



# Topics in AI (CPSC 532S): Multimodal Learning with Vision, Language and Sound



A decorative horizontal bar at the bottom of the slide, composed of four colored segments: light green, teal, light blue, and light purple.

**Lecture 9: Language Models and RNNs (Part 1)**

# Course Logistics

## — Assignment 3

# Representing a **Word**: One Hot Encoding

## Vocabulary

dog

cat

person

holding

tree

computer

using

# Representing a **Word**: One Hot Encoding

## Vocabulary

dog	1
cat	2
person	3
holding	4
tree	5
computer	6
using	7

# Representing a **Word**: One Hot Encoding

## Vocabulary

		one-hot encodings
dog	1	[ 1, 0, 0, 0, 0, 0, 0, 0, 0 ]
cat	2	[ 0, 1, 0, 0, 0, 0, 0, 0, 0 ]
person	3	[ 0, 0, 1, 0, 0, 0, 0, 0, 0 ]
holding	4	[ 0, 0, 0, 1, 0, 0, 0, 0, 0 ]
tree	5	[ 0, 0, 0, 0, 1, 0, 0, 0, 0 ]
computer	6	[ 0, 0, 0, 0, 0, 1, 0, 0, 0 ]
using	7	[ 0, 0, 0, 0, 0, 0, 1, 0, 0 ]

# Representing Phrases: Bag-of-Words

**bag-of-words** representation

dog    cat    person    holding    tree  
computer    using

Vocabulary	
dog	1
cat	2
person	3
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# Representing Phrases: Bag-of-Words

person holding dog

$$\{3, 4, 1\}$$

# bag-of-words representation

dog      cat      person      holding      tree      computer      using

Vocabulary	
dog	1
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# Representing Phrases: Bag-of-Words

person holding dog

$$\{3, 4, 1\} \quad [1, 0, 1, 1, 0, 0, 0, 0, 0, 0]$$

person holding cat

$$\{3, 4, 2\} \quad [1, 1, 0, 1, 0, 0, 0, 0, 0]$$

dog      cat      person      holding      tree      computer      using

Vocabulary	
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cat	2
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# Representing Phrases: Bag-of-Words

## bag-of-words representation

person holding dog

{3, 4, 1}

[ 1, 0, 1, 1, 0, 0, 0, 0, 0 ]

person holding cat

{3, 4, 2}

[ 1, 1, 0, 1, 0, 0, 0, 0, 0 ]

person using computer

{3, 7, 6}

[ 0, 0, 0, 1, 0, 1, 1, 0, 0 ]

dog    cat    person    holding    tree    computer    using

Vocabulary	
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person using computer

{3, 7, 6}

[ 0, 0, 0, 1, 0, 1, 1, 0, 0 ]

dog    cat    person    holding    tree    computer    using

person using computer

{3, 3, 7, 6, 2}

[ 0, 1, 2, 1, 0, 1, 1, 0, 0 ]

person holding cat

Vocabulary	
dog	1
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{3, 4, 2}

[ 1, 1, 0, 1, 0, 0, 0, 0, 0 ]

person using computer

{3, 7, 6}

[ 0, 0, 0, 1, 0, 1, 1, 0, 0, 0 ]

dog    cat    person    holding    tree    computer    using

person using computer

{3, 3, 7, 6, 2}

[ 0, 1, 2, 1, 0, 1, 1, 0, 0, 0 ]

person holding cat

What if we have large vocabulary?

# Representing Phrases: Sparse Representation

person holding dog

indices = [1, 3, 4]    values = [1, 1, 1]

person holding cat

indices = [2, 3, 4]    values = [1, 1, 1]

person using computer

indices = [3, 7, 6]    values = [1, 1, 1]

person using computer  
person holding cat

indices = [3, 7, 6, 2]    values = [2, 1, 1, 1]

Vocabulary	
dog	1
cat	2
person	3
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tree	5
computer	6
using	7

# Bag-of-Words Representations

- Really easy to use
- Can encode phrases, sentences, paragraph, documents
- Good for classification, clustering or to compute distance between text

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my friend makes a nice meal

my nice friend makes a meal

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**Problem:** hard to distinguish sentences that have same words

my friend makes a nice meal

These would be the same using bag-of-words

my nice friend makes a meal

# Bag-of-Bigrams

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**Problem:** hard to distinguish sentences that have same words

my friend makes a nice meal

{my nice, nice friend, friend makes, makes a, a meal}

my nice friend makes a meal

{my friend, friend makes, makes a, a nice, nice meal}

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- Good for classification, clustering or to compute distance between text

**Problem:** hard to distinguish sentences that have same words

my friend makes a nice meal

{my nice, nice friend, friend makes, makes a, a meal}

indices = [10132, 21342, 43233, 53123, 64233]

values = [1, 1, 1, 1, 1]

my nice friend makes a meal

{my friend, friend makes, makes a, a nice, nice meal}

indices = [10232, 43133, 21342, 43233, 54233]

values = [1, 1, 1, 1, 1]

# Word Representations

1. **One-hot encodings** — only non-zero at the index of the word

e.g., [ 0, 1, 0, 0, 0, ...., 0, 0, 0 ]

**Good:** simple

**Bad:** not compact, distance between words always same (e.g., synonyms vs. antonyms)

# Word Representations

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## 2. Word feature representations — manually define “good” features

e.g., [ 1, 1, 0, 30, 0, ...., 0, 0, 0 ] -> 300-dimensional irrespective of dictionary

e.g., word ends on -ing

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e.g., [ 1, 1, 0, 30, 0, ...., 0, 0, 0 ] -> 300-dimensional irrespective of dictionary

e.g., word ends on -ing

## 3. Learned word representations — vector should approximate “meaning” of the word

e.g., [ 1, 1, 0, 30, 0, ...., 0, 0, 0 ] -> 300-dimensional irrespective of dictionary

**Good:** compact, distance between words is semantic

# Distributional Hypothesis

[ Lenci, 2008 ]

- At least certain aspects of the meaning of lexical expressions depend on their distributional properties in the linguistic contexts
- The degree of semantic similarity between two linguistic expressions is a function of the similarity of the two linguistic contexts in which they can appear

# What is the meaning of “**bardiwac**”?

- He handed her glass of **bardiwac**.
- Beef dishes are made to complement the **bardiwacs**.
- Nigel staggered to his feet, face flushed from too much **bardiwac**.
- Malbec, one of the lesser-known **bardiwac** grapes, responds well to Australia’s sunshine.
- I dined off bread and cheese and this excellent **bardiwac**.
- The drinks were delicious: blood-red **bardiwac** as well as light, sweet Rhenish.

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**bardiwac** is an alcoholic beverage made from grapes

# Geometric Interpretation: Co-occurrence as feature

- Row vector describes usage of word in a corpus of text
- Can be seen as coordinates of the point in an n-dimensional Euclidian space

	get	see	use	hear	eat	kill
knife	51	20	84	0	3	0
cat	52	58	4	4	6	26
dog	115	83	10	42	33	17
boat	59	39	23	4	0	0
cup	98	14	6	2	1	0
pig	12	17	3	2	9	27
banana	11	2	2	0	18	0

Co-occurrence Matrix

# Geometric Interpretation: Co-occurrence as feature

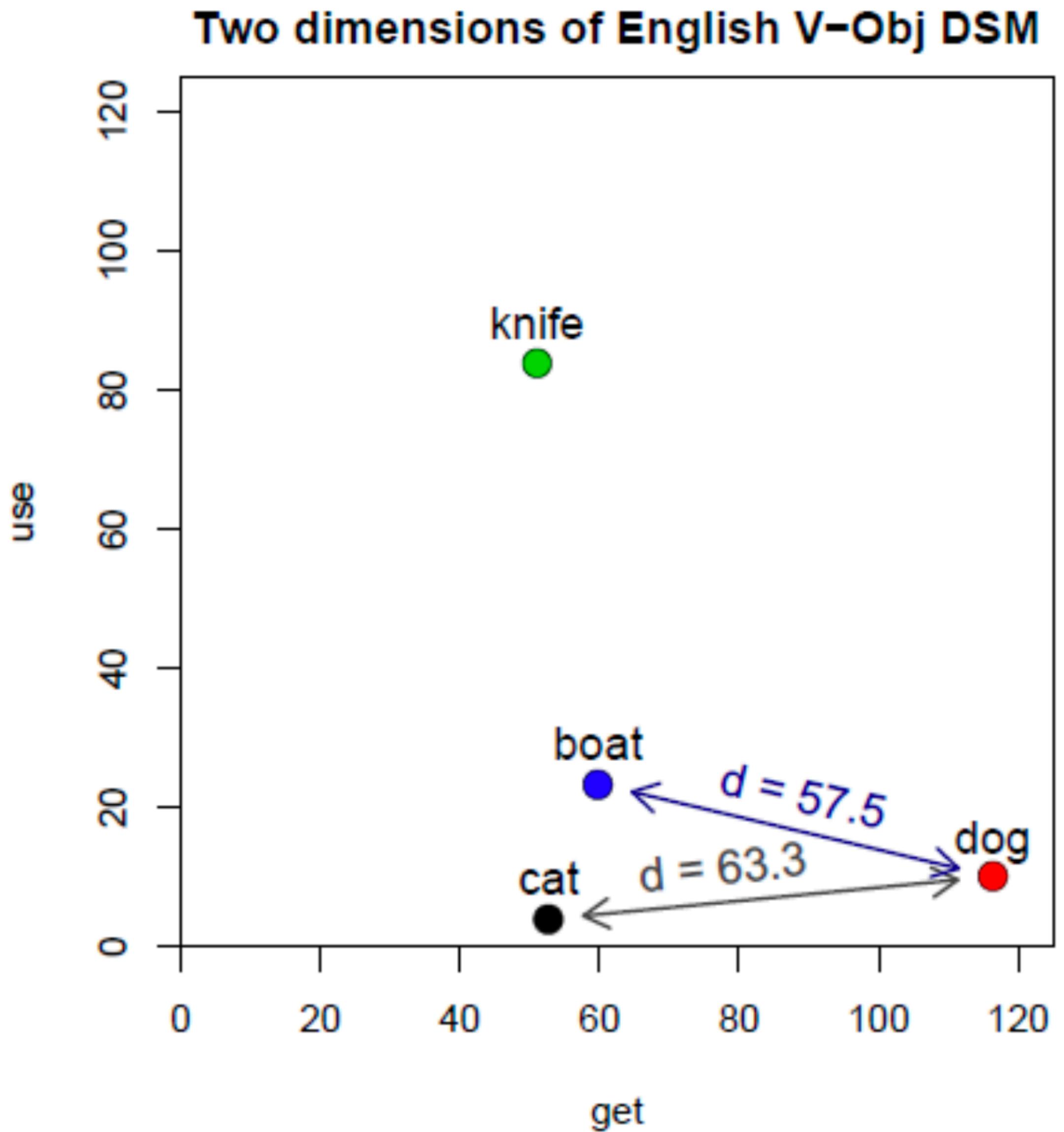
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Co-occurrence Matrix

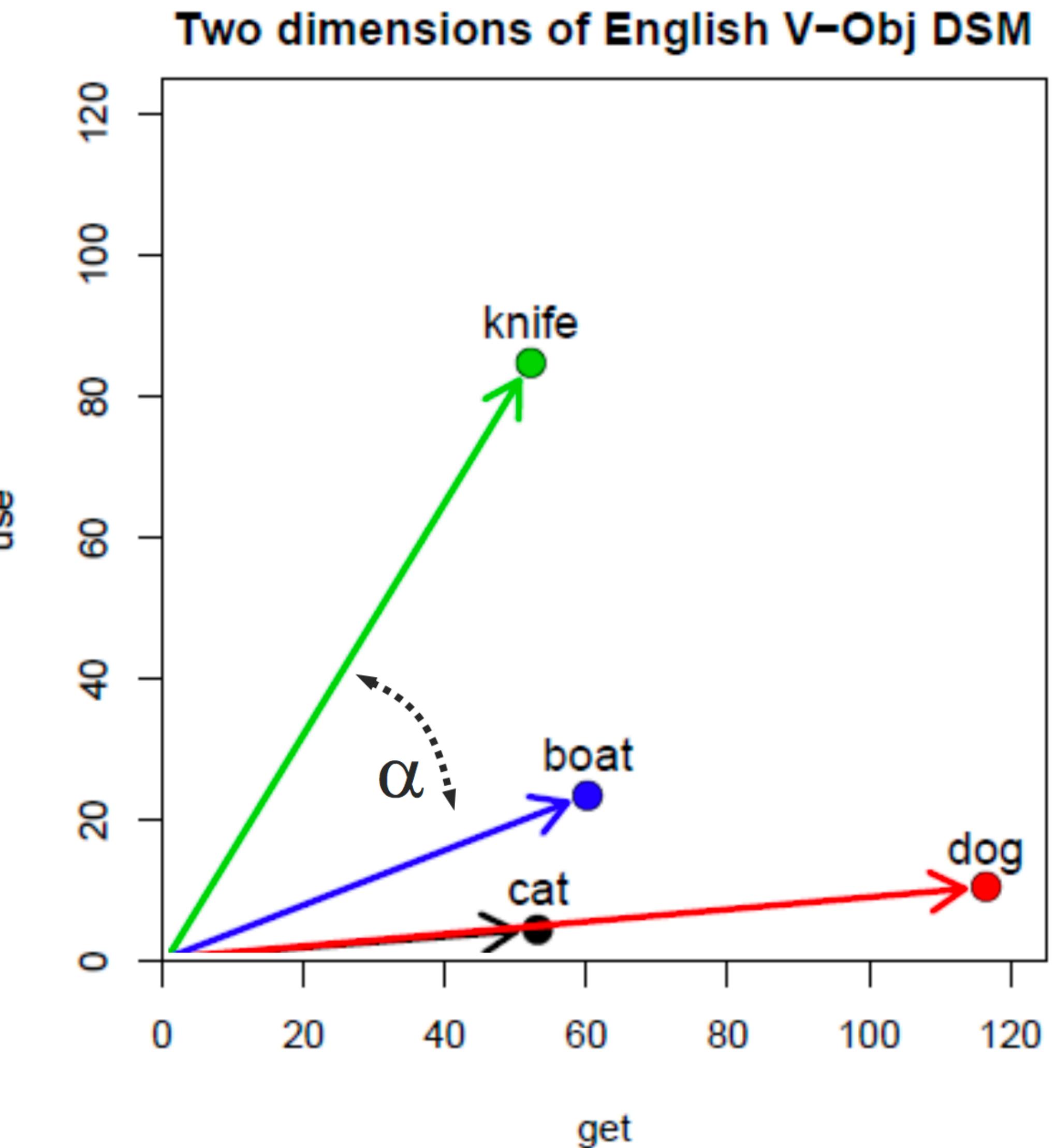
# Distance and Similarity

- Illustrated in two dimensions
- Similarity = spatial proximity  
(Euclidian distance)
- Location depends on frequency of a noun (dog is 27 times as frequent as cat)



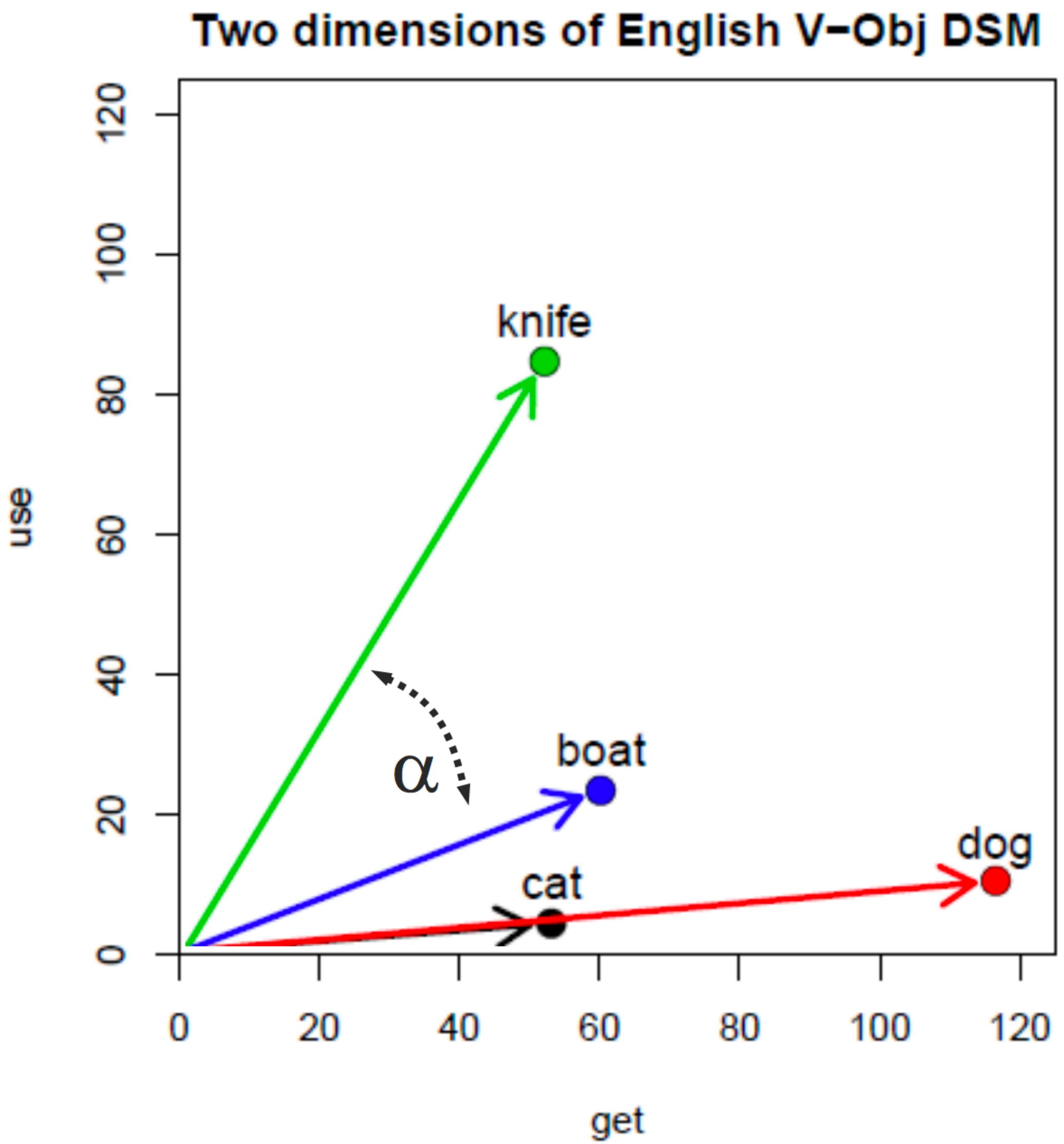
# Angle and Similarity

- direction is more important than location
- normalize length of vectors (or use angle as a distance measure)



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Way too high dimensional!

Co-occurrence Matrix

# SVD for Dimensionality Reduction

$$\begin{matrix} m \\ n \end{matrix} \boxed{X} = \begin{matrix} r \\ n \end{matrix} \boxed{U} \begin{matrix} r \\ r \end{matrix} \boxed{S} \begin{matrix} m \\ r \end{matrix} \boxed{V^T}$$

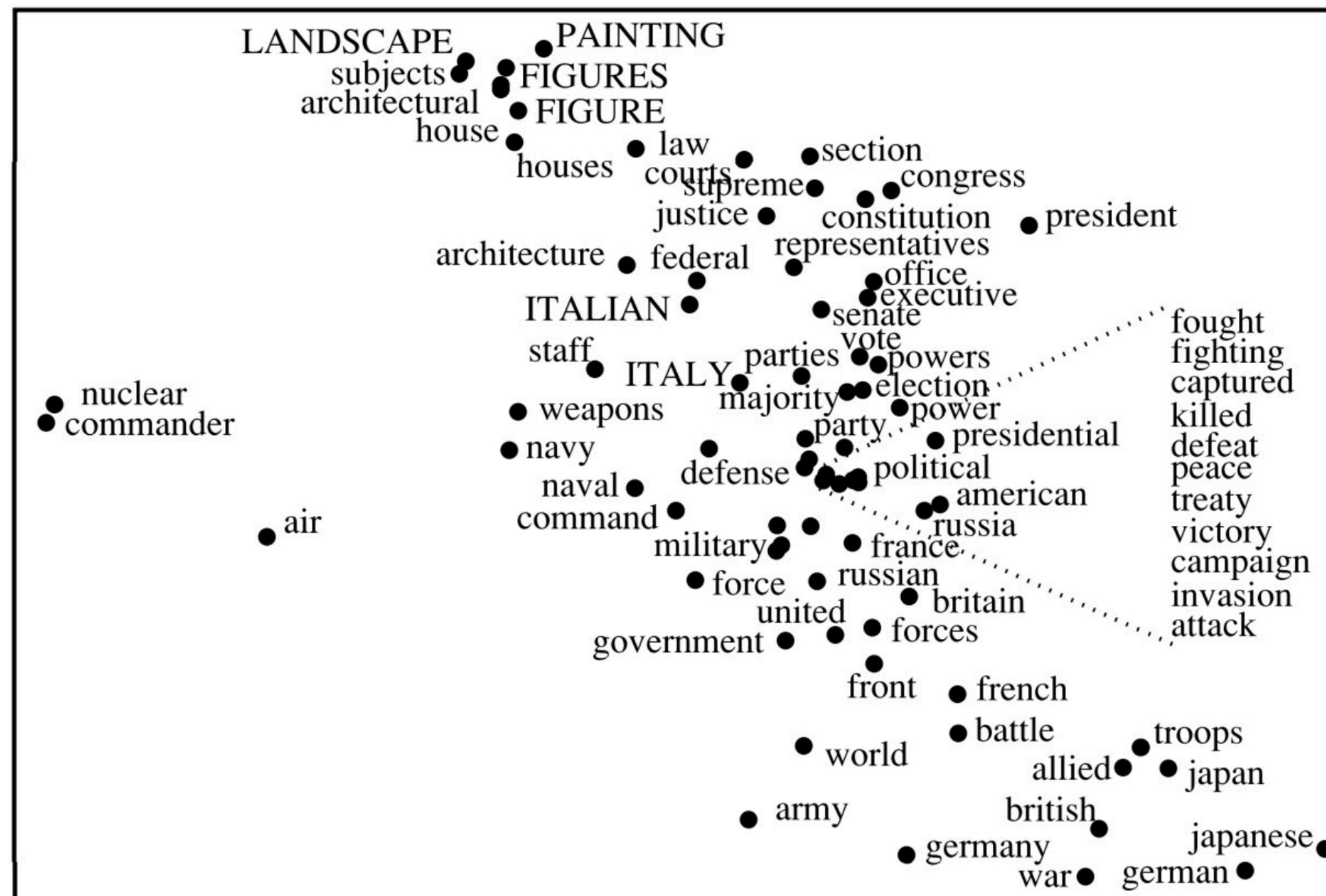
The diagram shows the full Singular Value Decomposition (SVD) of matrix  $X$ . Matrix  $X$  has dimensions  $m \times n$ . It is decomposed into  $U$ ,  $S$ , and  $V^T$ , where  $U$  is  $n \times n$ ,  $S$  is  $r \times r$  (with  $r \leq \min(m, n)$ ), and  $V^T$  is  $m \times r$ . The matrix  $U$  is shown with vertical lines separating its columns, and the matrix  $S$  is shown with vertical lines separating its rows.

$$\begin{matrix} m \\ n \end{matrix} \boxed{\hat{X}} = \begin{matrix} k \\ n \end{matrix} \boxed{\hat{U}} \begin{matrix} k \\ k \end{matrix} \boxed{\hat{S}} \begin{matrix} m \\ k \end{matrix} \boxed{\hat{V}^T}$$

The diagram shows the truncated Singular Value Decomposition (SVD) of matrix  $\hat{X}$ . Matrix  $\hat{X}$  has dimensions  $m \times n$ . It is decomposed into  $\hat{U}$ ,  $\hat{S}$ , and  $\hat{V}^T$ , where  $\hat{U}$  is  $n \times n$ ,  $\hat{S}$  is  $k \times k$  (with  $k \leq \min(m, n)$ ), and  $\hat{V}^T$  is  $m \times k$ . The matrix  $\hat{U}$  is shown with vertical lines separating its columns, and the matrix  $\hat{S}$  is shown with vertical lines separating its rows.

# Learned Word Vector Visualization

We can also use other methods, like LLE here:



Nonlinear dimensionality reduction by locally linear embedding. Sam Roweis & Lawrence Saul. Science, v.290,2000

[ Roweis and Saul, 2000 ]

# Issues with **SVD**

**Computational** cost for a  $d \times n$  matrix is  $\mathcal{O}(dn^2)$ , where  $d < n$

- Makes it not possible for large number of word vocabularies or documents

It is hard to incorporate out of sample (**new**) words or documents

# word2vec: Representing the Meaning of Words

[ Mikolov et al., 2013 ]

**Key idea:** Predict surrounding words  
of every word

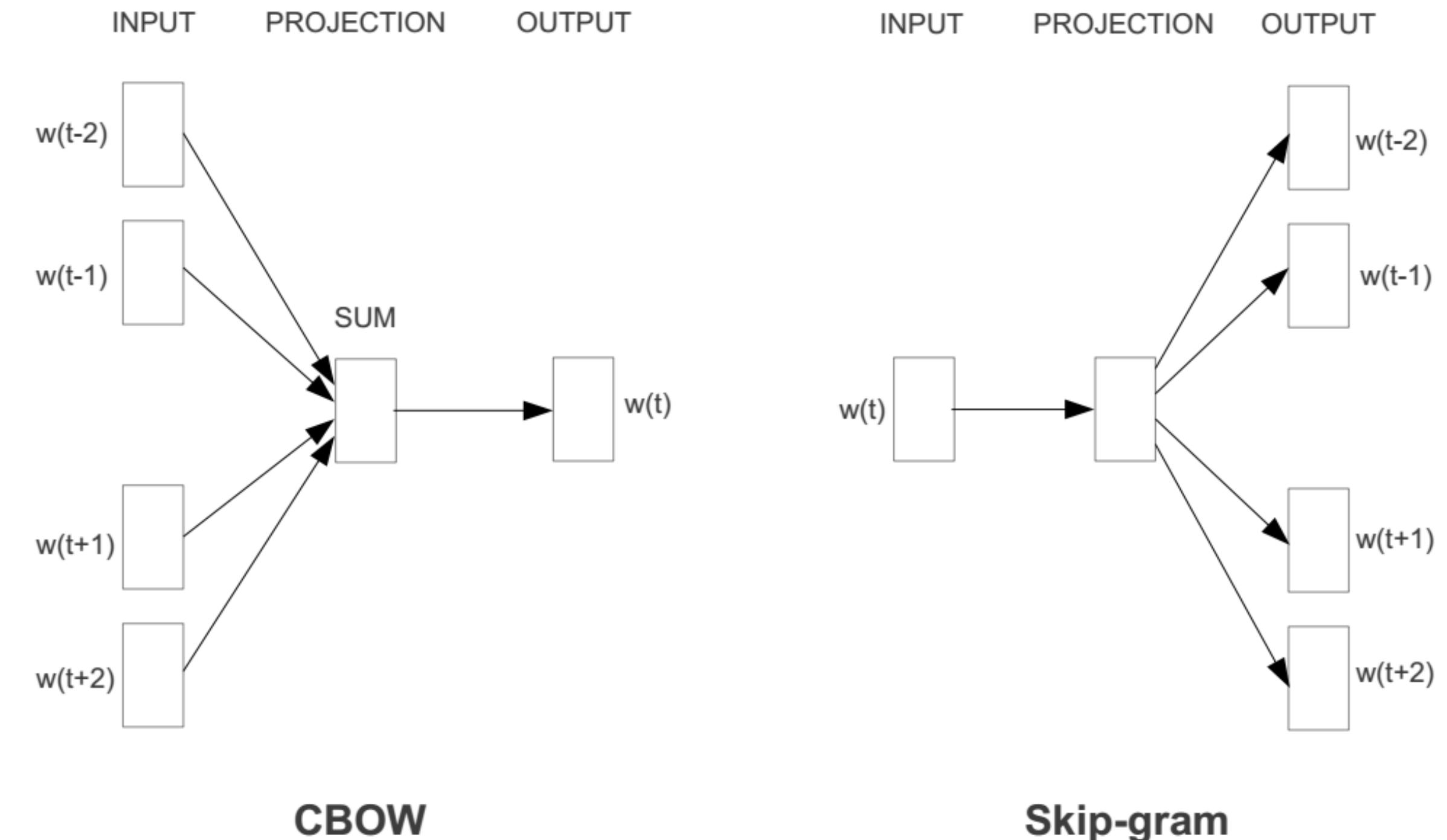
**Benefits:** Faster and easier to  
incorporate new document, words, etc.

# word2vec: Representing the Meaning of Words

[ Mikolov et al., 2013 ]

**Key idea:** Predict surrounding words of every word

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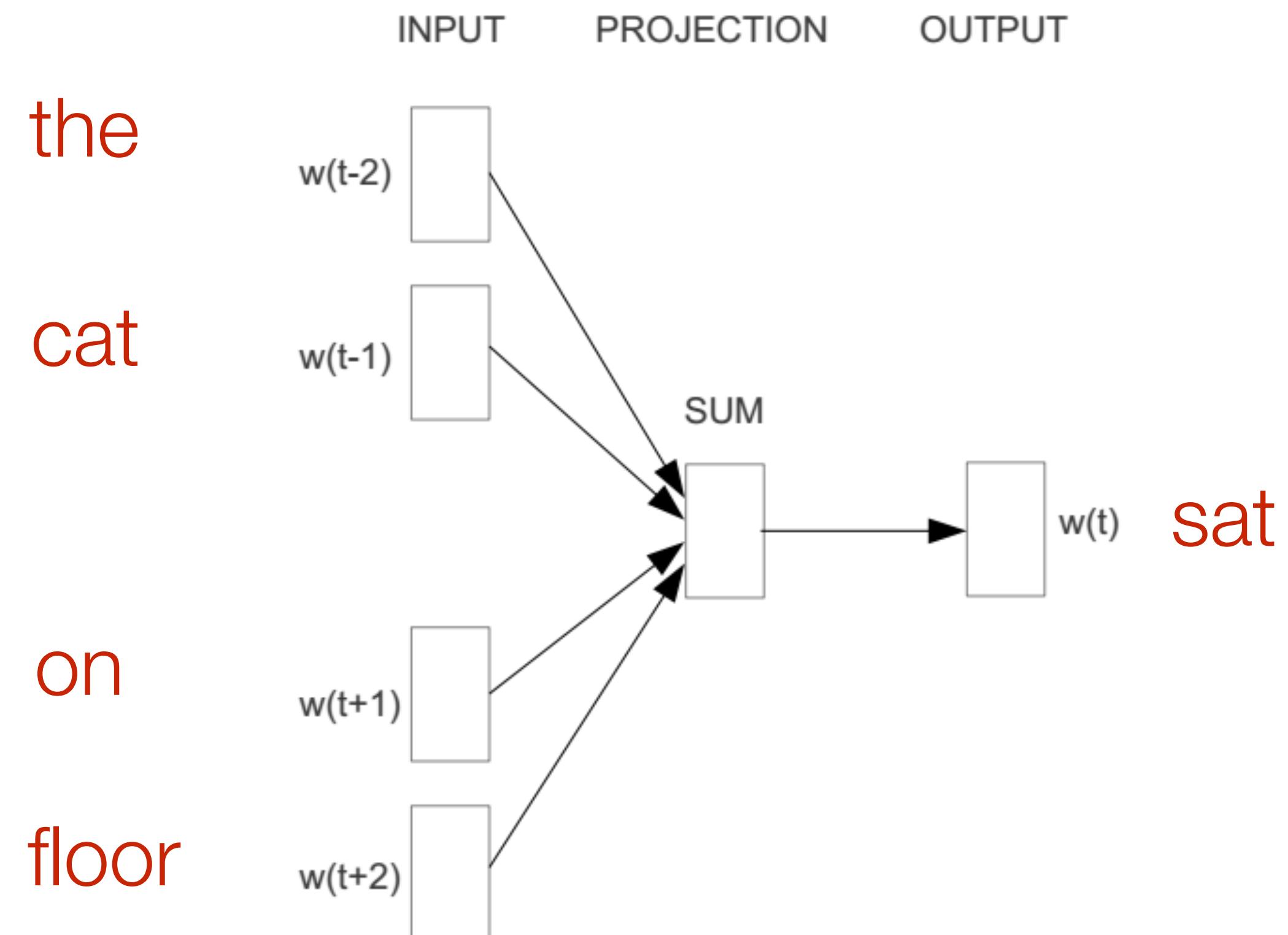
Continuous Bag of Words (**CBOW**): use context words in a window to predict middle word

**Skip-gram:** use the middle word to predict surrounding ones in a window

# CBOW: Continuous Bag of Words

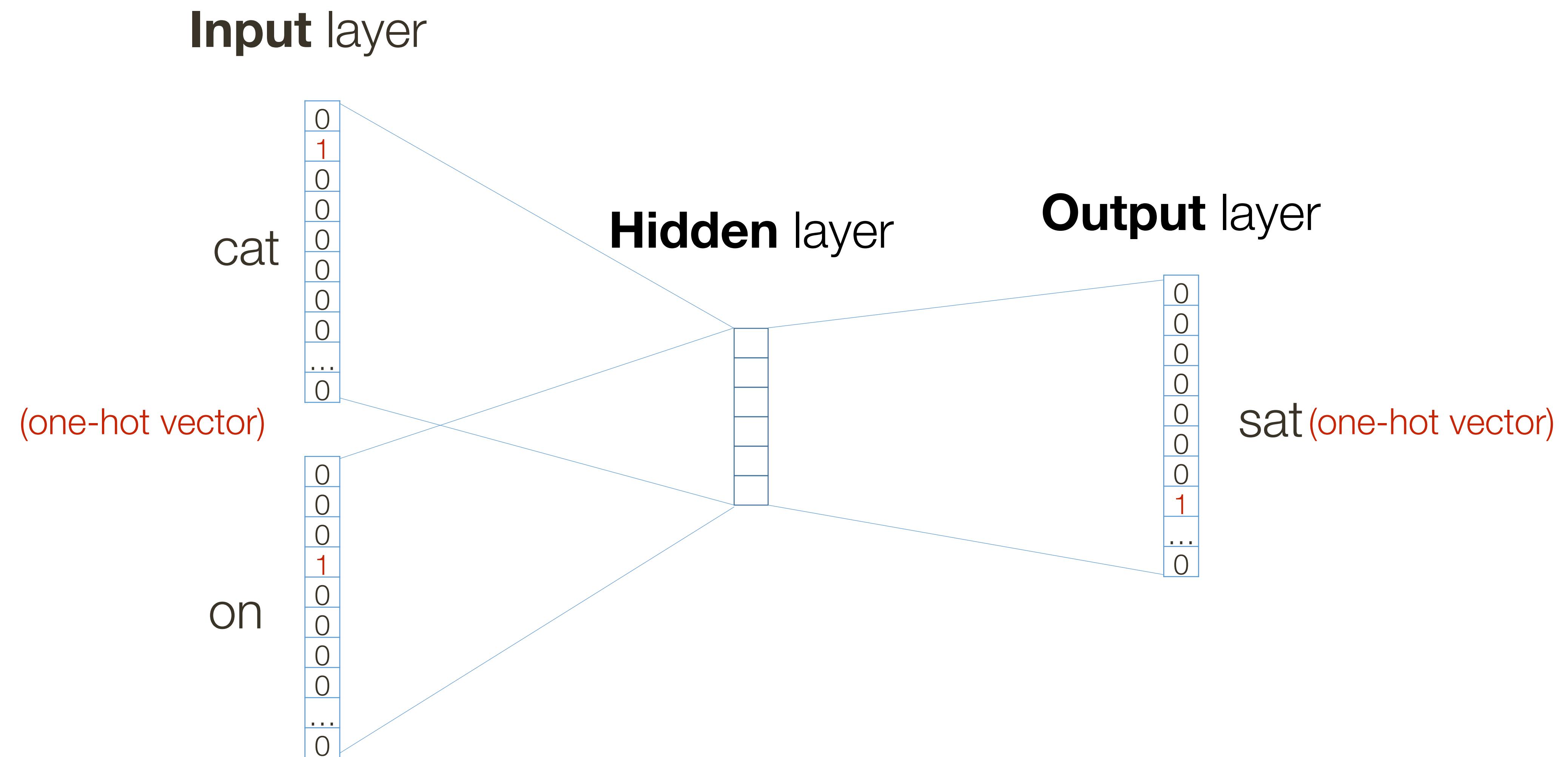
[ Mikolov et al., 2013 ]

**Example:** “The cat sat on floor” (window size 2)



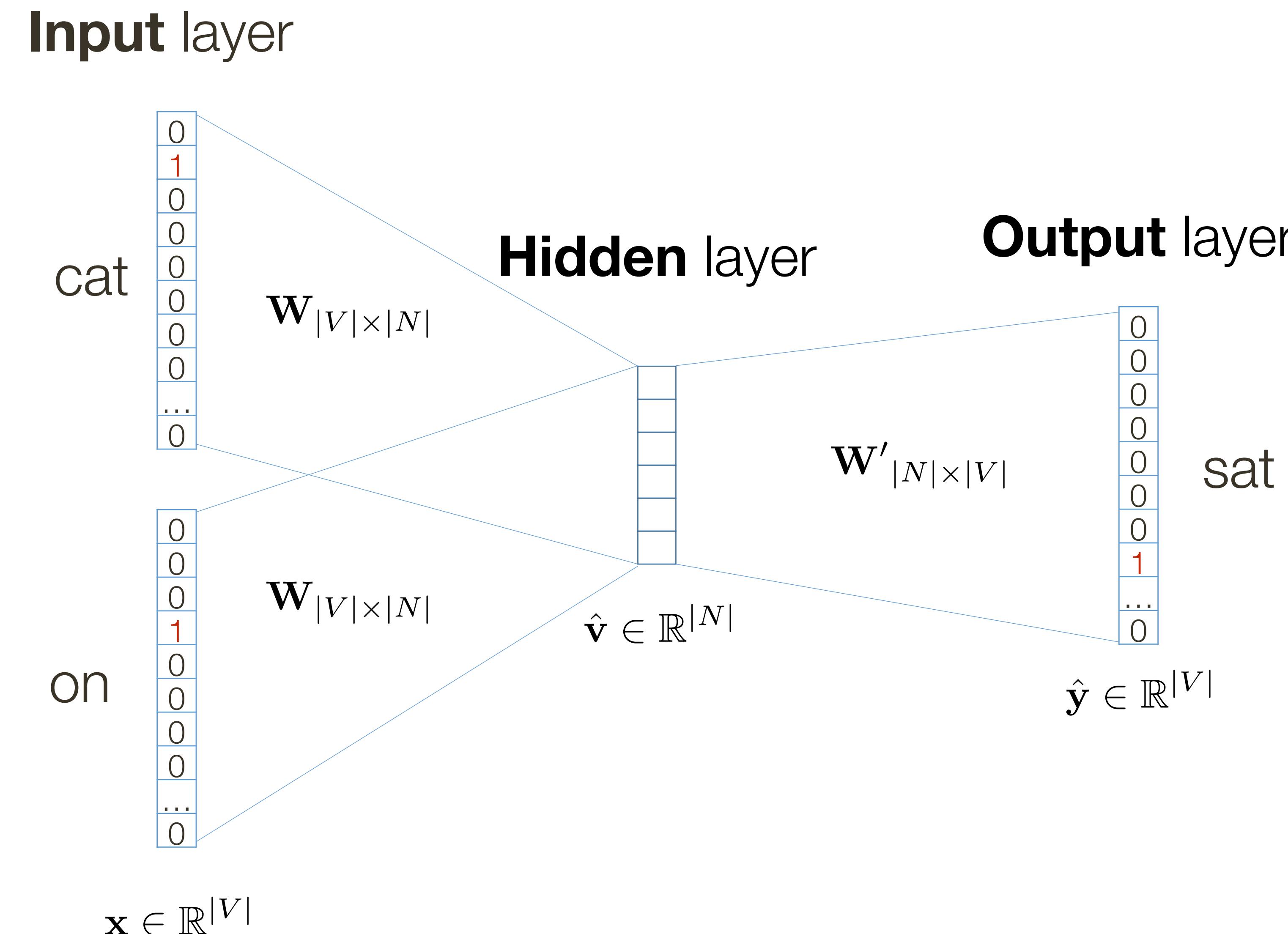
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# CBOW: Continuous Bag of Words

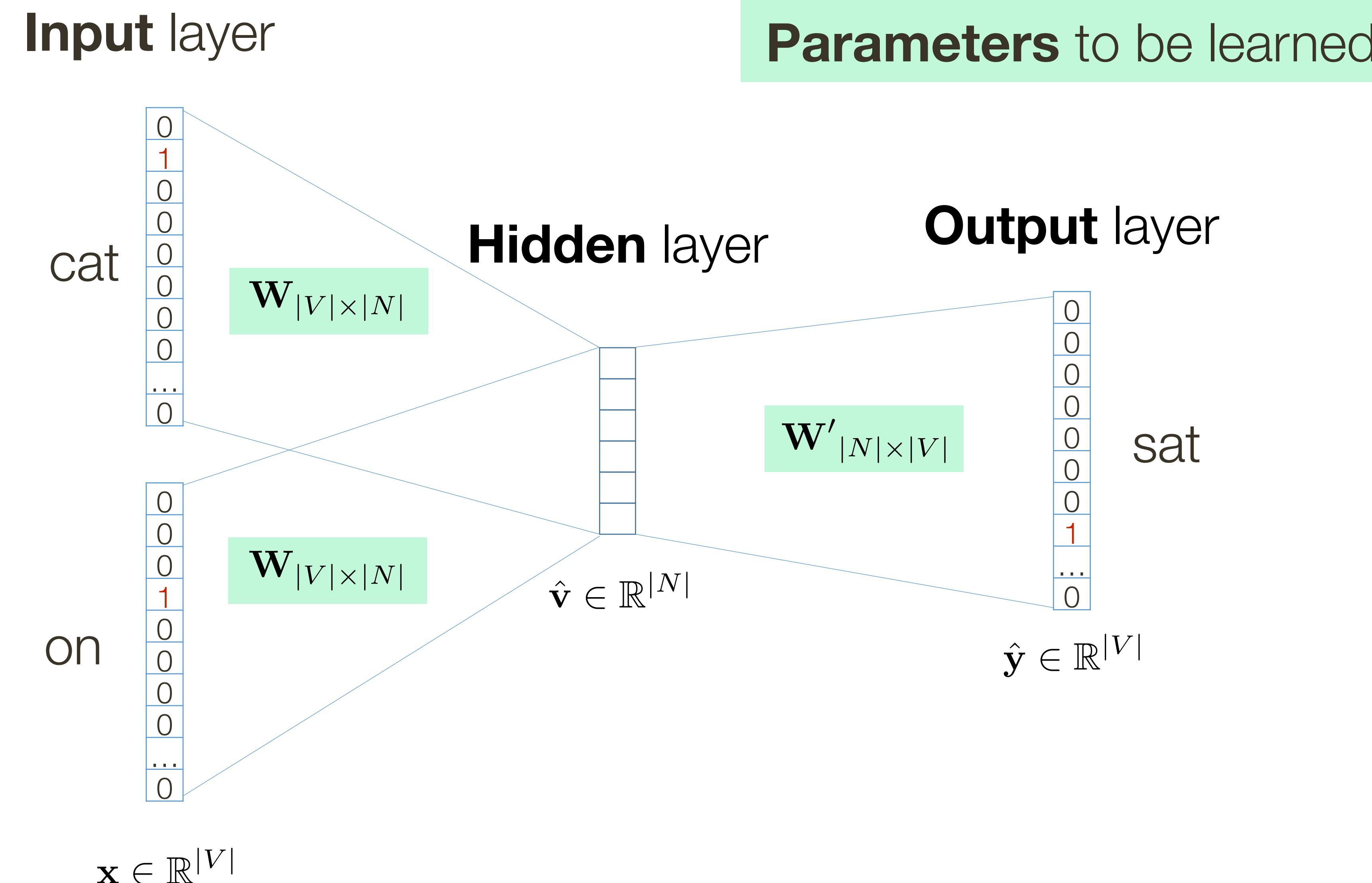
[ Mikolov et al., 2013 ]



\*slide from Vagelis Hristidis

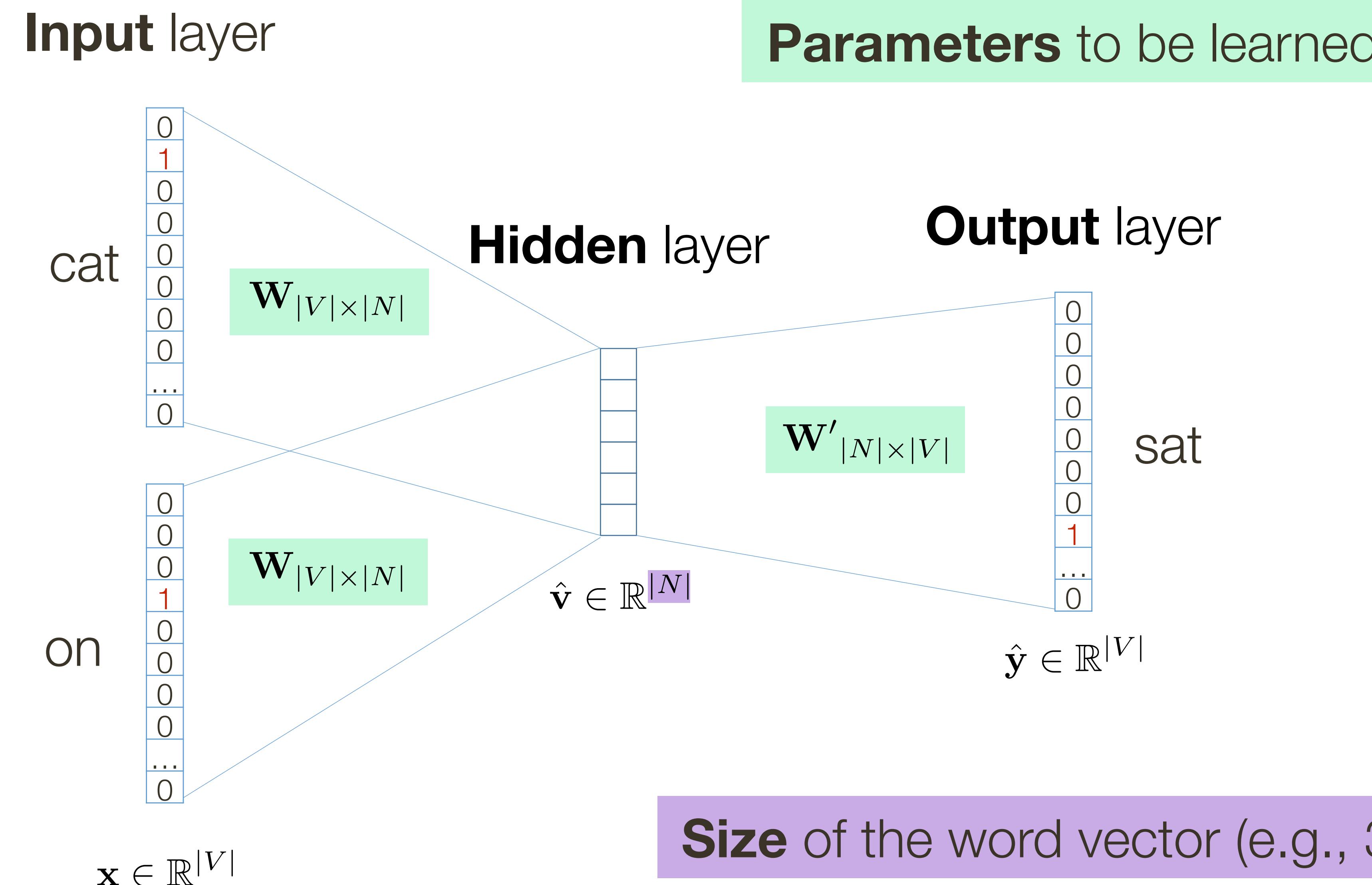
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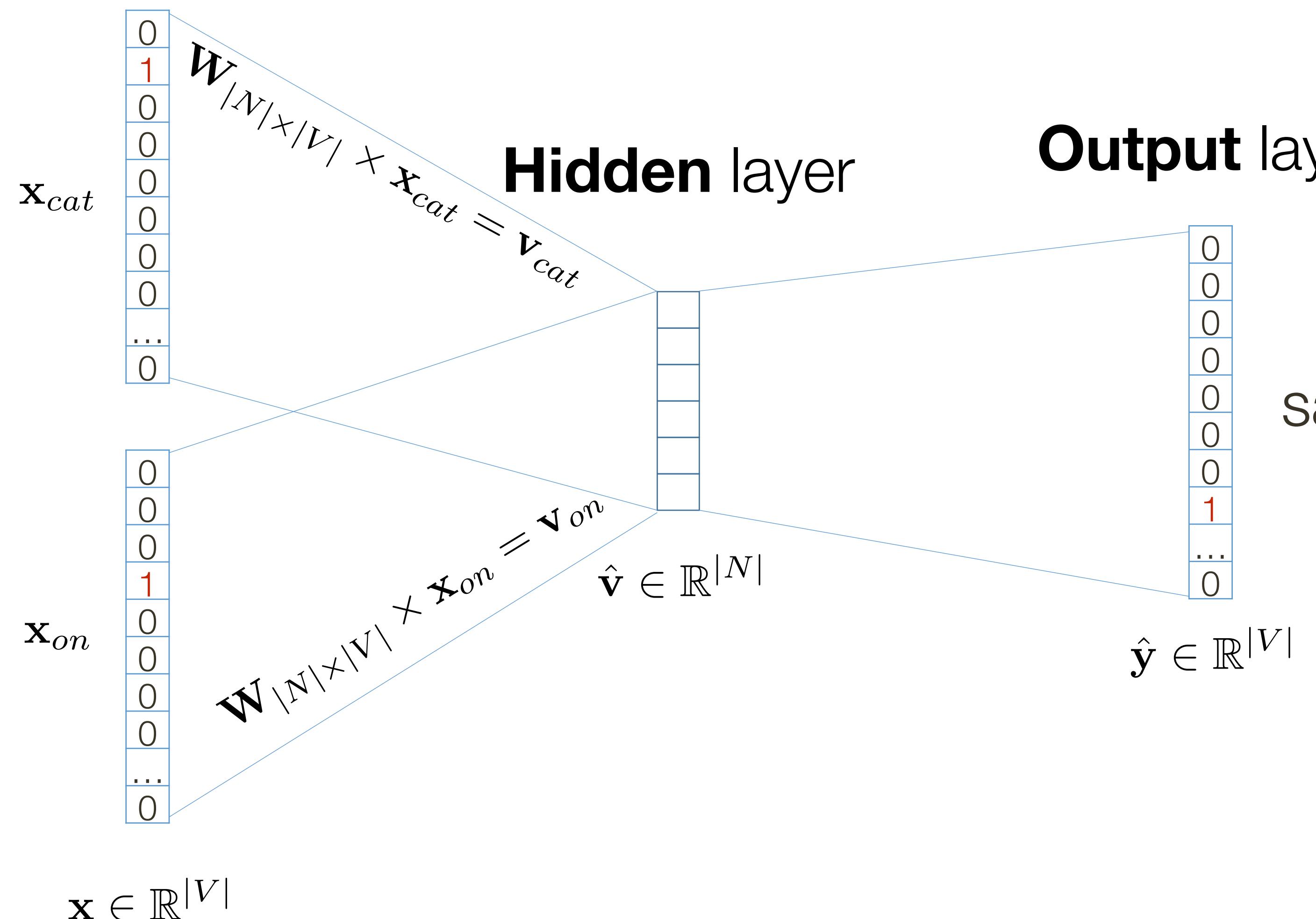
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[ Mikolov et al., 2013 ]

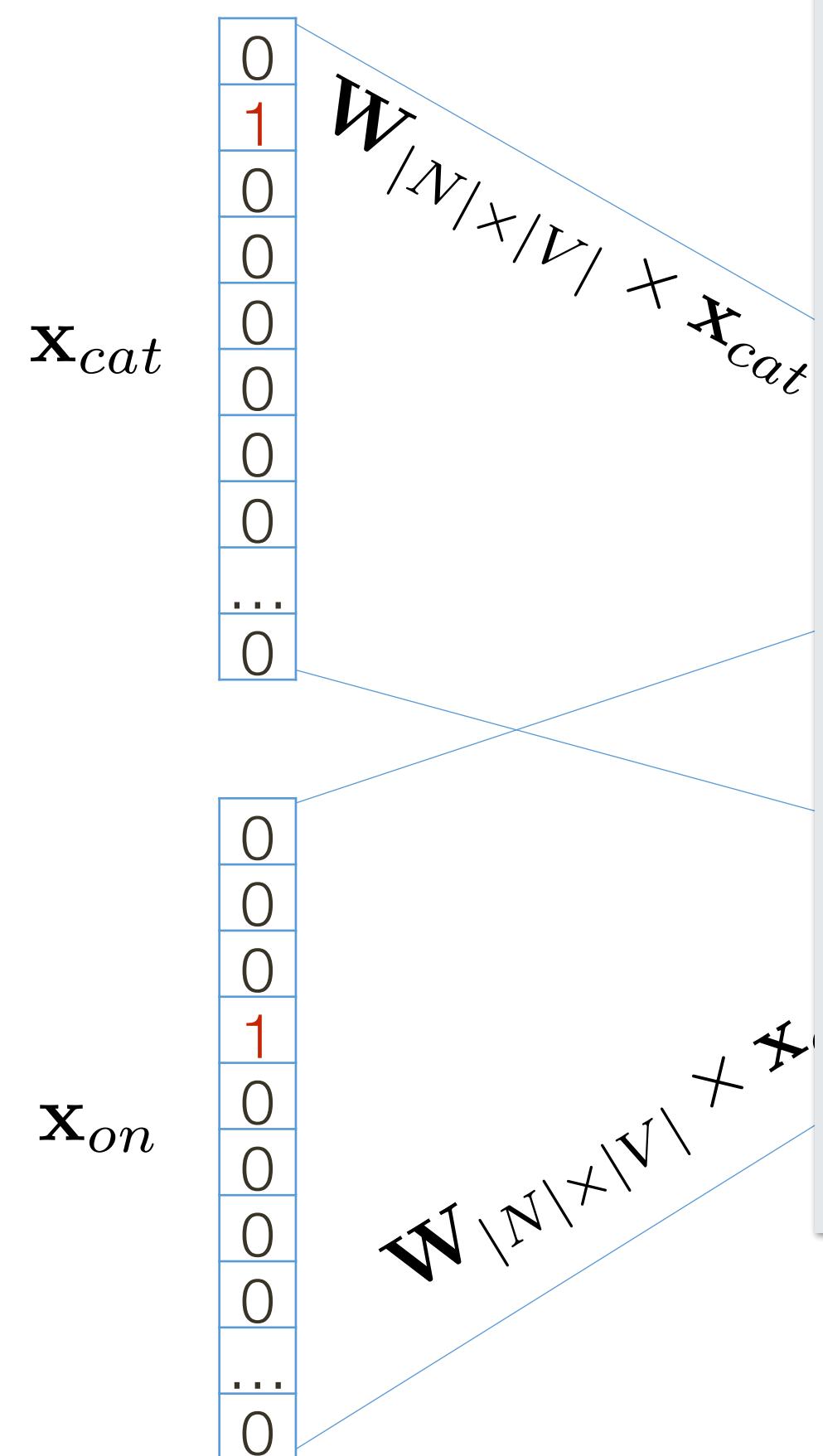
**Input layer**



# CBOW: Continuous Bag of Words

[ Mikolov et al., 2013 ]

## Input layer



$$\mathbf{x} \in \mathbb{R}^{|V|}$$

$$\mathbf{W}_{|V| \times |N|}^T \times \mathbf{x}_{cat} = \mathbf{v}_{cat}$$
$$\mathbf{W}_{|V| \times |N|}^T \times \mathbf{x}_{on} = \mathbf{v}_{on}$$

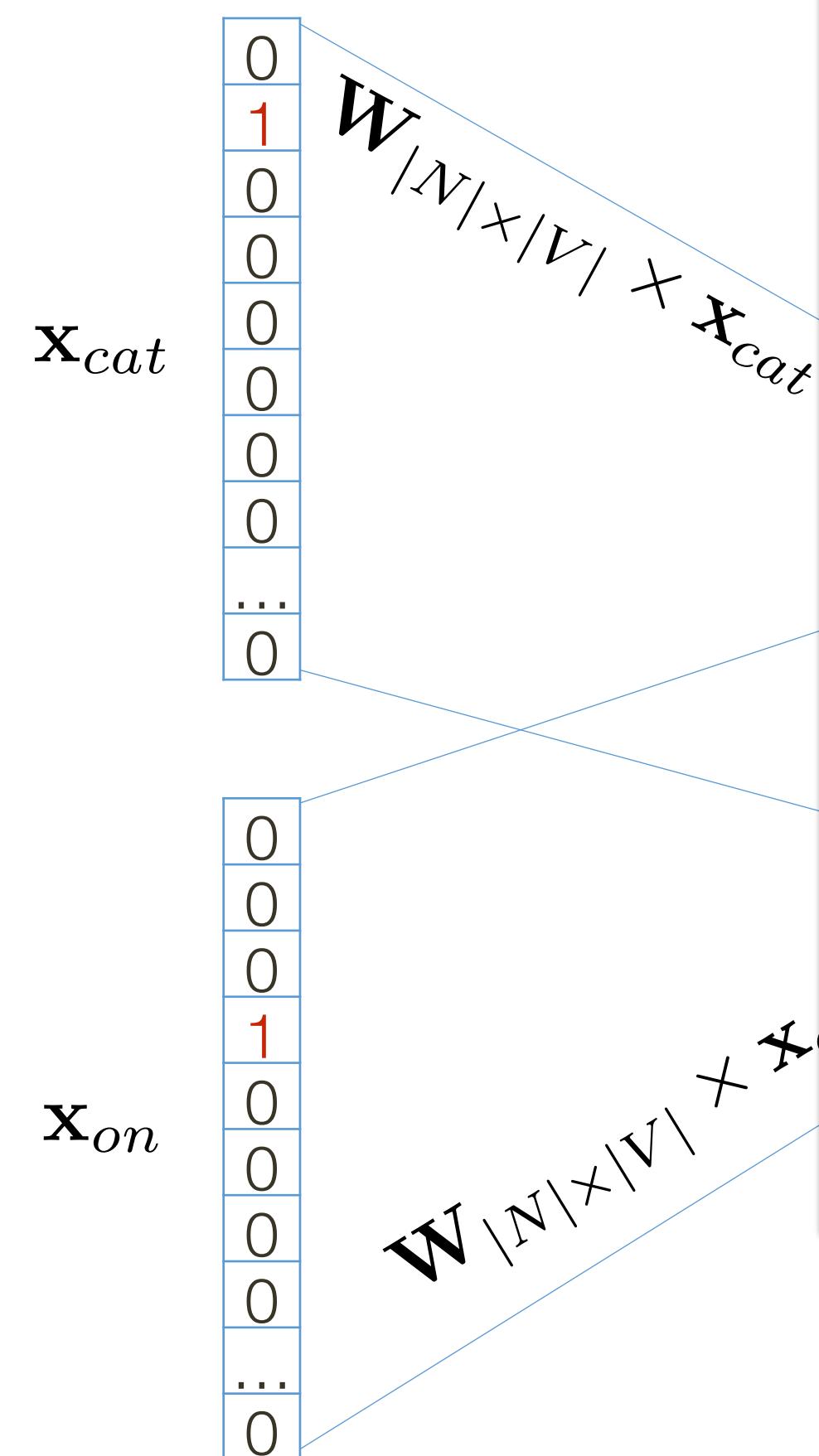
The diagram shows the calculation of the context vector  $\mathbf{v}_{cat}$  from the word vector  $\mathbf{x}_{cat}$ . The weight matrix  $\mathbf{W}_{|V| \times |N|}^T$  is a  $|V| \times |N|$  matrix where each row corresponds to a word vector. The element at position  $(i, j)$  is highlighted in red if  $x_{cat}(i) = 1$ . The resulting vector  $\mathbf{v}_{cat}$  has non-zero elements corresponding to the words present in the context window.

0.1	2.4	1.6	1.8	0.5	0.9	...	...	...	3.2
0.5	2.6	1.4	2.9	1.5	3.6	...	...	...	6.1
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...
0.6	1.8	2.7	1.9	2.4	2.0	...	...	...	1.2

# CBOW: Continuous Bag of Words

[ Mikolov et al., 2013 ]

## Input layer



$$\mathbf{x} \in \mathbb{R}^{|V|}$$

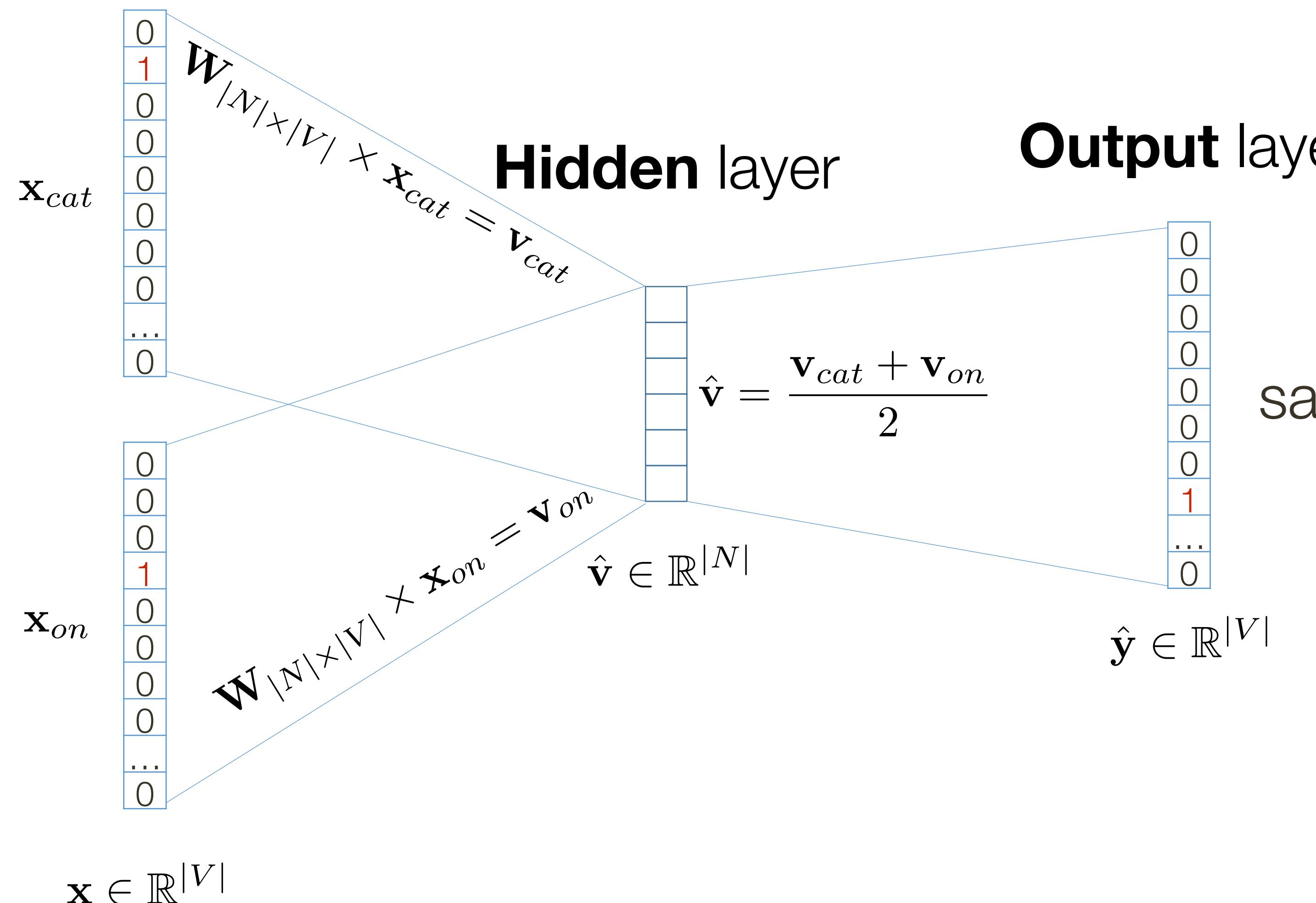
$$\mathbf{W}_{|V| \times |N|}^T \times \mathbf{x}_{on} = \mathbf{v}_{on}$$

The diagram shows the calculation of the output vector  $\mathbf{v}_{on}$  from the input vector  $\mathbf{x}_{on}$ . The weight matrix  $\mathbf{W}_{|V| \times |N|}^T$  is a  $|V| \times |N|$  matrix where each row corresponds to a word in the vocabulary. The input vector  $\mathbf{x}_{on}$  is a vertical array of zeros, with a single '1' at the position corresponding to the word 'on'. The result of the multiplication is the output vector  $\mathbf{v}_{on}$ , which has non-zero values only at the positions corresponding to the words 'cat' and 'on'.

# CBOW: Continuous Bag of Words

[ Mikolov et al., 2013 ]

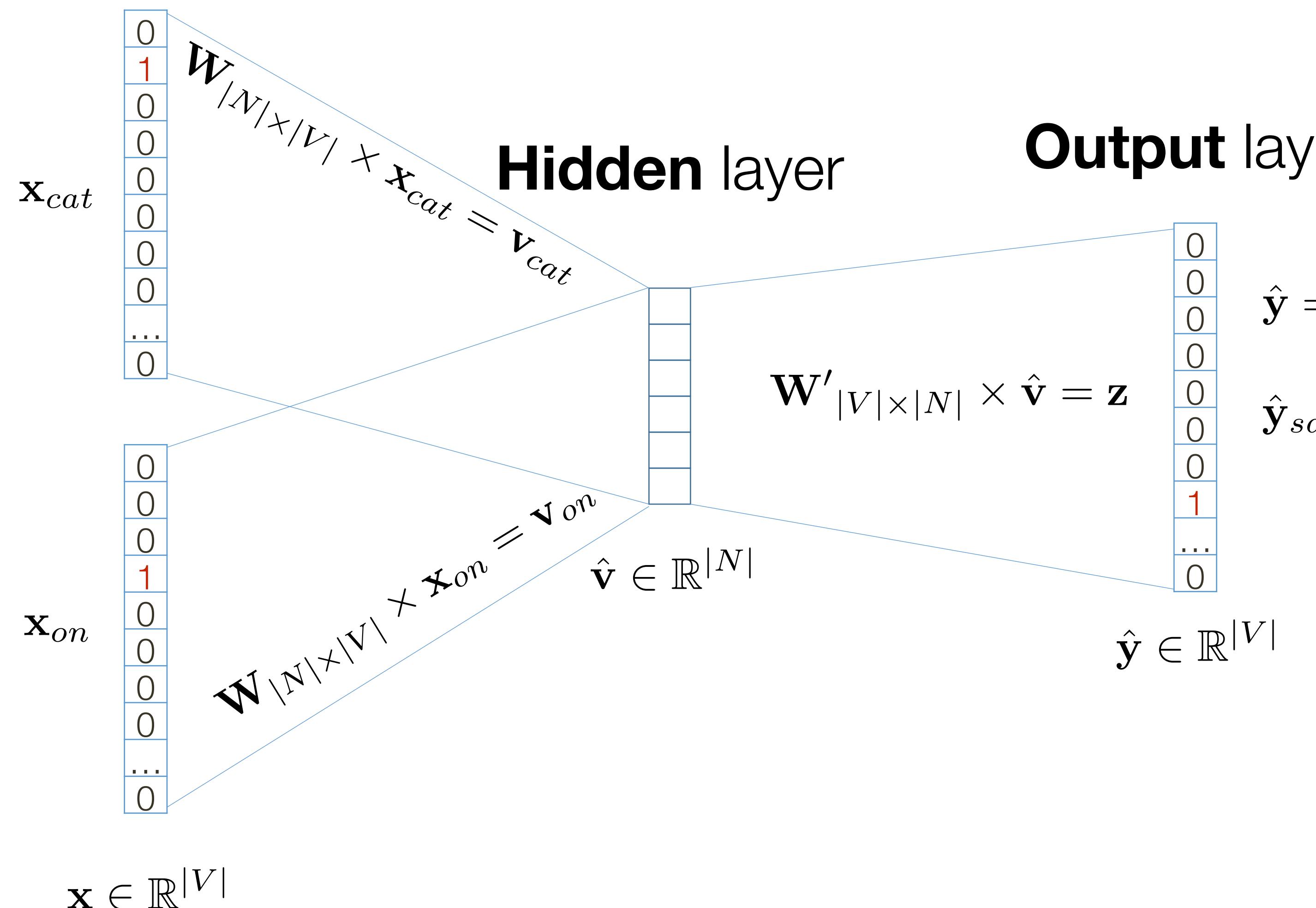
**Input layer**



# CBOW: Continuous Bag of Words

[ Mikolov et al., 2013 ]

**Input layer**

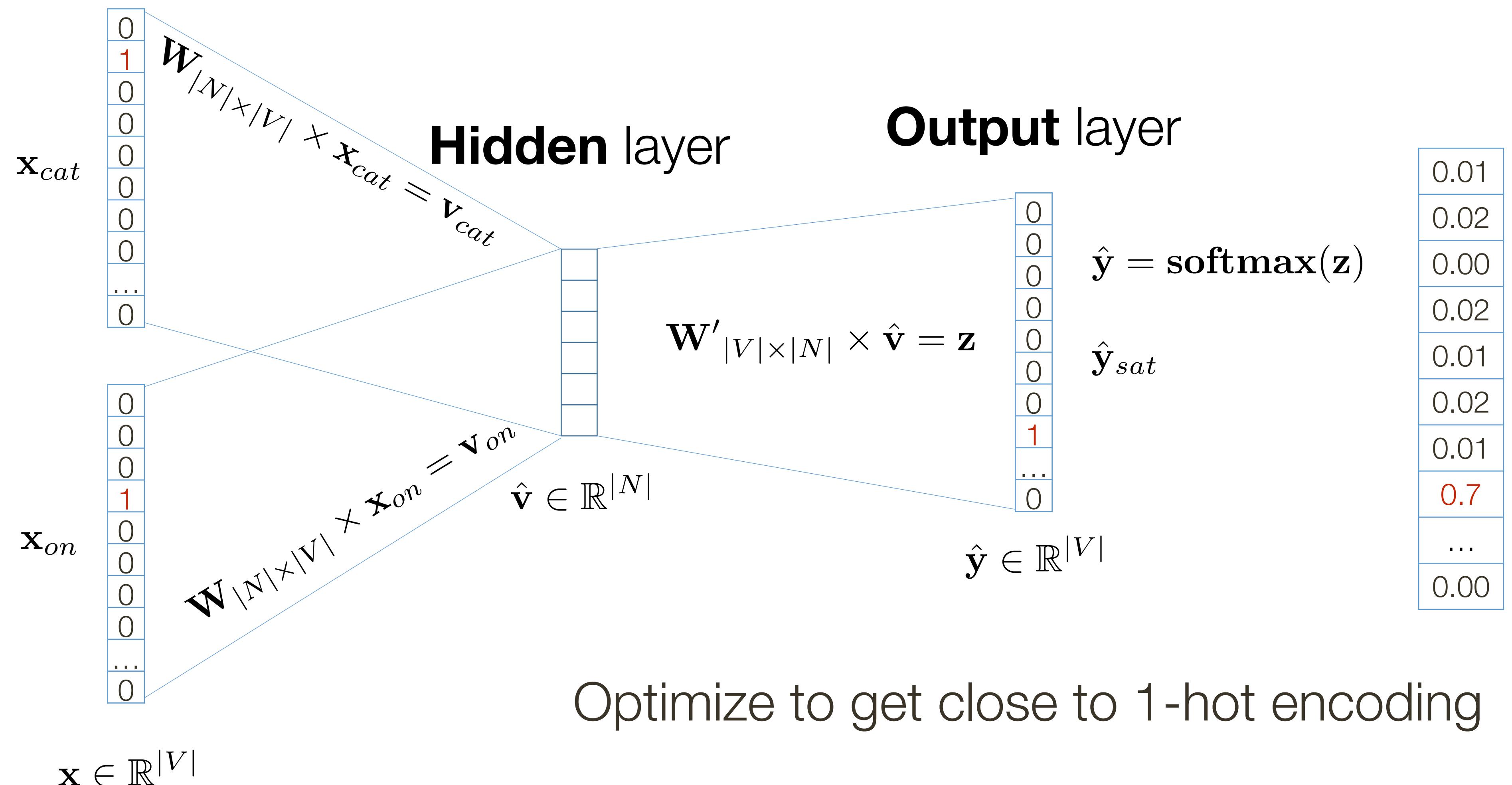


\*slide from Vagelis Hristidis

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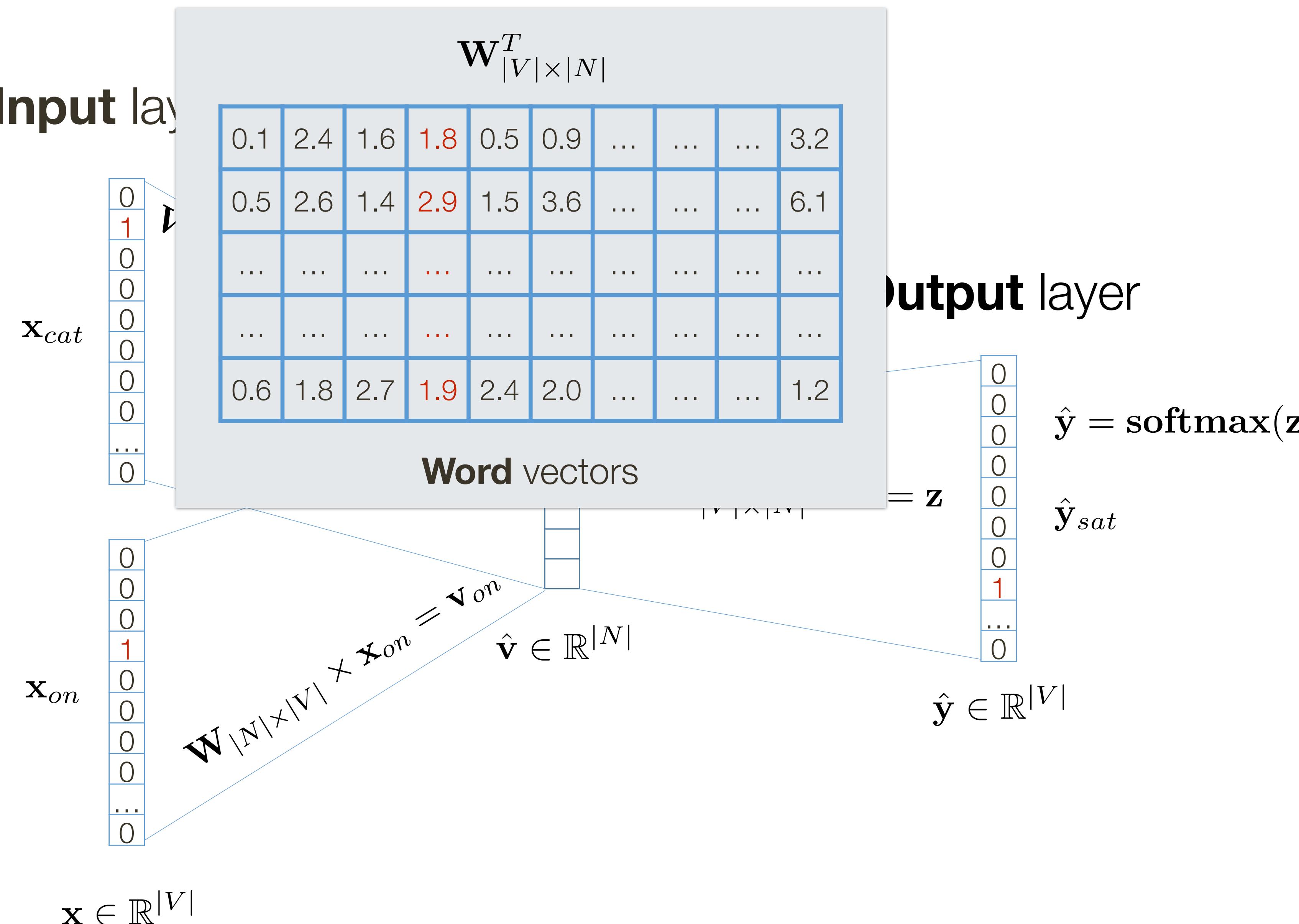
[ Mikolov et al., 2013 ]

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[ Mikolov et al., 2013 ]

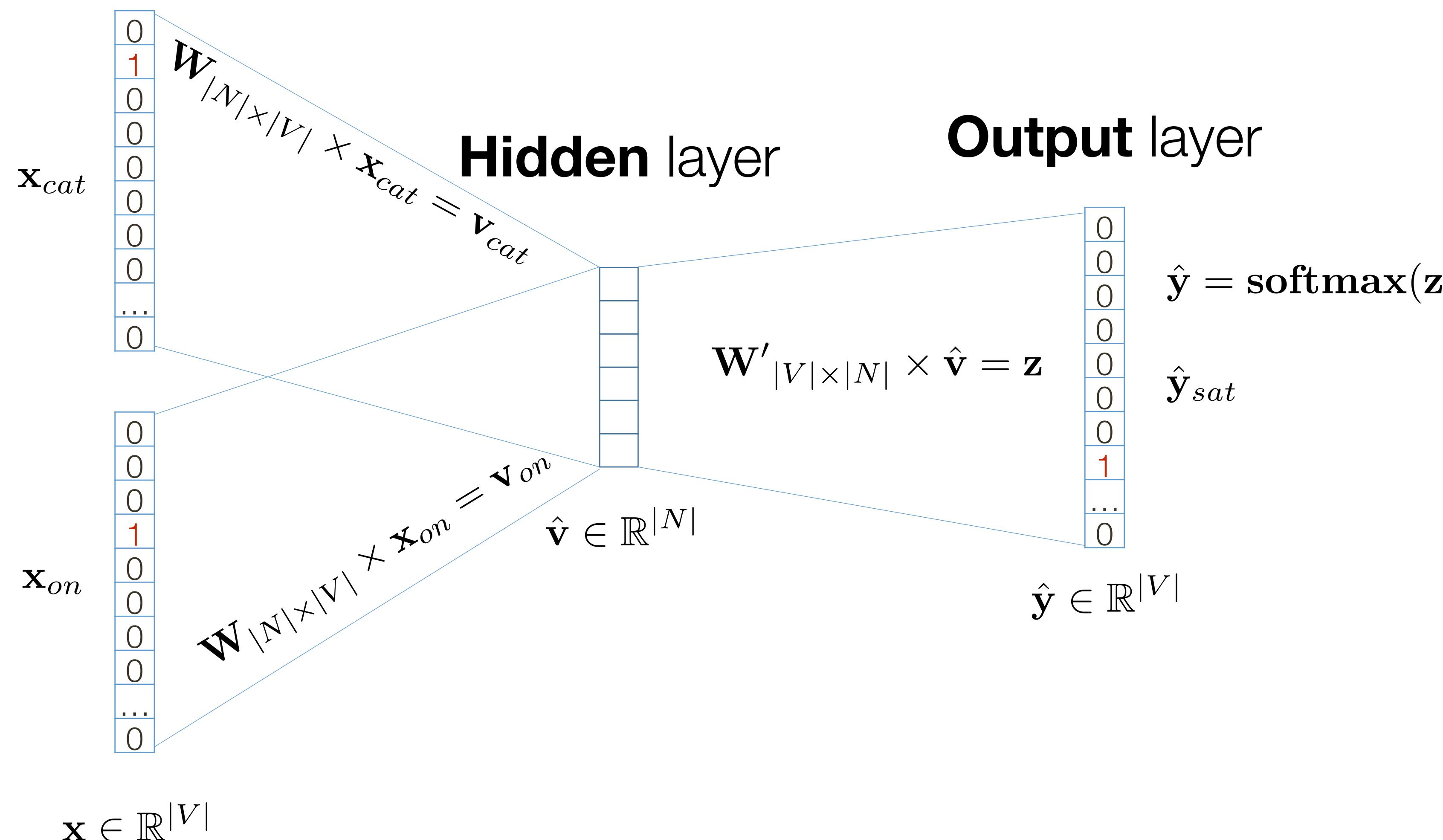


# CBOW: Interesting Observation

[ Mikolov et al., 2013 ]

**Input layer**

There are two representations for same word!



$$\mathbf{x} \in \mathbb{R}^{|V|}$$

# CBOW: Interesting Observation

[ Mikolov et al., 2013 ]

**Another way to look at it:** Maximize similarity between context word representation and the word representation itself

$$p(w|c) = \frac{\exp \left[ \left( \sum_c \mathbf{W} \mathbf{x}_c \right)^T (\mathbf{W} \mathbf{x}_w) \right]}{\sum_i^{|V|} \exp \left[ \left( \mathbf{W} \mathbf{x}_i \right)^T (\mathbf{W} \mathbf{x}_w) \right]}$$

# CBOW: Interesting Observation

[ Mikolov et al., 2013 ]

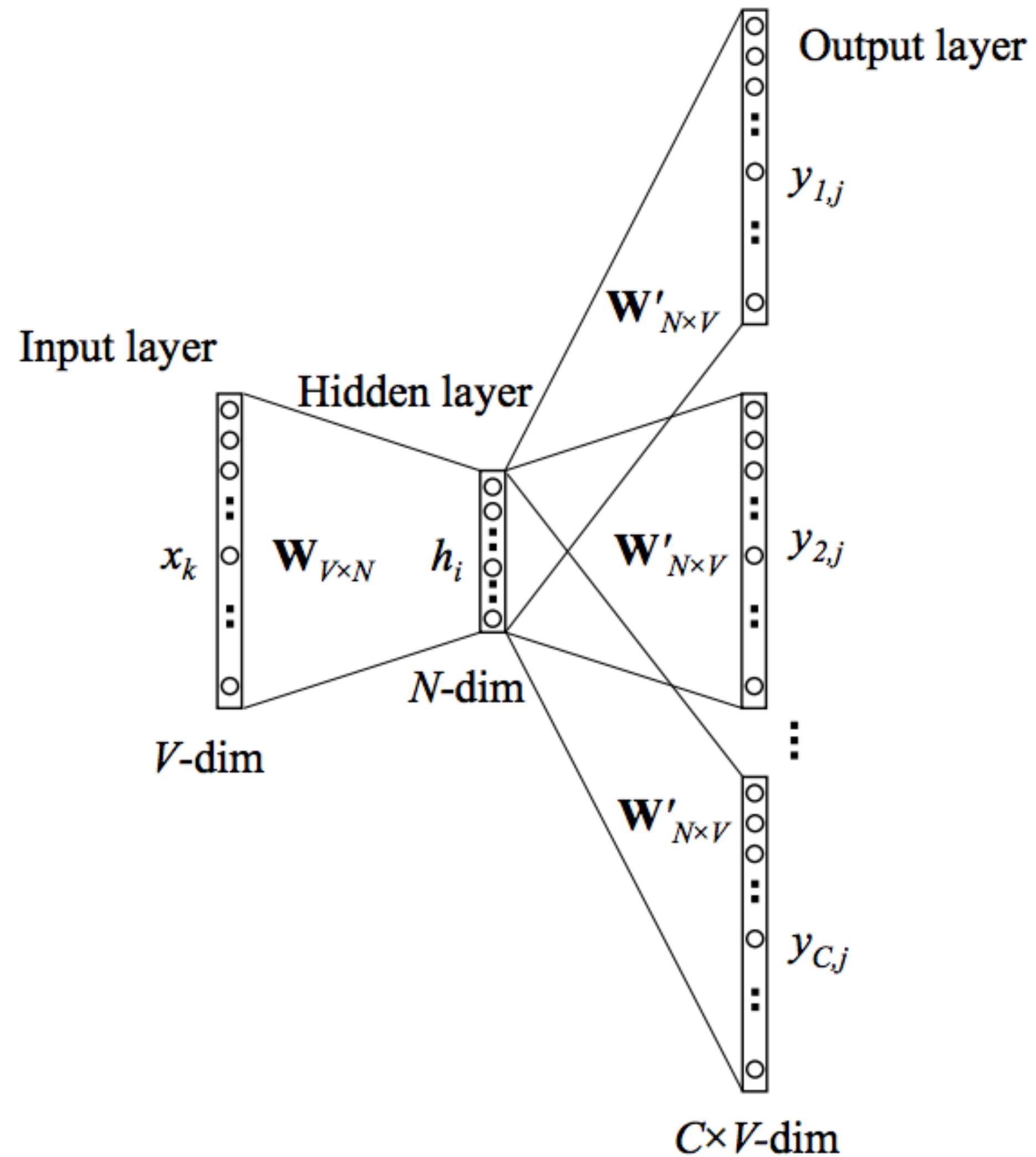
**Another way to look at it:** Maximize similarity between context word representation and the word representation itself

$$J(\mathbf{W}) = -\frac{1}{T} \sum_{t=1}^T \sum_{-m \leq j \leq m; j \neq 0} \log p(w_{t+j} | w_t)$$

$$p(w_{t+j} | w_t) = \frac{\exp(\mathbf{w}_{t+j}^T \mathbf{w}_t)}{\sum_{i=1}^{|V|} \exp(\mathbf{w}_i^T \mathbf{w}_t)}$$

# Skip-Gram Model

[ Mikolov et al., 2013 ]



# Comparison

[ Mikolov et al., 2013 ]

- **CBOW** is not great for rare words and typically needs less data to train
- **Skip-gram** better for rare words and needs more data to train the model

Model	Vector Dimensionality	Training words	Accuracy [%]		
			Semantic	Syntactic	Total
Collobert-Weston NNLM	50	660M	9.3	12.3	11.0
Turian NNLM	50	37M	1.4	2.6	2.1
Turian NNLM	200	37M	1.4	2.2	1.8
Mnih NNLM	50	37M	1.8	9.1	5.8
Mnih NNLM	100	37M	3.3	13.2	8.8
Mikolov RNNLM	80	320M	4.9	18.4	12.7
Mikolov RNNLM	640	320M	8.6	36.5	24.6
Huang NNLM	50	990M	13.3	11.6	12.3
Our NNLM	20	6B	12.9	26.4	20.3
Our NNLM	50	6B	27.9	55.8	43.2
Our NNLM	100	6B	34.2	<b>64.5</b>	50.8
CBOW	300	783M	15.5	53.1	36.1
Skip-gram	300	783M	<b>50.0</b>	55.9	<b>53.3</b>

# Interesting Results: Word Analogies

Test for linear relationships, examined by Mikolov et al. (2014)

$$a:b :: c:?$$



$$d = \arg \max_x \frac{(w_b - w_a + w_c)^T w_x}{\|w_b - w_a + w_c\|}$$

man:woman :: king:?

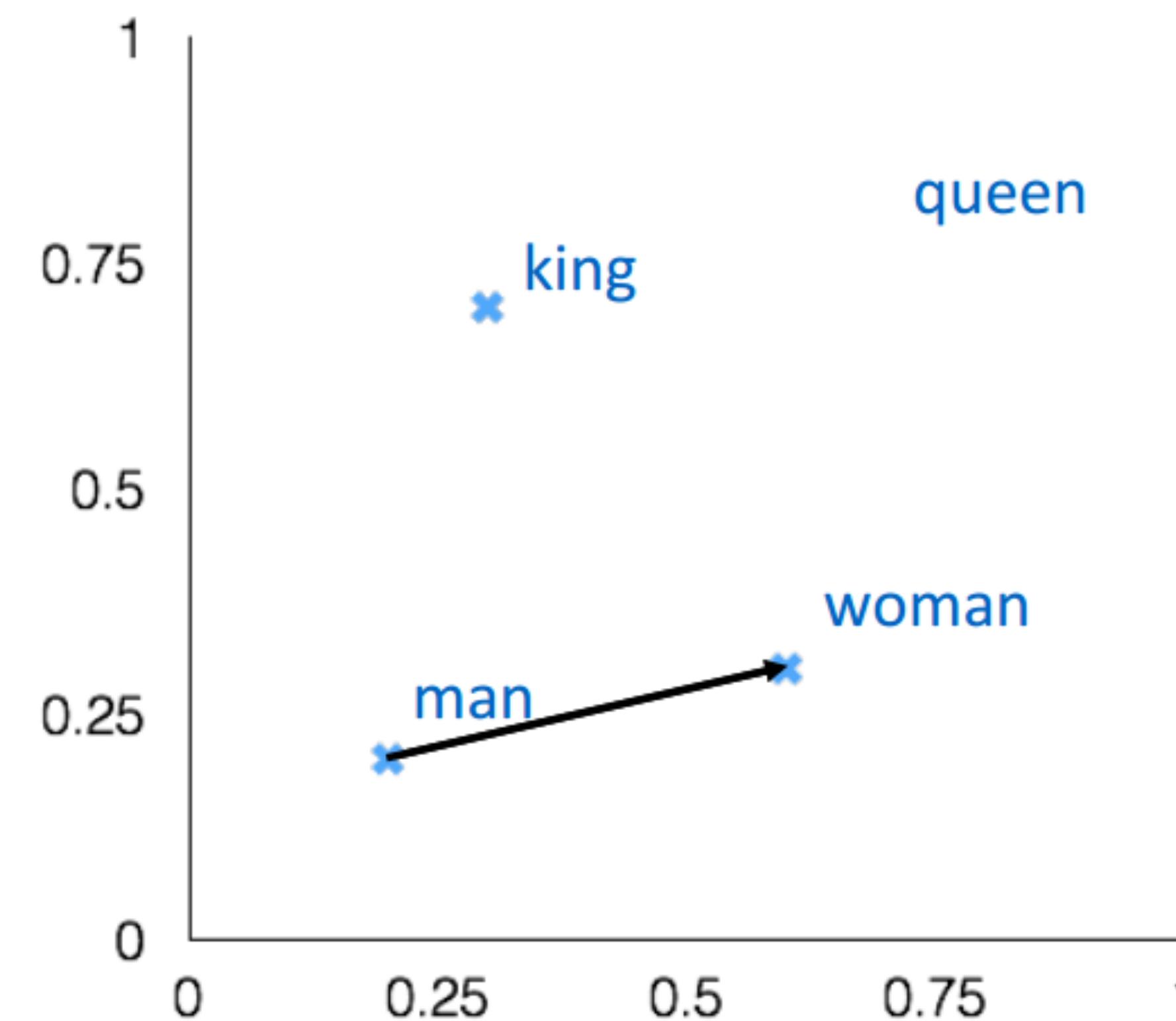
$$+ \text{ king} \quad [ 0.30 \ 0.70 ]$$

$$- \text{ man} \quad [ 0.20 \ 0.20 ]$$

$$+ \text{ woman} \quad [ 0.60 \ 0.30 ]$$

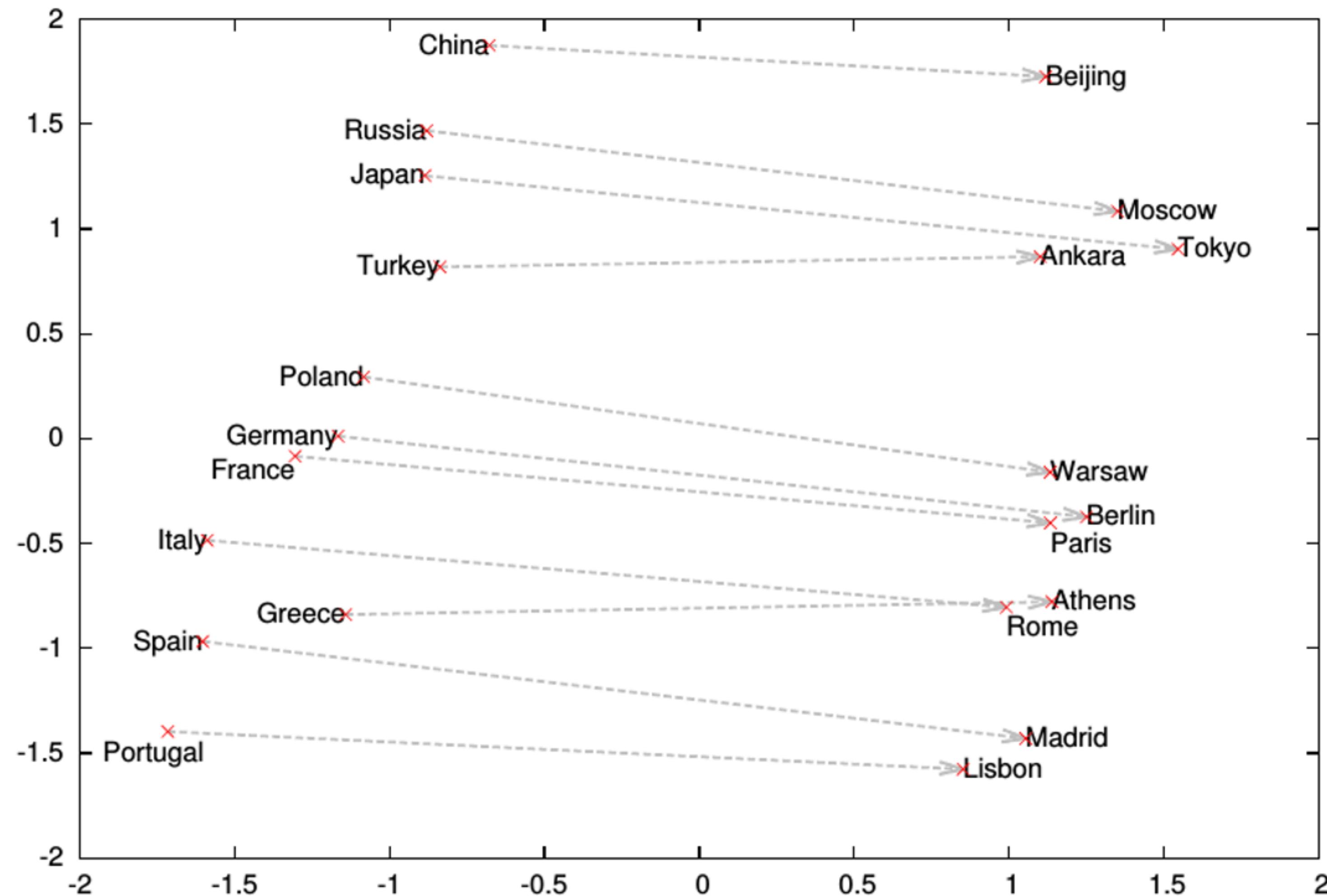
---

$$\text{queen} \quad [ 0.70 \ 0.80 ]$$



# Interesting Results: Word Analogies

[ Mikolov et al., 2013 ]



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# Simple Language Models: N-Grams

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**Bi-gram** Approximation:

$$p(w_{1:n}) = \prod_{k=1}^n p(w_k|w_{k-1})$$

**N-gram** Approximation:

$$p(w_{1:n}) = \prod_{k=1}^n p(w_k|w_{k-N+1:k-1})$$

# Estimating Probabilities

N-gram conditional probabilities can be estimated based on raw concurrence counts in the observed sequences

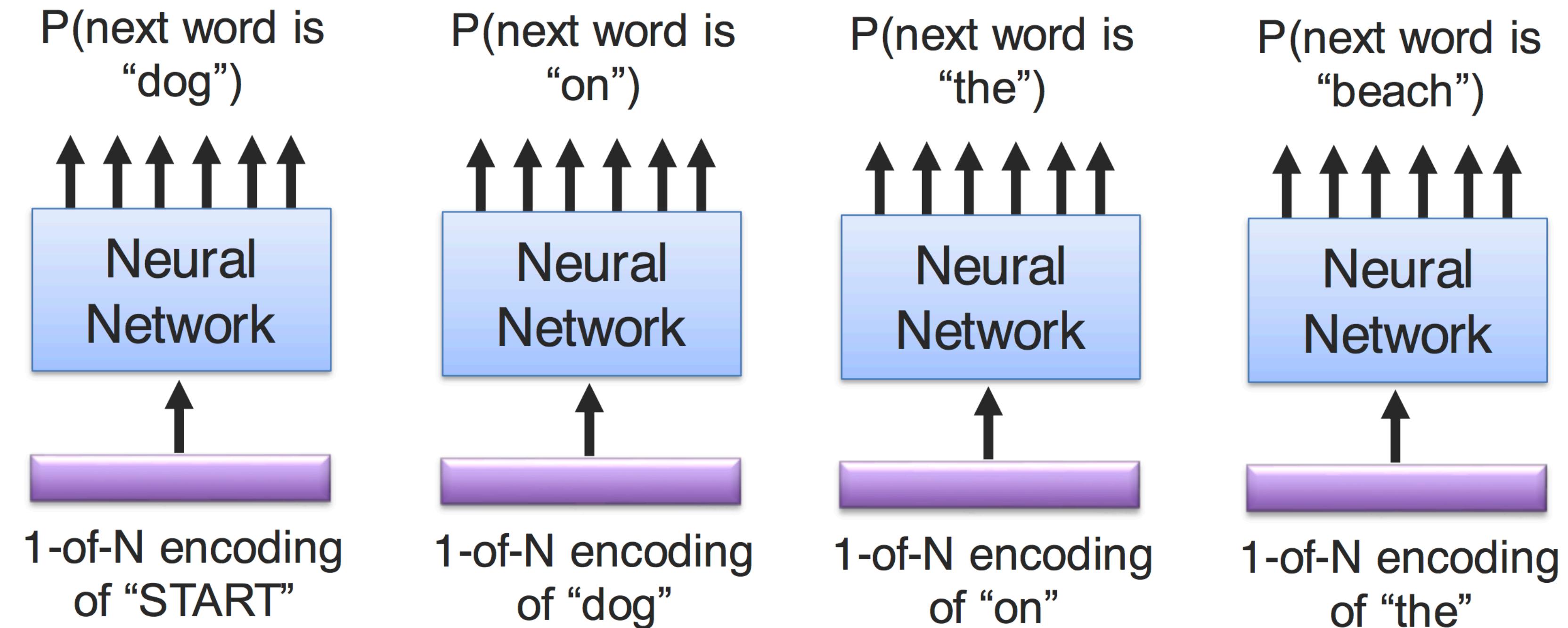
**Bi-gram:**

$$p(w_n | w_{n-1}) = \frac{C(w_{n-1} w_n)}{C(w_{n-1})}$$

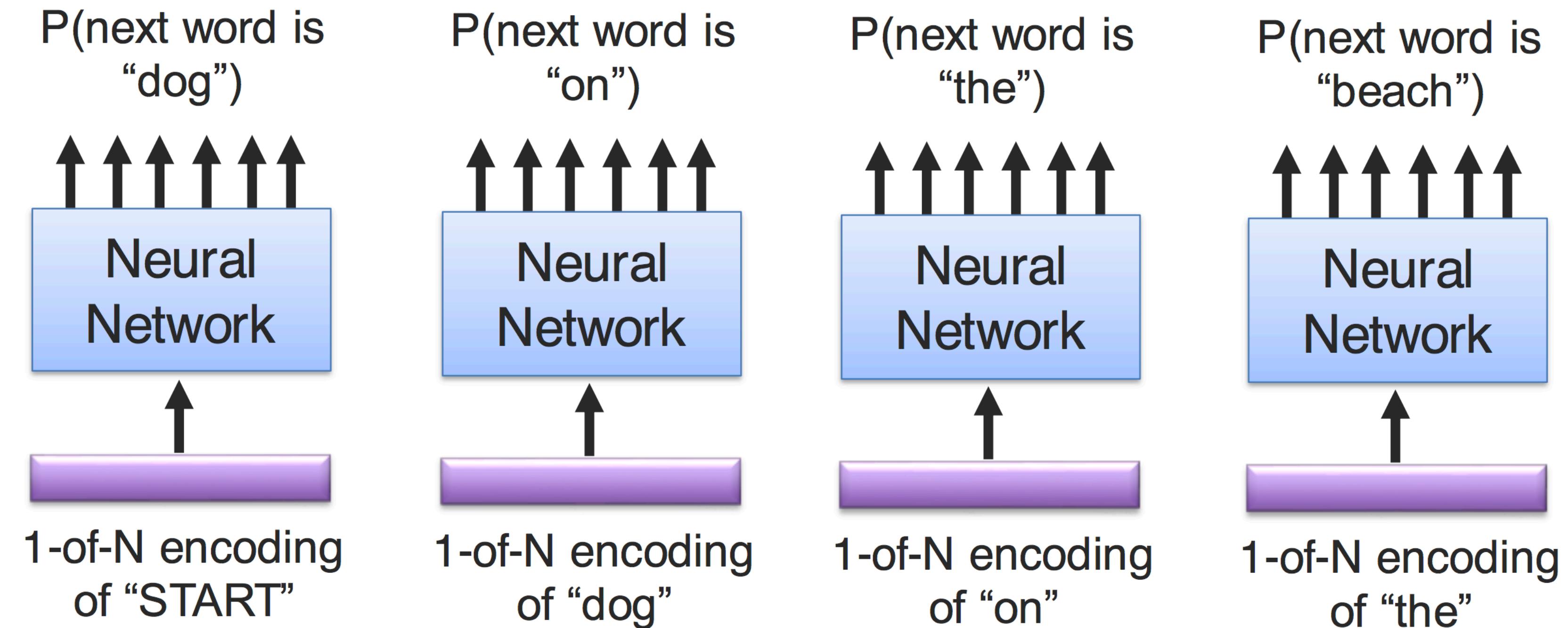
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$$p(w_n | w_{n-N-1:n-1}) = \frac{C(w_{n-N-1:n-1} w_n)}{C(w_{n-N-1:n-1})}$$

# Neural-based Unigram Language Mode

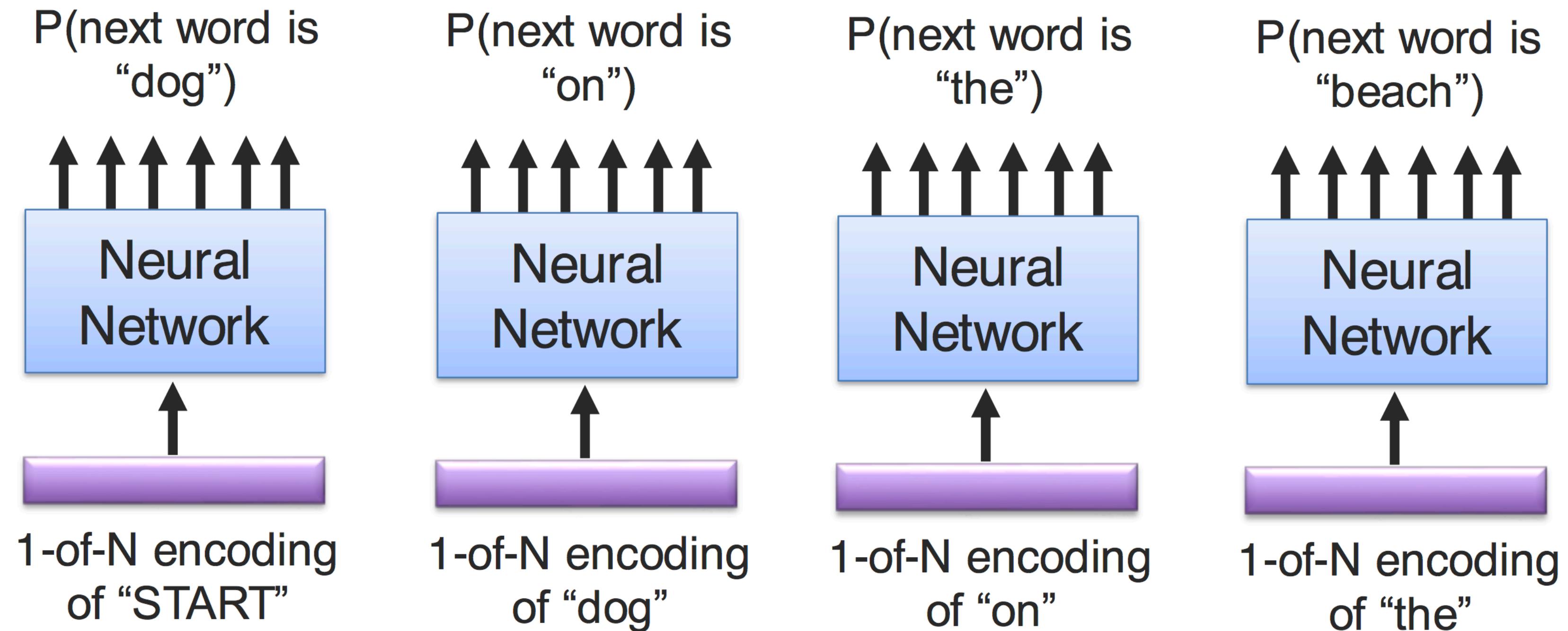


# Neural-based Unigram Language Mode



**Problem:** Does not model sequential information (too local)

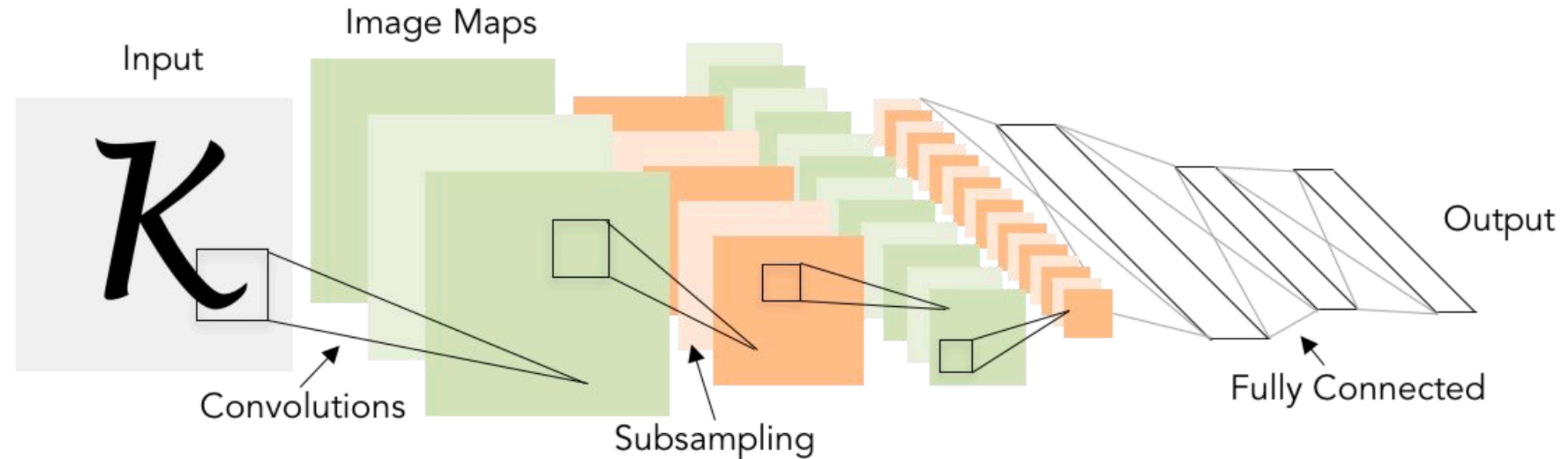
# Neural-based Unigram Language Mode



**Problem:** Does not model sequential information (too local)

We need sequence modeling!

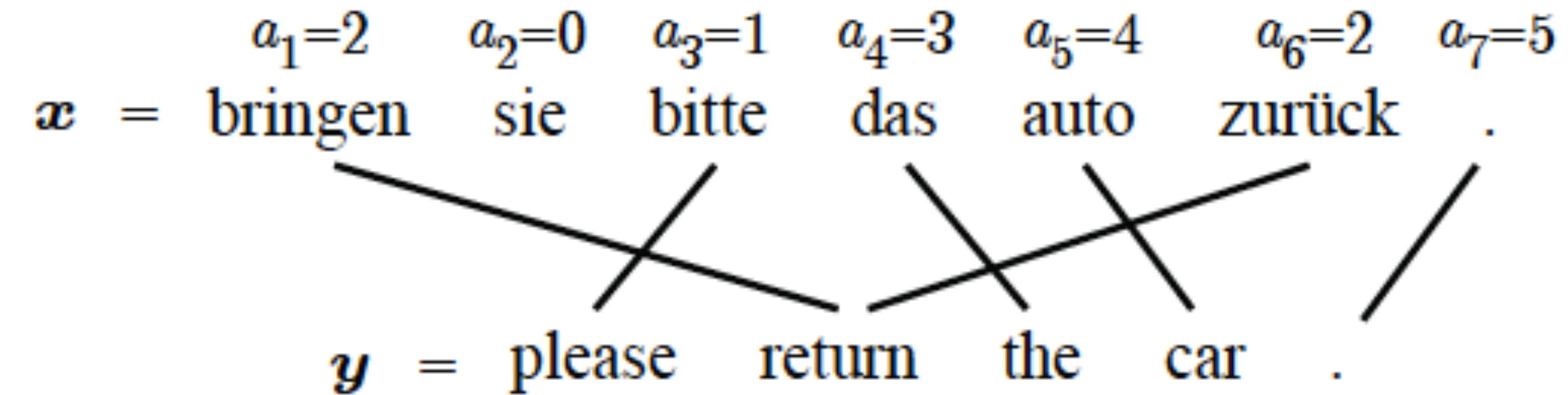
# Sequence Modeling



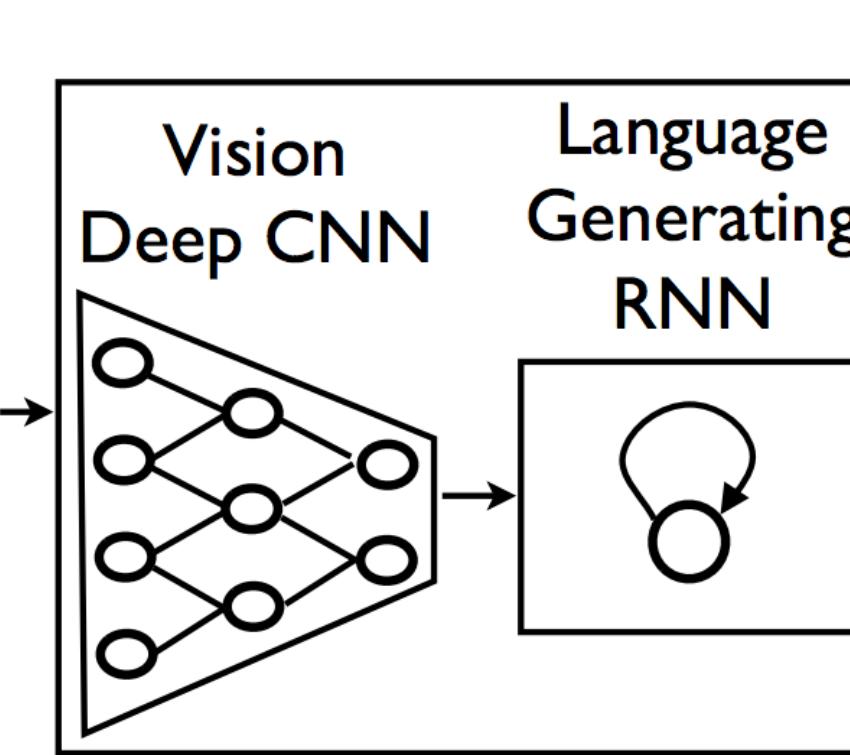
# Why Model Sequences?

*Foreign Minister.* → FOREIGN MINISTER.

→ THE SOUND OF


$$\begin{aligned} \mathbf{x} &= \begin{matrix} a_1=2 & a_2=0 & a_3=1 & a_4=3 & a_5=4 & a_6=2 & a_7=5 \\ \text{bringen} & \text{sie} & \text{bitte} & \text{das} & \text{auto} & \text{zurück} & . \end{matrix} \\ \mathbf{y} &= \begin{matrix} \text{please} & \text{return} & \text{the} & \text{car} & . \end{matrix} \end{aligned}$$


# Multi-modal tasks



**A group of people shopping at an outdoor market.**

**There are many vegetables at the fruit stand.**

[ Vinyals et al., 2015 ]

# Sequences where you don't expect them ...

Classify images by taking a series of “glimpses”

[ Gregor et al., ICML 2015 ]

[ Mnih et al., ICLR 2015 ]



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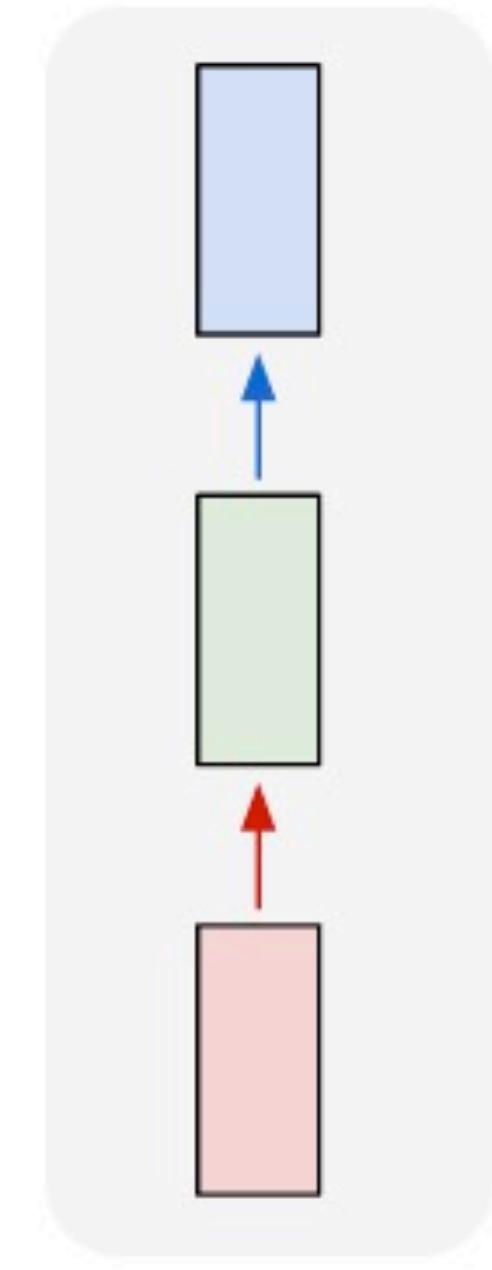
# Sequences where you don't expect them ...

Vision transformers



# Sequences in Inputs or Outputs?

one to one



**Input:** No sequence

**Output:** No seq.

**Example:**

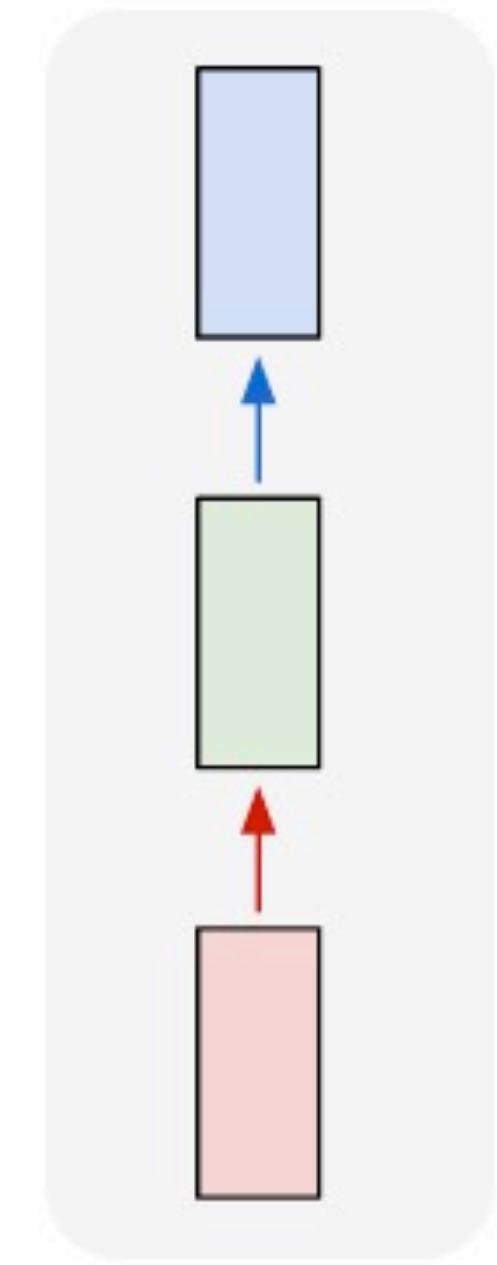
“standard”

classification /

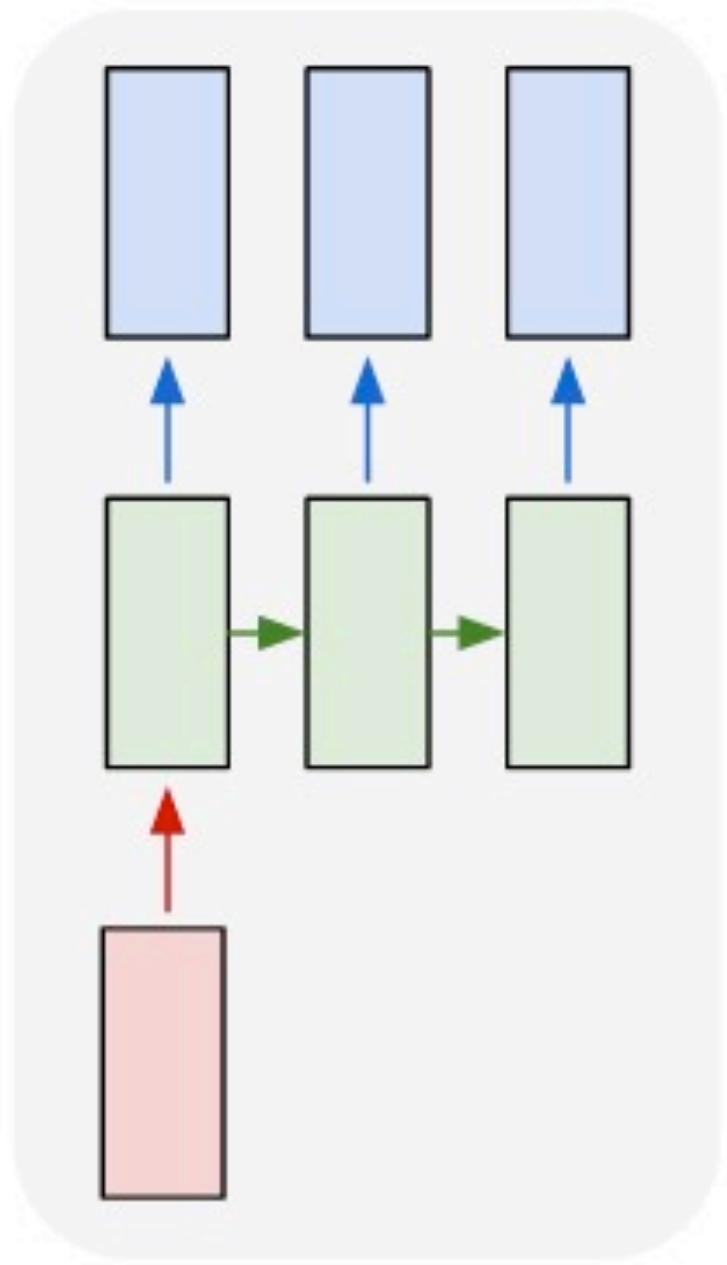
regression problems

# Sequences in Inputs or Outputs?

one to one



one to many



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**Example:**  
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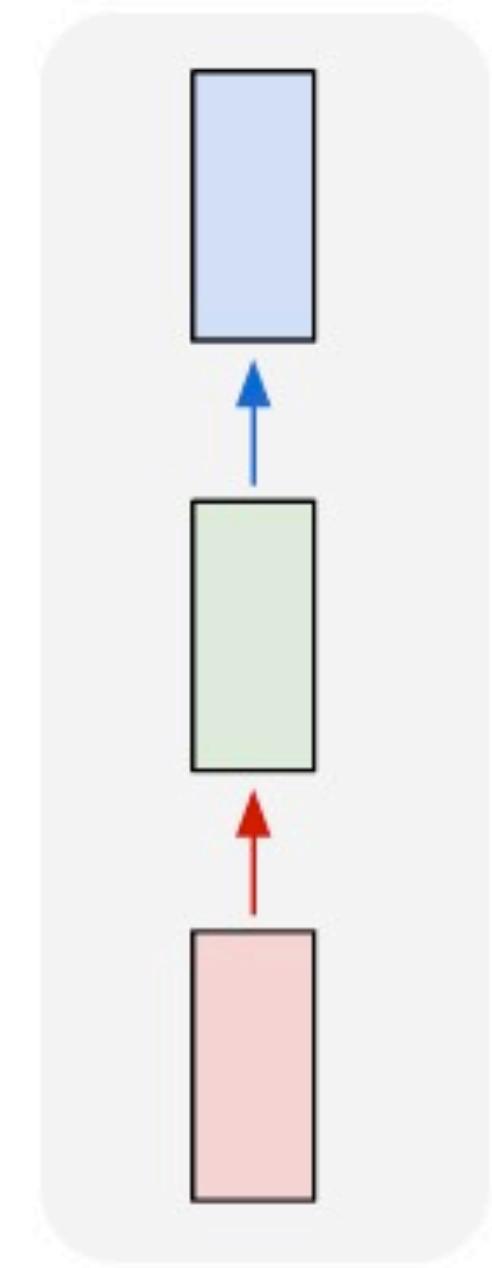
sequence

**Output:**  
Sequence

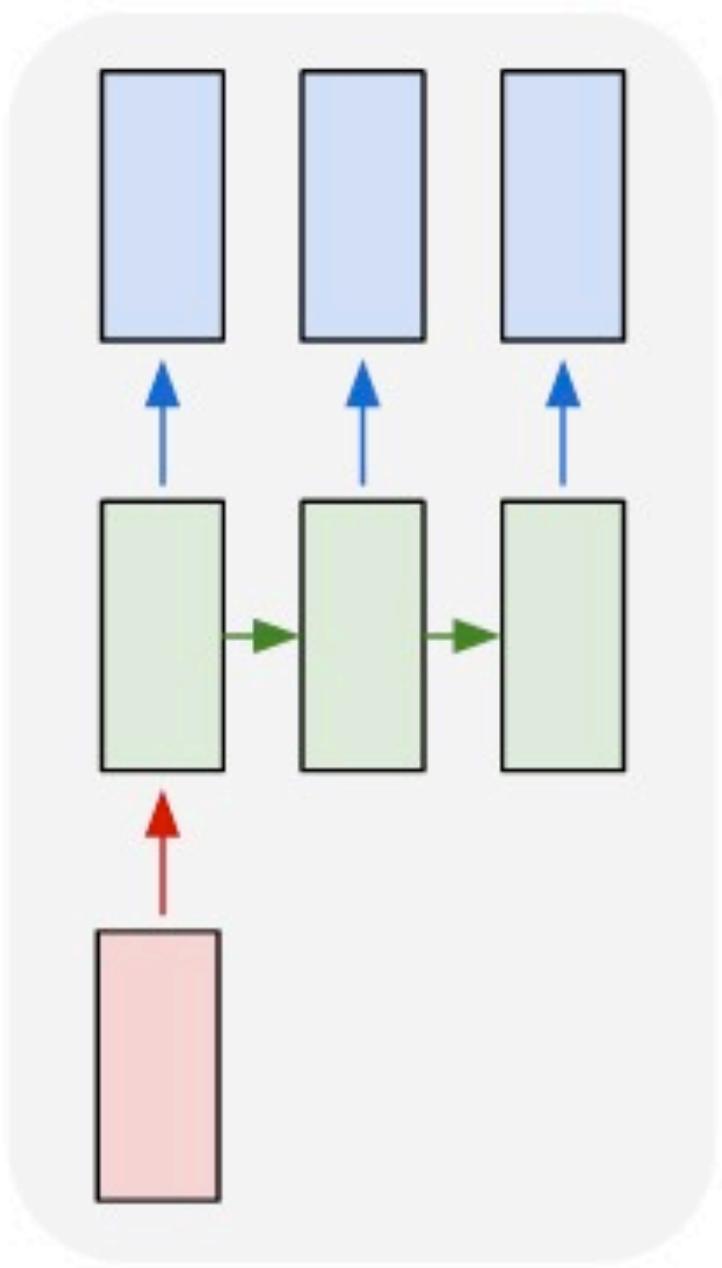
**Example:**  
Im2Caption

# Sequences in Inputs or Outputs?

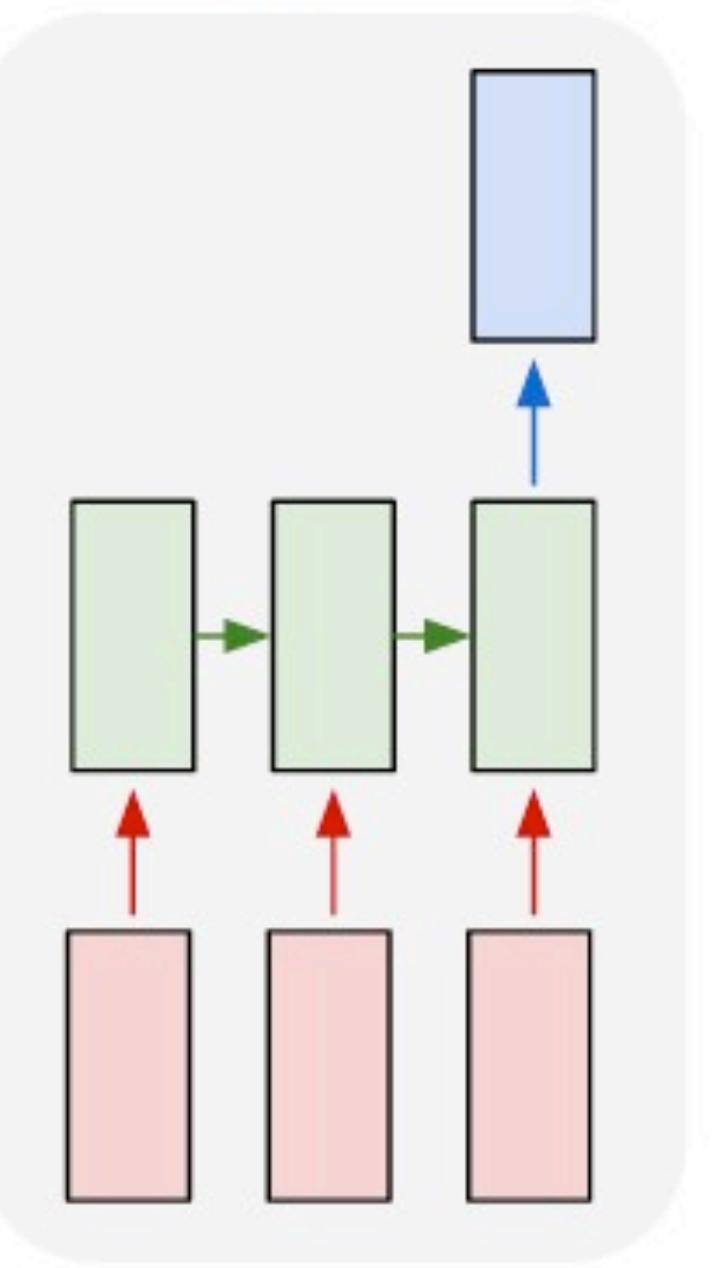
one to one



one to many



many to one



**Input:** No sequence

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**Example:**  
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sequence

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Sequence

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Im2Caption

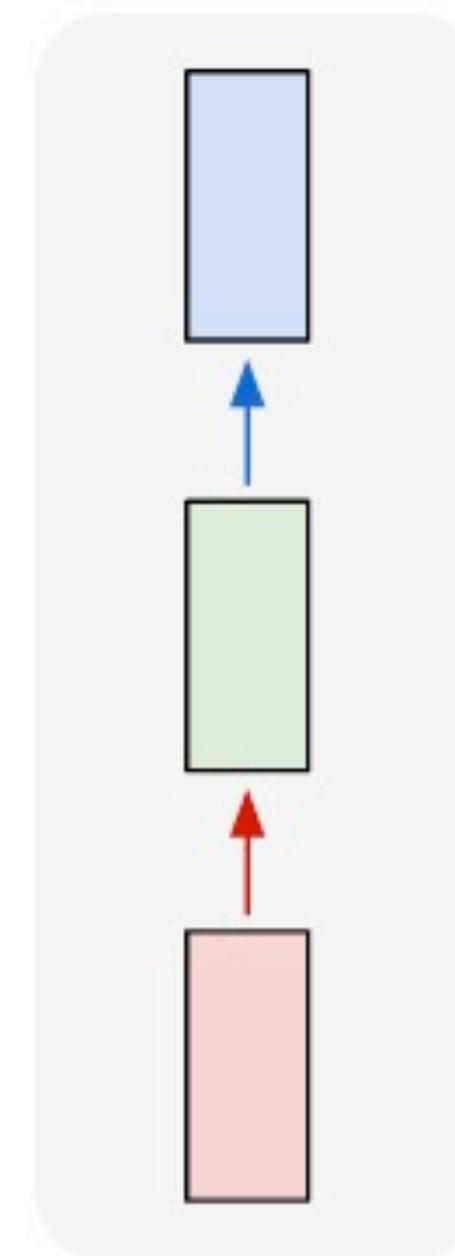
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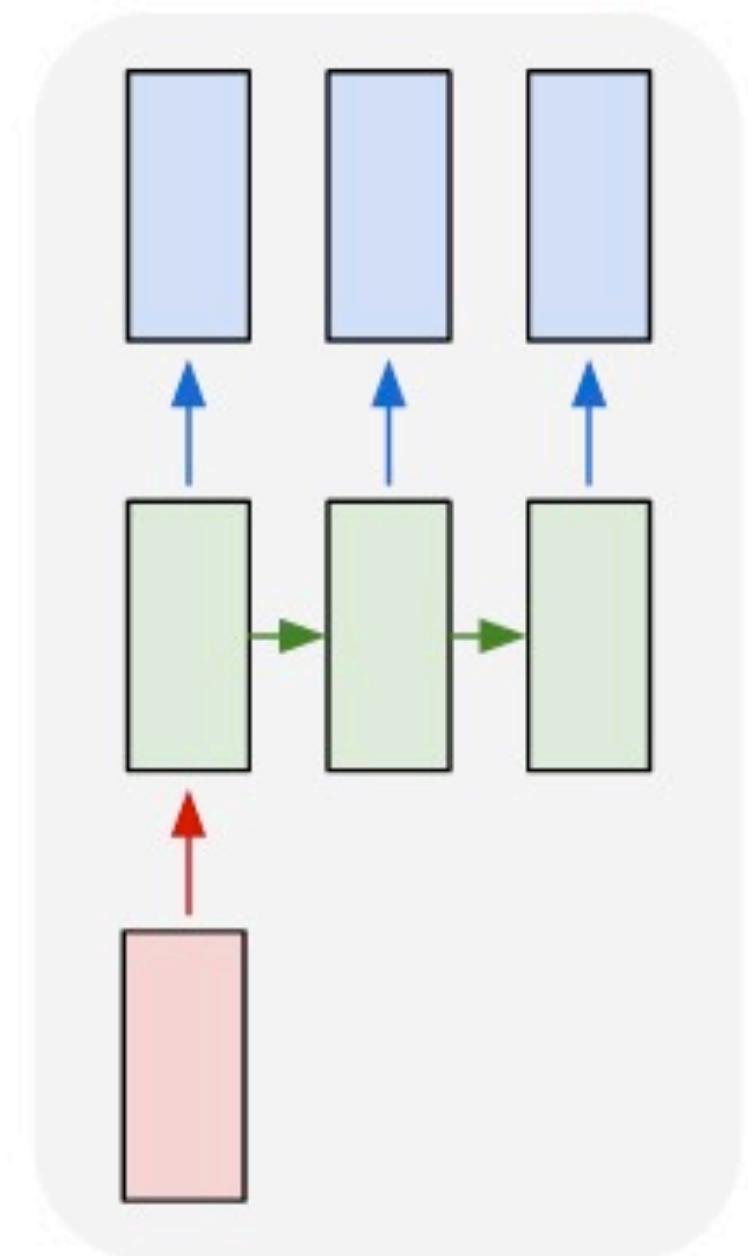
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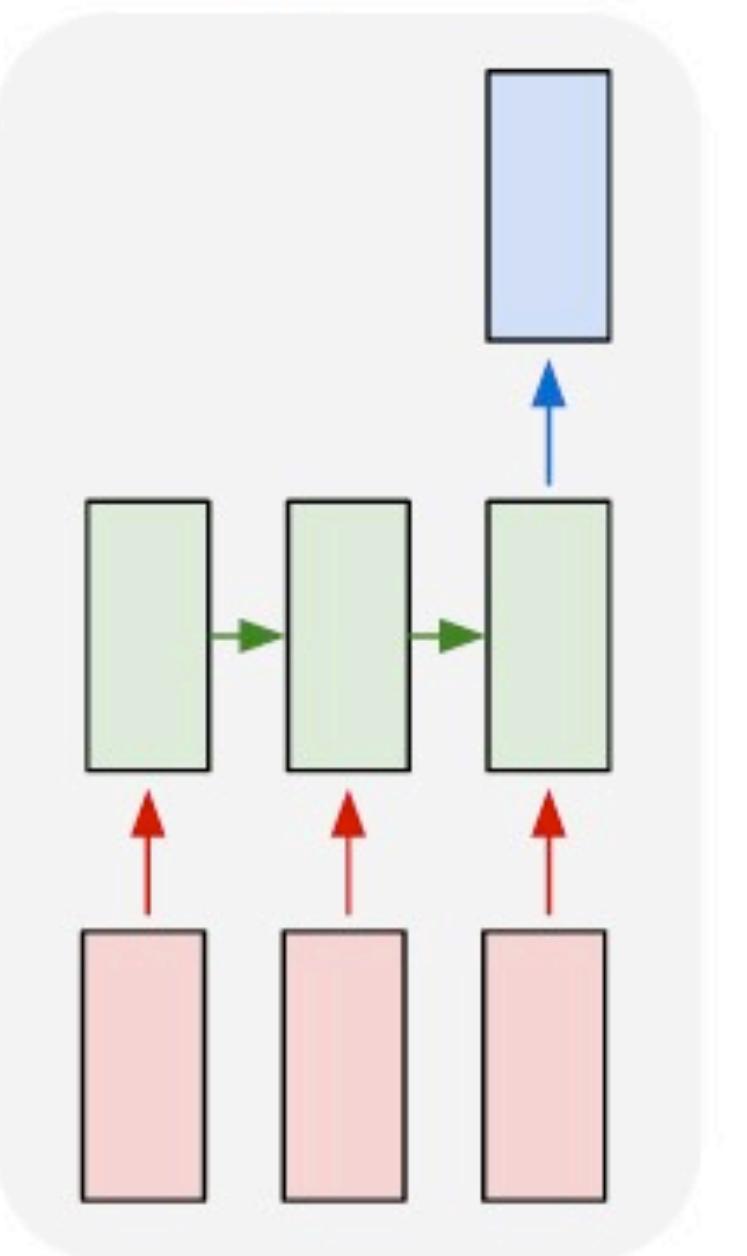
one to one



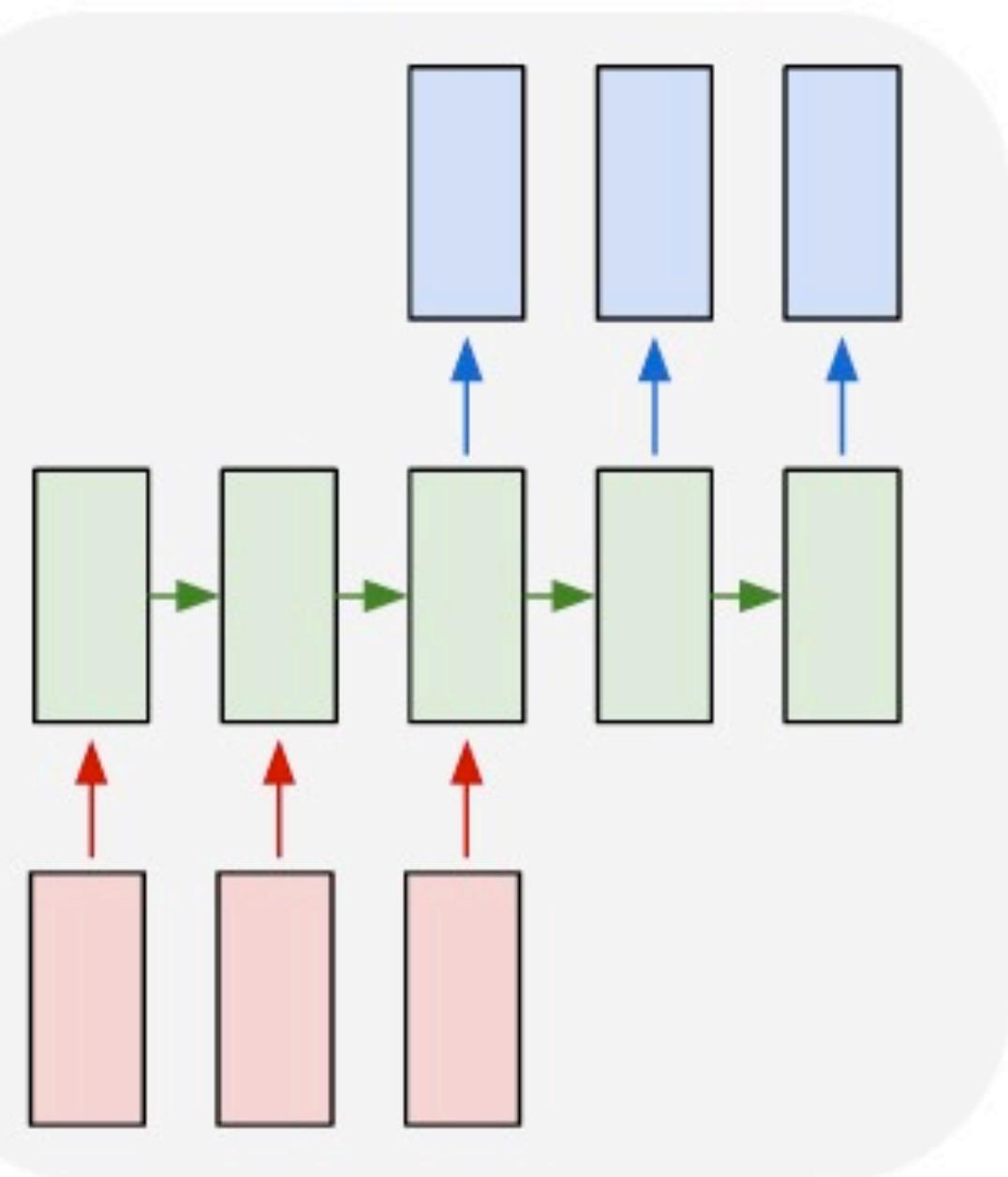
one to many



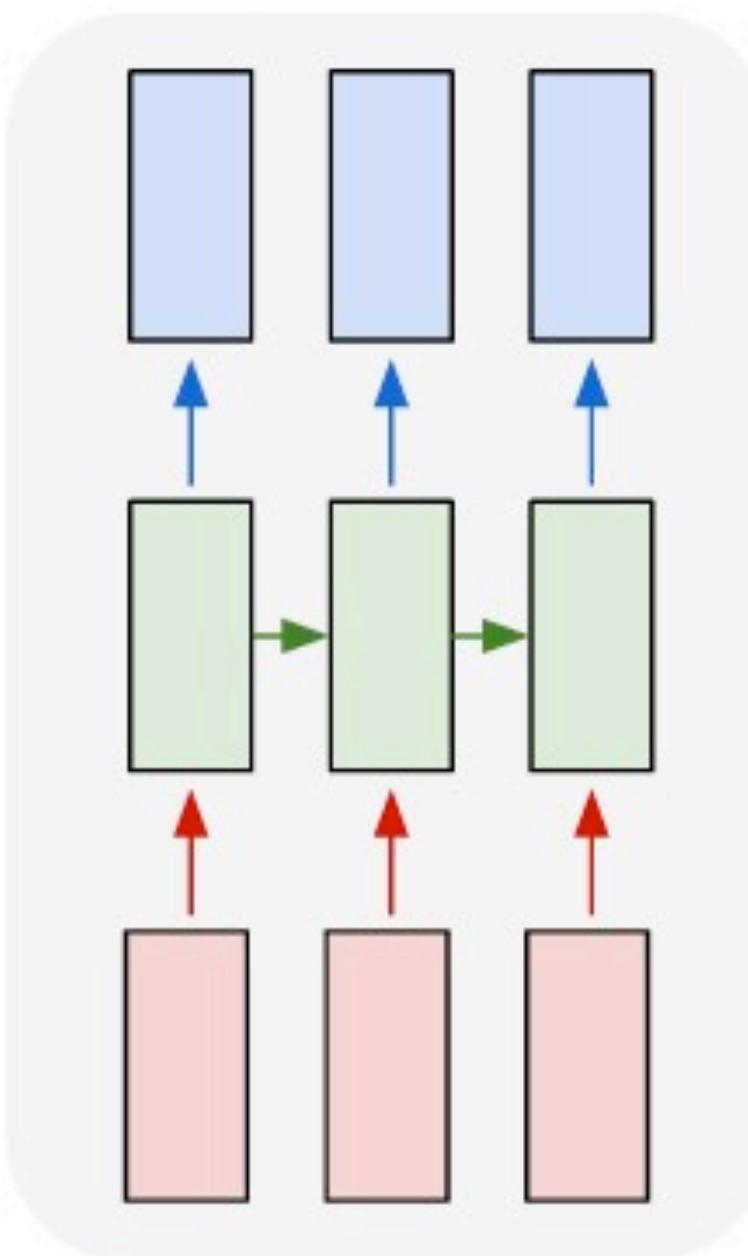
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**Example:** sentence  
classification,  
multiple-choice  
question answering

**Input:** Sequence

**Output:** Sequence

**Example:** machine translation, video captioning,  
open-ended question answering, video question  
answering

# Key Conceptual Ideas

## Parameter Sharing

- in computational graphs = adding gradients

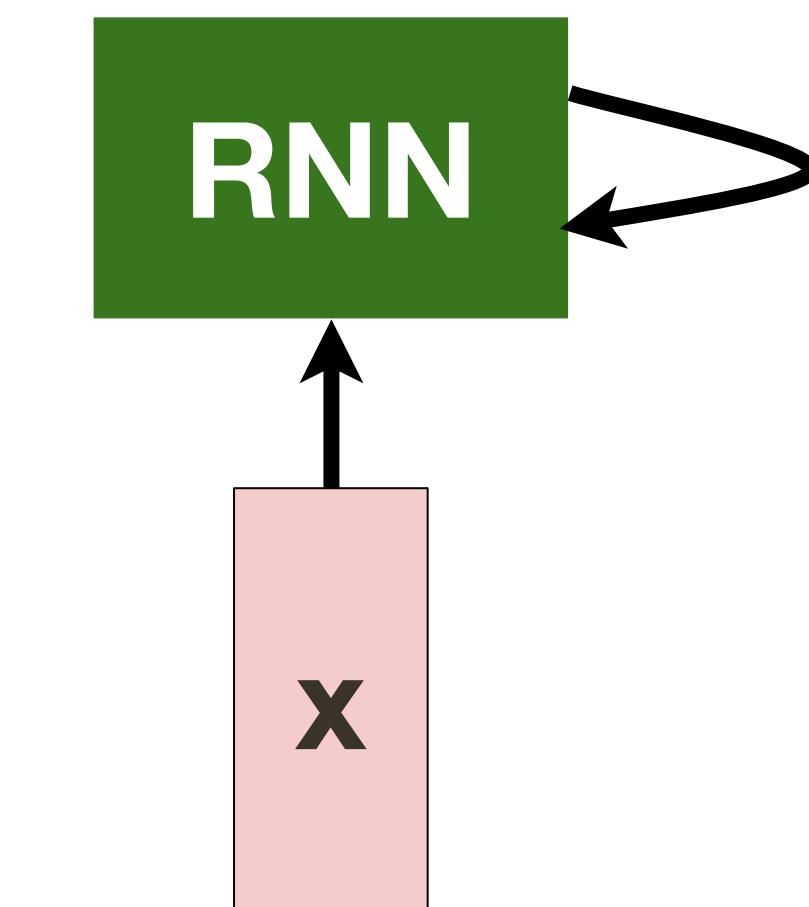
## “Unrolling”

- in computational graphs with parameter sharing

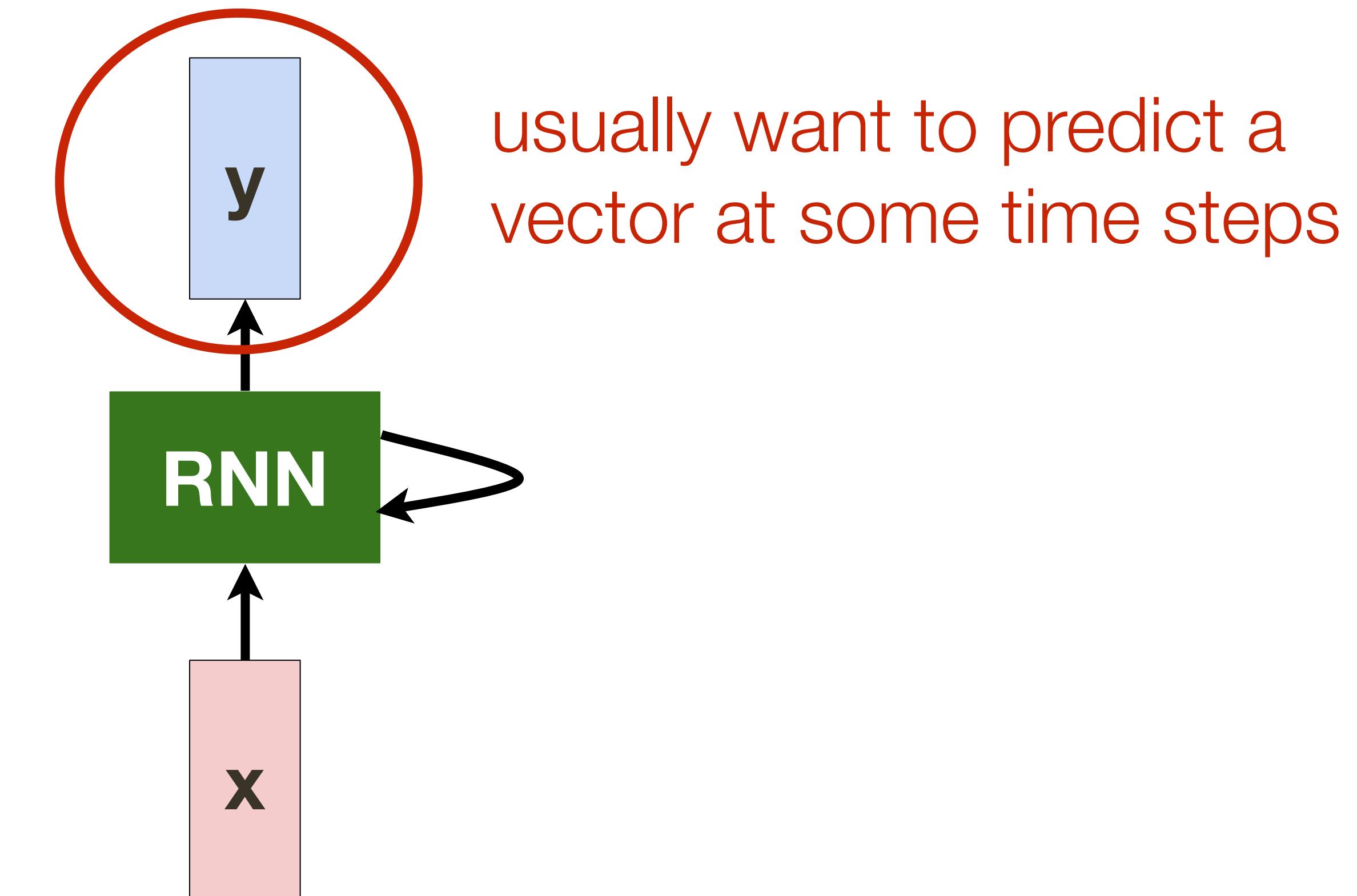
## Parameter Sharing + “Unrolling”

- Allows modeling **arbitrary length sequences!**
- Keeps number of parameters in check

# Recurrent Neural Network



# Recurrent Neural Network



# Recurrent Neural Network

We can process a sequence of vectors  $\mathbf{x}$  by applying a **recurrence formula** at every time step:

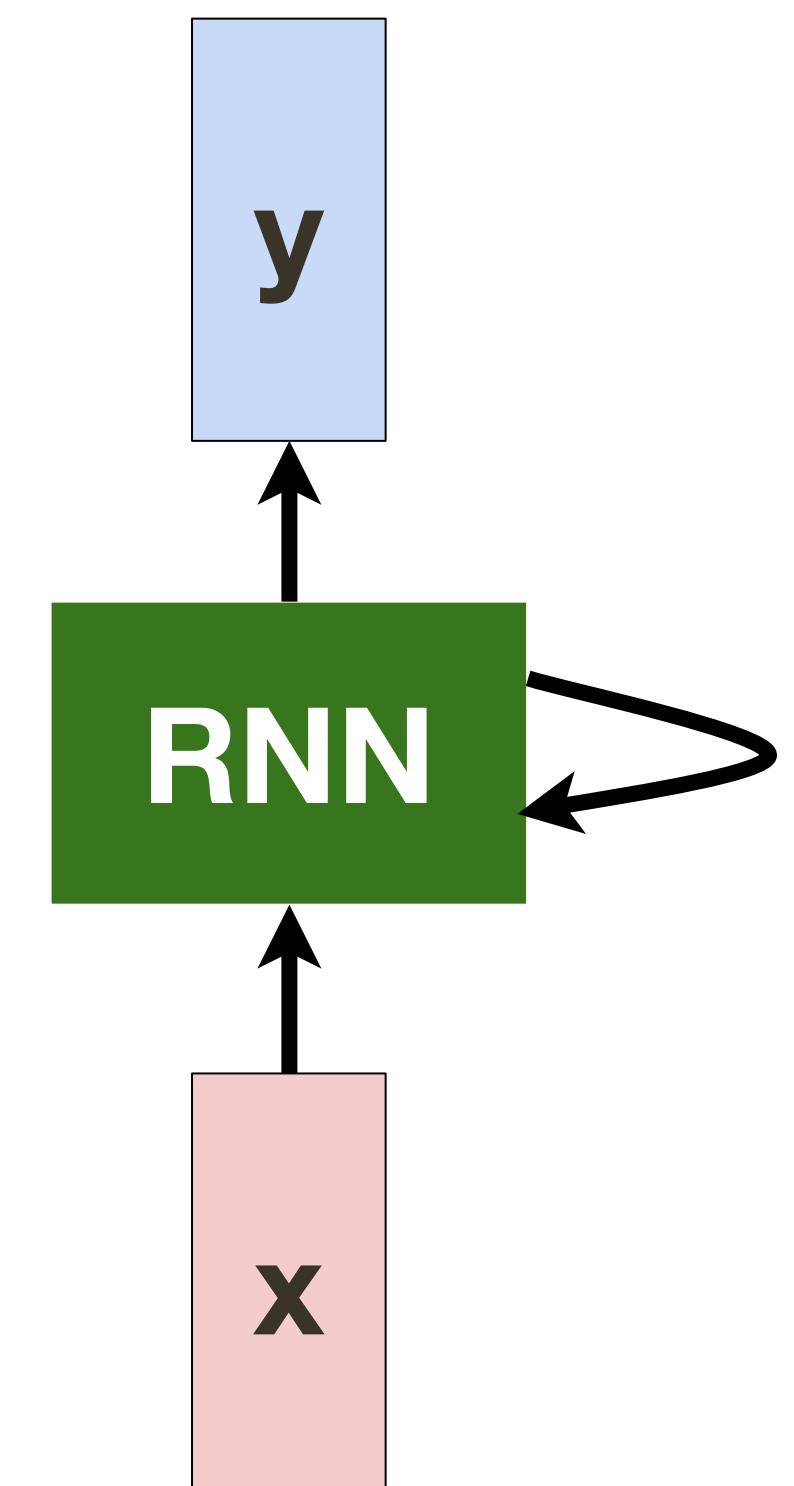
$$h_t = f_W(h_{t-1}, x_t)$$

new **state**

old **state**

some **function**  
with parameters W

input vector at  
some time step

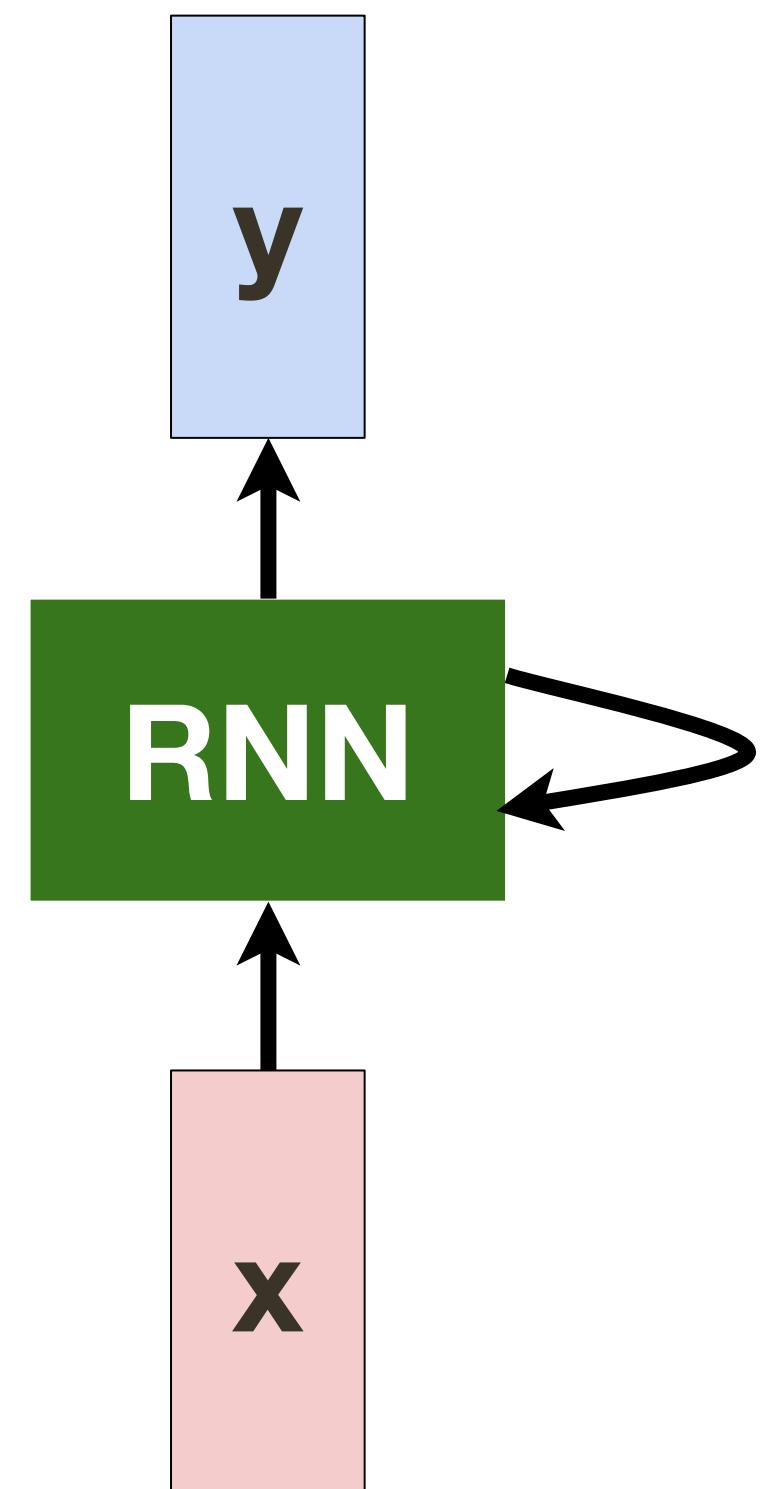


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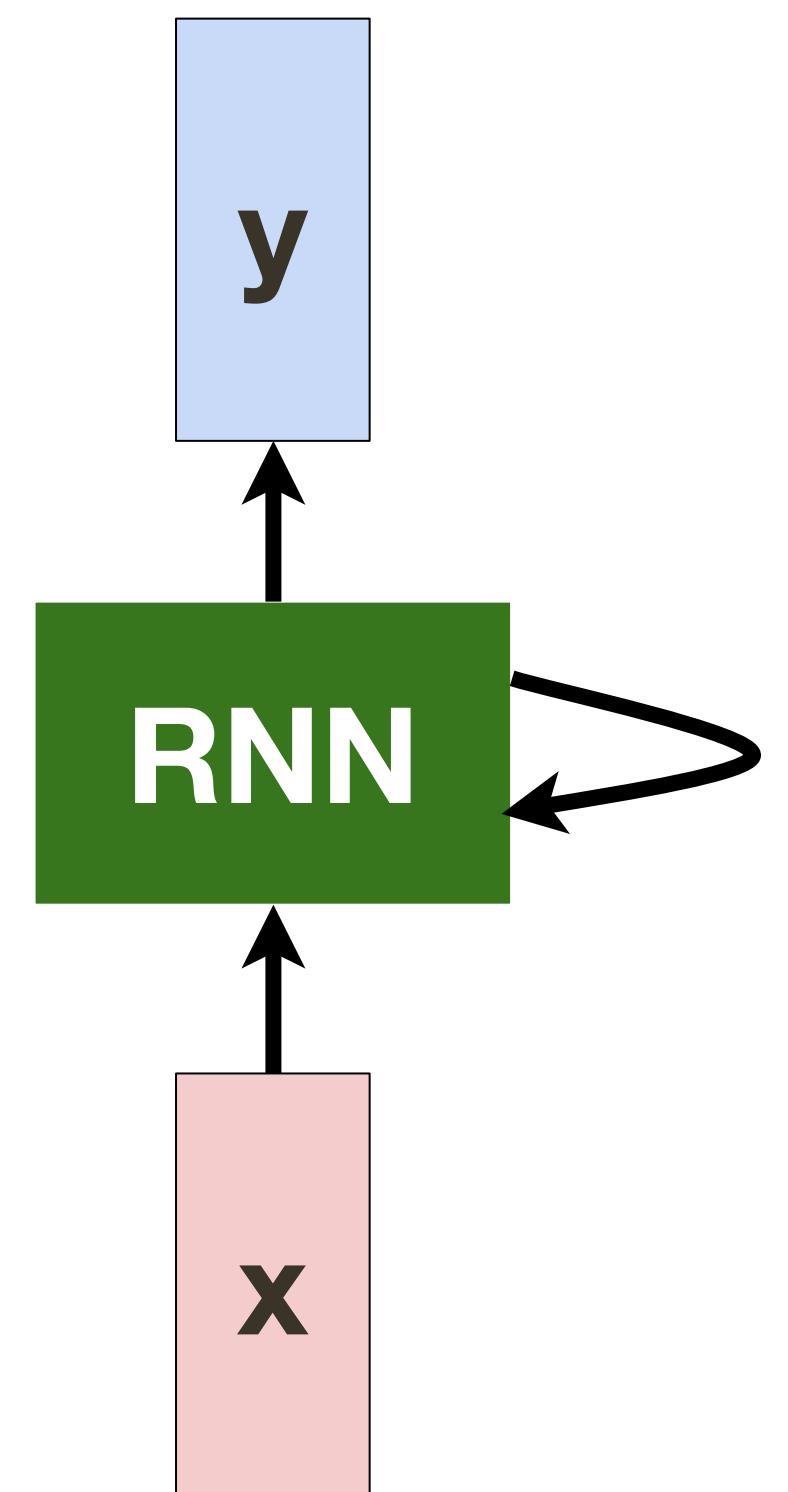
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**Note:** the same function and the same set of parameters are used at every time step



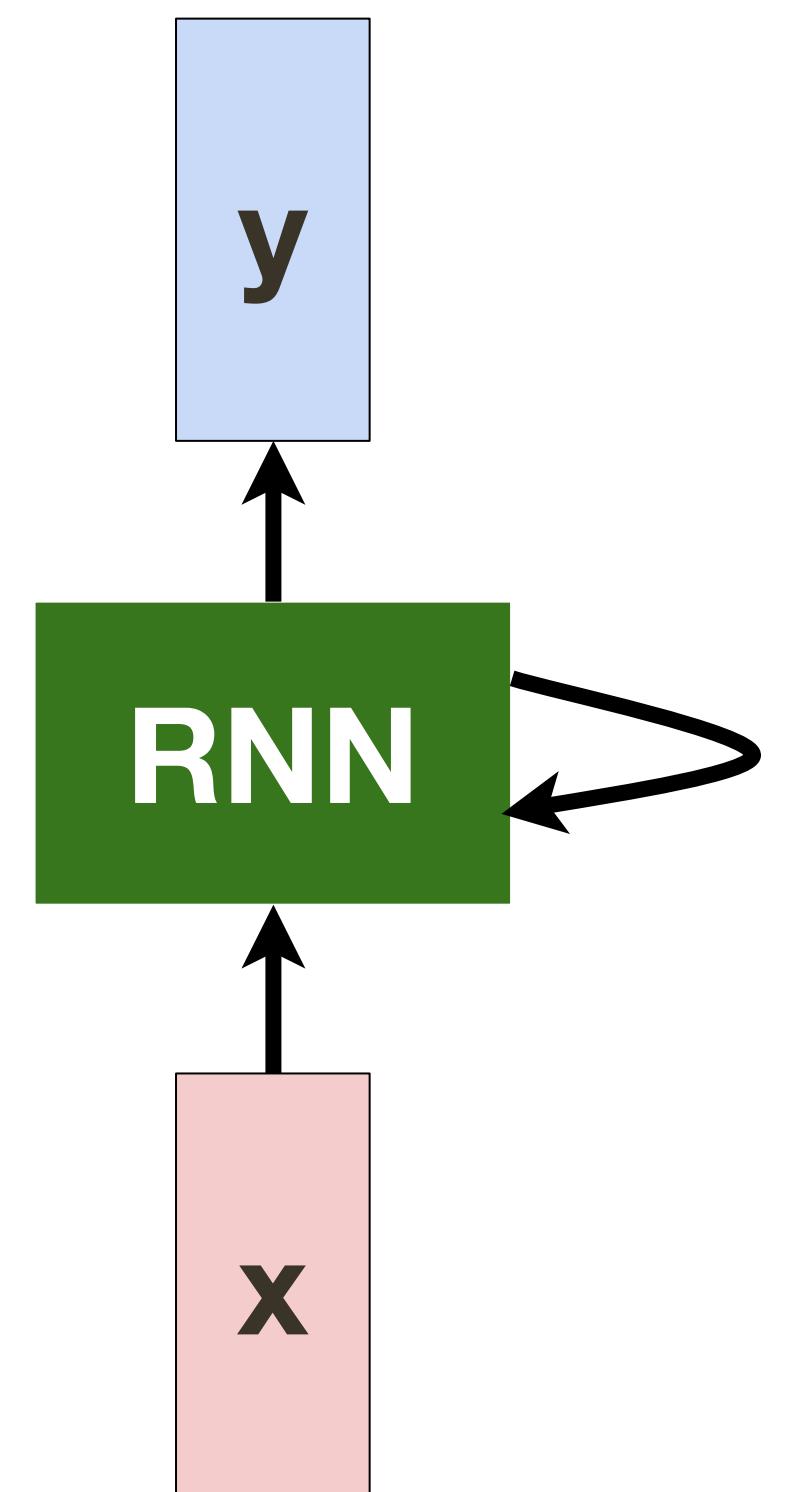
# (Vanilla) Recurrent Neural Network

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$$h_t = f_W(h_{t-1}, x_t)$$
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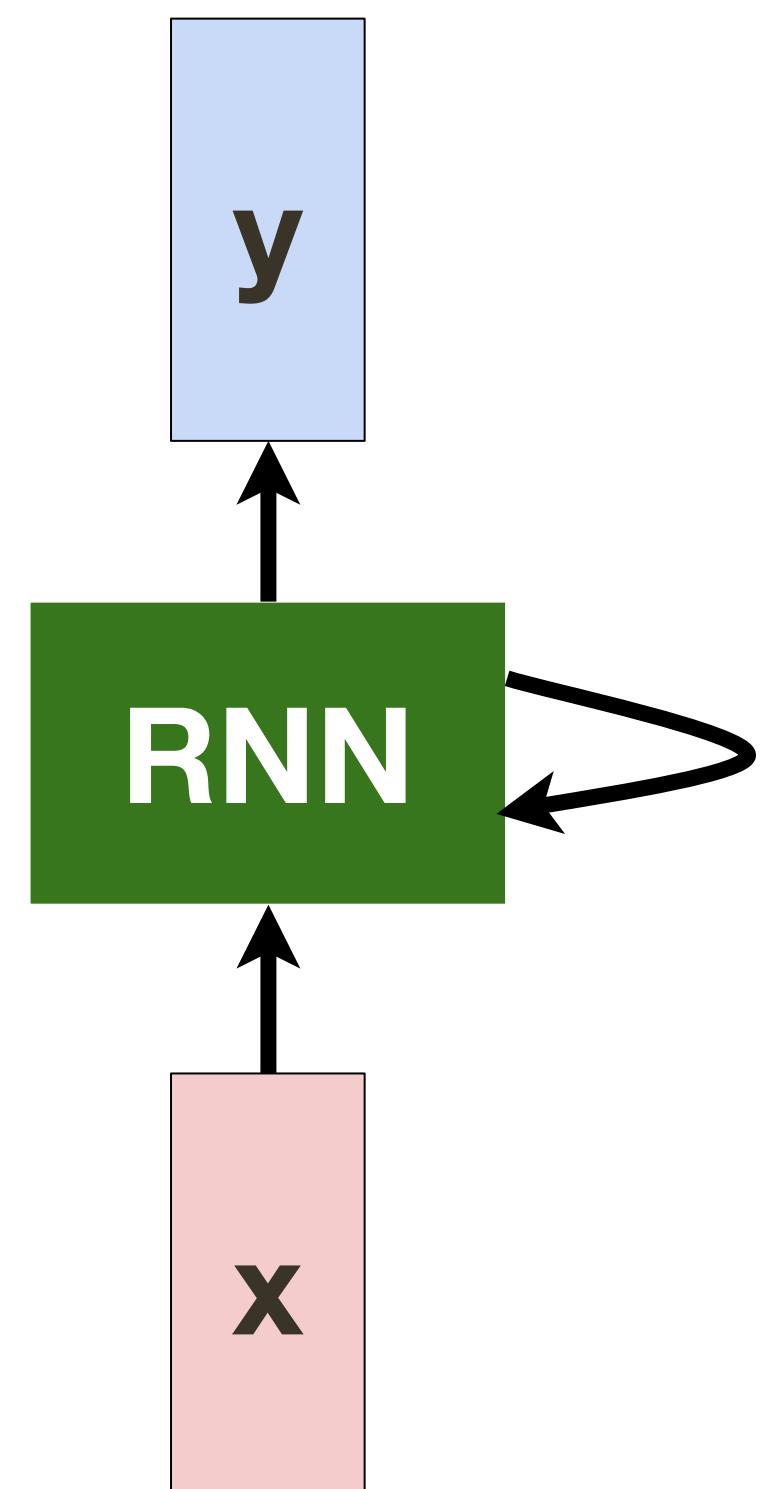
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$$y_t = W_{hy} h_t + b_y$$

$$h_t = f_W(h_{t-1}, x_t)$$

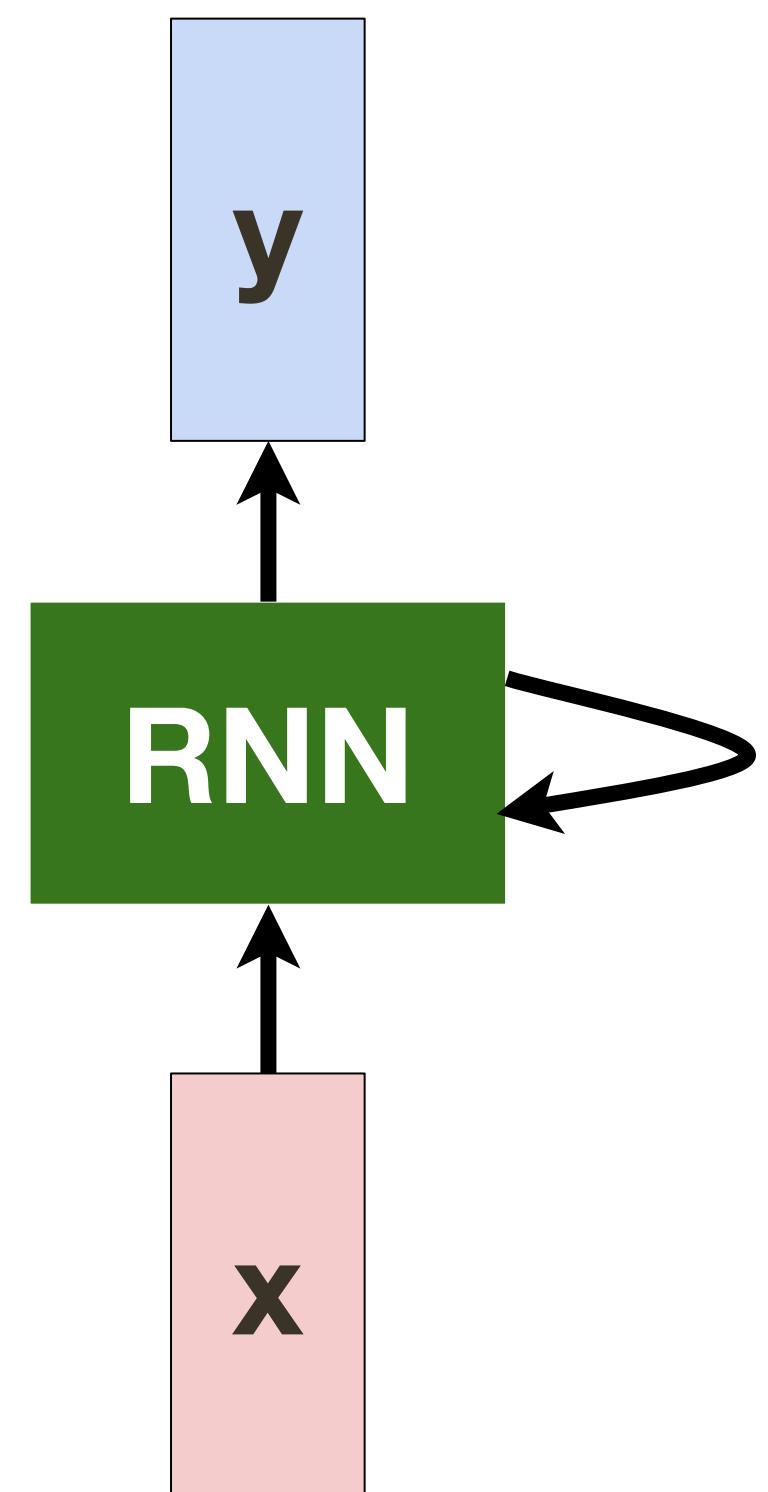


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**Intuition:** RNN incorporates one element of sequence at a time  
(e.g. letter, word, video frame, etc.)  
building up a representation of the sequence “so far”



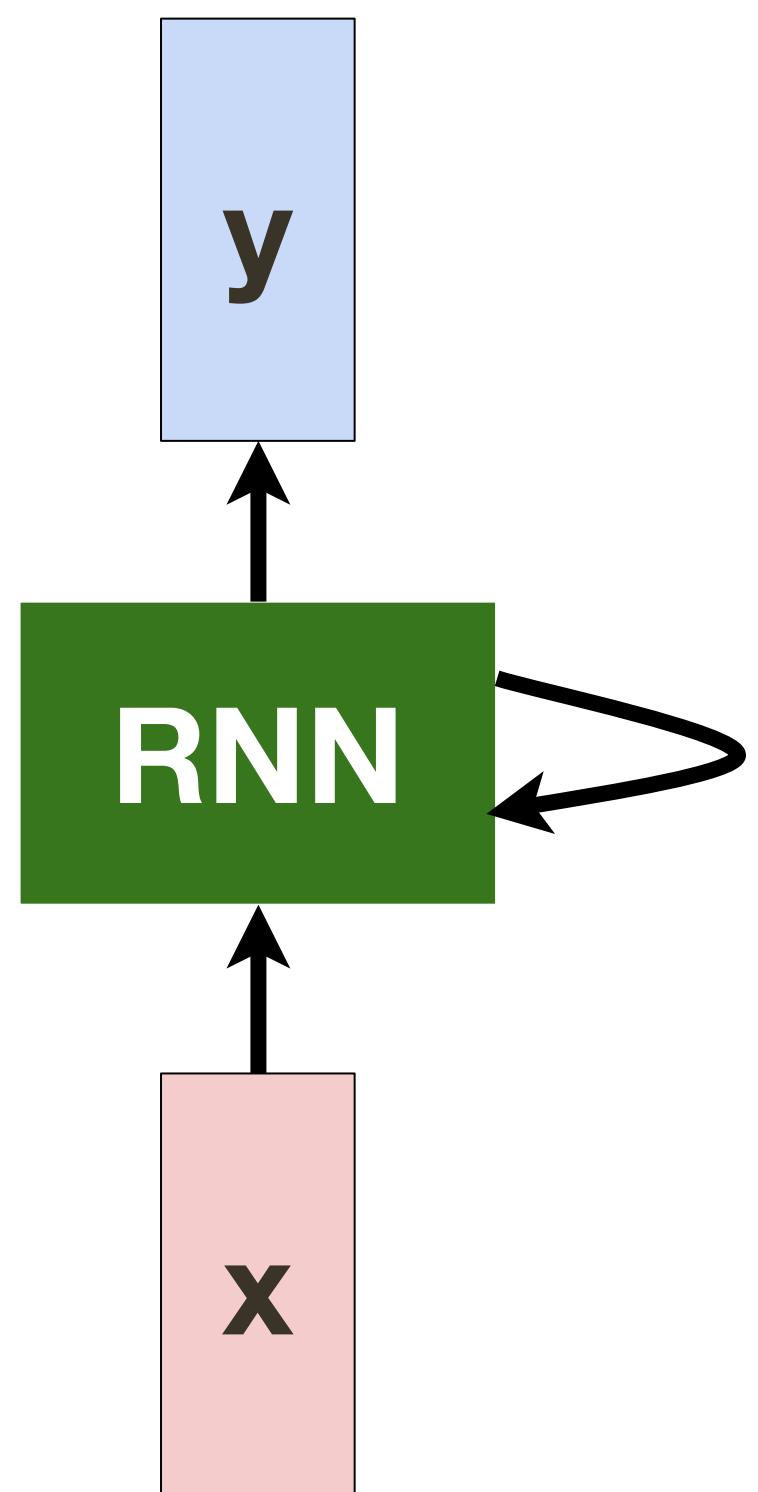
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**Alternative:** RNN computes a representation of sequence element  
(e.g. letter, word, video frame, etc.)  
with context provided by all previous processed elements

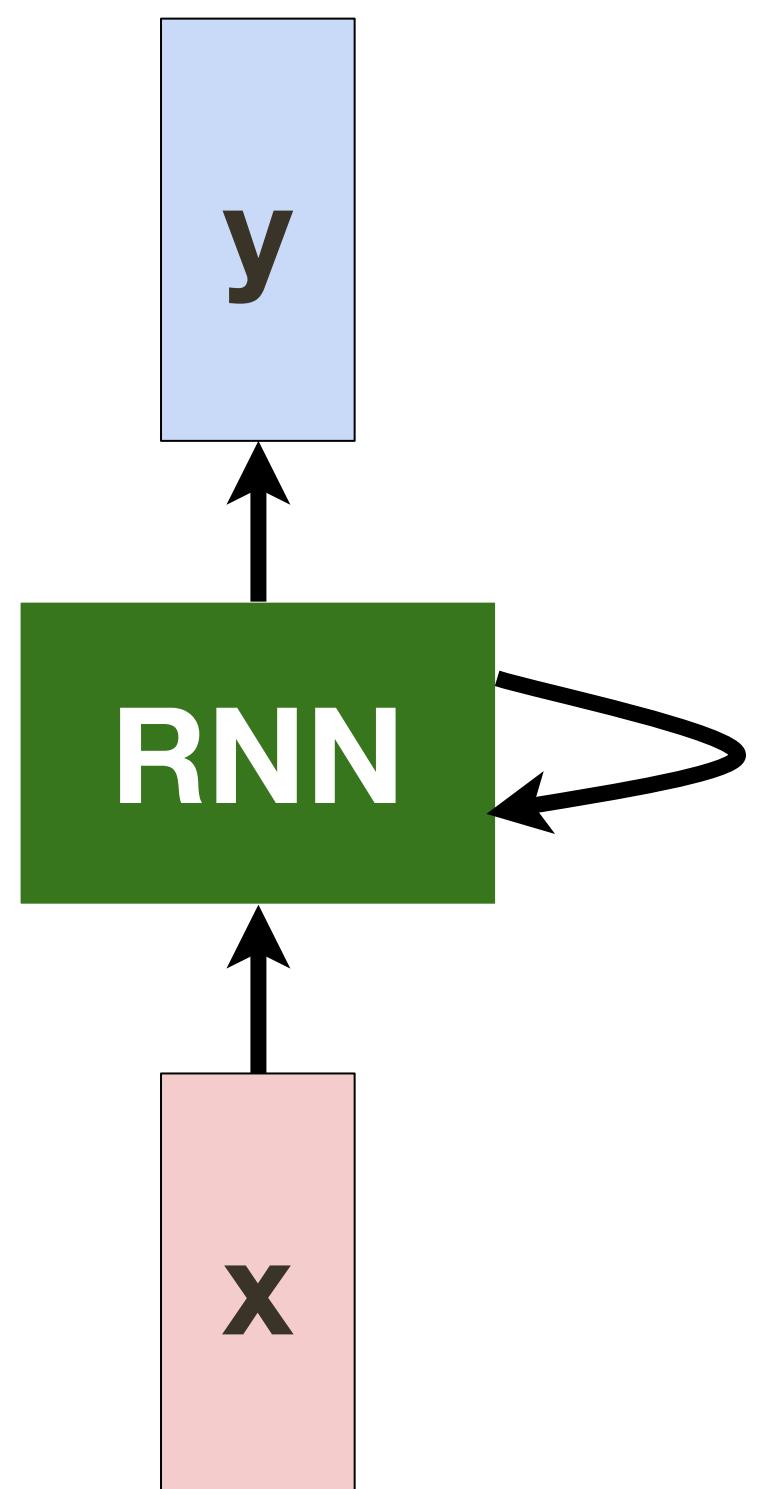
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cat	2	[ 0, 1, 0, 0, 0, 0, 0, 0, 0, 0 ]
person	3	[ 0, 0, 1, 0, 0, 0, 0, 0, 0, 0 ]
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person holding dog



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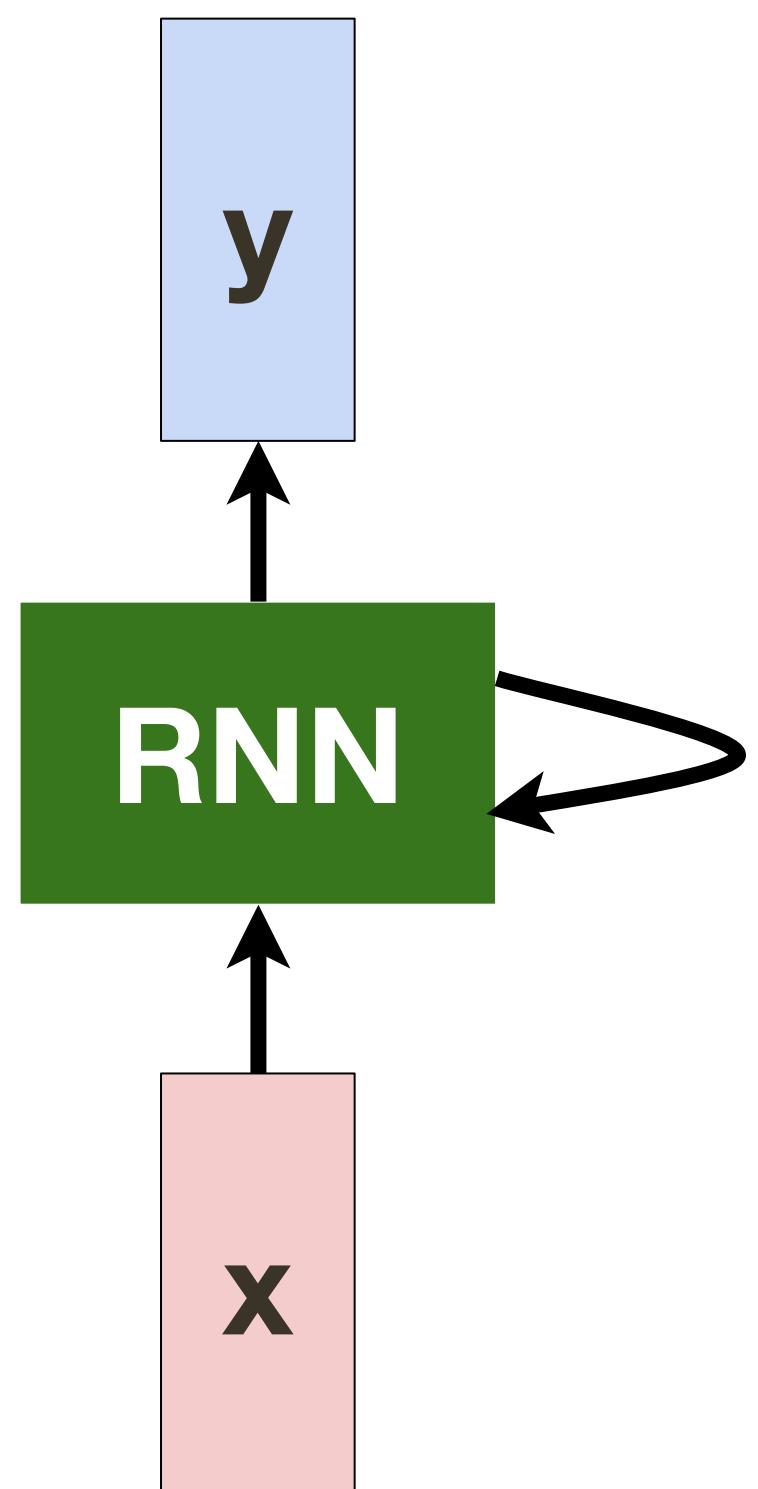
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person holding dog

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**Identity**      **Identity**      **zero**

A large green X is drawn over the terms  $W_{hh}h_{t-1}$ ,  $W_{xh}x_t$ , and  $b_h$ .



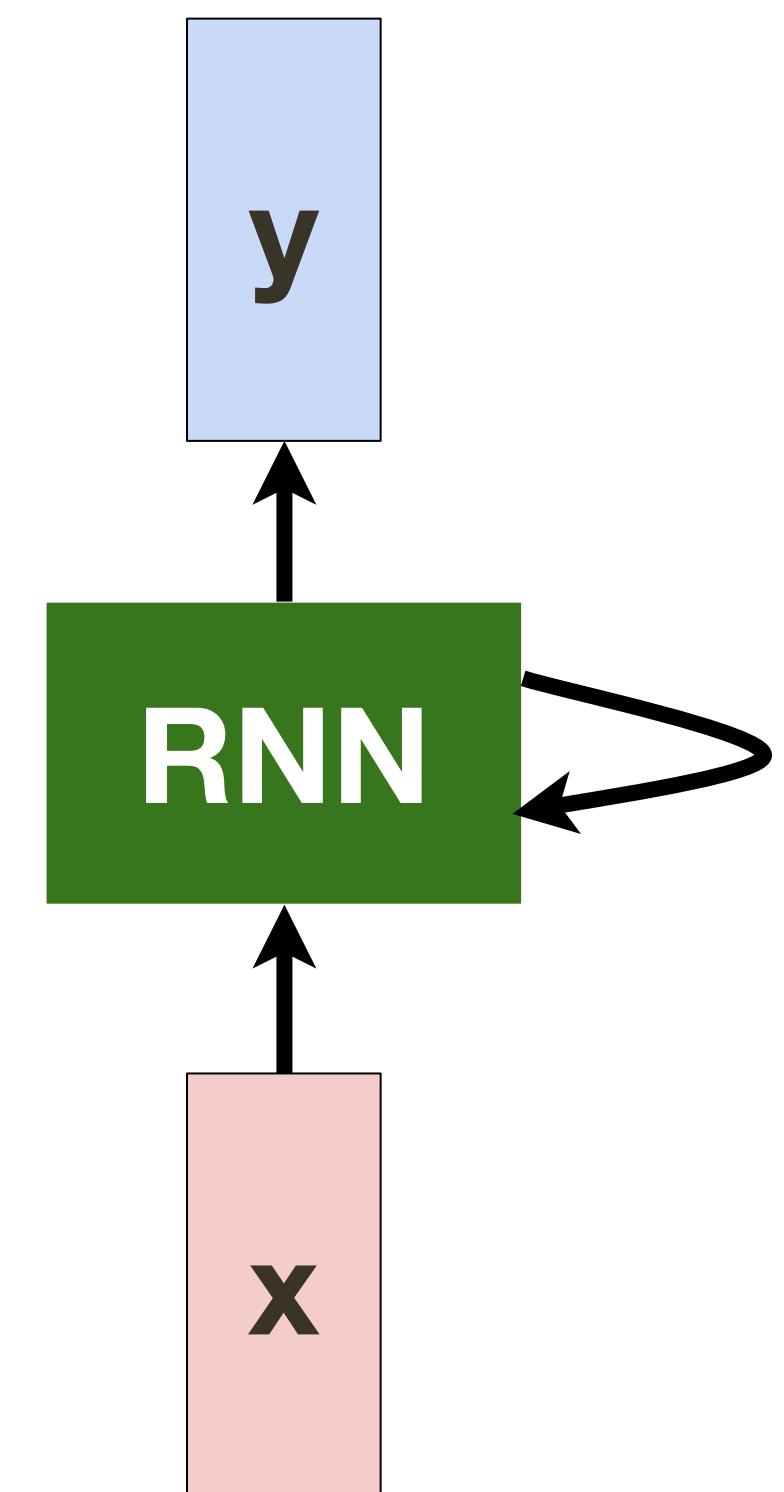
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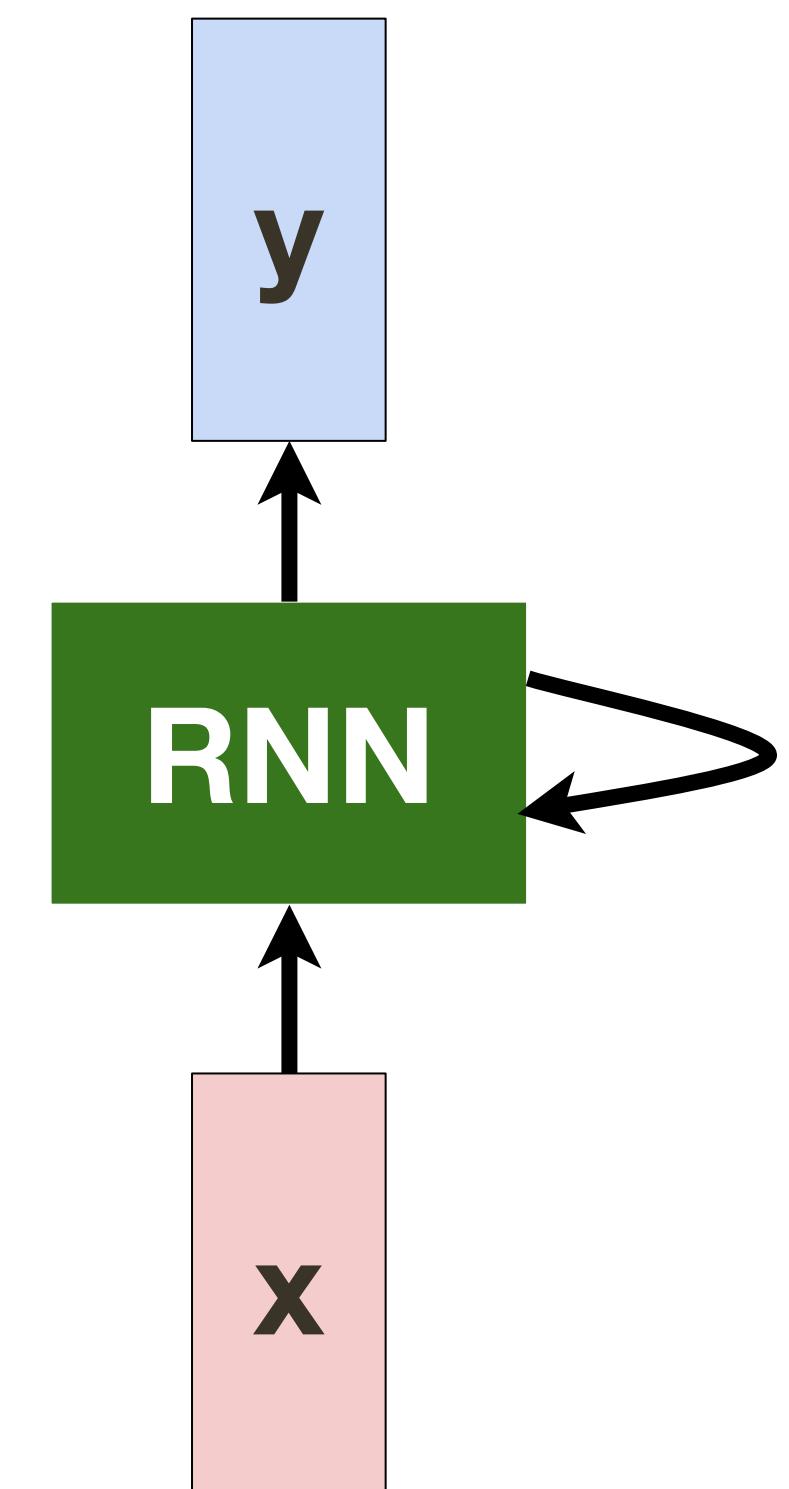
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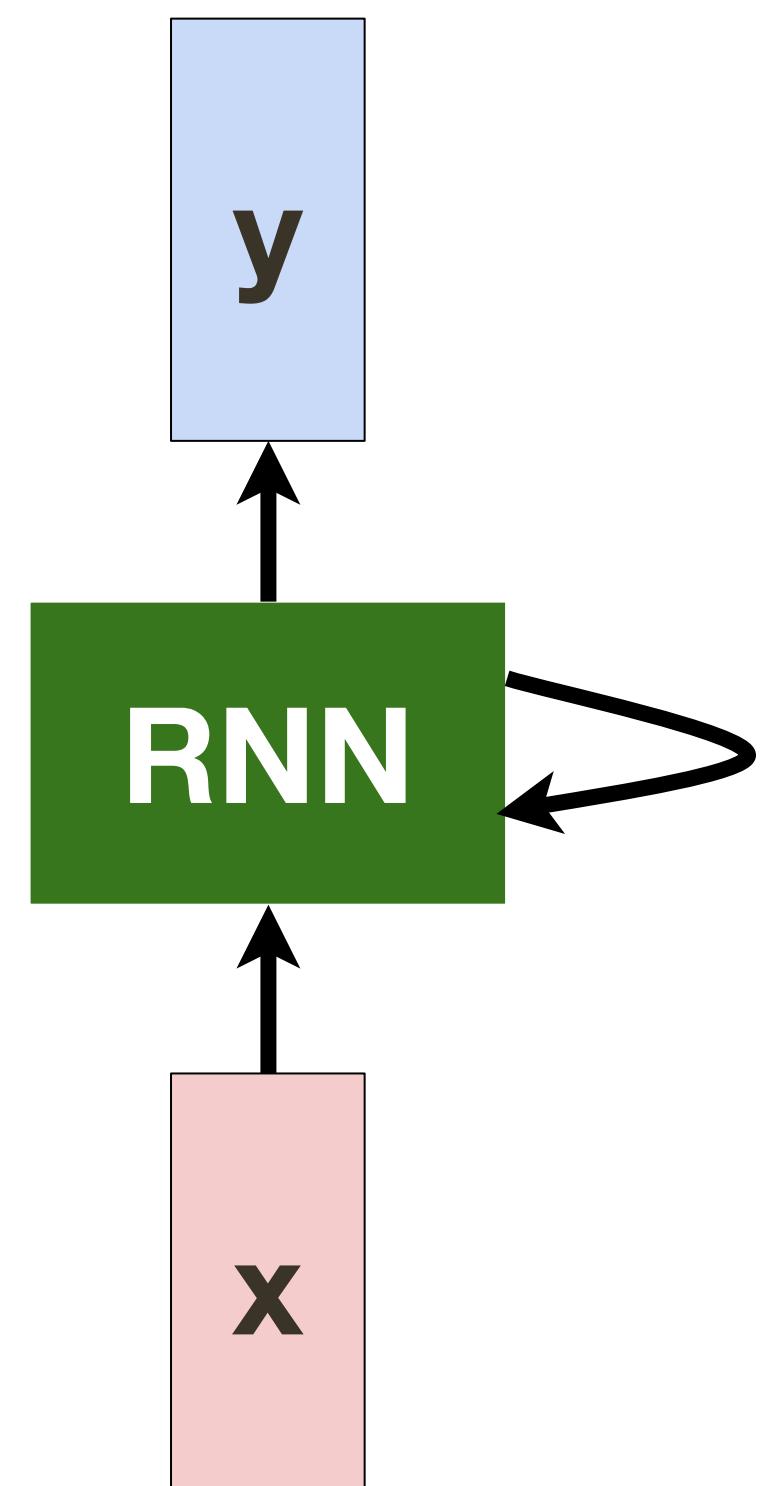
[ 0, 0, 0.76, 0, 0, 0, 0, 0, 0 ]

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**Identity**  ~~$W_{hh}$~~  **zero**  ~~$b_h$~~

[ 0, 0, 0, 0, 0, 0, 0, 0, 0 ]



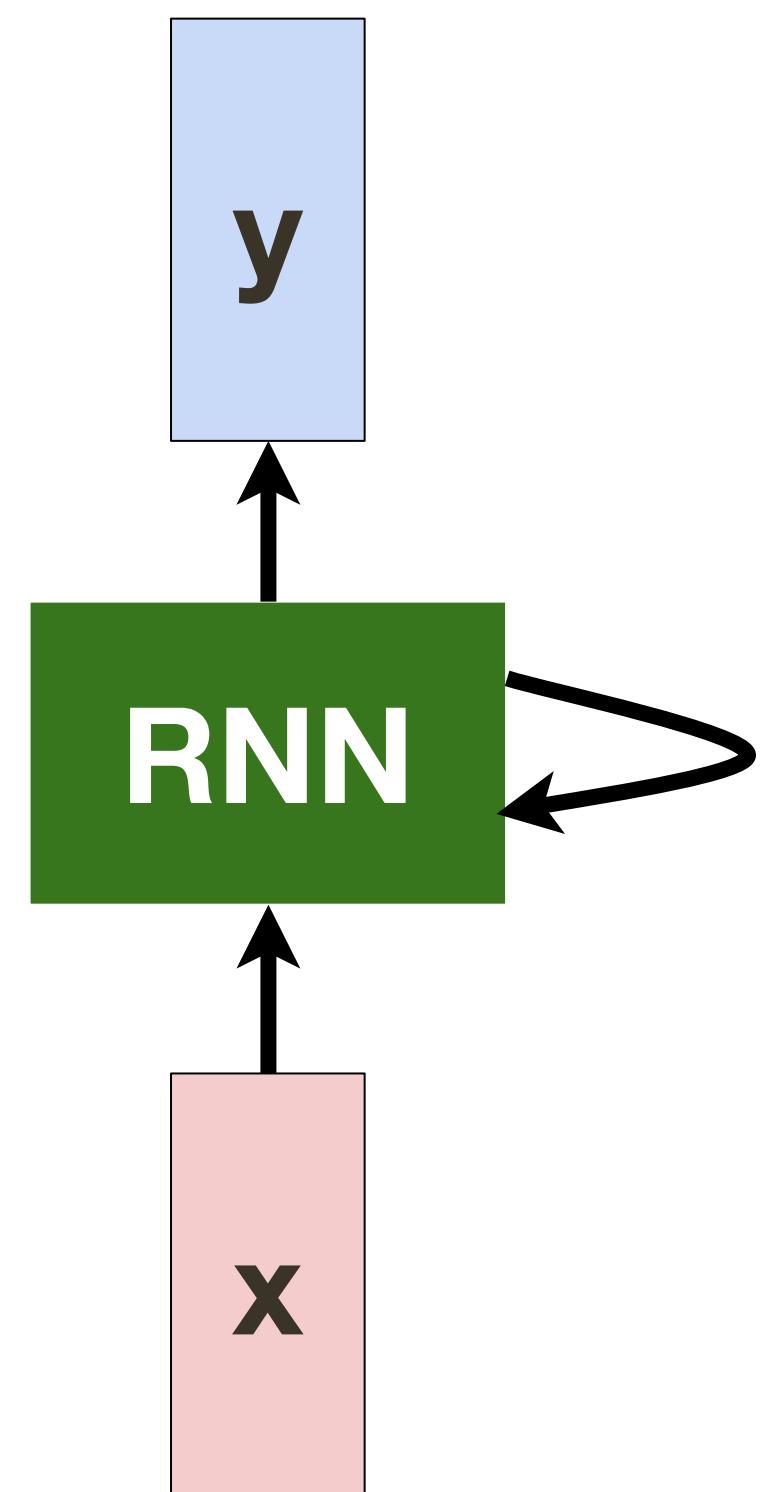
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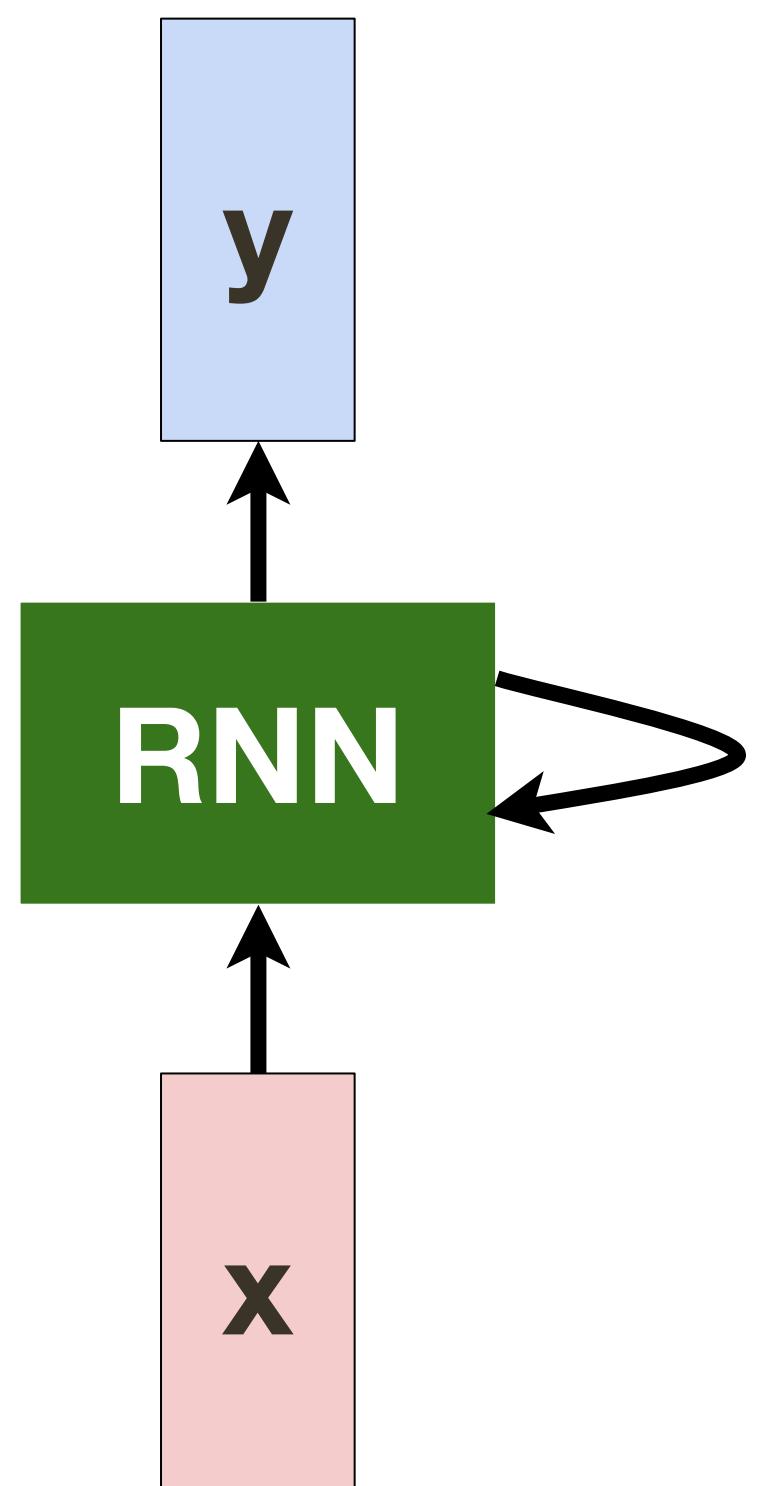
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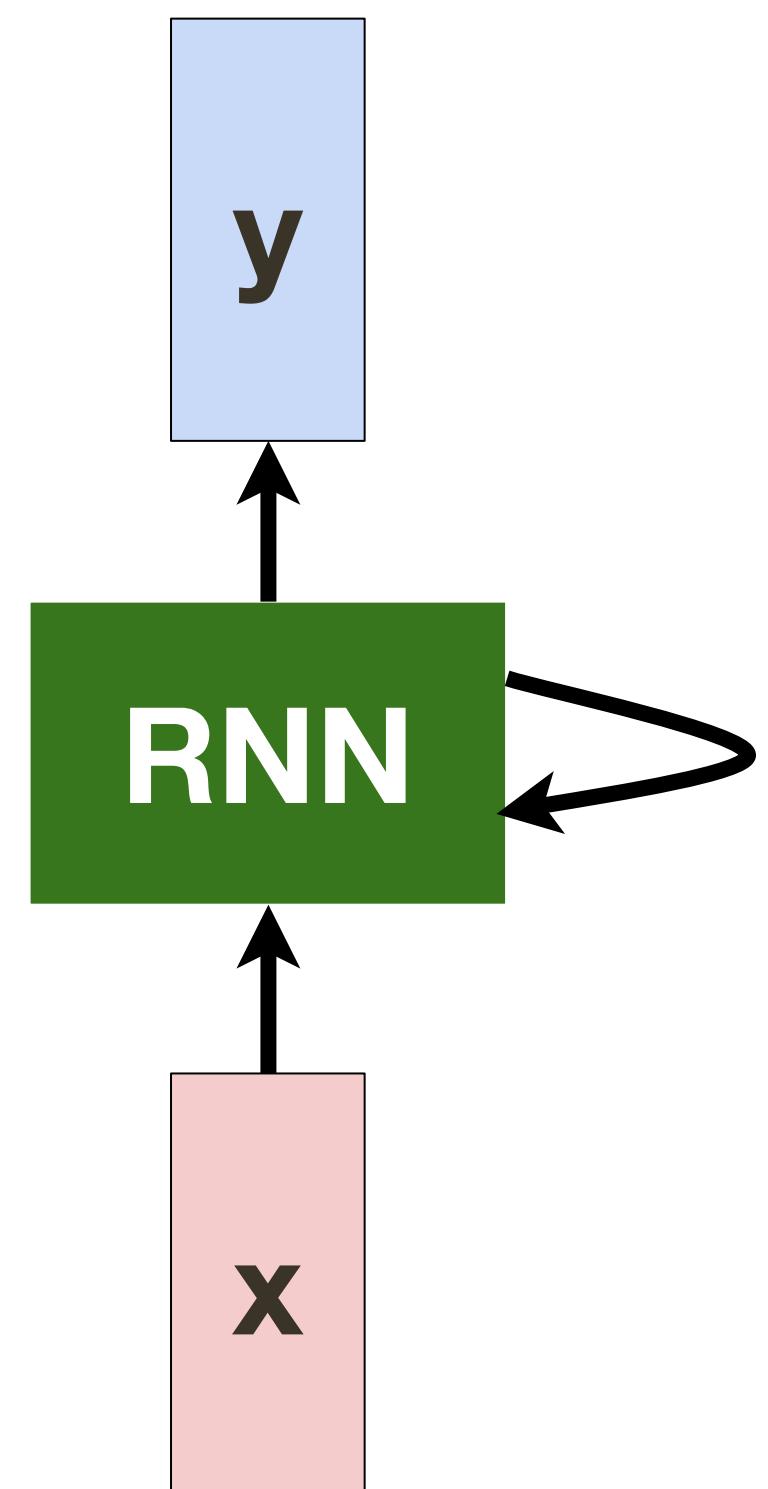
[ 0, 0, **0.64**, **0.76**, 0, 0, 0, 0, 0, 0 ]

[ 0, 0, 0, **1**, 0, 0, 0, 0, 0, 0 ]

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Like bag of words with some  
notion of recency

[ 0, 0, 0.64, 0.76, 0, 0, 0, 0, 0 ]

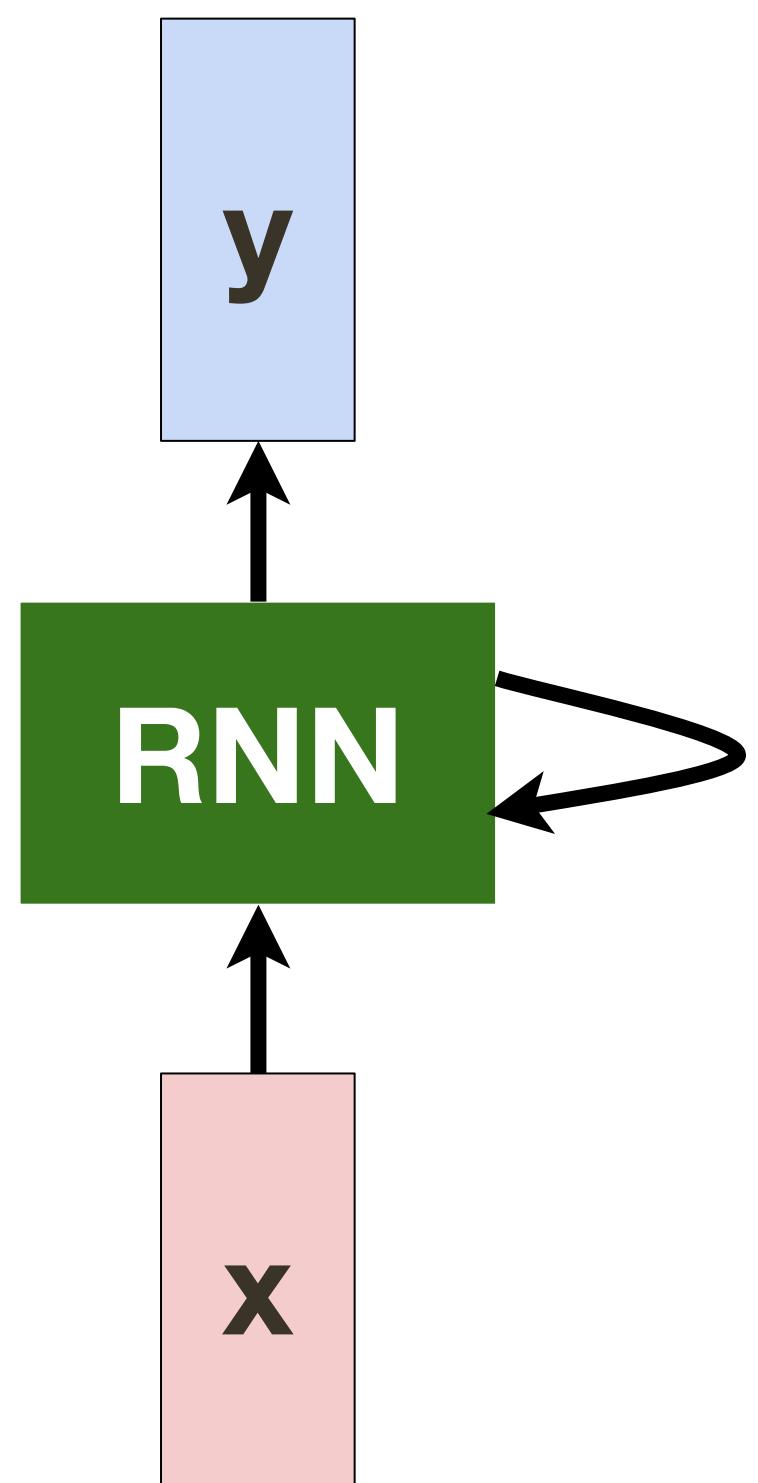
[ 0, 0, 0, 1, 0, 0, 0, 0, 0 ]

$$h_t = \tanh(W_{hh} h_{t-1} + W_{xh} x_t + b_h)$$

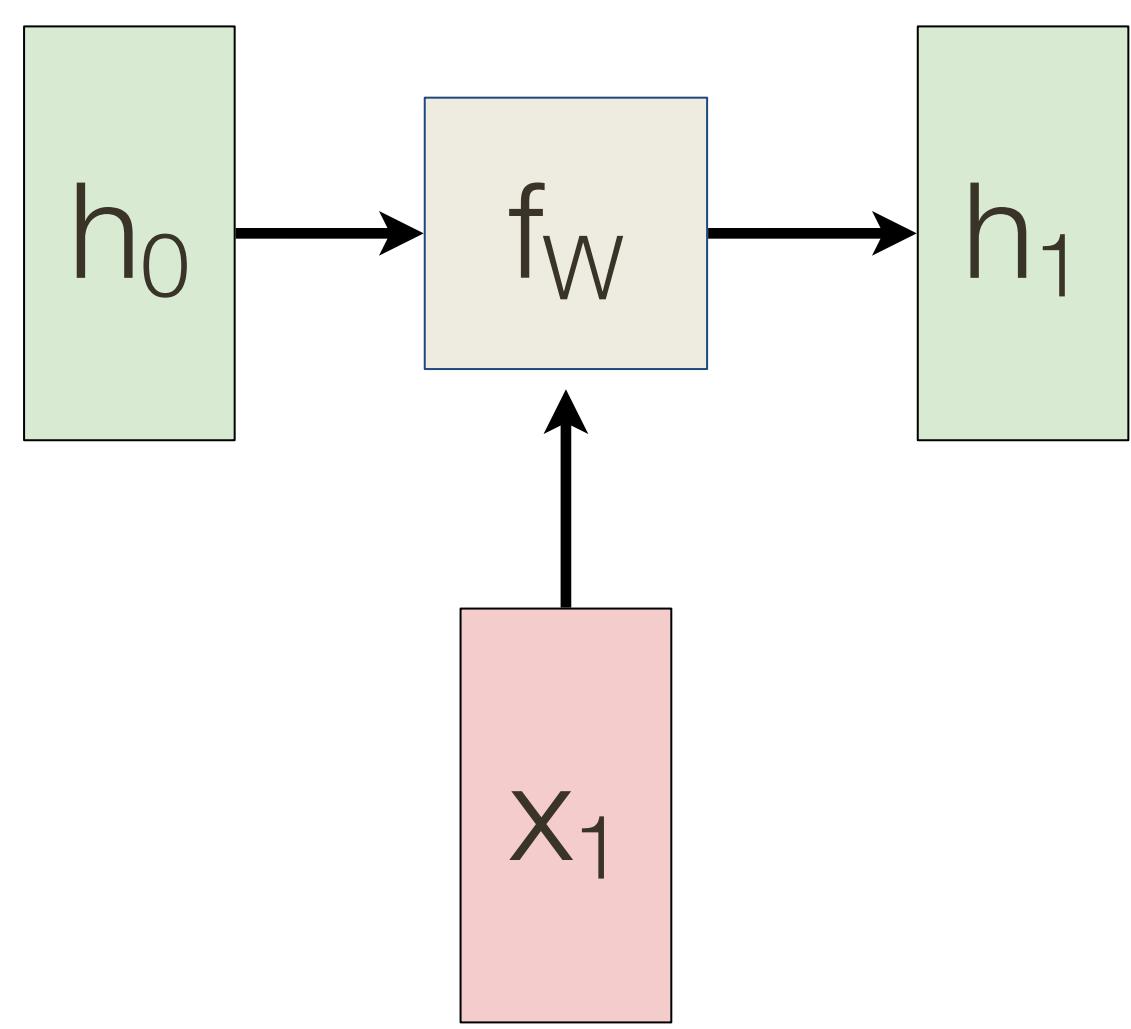
~~Identity~~

~~zero~~

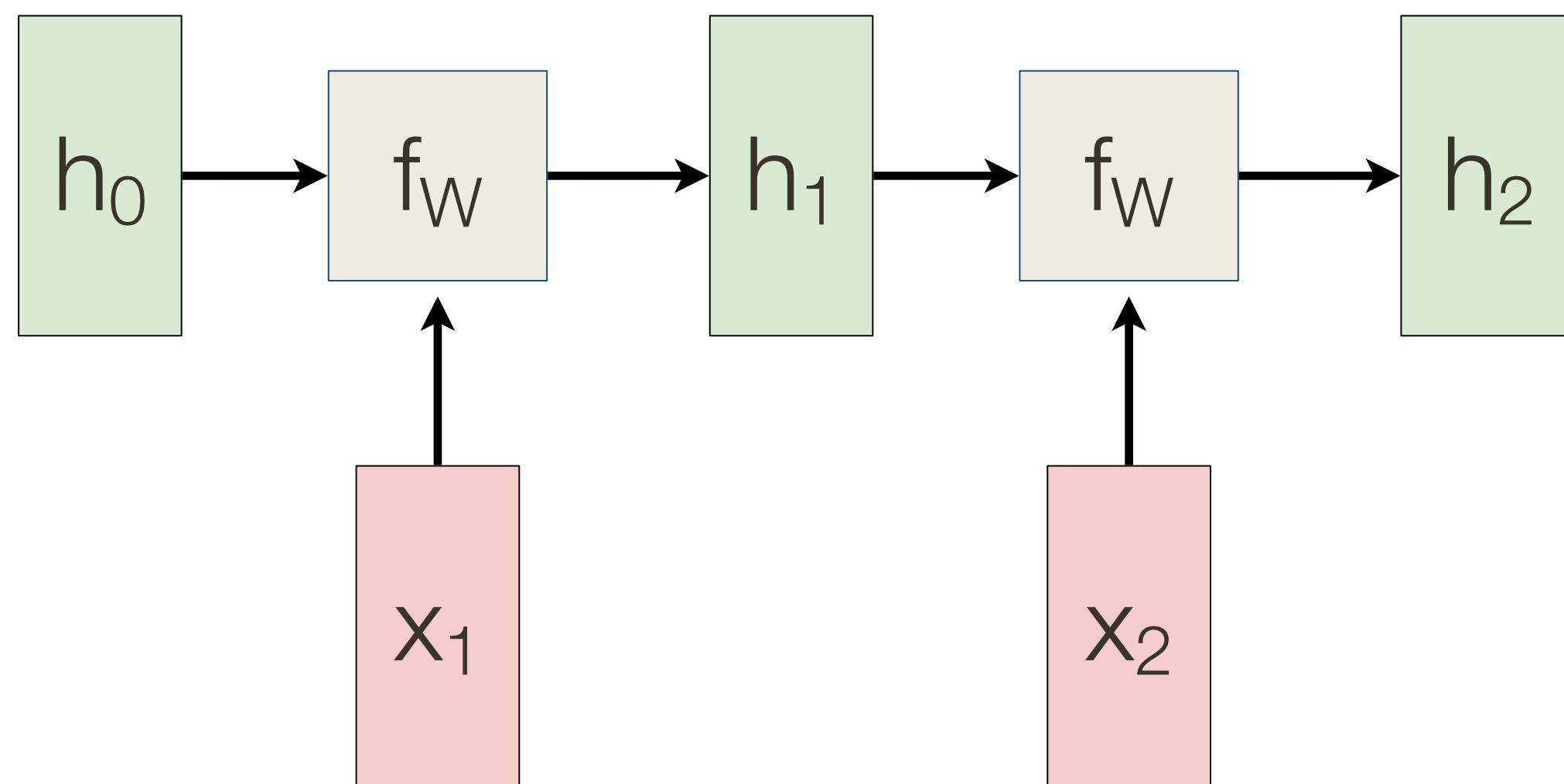
[ 0, 0, 0.76, 0, 0, 0, 0, 0, 0 ]



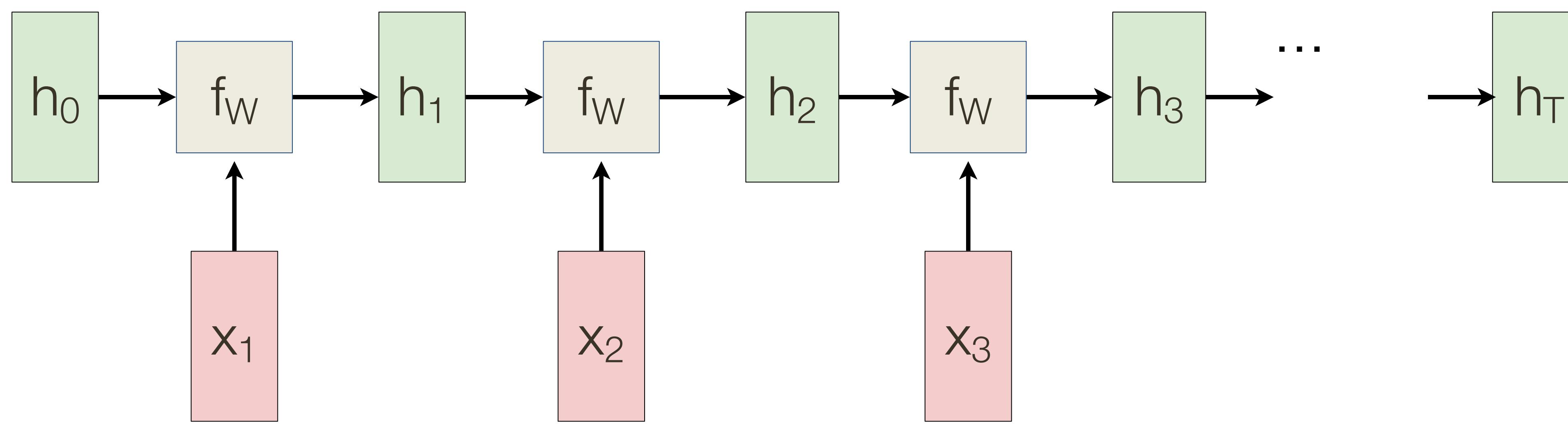
# RNN Computational Graph



# RNN Computational Graph

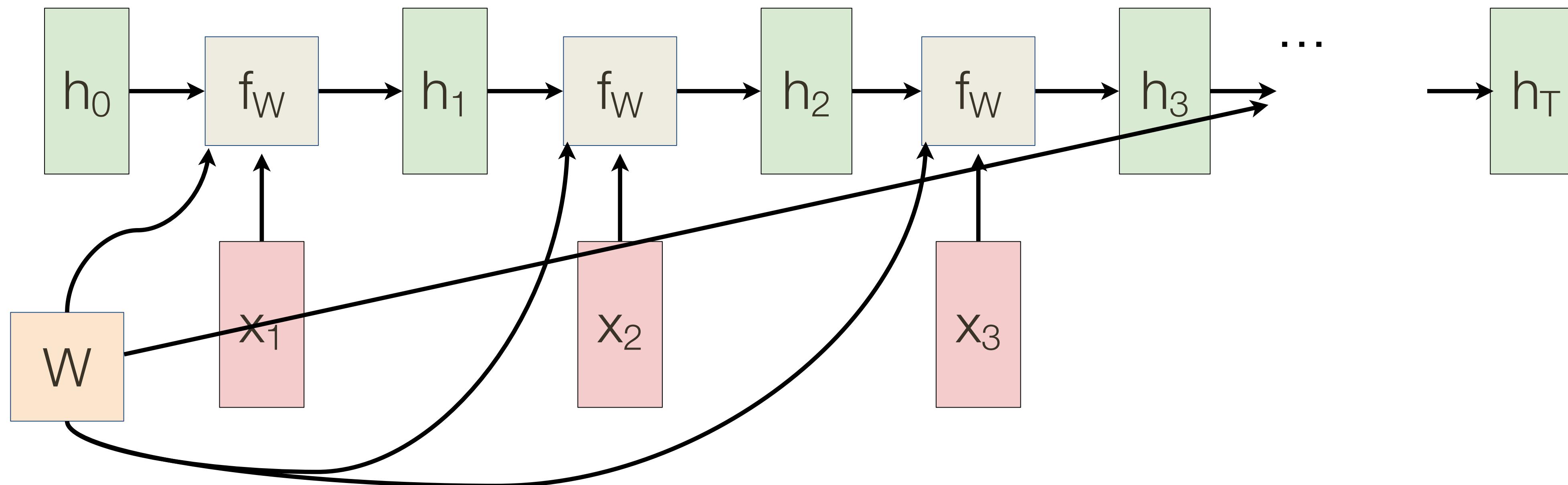


# RNN Computational Graph

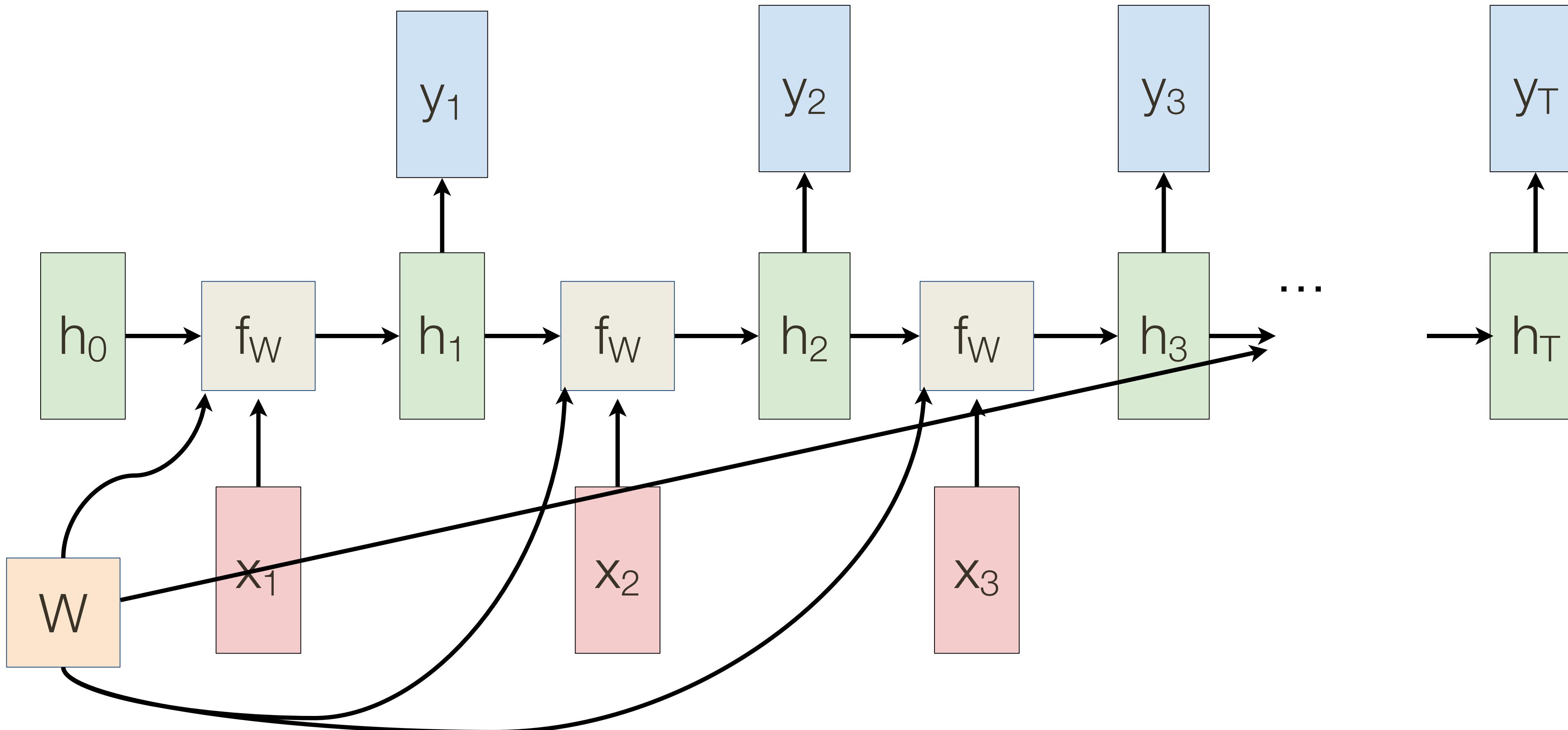


# RNN Computational Graph

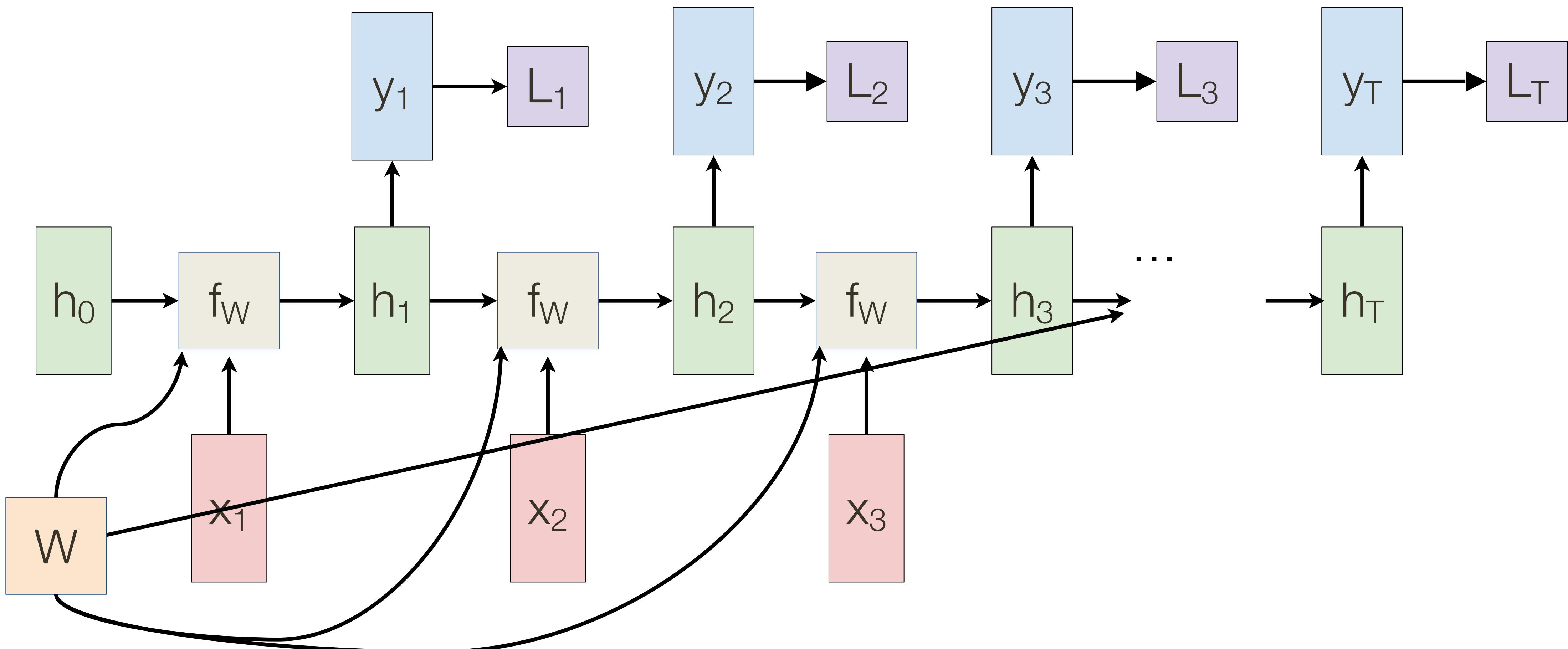
Re-use the same weight matrix at every time-step



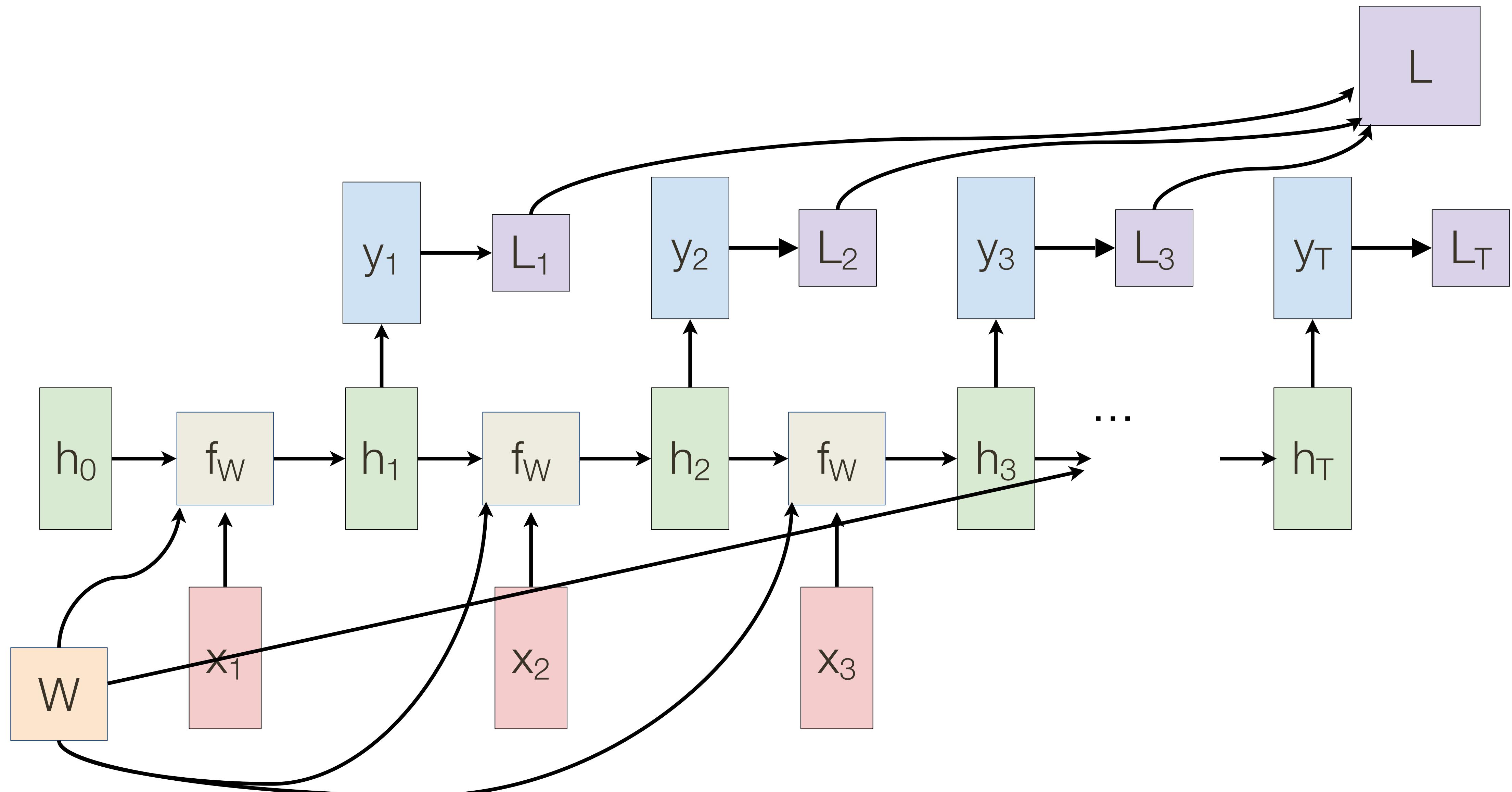
# RNN Computational Graph: Many to Many



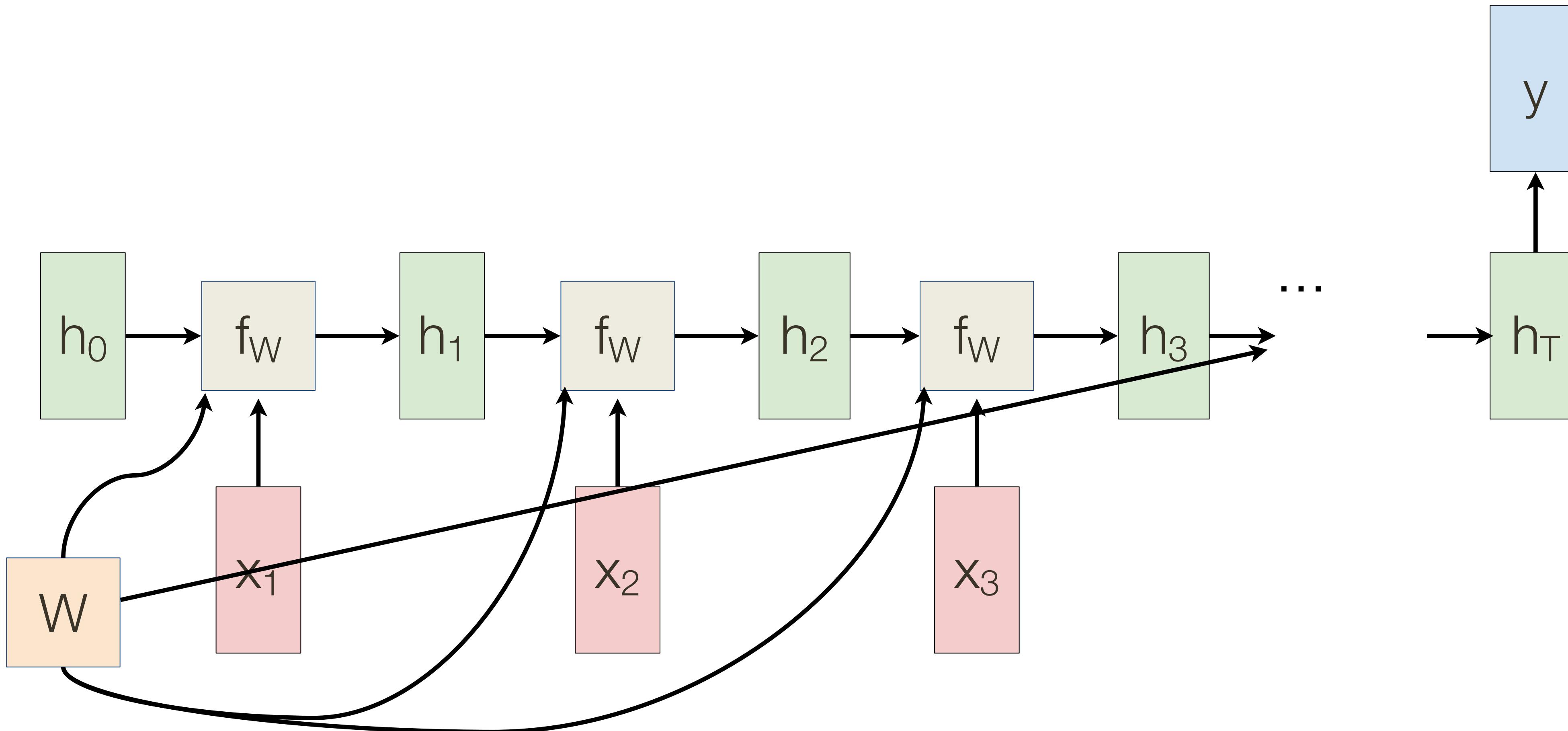
# RNN Computational Graph: Many to Many



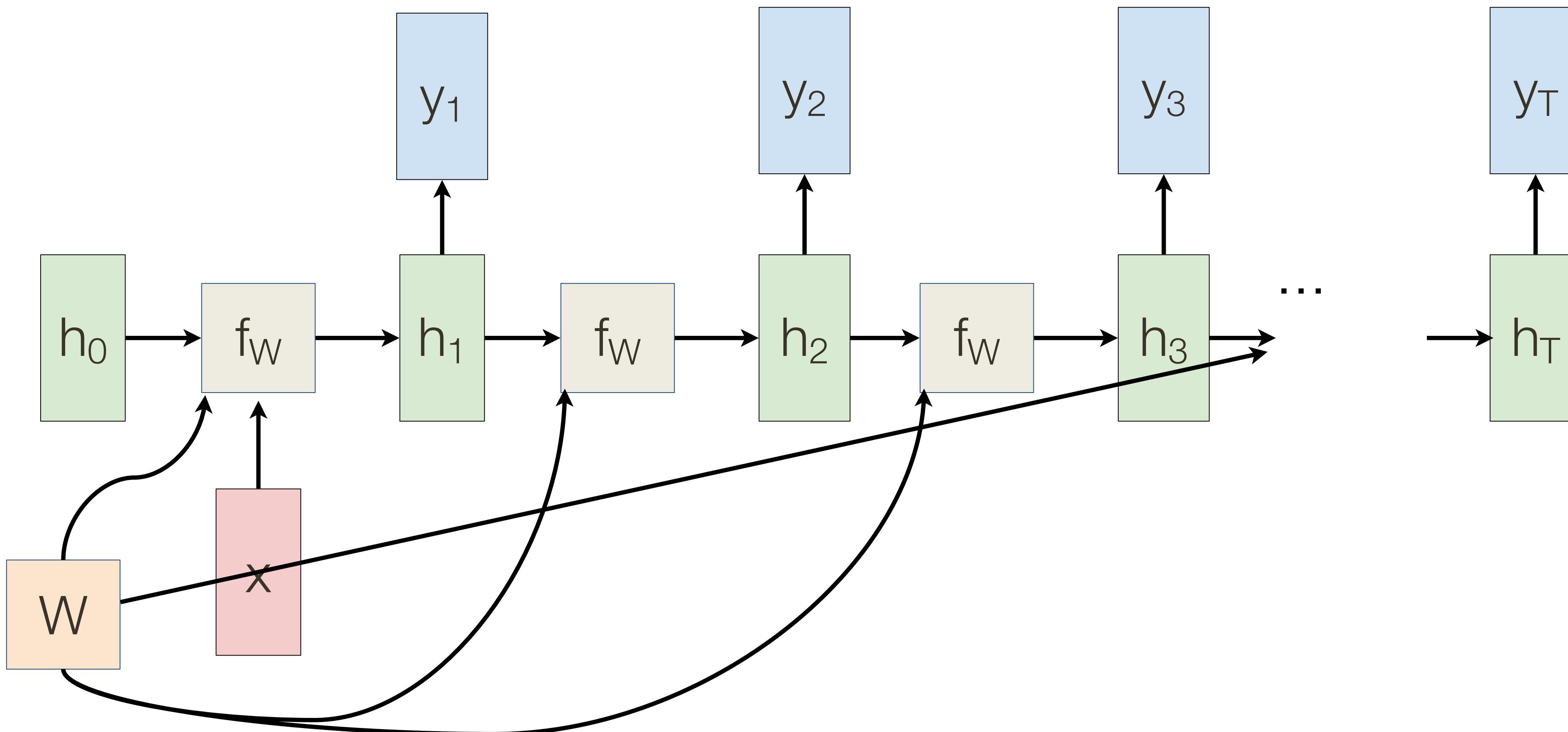
# RNN Computational Graph: Many to Many



# RNN Computational Graph: Many to One

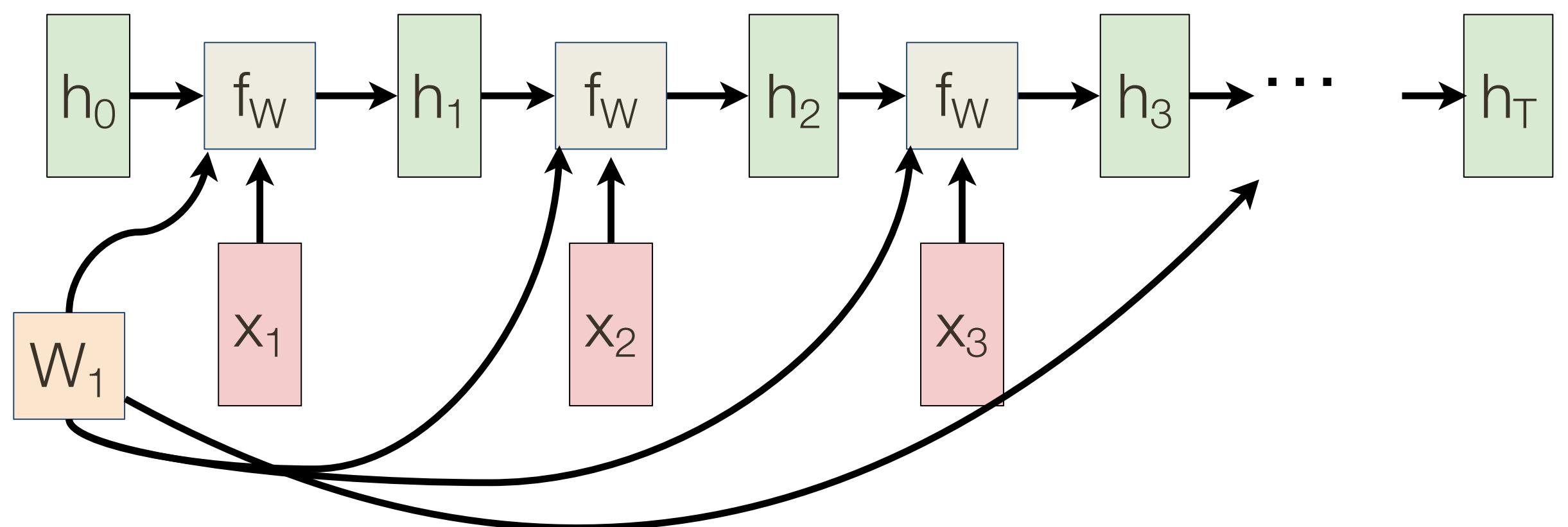


# RNN Computational Graph: One to Many



# Sequence to Sequence: Many to One + One to Many

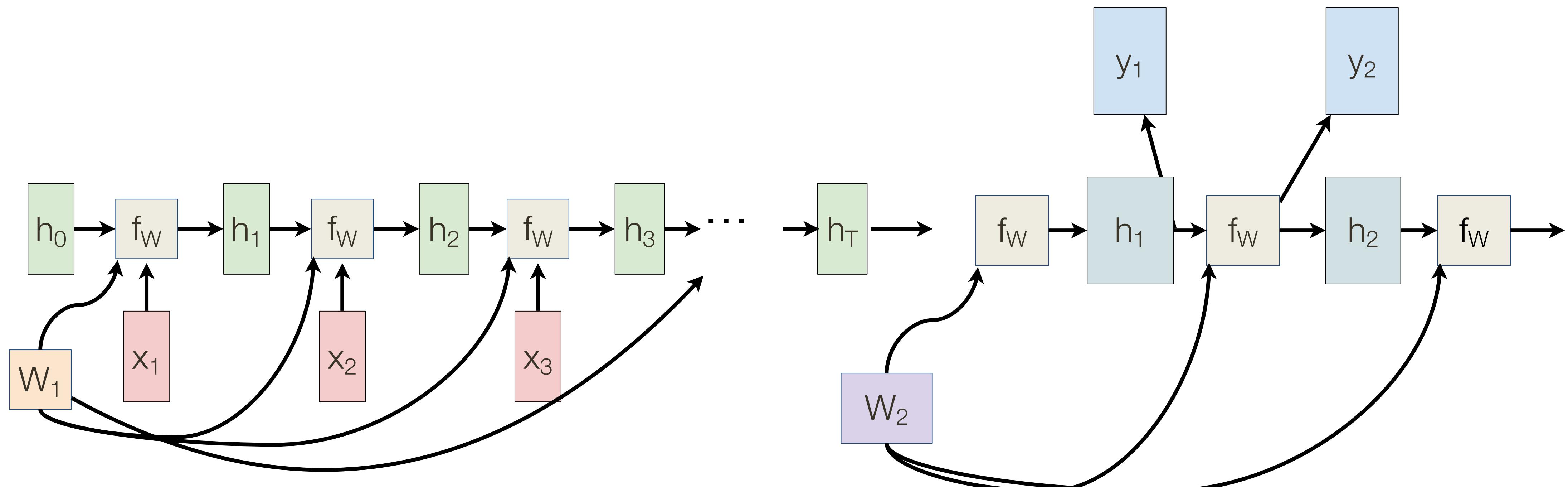
**Many to one:** Encode input sequence in a single vector



# Sequence to Sequence: Many to One + One to Many

**Many to one:** Encode input sequence in a single vector

**One to many:** Produce output sequence from single input vector



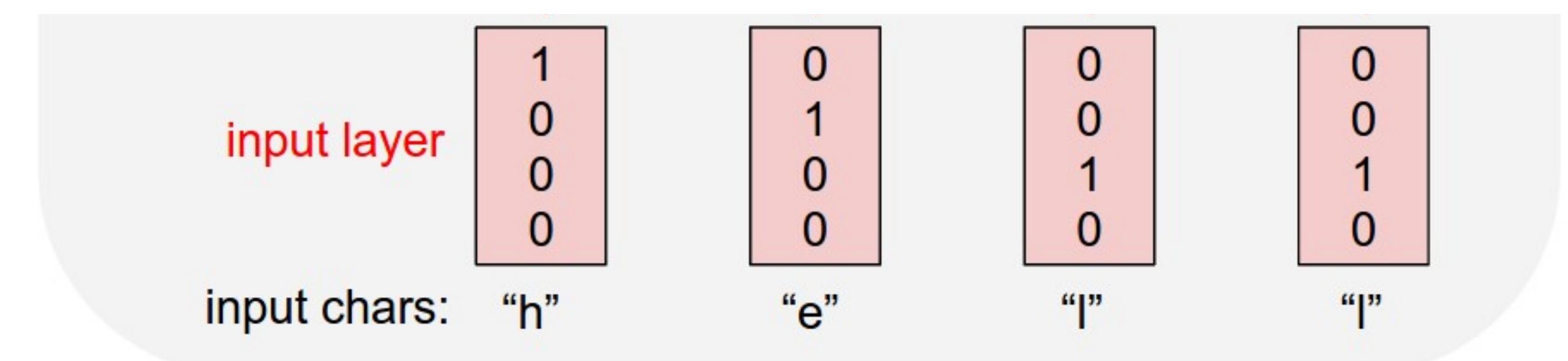
# Example: Character-level Language Model

## Vocabulary:

[‘h’, ‘e’, ‘l’, ‘o’]

Example training sequence:

“hello”



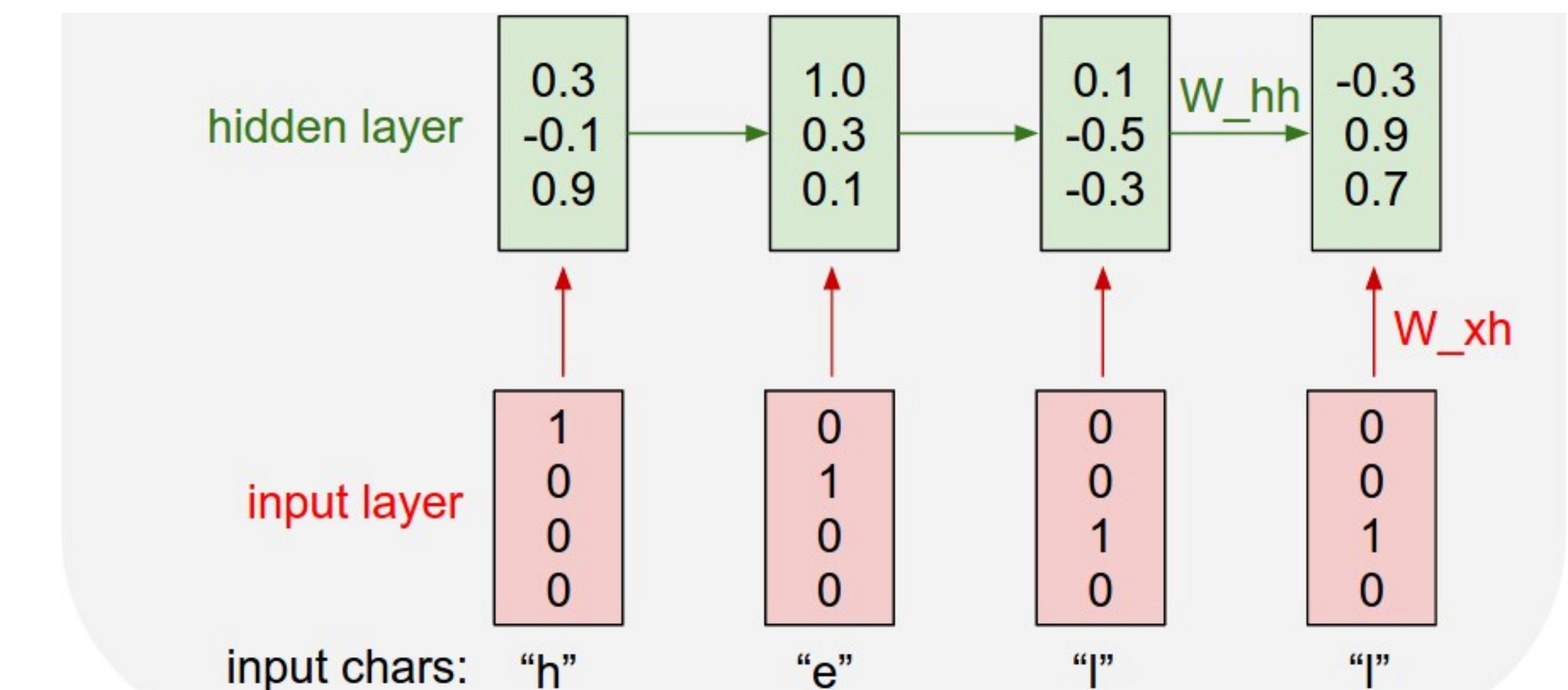
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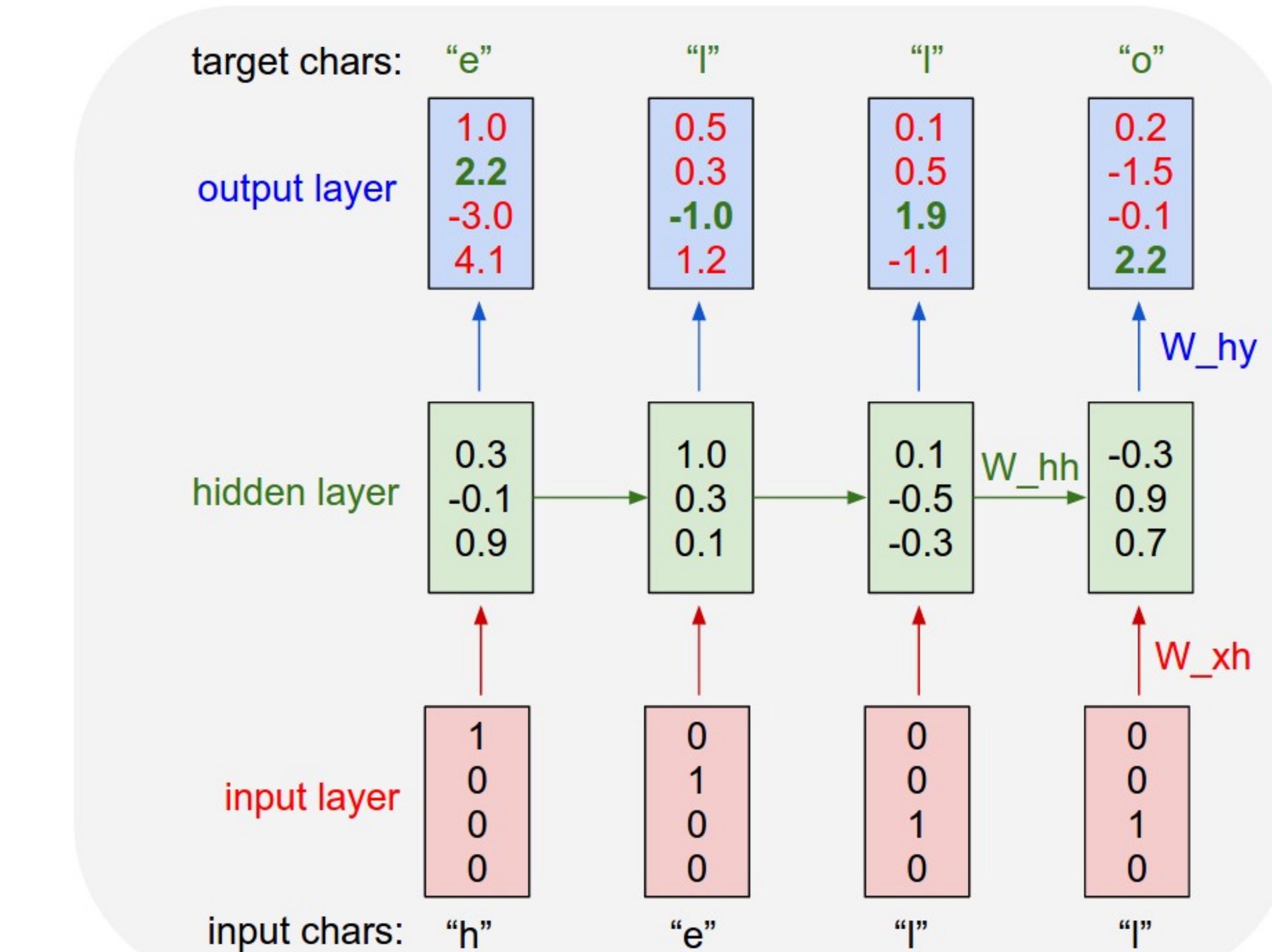


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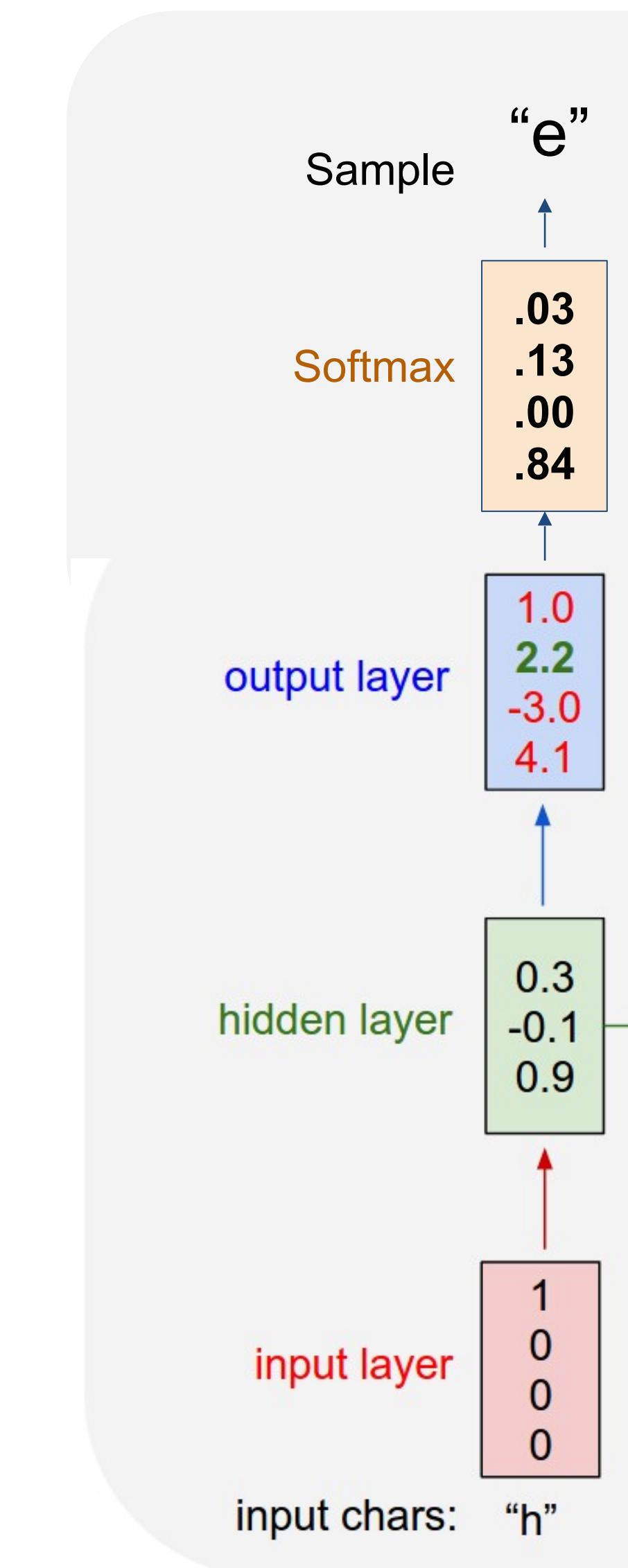


# Example: Character-level Language Model (**Sampling**)

## Vocabulary:

[‘h’, ‘e’, ‘l’, ‘o’]

At test time sample one character at a time and feed back to the model

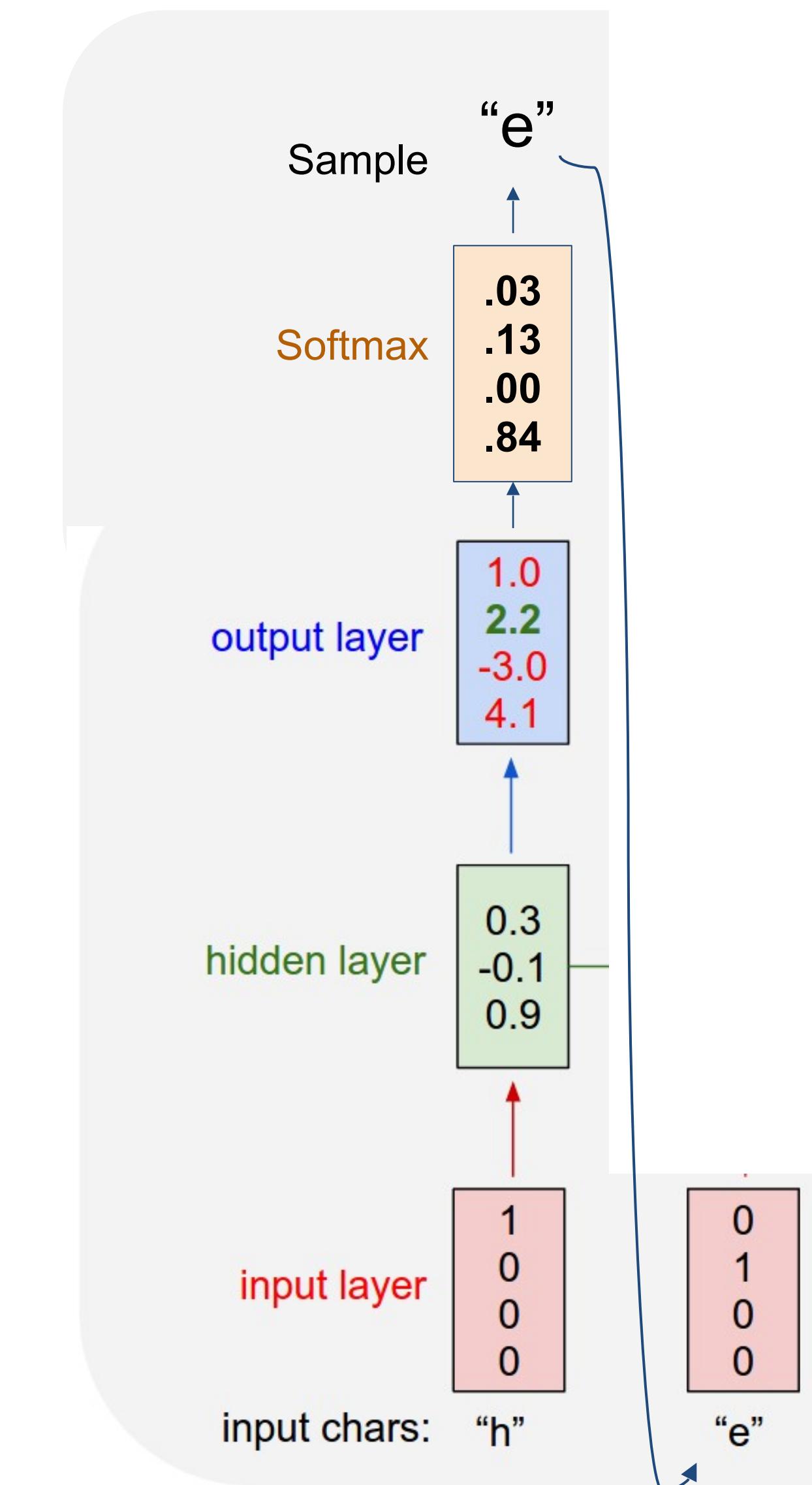


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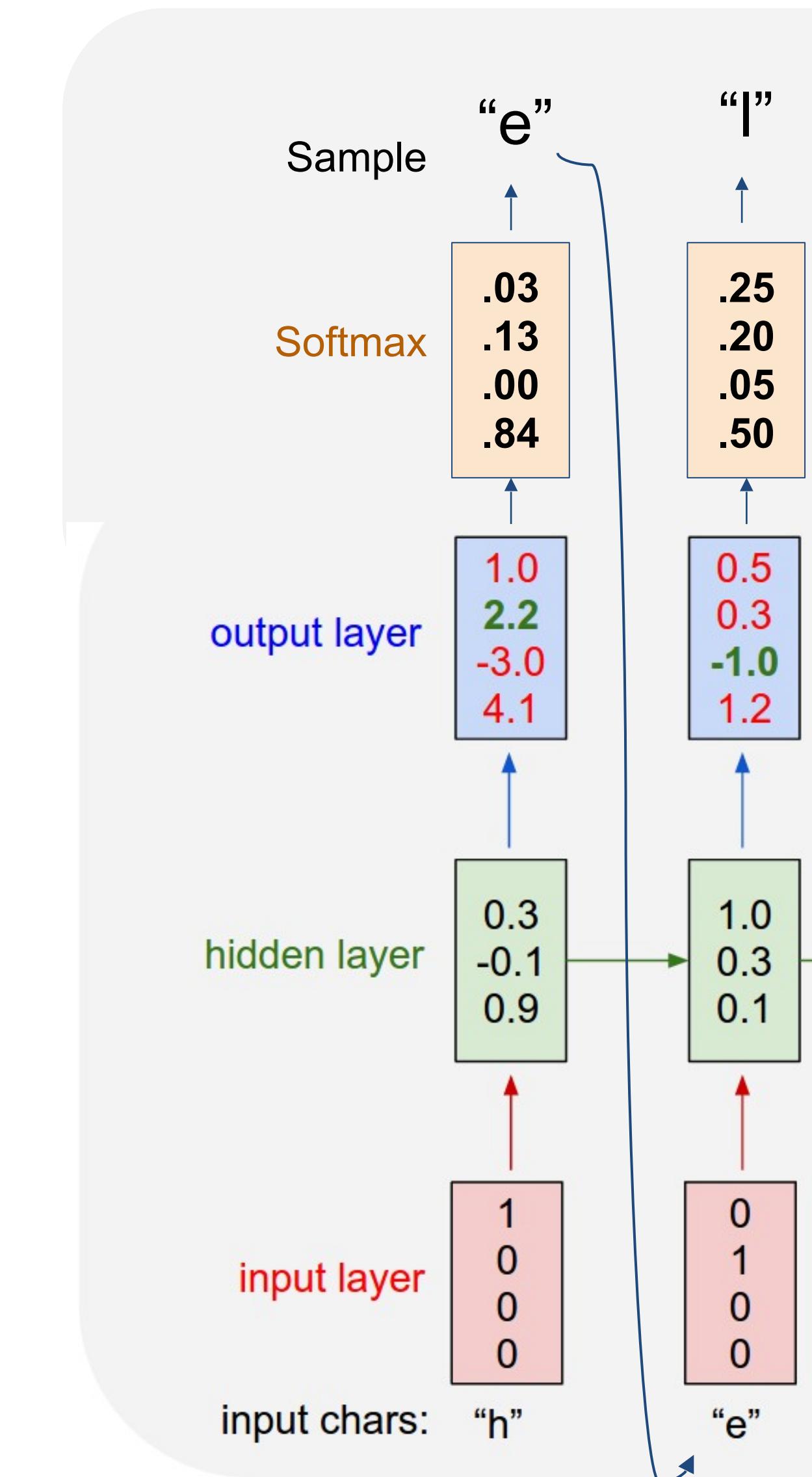


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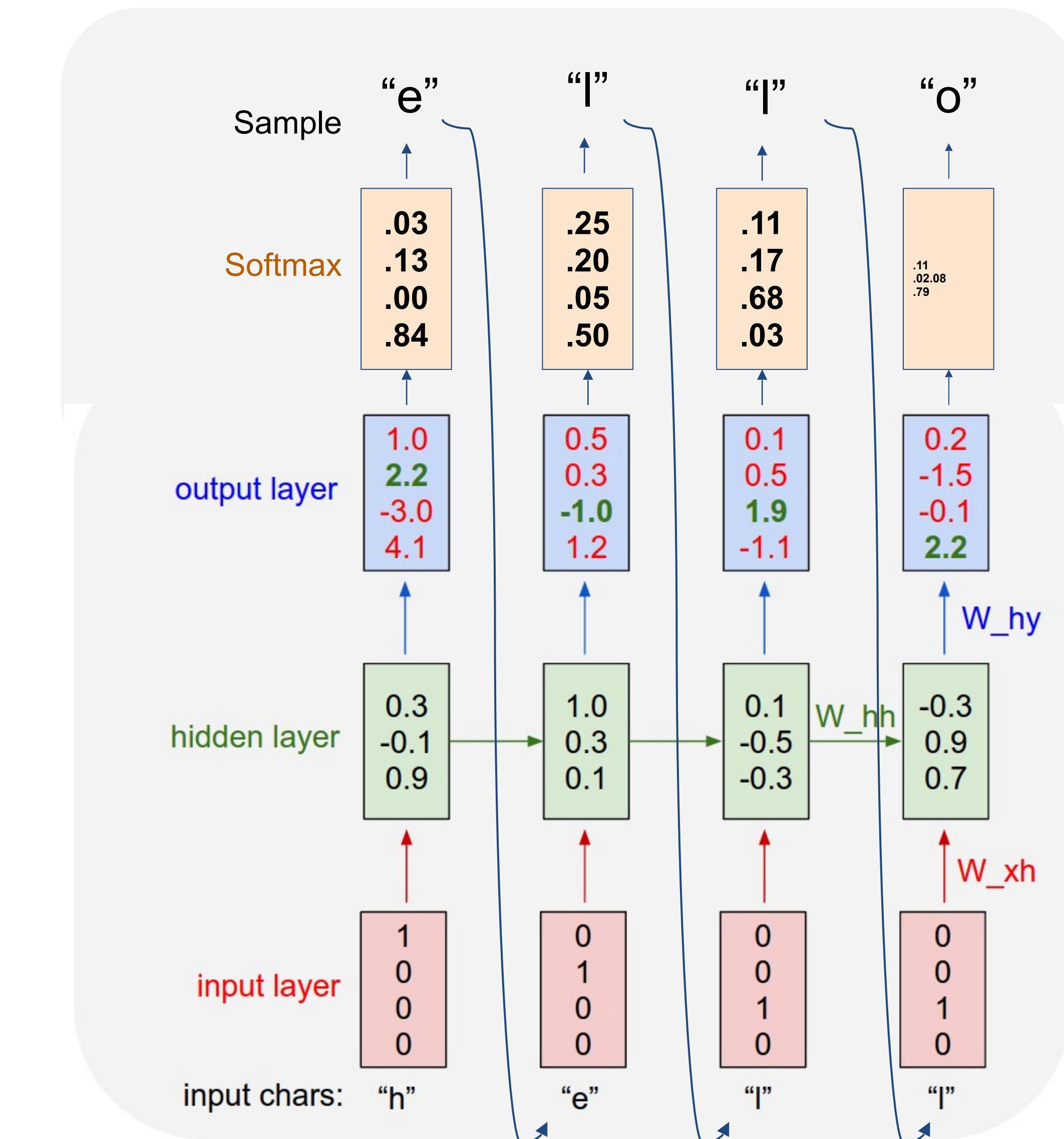


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**Vocabulary:**

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# Sampling vs. ArgMax vs. Beam Search

**Sampling:** allows to generate diverse outputs

**ArgMax:** could be more stable in practice

**Beam Search:** typically gets the best results

