Problem Set 8

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December 2, 2019

1.Economic Questions

I want to use dynamic model to study consumption and portfolio choices for individuals.

2.Model description

Individual has an initial wealth, w. The individual receives income, y, and makes consumption, c, every period. He can invest in two types of assets every period. One is risk-free asset with R_f as interest rate and another one is risky asset with R_m as interest rate. The rate of time preference is β. The individual wants to maximize his utility for infinite horizon.

Individual only know his current wealth (w),current income (y) and historical interest rate (R_{t-1}) at the start of each period. He needs to predict current interest rate R based on historical information and makes decisions on consumption (c) and how much he invests in risky asset (S_m) and how much he invests in risk-free asset (S_f) for this period.

3.Objective function

The objective function for the individual is

$$max_{c_t,S_{m,t},S_{f,t}} \sum_{t=0}^{\infty} E_t(\beta^t u(c_t)) \tag{1}$$

4.Bellman equation

The Bellman equation is

$$\begin{aligned} V(w,y,R_{m,t-1}) = max_{S_m,S_f} & u(w+y-S_m-S_f) + \beta E_{y',R|y,R_{m,t-1}}(V(S_m R_m + S_f R_f, y', R_{m,t})) \\ & S_m >= 0 \\ & S_f >= 0 \end{aligned} \tag{2}$$

State variable is (w,y,$R_{m,t-1}$), where w and y are wealth and income at the start of each period, $R_{m,t-1}$ is previous market return.

Control variable is (S_f,S_m), where S_m is the amount of wealth invested in risky asset during this period and S_f is the amount of wealth invested in risk-free

asset during this period.

5.Assumption

I assume there is no correlation between income (y) and market return (R_m). So that E(R_m)=R_f and I assume that market return (R_m) follow AR(1) process.

$$R_{m,t} = \rho R_{m,t-1} + (1-\rho)(1-R_f) + \epsilon_t \tag{3}$$

I assume that income (y) increase over time following AR(1) process. The start point is 0.1 and the stable point is 1.

$$y_t = \rho y_{t-1} + (1-\rho) * 1 + \epsilon_t \tag{4}$$

6.Solve Model

I use Value Function Iteration to solve model.

From transition function $w' = S_m R_m + S_f R_f$, I can derive that $S_f = (w' - Rm * Sm)/Rf$ and $c = w + y - S_m - S_f$.

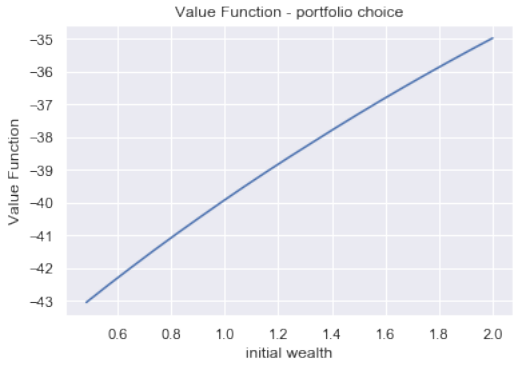
The model is converged after 378 iterations.

7.Model Solution

7.1 Value function

The first plot is value function as a function of initial wealth (w). This figure shows that total utility of individual increases with initial wealth.

Figure 1: value function as a function of initial wealth (w)



7.2 Policy function

The first plot is policy function of S_m as a function of initial wealth (w).This figure shows that amount invested in market asset increases with initial wealth.

The second plot is policy function of S_f as a function of initial wealth (w).This figure shows that amount invested in risk-free asset also increases with initial wealth.

The third plot is policy function of c as a function of initial wealth (w).This figure shows that consumption also increases with initial wealth.

Figure 2: Policy function as a function of initial wealth (w)

Figure 3: Policy function as a function of initial wealth (w)

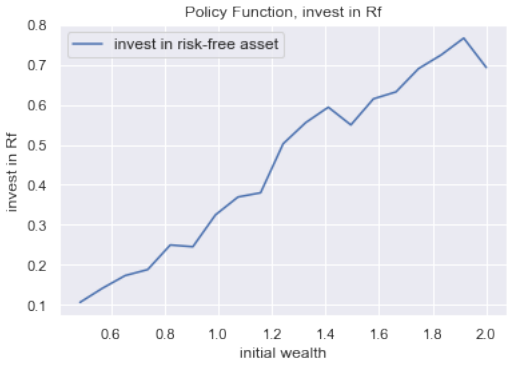


Figure 4: Policy function as a function of initial wealth (w)