Understanding Annuities

Regular Payments Over Time

What is an Annuity?

An annuity is a series of equal payments made at regular intervals:

Year 1	Year 2	Year 3	Year 4	Year 5
$1000	$1000	$1000	$1000	$1000

Each payment is:

- The same amount
- Made at regular intervals
- For a specified period

Present Value of an Annuity

The present value (PV) is what all future payments are worth today.

For each payment, we need to:

1. Discount it back to today
2. Add up all discounted values

$$PV = PMT \times \frac{1 - (1 + r)^{-n}}{r},$$

where:

- PMT = Payment amount
- r = Interest rate per period
- n = Number of payments

Understanding the Formula

If you receive \$1000 annually for 3 years at 5% interest:

1. First payment (1 year): $\frac{1000}{(1.05)^1} = 952.38$
2. Second payment (2 years): $\frac{1000}{(1.05)^2} = 907.03$
3. Third payment (3 years): $\frac{1000}{(1.05)^3} = 863.84$

Total Present Value = \$2,723.25

Geometric Series

A geometric series has the form:

$$S = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$$

Where:

- a is the first term
- r is the common ratio
- n is the number of terms

The sum of this finite geometric series is:

$$S = a \times \frac{1 - r^n}{1 - r}$$

For an annuity, we're calculating the present value of future payments:

$$PV = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \frac{PMT}{(1+r)^3} + \ldots + \frac{PMT}{(1+r)^n}$$

This is a geometric series where:

- $a = \frac{PMT}{(1+r)}$ (first term)
- Common ratio = $\frac{1}{(1+r)}$
- n = number of payments

Applying the geometric series formula:

$$PV = PMT \times \frac{1-(1+r)^{-n}}{r}$$

Which is our standard present value of an annuity formula.

Payment Formula

To find how much each payment should be:

$$PMT = PV \times \frac{r}{1-(1+r)^{-n}}$$

This is useful for:

- Planning savings goals
- Calculating loan payments
- Determining retirement withdrawals

Types of Annuities

1. **Ordinary Annuity (Annuity Immediate)**
    - Payments at END of each period
    - Example: Most loan payments
2. **Annuity Due**
    - Payments at START of each period
    - Example: Rent payments
    - Worth more because payments made earlier

Payment Frequency Matters!

Annual vs Monthly Payments:

- Annual: 1 payment per year
- Monthly: 12 payments per year

Need to adjust:

1. Interest rate: $r \to r/12$
2. Number of periods: $n \to n \times 12$

Example: 5% annual = 0.417% monthly

Basic Python Implementation

```
def calculate_pv_annuity(payment, rate, periods):
    """Calculate present value of an ordinary annuity"""
    if rate == 0:
        return payment * periods
    return payment * (1 - (1 + rate)**-periods) / rate

def calculate_payment(pv, rate, periods):
    """Calculate payment given present value"""
    if rate == 0:
        return pv / periods
    return pv * rate / (1 - (1 + rate)**-periods)
```

College Planning and Student Loans

Example 1: College Tuition Planning

Calculate monthly savings needed for future college expenses:

- Expected college cost: $200,000 in 10 years
- Annual interest rate: 5%
- Monthly savings plan

College Cost Considerations

1. **Rising Tuition Costs**
    - Average annual increase: 7-8%
    - Outpaces general inflation
    - Varies by institution type
2. **Additional Expenses**
    - Room and board
    - Books and supplies
    - Technology fees
    - Living expenses

Python Implementation

```python
future_college_cost = 200000
rate_annual = 0.05
rate_monthly = rate_annual/12
periods = 10 * 12  # 10 years of monthly savings

# Calculate present value needed
pv = future_college_cost / (1 + rate_monthly)**periods
# Calculate required monthly savings
monthly_savings = calculate_payment(pv, rate_monthly, periods)

print(f"Required monthly savings: ${monthly_savings:.2f}")
```

Example 2: Student Loan Payments

Calculate monthly payment for a $50,000 student loan:

- Federal loan rate: 4.99% (2023-2024), this is nominal rate, compounded monthly

$$i^{(12)} = 4.99\%$$

- Standard repayment term: 10 years
- Monthly payments

Python Implementation

```python
loan_amount = 50000
rate_annual = 0.0499
rate_monthly = rate_annual/12
periods = 10 * 12  # 10 years of monthly payments

monthly_payment = calculate_payment(loan_amount,
                                    rate_monthly,
                                    periods)
print(f"Monthly payment: ${monthly_payment:.2f}")
```

Questions for YOU to Consider

1. How much should your parents save monthly to afford your tuition?
2. How much should you borrow?
3. What's a reasonable monthly payment for your loan?

The above questions are based on making assumptions about the future, like:

- Which school you will attend?
- When do you first start college?
- Tuition amount (inflation?)
- Interest rate on loan
- Interest rate on savings
- Do you have other things to consider?

Making Assumptions

Making assumptions about the future is a key part of financial planning. It is a trade-off between:

- Oversimplifying and ignoring important details
- Overcomplicating and making it impossible to make a decision