ACTSC 632 Spring 2023

Data Science with Actuarial Applications

Week 7

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Last Week

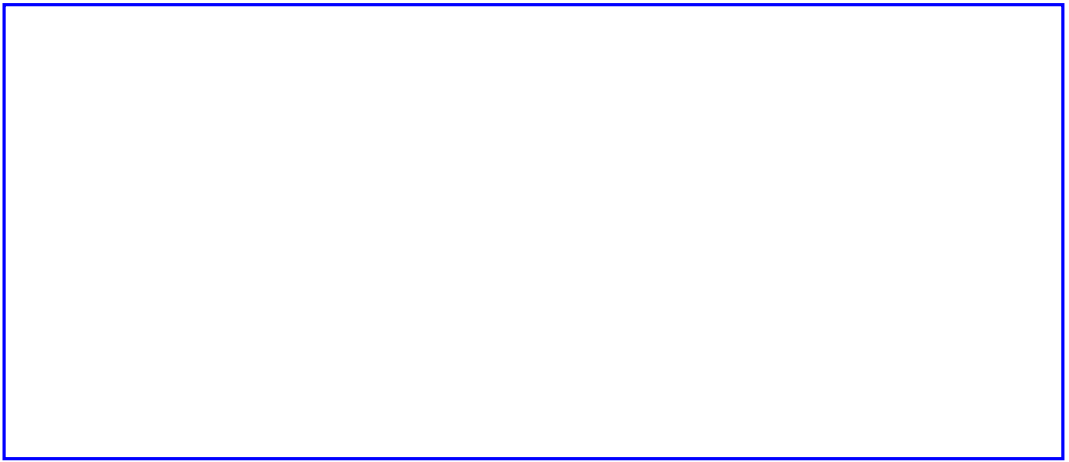
- Y is a key ratio: claim frequency or claim severity
- X is a vector of rating factors, modeled as categorical variables
- Exponential Dispersion Models (EDM)
- Multiplicative models (logarithmic link function)
- Claim frequency: Poisson regression
- Claim severity: Gamma regression

Today

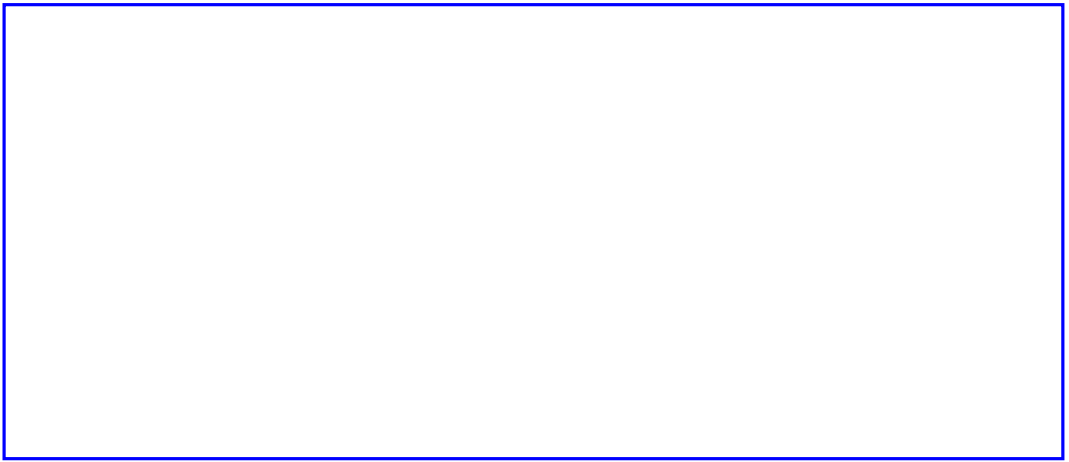
- Estimating the parameters of a GLM from:
- Writing the log-likelihood function
- Taking the derivative of the log-likelihood function
- Solving for the maximum likelihood estimators (MLE)

2.6 Parameter Estimation (MLE)

Taking derivative using the chain rule



Evaluating each partial derivative

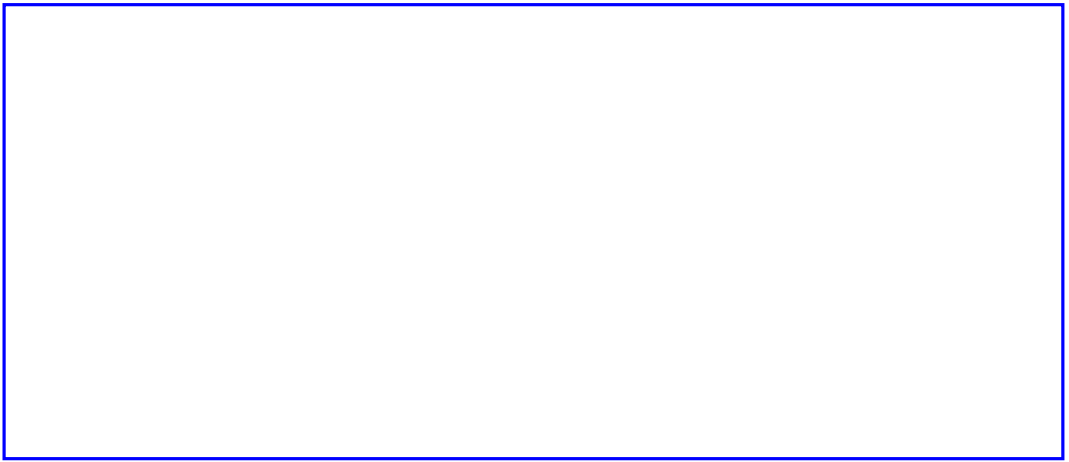


Remark: satured model

Corresponding GLM overfits. However, it is useful in the definition of deviance.

Multiplicative Poisson frequency model

Multiplicative gamma severity model



Summary of Theoretical Results

- **Goal:** estimate the parameters of a GLM using MLE
- The $(r+1)$ equations:

$$\sum_{i=1}^{n} w_i \frac{y_i - \mu_i}{v(\mu_i)\, g'(\mu_i)} x_{ij} = 0.$$

- For multiplicative Poisson frequency model:

$$\sum_{i=1}^{n} w_i (y_i - \mu_i) x_{ij} = 0.$$

- For multiplicative gamma severity model:

$$\sum_{i=1}^{n} w_i \frac{y_i - \mu_i}{\mu_i} x_{ij} = 0.$$

Example: Moped dataset

- **Goal:** Use everything we learned to build a GLM for the moped dataset.