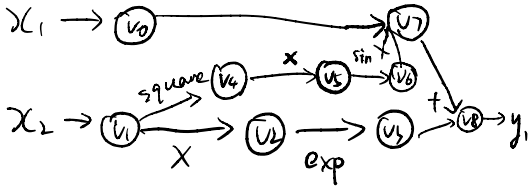
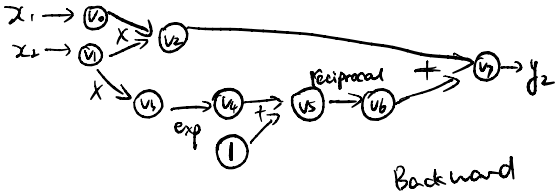
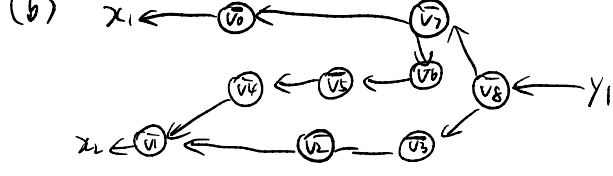
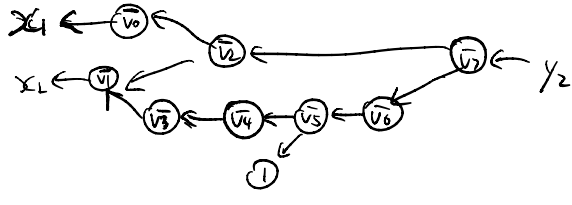
(a) Forward





(b) Backward





(c)

y_1	$f(2,1)$
$v_0 = x_1$	2
$v_1 = x_2$	1
$v_2 = 2v_1$	2
$v_3 = \exp(v_2)$	$\exp(2)$
$v_4 = v_1^2$	1
$v_5 = 3v_4$	3
$v_6 = \sin(v_5)$	$\sin(3)$
$v_7 = v_0 \cdot v_6$	$2\sin(3)$
$v_8 = v_3 + v_7$	$\exp(2) + 2\sin(3)$

y_2	$f(2,1)$
$v_0 = x_1$	2
$v_1 = x_2$	1
$v_2 = v_0 v_1$	2
$v_3 = -v_1$	-1
$v_4 = \exp(v_3)$	$\exp(-1)$
$v_5 = 1 + v_4$	$\exp(-1)+1$
$v_6 = \frac{1}{v_5}$	$\frac{1}{1+\exp(-1)}$
$v_7 = v_2 + v_6$	$2 + \frac{1}{1+\exp(-1)}$

a)

	$\frac{\partial f(x_1,x_2)}{\partial x_1}$	$\frac{\partial f(x_1,x_2)}{\partial x_2}$
$\frac{\partial v_0}{\partial x_1} = 1$	1	
$\frac{\partial v_0}{\partial x_2} = 0$		0
$\frac{\partial v_1}{\partial x_1} = 0$	0	
$\frac{\partial v_1}{\partial x_2} = 1$		1
$\frac{\partial v_2}{\partial x_1} = 2\frac{\partial v_1}{\partial x_1} = 0$	0	
$\frac{\partial v_2}{\partial x_2} = 2\frac{\partial v_1}{\partial x_2}$		2
$\frac{\partial v_3}{\partial x_1} = \exp(v_2)\frac{\partial v_2}{\partial x_1} = 0$	0	
$\frac{\partial v_3}{\partial x_2} = \exp(v_2)\frac{\partial v_2}{\partial x_2}$		$2\cdot\exp(2)$
$\frac{\partial v_4}{\partial x_1} = 2v_1\frac{\partial v_1}{\partial x_1} = 0$	0	
$\frac{\partial v_4}{\partial x_2} = 2v_1\frac{\partial v_1}{\partial x_2}$		2
$\frac{\partial v_5}{\partial x_1} = 3\frac{\partial v_4}{\partial x_1} = 0$	0	
$\frac{\partial v_5}{\partial x_2} = 3\cdot\frac{\partial v_4}{\partial x_2}$		6
$\frac{\partial v_6}{\partial x_1} = \cos(v_5)\frac{\partial v_5}{\partial x_1} = 0$	0	
$\frac{\partial v_6}{\partial x_2} = \cos(v_5)\cdot\frac{\partial v_5}{\partial x_2}$		$6\cos(3)$
$\frac{\partial v_7}{\partial x_1} = v_0\frac{\partial v_6}{\partial x_1} + v_6\cdot\frac{\partial v_0}{\partial x_1} = \sin(3)$	$\sin(3)$	
$\frac{\partial v_7}{\partial x_2} = v_0\cdot\frac{\partial v_6}{\partial x_2} + v_6\cdot\frac{\partial v_0}{\partial x_2}$		$12\cos(3)$
$\frac{\partial v_8}{\partial x_1} = \frac{\partial v_3}{\partial x_1} + \frac{\partial v_7}{\partial x_1} = \sin(3)$	$\sin(3)$	
$\frac{\partial v_8}{\partial x_2} = \frac{\partial v_3}{\partial x_2} + \frac{\partial v_7}{\partial x_2}$		$2\exp(2) + 12\cos(3)$

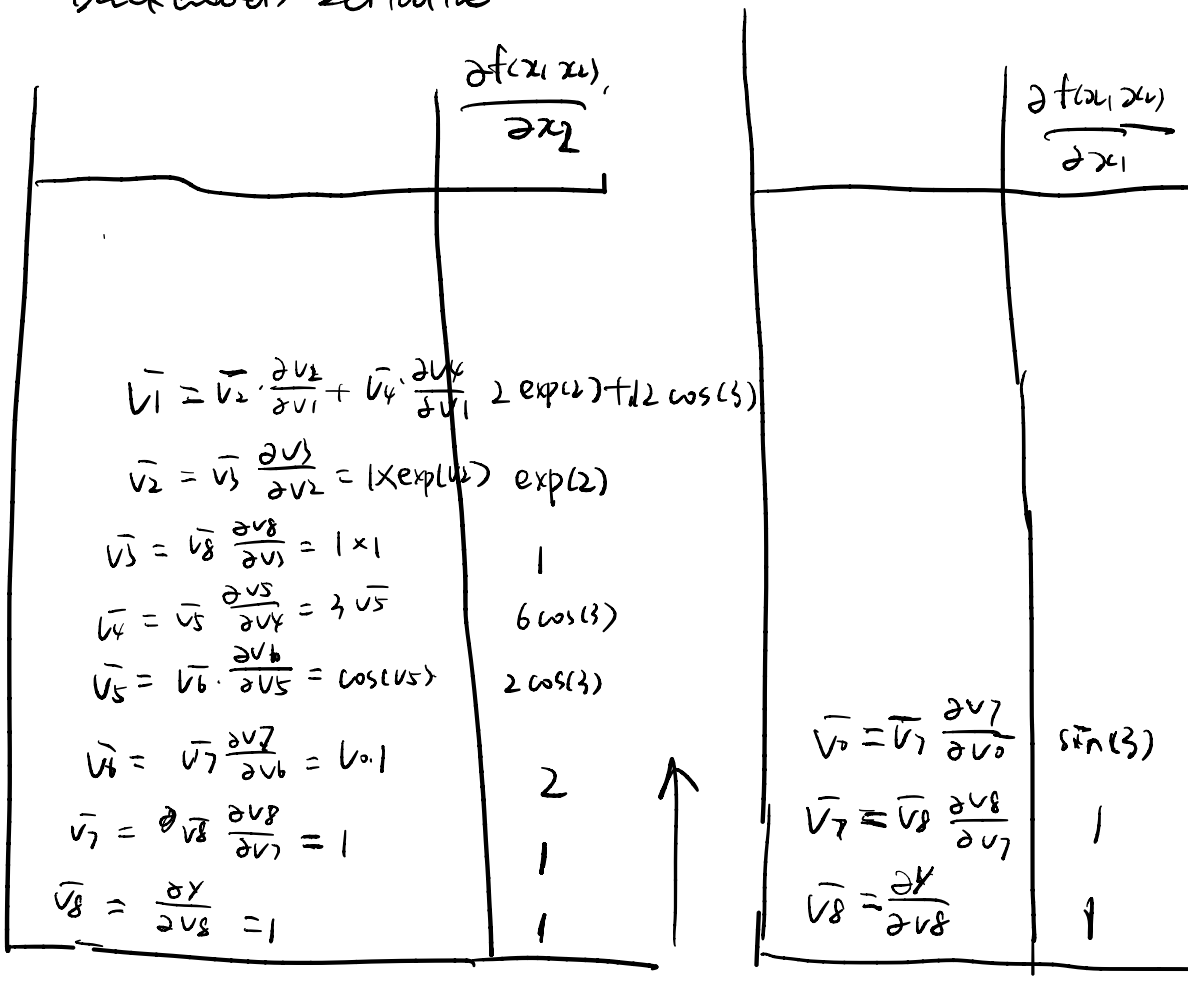
Forward

	$\frac{\partial f(x_1,x_2)}{\partial x_2}$
$\frac{\partial v_0}{\partial x_1} = 1$	1
$\frac{\partial v_1}{\partial x_1} = 0$	0
$\frac{\partial v_2}{\partial x_1} = v_0 \frac{\partial v_1}{\partial x_1} + v_1 \frac{\partial v_0}{\partial x_1}$	1
$\frac{\partial v_3}{\partial x_1} = -\frac{\partial v_1}{\partial x_1}$	0
$\frac{\partial v_4}{\partial x_1} = \exp(v_3) \cdot \frac{\partial v_3}{\partial x_1}$	0
$\frac{\partial v_5}{\partial x_1} = \frac{\partial v_4}{\partial x_1}$	0
$\frac{\partial v_6}{\partial x_1} = -\frac{1}{v_5^2} \frac{\partial v_5}{\partial x_1}$	0
$\frac{\partial v_7}{\partial x_1} = \frac{\partial v_2}{\partial x_1} + \frac{\partial v_6}{\partial x_1}$	1
$\frac{\partial y}{\partial x_1}$	

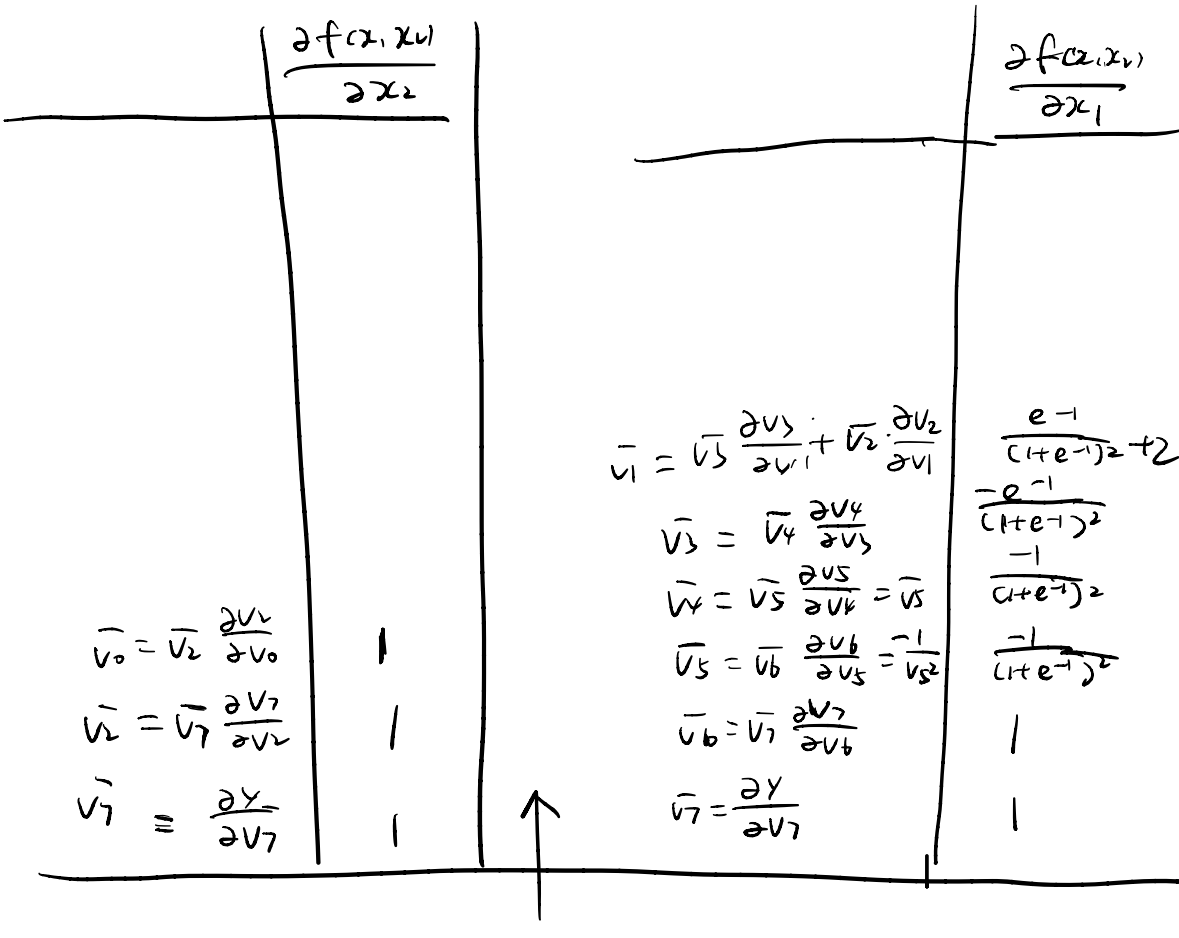
	$\frac{\partial f(x_1,x_2)}{\partial x_2}$
$\frac{\partial v_0}{\partial x_2} = 0$	0
$\frac{\partial v_1}{\partial x_2} = 1$	1
$\frac{\partial v_2}{\partial x_2} = v_0 \frac{\partial v_1}{\partial x_2} + v_1 \frac{\partial v_0}{\partial x_2}$	2
$\frac{\partial v_3}{\partial x_2} = -\frac{\partial v_1}{\partial x_2}$	-1
$\frac{\partial v_4}{\partial x_2} = \exp(v_3) \cdot \frac{\partial v_3}{\partial x_2}$	$-\exp(-1)$
$\frac{\partial v_5}{\partial x_2} = \frac{\partial v_4}{\partial x_2}$	$-\exp(-1)$
$\frac{\partial v_6}{\partial x_2} = -\frac{1}{v_5^2} \frac{\partial v_5}{\partial x_2}$	$\frac{e^{-1}}{(1+e^{-1})^2}$
$\frac{\partial v_7}{\partial x_2} = \frac{\partial v_2}{\partial x_2} + \frac{\partial v_6}{\partial x_2}$	$2 + \frac{e^{-1}}{(1+e^{-1})^2}$

Jacobian $\begin{pmatrix} \sin(3) & 2e^2 + 12\cos 3 \\ 1 & 2 + \frac{e^{-1}}{(1+e^{-1})^2} \end{pmatrix}$

Backwards Derivative



Backward


$\frac{\partial f(x_1, x_2)}{\partial x_2}$
$\frac{\partial f(x_1, x_2)}{\partial x_1}$
$\bar{v_0} = \bar{v_2} \frac{\partial v_2}{\partial v_0}$
$\bar{v_1} = \bar{v_7} \frac{\partial v_7}{\partial v_1}$
$\bar{v_7} = \frac{\partial \lambda e}{\partial v_7}$
$\bar{v_1} = \bar{v_5} \frac{\partial v_5}{\partial v_1} + \bar{v_2} \frac{\partial v_2}{\partial v_1}$
$\bar{v_3} = \bar{v_4} \frac{\partial v_4}{\partial v_3}$
$\bar{v_4} = \bar{v_5} \frac{\partial v_5}{\partial v_4} = \bar{v_5}$
$\bar{v_5} = \bar{v_6} \frac{\partial v_6}{\partial v_5} = \frac{1}{v_5^2}$
$\bar{v_6} = \frac{\partial \lambda e}{\partial v_6}$
$\bar{v_7} = \frac{\partial \lambda e}{\partial v_7}$
$\frac{e^{-1}}{(1+e^{-1})^2} + 2$
$\frac{-e^{-1}}{(1+e^{-1})^2}$
$\frac{-1}{(1+e^{-1})^2}$
$\frac{-1}{(1+e^{-1})^2}$
1
1
1
1