Artificial Intelligence Mathematical Foundation

人工智能与自然语言处理课/计算机视觉程组

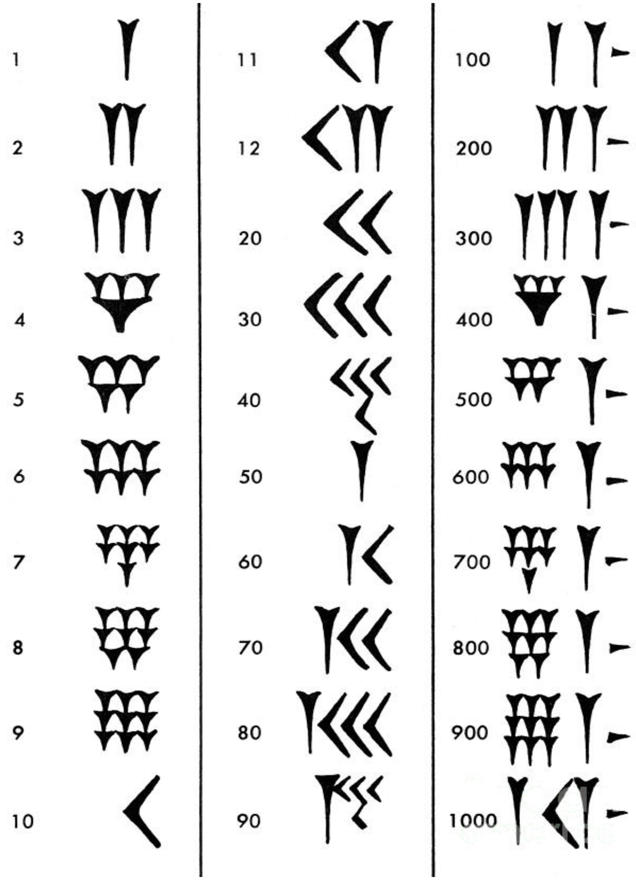
2019.March. 24

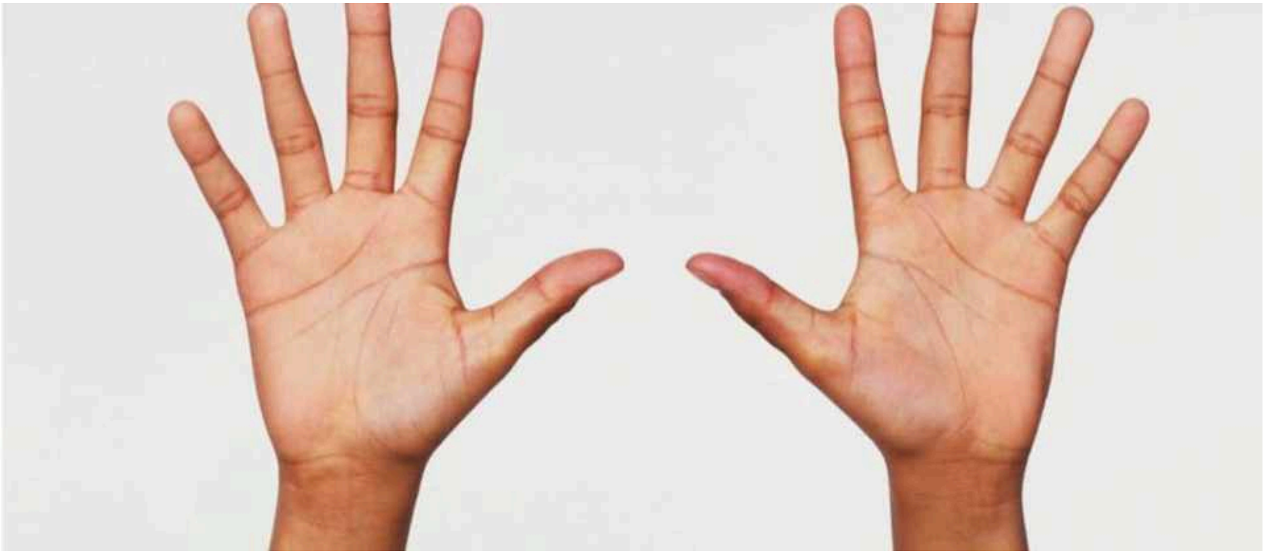
Outline

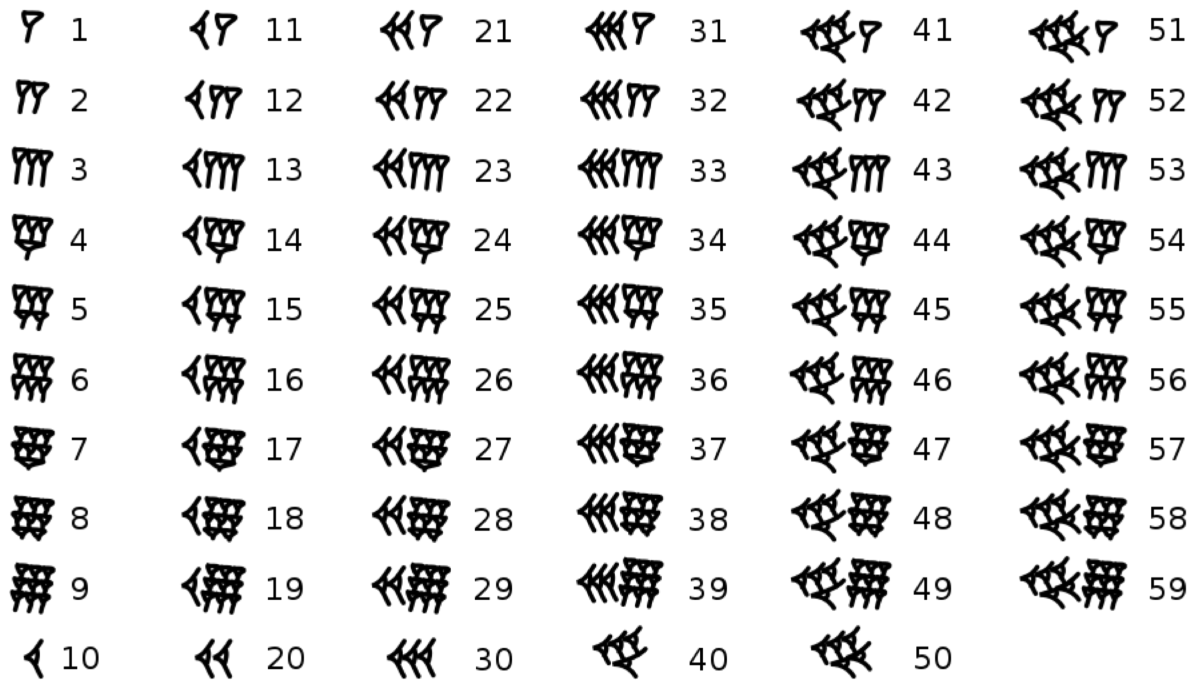
- 1. Representation
- 2. Calculus
- 3. Logic
- 4. Linear Algebra
- 5. Probability
- 7. Graph Theory
- 8. Dynamic Programming

1. Representation

1. Numerical System



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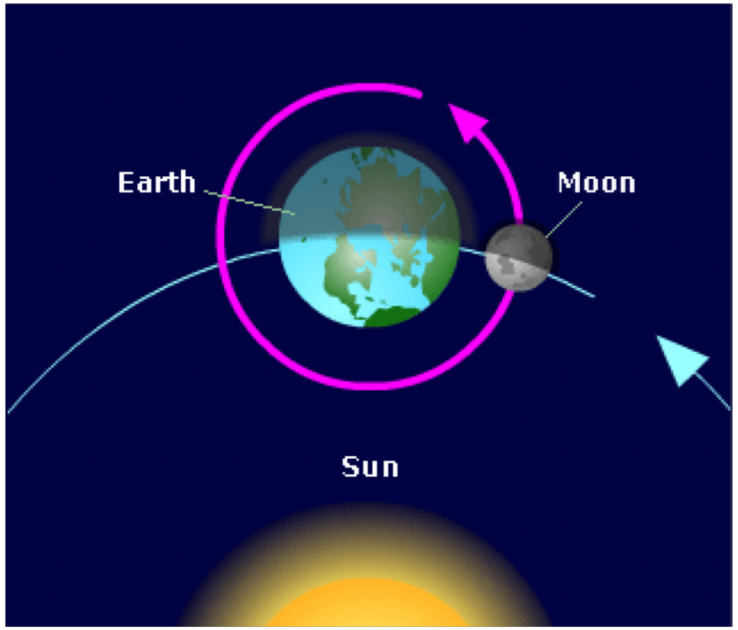
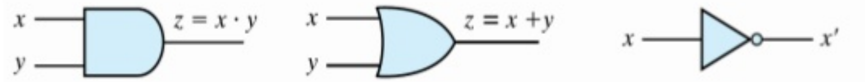

Earth
Moon
Sun

Binary Logic

- Logic gates
  - Graphic Symbols and Input-Output Signals for Logic gates:



(a) Two-input AND gate (b) Two-input OR gate (c) NOT gate or inverter

Fig. 1.4 Symbols for digital logic circuits

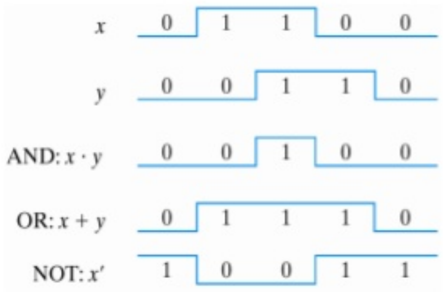
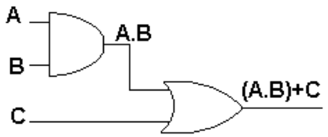
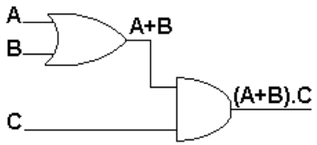


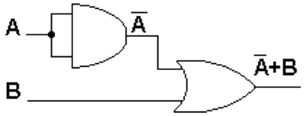
Fig. 1.5 Input-Output signals for gates



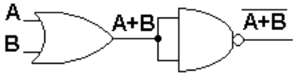
A and B high or C high
will make the output high.



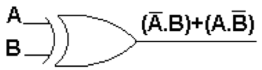
A or B high and C high
will make the output high.

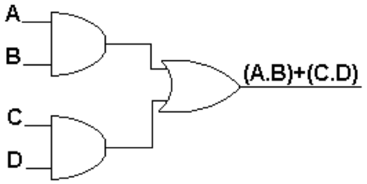


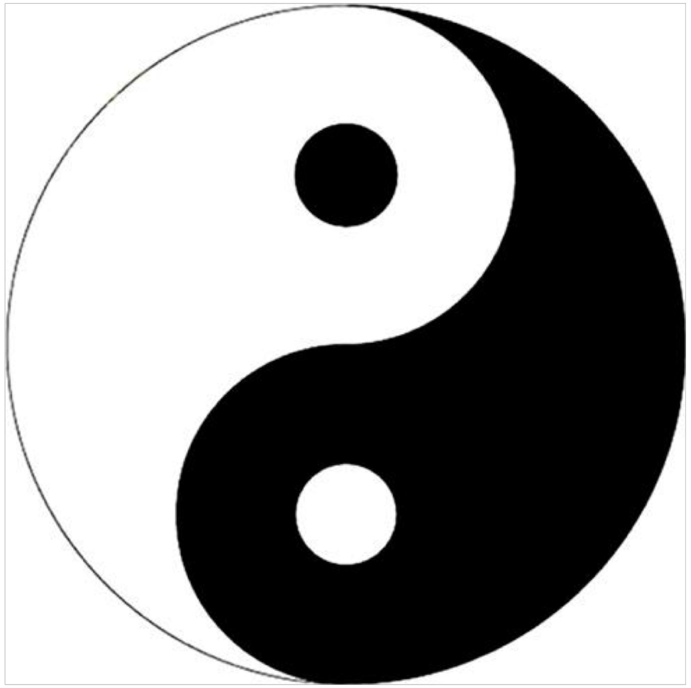
A low or B high
will make the ouput high.

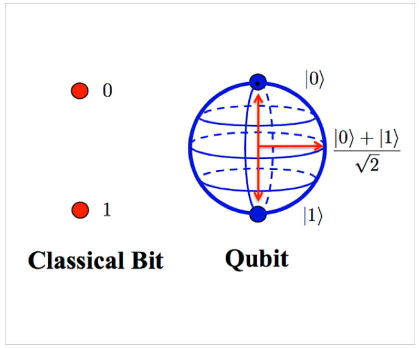


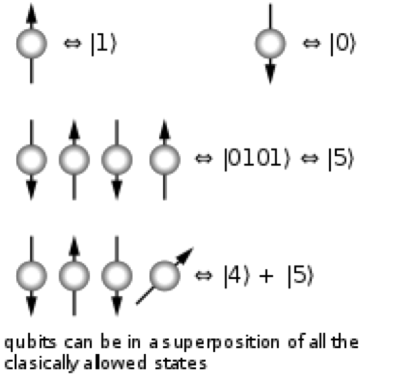
The long bar above the output means
that the output goes low when A or B
go high.



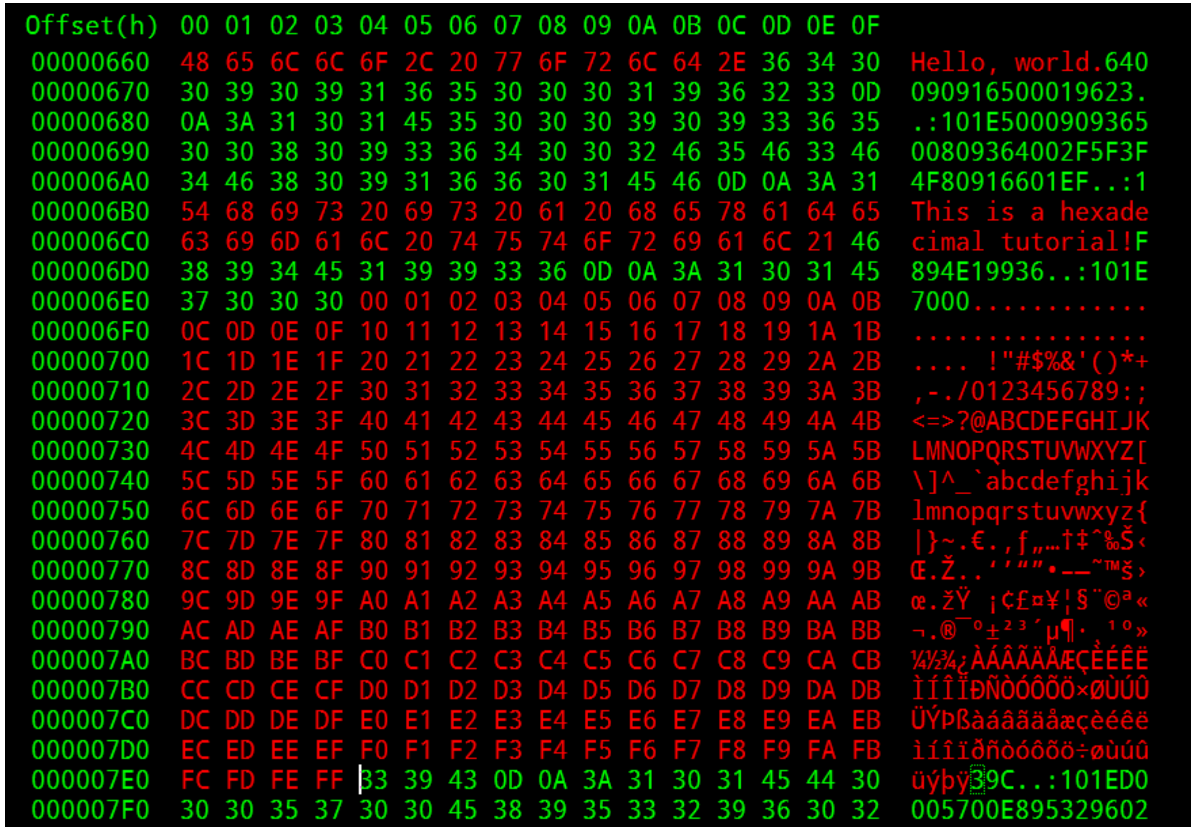





0
1
Classical Bit
Qubit
$|0\rangle$
$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$
$|1\rangle$


$\Leftrightarrow |1\rangle$
$\Leftrightarrow |0\rangle$
$\Leftrightarrow |0101\rangle \Leftrightarrow |5\rangle$
$\Leftrightarrow |4\rangle + |5\rangle$
qubits can be in a superposition of all the clasically allowed states

Hexadecimal	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111



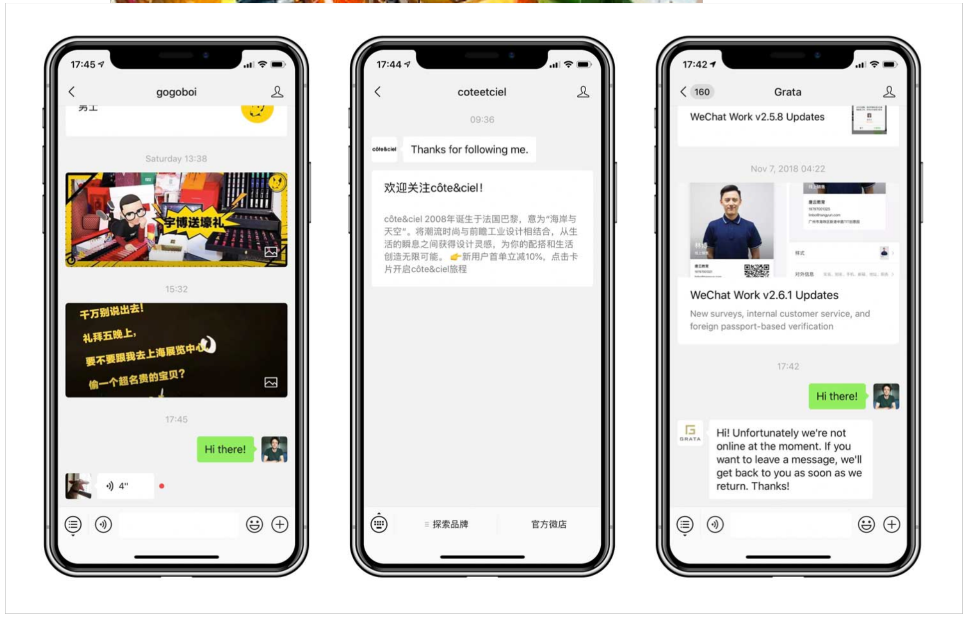
```
Offset(h) 00 01 02 03 04 05 06 07 08 09 0A 0B 0C 0D 0E 0F
00000660  48 65 6C 6C 6F 2C 20 77 6F 72 6C 64 2E 36 34 30  Hello, world.640
00000670  30 39 30 39 31 36 35 30 30 30 31 39 36 32 33 0D  090916500019623.
00000680  0A 3A 31 30 31 45 35 30 30 30 39 30 39 33 36 35  .:101E5000909365
00000690  30 30 38 30 39 33 36 34 30 30 32 46 35 46 33 46  00809364002F5F3F
000006A0  34 46 38 30 39 31 36 36 30 31 45 46 0D 0A 3A 31  4F80916601EF..:1
000006B0  54 68 69 73 20 69 73 20 61 20 68 65 78 61 64 65  This is a hexade
000006C0  63 69 6D 61 6C 20 74 75 74 6F 72 69 61 6C 21 46  cimal tutorial!F
000006D0  38 39 34 45 31 39 39 33 36 0D 0A 3A 31 30 31 45  894E19936..:101E
000006E0  37 30 30 30 00 01 02 03 04 05 06 07 08 09 0A 0B  7000............
000006F0  0C 0D 0E 0F 10 11 12 13 14 15 16 17 18 19 1A 1B  ................
00000700  1C 1D 1E 1F 20 21 22 23 24 25 26 27 28 29 2A 2B  .... !"#$%&'()*+
00000710  2C 2D 2E 2F 30 31 32 33 34 35 36 37 38 39 3A 3B  ,-./0123456789:;
00000720  3C 3D 3E 3F 40 41 42 43 44 45 46 47 48 49 4A 4B  <=>?@ABCDEFGHIJK
00000730  4C 4D 4E 4F 50 51 52 53 54 55 56 57 58 59 5A 5B  LMNOPQRSTUVWXYZ[
00000740  5C 5D 5E 5F 60 61 62 63 64 65 66 67 68 69 6A 6B  \]^_`abcdefghijk
00000750  6C 6D 6E 6F 70 71 72 73 74 75 76 77 78 79 7A 7B  lmnopqrstuvwxyz{
00000760  7C 7D 7E 7F 80 81 82 83 84 85 86 87 88 89 8A 8B  |}~.€.‚ƒ„…†‡ˆ‰Š‹
00000770  8C 8D 8E 8F 90 91 92 93 94 95 96 97 98 99 9A 9B  Œ.Ž..‘’“”•–—˜™š›
00000780  9C 9D 9E 9F A0 A1 A2 A3 A4 A5 A6 A7 A8 A9 AA AB  œ.žŸ ¡¢£¤¥¦§¨©ª«
00000790  AC AD AE AF B0 B1 B2 B3 B4 B5 B6 B7 B8 B9 BA BB  ¬.®¯°±²³´µ¶·¸¹º»
000007A0  BC BD BE BF C0 C1 C2 C3 C4 C5 C6 C7 C8 C9 CA CB  ¼½¾¿ÀÁÂÃÄÅÆÇÈÉÊË
000007B0  CC CD CE CF D0 D1 D2 D3 D4 D5 D6 D7 D8 D9 DA DB  ÌÍÎÏÐÑÒÓÔÕÖ×ØÙÚÛ
000007C0  DC DD DE DF E0 E1 E2 E3 E4 E5 E6 E7 E8 E9 EA EB  ÜÝÞßàáâãäåæçèéêë
000007D0  EC ED EE EF F0 F1 F2 F3 F4 F5 F6 F7 F8 F9 FA FB  ìíîïðñòóôõö÷øùúû
000007E0  FC FD FE FF 33 39 43 0D 0A 3A 31 30 31 45 44 30  üýþÿ39C..:101ED0
000007F0  30 30 35 37 30 30 45 38 39 35 33 32 39 36 30 32  005700E895329602
```

ASCII Table

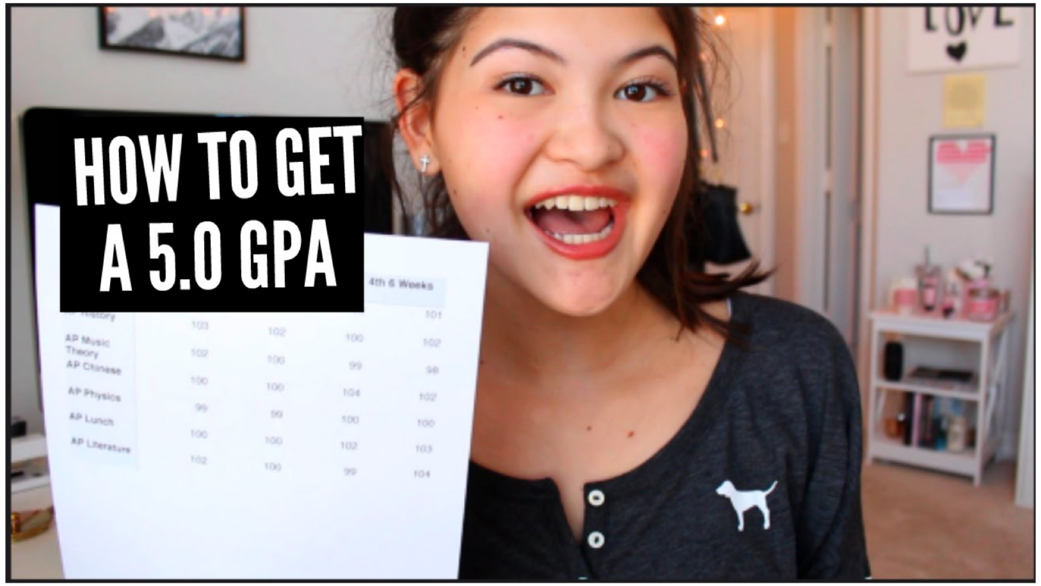
Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char
0	0	0		32	20	40	[space]	64	40	100	@	96	60	140	`
1	1	1		33	21	41	!	65	41	101	A	97	61	141	a
2	2	2		34	22	42	"	66	42	102	B	98	62	142	b
3	3	3		35	23	43	#	67	43	103	C	99	63	143	c
4	4	4		36	24	44	$	68	44	104	D	100	64	144	d
5	5	5		37	25	45	%	69	45	105	E	101	65	145	e
6	6	6		38	26	46	&	70	46	106	F	102	66	146	f
7	7	7		39	27	47	'	71	47	107	G	103	67	147	g
8	8	10		40	28	50	(	72	48	110	H	104	68	150	h
9	9	11		41	29	51	)	73	49	111	I	105	69	151	i
10	A	12		42	2A	52	*	74	4A	112	J	106	6A	152	j
11	B	13		43	2B	53	+	75	4B	113	K	107	6B	153	k
12	C	14		44	2C	54	,	76	4C	114	L	108	6C	154	l
13	D	15		45	2D	55	-	77	4D	115	M	109	6D	155	m
14	E	16		46	2E	56	.	78	4E	116	N	110	6E	156	n
15	F	17		47	2F	57	/	79	4F	117	O	111	6F	157	o
16	10	20		48	30	60	0	80	50	120	P	112	70	160	p
17	11	21		49	31	61	1	81	51	121	Q	113	71	161	q
18	12	22		50	32	62	2	82	52	122	R	114	72	162	r
19	13	23		51	33	63	3	83	53	123	S	115	73	163	s
20	14	24		52	34	64	4	84	54	124	T	116	74	164	t
21	15	25		53	35	65	5	85	55	125	U	117	75	165	u
22	16	26		54	36	66	6	86	56	126	V	118	76	166	v
23	17	27		55	37	67	7	87	57	127	W	119	77	167	w
24	18	30		56	38	70	8	88	58	130	X	120	78	170	x
25	19	31		57	39	71	9	89	59	131	Y	121	79	171	y
26	1A	32		58	3A	72	:	90	5A	132	Z	122	7A	172	z
27	1B	33		59	3B	73	;	91	5B	133	[	123	7B	173	{
28	1C	34		60	3C	74	<	92	5C	134	\	124	7C	174	|
29	1D	35		61	3D	75	=	93	5D	135	]	125	7D	175	}
30	1E	36		62	3E	76	>	94	5E	136	^	126	7E	176	~
31	1F	37		63	3F	77	?	95	5F	137	_	127	7F	177	

0020	0 0030	@ 0040	P 0050	` 0060	p 0070	00A0	° 00B0	À 00C0	Ð 00D0	à 00E0	ð 00F0
! 0021	1 0031	A 0041	Q 0051	a 0061	q 0071	¡ 00A1	± 00B1	Á 00C1	Ñ 00D1	á 00E1	ñ 00F1
" 0022	2 0032	B 0042	R 0052	b 0062	r 0072	¢ 00A2	² 00B2	Â 00C2	Ò 00D2	â 00E2	ò 00F2
# 0023	3 0033	C 0043	S 0053	c 0063	s 0073	£ 00A3	³ 00B3	Ã 00C3	Ó 00D3	ã 00E3	ó 00F3
$ 0024	4 0034	D 0044	T 0054	d 0064	t 0074	¤ 00A4	´ 00B4	Ä 00C4	Ô 00D4	ä 00E4	ô 00F4
% 0025	5 0035	E 0045	U 0055	e 0065	u 0075	¥ 00A5	µ 00B5	Å 00C5	Õ 00D5	å 00E5	õ 00F5
& 0026	6 0036	F 0046	V 0056	f 0066	v 0076	¦ 00A6	¶ 00B6	Æ 00C6	Ö 00D6	æ 00E6	ö 00F6
' 0027	7 0037	G 0047	W 0057	g 0067	w 0077	§ 00A7	· 00B7	Ç 00C7	× 00D7	ç 00E7	÷ 00F7
(0028	8 0038	H 0048	X 0058	h 0068	x 0078	¨ 00A8	¸ 00B8	È 00C8	Ø 00D8	è 00E8	ø 00F8
) 0029	9 0039	I 0049	Y 0059	i 0069	y 0079	© 00A9	¹ 00B9	É 00C9	Ù 00D9	é 00E9	ù 00F9
* 002A	: 003A	J 004A	Z 005A	j 006A	z 007A	ª 00AA	º 00BA	Ê 00CA	Ú 00DA	ê 00EA	ú 00FA
+ 002B	; 003B	K 004B	[005B	k 006B	{ 007B	« 00AB	» 00BB	Ë 00CB	Û 00DB	ë 00EB	û 00FB
, 002C	< 003C	L 004C	\ 005C	l 006C	| 007C	¬ 00AC	¼ 00BC	Ì 00CC	Ü 00DC	ì 00EC	ü 00FC
− 002D	= 003D	M 004D	] 005D	m 006D	} 007D	- 00AD	½ 00BD	Í 00CD	Ý 00DD	í 00ED	ý 00FD
. 002E	> 003E	N 004E	^ 005E	n 006E	~ 007E	® 00AE	¾ 00BE	Î 00CE	Þ 00DE	î 00EE	þ 00FE
/ 002F	? 003F	O 004F	_ 005F	o 006F	007F	¯ 00AF	¿ 00BF	Ï 00CF	ß 00DF	ï 00EF	ÿ 00FF

All are numbers



Categorical & Numerical

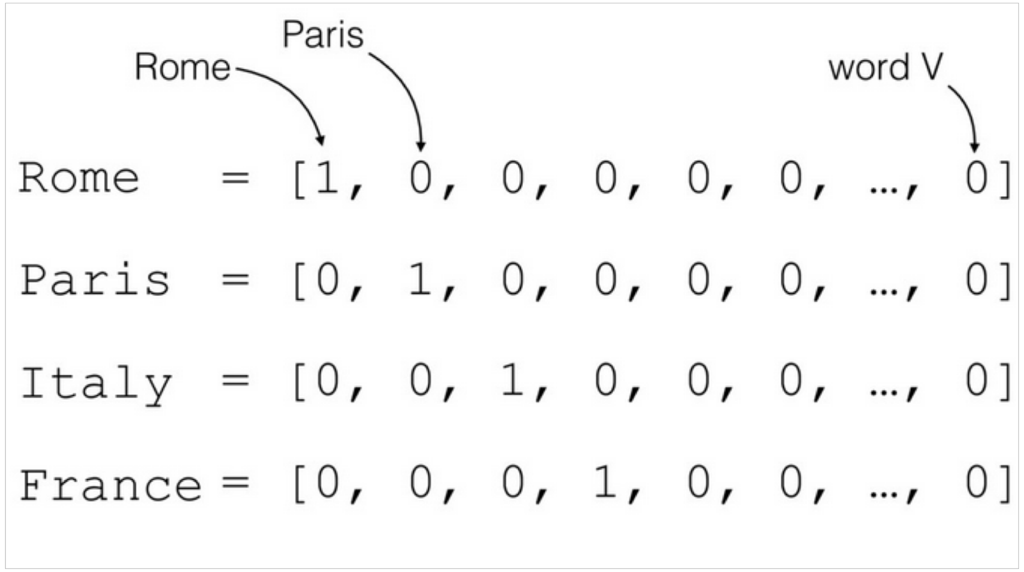


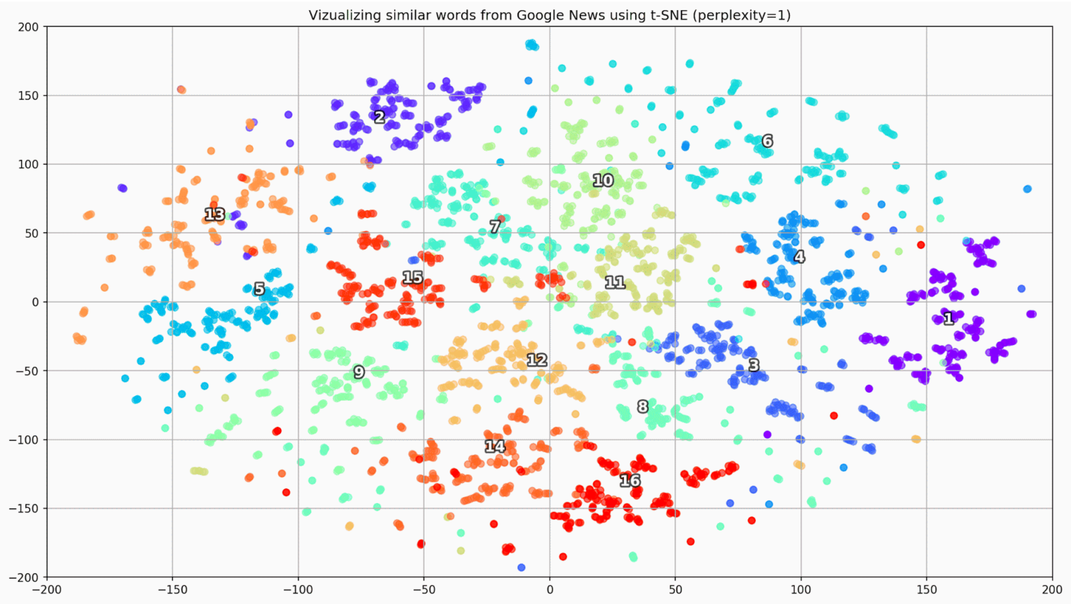
- Categorical
  - Could not be compared
  - Could not be sorted
  - The Number just is a representation

- Numerical
  - Could be compared
  - Could be sorted
  - The number with attribute

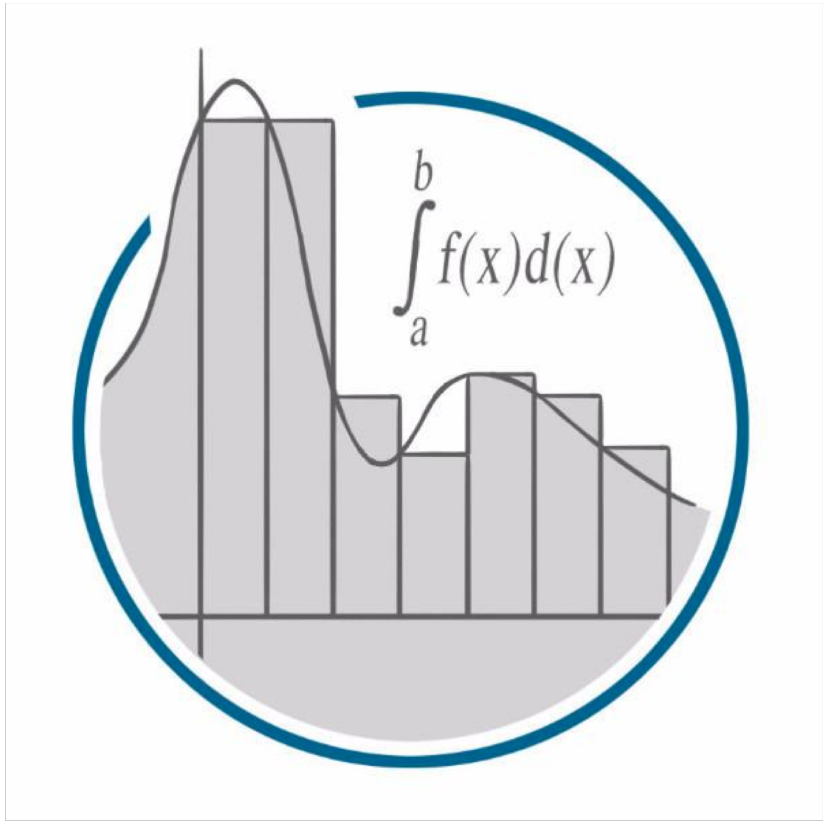
Why it counts?

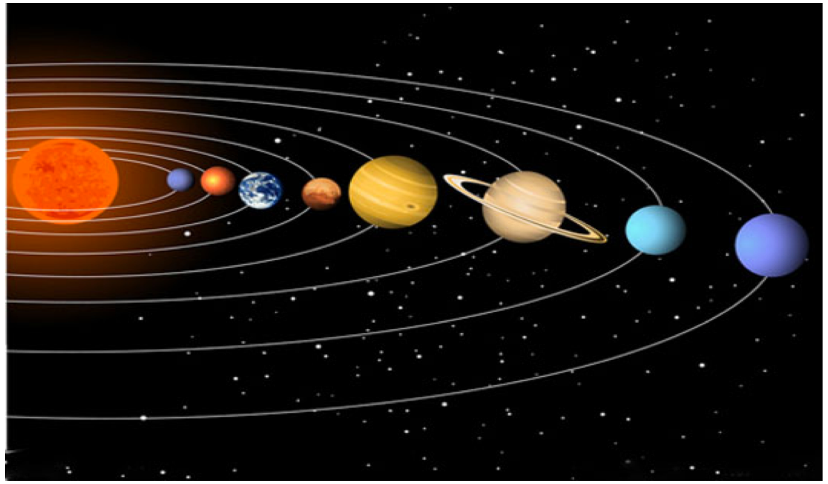
One-Hot and Embedding



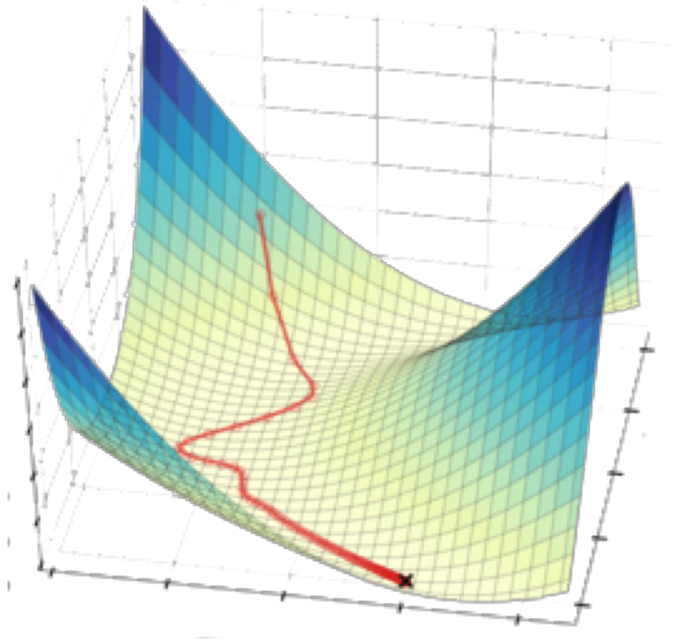


2. Calculus

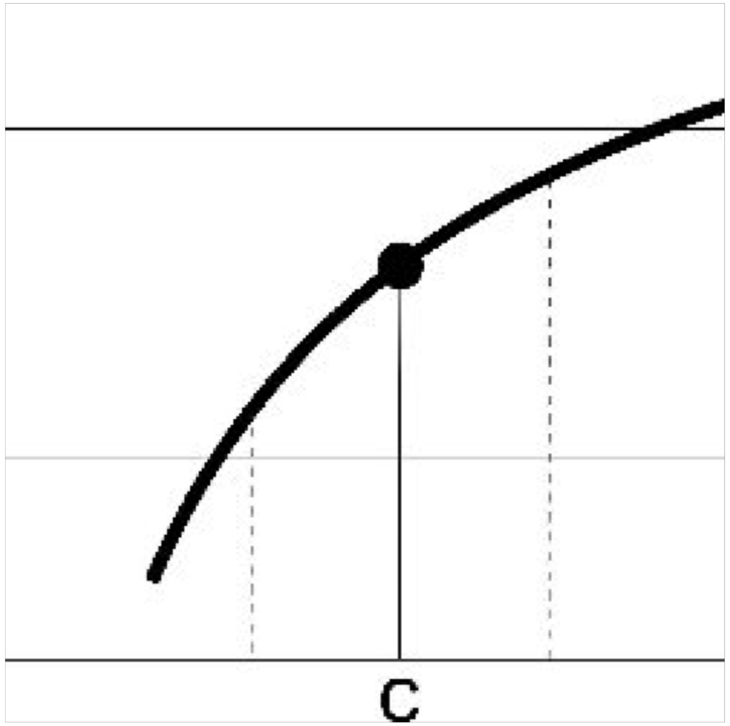




$$\int_a^b f(x)\,dx = F(b) - F(a)$$



$$\frac{df}{dt} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$



$$\frac{d}{dx}(a)=0$$

$$\frac{d}{dx}(x)=1$$

$$\frac{d}{dx}(au)=a\frac{du}{dx}$$

$$\frac{d}{dx}(u+v-w)=\frac{du}{dx}+\frac{dv}{dx}-\frac{dw}{dx}$$

$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{1}{v}\frac{du}{dx}-\frac{u}{v^2}\frac{dv}{dx}$$

$$\frac{d}{dx}(u^n)=nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}(\sqrt{u})=\frac{1}{2\sqrt{u}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u}\right)=-\frac{1}{u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u^n}\right)=-\frac{n}{u^{n+1}}\frac{du}{dx}$$

$$\frac{d}{dx}[f(u)]=\frac{d}{du}[f(u)]\frac{du}{dx}$$

$$\frac{d}{dx}[\ln u]=\frac{d}{dx}[\log_e u]=\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}[\log_a u]=\log_a e\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}e^u=e^u\frac{du}{dx}$$

$$\frac{d}{dx}a^u=a^u\ln a\frac{du}{dx}$$

$$\frac{d}{dx}(u^v)=vu^{v-1}\frac{du}{dx}+\ln u\ \ u^v\frac{dv}{dx}$$

$$\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u=-\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u=\sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\cot u=-\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\sec u=\sec u\tan u\frac{du}{dx}$$

$$\frac{d}{dx}\csc u=-\csc u\cot u\frac{du}{dx}$$

Exercise 1: Implicit Differentiation: Solutions

Use implicit differentiation to find $\frac{dy}{dx}$ where

(a) $x\,y = 1$

Differentiating both sides wrt x:

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

Using the product rule:

$$x\,.\,\frac{dy}{dx} + 1\,.\,y = 0$$

$$\frac{dy}{dx} = -\,\frac{y}{x}$$

(b) $x^2y = 1$

Differentiating both sides wrt x:

$$\frac{d}{dx}(x^2y) = \frac{d}{dx}(1)$$

Using the product rule:

$$x^2 .\,\frac{dy}{dx} + 2x\,.\,y = 0$$

$$\frac{dy}{dx} = -\,\frac{2xy}{x^2} = -\,\frac{2y}{x}$$

(c) $e^xy = 1$

Differentiating both sides wrt x:

$$\frac{d}{dx}(e^xy) = \frac{d}{dx}(1)$$

Using the product rule:

$$e^x .\,\frac{dy}{dx} + e^x .\,y = 0$$

$$\frac{dy}{dx} = -\,\frac{e^xy}{e^x} = -y$$

(d) $x\,y^2 = 1$

Differentiating both sides wrt x:

$$\frac{d}{dx}(xy^2) = \frac{d}{dx}(1)$$

Using the product and chain rules:

$$x\,.\,2y\,\frac{dy}{dx} + 1\,.\,y^2 = 0$$

$$\frac{dy}{dx} = -\,\frac{y^2}{2xy} = -\,\frac{y}{2x}$$

(e) $x^2y^2 = 1$

Differentiating both sides wrt x:

$$\frac{d}{dx}(x^2y^2) = \frac{d}{dx}(1)$$

Using the product and chain rules:

$$x^2 .\,2y\,\frac{dy}{dx} + 2x\,.\,y^2 = 0$$

$$\frac{dy}{dx} = -\,\frac{2xy^2}{2x^2y} = -\,\frac{y}{x}$$

(f) $e^xy^2 = 1$

Differentiating both sides wrt x:

$$\frac{d}{dx}(e^xy^2) = \frac{d}{dx}(1)$$

Using the product and chain rules:

$$e^x .\,2y\,\frac{dy}{dx} + e^x .\,y^2 = 0$$

$$\frac{dy}{dx} = -\,\frac{e^xy^2}{2e^xy} = -\,\frac{y}{2}$$

Note: (a) could be done by rearranging first to $y = 1/x$ and then differentiating

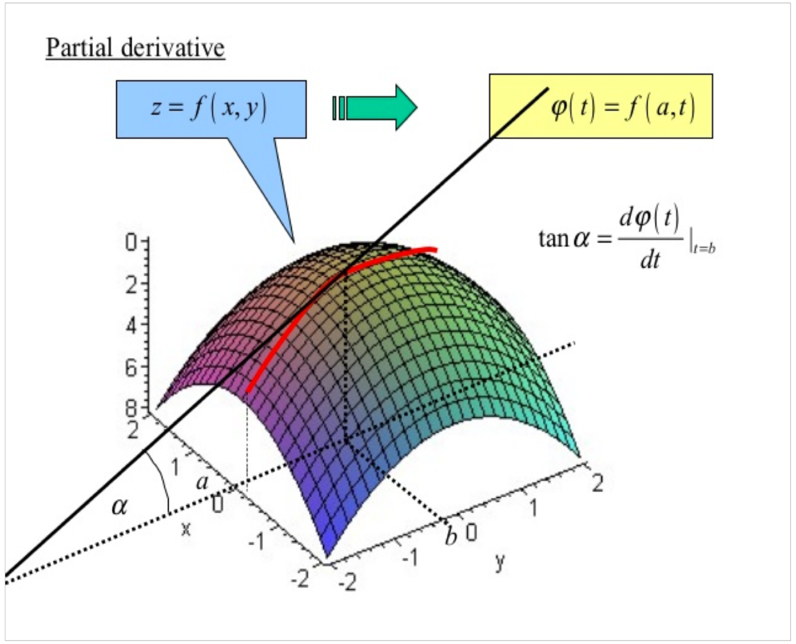
Note: (b) could be done by rearranging first to $y = 1/x^2$ and then differentiating

Note: (c) could be done by rearranging first to $y = e^{-x}$ and then differentiating

Find the Partial Derivatives: Example 4

$$f(x,y,z) = xz - 3x^2y^3z^4$$

$$\frac{\partial f}{\partial x} = ? \qquad \frac{\partial f}{\partial y} = ? \qquad \frac{\partial f}{\partial z} = ?$$



$$\sum_{k=1}^{n} a_k$$

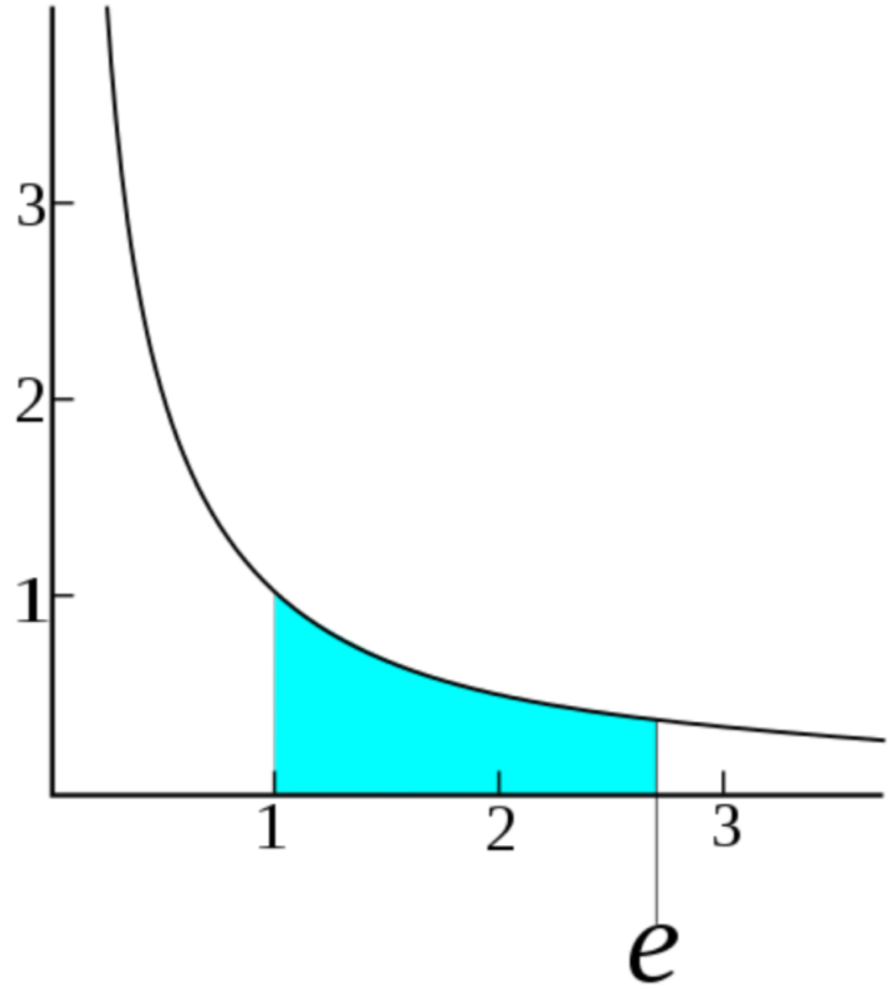
$$\sum_{k=1}^{\infty} a_k$$

$$\sum_{k=1}^{n}(ca_k + b_k) = c\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k .$$

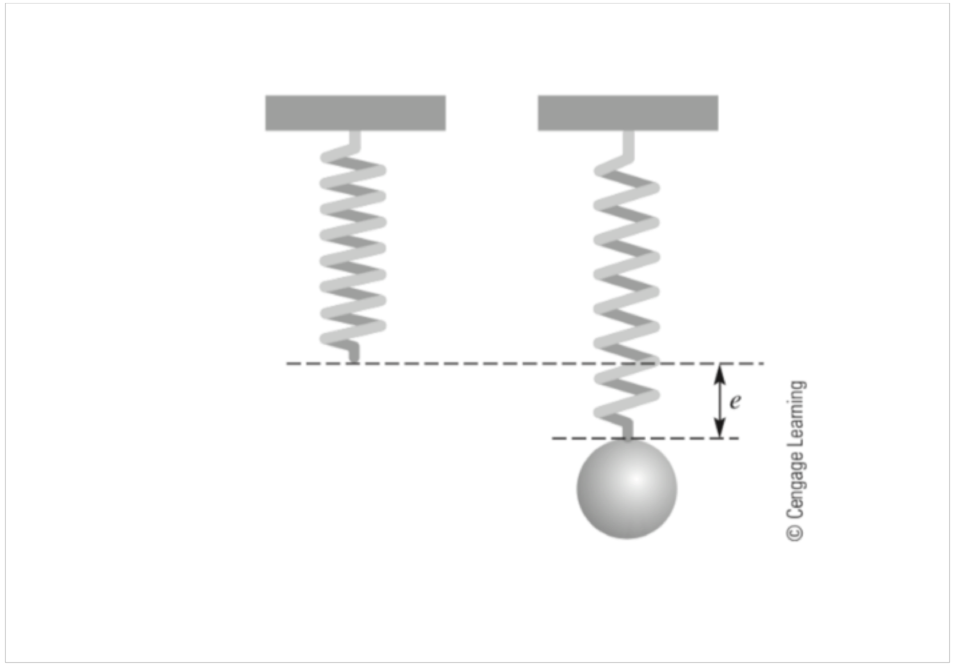
$$\sum_{k=1}^{n} k = 1 + 2 + \cdots + n$$

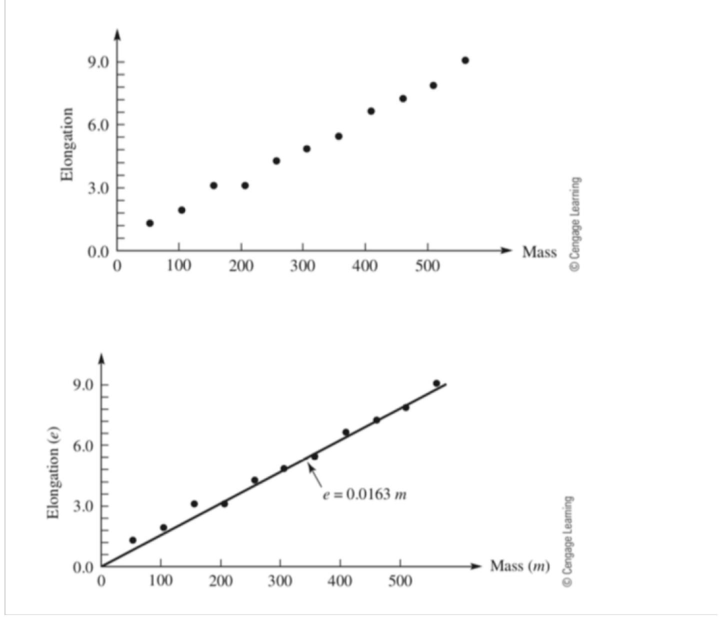
$$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \cdots$$



- We can input the math in jupyter notebook


e
© Cengage Learning


Elongation
9.0
6.0
3.0
0.0
0
100
200
300
400
500
Mass
© Cengage Learning
Elongation (e)
e = 0.0163 m
Mass (m)
© Cengage Learning

$y \propto x.$

Product:

$$\prod_{k=1}^{n} a_k$$

$$\lg\left(\prod_{k=1}^{n} a_k\right) = \sum_{k=1}^{n} \lg a_k \ .$$

3. Logic

Set, a collection of distinguishable objects.

$ \in $

S = {1, 2, 3} = {2, 3, 1} = {3, 1, 2}

- $\emptyset$ denotes the ***empty set***, that is, the set containing no members.
- $\mathbb{Z}$ denotes the set of ***integers***, that is, the set $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
- $\mathbb{R}$ denotes the set of ***real numbers***.
- $\mathbb{N}$ denotes the set of ***natural numbers***, that is, the set $\{0, 1, 2, \ldots\}$.[2]

- The ***intersection*** of sets A and B is the set

  $$A \cap B = \{x : x \in A \text{ and } x \in B\} \ .$$

- The ***union*** of sets A and B is the set

  $$A \cup B = \{x : x \in A \text{ or } x \in B\} \ .$$

- The ***difference*** between two sets A and B is the set

  $$A - B = \{x : x \in A \text{ and } x \notin B\} \ .$$

Associative laws:

$$\begin{aligned} A \cap (B \cap C) &= (A \cap B) \cap C \ , \\ A \cup (B \cup C) &= (A \cup B) \cup C \ . \end{aligned}$$

Distributive laws:

$$\begin{aligned} A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \ , \\ A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \ . \end{aligned}$$

Absorption laws:

$$\begin{aligned} A \cap (A \cup B) &= A \ , \\ A \cup (A \cap B) &= A \ . \end{aligned}$$

- $\exists$
- $\forall$
- s.t
- $\because$
- $\therefore$
- $argmax()$

- Running those in python (sum, mul, set, any, all)

Next

- March 29. PM 8:00 – 10:30

- 4. Linear Algebra
- 5. Probability
- 6. Graph Theory
- 7. Dynamic Programming