2012 AMC8 Problem 22

Let R be a set of nine distinct integers. Six of the elements are 2, 3, 4, 6, 9, and 14. What is the number of possible values of the median of R?

A. 4

B. 5

C. 6

D. 7

E. 8

Intuition: common strategy for all problems

When we don't know some of the quantities that are related to the problem, we can try assigning letters to the quantities.

For this problem

Let x, y, z be the three distinct integers that we don't know. Then the set becomes

$$R = \{2, 3, 4, 6, 9, 14, x, y, z\}$$

Step 1: Find the extreme cases

$$R = \{2, 3, 4, 6, 9, 14, x, y, z\}$$

When x, y, z are all **smaller** than 2, the median is 3.

When x, y, z are all **larger** than 14, the median is 9.

Observation:

The median of this set of integers is between 3 and 9.

Step 2: Having the freedom to choose x, y, z

Since we have the freedom to choose x, y, z to be any integer, the median can be any value between 3 and 9.

Therefore, the number of possible values of the median is $9 - 3 + 1 = 7$. The correct answer is $\boxed{D}$.

Homework:

Verify that the median can be any value between 3 and 9, that is, give an example of x, y, z such that the median of the numbers is 3, 4, 5, 6, 7, 8, or 9.