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Resilient Machine Learning Approaches for Fast Risk Evaluation and Management of Financial Portfolios and Variable Annuities

Supervised by Prof. Tony Wirjanto and Prof. Mingbin Feng

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Outline

Nested Simulation Procedures

Nested simulation procedures are necessary for **complex** financial derivatives and insurance products.

$$\rho(L) = \rho(L(X)), \quad L(X) = \mathbb{E}\left[Y|X=x\right]|_{x=X}.$$

Involves two levels of Monte Carlo simulations:

- Outer: underlying risk factors, $X_i \sim F_X$
- Inner: scenario-wise losses, $Y_{ij} \sim F_{Y|X_i}$

With an expensive total simulation budget $\Gamma = M \cdot N$:

$$\hat{L}_{N,i} = \frac{1}{N}\sum_{j=1}^{N} Y_{ij}; \quad Y_{ij} \sim F_{Y|X_i}$$

- Uses inner sample mean to estimate $L(X_i)$.

Metamodeling Approach

We focus on procedures that **pool** with **supervised learning metamodels**.

- Treat inner simulation as a black-box function
- Approximate $L(\cdot)$ with $\hat{L}^{\mathsf{SL}}_{M,N}(\cdot)$
- Train with feature-label pairs generated by simulation:

$$\{(X_i, \hat{L}_{N,i}) | i = 1, \ldots, M, j = 1, \ldots, N\}$$

- Use metamodel predictions to estimate risk measures

There are **computational costs** associated with pooling inner replications.

Asymptotic Convergence Rates of Different Procedures

$$\min_{M,N} \quad \mathbb{E}\left[(\hat{\rho}_{M,N} - \rho)^2\right]$$

$$\text{subject to } M \cdot N = \Gamma$$

Procedures	Smooth h	Hockey-Stick h	Indicator h
Standard Procedure	$\mathcal{O}(\Gamma^{-2/3})$	$\mathcal{O}(\Gamma^{-2/3})$	$\mathcal{O}(\Gamma^{-2/3})$
Regression	$\mathcal{O}(\Gamma^{-1})$	$\mathcal{O}(\Gamma^{-1+\delta})$	No Result
Kernel Smoothing	$\mathcal{O}(\Gamma^{-\min(1,4/(d+2))})$		
Kernel Ridge Regression[1]	$\mathcal{O}(\Gamma^{-1})$		
Likelihood Ratio	$\mathcal{O}(\Gamma^{-1})$		

Key Theoretical Results

Contribution: bridging the gap between MSE and absolute error convergence.

- Convergence in MSE:

$$\mathbb{E}\left[(\hat{\rho}_\Gamma - \rho)^2\right] = \mathcal{O}\left(\Gamma^{-\xi}\right)$$

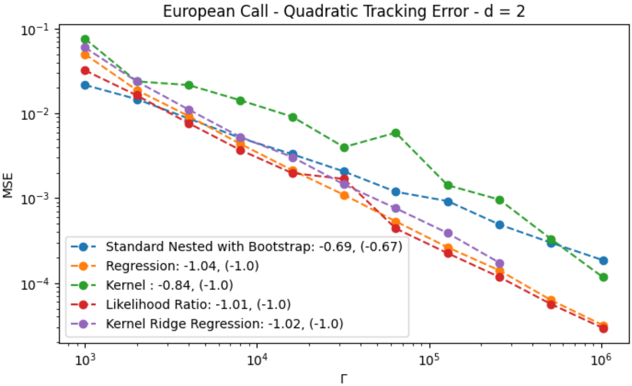
- Convergence in Probabilistic Order:

$$|\hat{\rho}_\Gamma - \rho| = \mathcal{O}_{\mathbb{P}}(\Gamma^{-\xi})$$

Theorem

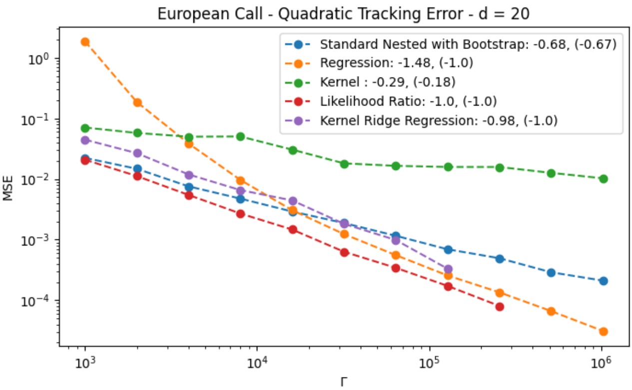
If $\hat{\rho}_\Gamma$ converges in MSE to ρ in order ξ, then $\hat{\rho}_\Gamma$ converges in probabilistic order to ρ in order $\frac{\xi}{2}$.

Finite-Sample Performance



- Standard, KRR, and likelihood ratio procedures are dimension-independent
- Kernel smoothing and regression show sensitivity to dimension, but in different ways

Sensitivity to Asset Dimension



- Regression and kernel converge faster than their asymptotic rates
- Results of other experiments are in the thesis (Figure 2.3 - Figure 2.11)

Additional Computational Costs

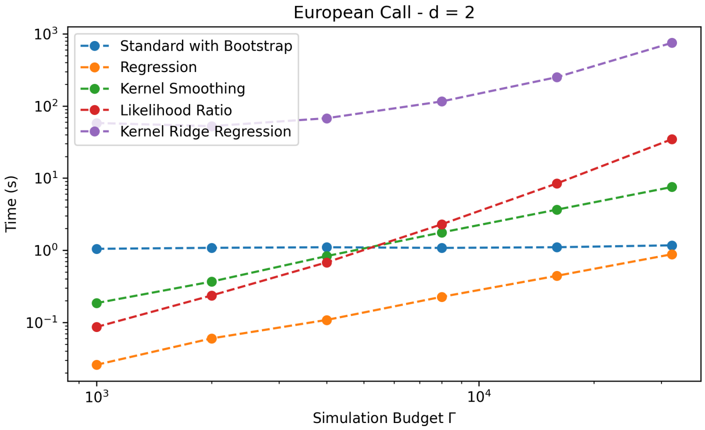


Figure: Additional computation time for different procedures

Conclusion

Regression-based nested simulation procedure:

- Most robust and stable for limited budgets
- Efficient to implement
- Fast empirical convergence for option portfolios

For high-dimensional or complex payoffs:

- Difficult to find a good regression basis
- Neural network-based procedures may be more suitable

Next project: examining performance of metamodel-based simulation procedures for variable annuities

Nested Simulation for Risk Management of VAs

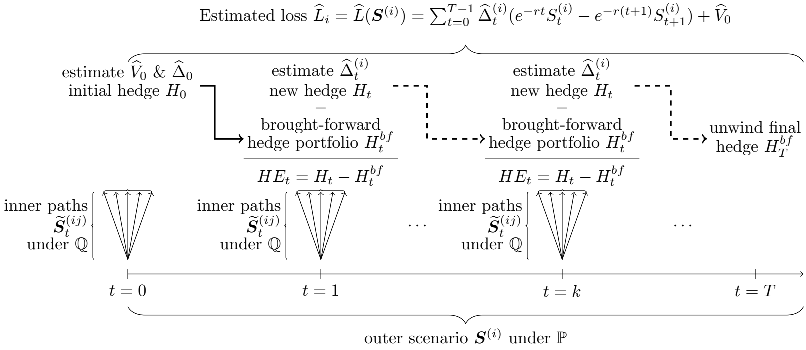


Figure: Illustration of nested simulation that estimates the P&L for one outer scenario

Metamodel-based Nested Simulation

We use deep neural networks (DNNs) as metamodels

- Use LSTMs for sequential data
- **Challenge**: lack of transparency and interpretability

Research Contributions:

1. Propose two generic DNN-based nested simulation procedures
   - Accurate tail scenario identification
   - Significant computational savings by **budget concentration**
2. Study noise tolerance of DNNs using simulated data
   - **Control noise levels** by adjusting simulation parameters
   - Provide direct evidence on transparency and interpretability

Two-Stage Metamodel-based Nested Simulation

Algorithm Two-Stage Metamodel-based Nested Simulation for VAs

1: **Generate training data for metamodels**
2: **Train metamodels**
3: **Estimate α-CVaR with extensive simulation on predicted tail scenarios**
- Concentrate simulation on predicted tails

Key Findings:

- Substantial computational savings ($70\% - 85\%$ reduction)
- Some DNN metamodels make **accurate loss predictions**

Single-Stage Metamodel-based Nested Simulation

Algorithm Single-Stage Metamodel-based Nested Simulation for VAs

1: **Generate training data for metamodels**
2: **Train metamodels**
3: **Estimate α-CVaR with metamodel predictions**
- Entirely avoids extensive simulation

Key advantages:

- more efficient than a two-stage procedure;
- avoids specifying a safety margin m.

Experiment Setting

We consider the following metamodel architectures:

Metamodel	Abbreviation	Capacity
Multiple Linear Regression	MLR	241
Quadratic Polynomial Regression	QPR	481
Feedforward Neural Network	FNN	35,009
Recurrent Neural Network	RNN	32,021
Long Short-Term Memory	LSTM	35,729

Table: Metamodel architectures for GMWB inner simulation model

In our experiment:

- 95%-CVaR of loss for a GMWB with a 240-month maturity
- 240-dimensional feature vector and 1-dimensional loss

Metamodel Performance

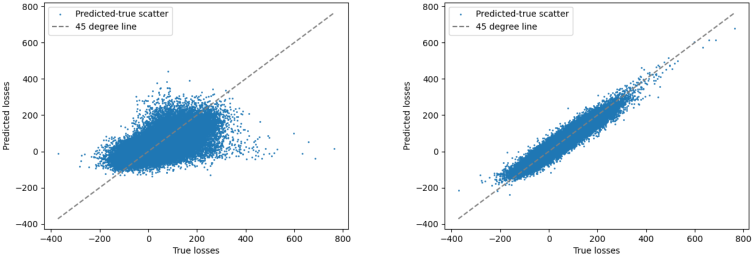
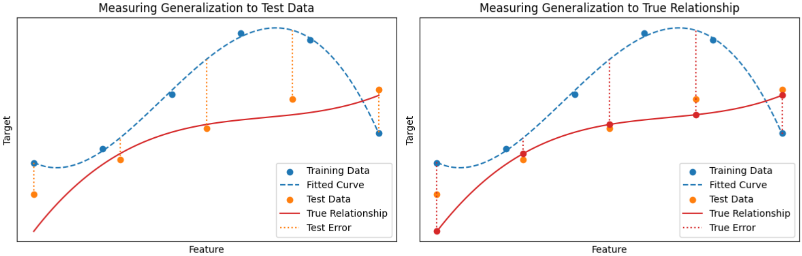


Figure: QQ plots between true and predicted loss labels for QPR and LSTM metamodels

- QPR make **inaccurate** loss predictions
- DNN metamodels are more flexible.

Research Questions

- What do DNNs learn from noisy data?
- How well do DNNs learn from noisy data?



Three datasets are considered:

- Training: 90,000 scenarios (100 inner replications).
- Test: 10,000 scenarios (100 inner replications).
- True: 100,000 scenarios (100,000 inner replications).

Metamodel Performance on Different Datasets

Metamodel	Training error	Test error	True error
MLR	0.706 $(\pm 8.3 \times 10^{-4})$	0.713 $(\pm 2.7 \times 10^{-2})$	0.706 $(\pm 3.4 \times 10^{-4})$
QPR	0.543 $(\pm 8.3 \times 10^{-4})$	0.554 $(\pm 2.7 \times 10^{-2})$	0.544 $(\pm 4.1 \times 10^{-4})$
FNN	0.129 $(\pm 6.0 \times 10^{-3})$	0.240 $(\pm 9.8 \times 10^{-3})$	0.132 $(\pm 5.8 \times 10^{-3})$
RNN	0.132 $(\pm 7.5 \times 10^{-3})$	0.137 $(\pm 7.6 \times 10^{-3})$	0.119 $(\pm 7.5 \times 10^{-3})$
LSTM	0.075 $(\pm 4.5 \times 10^{-3})$	0.079 $(\pm 5.4 \times 10^{-3})$	0.063 $(\pm 4.4 \times 10^{-3})$
RNN*[2]	0.109 $(\pm 5.2 \times 10^{-3})$	0.128 $(\pm 5.2 \times 10^{-3})$	0.109 $(\pm 5.2 \times 10^{-3})$

Table: Average MSEs and 95% confidence bands of metamodels for GMWB.

- RNN-based metamodels have lower true errors than their training errors.
- DNN metamodels with suitable architectures **cut through the noise** in training labels.

[2] This row summarizes the results of the well-trained RNNs.

Safety Margin

Consider estimating the 95% CVaR with 100,000 outer scenarios.

Figure: Illustration: a safety margin of 4% ($m = 9000$)

Trade-off between accuracy and efficiency

- Lower margin: not enough tail identified
- Higher margin: more budget spent on extensive simulation

Metamodel Performance

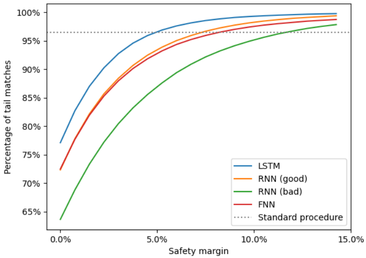


Figure: Tail Matches and CVaR Predictions for DNN Metamodels

- Traditional regression metamodels are **unable** to accurately identify tail scenarios even with high safety margins.
- LSTM metamodels surpasses the standard procedure with 5% safety margin.

Sensitivity Testing for DNNs

Simulation controls **noise level in training labels** and **number of training samples**.

N' varies the noise level.

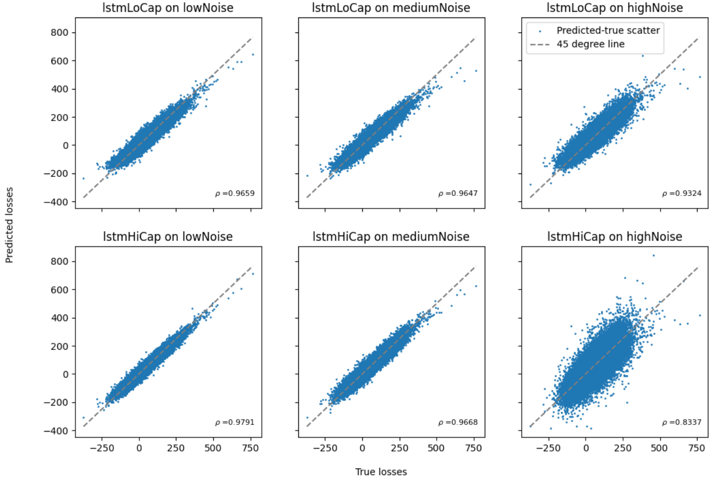
- **Low noise labels**: $N' = 100$
- **Medium noise labels**: $N' = 10$
- **High noise labels**: $N' = 1$

M varies the number of training samples.

- $M \in \{10^2, 10^3, 10^4, 10^5\}$

2 LSTMs of **different capacities** are examined based on their MSEs.

Noise Tolerance of DNNs



Sensitivity of High-capacity LSTM

	$N' = 1$	$N' = 10$	$N' = 100$	$N' = 1000$
$M = 100$	0.764	0.408	0.131	0.087
$M = 1000$	0.878	0.367	0.156	0.087
$M = 10000$	0.351	0.147	0.064	0.063
$M = 100000$	0.149	0.065	0.060	0.038

Table: MSE between high-capacity LSTM's predicted losses and true losses.

- Same color $\rightarrow$ same total simulation budget.
- $N' = 10$ is a reasonable budget allocation for LSTM metamodels.

Single-Stage Procedure

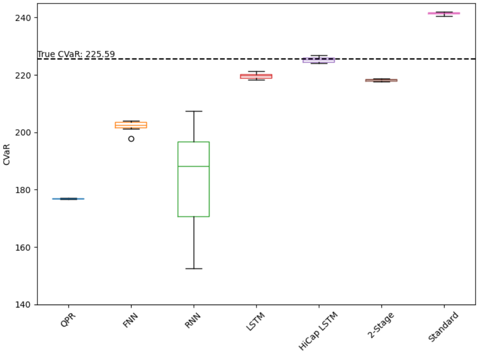


Figure: CVaR estimates of single-stage procedures with $N' = 10$.

- The single-stage procedure outperforms the two-stage procedure.
- $N' = 10$ is a reasonable budget allocation.

Convergence Analysis

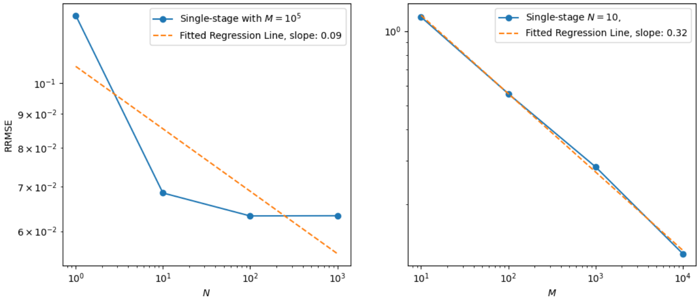


Figure: Empirical convergence of the single-stage procedure with a LSTM metamodel.

- Minimal effect of increasing N' on CVaR estimation.
- For a given Γ, set N' constant and allocate budget to outer simulations.

Conclusion

Key Findings:

- LSTMs are **resilient** to moderate levels of noise in training labels.
- DNNs can learn **true** complex dynamic hedging model.
- Two-stage procedure addresses regulatory concerns.
- Single-stage procedure is **efficient**.

Future Directions:

- Apply DNNs to other risk management tasks.
- Fast adaptation to new contracts/market conditions.

Transfer Learning for Rapid Adaptation of DNN Metamodels

Problem: Updating models for new conditions is expensive.

- Full retraining takes too much time.
- New VA contracts need quick model updates.
- Need balance between speed and accuracy.

Solution: Transfer learning for faster model adaptation.

- Train first on existing contract data.
- Update with small amount of new contract data.
- Reuse knowledge between similar contracts.
- Benefits:
  - Faster training.
  - Less data needed.

Transfer Learning Framework

Key Components:

- **Domain** $\mathcal{D}$: feature space $\mathcal{X}$ + probability distribution F
- **Task** $\mathcal{T}$: label space $\mathcal{Y}$ + predictive function $f : \mathcal{X} \to \mathcal{Y}$

Source vs. Target:

- **Source:** $\mathcal{D}_{\mathsf{So}} = \{\mathcal{X}_{\mathsf{So}}, F_{\mathsf{So}}(X)\}$
- **Target:** $\mathcal{D}_{\mathsf{Ta}} = \{\mathcal{X}_{\mathsf{Ta}}, F_{\mathsf{Ta}}(X)\}$

Our Goal:

- **Input features** X: risk factors from outer simulation
- **Output labels** L: contract losses
- **Source and target**: from VAs with abundant simulation data to new VAs with limited data
- **Goal**: improve $f_{\mathsf{Ta}}(\cdot)$ using knowledge from $\mathcal{D}_{\mathsf{So}}$ and $f_{\mathsf{So}}(\cdot)$

Transfer Learning Techniques

Common Techniques:

- **Fine-tuning:** a model pre-trained on a source task is used as a starting point for a target task.
- **Layer freezing:** only part of the model is fine-tuned.

Key considerations: similarity between source and target tasks

Experiment Design:

- Source domain (50000 samples): GMMB with no lapse and GMMB with static lapse
- Target domain (2000 samples): GMMB and GMWB with dynamic lapse

Results: Accuracy Comparison

Method and Setting	MSE
Extensive Training (Dynamic)	**0.0587**
Fine-tuning (Static)	0.0794
Layer Freezing (Static)	0.0763
Layer Freezing (No Lapse)	0.3361
Fine-tuning (No Lapse)	0.4894
Without TL (Dynamic)	0.2950

Table: Comparison of different TL methods on GMMB contracts (best MSE values)

Learning Curves: Effect of Similarity

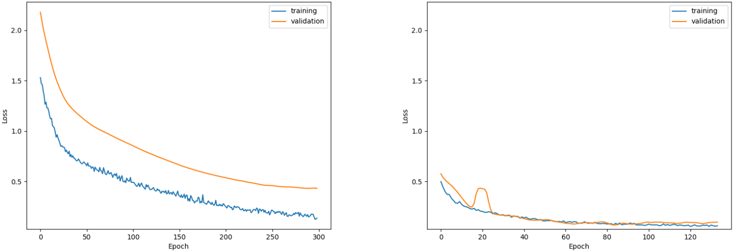
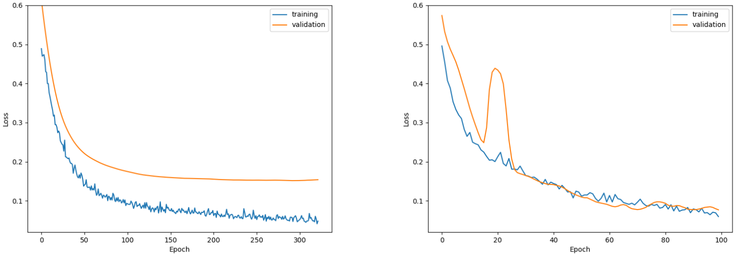


Figure: Fine-tuning performance on GMMB with dynamic lapse

- Similarity between source and target domain matters
- Negative transfer can happen for dissimilar tasks

Learning Curves: Effect of Freezing Layers



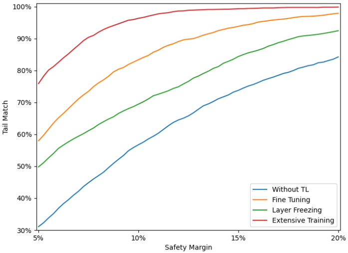
(a) Freezing LSTM layers

(b) Freezing fully connected layers

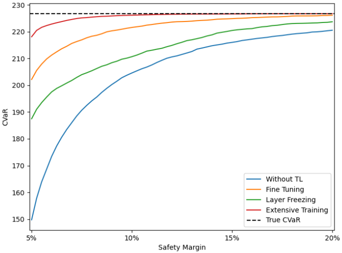
Figure: Learning dynamic lapse from a GMMB with static lapse

- When learning dynamic lapse, freezing the LSTM layers is suboptimal.
- Freezing the fully connected layers leads to better performance.

Metamodel Performance



(a) Identifying tail scenarios



(b) CVaR estimation

Figure: Comparison of different TL methods on transferring to GMWB contracts

Conclusions

- Transfer learning significantly improves metamodeling for VA contracts:
  - Faster training convergence
  - Better prediction accuracy
  - Reduced computational requirements
- Enables more frequent risk assessments and faster decision-making

Future Work:

- Incorporating domain knowledge into transfer process
- Extension to other insurance and financial products
- Multi-task learning

References I

Broadie, M., Du, Y., and Moallemi, C. C. (2015). Risk estimation via regression. *Operations Research*, 63(5):1077–1097.

Chung, J., Gulcehre, C., Cho, K., and Bengio, Y. (2014). Empirical evaluation of gated recurrent neural networks on sequence modeling. *arXiv preprint arXiv:1412.3555*. `https://arxiv.org/abs/1412.3555`. Accessed 7$^{\mathrm{th}}$ Sep 2023.

Dang, O. (2021). *Efficient Nested Simulation of Tail Risk Measures for Variable Annuities*. Ph.D. thesis, Department of Statistics and Actuarial Science, University of Waterloo, Waterloo, ON, Canada. `https://uwspace.uwaterloo.ca/handle/10012/17084`.

Feng, M. and Song, E. (2020). Optimal nested simulation experiment design via likelihood ratio method. *arXiv preprint arXiv:2008.13087*.

Gordy, M. B. and Juneja, S. (2010). Nested simulation in portfolio risk measurement. *Management Science*, 56(10):1833–1848.

Hardy, M. R. (2001). A regime-switching model of long-term stock returns. *North American Actuarial Journal*, 5(2):41–53.

References II

Hardy, M. R. (2003). *Investment Guarantees: Modeling and Risk Management for Equity-Linked Life Insurance*, volume 168. John Wiley & Sons.

Hochreiter, S. and Schmidhuber, J. (1997). Long short-term memory. *Neural Computation*, 9(8):1735–1780.

Hong, J. L., Juneja, S., and Liu, G. (2017). Kernel smoothing for nested estimation with application to portfolio risk measurement. *Operations Research*, 65(3):657–673.

Sutskever, I., Vinyals, O., and Le, Q. V. (2014). Sequence to sequence learning with neural networks. In Ghahramani, Z., Welling, M., Cortes, C., Lawrence, N., and Weinberger, K., editors, *Advances in Neural Information Processing Systems*, volume 27. Curran Associates, Inc.

Wang, W., Wang, Y., and Zhang, X. (2022). Smooth nested simulation: bridging cubic and square root convergence rates in high dimensions. *arXiv preprint arXiv:2201.02958*.

Yosinski, J., Clune, J., Bengio, Y., and Lipson, H. (2014). How transferable are features in deep neural networks? *Advances in neural information processing systems*, 27.

References III

Zhang, K., Feng, M., Liu, G., and Wang, S. (2022). Sample recycling for nested simulation with application in portfolio risk measurement. *arXiv preprint arXiv:2203.15929*.

Zhang, K., Liu, G., and Wang, S. (2021). Bootstrap-based budget allocation for nested simulation. *Operations Research*.