Photon Mapping with Participating Media

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CSE 528 Project Report

1 Introduction

Generally, rendering equation which is based on BRDF (Bidirectional reflectance distribution function) to compute the surface radiance assumes that the light only interacts with surfaces and the radiance leaving a surface remains unchanged before it hits another surfaces. However, it is only true in the case that surfaces are surrounded by a vacuum. In reality, the space between objects is typically occupied by some participating medium(e.g. air, water) which will obstructed or interact with the light when it travel through the medium. To produce realistic picture, we must take into account the interaction between light and participating media.

Jensen proposed an extension of the photon map method for computing global illuminations in scenes with participating media. The method is based on ray tracing and uses photon maps to increase efficiency and reduce noise. This method removes restrictions limiting the photon map method to surfaces by introducing a volume photon map containing photons in participating media. By introducing a new formula for estimating the radiance in a participating media, this method can simulate the effects such as multiple volume scattering, color bleeding between surfaces and volumes, and volume casutics. Photon map approach is as flexible as path tracing for complex scenes but it is more efficient because path tracing is very computing expensive.

2 Light transport in participating media

Light transport through a participating media is affected by emission, in-scattering, absorption and out-scattering. Taking these factors into account, the radiance at a position x in direction $\vec{\omega}$ is:

$$\frac{\partial L(x,\vec{\omega})}{\partial x} = \alpha(x)L_e(x,\vec{\omega}) + \sigma(x)L_i(x,\vec{\omega})$$

$$-\alpha(x)L(x,\vec{\omega}) + \sigma(x)L(x,\vec{\omega}) \tag{1}$$

where L_e is the emitted radiance, L_i is the in-scattered radiance, α is the absorption coefficient, and $\sigma$is the scattering coefficient. If the scattering and absorption coefficients are constant throughout the medium, we call the medium homogeneous or uniform; Otherwise, we call the medium non-homogeneous. The last two terms can be combined to an extinction term, where the extinction coefficient κ is defined as:

$$\kappa(\mathrm{x}) = \alpha(x) + \sigma(x) \tag{2}$$

The in-scattering radiance L_i depends on radiance L from all direction $\vec{\omega}'$ over the sphere Ω. The in-scattered radiance is

$$L_i(x,\vec{\omega}) = \int_\Omega f(x,\vec{\omega}',\vec{\omega})L(x,\vec{\omega}')d\omega' \tag{3}$$

where the normalized phase function f determines how much of the scattered light at x is scattered in direction $\vec{\omega}$. The normalized phase function integrates to 1 over the sphere. If the phase function is constant in all direction, we call the scattering isotropic or diffuse; If not, we call the scattering anisotropic or directional.

Inserting equation (2) and (3) into equation (1) and integrating both sides of the equation along a straight

path from x_o to x, we get the following equation:

$$L(x,\vec{\omega}) = \int_{xo}^{x} \tau(x',x)\alpha(x')\, L_e(x',\vec{\omega})dx' + \int_{xo}^{x} \tau(x',x)\sigma(x') \int_{\Omega} f(x,\vec{\omega}',\vec{\omega})L(x,\vec{\omega}')d\omega' dx' + \tau(x_o,x)L(x_o,\vec{\omega}) \quad (4)$$

where $\tau(x',x)$ is the transmittance along the line segment from x' to x,

$$\tau(x',x) = e^{-\int_{x'}^{x} \kappa(\varepsilon)d\varepsilon} \quad (5)$$

3 Extending the photon map to participating media

Photon map method is very easily to be extended to rendering the participating media in volume. In order to simulate the light transport in medium, several modifications have been considered.

3.1 Volume Photon Map

One thing that should be considered is that photon will be scattered and absorbed by the media. It means that in order to compute the volume radiance, we need to create volume photon map to store the photons either be absorbed or scattered. The volume photon map is used to compute radiance inside the participating media. Meanwhile, global photon map is used as before to compute the radiance on surface.

3.2 Photon Tracing in medium

When photon trace through the media, it can either pass unaffected, or it can interact with it. The probability of interaction, F(x), can be expressed as follows:

$$F(x) = 1 - e^{-\int_{x_o}^{x} \kappa(\varepsilon)d\varepsilon} \quad (6)$$

where x_o is the point where the photon enters the medium. For homogeneous media, the scattering and absorption coefficients are constant, the integration can be easily computed. For non-homogeneous media, the integration can be solved by numeric method approximately. After we get F(x), we can sample the distance which simulate the distance photon traverse before interacting with the medium.

If a photon interacts with the medium, we store the photon in the volume photon map, and then use Russian roulette decides whether the photon is scattered or absorbed. The probability of a photon being scattering is given by $\sigma(x)/\kappa(x)$. The new direction of scattered photon is chosen using importance sampling based on the phase function at x.

3.3 Volume radiance estimation

$$L_i(x,\vec{\omega}) = \frac{1}{\sigma(x)} \sum_{p=1}^{n} f(x,\vec{\omega}',\vec{\omega}) \frac{\Delta\phi_p(x,\vec{\omega}')}{\frac{4}{3}\pi r^3} \quad (7)$$

Using the equation (7), we can compute a radiance estimate at any given point inside a participating medium. The volume is approximated by $\frac{4}{3}\pi r^3$, which is the smallest sphere containing the n nearest photons. The main difference from the surface estimation equation is that the density on surface is computed using the projected area, whereas the density in a medium is computed using the full volume, as shown in Figure 2.

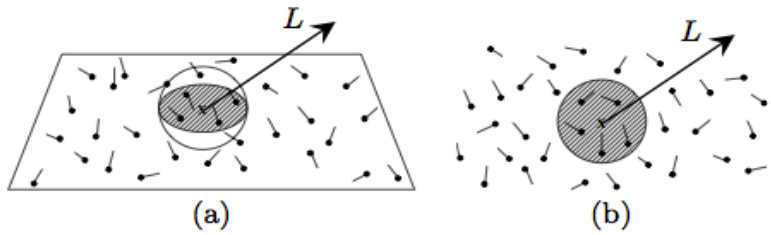


Figure 1: Radiance estimate for (a) surfaces and (b) volumes.

3.4 Volume Rendering

The radiance that a ray traversing a medium is computed with adaptive ray marching algorithm that iteratively computes radiance along the ray. In each step, the radiance from the previous step is attenuated and the contribution from emission and in-scattering within the step is added. The emitted and in-scattered radiance is approximated as being constant within each step. With this approximation, the radiance at point x_k along a ray in direction $\vec{\omega}$is computed as:

$$L(x_k, \vec{\omega}) = \alpha(x_k) L_e(x_k, \vec{\omega}) \Delta x_k$$

$$+ \sigma(x_k) L_i(x_k, \vec{\omega}) \Delta x_k$$

$$+ e^{-\kappa(x_k)\Delta x_k} L(x_{k-1}, \vec{\omega}) \tag{8}$$

where $\Delta x_k = |x_k - x_{k-1}|$ is the step size and x_0 is the intersection point of the ray with a surface.

4 Project

For this project I have implemented photon map algorithm to render the surfaces. This algorithm includes the photon tracing, photon storing and surface radiance estimation. A simple ray tracer is implemented in order to simulate the reflection and refraction effects at rendering process. The photon map algorithm for surfaces supports soft shadowing, caustics and diffuse reflection. Beyond that, the algorithm is accompanied by a photon visualization which will display where the photon are stored on the surface. To simulate the effects when light traverse through participating media, I extended the algorithm and created a new volume photon map to store photons in the volume. Photon tracing and radiance estimation are also been modified to adapt to the rendering requirement for participating media. A ray marching algorithm has been used to compute the radiance of a ray traversing a participating medium. The extended photon mapping method makes it easy to simulate homogeneous as well as nonhomogeneous media and also makes it possible to directly render the media with isotropic and anisotropic scattering. The scene used in this project is “Cornell Box”. The box has diffuse reflection on the walls, floor, back and ceiling. It contains a glass sphere and a mirror sphere. I render the “Cornell Box” with participating media and without participating media respectively.

4.1 Photon Visualization

During the course of creating photon map, it is important to verify the distribution of photons in the scene. Thus, I implemented a visualization of photons to see their locations and color values. The photon visualization is shown in Figure 2 and Figure 3. Figure 2 shows that the photons are stored in the surface and total number of photons vary from 1k in (a) to 64k in (d). Figure 3 shows that the photons are stored in the volume and surface respectively and the total number of photon in the surface and volume vary from 1.6k in (a) to 51.2k in (d).

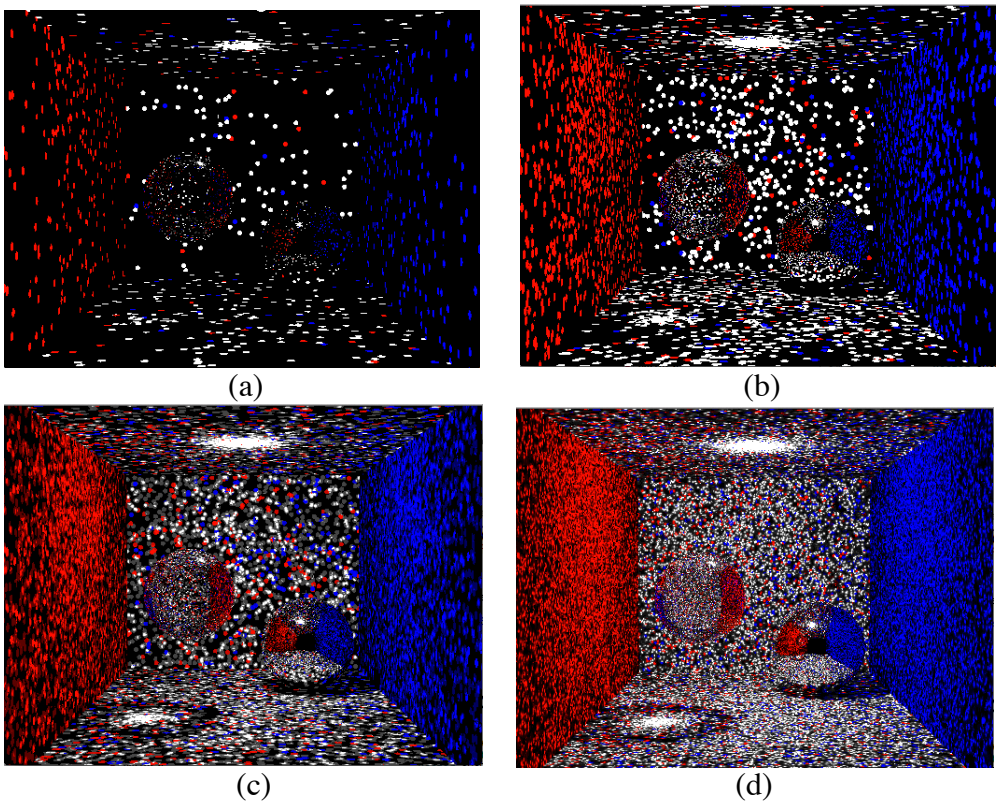


Figure 2: Photon Visualization of Surface: (a) 1k photons; (b) 4k photons; (c) 16k photons; (d)64k photons

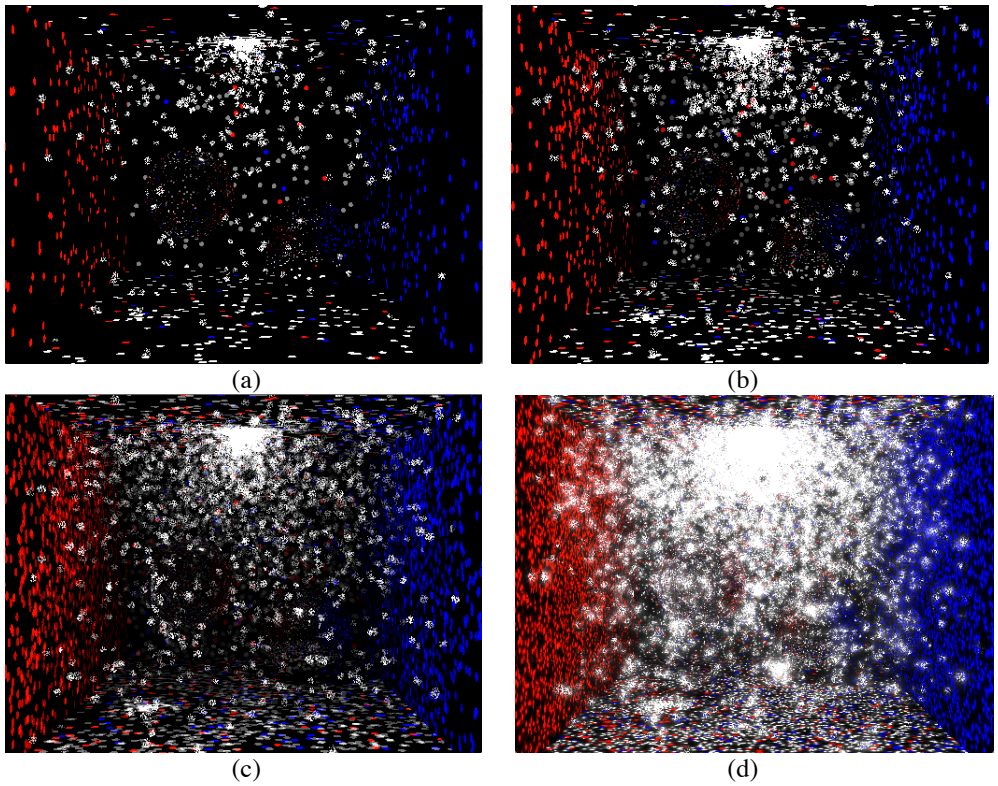


Figure 3: Photon Visualization of Volume: (a)1.6k photons; (b) 3.2kphotons; (c) 12.8k photons; (d)51.2k

photons

4.2 Cornell Box without Participating Media

Figure 3 shows direct illumination and indirect illumination on surfaces. The caustic is created as light is focused by a glass sphere. Due to the area light source, the shadow of the sphere is soft without sharp boundaries. The picture below is rendered using 10million photons and 40k photons are used to estimate the surface radiance.

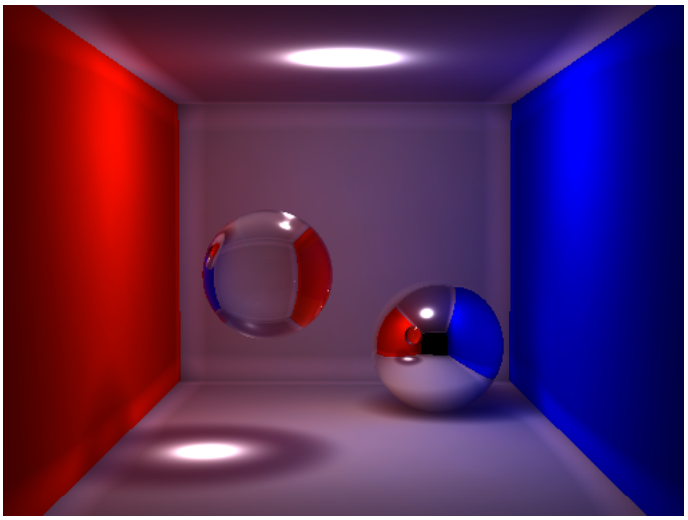
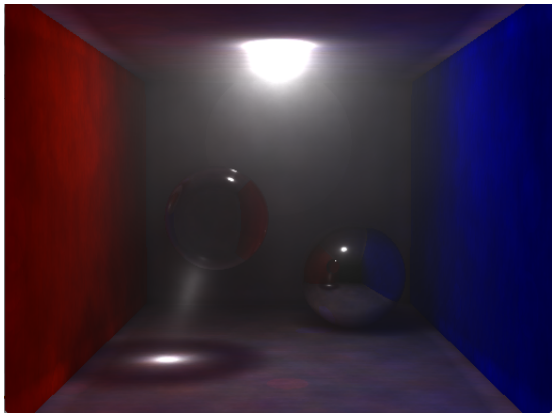


Figure 3: Cornell Box without participating media

4.3 Jittering

During rendering, I found that a fixed step size would introduce aliasing in the image in the form of clearly defined bands of intensity in the light source region. To counteract this, I added jittering to the ray marching at the rendering stage. In the algorithm, the ray would march a random distance rather than fixed step size. Below are images showing before and after the jittering algorithm. In the left image, there is obvious banding in the volume around the light source. While after introducing jittering technique, it effectively removed this banding, as shown in the right image.



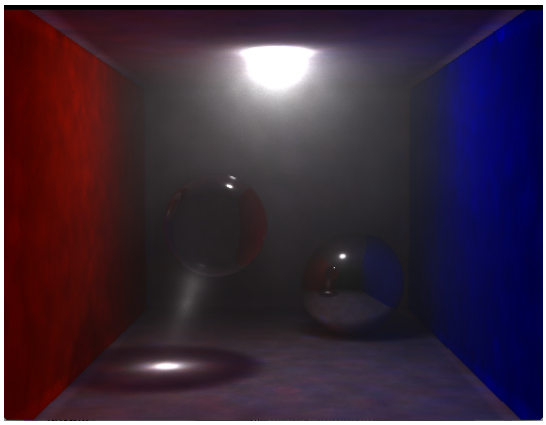


Figure 4: Comparison of before and after effects of jittering

4.4 Cornell Box with Participating Media

The following pictures show a variation of the "Cornell Box" with different participating media. To easily compare the quality of each picture, all pictures are rendered using 1 million photons, in which 500,000 photons were used for the volume while 500,000 photons were used for surfaces.

Figure 5(a) has full global illumination on the surfaces and in the isotropic, homogeneous participating media. Notice that the volume caustic created by the glass sphere becomes visible in the medium and the halo around the light source is created due to the multiple scattering. The phase function for the volume rendering is constant ($p(\theta) = 1/4\pi$) and using importance sampling technique we can simulate the scattering direction for every photon.

Figure 5(b) the media has anisotropic scattering. The scattering is modeled using Schlick phase function to model with a single lobe of eccentricity k = 0.8. The Schlick phase function shows below:

$$p(\theta) = \frac{1-k^2}{4\pi(1+k\cos\theta)^2} \tag{9}$$

and the sampled direction of θ is very simple to importance-sample and the new scattered direction is given by:

$$\cos\theta = \frac{2\xi+k-1}{2k\xi-k+1} \tag{10}$$

where ξ is random number between zero and one. We can see the halo around the light source changes shape. And it looks brighter than the previous one.

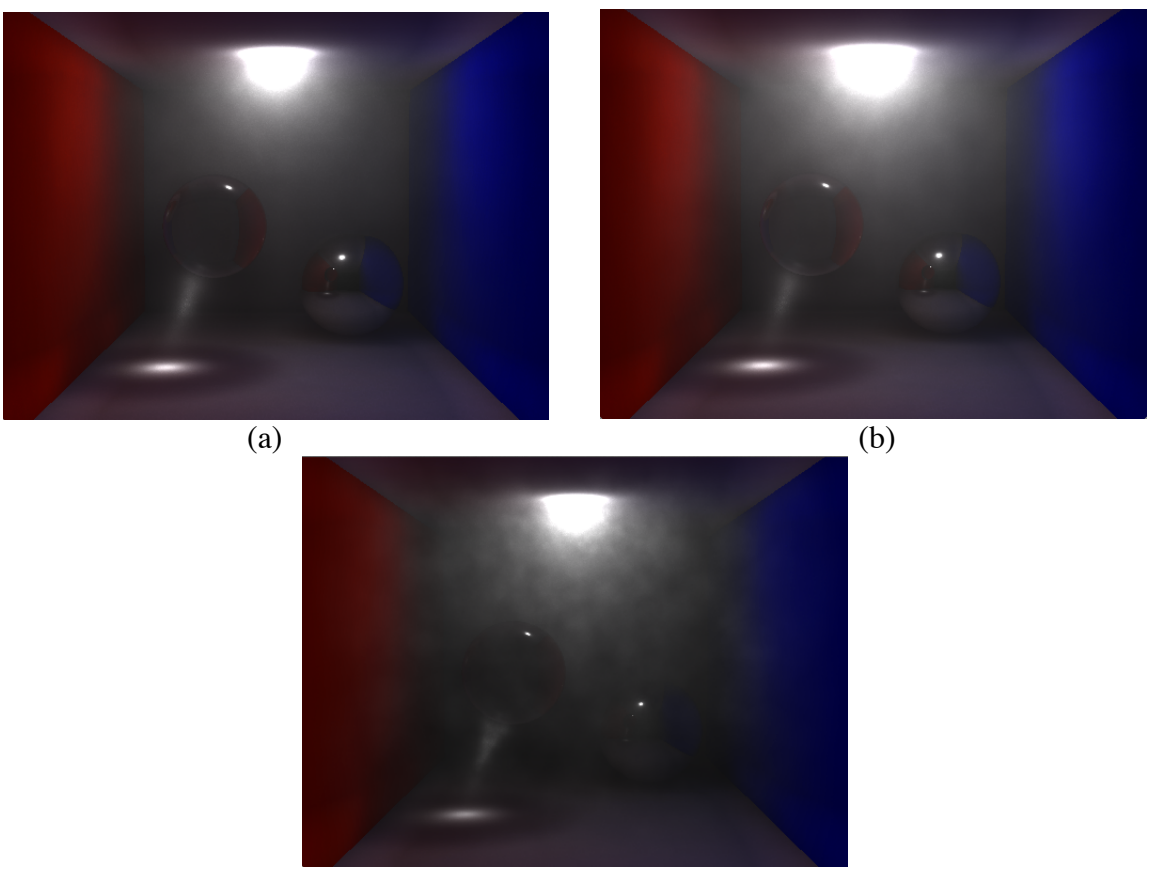


Figure 5: "Cornell boxes": (a) isotropic, homogeneous. (b)anisotropic, homogeneous medium. (c) isotropic,

nonhomogeneous medium

Figure 5(c) the participating media is nonhomogeneous. Perlin noise was used to model the variation of scattering and extinction coefficients. As a result, smoke effect is simulated and the picture looks very dark overall.

5 Conclusion

Light transport in participating media is an important effect in producing realistic computer image. Based on Jensen's propose, I implemented the photon mapping method and extended it to simulate the global illumination with participating media. This extension is simple though, it can efficiently handle both homogeneous and nonhomogeneous media with isotropic or anisotropic property. As a result, my project can simulate the effects such as caustics, volume caustics and multiple scattering.

The results demonstrated in the previous section shows that the photon mapping method for participating media works very well. During the development, I found that even a low number of photons can give good results. When computing the radiance of volume along the ray, 50-100 photons per estimation can yield good result. This is a trade-off between efficiency and high quality.

However, there are two main limitations in my project. One is that my project currently only support "Cornell Box". To support more scenes, more work should be done. The other limitation is that it is computationally expensive to render the participating media using this method. Especially, using ray march algorithm requires to estimate the radiance for every step.

Reference

Henrik Wann Jensen and Per H. Christensen. "Efficient simulation of light transport in scenes with participating media using photon maps." In *Proceedings of SIGGRAPH'98, Computer Graphics Proceedings, Annual Conference Series*, pp. 311-320, 1998.