Chapter 2 - Sets and Venn Diagrams

2.1 - Number Sets

Definition 2.1.1 (Set)

- A **set** is a collection of objects.

Example 2.1.2

- The set of vowels: $V = \{a, e, i, o, u\}$
- The set of all positive integers: $\mathbb{Z}^+ = \{1, 2, 3, 4, \ldots\}$
- The set of all positive even integers: $E = \{2, 4, 6, 8, \ldots\}$

Question: Can you think of a set that contains only one element?

Definition 2.1.3 (Element)

- The objects in a set are called **elements** or **members** of the set.

In mathematics, we use $\in$ to denote that an element is in a set, and $\notin$ to denote that an element is not in a set.

Example 2.1.4

Let $V = \{a, e, i, o, u\}$ be the set of vowels.

- a $\in$ V
- b $\notin$ V

Let $\mathbb{Z}^+$ be the set of all positive integers.

- 1 $\in \mathbb{Z}^+$
- -1 $\notin \mathbb{Z}^+$

Let $E = \{2, 4, 6, 8, \ldots\}$ be the set of all positive even integers.

- 2 $\in$ E

- 3 $\notin$ E

Definition 2.1.5 (Empty Set)

- The set that contains no elements is called the **empty set**.

We use $\emptyset$ or $\{\}$ to denote the empty set.

Exercise 2.1.6

- Is 1 an element of the empty set?
- Is anythng an element of the empty set?

Definition 2.1.7 (Cardinality)

- The **cardinality** of a set is the number of elements in the set.

We use $|S|$ or $n(S)$ to denote the cardinality of a set S.

Example 2.1.8

Let $V = \{a, e, i, o, u\}$ be the set of vowels.

- $|V| = 5$
- $n(V) = 5$
- The cardinality of the set of vowels is 5.

We say that V is a **finite set**.

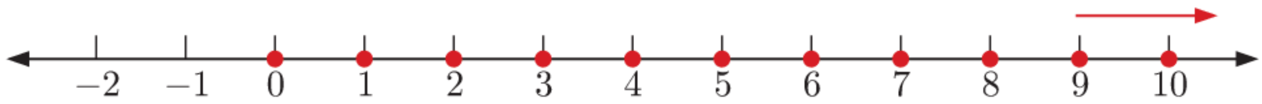
Let $\mathbb{Z}^+$ be the set of all positive integers.

- $|\mathbb{Z}^+| = \infty$
- $n(\mathbb{Z}^+) = \infty$
- The cardinality of the set of all positive integers is infinity.

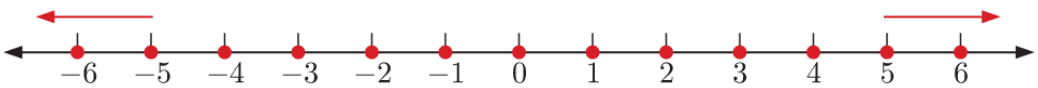
We say that $\mathbb{Z}^+$ is an **infinite set**.

Example 2.1.9 (Special Sets)

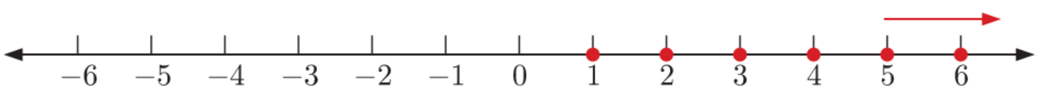
- $\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, 7, ...\}$ is the set of all **natural** or **counting numbers**.



- $\mathbb{Z} = \{0, \pm1, \pm2, \pm3, \pm4, ...\}$ is the set of all **integers**.



- $\mathbb{Z}^+ = \{1, 2, 3, 4, 5, 6, 7, ...\}$ is the set of all **positive integers**.

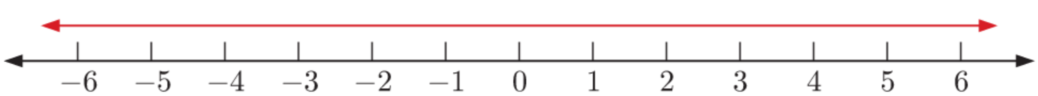


- $\mathbb{Q}$ is the set of all **rational numbers**, or numbers which can be written in the form $\frac{p}{q}$ where p and q are integers, $q \neq 0$.

For example: $\frac{15}{4}$, $10\ (= \frac{10}{1})$, $0.5\ (= \frac{1}{2})$, and $-\frac{3}{8}$ are all rational numbers.
We cannot represent the rational numbers on a number line, because there are infinitely many of them, and in between them are **irrational numbers** which cannot be written in rational form.
For example:

  - Radicals or surds such as $\sqrt{2}$ and $\sqrt{7}$ are irrational.
  - $\pi \approx 3.141\,592\,65$ is an irrational number.
  - Decimal numbers which neither terminate nor recur are irrational.

- $\mathbb{R}$ is the set of all **real numbers**, which are all numbers which can be placed on the number line.



$\mathbb{R}$ includes all rational and irrational numbers.
$\frac{2}{0}$ and $\sqrt{-2}$ are not real numbers because we cannot write them in decimal form or place them on a number line.

Exercise 2.1.10

EXERCISE 2A

1 Write using set notation:

- **a** 8 is an element of set P.
- **b** k is not an element of set S.
- **c** 14 is not an element of the set of all odd numbers.
- **d** There are 9 elements in set Y.

2 True or false?

- **a** $3 \in \mathbb{Z}^+$
- **b** $6 \in \mathbb{Z}$
- **c** $\frac{3}{4} \in \mathbb{Q}$
- **d** $\sqrt{2} \notin \mathbb{Q}$
- **e** $-\frac{1}{4} \notin \mathbb{Q}$
- **f** $2\frac{1}{3} \in \mathbb{Z}$
- **g** $0.3684 \in \mathbb{R}$
- **h** $\dfrac{1}{0.1} \in \mathbb{Z}$

3 Determine whether each of the following numbers is rational, irrational, or neither:

- **a** 8
- **b** -8
- **c** $2\frac{1}{3}$
- **d** $-3\frac{1}{4}$
- **e** $\sqrt{3}$
- **f** $\sqrt{-3}$
- **g** $\sqrt{400}$
- **h** 9.176
- **i** $\frac{1}{0}$
- **j** $\pi - \pi$

4 For each of the following sets:

- **i** list the elements of the set
- **ii** determine whether the set is finite or infinite
- **iii** if the set is finite, find the number of elements in the set.

- **a** $A = \{\text{factors of } 6\}$
- **b** $B = \{\text{multiples of } 6\}$
- **c** $C = \{\text{factors of } 17\}$
- **d** $D = \{\text{multiples of } 17\}$
- **e** $E = \{\text{prime numbers less than } 20\}$
- **f** $F = \{\text{composite numbers between } 10 \text{ and } 30\}$

5 Show that each of the following numbers is rational:

- **a** $0.\overline{7}$
- **b** $0.\overline{41}$
- **c** $0.\overline{324}$

6 Explain why 0.527 is a rational number.

7 Explain why $0.\overline{9} \in \mathbb{Z}$.

8 Give examples to show that these statements are false:

- **a** The sum of two irrationals is irrational.
- **b** The product of two irrationals is irrational.

2.2 - Interval Notation

Definition 2.2.1 (Interval)

An **interval** is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set.

- **Open Interval:** $(a, b) = \{x | a < x < b, x \in \mathbb{R}\}$
- **Closed Interval:** $[a, b] = \{x | a \leq x \leq b, x \in \mathbb{R}\}$
- **Half-Open Interval:** $[a, b) = \{x | a \leq x < b, x \in \mathbb{R}\}$
- **Half-Open Interval:** $(a, b] = \{x | a < x \leq b, x \in \mathbb{R}\}$

where $\{x \in \mathbb{R} | a < x < b\}$ is read as "the set of all x in the real numbers such that $a < x < b$".

Example 2.2.2

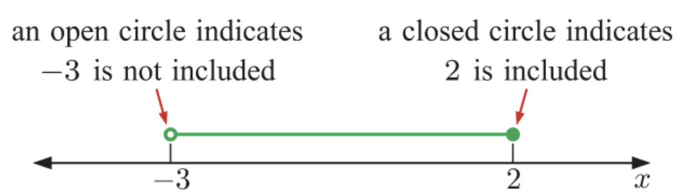
There can also be intervals where x is a integer, or a natural number, or a rational number, etc.

- $\{x | a \leq x \leq b, x \in \mathbb{N}\}$
- $\{x | a \leq x \leq b, x \in \mathbb{Q}\}$

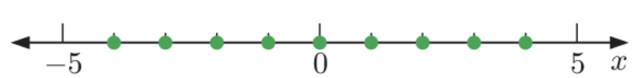
However, there are no simple notations for these intervals.

For example:

- $\{x \mid -3 < x \leqslant 2, \;\; x \in \mathbb{R}\}$
  reads "the set of all real x such that x lies between minus 3 and 2, including 2".
  We can represent the set on a number line as:



  Unless stated otherwise, we *assume* we are dealing with *real* numbers. Thus, the set can also be written as $\{x \mid -3 < x \leqslant 2\}$.
- $\{x \mid -5 < x < 5, \;\; x \in \mathbb{Z}\}$
  reads "the set of all integers x such that x lies between minus 5 and 5".
  We can represent the set on a number line as:

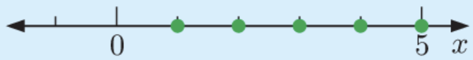


Exercise 2.2.3

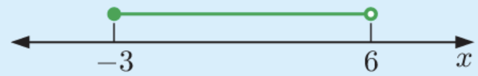
Example 2 **Self Tutor**

Write using interval notation:

a



b



a $\{x \mid 1 \leqslant x \leqslant 5,\ \ x \in \mathbb{N}\}$
or $\{x \mid 1 \leqslant x \leqslant 5,\ \ x \in \mathbb{Z}\}$

b $\{x \mid -3 \leqslant x < 6\}$

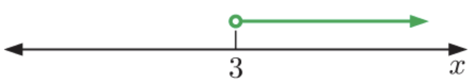
EXERCISE 2B

1 Explain the meaning of:

a $\{x \mid x > 4\}$
b $\{x \mid x \leqslant 5,\ \ x \in \mathbb{Z}\}$
c $\{y \mid 0 < y < 8\}$
d $\{x \mid 1 \leqslant x \leqslant 4,\ \ x \in \mathbb{Z}\}$
e $\{t \mid 2 < t < 7,\ \ t \in \mathbb{R}\}$
f $\{n \mid n \leqslant 3 \ \text{ or } \ n > 6\}$

2 Write using interval notation:

a



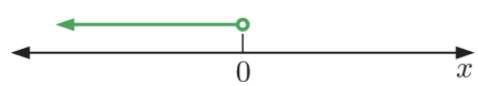
b

c

d

e

f



3 Represent each of the following number sets on a number line:

a $\{x \mid 4 \leqslant x < 8,\ \ x \in \mathbb{N}\}$
b $\{x \mid -5 < x \leqslant 4,\ \ x \in \mathbb{Z}\}$
c $\{x \mid -3 < x \leqslant 5,\ \ x \in \mathbb{R}\}$
d $\{x \mid x > -5,\ \ x \in \mathbb{Z}\}$
e $\{x \mid x \leqslant 6\}$
f $\{x \mid -5 \leqslant x \leqslant 0\}$

4 Write in interval notation:

a the set of all real numbers greater than 7
b the set of all integers between -8 and 15
c the set of all rational numbers between 4 and 6, including 4.

2.3 - Subsets and Complements

Definition 2.3.1 (Subset)

- A set A is a **subset** of a set B if every element of A is also an element of B.
- We use $A \subseteq B$ to denote that A is a subset of B.
- If not every element of A is an element of B, then A is not a subset of B, denoted by $A \nsubseteq B$.

Example 2.3.2

Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$, and $C = \{2, 4, 6\}$.

- $A \subseteq B$
- $C \nsubseteq B$
- $B \subseteq \mathbb{Z}^+$
- $B \subseteq B$

Exercise 2.3.3

1 For each of the following sets A and B, decide whether $A \subseteq B$:

- **a** $A = \{2, 5, 6\}, \quad B = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- **b** $A = \{4, 8, 11, 12\}, \quad B = \{2, 4, 6, 8, 10, 12, 14, 16\}$
- **c** $A = \varnothing, \quad B = \{1, 4, 7, 10\}$
- **d** $A = \{5, 10, 15, 20, 25, 30\}, \quad B = \{10, 15, 20\}$
- **e** $A = \{6, 7, 8\}, \quad B = \mathbb{N}$

Definition 2.3.4 (Complement)

Let U denote the **universal set**, which usually refers to the set of all elements under consideration.

- The **complement** of a set A is the set of all elements in U that are not in A.
- We use A' to denote the complement of A, hence

$$A' = \{x | x \notin A, x \in U\}$$

Example 2.3.5

Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 3\}$, and $B = \{2, 4, 6\}$.

- $A' = \{4, 5, 6\}$
- $B' = \{1, 3, 5\}$

Exercise 2.3.5

5 Suppose $U = \{x \mid x \leqslant 9,\ \ x \in \mathbb{Z}^+\}$. Find the complement of:

- **a** $A = \{2,\ 5,\ 6\}$
- **b** $B = \{\text{prime numbers in } U\}$
- **c** $C = \{\text{odd numbers in } U\}$
- **d** $D = \{\text{multiples of 4 in } U\}$
- **e** $E = \varnothing$
- **f** $F = \{x \mid x < 3,\ \ x \in \mathbb{Z}^+\}$.

6 Suppose $U = \{\text{letters of the English alphabet}\}$. Find the complement of:

- **a** $P = \{\text{C, F, J, M, P, U, Y, Z}\}$
- **b** $Q = \{\text{consonants}\}$
- **c** $R = \{\text{letters in the word HOSPITAL}\}$
- **d** $S = \{\text{letters after J in the alphabet}\}$.

2.4 - Venn Diagrams

Definition 2.4.1 (Venn Diagram)

A **Venn diagram** is a diagram that uses circles to represent sets and their relationships.
It consists of a rectangle that represents the universal set U and circles that represent sets that are subsets of U.

Example 2.4.2

Example 5 | Self Tutor

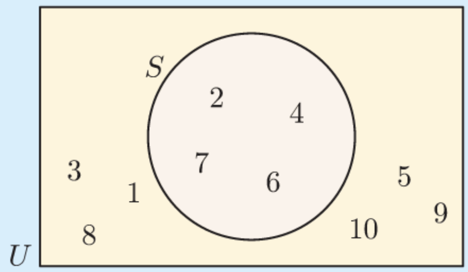
Consider the set $S = \{2, 4, 6, 7\}$ within the universal set $U = \{x \mid x \leqslant 10,\ x \in \mathbb{Z}^+\}$.

a Draw a Venn diagram to show S.

b List the elements of the complement set S'.

c Find: **i** $n(S)$ **ii** $n(S')$ **iii** $n(U)$

a



b $S' = \{1, 3, 5, 8, 9, 10\}$

c **i** $n(S) = 4$

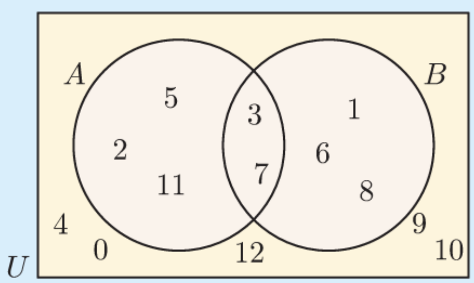
ii $n(S') = 6$

iii $n(U) = 10$

Example 7 | Self Tutor

Consider $U = \{x \mid 0 \leqslant x \leqslant 12,\ x \in \mathbb{Z}\}$, $A = \{2, 3, 5, 7, 11\}$, and $B = \{1, 3, 6, 7, 8\}$.

Illustrate A and B on a Venn diagram.



3 and 7 are in both A and B, so the circles representing A and B must overlap.

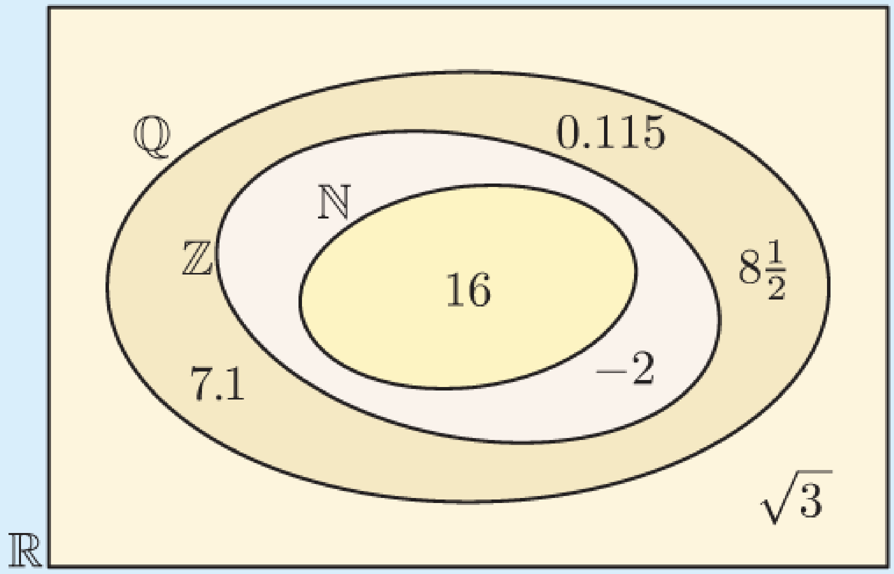
We place 3 and 7 in the overlap, then fill in the rest of A and the rest of B.

The remaining elements of U are placed outside the two circles.

Example 6 **Self Tutor**

Illustrate the following numbers on a Venn diagram:

$\sqrt{3}$, $8\frac{1}{2}$, -2, 7.1, 16, 0.115



Exercise 2.4.3

1 Suppose $U = \{x \mid x \leqslant 8,\ x \in \mathbb{Z}^+\}$ and $A = \{\text{prime numbers} \leqslant 8\}$.

a Show set A on a Venn diagram.

b List the set A'.

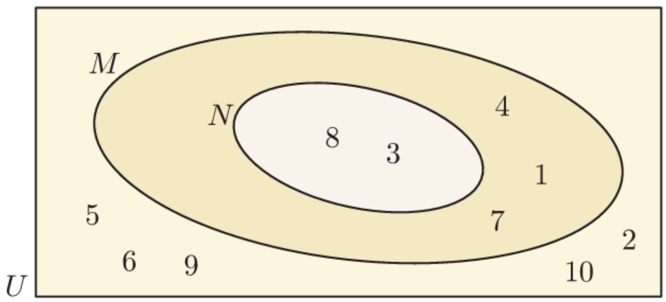
c Find: **i** $n(A)$ **ii** $n(A')$ **iii** $n(U)$

2 Suppose $U = \{\text{letters of the English alphabet}\}$ and
$V = \{\text{letters of the English alphabet which are vowels}\}$.

a Show these sets on a Venn diagram. **b** List the set V'.

c Find: **i** $n(V)$ **ii** $n(V')$ **iii** $n(U)$

3



a List the elements of:
i U **ii** N **iii** M

b Find $n(N)$ and $n(M)$.

c Is $M \subseteq N$?

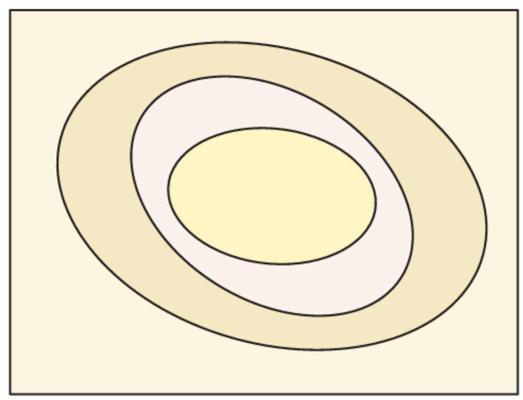
4 Illustrate A and B on a Venn diagram if:

a $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$

b $U = \{4, 5, 6, 7, 8, 9, 10\}$, $A = \{6, 7, 9, 10\}$, $B = \{5, 6, 8, 9\}$

c $U = \{3, 4, 5, 6, 7, 8, 9\}$, $A = \{3, 5, 7, 9\}$, $B = \{4, 6, 8\}$

5 Suppose the universal set is $U = \mathbb{R}$, the set of all real numbers.
$\mathbb{Q}$, $\mathbb{Z}$, and $\mathbb{N}$ are all subsets of $\mathbb{R}$.



a Copy the given Venn diagram and label the sets U, $\mathbb{Q}$, $\mathbb{Z}$, and $\mathbb{N}$.

b Place these numbers on the Venn diagram:
$\frac{2}{3}$, $\sqrt{7}$, $0.\overline{4}$, -1, $-8\frac{1}{3}$, 0, 4, and
$\alpha = 0.564\,105\,923\,6\ldots.$ which does not terminate or recur.

c Shade the region representing the set of irrationals $\mathbb{Q}'$.

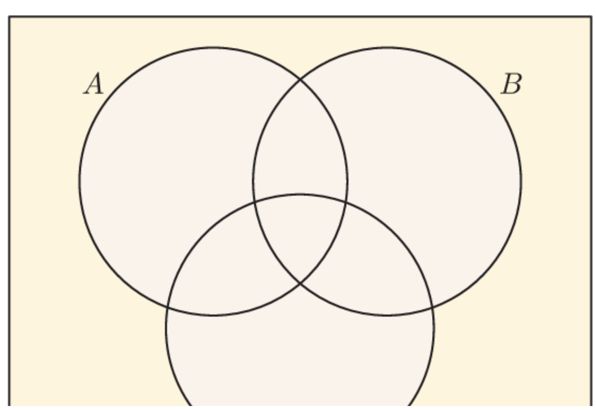
6 Show the following information on a Venn diagram:

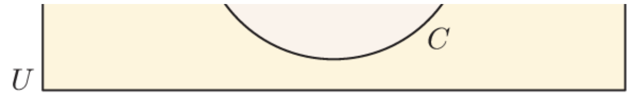
a $U = \{\text{triangles}\}$, $E = \{\text{equilateral triangles}\}$, $I = \{\text{isosceles triangles}\}$

b $U = \{\text{quadrilaterals}\}$, $P = \{\text{parallelograms}\}$, $R = \{\text{rectangles}\}$

7 Suppose $U = \{x \mid x \leqslant 30,\ x \in \mathbb{Z}^+\}$,
$A = \{\text{prime numbers} \leqslant 30\}$,
$B = \{\text{multiples of } 5 \leqslant 30\}$,
and $C = \{\text{odd numbers} \leqslant 30\}$.

Use the Venn diagram shown to display the elements of the sets.





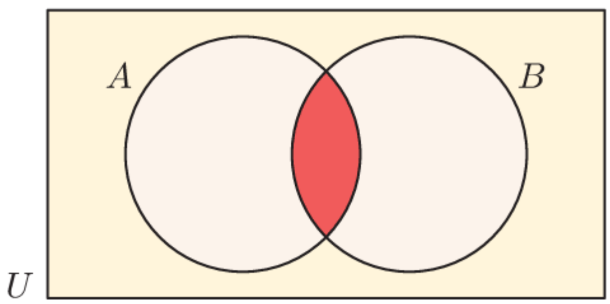
2.5 - Set Operations

Definition 2.5.1 (Union)

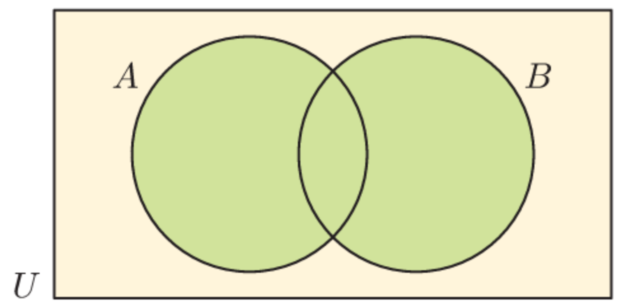
- The **union** of two sets A and B is the set of all elements that are in A or B or both.
- We use $A \cup B$ to denote the union of A and B.

Definition 2.5.2 (Intersection)

- The **intersection** of two sets A and B is the set of all elements that are in both A and B.
- We use $A \cap B$ to denote the intersection of A and B.



$A \cap B$ is shaded red.



$A \cup B$ is shaded green.

Example 8 Self Tutor

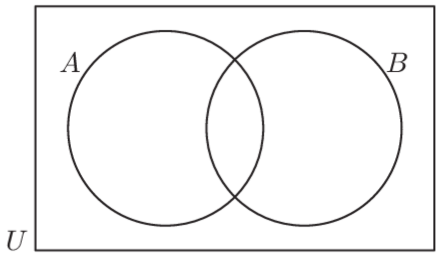
Suppose $U = \{\text{positive integers} \leqslant 12\}$, $A = \{\text{primes} \leqslant 12\}$, and $B = \{\text{factors of } 12\}$.

- **a** List the elements of the sets A and B.
- **b** Show the sets A, B, and U on a Venn diagram.
- **c** List the elements in: **i** A' **ii** $A \cap B$ **iii** $A \cup B$
- **d** Find: **i** $n(A \cap B)$ **ii** $n(A \cup B)$ **iii** $n(B')$

Definition 2.5.3 (Disjoint)

- Two sets A and B are **disjoint** or **mutually exclusive** if $A \cap B = \emptyset$.
- It means that A and B have no elements in common.

EXERCISE 2E.2

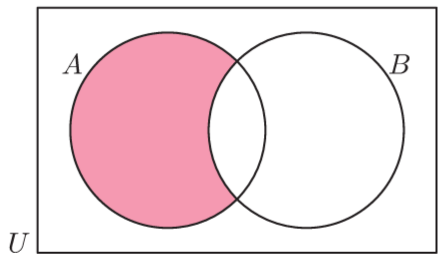
1 

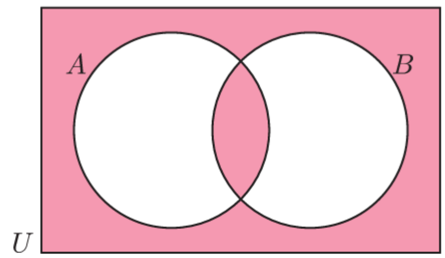
On separate Venn diagrams, shade regions for:

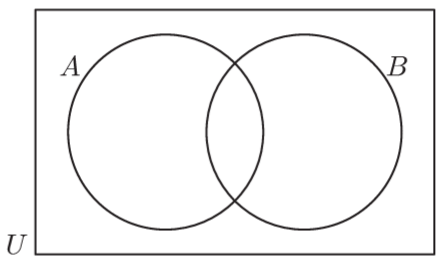
a $A \cap B$ **b** $A \cap B'$
c $A' \cup B$ **d** $A \cup B'$
e $A' \cap B$ **f** $A' \cap B'$

PRINTABLE VENN DIAGRAMS (OVERLAPPING)

2 Describe in words, the shaded region of:

a 

b 

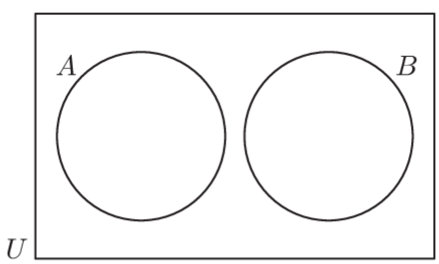
3 

a On separate Venn diagrams, shade regions for:

i $A \cup B$ **ii** $(A \cup B)'$ **iii** $A' \cap B'$
iv $A \cap B'$ **v** $(A \cap B)'$ **vi** $A' \cup B'$
vii $(A' \cup B')'$

b Hence verify that:

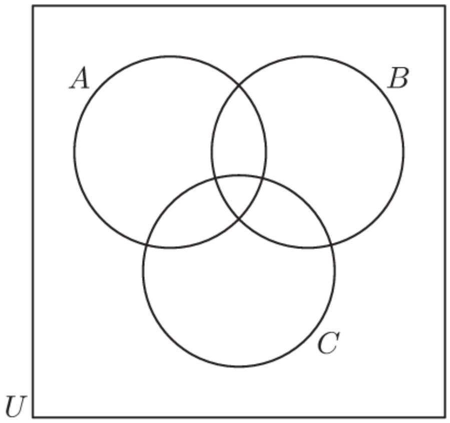
i $(A \cap B)' = A' \cup B'$ **ii** $(A \cup B)' = A' \cap B'$

4 

Suppose A and B are two disjoint sets. Shade on separate Venn diagrams:

a A **b** B'
c $A \cup B$ **d** $A' \cap B$
e $(A \cap B)'$

PRINTABLE VENN DIAGRAMS (DISJOINT)

5 

This Venn diagram consists of three intersecting sets.

a Shade on separate Venn diagrams:

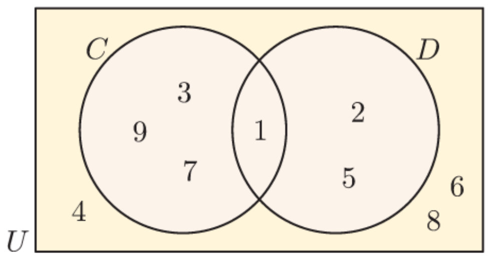
PRINTABLE VENN DIAGRAMS (3 SETS)

- **i** A
- **ii** B'
- **iii** $B \cap C$
- **iv** $A \cup B$
- **v** $A \cap B \cap C$
- **vi** $A \cup B \cup C$
- **vii** $(A \cap B \cap C)'$
- **viii** $(B \cap C) \cup A$
- **ix** $(A \cup B) \cap C$
- **x** $(A \cap C) \cup (B \cap C)$
- **xi** $(A \cap B) \cup C$
- **xii** $(A \cup C) \cap (B \cup C)$

b Verify that:

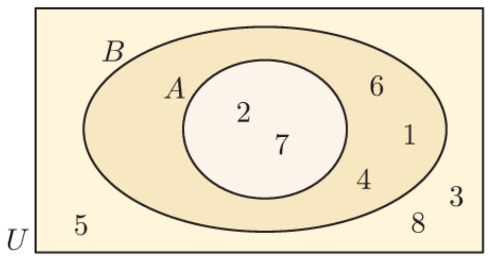
- **i** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- **ii** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

EXERCISE 2E.1

1 

a List the elements of set:
i C **ii** D **iii** U
iv $C \cap D$ **v** $C \cup D$

b Find:
i $n(C)$ **ii** $n(D)$ **iii** $n(U)$
iv $n(C \cap D)$ **v** $n(C \cup D)$

2 

a List the elements of set:
i A **ii** B **iii** U
iv $A \cap B$ **v** $A \cup B$

b Find:
i $n(A)$ **ii** $n(B)$ **iii** $n(U)$
iv $n(A \cap B)$ **v** $n(A \cup B)$

3 Consider $U = \{x \mid x \leqslant 12,\ x \in \mathbb{Z}^+\}$, $A = \{2, 7, 9, 10, 11\}$, and $B = \{1, 2, 9, 11, 12\}$.

a Show these sets on a Venn diagram.
b List the elements of: **i** $A \cap B$ **ii** $A \cup B$ **iii** B'
c Find: **i** $n(A)$ **ii** $n(B')$ **iii** $n(A \cap B)$ **iv** $n(A \cup B)$

4 If A is the set of all factors of 36 and B is the set of all factors of 63, find:

a $A \cap B$ **b** $A \cup B$

5 If $X = \{\text{A, B, D, M, N, P, R, T, Z}\}$ and $Y = \{\text{B, C, M, T, W, Z}\}$, find:

a $X \cap Y$ **b** $X \cup Y$

6 Suppose $U = \{x \mid x \leqslant 30,\ x \in \mathbb{Z}^+\}$, $A = \{\text{factors of } 30\}$, and $B = \{\text{prime numbers} \leqslant 30\}$.

a Find: **i** $n(A)$ **ii** $n(B)$ **iii** $n(A \cap B)$ **iv** $n(A \cup B)$
b Show that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

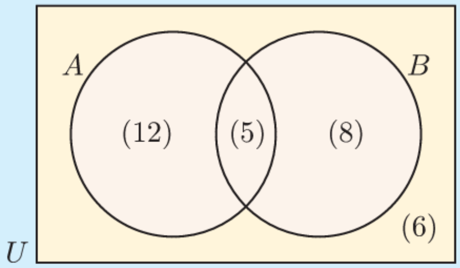
7 Simplify:

a $X \cap Y$ for $X = \{1, 3, 5, 7\}$ and $Y = \{2, 4, 6, 8\}$
b $A \cup A'$ for any set $A \in U$.
c $A \cap A'$ for any set $A \in U$.

2.6 - Expressing Number of Elements in a Set using Venn Diagrams

Sometimes it is too difficult to count the number of elements in a set by listing them out.
In this case, we can just put the number of elements in the set in the corresponding region of the Venn diagram.

Example 10 **Self Tutor**



In the Venn diagram given, (5) means that there are 5 elements in the set $A \cap B$.

How many elements are there in:

a A **b** B' **c** $A \cup B$

d A, but not B **e** B, but not A **f** neither A nor B?

a $n(A) = 12 + 5 = 17$

b $n(B') = 12 + 6 = 18$

c $n(A \cup B) = 12 + 5 + 8 = 25$

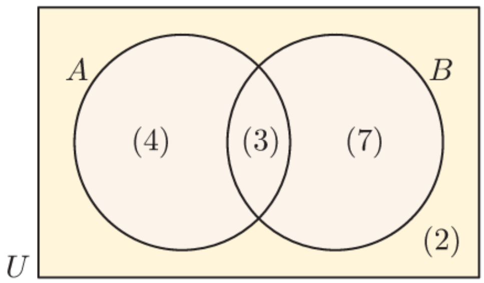
d $n(A, \text{ but not } B) = 12$

e $n(B, \text{ but not } A) = 8$

f $n(\text{neither } A \text{ nor } B) = 6$

EXERCISE 2F

1



How many elements are there in:

a B

b A'

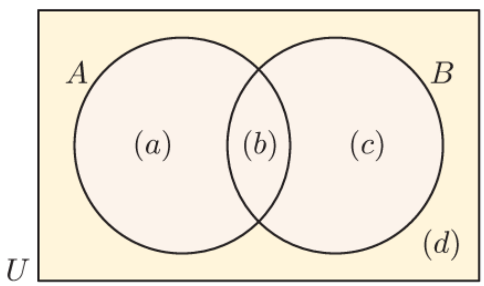
c $A \cup B$

d A, but not B

e B, but not A

f neither A nor B?

2 In the Venn diagram below, (a) means that there are a elements in that region.



a Write an expression for:

i $n(A)$

ii $n(B)$

iii $n(A \cap B)$

iv $n(A \cup B)$

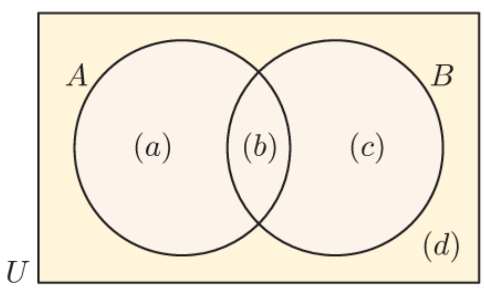
b Show that:

i $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

ii $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

iii if A and B are disjoint, then $n(A \cup B) = n(A) + n(B)$.

3



Use the Venn diagram to show that:

a $n(A \cap B') = n(A) - n(A \cap B)$

b $n(A \cup B') = n(U) - n(A' \cap B)$

4 Given $n(U) = 20$, $n(A) = 12$, $n(B) = 13$, and $n(A \cap B) = 8$, find:

a $n(A \cup B)$

b $n(B$, but not $A)$

5 Given $n(U) = 28$, $n(M) = 14$, $n(M \cap N) = 3$, and $n(M \cup N) = 18$, find:

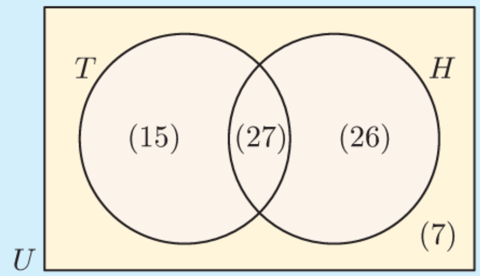
a $n(N)$

b $n((M \cup N)')$

2.7 - Solving Set Problems using Venn Diagrams

Example 12 **Self Tutor**

The Venn diagram alongside illustrates the number of people in a sporting club who play tennis (T) and hockey (H).



Determine the number of people:

a in the club
b who play hockey
c who play both sports
d who play neither sport
e who play at least one sport.

a $n(U) = 15 + 27 + 26 + 7 = 75$
There are 75 people in the club.

b $n(H) = 27 + 26 = 53$
53 people play hockey.

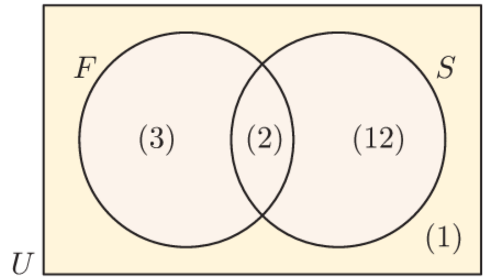
c $n(T \cap H) = 27$
27 people play both sports.

d $n(T' \cap H') = 7$
7 people play neither sport.

e $n(T \cup H) = 15 + 27 + 26 = 68$
68 people play at least one sport.

EXERCISE 2G

1 The Venn diagram alongside illustrates the number of students in a particular class who study French (F) and Spanish (S).



Determine the number of students:

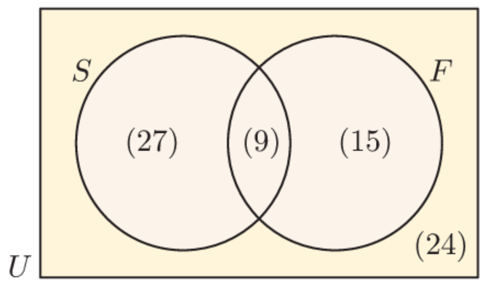
a in the class

b who study both subjects

c who study at least one of the subjects

d who only study Spanish.

2 In a survey at a resort, people were asked whether they went sailing (S) or fishing (F) during their stay.



Use the Venn diagram to determine the number of people:

a in the survey

b who did both activities

c who did neither activity

d who did exactly one of the activities.

3 In a class of 30 students, 19 study Physics, 17 study Chemistry, and 15 study both subjects. Determine the number of students who study:

a at least one of the subjects

b Physics, but not Chemistry

c exactly one of the subjects

d neither subject.

4 In a class of 40 students, 19 play tennis, 20 play netball, and 8 play neither sport. Determine the number of students in the class who:

a do not play netball

b play at least one of the sports

c play exactly one of the sports

d play netball, but not tennis.

5 In a class of 25 students, 15 play hockey, 16 play basketball, and 4 play neither sport. Determine the number of students who play:

a both sports

b hockey but not basketball.

6 In a class of 40 students, 34 like bananas, 22 like pineapples, and 2 dislike both fruits. Find the number of students who:

a like both fruits

b like at least one fruit.

7 In a class of 40 students, 23 have dark hair, 18 have brown eyes, and 26 have dark hair, brown eyes or both. How many students have:

a dark hair and brown eyes

b neither dark hair nor brown eyes

c dark hair but not brown eyes?

Review Exercises

REVIEW SET 2A

1 Explain why 1.3 is a rational number.

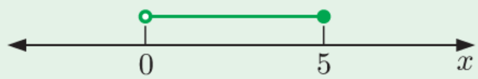
2 Is $\sqrt{4000} \in \mathbb{Q}$?

3 Let P be the set of all prime numbers between 20 and 40.

a Is $37 \in P$? **b** Find $n(P)$.

4 Write a statement describing the meaning of $S = \{t \mid -1 \leqslant t < 3\}$.

5 Write using interval notation:



6 For each of the following sets P and Q, decide whether $P \subseteq Q$:

a $P = \{5, 6, 7, 8\}$, $Q = \{1, 2, 3, 4, 5, 6, 7\}$

b $P = \{\text{multiples of 4 between 10 and 30}\}$, $Q = \{\text{even numbers between 0 and 40}\}$

7 Suppose $U = \{x \mid x \leqslant 10,\ x \in \mathbb{Z}^+\}$. Find the complement of:

a $A = \{3, 7, 9\}$ **b** $B = \{\text{composite numbers in } U\}$.

8 Suppose $U = \{x \mid x \leqslant 12,\ x \in \mathbb{Z}^+\}$ and $A = \{\text{multiples of } 3 \leqslant 12\}$.

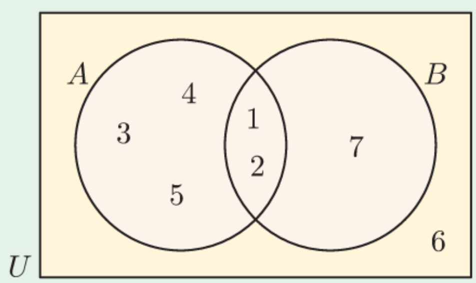
a Show A on a Venn diagram. **b** List the set A'.

c Find $n(A')$.

9 True or false?

a $\mathbb{N} \subseteq \mathbb{Z}^+$ **b** $\mathbb{Q} \subseteq \mathbb{Z}$

10



a List the elements of set:

i A **ii** B **iii** U

iv $A \cup B$ **v** $A \cap B$

b Find:

i $n(A)$ **ii** $n(B)$ **iii** $n(A \cup B)$

11 Consider $U = \{x \mid x \leqslant 10,\ x \in \mathbb{Z}^+\}$, $P = \{2, 3, 5, 7\}$, and $Q = \{2, 4, 6, 8\}$.

a Show these sets on a Venn diagram.

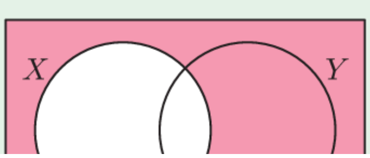
b List the elements of: **i** $P \cap Q$ **ii** $P \cup Q$ **iii** Q'

c Find: **i** $n(P')$ **ii** $n(P \cap Q)$ **iii** $n(P \cup Q)$

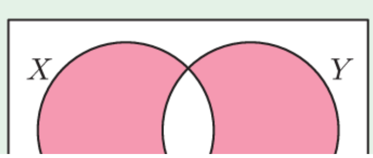
d Is $P \cap Q \subseteq P$?

12 Describe in words the shaded region:

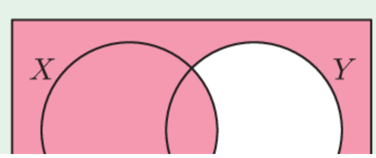
a

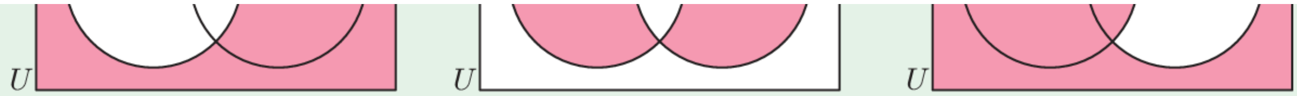


b



c

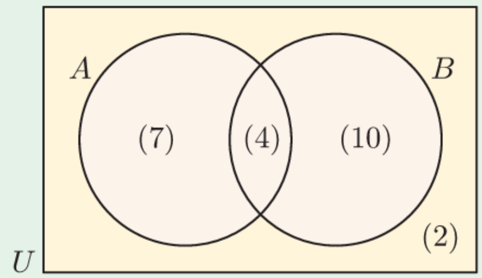




13 How many elements are there in:

a A **b** B

c $A \cup B$ **d** neither A nor B?



14 400 families were surveyed. It was found that 90% had a TV set, and 60% had a computer. Every family had at least one of these items. How many families had both a TV set and a computer?

REVIEW SET 2B

1 Is $-2 \in \mathbb{Z}^+$?

2 Show that $0.\overline{51}$ is a rational number.

3 Sketch the number set $\{x \mid x \leqslant 3 \quad \text{or} \quad x > 7, \ x \in \mathbb{R}\}$.

4 For each of the following sets:

- **i** list the elements of the set
- **ii** determine whether the set is finite or infinite
- **iii** if the set is finite, find the number of elements in the set.

a $A = \{\text{factors of } 15\}$ **b** $B = \{\text{multiples of } 8\}$

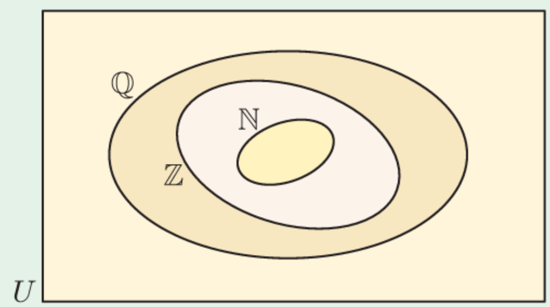
c $C = \{\text{odd numbers between } 30 \text{ and } 50\}$ **d** $D = \{\text{prime numbers less than } 30\}$

5 Suppose $P = \{3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $Q = \{4, 9, 10\}$, and $R = \{5, 6, 12\}$. Decide whether Q and R are subsets of P.

6 Suppose $U = \{x \mid x \leqslant 12, \ x \in \mathbb{Z}^+\}$ and $A = \{\text{prime numbers less than } 12\}$. Find:

a A **b** A' **c** $n(A)$ **d** $n(A')$ **e** $n(U)$

7 Illustrate these numbers on a Venn diagram like the one shown:
$-1, \sqrt{2}, 2, 3.1, \pi, 4.\overline{2}$



8 Show this information on a Venn diagram:

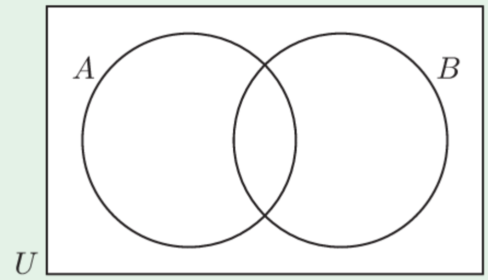
a $U = \{10, 11, 12, 13, 14, 15\}$, $A = \{10, 12, 14\}$, $B = \{11, 12, 13\}$

b $U = \{\text{quadrilaterals}\}$, $S = \{\text{squares}\}$, $R = \{\text{rectangles}\}$

9 If A is the set of all factors of 24 and B is the set of all factors of 18, find:

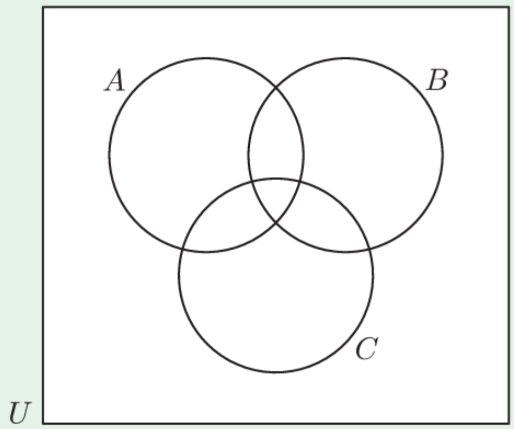
a $A \cap B$ **b** $A \cup B$

10 On separate Venn diagrams like the one shown, shade the region representing:



a B' **b** in A and in B

c $(A \cup B)'$

11 

Using separate Venn diagrams like the one shown, shade regions to verify that $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

12 Given $n(U) = 30$, $n(A) = 14$, $n(B) = 10$, and $n(A \cap B) = 6$, find:

a $n(A \cup B)$ **b** $n(B$, but not $A)$

13 In a certain town, three newspapers are published. 20% of the population read A, 16% read B, 14% read C, 8% read A and B, 5% read A and C, 4% read B and C, and 2% read all 3 newspapers. What percentage of the population read:

a none of the papers **b** at least one of the papers

c exactly one of the papers **d** either A or B

e A only?