NUCLEAR INSTRUMENTS AND METHODS 129 (1975) 497–499; © NORTH-HOLLAND PUBLISHING CO.

SOME PRACTICAL REMARKS ON MULTIPLE SCATTERING*

VIRGIL L. HIGHLAND

Physics Department, Temple University, Philadelphia, Pennsylvania 19121, U.S.A.

Received 22 August 1975

The experimentalist's familiar formula for multiple scattering is investigated in terms of the more exact theory. A new value for the constant is suggested: $E_S = 17.5$ MeV. Convenient formulas for a thickness-dependent correction and a center-of-mass correction are given.

1. Introduction

The theory of multiple Coulomb scattering of particles by material has been developed in considerable detail and elegance in many papers[1–4]). But the average experimentalist, when pressed for a quick decision in the design or execution of an experiment, usually relies on the simple formula of Rossi and Greisen[5]). That formula gives an estimate of the rms scattering angle $\theta_{1/e}$ in terms of the radiation length L_R:

$$\theta_{1/e} = \frac{E_s}{p\beta c}\sqrt{\frac{L}{L_R}}, \qquad (1)$$

where L is the thickness of material, p is the momentum and βc the velocity of the incident particle in the laboratory. The scattering distribution has a long non-Gaussian tail below the 5% level, so $\theta_{1/e}$ is better considered to be the angle at which the distribution has fallen by 1/e. E_s is supposed to be a constant for which Rossi and Greisen gave the value $E_s = 21$ MeV. The value $E_{sx} = 15$ MeV given on the Particle Data Group wallet card[6]) is just $\frac{1}{2}\sqrt{2}E_s$, and is appropriate for the projected angle. The wallet card also suggests an additional dependence on L given by a correction factor $1+\varepsilon(L)$, but no prescription is given for computing $\varepsilon(L)$.

The main purpose of this note is to investigate the degree of accuracy of eq. (1), and to suggest an improved value for E_s (and E_{sx}). Although such a simple rule of thumb should not be taken too seriously, its everyday utility suggests that it might as well be as unbiased as is practical. In addition a formula for the correction term ε is given. Finally, a simple form for a center-of-mass correction is given.

* Work supported in part by U.S. Energy Research and Development Administration.

2. Calculations

Multiple scattering has little to do with the radiation length except that both involve a dependence on Z^2/A (Z and A are the atomic number and weight of the scatterer). The success of eq. (1) is due to that fact and to the fortuitous near-cancellation of the effects of several complicated additional Z dependences in multiple scattering theory[2]) and in the modern definition of the radiation length. It is possible to gather all these Z dependences together and evaluate them semi-analytically. Here a more straightforward method is adopted: $\theta_{1/e}$ is evaluated from the theory for various values of Z, L and β. Then for each point we compute the value

$$E_s = \theta_{1/e}\, p\beta c\, \sqrt{(L_R/L)},$$

using the wallet card value of L_R in every case[6, 7]).

The theory of Molière[1]) is used in the calculation since it is relatively simple to apply and is in essential agreement with most other formulations within the accuracy needed here. In addition the Rice–Houston

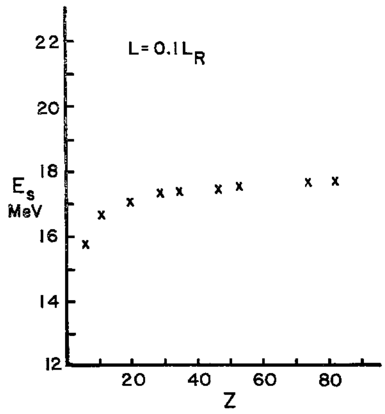

Fig. 1. Value of E_S as a function of Z, for $L = 0.1\,L_R$ and $\beta = 1$.

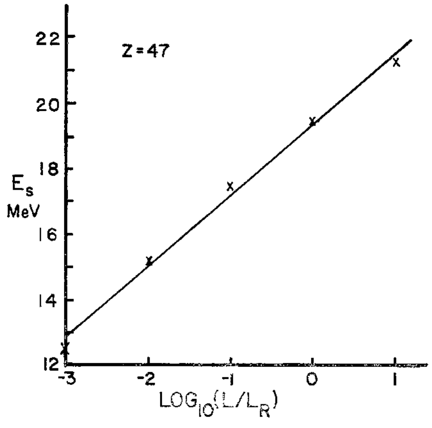

Fig. 2. Value of E_S as a function of L, for $Z = 47$.

group has observed the scattering of energetic protons[8]) and pions[9]). They find the Molière theory fits the data well, provided a center of mass correction is made (see section 3). The fit can be improved, especially at lower Z, by an empirical adjustment of an interpolation formula of Molière.

In terms of the standard parameters χ_c and B of Molière's theory[1–3]), the angle θ was calculated as

$$\theta_{1/e} = \chi_c \sqrt{(B-1.2)}.$$

The subtraction of 1.2 follows the suggestion of Hanson et al.[10,2]) to give the best approximation to the 1/e point of the distribution, taking into account its non-Gaussian character.

In fig. 1 is presented the variation of E_s with Z for $L = 0.1\,L_R$ and $\beta \approx 1$. For $Z > 20$ the variation of E_s is less than $\pm 2\%$. Thus E_s is essentially independent of Z. Since $L = 0.1\,L_R$ is probably the center of the most interesting range of L, I suggest adopting the value $E_s = 17.5$ MeV. (If the original Molière formula were used the value would be 17.0 MeV.)

For $Z < 20$ it can be seen that the value of E_s is not so constant. It is well known that Molière's theory is less accurate at low Z, probably due to inaccuracies in the screening calculation. Mayes et al. find that the theory underestimates the actual scattering at low Z. The higher value of E_s is at least in the proper direction to tend to compensate that underestimation.

Fig. 2 shows the variation of E_s with L for a central value of Z. Analytical considerations suggest that there will be a linear dependence of E_s on ${}^{10}\log\,(L/L_R)$. This expectation is confirmed by the good fit of the straight line to the data. Thus I suggest for the L dependent correction term ε:

$$\varepsilon(L) = 0.125\ {}^{10}\log\left(\frac{L}{0.1\,L_R}\right) \approx 0.1\ {}^{10}\log\left(\frac{L}{0.1\,L_R}\right), \tag{2}$$

the latter form being sufficiently accurate and easier to remember.

There is a residual β dependence in the Molière theory besides that shown in eq. (1). (If the charge of the projectile is ze instead of e, replace β everywhere by β/z.) The calculations show that as β decreases from 1 to 0.8, the value of E_s increases by about 1%. This is entirely negligible in the present context. For larger decreases in β the change in E_s becomes more significant, depending on Z. As $\beta \to 0$ the increase is 20% for $Z=6$, 10% for $Z=20$ and 3% for $Z=82$.

3. Center–of–Mass Correction

Hungerford et al.[8]) find good agreement with Molière's theory provided that the theory is applied in the center-of-mass system. That is, $p\beta$ should be evaluated in the c.m. and a solid angle transformation from c.m. to laboratory should be made.

All the necessary kinematics are presented by Hungerford et al. But the equations are sufficiently cumbersome-looking to conceal the fact that they reduce to the following simple result (in small angle approximation):

$$\theta = \theta_0 \left(1 + \frac{m_p^2}{E_p m_t}\right), \tag{3}$$

where m_p, E_p are the mass and total energy of the projectile and m_t is the mass of the target atom. The angle θ is the corrected value and θ_0 is the uncorrected value calculated using laboratory system values.

4. Summary

It is suggested that the 1/e multiple scattering angle be calculated from:

$$\theta_{1/e} = \frac{17.5}{p\beta c}\sqrt{\frac{L}{L_R}}\,(1+\varepsilon) \tag{4}$$

where ε is given by eq. (2) and L_R is taken from the wallet card[6,7]). The accuracy is about 5% except for very light elements or low velocity where it is 10–20%. For the projected angle the corresponding constant E_{sx} equals 12.5 MeV. For heavy projectiles or light targets, the center-of-mass correction is given by eq. (3).

As an overall check, representative values computed from eq. (4) were compared to the scattering angle tables of Hungerford and Mayes[11]). The results were in agreement within the stated accuracy.

I would like to thank Dr W. Frati for bringing this topic to my attention and Dr W. Scott for reading the manuscript.

References

[1]) G. Molière, Z. Naturforsch. **2a** (1947) 133; **3a** (1948) 78.

[2]) H. A. Bethe, Phys. Rev. **89** (1953) 1256.

[3]) W. T. Scott, Rev. Mod. Phys. **35** (1963) 231. This article is a complete review of the subject.

[4]) J. B. Marion and B. A. Zimmerman, Nucl. Instr. and Meth. **51** (1967) 93. This paper presents graphs which expedite application.

[5]) B. Rossi and K. Greisen, Rev. Mod. Phys. **13** (1941) 240; B. Rossi, *High energy particles* (Prentice-Hall, Englewood Cliffs, N.J., 1961) p. 67.

[6]) Particle Data Group, Rev. Mod. Phys. **45** (1973) S35.

[7]) O. I. Dovzhenko and A. A. Pomanskii, Sov. Phys. JETP **18** (1964) 187. This article is the source of the radiation lengths quoted in ref. 6.

[8]) E. V. Hungerford, G. S. Mutchler, G. C. Philips, M. L. Scott, J. C. Allred, L. Y. Lee, B. W. Mayes and C. Goodman, Nucl. Phys. **A197** (1972) 515.

[9]) B. W. Mayes, L. Y. Lee, J. C. Allred, C. Goodman, G. S. Mutchler, E. V. Hungerford, M. L. Scott and G. C. Phillips, Nucl. Phys. **A230** (1974) 515.

[10]) A. O. Hanson, L. H. Lanzl, E. M. Lyman and M. B. Scott, Phys. Rev. **84** (1951) 634.

[11]) E. V. Hungerford and B. W. Mayes, At. Data Nucl. Tables **15** (1975) 478.