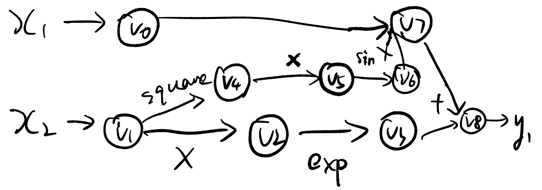
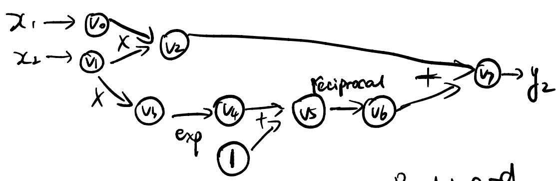
(a)

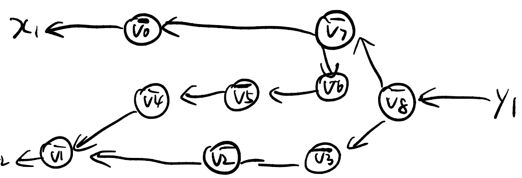
Forward

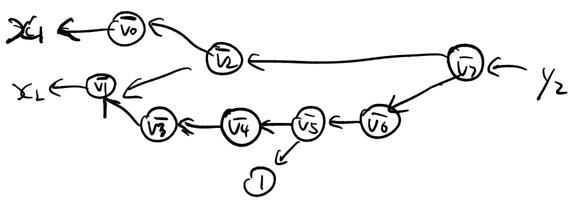




Backward

(b)





(c)

y_1	$f(2,1)$
$v_0 = x_1$	2
$v_1 = x_2$	1
$v_2 = 2v_1$	2
$v_3 = \exp(v_2)$	$\exp(2)$
$v_4 = v_1^2$	1
$v_5 = 3v_4$	3
$v_6 = \sin(v_5)$	$\sin(3)$
$v_7 = v_0 \cdot v_6$	$2\sin(3)$
$v_8 = v_3 + v_7$	$\exp(2) + 2\sin(3)$

y_2	$f(2,1)$
$v_0 = x_1$	2
$v_1 = x_2$	1
$v_2 = v_0 v_1$	2
$v_3 = -v_1$	-1
$v_4 = \exp(v_3)$	$\exp(-1)$
$v_5 = 1 + v_4$	$\exp(-1)+1$
$v_6 = \frac{1}{v_5}$	$\frac{1}{1+\exp(-1)}$
$v_7 = v_2 + v_6$	$2 + \frac{1}{1+\exp(-1)}$

d)

	$\frac{\partial f(x_1,x_2)}{\partial x_1}$
$\frac{\partial v_0}{\partial x_1} = 1$	1
$\frac{\partial v_1}{\partial x_1} = 0$	0
$\frac{\partial v_2}{\partial x_1} = 2\frac{\partial v_1}{\partial x_1} = 0$	0
$\frac{\partial v_3}{\partial x_1} = \exp(v_2)\frac{\partial v_2}{\partial x_1} = 0$	0
$\frac{\partial v_4}{\partial x_1} = 2v_1\frac{\partial v_1}{\partial x_1} = 0$	0
$\frac{\partial v_5}{\partial x_1} = 3\frac{\partial v_4}{\partial x_1} = 0$	0
$\frac{\partial v_6}{\partial x_1} = \cos(v_5)\frac{\partial v_5}{\partial x_1} = 0$	0
$\frac{\partial v_7}{\partial x_1} = v_0\frac{\partial v_6}{\partial x_1} + v_6 \cdot \frac{\partial v_0}{\partial x_1} = \sin(3)$	$\sin(3)$
$\frac{\partial v_8}{\partial x_1} = \frac{\partial v_3}{\partial x_1} + \frac{\partial v_7}{\partial x_1} = \sin(3)$	$\sin(3)$

	$\frac{\partial f(x_1,x_2)}{\partial x_2}$
$\frac{\partial v_0}{\partial x_2} = 0$	0
$\frac{\partial v_1}{\partial x_2} = 1$	1
$\frac{\partial v_2}{\partial x_2} = 2\frac{\partial v_1}{\partial x_2}$	2
$\frac{\partial v_3}{\partial x_2} = \exp(v_2)\cdot\frac{\partial v_2}{\partial x_2}$	$2\cdot\exp(2)$
$\frac{\partial v_4}{\partial x_2} = 2v_1\frac{\partial v_1}{\partial x_2}$	2
$\frac{\partial v_5}{\partial x_2} = 3\cdot\frac{\partial v_4}{\partial x_2}$	6
$\frac{\partial v_6}{\partial x_2} = \cos(v_5)\cdot\frac{\partial v_5}{\partial x_2}$	$6\cos(3)$
$\frac{\partial v_7}{\partial x_2} = v_0\cdot\frac{\partial v_6}{\partial x_2} + v_6\cdot\frac{\partial v_0}{\partial x_2}$	$12\cos(3)$
$\frac{\partial v_8}{\partial x_2} = \frac{\partial v_3}{\partial x_2} + \frac{\partial v_7}{\partial x_2}$	$2\exp(2) + 12\cos(3)$

Forward

	$\frac{\partial f(x_1,x_2)}{\partial x_2}$
$\frac{\partial v_0}{\partial x_1} = 1$	1
$\frac{\partial v_1}{\partial x_1} = 0$	0
$\frac{\partial v_2}{\partial x_1} = v_0 \frac{\partial v_1}{\partial x_1} + v_1 \frac{\partial v_0}{\partial x_1}$	1
$\frac{\partial v_3}{\partial x_1} = -\frac{\partial v_1}{\partial x_1}$	0
$\frac{\partial v_4}{\partial x_1} = \exp(v_3) \cdot \frac{\partial v_3}{\partial x_1}$	0
$\frac{\partial v_5}{\partial x_1} = \frac{\partial v_4}{\partial x_1}$	0
$\frac{\partial v_6}{\partial x_1} = -\frac{1}{v_5^2} \frac{\partial v_5}{\partial x_1}$	0
$\frac{\partial v_7}{\partial x_1} = \frac{\partial v_2}{\partial x_1} + \frac{\partial v_6}{\partial x_1}$	1
$\frac{\partial y}{\partial x_1}$	

	$\frac{\partial f(x_1,x_2)}{\partial x_2}$
$\frac{\partial v_0}{\partial x_2} = 0$	0
$\frac{\partial v_1}{\partial x_2} = 1$	1
$\frac{\partial v_2}{\partial x_2} = v_0 \frac{\partial v_1}{\partial x_2} + v_1 \frac{\partial v_0}{\partial x_2}$	2
$\frac{\partial v_3}{\partial x_2} = -\frac{\partial v_1}{\partial x_2}$	-1
$\frac{\partial v_4}{\partial x_2} = \exp(v_3) \cdot \frac{\partial v_3}{\partial x_2}$	$-\exp(-1)$
$\frac{\partial v_5}{\partial x_2} = \frac{\partial v_4}{\partial x_2}$	$-\exp(-1)$
$\frac{\partial v_6}{\partial x_2} = -\frac{1}{v_5^2} \frac{\partial v_5}{\partial x_2}$	$\frac{e^{-1}}{(1+e^{-1})^2}$
$\frac{\partial v_7}{\partial x_2} = \frac{\partial v_2}{\partial x_2} + \frac{\partial v_6}{\partial x_2}$	$2+\frac{e^{-1}}{(1+e^{-1})^2}$

$$\text{Jacobian} \begin{pmatrix} \sin(3) & 2e^2 + 12\cos 3 \\ 1 & 2+\frac{e^{-1}}{(1+e^{-1})^2} \end{pmatrix}$$

Backwards Deriatie

	$\frac{\partial f(x_1, x_2)}{\partial x_2}$
$\bar{v_1} = \bar{v_2} \cdot \frac{\partial v_2}{\partial v_1} + \bar{v_4} \cdot \frac{\partial v_4}{\partial v_1}$	$2\exp(2) + 12\cos(3)$
$\bar{v_2} = \bar{v_3} \frac{\partial v_3}{\partial v_2} = 1 \times \exp(v_2)$	$\exp(2)$
$\bar{v_3} = \bar{v_8} \frac{\partial v_8}{\partial v_3} = 1 \times 1$	1
$\bar{v_4} = \bar{v_5} \frac{\partial v_5}{\partial v_4} = 3\bar{v_5}$	$6\cos(3)$
$\bar{v_5} = \bar{v_6} \cdot \frac{\partial v_6}{\partial v_5} = \cos(v_5)$	$2\cos(3)$
$\bar{v_6} = \bar{v_7} \frac{\partial v_7}{\partial v_6} = v_0 \cdot 1$	2
$\bar{v_7} = \bar{v_8} \frac{\partial v_8}{\partial v_7} = 1$	1
$\bar{v_8} = \frac{\partial y}{\partial v_8} = 1$	1

	$\frac{\partial f(x_1, x_2)}{\partial x_1}$
$\bar{v_0} = \bar{v_7} \frac{\partial v_7}{\partial v_0}$	$\sin(3)$
$\bar{v_7} = \bar{v_8} \frac{\partial v_8}{\partial v_7}$	1
$\bar{v_8} = \frac{\partial y}{\partial v_8}$	1

Backward

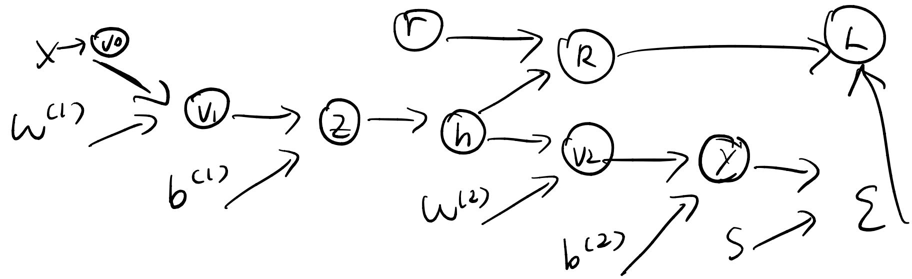
	$\frac{\partial f(x_1, x_2)}{\partial x_2}$
$\bar{v}_0 = \bar{v}_2 \frac{\partial v_2}{\partial v_0}$	1
$\bar{v}_2 = \bar{v}_7 \frac{\partial v_7}{\partial v_2}$	1
$\bar{v}_7 = \frac{\partial y}{\partial v_7}$	1

	$\frac{\partial f(x_1, x_2)}{\partial x_1}$
$\bar{v}_1 = \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_2 \cdot \frac{\partial v_2}{\partial v_1}$	$\frac{e^{-1}}{(1+e^{-1})^2} + 2$
$\bar{v}_3 = \bar{v}_4 \frac{\partial v_4}{\partial v_3}$	$\frac{-e^{-1}}{(1+e^{-1})^2}$
$\bar{v}_4 = \bar{v}_5 \frac{\partial v_5}{\partial v_4} = \bar{v}_5$	$\frac{-1}{(1+e^{-1})^2}$
$\bar{v}_5 = \bar{v}_6 \frac{\partial v_6}{\partial v_5} = \frac{-1}{v_5^2}$	$\frac{-1}{(1+e^{-1})^2}$
$\bar{v}_6 = \bar{v}_7 \frac{\partial v_7}{\partial v_6}$	1
$\bar{v}_7 = \frac{\partial y}{\partial v_7}$	1

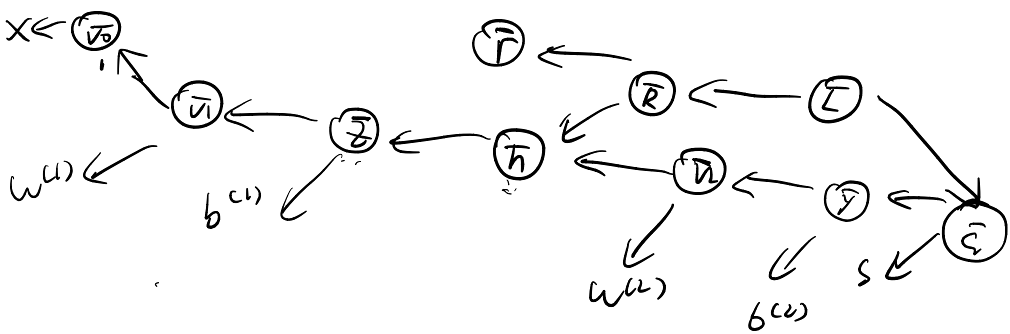
a)

computational graph

Forward



BackForward



b) Forwards

$x = v_0$

$v_1 = W^{(1)} v_0$

$z = v_1 + b^{(1)}$

$h = \sigma(z)$

$R = r^T h$

$v_2 = W^{(2)} h$

$y = v_2 + b^{(2)}$

$\mathcal{E} = \frac{1}{2} \|y - s\|^2$

$L = \mathcal{E} + R$.

$\bar{v}_0 = \bar{v}_1 \frac{\partial v_1}{\partial v_0} = W^{(1)T} \bar{h} \circ \sigma'(z)$

$\bar{v}_1 = \bar{z} \frac{\partial v_1}{\partial z} = \bar{h} \circ \sigma'(z)$

$\bar{z} = \bar{h} \frac{\partial h}{\partial z} = \bar{h} \circ \sigma'(z) = \bar{h} \circ \sigma'(z)$

$\bar{h} = \bar{R} \frac{\partial R}{\partial h} + \bar{v}_2 \frac{\partial v_2}{\partial h} = r + W^{(2)T} \bar{y}$

$\bar{v}_2 = \bar{y} \frac{\partial y}{\partial v_2} = \bar{y}$

$\bar{y} = \bar{\mathcal{E}} \frac{\partial \mathcal{E}}{\partial y} = y - s$

$\bar{\mathcal{E}} = \bar{L} \frac{\partial L}{\partial \mathcal{E}} = 1$

$\bar{R} = \bar{L} \frac{\partial L}{\partial R} = 1$

$\bar{L} = 1$

Backward

Dim:

$x: N \times 1$

$y: M \times 1$

$h: K \times 1$

$b^{(2)}: M \times 1$

$b^{(1)}: K \times 1$

$W^{(1)}: K \times N$

$W^{(2)}: M \times K$

$r: K \times 1$

$z: K \times 1$

$$\frac{\partial L}{\partial x} = W^{(1)T} \bar{h} \circ \sigma'(z)$$

where

$$\bar{h} = r + W^{(2)T}(y - s)$$