Understanding Modulo

What is Modulo? 🤔

- Modulo is the remainder when you divide one number by another
- Example: $7 \mod 3 = 1$
- $7 \div 3 = 2$ remainder 1

How to Use Modulo 🧠

- Modulo is useful for checking if two numbers are congruent, i.e. they have the same remainder when divided by a certain number
- Example: $7 \mod 3 = 1$ and $10 \mod 3 = 1$
- Therefore,

$$7 \equiv 10 \mod 3$$

Example Exam Question 🔥

What is the tens digit of 7^{2011}?

Explanation 🧠

The tens digit of a number is the second-to-last digit of the number.

Solution 🧠

- $7^1 \mod 100 = 7$
- $7^2 \mod 100 = 49$
- $7^3 \mod 100 = 343 \mod 100 = 43$
- $7^4 \mod 100 = 7 \times 343 = 2401 \mod 100 = 1$

Only the last two digits of the product are relevant, so we can ignore the rest of the digits.

- $7^5 \mod 100 = 7 \times 1 \mod 100 = 7$
- $7^6 \mod 100 = 7 \times 7 \mod 100 = 49$
- $7^7 \mod 100 = 7 \times 49 \mod 100 = 343 \mod 100 = 43$
- $7^8 \mod 100 = 7 \times 343 \mod 100 = 2401 \mod 100 = 1$

We can see that the last two digits of $7^n \mod 100$ repeat every 4 powers.

Answer 🤖

- $7^{2011} \mod 100 = 7^{4\times 502+3} \mod 100 = 7^3 \mod 100 = 43$
- Therefore, the tens digit of 7^{2011} is $\boxed{4}$

Rationale Behind Repeating Patterns 🧠

When multiplying long-digit numbers:

- The last few digits of the product are only affected by the last few digits of the numbers being multiplied
- This means that the last few digits of the product will repeat in a cycle
- This cycle can be used to find the last few digits of large powers

Practice Problems 📝

1. What is the last two digits of 3^{2024}?
2. What is the last three digits of 7^{2024}?

Advanced Topics 🧠

Does the same pattern hold for other bases?

- What if we used a different base, like 12?
- **Hint:** Try to find a pattern for $7^n \mod 12$

Further Reading:

- Modular Arithmetic
- Modular Exponentiation