTATATATATGA AGTACCAATGACTTCCTTTTCCAGAGCAATAATGAAATTTCACAGTATGAAA ATGGAAGAATCAATAAAATTATACGTGACCTGTGGCGAAGTACCTATCGTG GACAAGGTGAGTACCATGGTGTATCACAAATGCTCTTTCCAAAGCCCTCTCC GCAGCTCTTCCCCTTATGACCTCTCATCATGCCAGCATTACCTCCCTGGACCC CTTTCTAAGCATGTCTTTGAGATTTTCTAAGAATTCTTATCTTGGCAACATCTT GTAGCAAGAAATGTAAAGTTTTCTGTTCCAGAGCCTAACAGGACTTACATA TTTGACTGCAGTAGGCATTATATTTAGCTGATGACATAATAGGTTCTGTCATA GTGTAGATAGGGATAAGCCAAAATGCAATAAGAAAAACCAT CCAGAGGAA ACTCTTTTTTTTTTTTTTTTTTTTTTTCCAGATGGAGTCTCGCACTTC TCTGTCACCCGGGCTGGAGCGCAGTGGTGCAATCTTGGCTCACTGCAACCT CCACCTCCTGGGTTCAGGTGATTCTCCCACCTCAGCCTCCCGAGTAGTAGCT GGAATTACAGGTGCGCGCTCCCACACCTGGCTAATTTTTTGTATTCTTAGTA GAGATGGGGTTTCACCATGTTGGCCAGGCTGGTCTCAAACTCCTGCCCTCA GGTGATCTGCCCACCTTGGCCTCCCAGTGTTGGGTTTACAGGCGTGAGCCA AGGCTGAGGAACTGGGGCATCTGGGTTGCTTCTGGCCAGACCACCAGGCT CTTGAATCCTCCCAGCCAGAGAAAGAGTTTCCACACCAGCCATTGTTTTCCT CTGGTAATGTCAGCCTCATCTGTTGTTCCTAGGCTTACTTGATATGTTTGTAA ATGACAAAAGGCTACAGAGCATAGGTTCCTCTAAAATATTCTTCTTCCTGTGT CAGATATTGAATACATAGAAATACGGTCTGATGCCGATGAAAATGTATCAGCT TCTGATAAAAGGCGGAATTATAACTACCGAGTGGTGATGCTGAAGGGAGAC ACAGCCTTGGATATGCGAGGACGATGCAGTGCTGGACAAAAGGCAGGTAT CTCAAAAGCCTGGGGAGCCAACTCACCCAAGTAACTGAAAGAGAGAAACA AACATCAGTGCAGTGGAAGCACCCAAGGCTACACCTGAATGGTGGGAAGC TCTTTGCTGCTATATAAAATGAATCAGGCTCAGCTACTATTATT

Functional genomics: Decoding genome function

Function?

~ 3 billion nucleotides

Mapping reads to reference genome

Naïve method

- Scan whole genome with every read
- Problem: Too slow

Indexing + Alignment approach

- Create a compressed reference 'genome index'
 - a map of where each short subsequence of length 'k' hits the genome
- Map reads using index via smart alignment algorithms and data structures (e.g suffix array)
- Allow for errors: insertions, deletions, mismatches in alignments

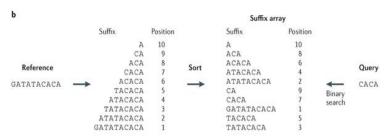
Run times for indexing alignment

- Indexing human genome ~ 3 hours
- Alignment speed: 2 million 35 bp reads on 1 processor ~20 mins
- Alignment speed depends on error rate

ACGTTACCGAATCGATCAAGTCGA

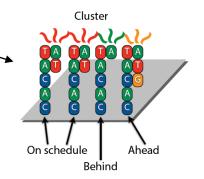




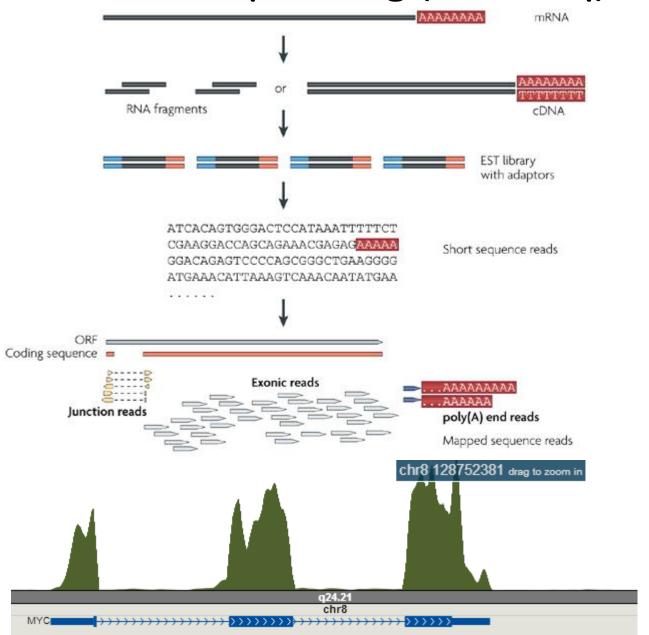


Nature Reviews | Genetics

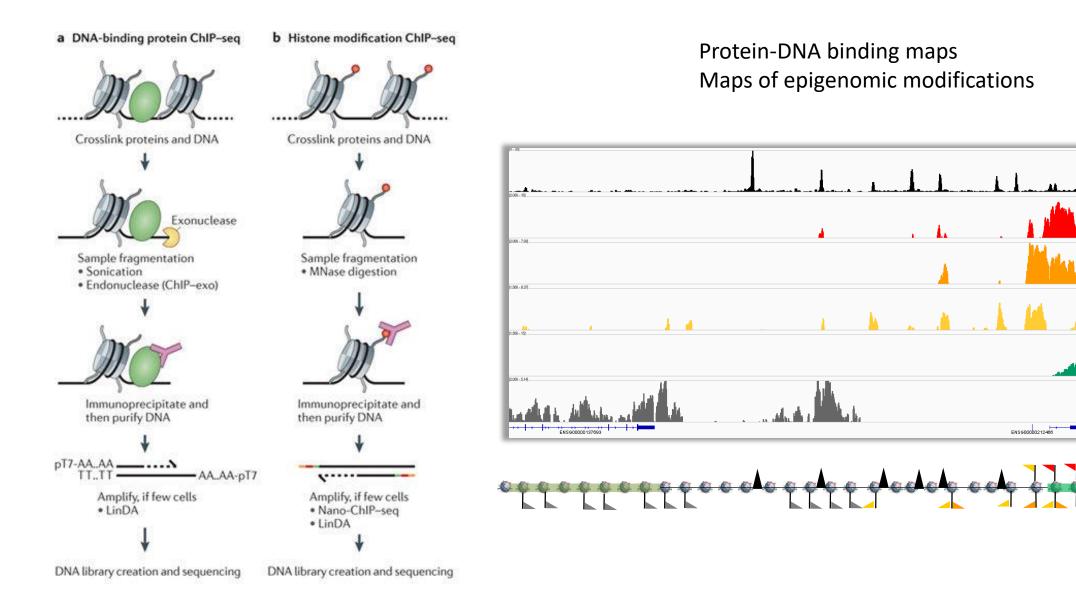
http://www.nature.com/nrg/journal/v14/n5/box/nrg3433_BX2.html



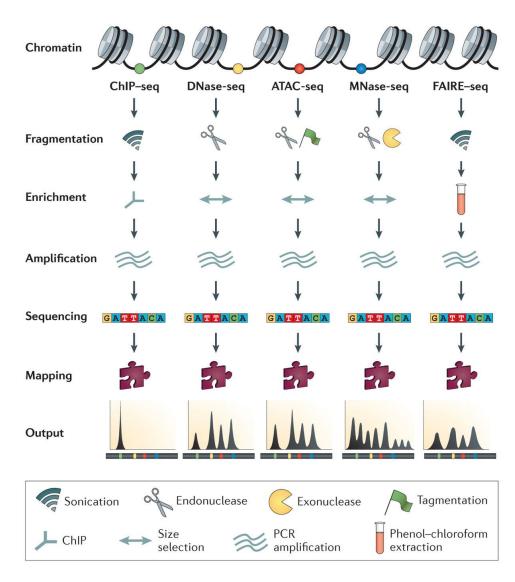
RNA sequencing (RNA-seq)

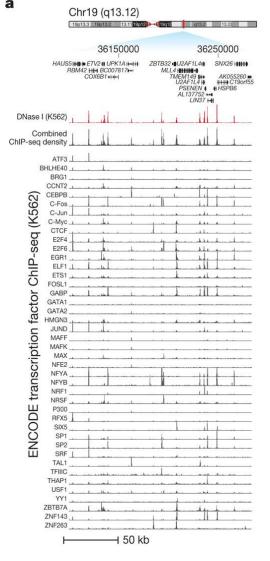


Chromatin immunoprecipitation (ChIP-seq)

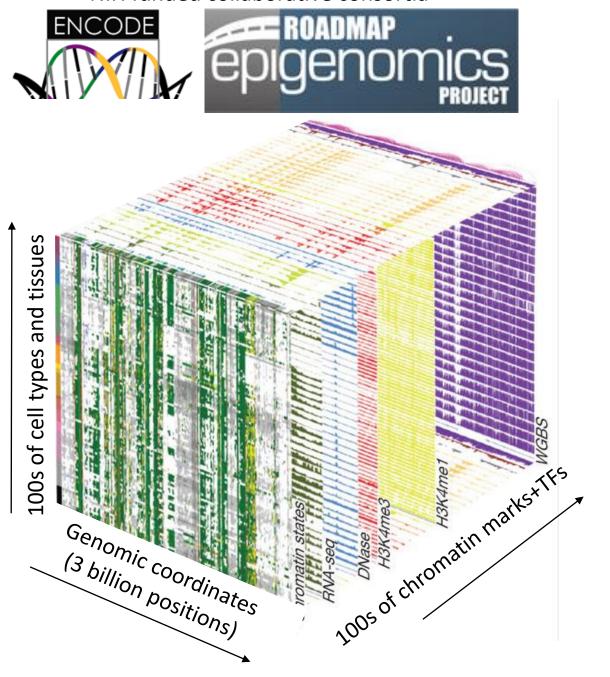


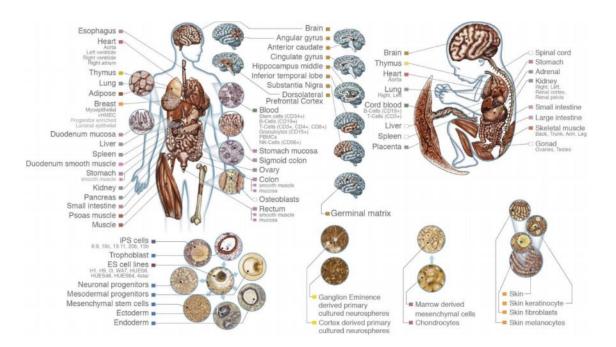
Chromatin accessibility (DNase-seq, ATAC-seq) and nucleosome sequencing (MNase-seq)





NIH funded collaborative consortia



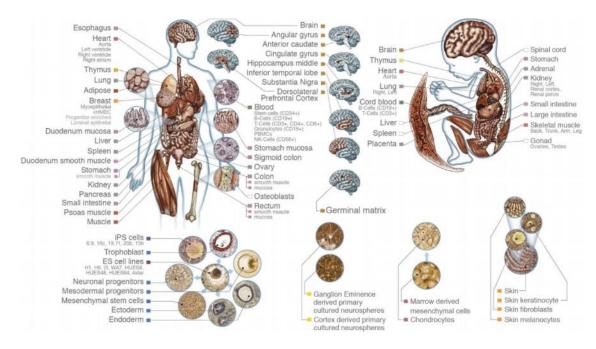


100s of Cell-Types/Tissues

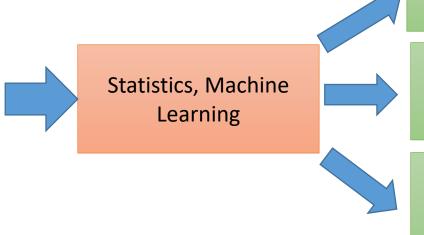
NIH funded collaborative consortia ENCODE tissue and types cell of 100s of chromatin markstris 100s

Genomic coordinates

(3 billion positions)



100s of Cell-Types/Tissues



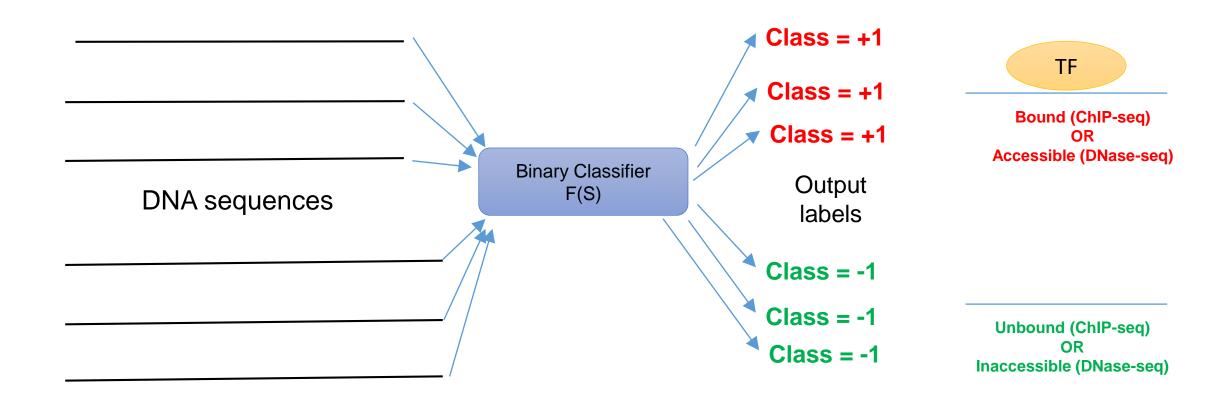
Denoise data

Integrate and impute missing data

Identify, interpret, understand properties of reg. elements & variants

Dense neural networks

Classifying regulatory DNA sequences



How to represent DNA sequence?

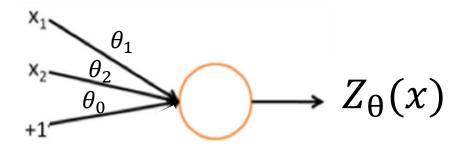
Bag of words

K-mer features $(x_1, x_2, x_3 \dots x_n)$

	AGA	GAT	ATC	TCG	CGA	GAG	AGT	GTG	Label	
Sequence1	1	4	5	2	0	0	0	0		1
Sequence2	4	1	1	0	0	2	8	8		1
Sequence3	6	3	3	2	2	4	10	10		1
Sequence4	5	5	6	2	0	2	8	8		-1
Sequence5	10	4	4	2	2	6	18	18		-1

 What are the advantages and disadvantages of bag of k-mers representation?

Logistic regression

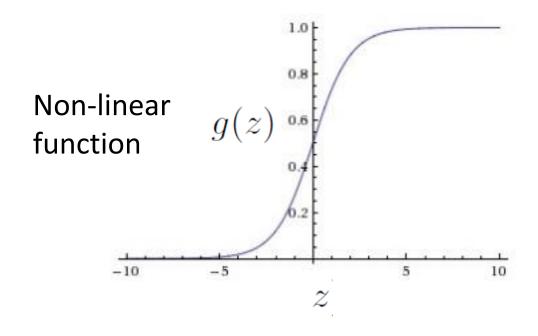


$$Z_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

$$g(z) = \frac{1}{1 + e^{-z}}$$

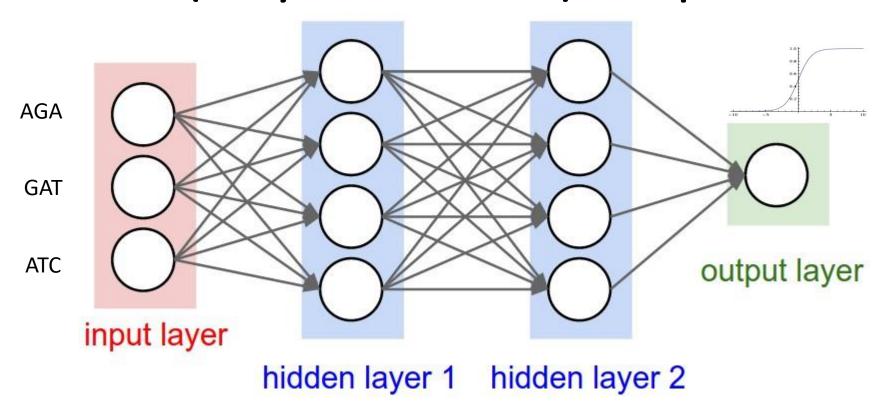
$$h_{\theta}(x) = g(\theta^T x) = \frac{1}{1 + e^{-\theta^T x}},$$

Logistic / Sigmoid
Useful for predicting probabilities



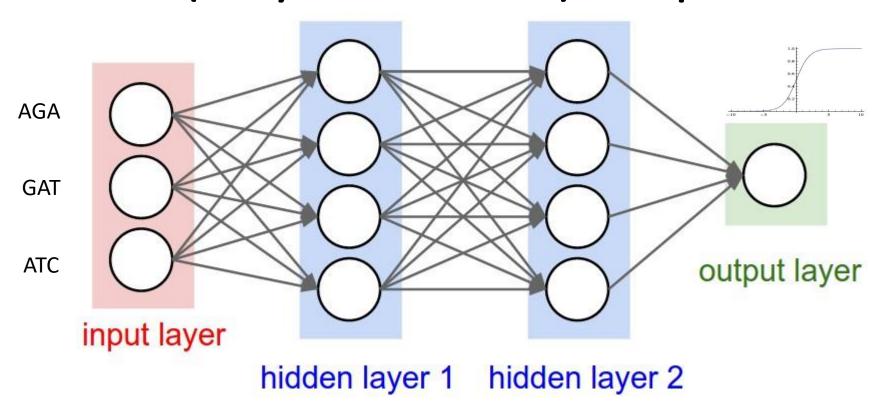
Training the model means learning the parameters to minimize some logistic loss

Dense (fully-connected) deep neural network



How many parameters does this DNN have?

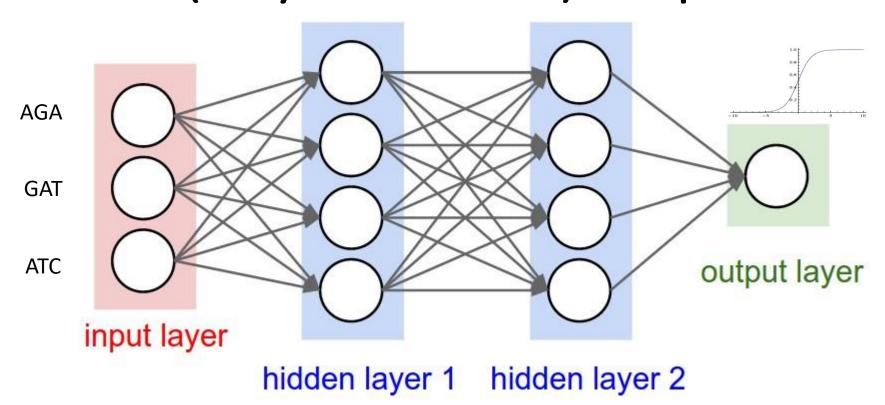
Dense (fully-connected) deep neural network



How many parameters does this DNN have?

3x4 + 4x4 + 4x1 = 12 + 16 + 4 = 32

Dense (fully-connected) deep neural network



How many parameters does this DNN have?

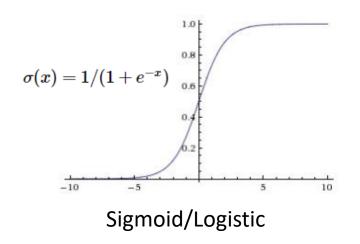
$$3x4 + 4x4 + 4x1 = 12 + 16 + 4 = 32$$

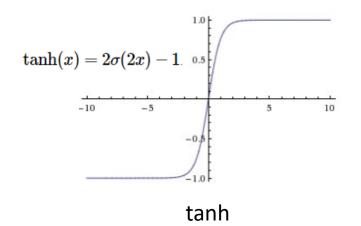
Architecture and hyperparameters of DNN

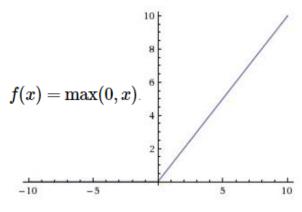
- How many layers?
- How many neurons per layer?
- What types of activations to use?

Search over architectures and identify optimal architecture by evaluating performance on validation set

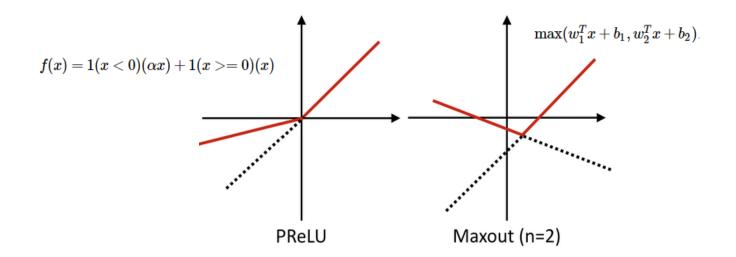
Types of activations



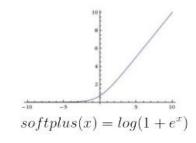


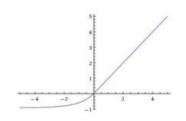


ReLU (rectified linear unit)



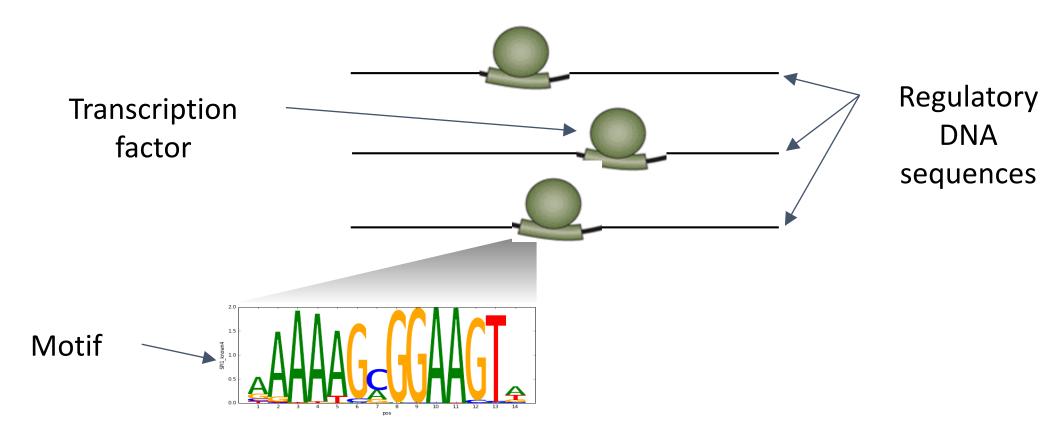
Softplus and Exponential Linear Unit (ELU)





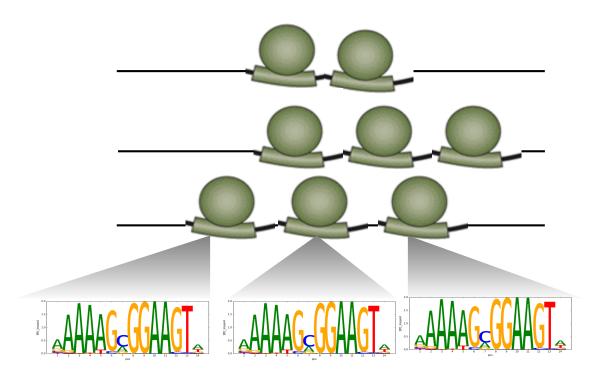
$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha & (\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

Convolutional architectures



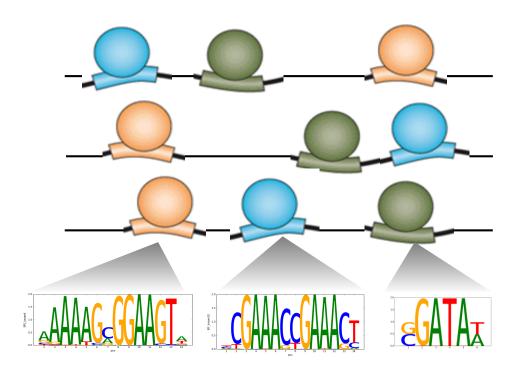
TRANSCRIPTION FACTOR BINDING

Regulatory proteins called <u>transcription factors</u> (TFs) bind to high affinity sequence patterns (<u>motifs</u>) in regulatory DNA



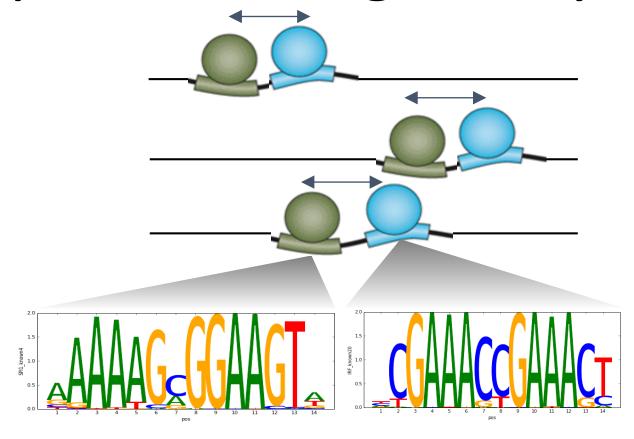
HOMOTYPIC MOTIF DENSITY

Regulatory sequences often contain more than one binding instance of a TF resulting in homotypic clusters of motifs of the same TF



HETEROTYPIC MOTIF COMBINATIONS

Regulatory sequences often bound by <u>combinations of TFs</u> resulting in <u>heterotypic clusters of motifs of different TFs</u>

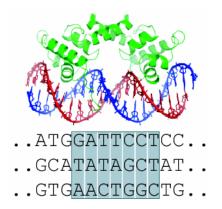


SPATIAL GRAMMARS OF HETEROTYPIC MOTIF COMBINATIONS

Regulatory sequences are often bound by <u>combinations of TFs</u> with specific <u>spatial and</u> <u>positional constraints</u> resulting in distinct <u>motif grammars</u>

GGATAA CGATAT CGATAT

Set of aligned sequences
Bound by TF



$$p_i(x_i = a_i)$$

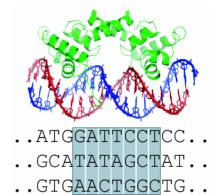
GGATAA CGATAT CGATAT

Bound by TF

	GGATAT		
Set of	aligned se	quences	

А	0	0	1	0	1	0.5
С	0.5	0	0	0	0	0
G	0.5	1	0	0	0	0
Т	0	0	0	1	0	0.5

Position weight matrix (PWM)

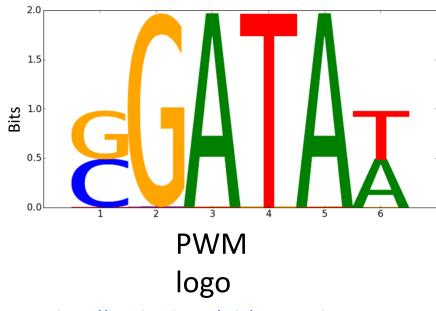


$$p_i(x_i=a_i)$$

Set of	ali	igned	se	que	ences
	Bo	ound	by	TF	

А	0	0	1	0	1	0.5
С	0.5	0	0	0	0	0
G	0.5	1	0	0	0	0
Т	0	0	0	1	0	0.5

Position weight matrix (PWM)



https://en.wikipedia.org/wiki/Sequence logo

The information content (y-axis) of position i is given by: [2]

$$R_i = \log_2(4) - (H_i + e_n)$$

where H_i is the uncertainty (sometimes called the Shannon entropy) of position i

$$H_i = -\sum f_{a,i} imes \log_2 f_{a,i}$$

. The height of letter $m{a}$ in column $m{i}$ is given by

$$\text{height} = f_{a,i} \times R_i$$



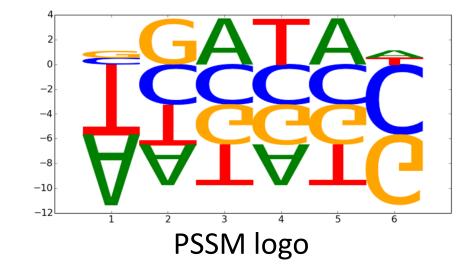
..GTGAACTGGCTG..

Accounting for genomic background nucleotide distribution

Position-specific scoring matrix (PSSM)

$$\log_2\left(\frac{p_i(x_i=a_i)}{p_{ba}(x_i=a_i)}\right)$$

А	-5.7	-3.2	3.7	-3.2	3.7	0.6
С	0.5	-3.2	-3.2	-3.2	-3.2	-5.7
G	0.5	3.7	-3.2	-3.2	-3.2	-5.7
Т	-5.7	-3.2	-3.2	3.7	-3.2	0.5



Scoring a sequence with a motif PSSM

PSSM parameters

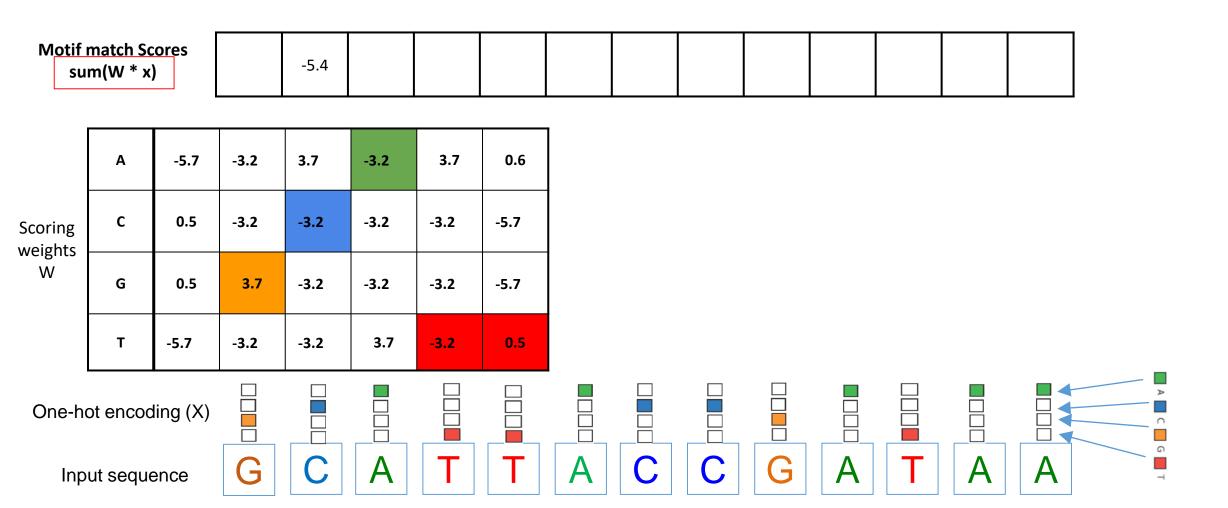


One-hot encoding (X)
Input sequence

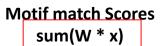
G
C
A
T
T
A
C
C
G
A
T
A
A

G

Convolution: Scoring a sequence with a PSSM



Convolution





Scoring weights W

Α	-5.7	-3.2	3.7	-3.2	3.7	0.6
С	0.5	-3.2	-3.2	-3.2	-3.2	-5.7
G	0.5	3.7	-3.2	-3.2	-3.2	-5.7
т	-5.7	-3.2	-3.2	3.7	-3.2	0.5

One-hot encoding (X)

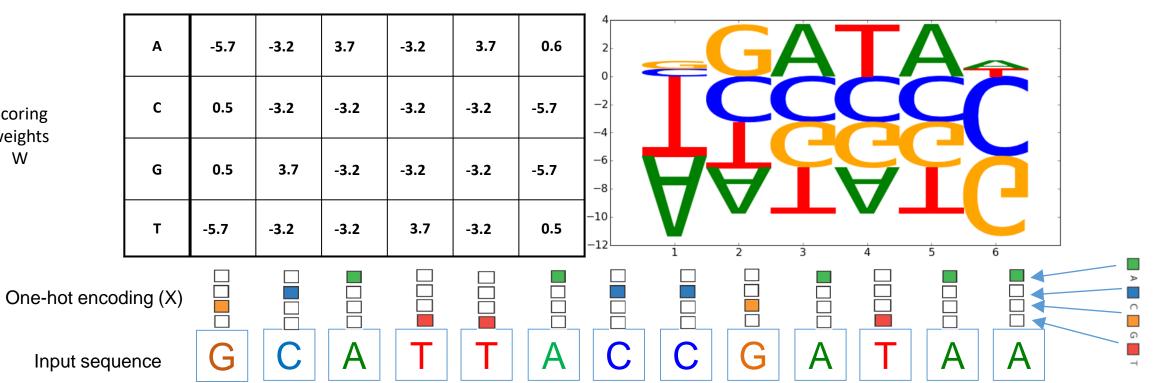
Input sequence

Convolution

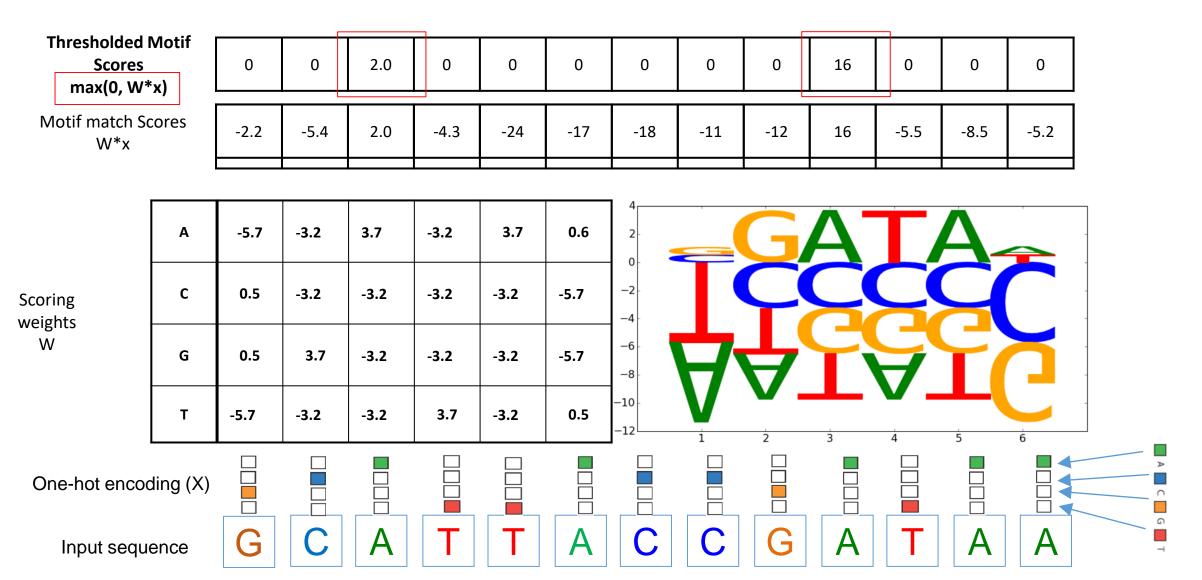
Motif match Scores sum(W * x)

-2.2 -5.4 2.0 -4.3 -24 -17 -18 -11 -12 16 -5.5 -8.5

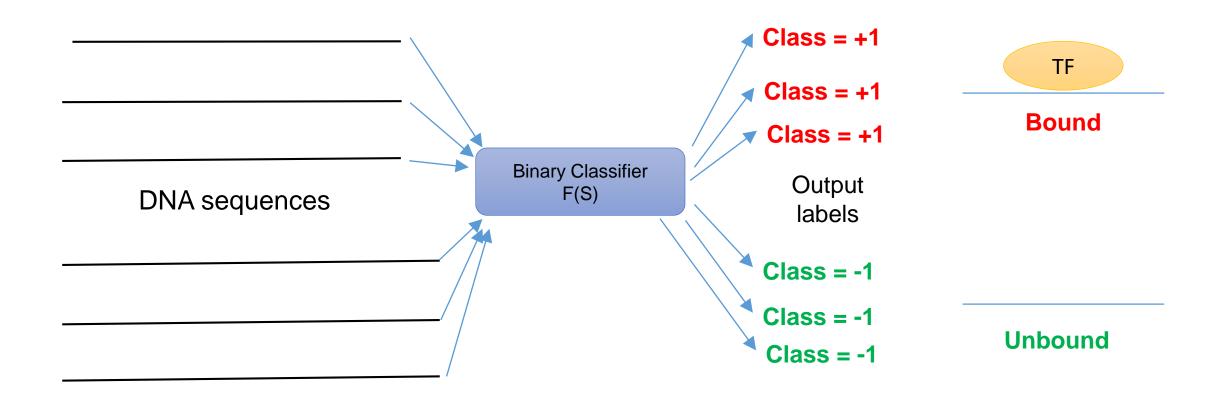
Scoring weights W



Thresholding scores



Binary classification of regulatory DNA sequences

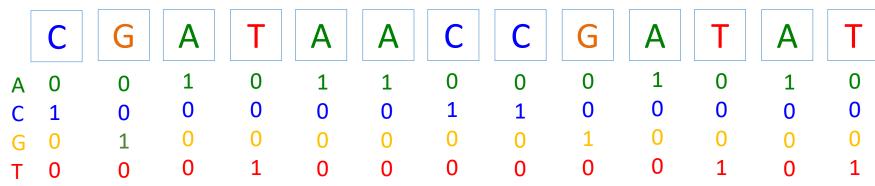


An artificial neuron on DNA sequence is a motif pattern detector

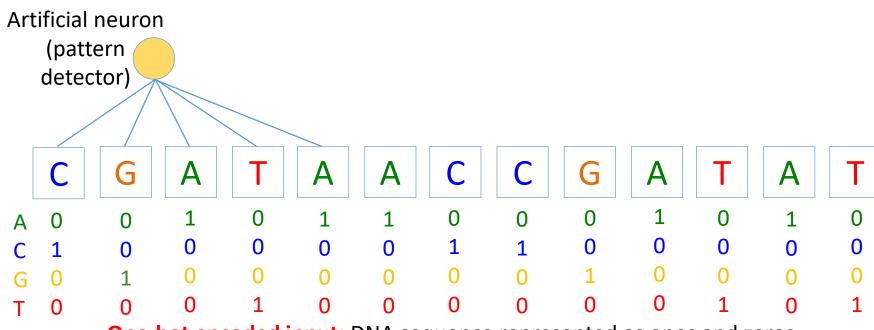
Binary Output:

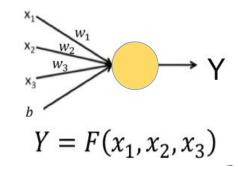
Yes (1) vs No (0)

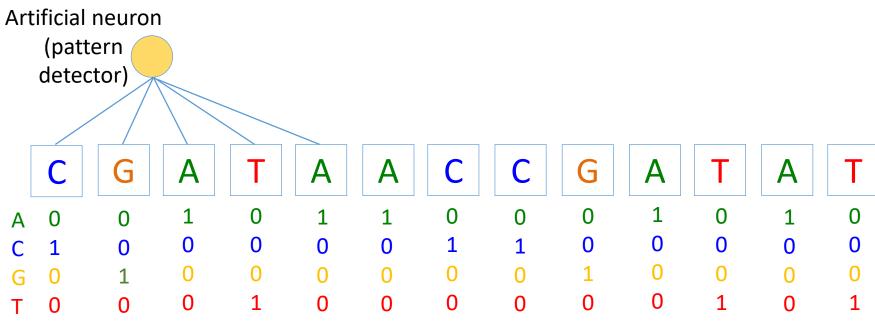
Is TF bound?

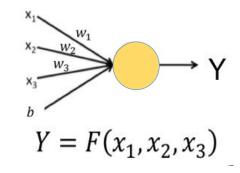


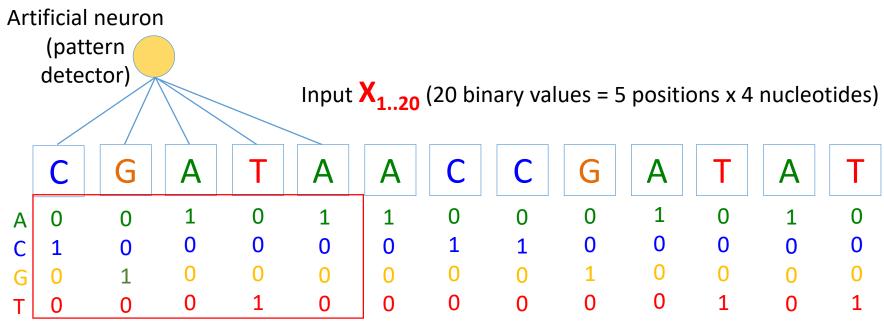
One-hot encoded input: DNA sequence represented as ones and zeros

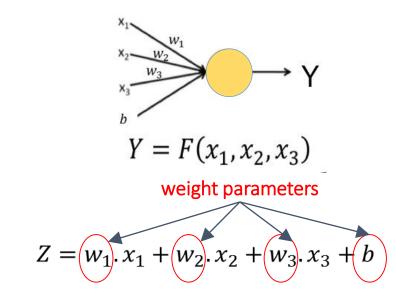


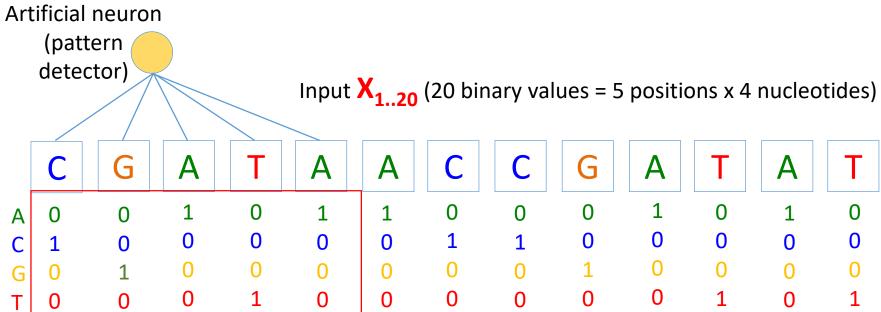




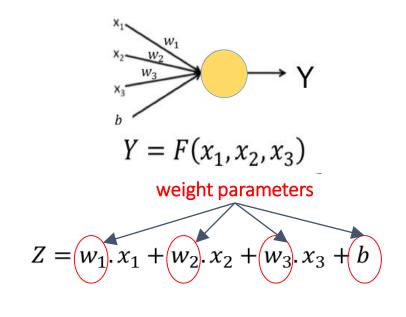


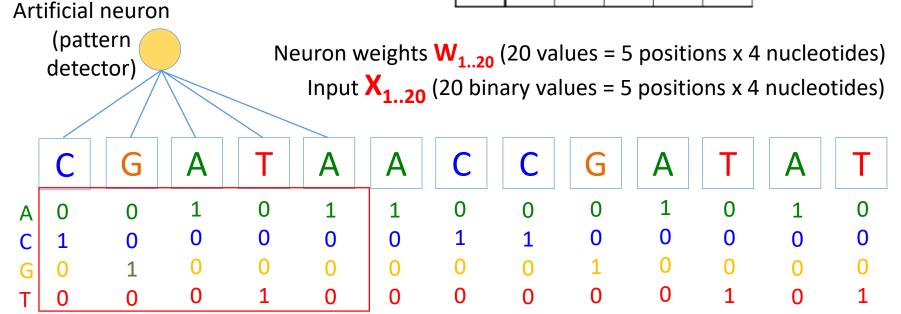


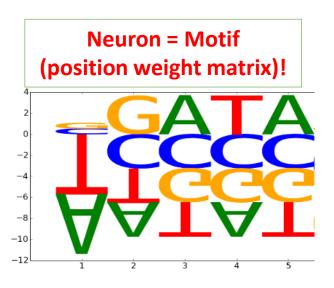




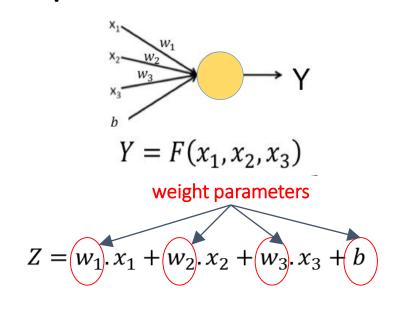
А	-5.7	-3.2	3.7	-3.2	3.7
С	0.5	-3.2	-3.2	-3.2	-3.2
G	0.5	3.7	-3.2	-3.2	-3.2
т	-5.7	-3.2	-3.2	3.7	-3.2

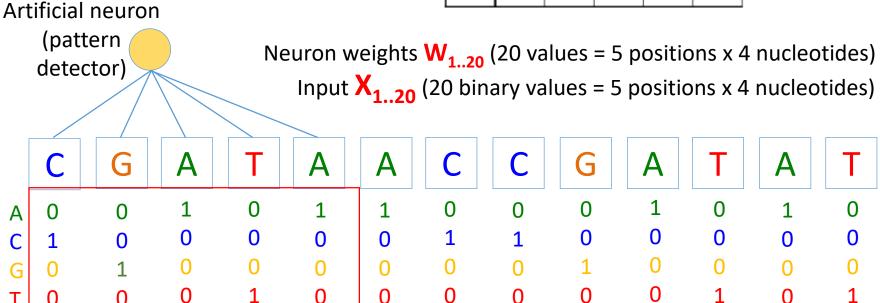


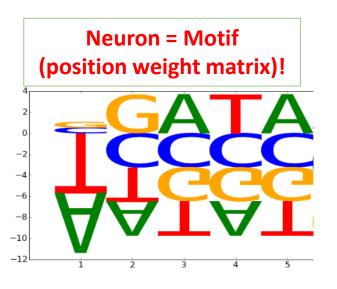


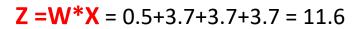


А	-5.7	-3.2	3.7	-3.2	3.7
С	0.5	-3.2	-3.2	-3.2	-3.2
G	0.5	3.7	-3.2	-3.2	-3.2
т	-5.7	-3.2	-3.2	3.7	-3.2

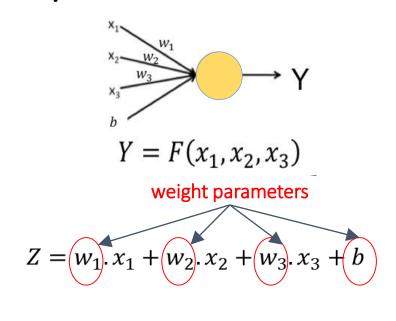


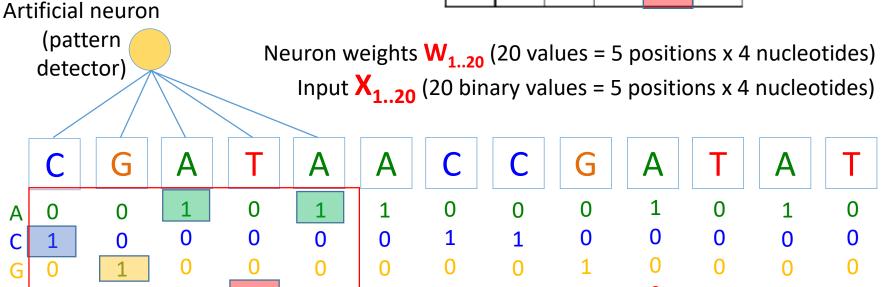


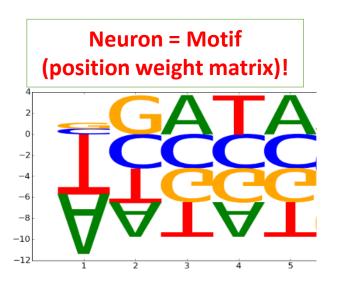


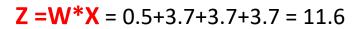


А	-5.7	-3.2	3.7	-3.2	3.7
С	0.5	-3.2	-3.2	-3.2	-3.2
G	0.5	3.7	-3.2	-3.2	-3.2
Т	-5.7	-3.2	-3.2	3.7	-3.2

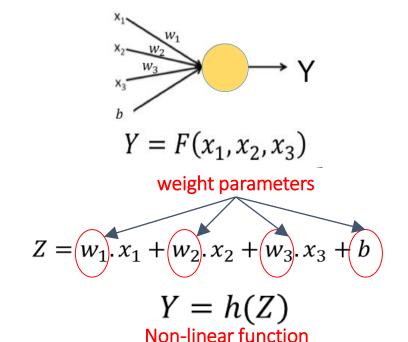


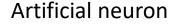






А	-5.7	-3.2	3.7	-3.2	3.7
С	0.5	-3.2	-3.2	-3.2	-3.2
G	0.5	3.7	-3.2	-3.2	-3.2
Т	-5.7	-3.2	-3.2	3.7	-3.2

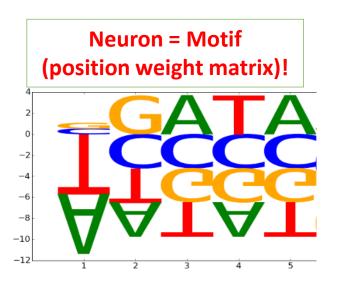




(pattern detector)

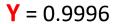
Neuron weights $W_{1..20}$ (20 values = 5 positions x 4 nucleotides) Input $X_{1..20}$ (20 binary values = 5 positions x 4 nucleotides)

	C	G	Α	T	A	Α	C	C	G	Α	T	Α	Т
Α	0	0	1	0	1	1	0	0	0	1	0	1	0
С	1	0	0	0	0	0	1	1	0	0	0	0	0
					0								
т	0	0	0	1	0	0	0	0	0	0	1	0	1



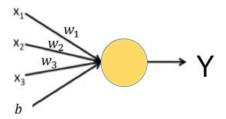
Artificial neuron

(pattern



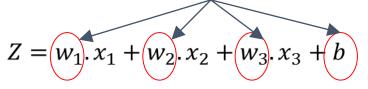
$$Z = W*X = 0.5+3.7+3.7+3.7 = 11.6$$

А	-5.7	-3.2	3.7	-3.2	3.7
С	0.5	-3.2	-3.2	-3.2	-3.2
G	0.5	3.7	-3.2	-3.2	-3.2
т	-5.7	-3.2	-3.2	3.7	-3.2



$$Y = F(x_1, x_2, x_3)$$

weight parameters

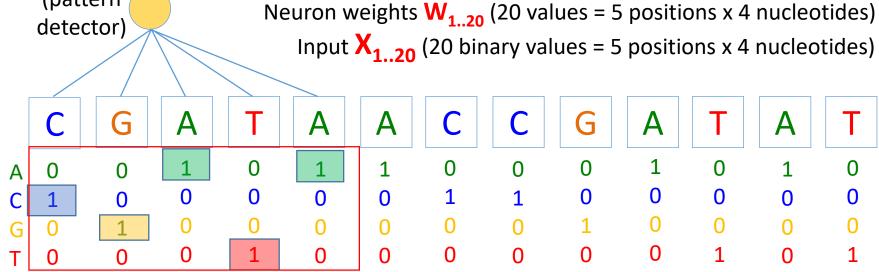


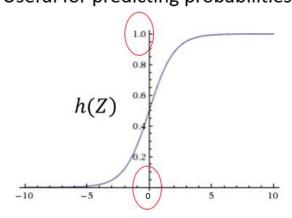
$$Y = h(Z)$$

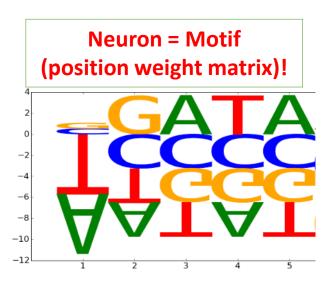
Non-linear function

1 . . . / 6: . .

Logistic / Sigmoid
Useful for predicting probabilities



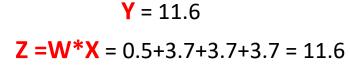




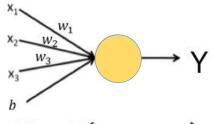
Artificial neuron

(pattern

detector)

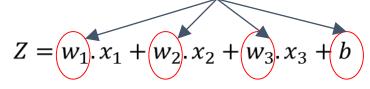


А	-5.7	-3.2	3.7	-3.2	3.7
С	0.5	-3.2	-3.2	-3.2	-3.2
G	0.5	3.7	-3.2	-3.2	-3.2
Т	-5.7	-3.2	-3.2	3.7	-3.2



$$Y = F(x_1, x_2, x_3)$$

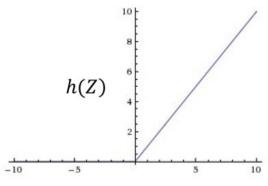
weight parameters



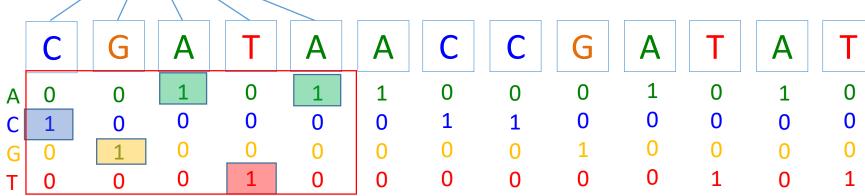
$$Y = h(Z)$$

Non-linear function

ReLu (Rectified Linear Unit) Useful for thresholding



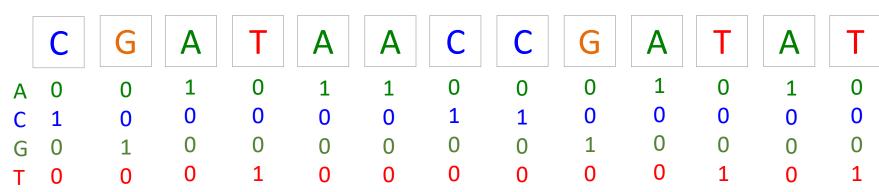
Neuron weights $W_{1,20}$ (20 values = 5 positions x 4 nucleotides) Input $X_{1,20}$ (20 binary values = 5 positions x 4 nucleotides)



Binary Output:

Yes (1) vs No (0)

Is TF bound?

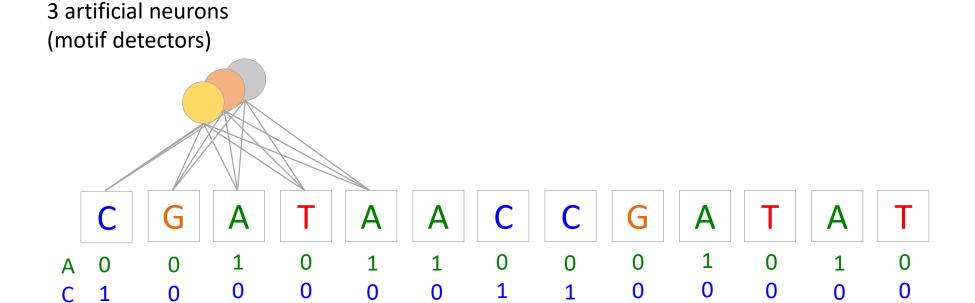


Binary Output:

Yes (1) vs No (0)

0

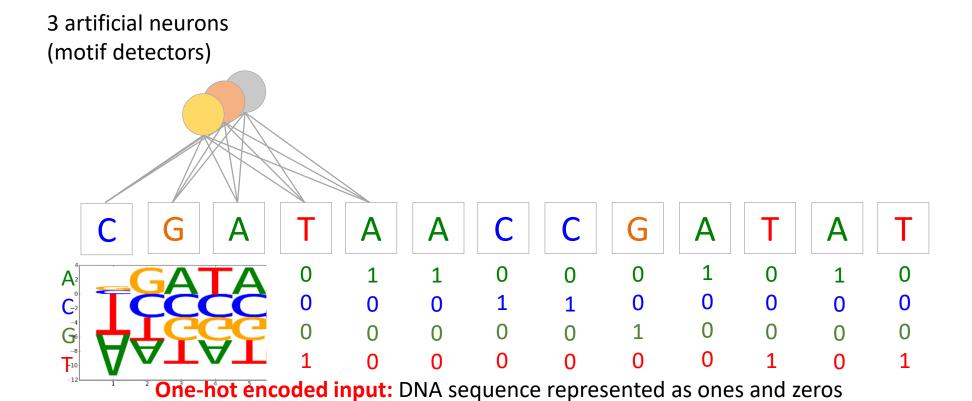
Is TF bound?



Binary Output:

Yes (1) vs No (0)

Is TF bound?

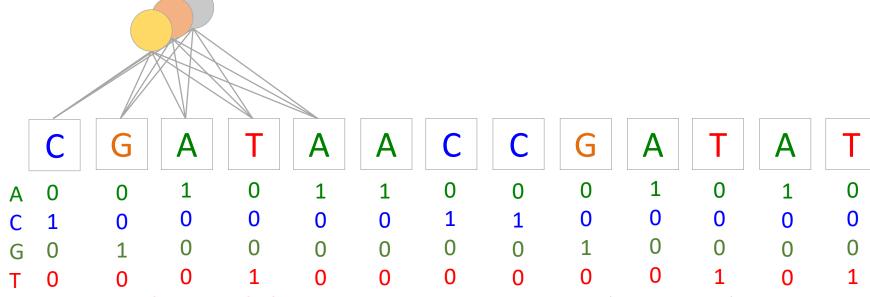


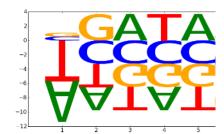
Binary Output:

Yes (1) vs No (0)

Is TF bound?

3 artificial neurons (motif detectors)





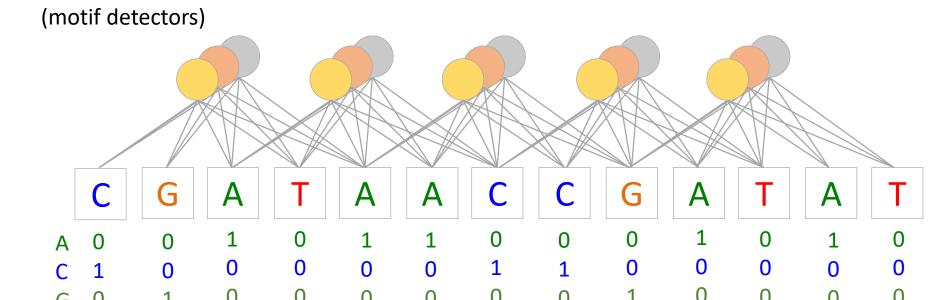
Binary Output:

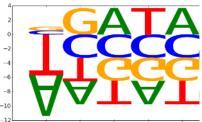
Yes (1) vs No (0)

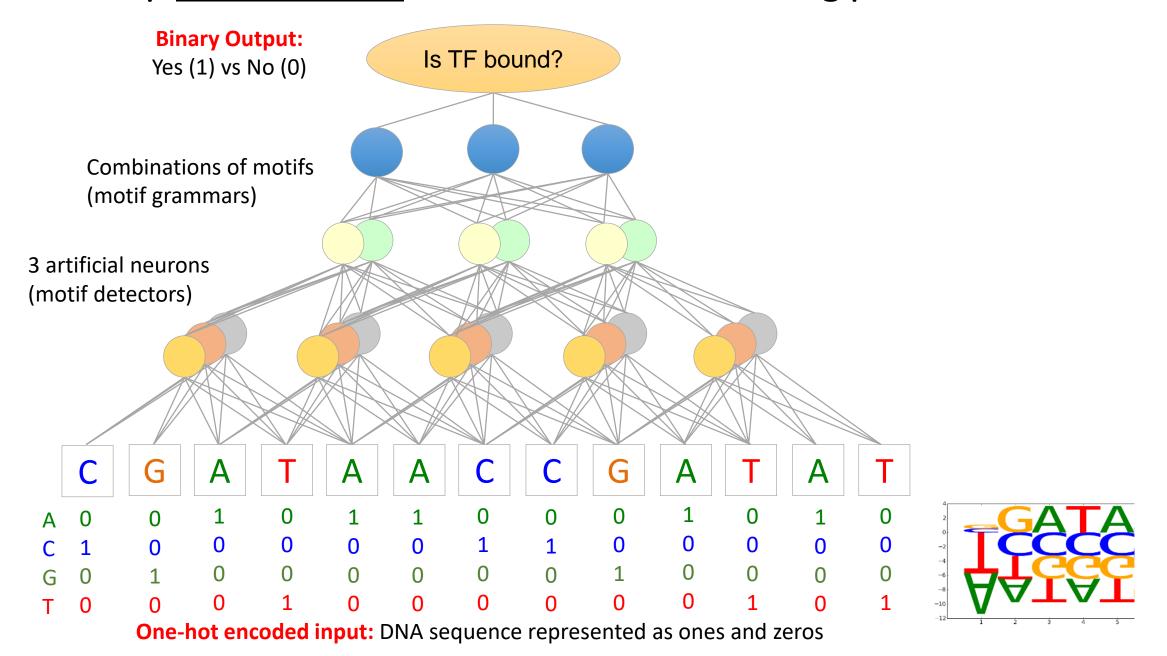
3 artificial neurons

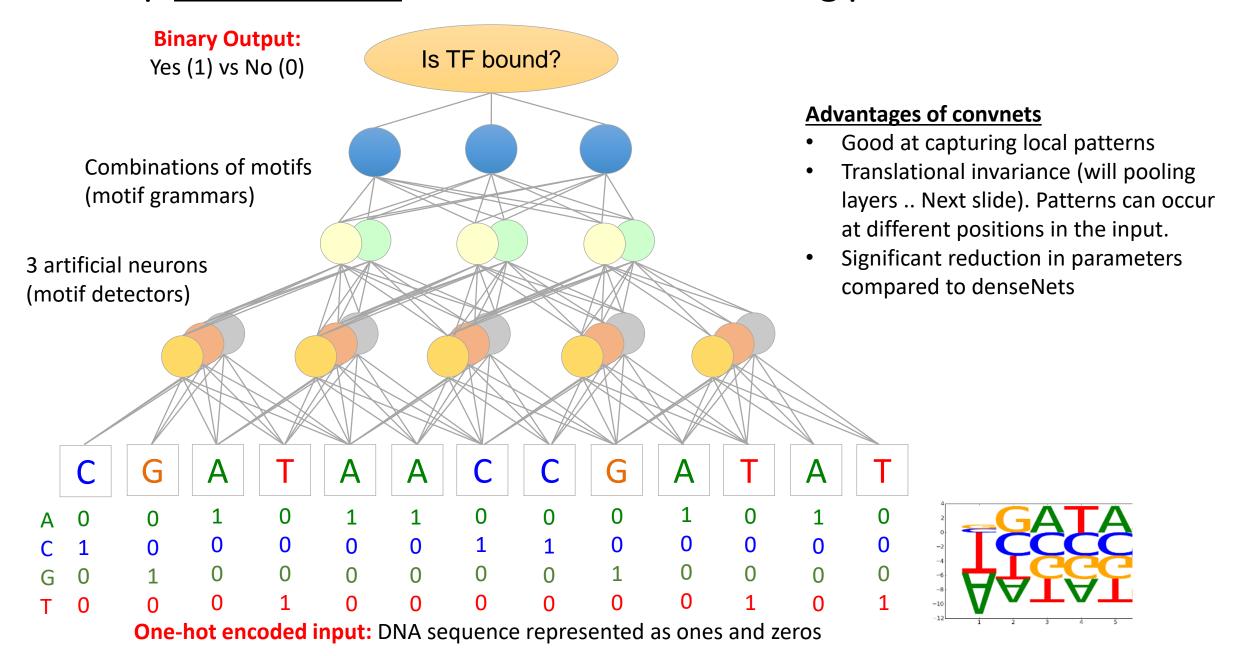
0

Is TF bound?

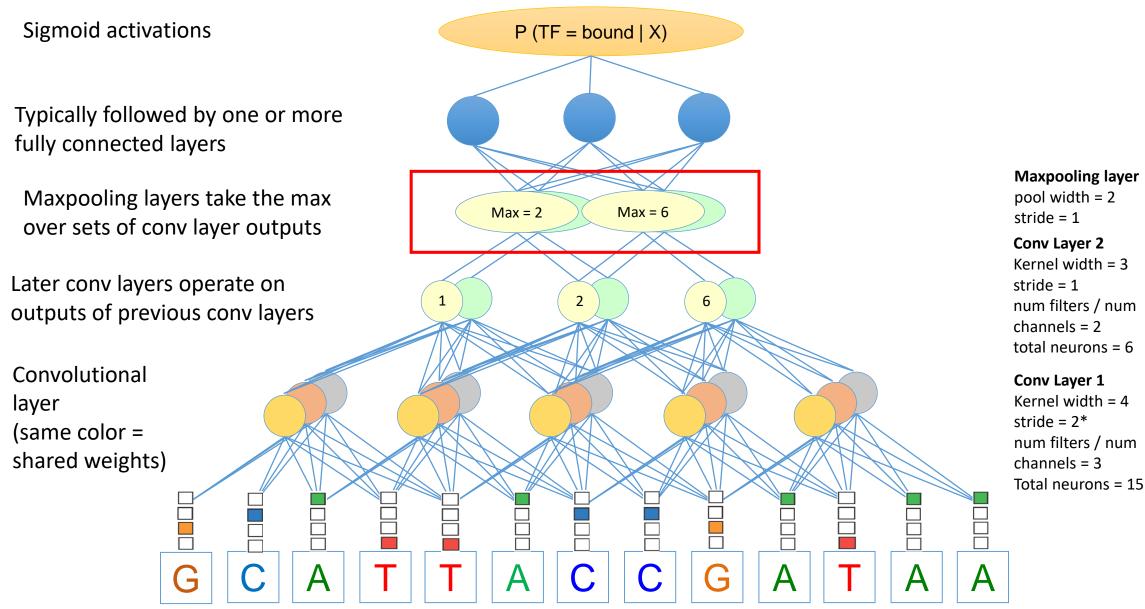






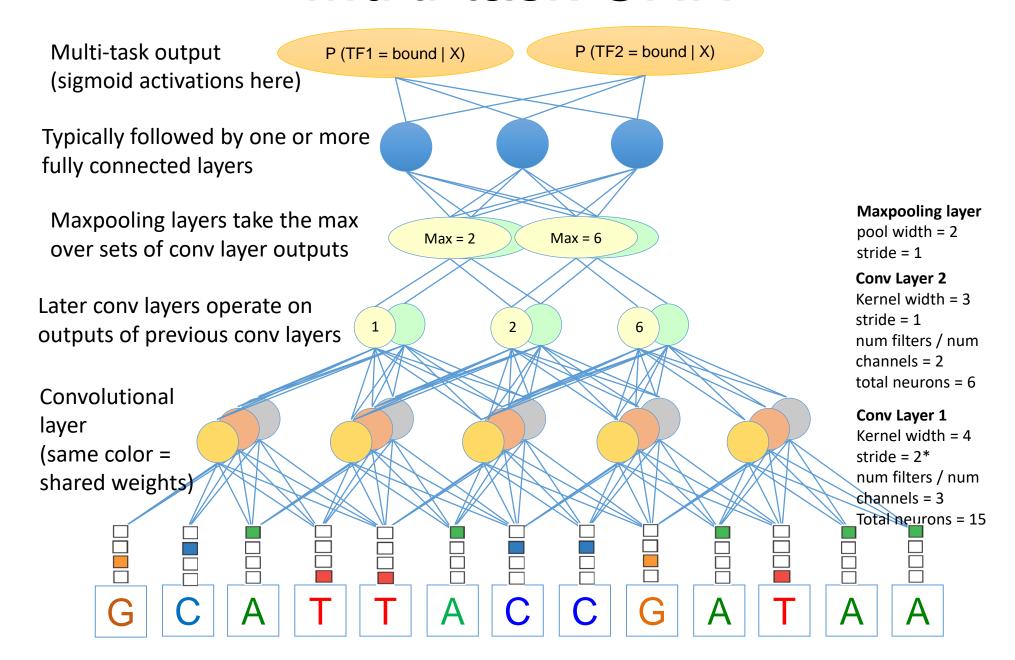


Pooling operations allow positional invariance

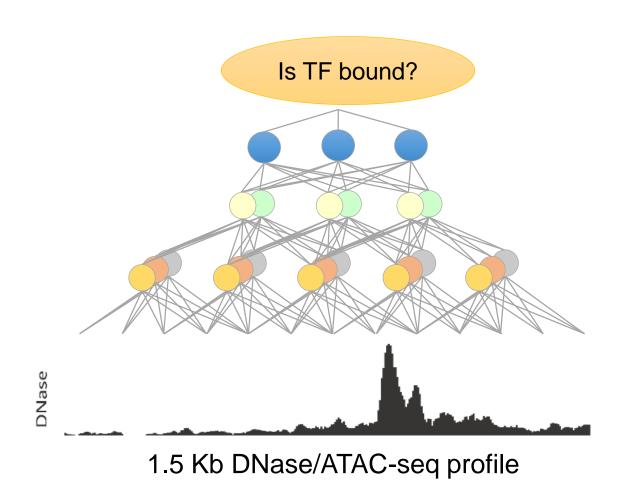


^{*}for genomics, a stride of 1 for conv layers is recommended

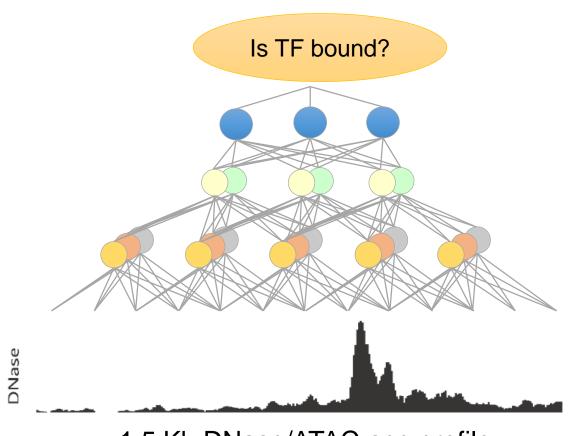
Multi-task CNN



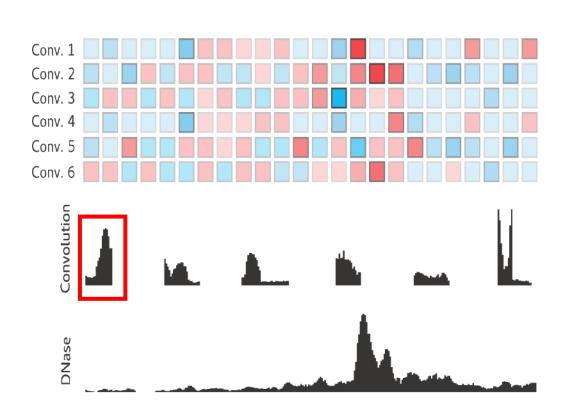
Learning predictive patterns from continuous chromatin accessibility (DNase-seq/ATAC-seq) profiles



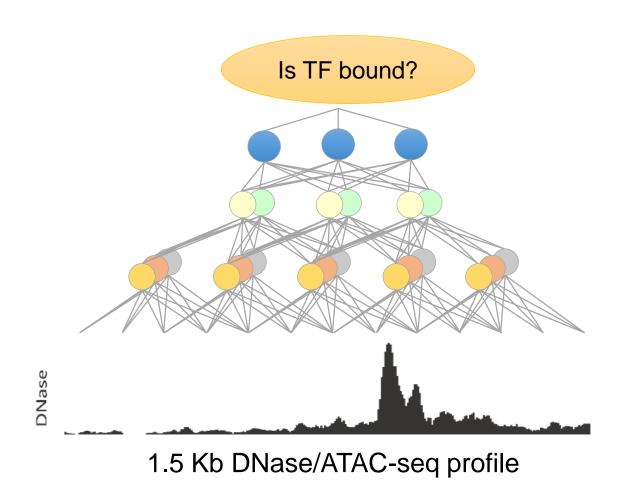
Learning predictive patterns from continuous chromatin accessibility (DNase-seq/ATAC-seq) profiles

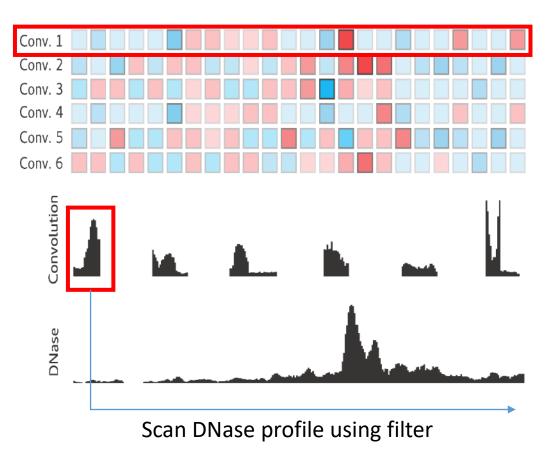






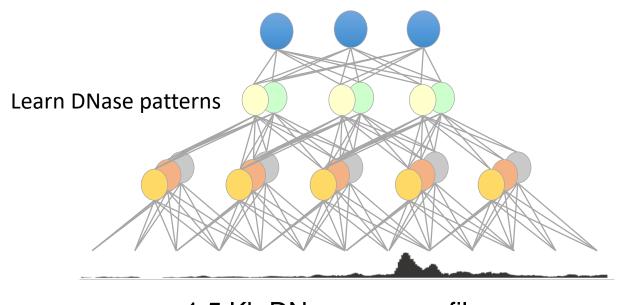
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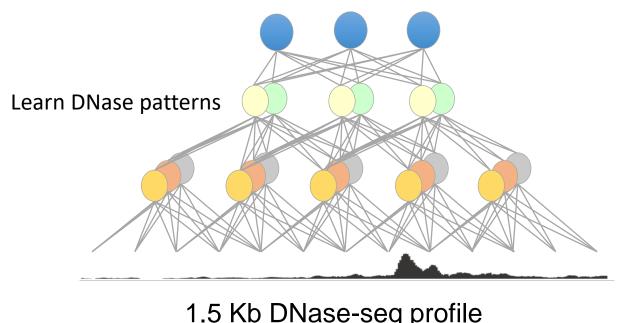


Multi-modal, multi-task CNN: Integrating raw seq + continuous DNase profiles to predict binding

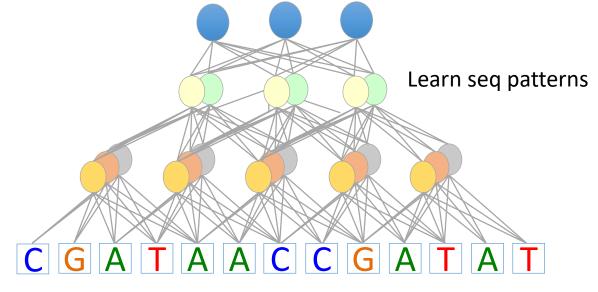
Multi-modal, multi-task CNN: Integrating raw seq + continuous DNase profiles to predict binding



Multi-modal, multi-task CNN: Integrating raw seq + continuous DNase profiles to predict binding

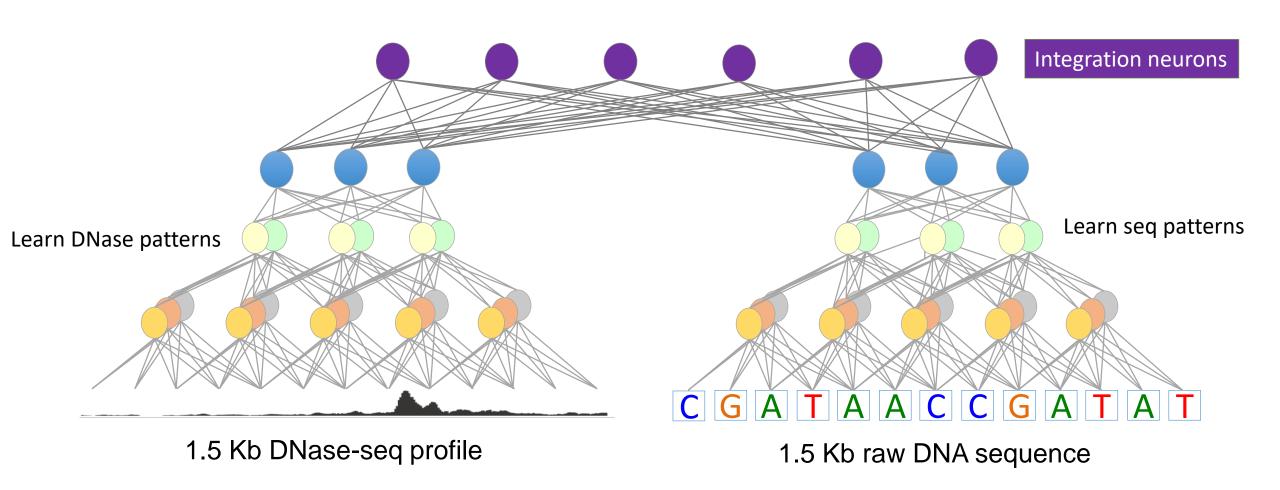


1.5 Kb DNase-seq profile

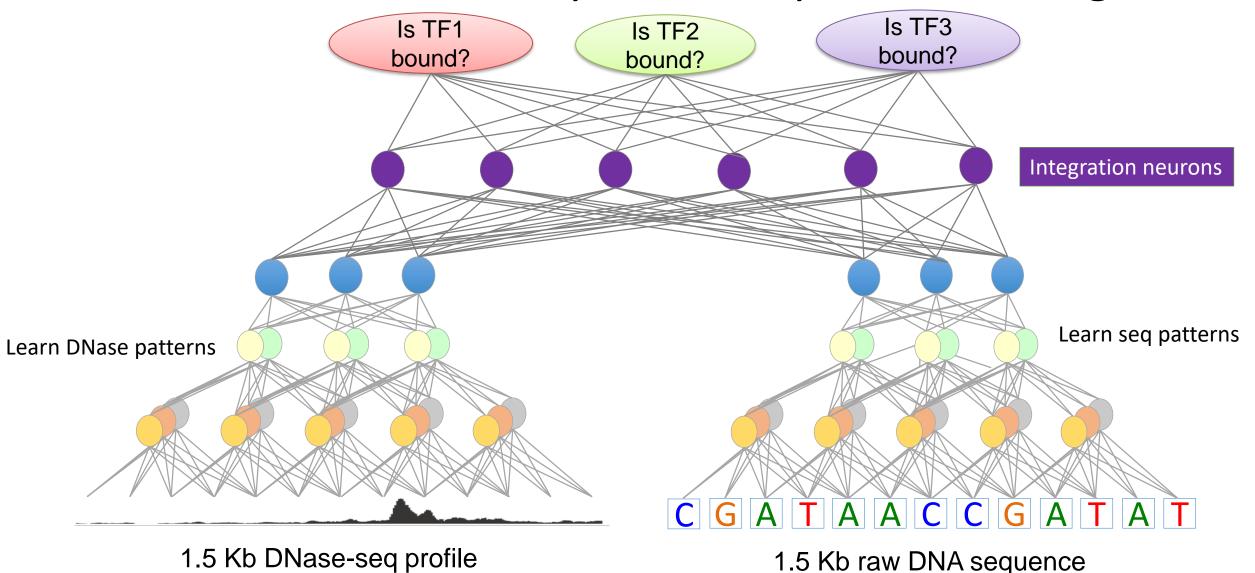


1.5 Kb raw DNA sequence

Multi-modal, multi-task CNN: Integrating raw seq + continuous DNase profiles to predict binding



Multi-modal, multi-task CNN: Integrating raw seq + continuous DNase profiles to predict binding



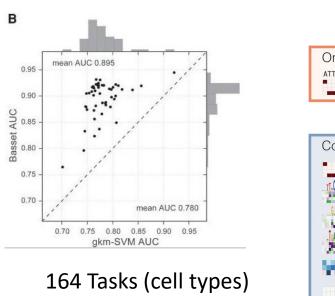
Basset: learning the regulatory code of the accessible genome with deep convolutional neural

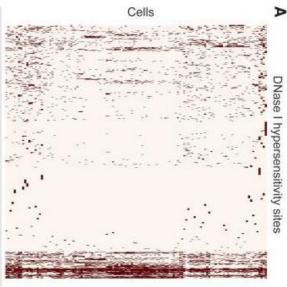
Kelley DR1, Snoek J2, Rinn JL1.

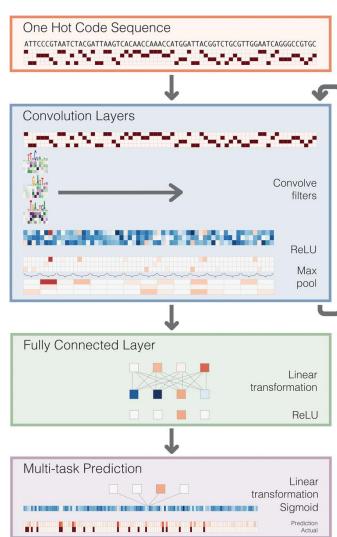
networks.

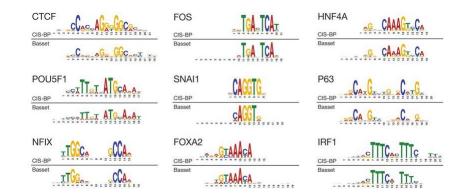
examples

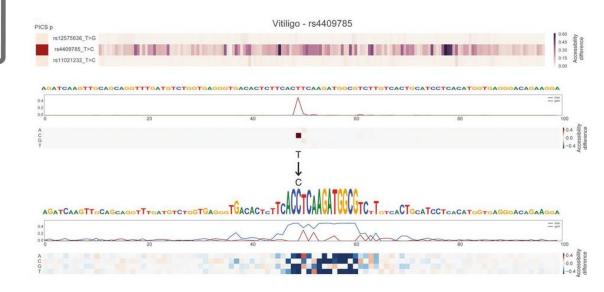
million





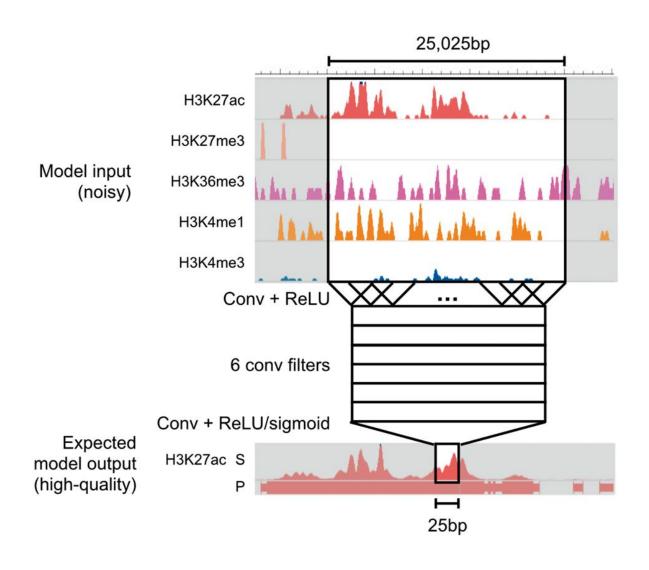






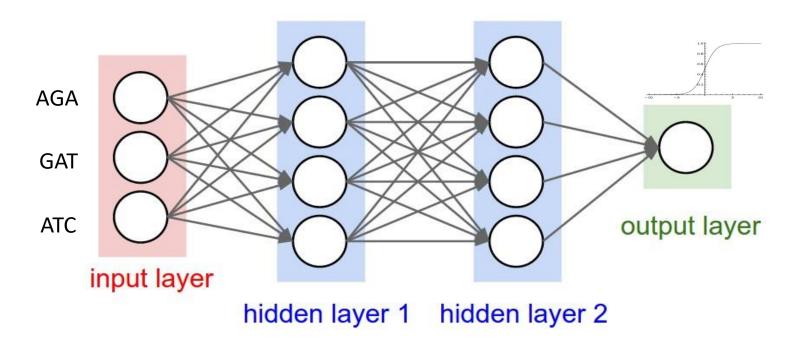
Denoising genome-wide histone ChIP-seq with convolutional neural networks.

Koh PW^{1,2}, Pierson E¹, Kundaje A^{1,2}.



How to train: SGD with backpropagation to compute gradients

Stochastic gradient descent



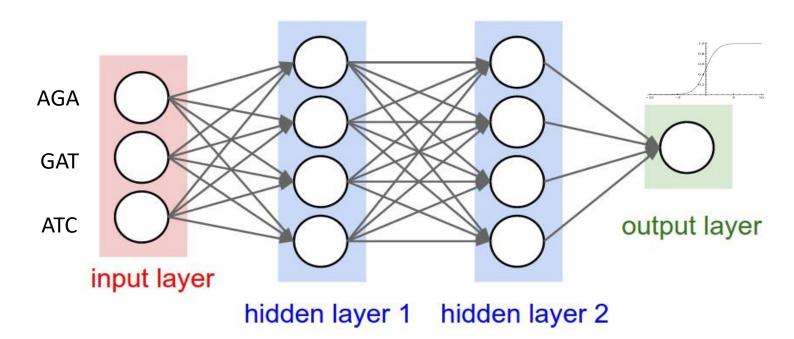
$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta).$$

One pass of updates over all training examples = 1 Epoch

Run through multiple Epochs until 'convergence'

Early stopping: Stop when validation set error stops decreasing

Stochastic gradient descent



$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta).$$

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Run through multiple Epochs until 'convergence'

Early stopping: Stop when validation set error stops decreasing

Challenge: How to compute gradients of loss wrt. parameters in a deep network

Ideas behind backpropagation: Chain rule of derivatives and reverse-mode differentiation

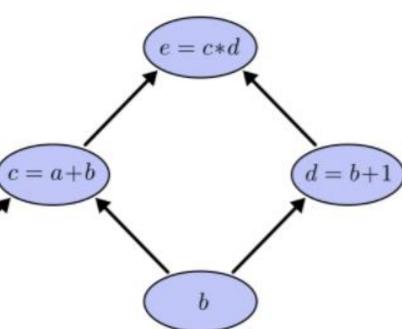
Computational graphs are a nice way to think about mathematical expressions. For example, consider the expression e = (a + b) * (b + 1). There are three operations: two additions and one multiplication. To help us talk about this, let's introduce two intermediary variables, c and d so that every function's output has a variable. We now have:

c = a + b

d = b + 1

e = c * d

To create a computational graph, we make each of these operations, along with the input variables, into nodes. When one node's value is the input to another node, an arrow goes from one to another



Derivatives on Computational Graphs

If one wants to understand derivatives in a computational graph, the key is to understand derivatives on the edges. If a directly affects c, then we want to know how it affects c. If a changes a little bit, how does c change? We call this the partial derivative of c with respect to a.

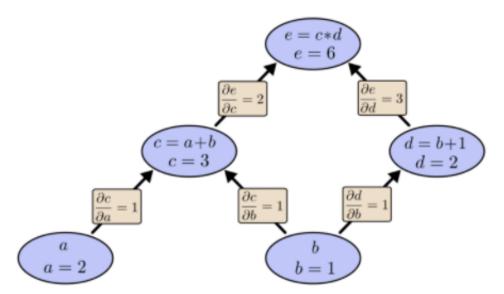


To evaluate the partial derivatives in this graph, we need the sum rule and the product rule:

$$\frac{\partial}{\partial a}(a+b) = \frac{\partial a}{\partial a} + \frac{\partial b}{\partial a} = 1$$

$$rac{\partial}{\partial u}uv=urac{\partial v}{\partial u}+vrac{\partial u}{\partial u}=v$$

Below, the graph has the derivative on each edge labeled.



http://colah.github.io/posts/2015-08-Backprop/

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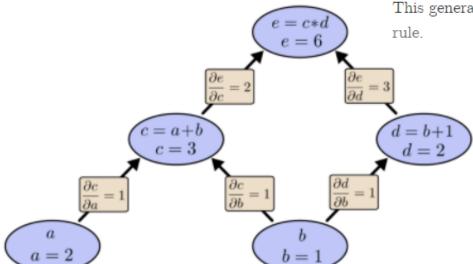
$$rac{\partial}{\partial u}uv=urac{\partial v}{\partial u}+vrac{\partial u}{\partial u}=v$$

The general rule is to sum over all possible paths from one node to the other, multiplying the derivatives on each edge of the path together. For example, to get the derivative of e with respect to b we get:

$$\frac{\partial e}{\partial b} = 1 * 2 + 1 * 3$$

This accounts for how b affects e through c and also how it affects it through d.

This general "sum over paths" rule is just a different way of thinking about the multivariate chain rule

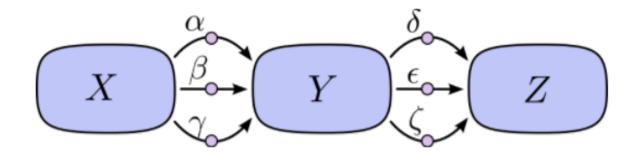


http://colah.github.io/posts/2015-08-Backprop/

Below, the graph has the derivative on each edge labeled.

Factoring Paths

The problem with just "summing over the paths" is that it's very easy to get a combinatorial explosion in the number of possible paths.



In the above diagram, there are three paths from X to Y, and a further three paths from Y to Z. If we want to get the derivative $\frac{\partial Z}{\partial X}$ by summing over all paths, we need to sum over 3*3=9 paths:

$$\frac{\partial Z}{\partial X} = \alpha \delta + \alpha \epsilon + \alpha \zeta + \beta \delta + \beta \epsilon + \beta \zeta + \gamma \delta + \gamma \epsilon + \gamma \zeta$$

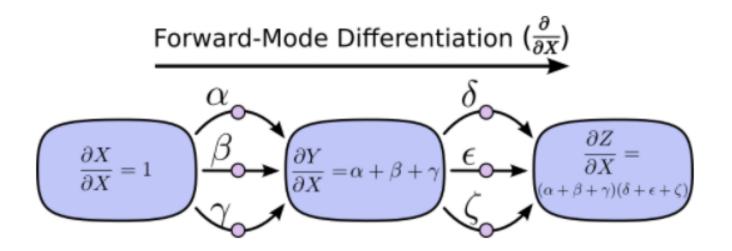
The above only has nine paths, but it would be easy to have the number of paths to grow exponentially as the graph becomes more complicated.

Instead of just naively summing over the paths, it would be much better to factor them:

$$\frac{\partial Z}{\partial X} = (\alpha + \beta + \gamma)(\delta + \epsilon + \zeta)$$

This is where "forward-mode differentiation" and "reverse-mode differentiation" come in. They're algorithms for efficiently computing the sum by factoring the paths. Instead of summing over all of the paths explicitly, they compute the same sum more efficiently by merging paths back together at every node. In fact, both algorithms touch each edge exactly once!

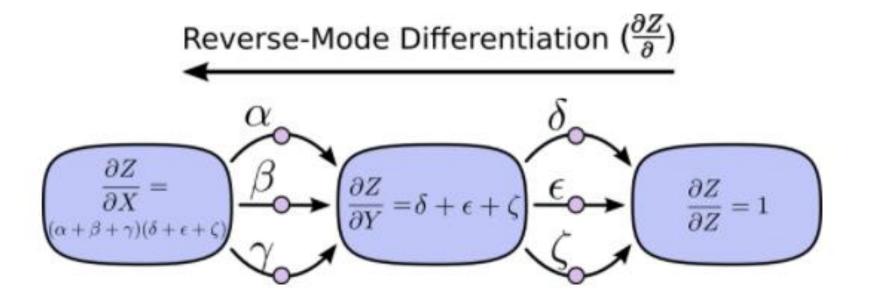
Forward-mode differentiation starts at an input to the graph and moves towards the end. At every node, it sums all the paths feeding in. Each of those paths represents one way in which the input affects that node. By adding them up, we get the total way in which the node is affected by the input, it's derivative.



Though you probably didn't think of it in terms of graphs, forward-mode differentiation is very similar to what you implicitly learned to do if you took an introduction to calculus class.

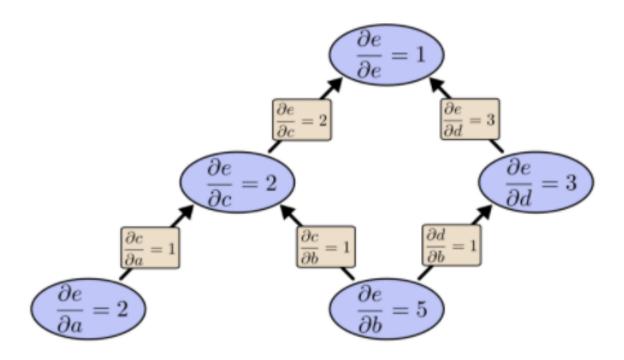
http://colah.github.io/posts/2015-08-Backprop/

Reverse-mode differentiation, on the other hand, starts at an output of the graph and moves towards the beginning. At each node, it merges all paths which originated at that node.



Forward-mode differentiation tracks how one input affects every node. Reverse-mode differentiation tracks how every node affects one output. That is, forward-mode differentiation applies the operator $\frac{\partial}{\partial X}$ to every node, while reverse mode differentiation applies the operator $\frac{\partial Z}{\partial X}$ to every node.

What if we do reverse-mode differentiation from e down? This gives us the derivative of e with respect to every node:



When I say that reverse-mode differentiation gives us the derivative of e with respect to every node, I really do mean every node. We get both $\frac{\partial e}{\partial a}$ and $\frac{\partial e}{\partial b}$, the derivatives of e with respect to both inputs. Forward-mode differentiation gave us the derivative of our output with respect to a single input, but reverse-mode differentiation gives us all of them.

http://colah.github.io/posts/2015-08-Backprop/

When training neural networks, we think of the cost (a value describing how bad a neural network performs) as a function of the parameters (numbers describing how the network behaves). We want to calculate the derivatives of the cost with respect to all the parameters, for use in gradient descent. Now, there's often millions, or even tens of millions of parameters in a neural network. So, reverse-mode differentiation, called backpropagation in the context of neural networks, gives us a massive speed up!