Efficient Nested Simulation of Tail Risk Measures with Machine Learning Proxies

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Nested simulation procedures are commonly required for estimating tail risk measures for complex financial contracts or portfolios. In a standard nested simulation procedure, an outer simulation simulates M risk scenarios under a real-world measure to reflect real-world uncertainties in portfolio values in different risk scenarios. For each outer scenario, inner simulation estimates the corresponding portfolio loss based on N sample paths simulated under a risk-neutral measure according to the risk-neutral pricing theory. This standard procedure is computationally expensive. For example, the simulation budget, i.e., the total number of inner sample paths in all outer scenarios, is a common way to measure the computation required for a simulation procedure. For the standard nested simulation procedure, the simulation budget is $\Gamma = MN$. To alleviate computational burden of the standard procedure, many studies proposed to replace the inner simulation with alternative such as regression, kernel smoothing, likelihood ratio method, kernel ridge regression, etc. We refer to these alternatives as proxies. The above proxies have been demonstrated to have superior performance than the standard procedure in different examples.

The three parts of the thesis have different research focuses but serve one common theme. Chapter 2 compares four existing nested simulation methods on their theoretical convergence properties and practical performance. Extensive numerical experiments are conducted for fair comparisons of proxy models on their practical performance in different angles. Proxies in chapter 2 fail to perform for high dimensional scenarios. In chapter 3, machine learning proxies are considered to model variable annuity contract losses. Numerical experiments show that, for scenarios with temporal structure, time-series machine learning models are better proxies than generic machine learning models. The machine learning proxies are applied in a 2-stage nested simulation design. Experiments show that machine learning proxies outperform payoff-specific proxies. Chapter 4 proposes direction for future research. In the 2-stage design, inner simulation in the second stage is still costly. We propose replacing stage 2 with machine learning proxy predictions. We test the accuracy, sensitivity, and resilience of such proposal.

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Chapter 1

Introduction

Quantitative risk management is a key component of modern financial systems, ensuring the stability and resilience of markets, institutions, and portfolios against an array of risks. In the era of complex financial products, such as option portfolios and variable annuity contracts, traditional risk assessment methods often fall short in accurately capturing the multifaceted nature of market, credit, and operational risks. This is where advanced simulation techniques, particularly nested simulation procedures, become indispensable.

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