First of all, the proof on Pages 29-30 does not justify the convergence in probability of the order /xi/2, because the constant C^\* in the proof depends on epsilon while the constant C in Definition 5 needs to be independent of epsilon.

Scond, while I donot know whether one can fix the proof with the same condition, it would be a trivial implication if we impose a bit more order for the convergence in MSE as the condition. The proof is a trivial application of a well-known inequality as shown in the photo I attached below. The proof is the same as how one could use to prove the well-known result in probabiliyt theory that convergence in MSE implies convergence in probability.