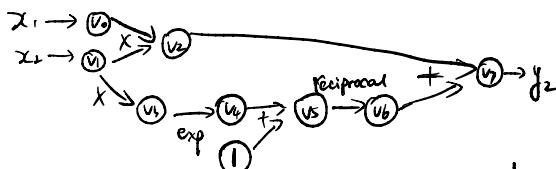
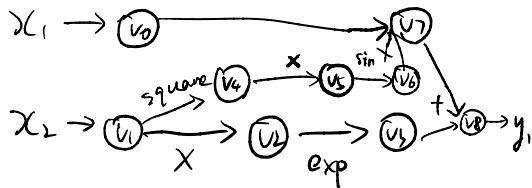


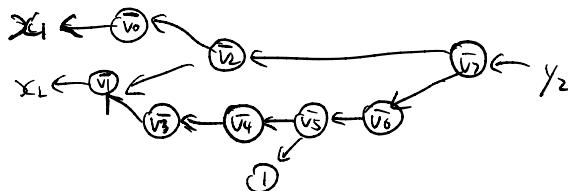
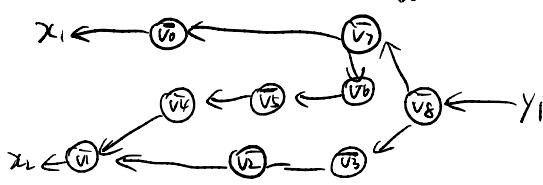
(a)

Forward



(b)

Backward



(c)

y_1	$f(2,1)$
$v_0 = x_1$	2
$v_1 = x_2$	1
$v_2 = 2v_1$	2
$v_3 = \exp(v_5)$	$\exp(2)$
$v_4 = v_1^2$	1
$v_5 = 3v_4$	3
$v_6 = \sin(v_5)$	$\sin(3)$
$v_7 = v_0 \cdot v_6$	$2\sin(3)$
$v_8 = v_5 + v_7$	$\exp(2) + 2\sin(3)$

 y_2 $f(2,1)$

$v_0 = x_1$	2
$v_1 = x_2$	1
$v_2 = v_0 \cdot v_1$	2
$v_3 = -v_1$	-1
$v_4 = \exp(v_3)$	$\exp(-1)$
$v_5 = 1 + v_4$	$\exp(-1) + 1$
$v_6 = \frac{1}{v_5}$	$\frac{1}{1 + \exp(-1)}$
$v_7 = v_2 + v_6$	$2 + \frac{1}{1 + \exp(-1)}$

d)

$$\frac{\partial v_0}{\partial x_1} = 1$$

$$\frac{\partial v_1}{\partial x_1} = 0$$

$$\frac{\partial v_2}{\partial x_1} = 2 \frac{\partial v_1}{\partial x_1} = 0$$

$$\frac{\partial v_3}{\partial x_1} = \exp(v_2) \frac{\partial v_2}{\partial x_1} = 0$$

$$\frac{\partial v_4}{\partial x_1} = 2v_1 \frac{\partial v_1}{\partial x_1} = 0$$

$$\frac{\partial v_5}{\partial x_1} = 3 \frac{\partial v_4}{\partial x_1} = 0$$

$$\frac{\partial v_6}{\partial x_1} = \cos(v_5) \frac{\partial v_5}{\partial x_1} = 0$$

$$\begin{aligned} \frac{\partial v_7}{\partial x_1} &= v_0 \frac{\partial v_6}{\partial x_1} + v_6 \cdot \frac{\partial v_0}{\partial x_1} \\ &= \sin(3) \end{aligned}$$

$$\frac{\partial v_8}{\partial x_1} = \frac{\partial v_3}{\partial x_1} + \frac{\partial v_7}{\partial x_1} = \sin(3)$$

1

0

0

0

0

0

0

 $\sin(3)$ $\sin(3)$

$$\frac{\partial f(x_1, x_2)}{\partial x_1}$$

$$\frac{\partial f(x_1, x_2)}{\partial x_2}$$

$$\frac{\partial v_0}{\partial x_2} = 0$$

$$\frac{\partial v_1}{\partial x_2} = 1$$

$$\frac{\partial v_2}{\partial x_2} = 2 \frac{\partial v_1}{\partial x_2}$$

$$\frac{\partial v_3}{\partial x_2} = \exp(v_2) \frac{\partial v_2}{\partial x_2} \quad 2 \cdot \exp(2)$$

$$\frac{\partial v_4}{\partial x_2} = 2v_1 \frac{\partial v_1}{\partial x_2}$$

$$\frac{\partial v_5}{\partial x_2} = 3 \cdot \frac{\partial v_4}{\partial x_2}$$

$$\frac{\partial v_6}{\partial x_2} = \cos(v_5) \frac{\partial v_5}{\partial x_2} \quad 6 \cos(3)$$

$$\begin{aligned} \frac{\partial v_7}{\partial x_2} &\equiv v_0 \cdot \frac{\partial v_6}{\partial x_2} \\ &\quad + v_6 \cdot \frac{\partial v_0}{\partial x_2} \end{aligned} \quad 12 \cos(3)$$

$$\frac{\partial v_8}{\partial x_2} = \frac{\partial v_3}{\partial x_2} + \frac{\partial v_7}{\partial x_2} \quad 2 \exp(2) + 12 \cos(3)$$

Forward

$\frac{\partial f(x_1, x_2)}{\partial x_2}$		$\frac{\partial f(x_1, x_2)}{\partial x_2}$	
$\frac{\partial v_0}{\partial x_1} = 1$	1	$\frac{\partial v_0}{\partial x_2} = 0$	0
$\frac{\partial v_1}{\partial x_1} = 0$	0	$\frac{\partial v_1}{\partial x_2} = 1$	1
$\frac{\partial v_2}{\partial x_1} = v_0 \frac{\partial v_1}{\partial x_1} + v_1 \frac{\partial v_0}{\partial x_1}$	1	$\frac{\partial v_2}{\partial x_1} = v_0 \frac{\partial v_1}{\partial x_2} + v_1 \frac{\partial v_0}{\partial x_2}$	2
$\frac{\partial v_3}{\partial x_1} = -\frac{\partial v_1}{\partial x_1}$	0	$\frac{\partial v_3}{\partial x_2} = -\frac{\partial v_1}{\partial x_2}$	-1
$\frac{\partial v_4}{\partial x_1} = \exp(v_3) \cdot \frac{\partial v_3}{\partial x_1}$	0	$\frac{\partial v_4}{\partial x_2} = \exp(v_3) \cdot \frac{\partial v_3}{\partial x_2}$	$-\exp(-1)$
$\frac{\partial v_5}{\partial x_1} = \frac{\partial v_4}{\partial x_1}$	0	$\frac{\partial v_5}{\partial x_2} = \frac{\partial v_4}{\partial x_2}$	$-\exp(-1)$
$\frac{\partial v_6}{\partial x_1} = -\frac{1}{v_5^2} \frac{\partial v_5}{\partial x_1}$	0	$\frac{\partial v_6}{\partial x_2} = -\frac{1}{v_5^2} \frac{\partial v_5}{\partial x_2}$	$\frac{e^{-1}}{(1+e^{-1})^2}$
$\frac{\partial v_7}{\partial x_1} = \frac{\partial v_6}{\partial x_1} + \frac{\partial v_6}{\partial x_2}$	1	$\frac{\partial v_7}{\partial x_2} = \frac{\partial v_6}{\partial x_1} + \frac{\partial v_6}{\partial x_2}$	$2 + \frac{e^{-1}}{(1+e^{-1})^2}$
$\frac{\partial y}{\partial x_1}$			

Jacobian

$$\begin{pmatrix} \sin(3) & 2e^2 + 12\cos 3 \\ 1 & 2 + \frac{e^{-1}}{(1+e^{-1})^2} \end{pmatrix}$$

Backwards Derivative

$$\frac{\partial f(x_1, x_2)}{\partial x_2}$$

$$\bar{v}_1 = \bar{v}_2 \cdot \frac{\partial v_2}{\partial v_1} + \bar{v}_4 \cdot \frac{\partial v_4}{\partial v_1} \quad 2 \exp(2) + 12 \cos(3)$$

$$\bar{v}_2 = \bar{v}_3 \cdot \frac{\partial v_3}{\partial v_2} = 1 \times \exp(4) \quad \exp(2)$$

$$\bar{v}_3 = \bar{v}_8 \cdot \frac{\partial v_8}{\partial v_3} = 1 \times 1$$

$$\bar{v}_4 = \bar{v}_5 \cdot \frac{\partial v_5}{\partial v_4} = 3 \bar{v}_5 \quad 6 \cos(3)$$

$$\bar{v}_5 = \bar{v}_6 \cdot \frac{\partial v_6}{\partial v_5} = \cos(v_5) \quad 2 \cos(3)$$

$$\bar{v}_6 = \bar{v}_7 \cdot \frac{\partial v_7}{\partial v_6} = v_0 \cdot 1$$

$$\bar{v}_7 = \bar{v}_8 \cdot \frac{\partial v_8}{\partial v_7} = 1$$

$$\bar{v}_8 = \frac{\partial y}{\partial v_8} = 1$$

1

2

1

1

$$\frac{\partial f(x_1, x_2)}{\partial x_1}$$

$$\bar{v}_0 = \bar{v}_1 \cdot \frac{\partial v_1}{\partial v_0} \quad \sin(3)$$

$$\bar{v}_1 = \bar{v}_2 \cdot \frac{\partial v_2}{\partial v_1} \quad 1$$

$$\bar{v}_2 = \frac{\partial y}{\partial v_2} \quad 1$$

backward

$$\frac{\partial f(x, x_u)}{\partial x_2}$$

$$\frac{\partial f(x, x_v)}{\partial x_1}$$

$$\bar{v}_0 = \bar{v}_2 \frac{\partial v_2}{\partial v_0}$$

$$\bar{v}_2 = \bar{v}_7 \frac{\partial v_7}{\partial v_2}$$

$$\bar{v}_7 = \frac{\partial y}{\partial v_7}$$

$$\bar{v}_1 = \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_2 \frac{\partial v_2}{\partial v_1}$$

$$\frac{e^{-1}}{(1+e^{-1})^2} + 2$$

$$\bar{v}_3 = \bar{v}_4 \frac{\partial v_4}{\partial v_3}$$

$$\frac{-e^{-1}}{(1+e^{-1})^2}$$

$$\bar{v}_4 = \bar{v}_5 \frac{\partial v_5}{\partial v_4} = \bar{v}_5$$

$$\frac{-1}{(1+e^{-1})^2}$$

$$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \frac{-1}{v_5^2}$$

$$\frac{-1}{(1+e^{-1})^2}$$

$$\bar{v}_6 = \bar{v}_7 \frac{\partial v_7}{\partial v_6}$$

$$|$$

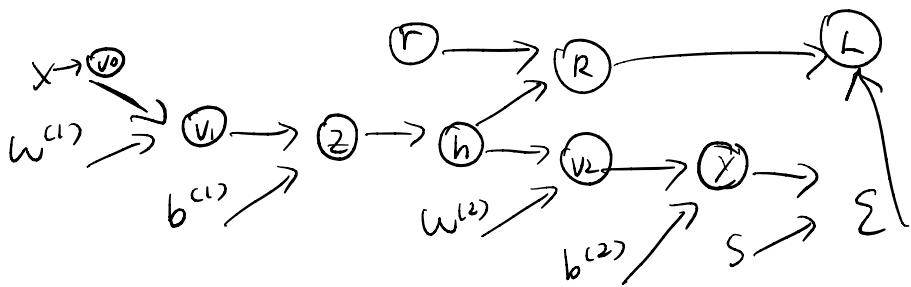
$$\bar{v}_7 = \frac{\partial y}{\partial v_7}$$

$$|$$

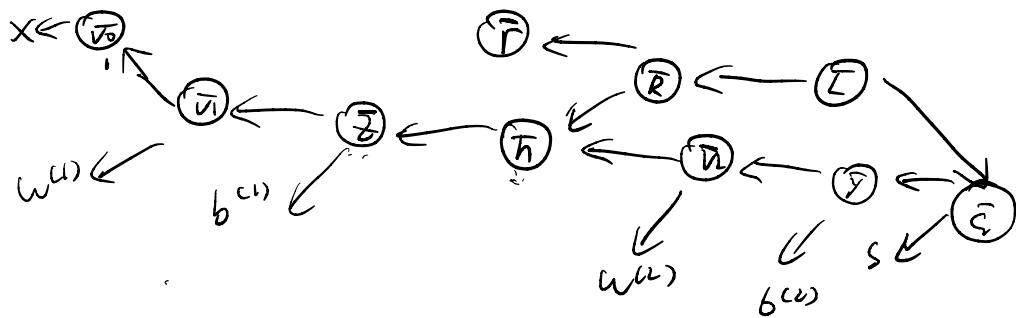
a)

computational graph

Forward



BackForward



b)

Forwards

$$x = v_0$$

$$v_1 = w^{(1)T} v_0$$

$$z = v_1 + b^{(1)}$$

$$h = \sigma(z)$$

$$R = r^T h$$

$$v_2 = w^{(2)} h$$

$$y = v_2 + b^{(2)}$$

$$\xi = \frac{1}{2} \|y - s\|^2$$

$$L = \xi + R.$$

$$\bar{v}_0 = \bar{v}_1 \frac{\partial v_1}{\partial v_0} = w^{(1)T} \bar{\sigma}'(z) \bar{h}$$

$$\bar{v}_1 = \bar{z} \frac{\partial v_1}{\partial z} = \sigma'(z) \bar{h}$$

$$\bar{z} = \bar{h} \frac{\partial h}{\partial z} = \bar{h} \sigma'(z) = \bar{h} \circ \sigma'(z)$$

$$\bar{h} = \bar{R} \frac{\partial R}{\partial h} + \bar{v}_2 \frac{\partial v_2}{\partial h} = r + w^{(2)T} \bar{y}$$

$$\bar{v}_2 = \bar{y} \frac{\partial y}{\partial v_2} = \bar{y}$$

$$\bar{y} = \bar{s} \frac{\partial s}{\partial y} = y - s$$

$$\bar{\xi} = \bar{I} \frac{\partial I}{\partial \xi} = 1$$

$$\bar{R} = \bar{I} \frac{\partial I}{\partial R} = 1$$

$$\bar{I} = 1$$

Backward

Dim:

$$x: N \times 1 \quad w^{(1)}: M \times K$$

$$y: M \times 1 \quad r: K \times 1$$

$$h: K \times 1 \quad z: K \times 1$$

$$b^{(2)}: M \times 1$$

$$b^{(1)}: K \times 1$$

$$w^{(1)}: K \times N$$

$$\frac{\partial L}{\partial x} = w^{(1)T} \sigma'(z) \bar{h}$$

where $N \times K$ $K \times K$ $K \times 1$ $M \times 1$

$$\bar{h} = r + w^{(2)T} (y - s)$$