Modeling Temporal Correlations and Dynamics in Spatiotemporal Data Systems

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Outline

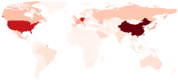
A quick look:

- Motivation (data, task, and models)
- Traffic data imputation with global/local trend modeling
  - Traffic flow imputation, speed field reconstruction, and network traffic state estimation
- Unsupervised pattern discovery from spatiotemporal systems
  - Time-varying autoregression & tensor factorization
  - Applications to fluid flow (benchmark), sea surface temperature, USA climate, NYC taxi, international trade, and Chicago ridesharing
- Possible directions & collaboration:
  - Clifford product & convolution
  - Graphs, hypergraphs, and higher-order motifs

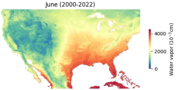
Motivation

- Spatiotemporal systems & data scenarios

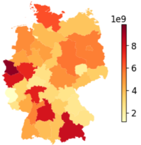
Transportation



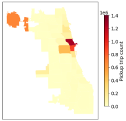
International trade



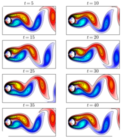
Climate



Energy



Mobility



Fluid flow

- Challenges: Sparsity, time-varying system, multidimensional system (e.g., human mobility)

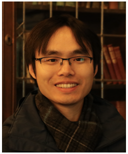
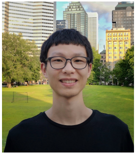
Prior Art

- Sequence models: Time series autoregression, LSTM, attention-based sequence models, etc.
- Machine learning problems:
  - **Imputation/Interpolation**: Time series models, sparse learning (e.g., matrix/tensor factorization), deep learning (e.g., generative models), etc.
  - **Unsupervised pattern discovery**: Dynamic mode decomposition in dynamical systems, matrix/tensor factorization, etc.
  - **Prediction**: Almost deep learning, but depending on scenarios

Laplacian Convolutional Representation for Traffic Time Series Imputation

IEEE Transactions on Knowledge and Data Engineering, 2024

https://doi.org/10.1109/TKDE.2024.3419698

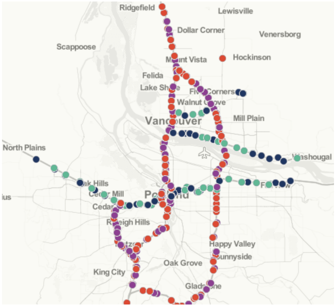

Xinyu Chen Zhanhong Cheng HanQin Cai Nicolas Saunier Lijun Sun

Materials:

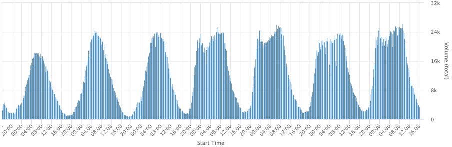
- PDF: https://xinychen.github.io/papers/Laplacian_convolution.pdf
- GitHub: https://github.com/xinychen/transdim (1.1k+ stars)
- Blog: https://spatiotemporal-data.github.io/posts/LCR/ (coming soon)

Traffic Flow Data

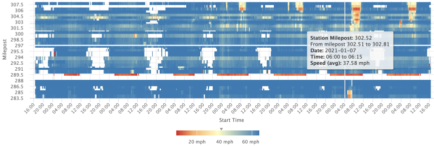
- Portland highway traffic data[1]



Highway network & sensor locations



Traffic volume



Traffic speed

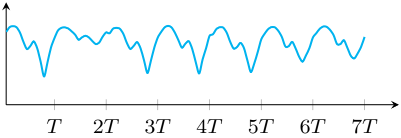
- $\boldsymbol{X} \in \mathbb{R}^{N \times T}$ with N spatial locations $\times$ T time steps
- Traffic volume/speed shows strong spatial/temporal dependencies

[1] https://portal.its.pdx.edu/home

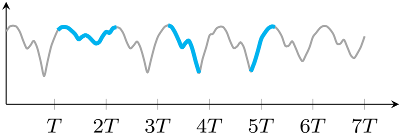
Time Series Imputation

Motivation: Traffic imputation

- Global trends (e.g., long-term quasi-seasonality & daily/weekly rhythm):



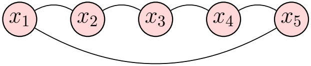
- Local trends (e.g., short-term time series trends):



How to characterize both global and local trends in sparse time series?

Local Trend Modeling

- Intuition of (circulant) Laplacian matrix



Undirected and circulant graph

$$\xrightarrow{\text{Modeling}} \quad \boldsymbol{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Define Laplacian kernel:

$$\boldsymbol{\ell} \triangleq (2, -1, 0, 0, -1)^\top$$

$$\Downarrow$$

$$\boldsymbol{\ell} \triangleq (\underbrace{2\tau}_{\text{degree}}, \underbrace{-1, \cdots, -1}_{\tau}, 0, \cdots, 0, \underbrace{-1, \cdots, -1}_{\tau})^\top \in \mathbb{R}^T$$

for any time series $\boldsymbol{x} = (x_1, \ldots, x_T)^\top \in \mathbb{R}^T$.

- Temporal regularization (w/ circular convolution $\star$):

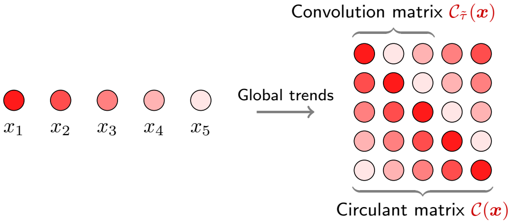
$$\mathcal{R}_\tau(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{L}\boldsymbol{x}\|_2^2 = \frac{1}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2$$

"... The circulant graph has an adjacency matrix that is a circulant matrix."

— Circulant graph on Wikipedia

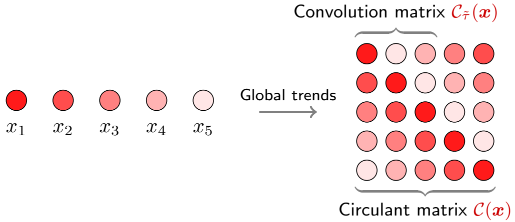
Global Trend Modeling

Circulant matrix $\mathcal{C}(\boldsymbol{x})$ vs. convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})$



Global Trend Modeling

Circulant matrix $\mathcal{C}(\boldsymbol{x})$ vs. convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})$



- Circulant/Convolution nuclear norm minimization
  - A balance between global and local trends modeling?

CircNNM (Liu'22, Liu & Zhang'23)

Estimating $\boldsymbol{x}$:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_*$$
$$\text{s.t. } \|\mathcal{P}_{\Omega}(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

ConvNNM (Liu'22, Liu & Zhang'23)

Estimating $\boldsymbol{x}$:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})\|_*$$
$$\text{s.t. } \|\mathcal{P}_{\Omega}(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

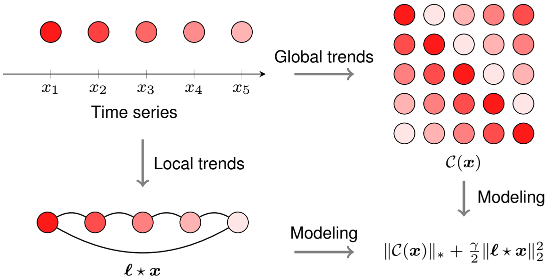
Global + Local Trends?

Laplacian Convolutional Representation (LCR)

For any partially observed time series $\boldsymbol{y} \in \mathbb{R}^T$ with observed index set Ω, LCR utilizes circulant matrix and Laplacian kernel to characterize global and local trends in time series, respectively, i.e.,

$$\min_{\boldsymbol{x}} \ \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{local}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon$$



Laplacian Convolutional Representation

- Augmented Lagrangian function:[2]

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) = \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{z}\|_2^2 + \langle\boldsymbol{w},\boldsymbol{x}-\boldsymbol{z}\rangle + \frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{z}-\boldsymbol{y})\|_2^2$$

- The ADMM scheme:

$$\begin{cases} \boldsymbol{x} := \arg\min\limits_{\boldsymbol{x}} \ \mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) & \text{(Nuclear norm minimization)} \\ \boldsymbol{z} := \arg\min\limits_{\boldsymbol{z}} \ \mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) & \text{(Closed-form solution)} \\ \boldsymbol{w} := \boldsymbol{w} + \lambda(\boldsymbol{x}-\boldsymbol{z}) & \text{(Standard update)} \end{cases}$$

- Optimize $\boldsymbol{x}$?

$$\underbrace{\|\mathcal{C}(\boldsymbol{x})\|_* = \|\mathcal{F}(\boldsymbol{x})\|_1}_{\text{property of circulant matrix}} \qquad \& \qquad \underbrace{\frac{1}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 = \frac{1}{2T}\|\mathcal{F}(\boldsymbol{\ell})\circ\mathcal{F}(\boldsymbol{x})\|_2^2}_{\text{property of circular convolution}}$$

Nuclear norm minimization $\Rightarrow$ ℓ_1**-norm minimization with FFT** in $\mathcal{O}(T\log T)$ time.

[2] $\boldsymbol{w}\in\mathbb{R}^T$ (Lagrange multiplier); $\langle\cdot,\cdot\rangle$ (inner product).

Laplacian Convolutional Representation

- Optimize $\boldsymbol{x}$ via FFT (in $\mathcal{O}(T\log T)$ time):

$$\boldsymbol{x} := \arg\min_{\boldsymbol{x}} \; \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{z}+\boldsymbol{w}/\lambda\|_2^2$$

$$\Longrightarrow \hat{\boldsymbol{x}} := \arg\min_{\hat{\boldsymbol{x}}} \; \|\hat{\boldsymbol{x}}\|_1 + \frac{\gamma}{2T}\|\hat{\boldsymbol{\ell}}\circ\hat{\boldsymbol{x}}\|_2^2 + \frac{\lambda}{2T}\|\hat{\boldsymbol{x}}-\hat{\boldsymbol{z}}+\hat{\boldsymbol{w}}/\lambda\|_2^2$$

where we introduce $\{\hat{\boldsymbol{\ell}},\hat{\boldsymbol{x}},\hat{\boldsymbol{z}},\hat{\boldsymbol{w}}\} \triangleq \mathcal{F}\{\boldsymbol{\ell},\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}\}$ (i.e., FFT).

ℓ_1-norm Minimization in Complex Space (Liu & Zhang'23)

For any optimization problem in the form of ℓ_1-norm minimization in complex space:

$$\min_{\hat{\boldsymbol{x}}} \; \|\hat{\boldsymbol{x}}\|_1 + \frac{\delta}{2}\|\hat{\boldsymbol{x}}-\hat{\boldsymbol{h}}\|_2^2$$

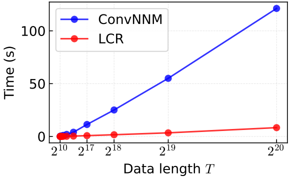
with complex-valued $\hat{\boldsymbol{x}},\hat{\boldsymbol{h}}\in\mathbb{C}^T$ and weight parameter δ, element-wise, the solution is given by

$$\hat{x}_t := \frac{\hat{h}_t}{|\hat{h}_t|}\cdot\max\{0,|\hat{h}_t|-1/\delta\}, t\in[T].$$

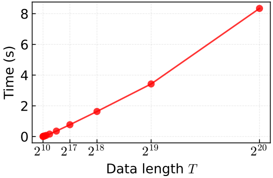
Empirical time complexity

On the synthetic data $\boldsymbol{y} \in \mathbb{R}^T$ with $T \in \{2^{10}, 2^{11}, \ldots, 2^{20}\}$

- Ours: **LCR**
  - An FFT implementation in $\mathcal{O}(T \log T)$
  - The logarithmic factor $\log T$ makes the FFT highly efficient
- Baseline: **ConvNNM** (Liu'22, Liu & Zhang'23)
  - Convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{y}) \in \mathbb{R}^{T \times \tilde{\tau}}$ with kernel size $\tilde{\tau} = 2^4$
  - Singular value thresholding in $\mathcal{O}(\tilde{\tau}^2 T)$

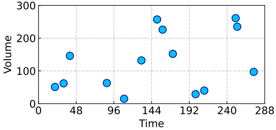


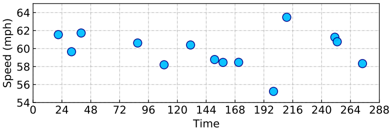
ConvNNM vs. LCR



LCR

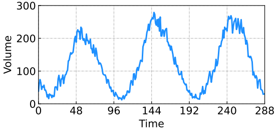
Experiments

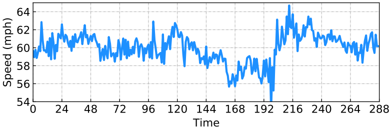




Reconstruct
traffic volume?

Reconstruct
traffic speed?

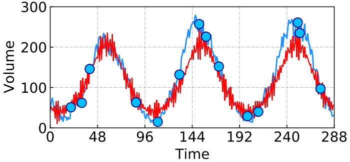




- How to utilize the global trends of traffic time series?
- How to produce local consistency of traffic data?

Experiments

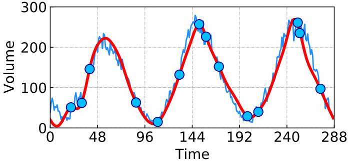
- Substantial performance gains?



CircNNM:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* \quad \text{s. t.} \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

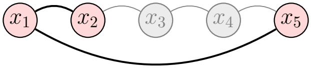
$\Downarrow$ Plus **local** time series trends



LCR:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 \quad \text{s. t.} \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

- The start data points and end data points are connected?



Undirected and circulant graph

$$\xrightarrow{\text{Modeling}} \quad \boldsymbol{L} = \begin{bmatrix} \mathbf{2} & -1 & 0 & 0 & -1 \\ \mathbf{-1} & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ \mathbf{-1} & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Flipping operation on $\boldsymbol{x} \in \mathbb{R}^5$:

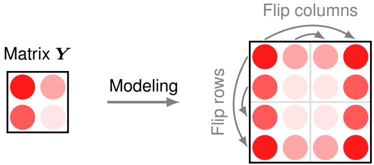
$$\boldsymbol{x}_{\text{new}} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{J}\boldsymbol{x} \end{bmatrix} = (\underbrace{x_1, x_2, x_3, x_4, x_5,}_{\text{original time series}} \; \underbrace{x_5, x_4, x_3, x_2, x_1}_{\text{flipped time series}})^\top \in \mathbb{R}^{10}$$

where $\boldsymbol{J} \in \mathbb{R}^{5\times 5}$ is the exchange matrix.

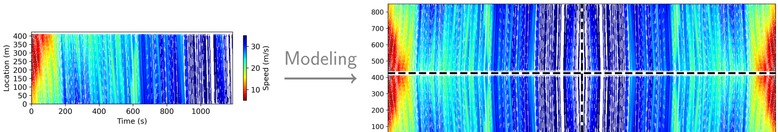
- Potential applications: Passenger flow prediction with strong global/local trends

Speed field reconstruction[3]

- Flipping operation on a matrix:



- Flipping operation on a speed field of vehicular traffic flow:



Original speed field

Flipped speed field

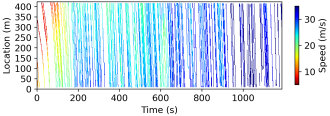
[3]Highway Drone (HighD) dataset at https://www.highd-dataset.com/

Speed field reconstruction[4]

- Scenario: Mask trajectories of 70% vehicles
- LCR-2D on partially observed $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

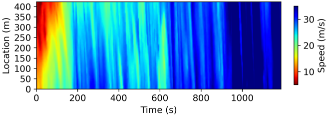
$$\min_{\boldsymbol{X}} \quad \underbrace{\|\mathcal{C}(\boldsymbol{X})\|_*}_{\text{global trend}} + \frac{\gamma}{2} \underbrace{\|(\boldsymbol{\ell}_s \boldsymbol{\ell}^\top) \star \boldsymbol{X}\|_F^2}_{\text{local trend}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{X} - \boldsymbol{Y})\|_F \leq \epsilon$$

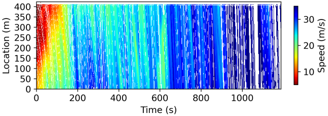


Sparse speed field

LCR-2D



Reconstructed speed field



Ground-truth speed field

[4] Highway Drone (HighD) dataset at https://www.highd-dataset.com/

Contributions

Matrix nuclear norm ($\|\boldsymbol{X}\|_*$) minimization

Candès & Recht'09

Singular value thresholding

Cai et al.'10

Truncated nuclear norm ($\|\boldsymbol{X}\|_{r,*}, r \in \mathbb{Z}^+$) minimization

Zhang et al.'12

Hu et al.'12

Tensor nuclear norm ($\|\boldsymbol{\mathcal{X}}\|_*$) minimization

Liu et al.'13

Generalized nonconvex nonsmooth low-rank minimization

Lu et al.'14

Tensor nuclear norm minimization with linear transform

Lu et al.'19

Low-rank Hankel matrix/tensor ($\mathcal{H}_\tau(\cdot)$) completion

Yokota et al.'18

Sedighin et al.'20

Cai et al.'21

Yamamoto et al.'22

Circulant/Convolution nuclear norm ($\|\mathcal{C}(\boldsymbol{x})\|_*$ or $\|\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})\|_*$) minimization

Liu'22

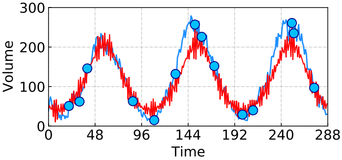
Liu & Zhang'23

(Ours) **LCR**:

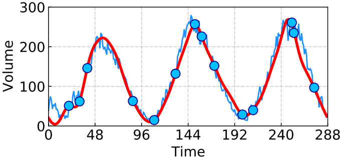
✓ Local trend modeling

✓ An FFT implementation

Vision & Insight



⇓ Plus **local** time series trends



Highlights:

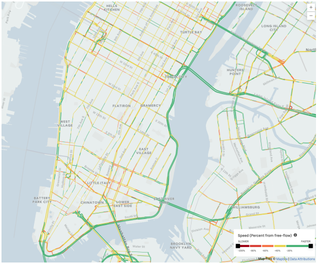
- Rethinking the importance of local trend modeling in traffic data imputation tasks.
- Finding a unified global and local trend modeling framework whose optimization can be efficiently solved by FFT:

$$\min_{\boldsymbol{x}} \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2}\underbrace{\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2}_{\text{local}}$$

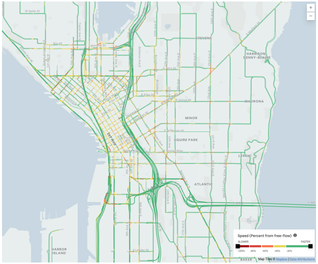
$$\text{s.t.} \ \|\mathcal{P}_{\Omega}(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

Vision & Insight

- Uber (hourly) movement speed data[5]



NYC movement



Seattle movement

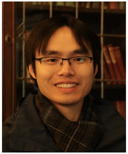
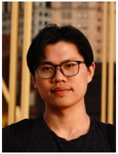
- {road segment, time slot (hour), average speed}
- Computing hourly speed: Road segments have 5+ unique trips.
- Estimating network-wide traffic states for traffic planning & management.
  (Data/model biases/fairness concerns for imputation, interpolation, and prediction.)

[5] https://movement.uber.com/ (not available now)

Discovering Dynamic Patterns from Spatiotemporal Data with Time-Varying Low-Rank Autoregression

IEEE Transactions on Knowledge and Data Engineering, 2024

https://doi.org/10.1109/TKDE.2023.3294440

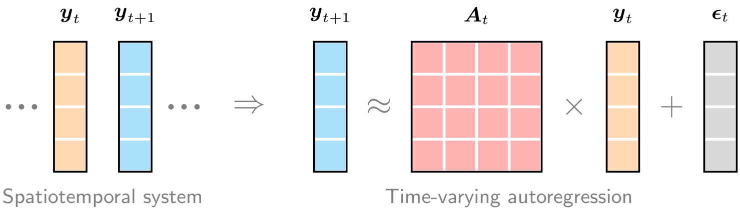

Xinyu Chen   Chengyuan Zhang*   Xiaoxu Chen   Nicolas Saunier   Lijun Sun

Materials:

- PDF: https://xinychen.github.io/papers/time_varying_model.pdf
- GitHub: https://github.com/xinychen/vars
- Blog: https://spatiotemporal-data.github.io/posts/time_varying_model

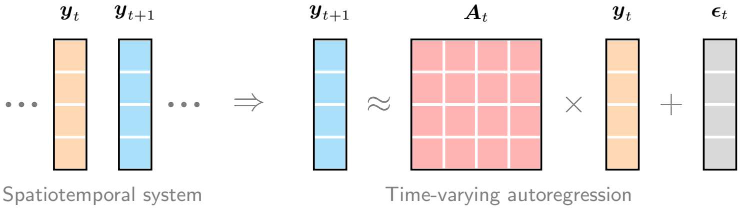
Autoregression

- How to characterize dynamical systems?



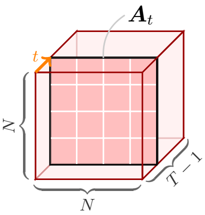
Autoregression

- How to characterize dynamical systems?

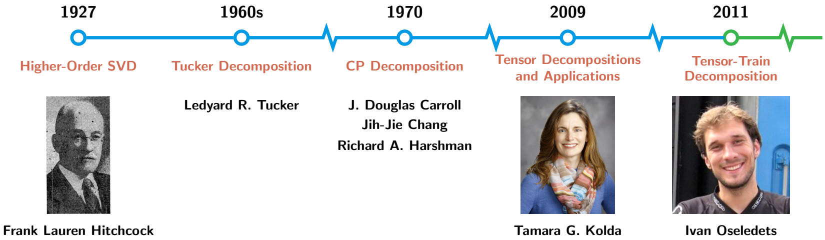


- On spatiotemporal systems $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

$$\underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-invariant (e.g., DMD)}} \quad \text{v.s.} \quad \underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}_t\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-varying}}$$



- How to discover spatial/temporal modes (patterns) from the tensor $\boldsymbol{\mathcal{A}} \triangleq \{\boldsymbol{A}_t\}_{t\in[T-1]}$?


1927
Higher-Order SVD
Frank Lauren Hitchcock
1960s
Tucker Decomposition
Ledyard R. Tucker
1970
CP Decomposition
J. Douglas Carroll
Jih-Jie Chang
Richard A. Harshman
2009
Tensor Decompositions and Applications
Tamara G. Kolda
2011
Tensor-Train Decomposition
Ivan Oseledets

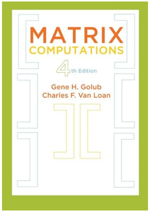
Time-Varying Autoregression

- Tensor factorization[6]:

$$\boldsymbol{\mathcal{A}} = \underbrace{\boldsymbol{\mathcal{G}} \times_1 \boldsymbol{W} \times_2 \boldsymbol{V} \times_3 \boldsymbol{X}}_{\text{Tucker decomposition}}$$

$$\Updownarrow$$

$$\boldsymbol{A}_t = \boldsymbol{\mathcal{G}} \times_1 \underbrace{\boldsymbol{W}}_{\text{spatial modes}} \times_2 \boldsymbol{V} \times_3 \underbrace{\boldsymbol{x}_t^\top}_{\text{temporal modes}}$$



- **(Ours)** Time-varying low-rank autoregression:

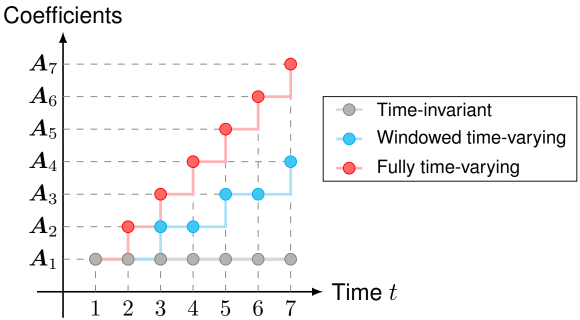
$$\min_{\boldsymbol{\mathcal{G}},\boldsymbol{W},\boldsymbol{V},\boldsymbol{X}} \ \frac{1}{2}\sum_{t\in[T-1]} \left\|\boldsymbol{y}_{t+1} - (\boldsymbol{\mathcal{G}} \times_1 \boldsymbol{W} \times_2 \boldsymbol{V} \times_3 \boldsymbol{x}_t^\top)\boldsymbol{y}_t\right\|_2^2$$

- Alternating minimization: $\boldsymbol{\mathcal{G}}$ (LS) $\rightarrow$ $\boldsymbol{W}$ (LS) $\rightarrow$ $\boldsymbol{V}$ (CG) $\rightarrow$ $\boldsymbol{x}_t$ (LS)

[6] $\times_k, \forall k$ is the mode-k product between tensor and matrix/vector.

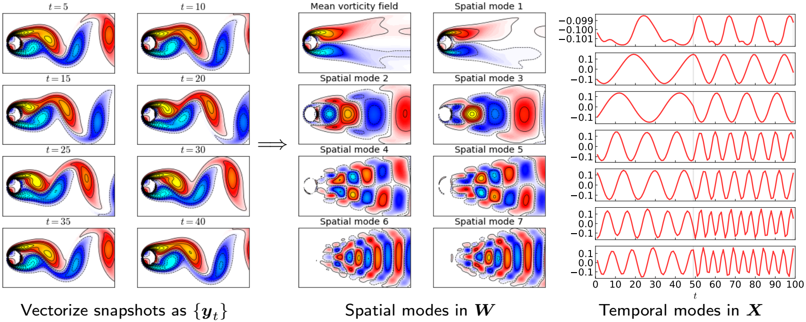
- On the data $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

$$\underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-invariant (e.g., DMD)}} \quad \text{v.s.} \quad \underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}_t\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{fully time-varying (ours)}}$$



Fluid Flow

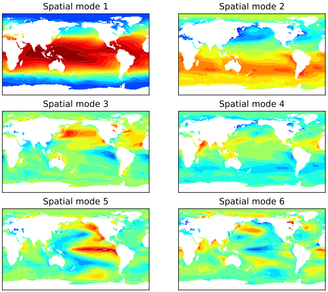
- **Multiresolution fluid flow dataset** (the first 50 snapshots + 50 snapshots randomly selected from the last 100 snapshots)



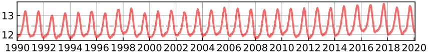
- Produce interpretable patterns and identify the system of different frequencies.

Sea Surface Temperature

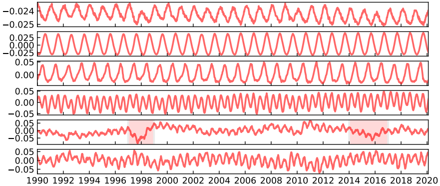
- Sea surface temperature (**SST**) dataset



Spatial modes in $\boldsymbol{W}$



Time series of mean temperature

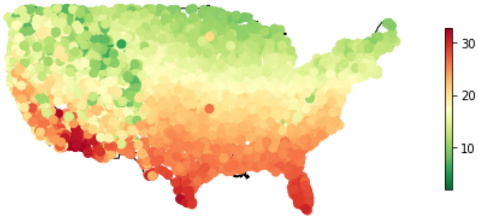


Temporal modes in $\boldsymbol{X}$

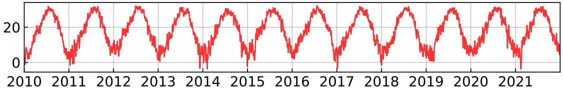
- Identify two strongest El Nino events (on 1997-98 & 2014-16).

USA Temperature

- Data[7]: 5,380 stations & 12 years with the day resolution
- Spatial distribution of mean temperature

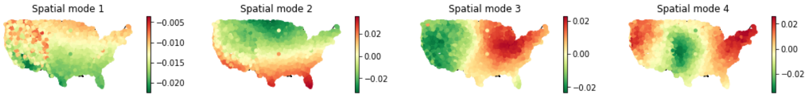


- Time series of mean temperatures

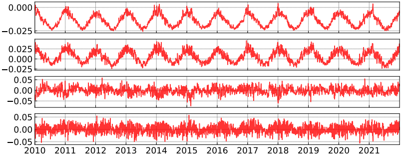


[7] https://daac.ornl.gov/DAYMET

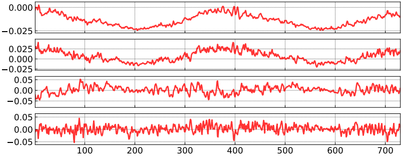
USA Temperature



Spatial patterns in $\boldsymbol{W}$



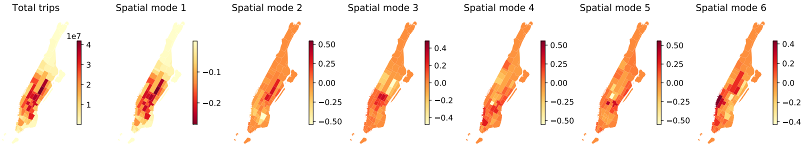
Temporal patterns over 12 years



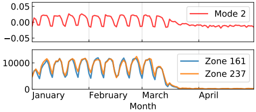
Temporal patterns over 2 years

NYC Taxi Data

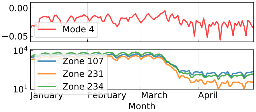
- **NYC taxi dataset** (pickup)



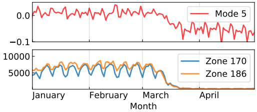
Spatial patterns in $\boldsymbol{W}$



Pattern #2 & taxi trips (2020)



Pattern #4 & taxi trips (2020)



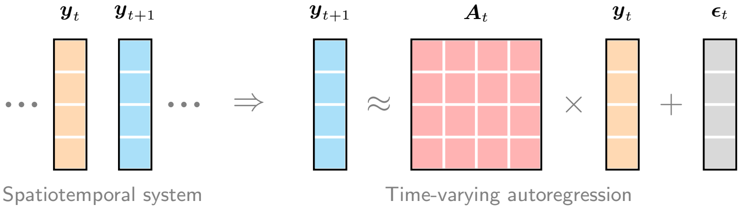
Pattern #5 & taxi trips (2020)

Dynamic Autoregressive Tensor Factorization for Pattern Discovery of Spatiotemporal Systems

International Trade & Ridesharing Mobility

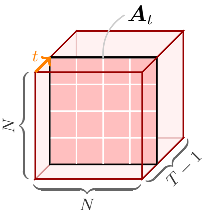
Autoregression

- How to characterize dynamical systems?



- On spatiotemporal systems $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

$$\underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-invariant (e.g., DMD)}} \quad \text{v.s.} \quad \underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}_t\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-varying}}$$



- How to discover spatial/temporal modes (patterns) from the tensor $\boldsymbol{\mathcal{A}} \triangleq \{\boldsymbol{A}_t\}_{t \in [T-1]}$?

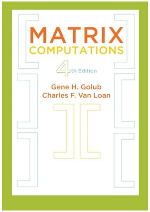
DATF

- Tensor factorization:

$$\mathcal{A} = \underbrace{\mathcal{G} \times_1 \boldsymbol{W} \times_2 \boldsymbol{V} \times_3 \boldsymbol{X}}_{\text{Tucker decomposition}}$$

$$\Updownarrow$$

$$\boldsymbol{A}_t = \mathcal{G} \times_1 \underbrace{\boldsymbol{W}}_{\text{spatial modes}} \times_2 \boldsymbol{V} \times_3 \underbrace{\boldsymbol{x}_t^\top}_{\text{temporal modes}}$$



- **(Ours)** Dynamic autoregressive tensor factorization (DATF):

$$\min_{\mathcal{G}, \boldsymbol{W}, \boldsymbol{V}, \boldsymbol{X}} \frac{1}{2} \sum_{t \in [T-1]} \left\| \boldsymbol{y}_{t+1} - (\mathcal{G} \times_1 \boldsymbol{W} \times_2 \boldsymbol{V} \times_3 \boldsymbol{x}_t^\top) \boldsymbol{y}_t \right\|_2^2$$

$$\text{s.t.} \quad \underbrace{\boldsymbol{W}^\top \boldsymbol{W} = \boldsymbol{I}_R}_{\text{orthogonal spatial modes}}$$

- Solution: $\mathcal{G}$ (LS) $\rightarrow$ $\boldsymbol{W}$ **(OPP)** $\rightarrow$ $\boldsymbol{V}$ (CG) $\rightarrow$ $\boldsymbol{x}_t$ (LS)

OPP

- **Orthogonal Procrustes problem (OPP)**: For any $\boldsymbol{Q} \in \mathbb{R}^{m\times r}$, $m \geq r$, the solution to

$$\min_{\boldsymbol{F}} \ \|\boldsymbol{F} - \boldsymbol{Q}\|_F^2$$

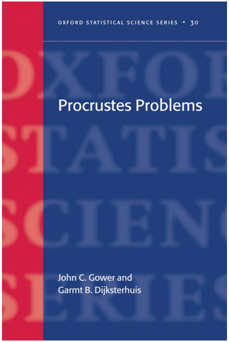
$$\text{s. t.} \ \underbrace{\boldsymbol{F}^\top \boldsymbol{F} = \boldsymbol{I}_r}_{\text{orthogonal}}$$

is

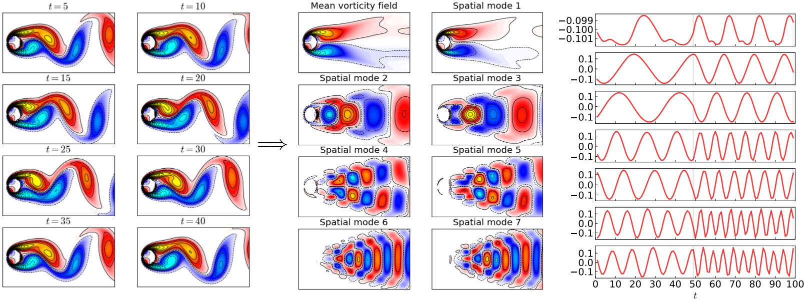
$$\boldsymbol{F} := \boldsymbol{U}\boldsymbol{V}^\top$$

where

$$\underbrace{\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^\top}_{\text{singular value decomposition}}$$



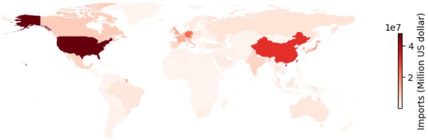
- **Multi-resolution fluid flow dataset** (the first 50 snapshots + 50 snapshots randomly selected from the last 100 snapshots)
    - Produce interpretable patterns: Low-frequency modes (dominant patterns) & high-frequency modes (e.g., secondary patterns, outliers)
    - Identify the system of different frequencies (i.e., at $t = 50$)



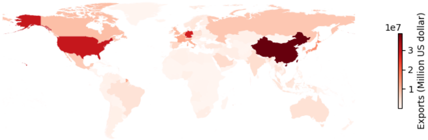
Vectorize snapshots as $\{\boldsymbol{y}_t\}_t$ Spatial modes in $\boldsymbol{W}$ Temporal modes in $\boldsymbol{X}$

International Trade

- **Import/Export merchandise trade values (annual)**[8] (215 countries/regions & period of 2000-2022)
  - Total merchandise trade values
  - Represent import/export trade data as a 215-by-23 matrix

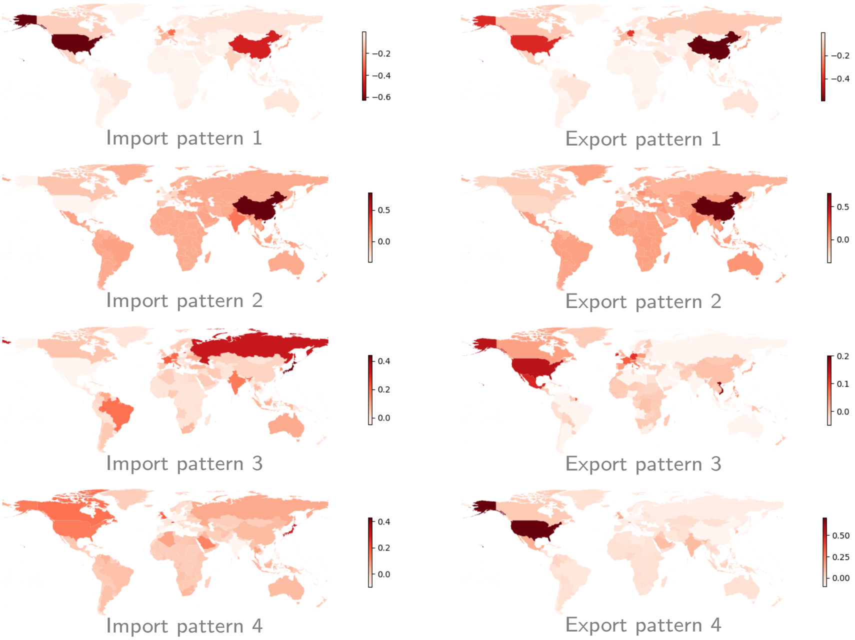


Imports from 2000 to 2022



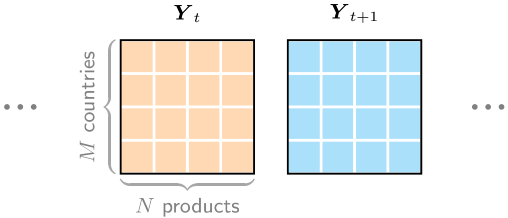
Exports from 2000 to 2022

[8] The dataset is available at https://stats.wto.org.


Import pattern 1
−0.2
−0.4
−0.6
Export pattern 1
−0.2
−0.4
Import pattern 2
0.5
0.0
Export pattern 2
0.5
0.0
Import pattern 3
0.4
0.2
0.0
Export pattern 3
0.2
0.1
0.0
Import pattern 4
0.4
0.2
0.0
Export pattern 4
0.50
0.25
0.00

International Trade

- Three-dimensional trade (Economy, Product, Year)



- On spatiotemporal systems $\boldsymbol{\mathcal{Y}} \in \mathbb{R}^{M \times N \times T}$:

$$\underbrace{\boldsymbol{y}_{n,t+1} = \boldsymbol{A}_{n,t}\boldsymbol{y}_{n,t} + \boldsymbol{\epsilon}_{n,t}}_{\text{time-varying \& product-varying}}$$

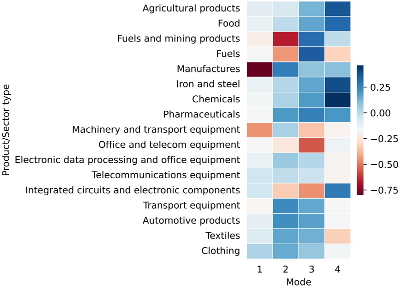
- Optimization problem of DATF:

$$\min_{\boldsymbol{\mathcal{G}},\boldsymbol{W},\boldsymbol{U},\boldsymbol{V},\boldsymbol{X}} \frac{1}{2}\sum_{n\in[N]}\sum_{t\in[T-1]}\left\|\boldsymbol{y}_{n,t+1} - (\boldsymbol{\mathcal{G}} \times_1 \boldsymbol{W} \times_2 \boldsymbol{U} \times_3 \boldsymbol{V} \times_4 \boldsymbol{x}_t^\top)\boldsymbol{y}_{n,t}\right\|_2^2$$

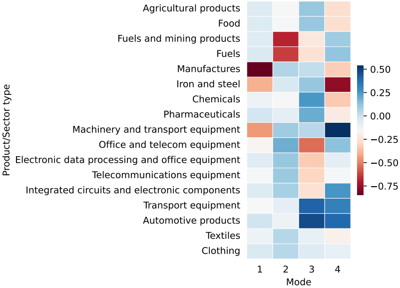
$$\text{s.t.} \quad \underbrace{\boldsymbol{W}^\top\boldsymbol{W} = \boldsymbol{I}_R}_{\text{orthogonal country patterns}}$$

Product Patterns

- On 17 merchandise types



Imports



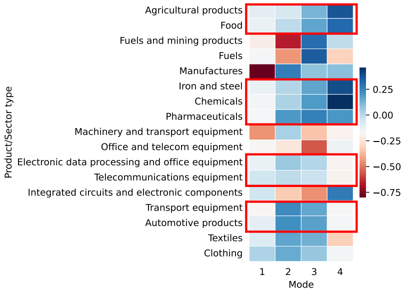
Exports

- Classify import/export merchandise according to product patterns
- Basic principle:

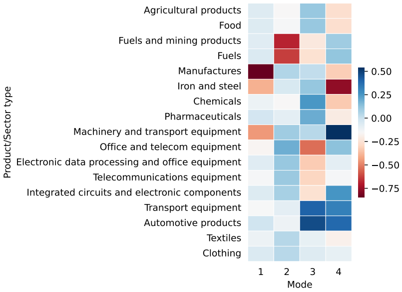
Import: What we buy? (demand) vs. Export: What we sell? (supply)

Product Patterns

- On 17 merchandise types



Imports



Exports

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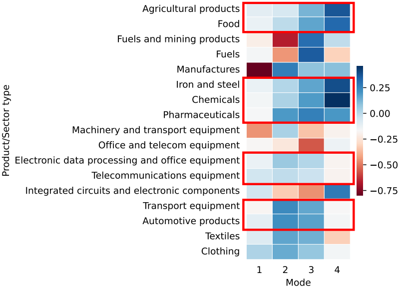
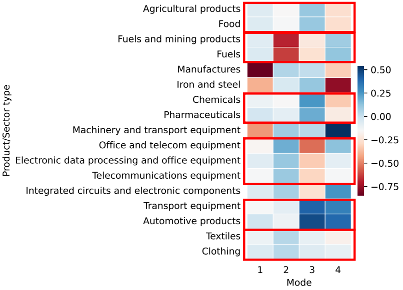
Product Patterns

- On 17 merchandise types



Imports



Exports

- Classify import/export merchandise according to product patterns
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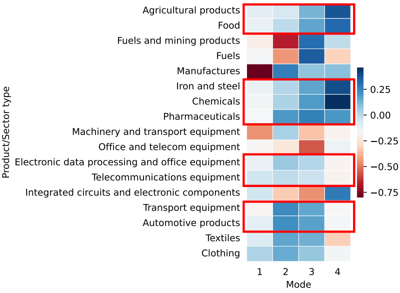
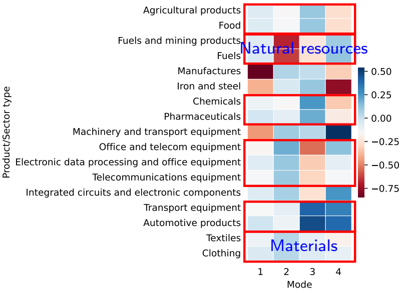
Product Patterns

- On 17 merchandise types



Imports



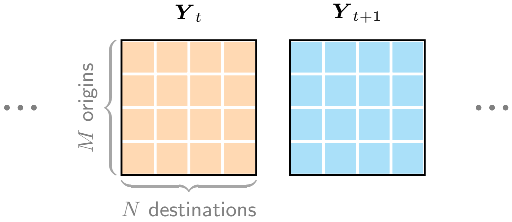
Exports

- Classify import/export merchandise according to product patterns
- Basic principle:

Import: What we buy? (demand) vs. Export: What we sell? (supply)

Human Mobility

- Origin-Destination (OD) matrices



- On spatiotemporal systems $\boldsymbol{\mathcal{Y}} \in \mathbb{R}^{M\times N\times T}$:

$$\underbrace{\boldsymbol{y}_{n,t+1} = \boldsymbol{A}_{n,t}\boldsymbol{y}_{n,t} + \boldsymbol{\epsilon}_{n,t}}_{\text{time-varying \& destination-varying}}$$

- Optimization problem of DATF:

$$\min_{\boldsymbol{\mathcal{G}},\boldsymbol{W},\boldsymbol{U},\boldsymbol{V},\boldsymbol{X}} \frac{1}{2}\sum_{n\in[N]}\sum_{t\in[T-1]}\left\|\boldsymbol{y}_{n,t+1} - (\boldsymbol{\mathcal{G}}\times_1\boldsymbol{W}\times_2\boldsymbol{U}\times_3\boldsymbol{V}\times_4\boldsymbol{x}_t^\top)\boldsymbol{y}_{n,t}\right\|_2^2$$

$$\text{s.t.} \quad \underbrace{\boldsymbol{W}^\top\boldsymbol{W} = \boldsymbol{I}_R}_{\text{orthogonal origin patterns}}$$

Human Mobility

- Chicago taxi/ridesharing data

Matching Taxi Trips with Community Areas

There are three basic steps to follow for processing taxi trip data:

- Download taxi trips in 2022 in the `.csv` format, e.g., `Taxi_Trips_-_2022.csv`.
- Use the `pandas` package in Python to process the raw trip data.
- Match trip pickup/dropoff locations with boundaries of the community area.

```python
import pandas as pd

data = pd.read_csv('Taxi_Trips_-_2022.csv')
data.head()
```

For each taxi trip, one can select some important information:

- `Trip Start Timestamp`: When the trip started, rounded to the nearest 15 minutes.
- `Trip Seconds`: Time of the trip in seconds.
- `Trip Miles`: Distance of the trip in miles.
- `Pickup Community Area`: The Community Area where the trip began. This column will be blank for locations ourside Chicago.
- `Dropoff Community Area`: The Community Area where the trip ended. This column will be blank for locations outside Chicago.

```python
df = pd.DataFrame()
df['Trip Start Timestamp'] = data['Trip Start Timestamp']
df['Trip Seconds'] = data['Trip Seconds']
df['Trip Miles'] = data['Trip Miles']
df['Pickup Community Area'] = data['Pickup Community Area']
df['Dropoff Community Area'] = data['Dropoff Community Area']
del data
df
```

Figure 2 shows taxi pickup and dropoff trips (2022) on 77 community areas in the City of Chicago. Note that the average trip duration is **1207.75 seconds** and the average trip distance is **6.16 miles**.

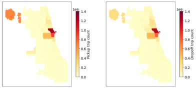


Figure 2. Taxi pickup and dropoff trips (2022) in the City of Chicago, USA. There are 4,763,961 remaining trips after the data processing.

For comparison, Figure 3 shows taxi pickup and dropoff trips (2019) on 77 community areas in the City of Chicago. Note that the average trip duration is **915.62 seconds** and the average trip distance is **3.93 miles**.

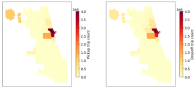


Figure 3. Taxi pickup and dropoff trips (2019) in the City of Chicago, USA. There are 12,484,572 remaining trips after the data processing. See the data processing codes.

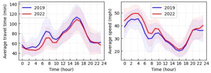


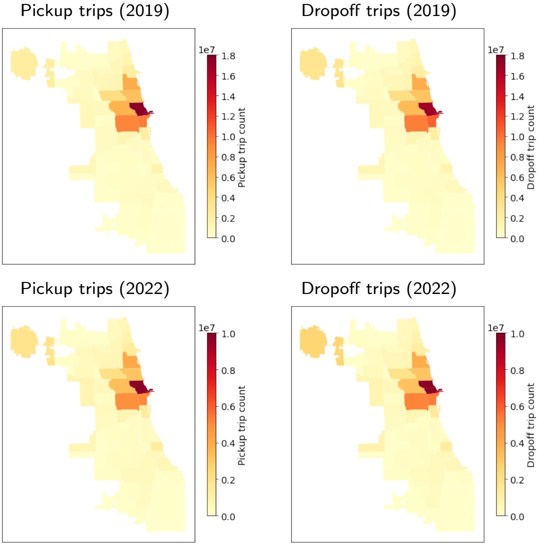
Figure 6. Average travel time and speed from area 32 (i.e., Downtown) to area 76 (i.e., Airport) in both 2019 and 2022.

```python
import numpy as np
import matplotlib.pyplot as plt

fig = plt.figure(figsize = (8, 2.5))
ax = fig.add_subplot(1, 2, 1)
# Average travel time in 2019
m1 = df1.groupby(['hour'])['Trip Seconds'].mean().values / 30
s1 = df1.groupby(['hour'])['Trip Seconds'].std().values / 30
plt.plot(m1, color = 'blue', linewidth = 1.8, label = '2019')
upper = m1 + s1
lower = m1 - s1
x_bound = np.append(np.append(np.append(np.array([0, 0]), np.arange(0, 24)),
                              np.array([24 - 1, 24 - 1])), np.arange(24 - 1, -1,
y_bound = np.append(np.append(np.append(np.array([upper[0], lower[0]]), lower),
                              np.array([lower[-1], upper[-1]])), np.flip(upper))
plt.fill(x_bound, y_bound, color = 'blue', alpha = 0.05)

# Average travel time in 2022
m1 = df2.groupby(['hour'])['Trip Seconds'].mean().values / 30
s1 = df2.groupby(['hour'])['Trip Seconds'].std().values / 30
plt.plot(m1, color = 'red', linewidth = 1.8, label = '2022')
upper = m1 + s1
lower = m1 - s1
```

Source: https://spatiotemporal-data.github.io/Chicago-mobility/taxi-data

Human Mobility

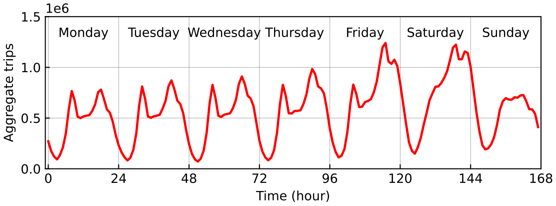
- **Ridesharing:** 96,642,881 trips in 2019 vs. 57,290,954 trips in 2022



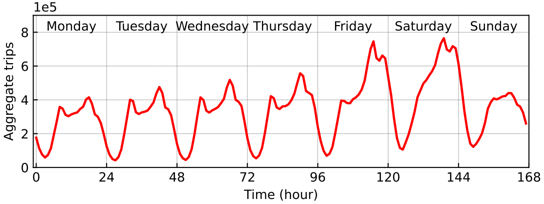
Human Mobility

- **Ridesharing:** 96,642,881 trips in 2019 vs. 57,290,954 trips in 2022

Pickup trips aggregated over 52 weeks in 2019

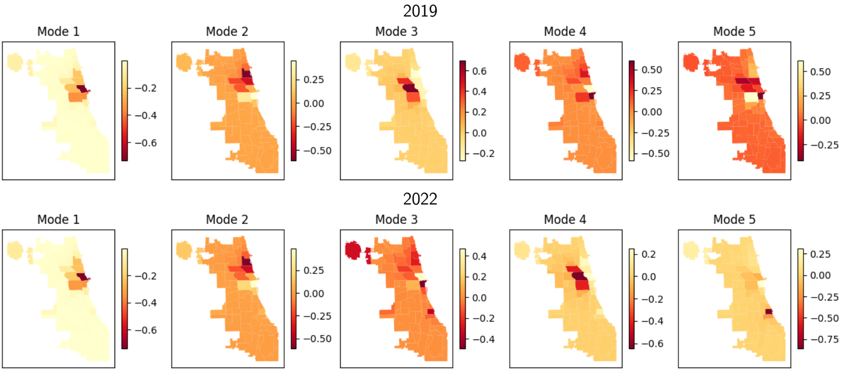


Pickup trips aggregated over 52 weeks in 2022



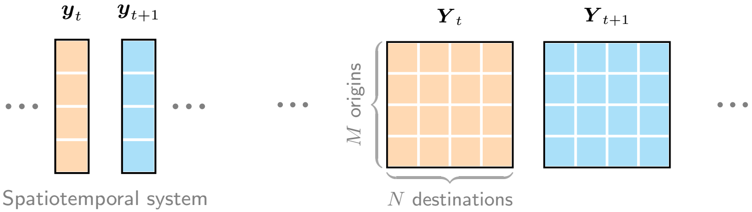
Human Mobility

- Ridesharing trip data: 77 origins $\times$ 77 destinations $\times$ 168 hours
- Our model Identifies the changes in pickup zones before and after COVID-19



Concluding Remark

- Discovering spatial/temporal patterns from 2D and 3D spatiotemporal systems with unsupervised learning:
  - Time-varying autoregression on the data
  - Tensor factorization on the coefficients



Next Stage?

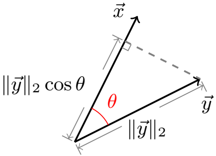
1st Direction: Clifford Product

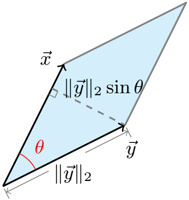
- Grassmann algebra[9]:
  - Inner product

$$\langle \vec{x}, \vec{y} \rangle = \|\vec{x}\|_2 \underbrace{\|\vec{y}\|_2 \cos\theta}_{\text{projection}}$$

  - Wedge product

$$\vec{x} \wedge \vec{y} = \underbrace{(\vec{e}_1 \wedge \vec{e}_2)}_{\text{orientation } (\pm 1)} \underbrace{\|\vec{x}\|_2 \|\vec{y}\|_2 \sin\theta}_{\text{area/determinant}}$$





- Clifford algebra (w/ Clifford product)

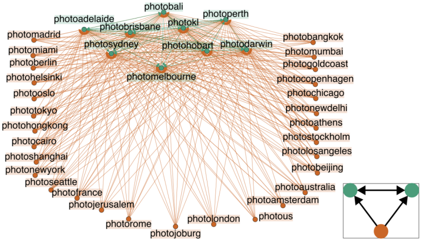
$$\vec{x} \cdot \vec{y} = \underbrace{\langle \vec{x}, \vec{y} \rangle}_{\text{inner prod.}} + \underbrace{\vec{x} \wedge \vec{y}}_{\text{Wedge prod.}}$$

- On PDE: Vector fields such as wind field and fluid dynamics

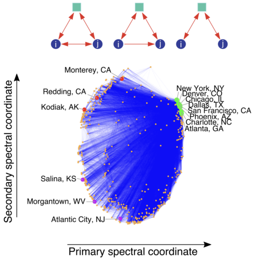
[9] On vectors $\vec{x} = x_1\vec{e}_1 + x_2\vec{e}_2$ and $\vec{y} = y_1\vec{e}_1 + y_2\vec{e}_2$.

2nd Direction: Learning from Graphs

- Graph **topology**: Graph & higher-order graph (e.g., Twitter)[10, 11]
- Graph **community**: Hypergraph & higher-order motif (e.g., transport net)



Social media (Twitter)



Transport reachability net

- Connecting anything? Graph theory, signal processing, network science, data science, machine learning, ...
- Application: System evolution analysis on dynamic graphs

[10] Picture source: https://snap.stanford.edu/higher-order/

[11] Notes: https://spatiotemporal-data.github.io/bib/

Thanks for your attention!

Any Questions?

Slides: https://xinychen.github.io/slides/temporal_modeling.pdf

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