

# Interest Rates

## How to Compare Different Interest Rates

# Types of Interest Rates

- **Effective Rates**

- Compound rates with interest paid once per unit time
- Interest is paid at the end of the period
- Used for future value and present value calculations

- **Nominal Rates**

- Compound rates with interest paid multiple times per unit time
- Example: Annual rate with monthly payments

# Common Nominal Rate Notations

Notation	Meaning	Payment Frequency
$i^{(12)} \text{ pa}$	Nominal annual rate	Monthly payments
$i^{(4)} \text{ pa}$	Nominal annual rate	Quarterly payments
$i^{(1)} \text{ pa}$	Nominal annual rate	Yearly payments

## Nominal Rate Example: 12% per annum

Assume you have \$1000 in the bank and the bank offers a **nominal interest rate of 12% per annum**.

Notation	Payment Frequency	Effective Rate per Payment
$i^{(12)}$ pa	12 times per year	1% (12% ÷ 12)
$i^{(4)}$ pa	4 times per year	3% (12% ÷ 4)
$i^{(1)}$ pa	Once per year	12% (12% ÷ 1)

Month	1	2	3	4	5	6	7	8	9	10	11	12
Monthly ( $i^{(12)}$ )	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%
Quarterly ( $i^{(4)}$ )	-	-	3%	-	-	3%	-	-	3%	-	-	3%
Yearly ( $i^{(1)}$ )	-	-	-	-	-	-	-	-	-	-	-	12%

If your have \$1000 initially, how much will you have in 6 months?

Month	1	2	3	4	5	6
Monthly ( $i^{(12)}$ )	\$1010.00	\$1020.10	\$1030.30	\$1040.60	\$1051.01	\$1061.52
Quarterly ( $i^{(4)}$ )	\$1000.00	\$1000.00	\$1030.00	\$1030.00	\$1030.00	\$1060.90
Yearly ( $i^{(1)}$ )	\$1000.00	\$1000.00	\$1000.00	\$1000.00	\$1000.00	\$1000.00

# Interest Accumulation on \$1,000

Time Period	Monthly ( $i^{(12)}$ )	Quarterly ( $i^{(4)}$ )	Yearly ( $i^{(1)}$ )
Initial	\$1,000.00	\$1,000.00	\$1,000.00
3 months	\$1,030.42	\$1,030.00	\$1,000.00
6 months	\$1,061.52	\$1,060.90	\$1,000.00
9 months	\$1,093.33	\$1,092.73	\$1,000.00
12 months	\$1,126.83	\$1,125.51	\$1,120.00

# Converting Nominal to Effective Rates

With \$1 in the bank and the bank offers a **nominal interest rate of 12% per annum**.

1. Effective rate per payment =  $(12\% \div 12) \rightarrow 1\%$

2. Compound 12 times per year:

$$(1 + 0.01)^{12} = 1.1268$$

3. Equivalent effective rate = 12.68% per annum

**Conclusion:** 12% pa nominal with monthly payments ( $i^{(12)}$ ) is equivalent to an annual effective rate of 12.68%.

## Practice Problems

Calculate the equivalent effective rate per annum for:

1. 6% per annum, monthly payments (once per month,  $i^{(12)}$ )
2. 8% per annum, quarterly payments (once per quarter,  $i^{(4)}$ )
3. 10% per annum, semiannual payments (once per 6 months,  $i^{(2)}$ )
4. 12% per annum, yearly payments (once per year,  $i^{(1)}$ )

*Hint: Use the formula  $(1 + \frac{r}{n})^n - 1$  where:*

- $r$  = nominal rate
- $n$  = number of payments per year



# Key Takeaways

## 1. Effective Rates

- Interest paid once per period
- Actual rate of return

## 2. Nominal Rates

- Interest paid multiple times per period
- Must be converted to effective rates
- Common in banking and finance

## 3. Conversion Formula

$$i_{eff} = \left(1 + \frac{i^{(n)}}{n}\right)^n - 1$$