Understanding Recurrence Relations

What is a Recurrence Relation? 🤔

- A recurrence relation is an equation that defines a sequence

- It relates each term to previous **terms**

- Example: Fibonacci sequence
  - $f(n) = f(n-1) + f(n-2)$
  - $f(1) = 1, f(2) = 1$

How to Solve Recurrence Relations 🧠

- Find the first few terms of the sequence

- Set up the recurrence relation

- Going back in the sequence until you get to the terms you know

- Going forward in the sequence until you get to the term you want

Example: Fibonacci Sequence 🔥

- $f(n) = f(n - 1) + f(n - 2)$
- $f(1) = 1, f(2) = 1$

Question: What is $f(5)$?

Example: Fibonacci Sequence 🔥

Going back in the sequence:

- In the sequence, $f(5) = f(4) + f(3)$
- $f(4) = f(3) + f(2)$
- $f(3) = f(2) + f(1)$
- $f(2) = 1$
- $f(1) = 1$

Example: Fibonacci Sequence 🔥

Going forward in the sequence:

- $f(3) = f(2) + f(1) = 1 + 1 = 2$
- $f(4) = f(3) + f(2) = 2 + 1 = 3$
- $f(5) = f(4) + f(3) = 3 + 2 = 5$

Answer: $f(5) = 5$

Exam Question 📝

Everyday at school, Jo climbs a flight of 6 stairs. Jo can take the stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2. In how many ways can Jo climb the stairs?

Finding the Recurrence Relation 📝

The question is asking for the number of ways to climb the stairs. We can use a recurrence relation to solve this problem.

Let $f(n)$ be the number of ways to climb n stairs.The recurrence relation is
$$f(n) = f(n-1) + f(n-2) + f(n-3)$$

Here is why:

When there are n stairs left, Jo can either:

- Take 1 step, then $f(n-1)$ ways to climb the remaining stairs
- Take 2 steps, then $f(n-2)$ ways to climb the remaining stairs
- Take 3 steps, then $f(n-3)$ ways to climb the remaining stairs

So, $f(n) = f(n-1) + f(n-2) + f(n-3)$

Finding the first few terms 📝

We just need to know the first three terms:

- When there is 1 stair left, there is 1 way to climb it

- When there is 2 stairs left, there are 2 ways to climb it (1+1, 2)

- When there is 3 stairs left, there are 4 ways to climb it (1+1+1, 1+2, 2+1, 3)

So, $f(1) = 1, f(2) = 2, f(3) = 4$

Finding the number of ways to climb 6 stairs 📝

We can use the recurrence relation to find the number of ways to climb 6 stairs:

- $f(4) = f(3) + f(2) + f(1) = 4 + 2 + 1 = 7$
- $f(5) = f(4) + f(3) + f(2) = 7 + 4 + 2 = 13$
- $f(6) = f(5) + f(4) + f(3) = 13 + 7 + 4 = 24$

Answer: $f(6) = 24$