



UNIVERSITY OF
CENTRAL FLORIDA

Machine Learning and Optimization for Understanding Spatiotemporal Systems

Time Series Imputation & Periodicity Quantification

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May 22, 2025

Orlando, USA

Spatiotemporal Data

- Transport & mobility application scenarios



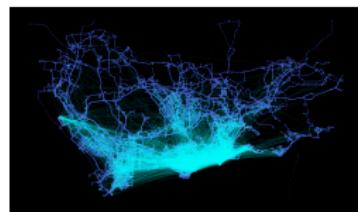
Highway (Portland)



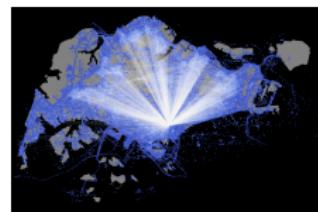
Uber movement (NYC)



Uber movement (Seattle)



Taxi trajectory (Shenzhen)



Human movement (Singapore)

- Challenges: Sparsity, high-dimensionality (network-scale), and multi-dimensionality (complicated data structure), time-varying systems

Spatiotemporal Data Imputation

- Convolution Fast Fourier transform Optimization w/ ℓ_1 -norm
- Time series imputation Speed field reconstruction



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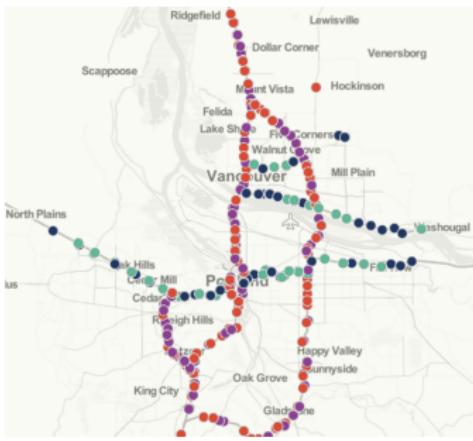


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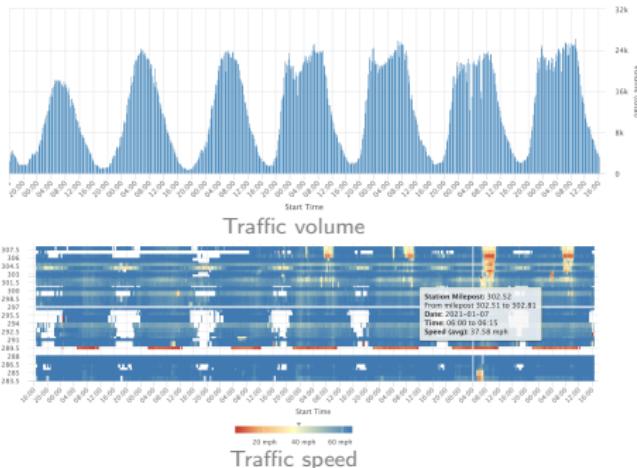
- **Xinyu Chen**, Zhanhong Chen, HanQin Cai, Nicolas Saunier, Lijun Sun (2024). “Laplacian Convolutional Representation for Traffic Time Series Imputation”. *IEEE Transactions on Knowledge and Data Engineering*, 36 (11): 6490–6502.
- Blog post: Understanding time series convolution.
https://spatiotemporal-data.github.io/posts/ts_conv

Motivation

- Portland highway traffic data¹



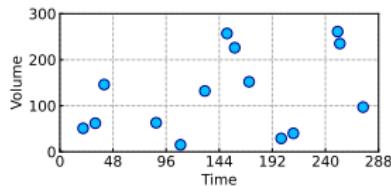
Highway network & sensor locations



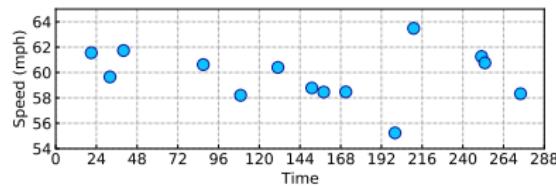
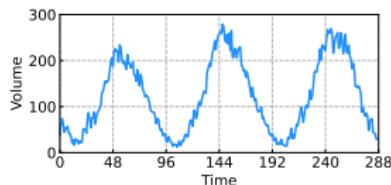
- $\mathbf{X} \in \mathbb{R}^{N \times T}$ with N spatial locations \times T time steps
- Traffic volume/speed shows strong spatial/temporal dependencies
- Missing data are there, how to improve data quality?

¹<https://portal.its.pdx.edu/home>

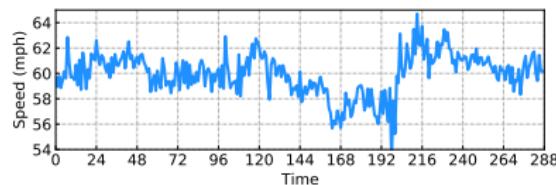
Motivation



↓
Reconstruct
traffic volume?

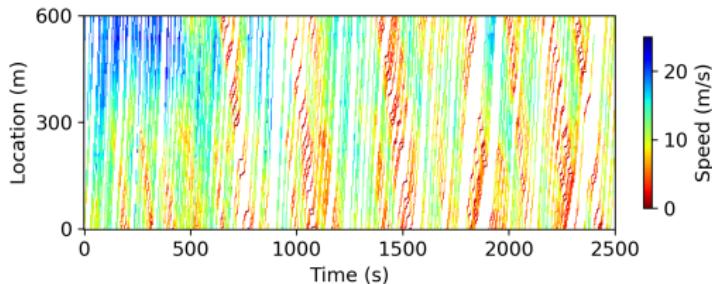


↓
Reconstruct
traffic speed?



- How to utilize the global trends of traffic time series?
- How to produce local consistency of traffic data?

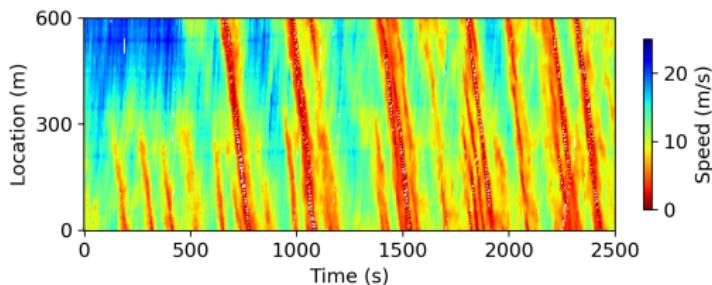
Motivation



200-by-500 matrix
(NGSIM)



Reconstruct speed field from
20% sparse trajectories?

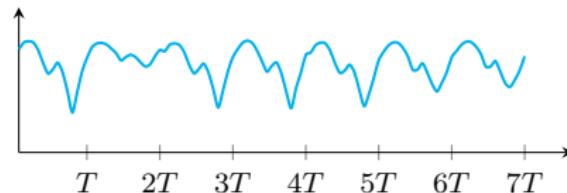


- How to learn from sparse spatiotemporal data?
- How to characterize spatial/temporal local dependencies?

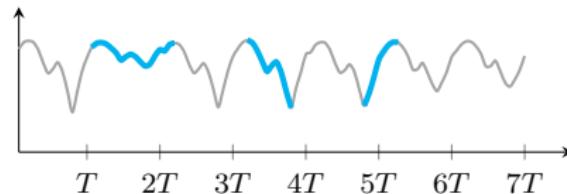
Time Series Imputation

Global/local trends in sparse data?

- Global trends (e.g., daily/weekly periodicity):



- Local trends (e.g., short-term time series trends):



Local Trend Modeling

- Intuition of Laplacian matrix

Undirected and circulant graph

Modeling \longrightarrow

$$\mathbf{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Laplacian kernel:

$$\boldsymbol{\ell} \triangleq \underbrace{(2, -1, 0, 0, -1)}_{\text{first column of } \mathbf{L}}^\top$$

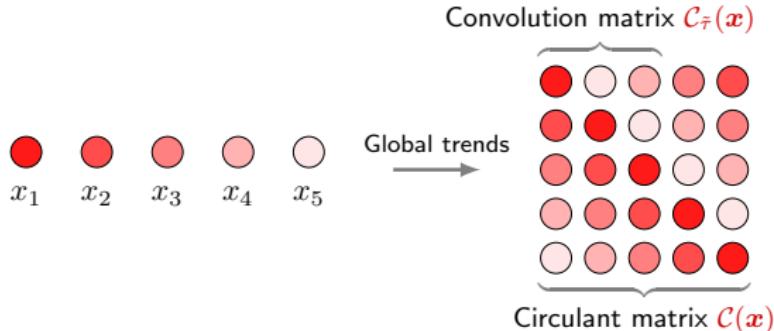
extending to the degree 2τ (i.e., graph connectivity) for $\mathbf{x} \in \mathbb{R}^T$.

- Temporal regularization:

$$\mathcal{R}(\mathbf{x}) = \underbrace{\frac{1}{2} \|\mathbf{L}\mathbf{x}\|_2^2}_{\text{mat-vec mul.}} = \underbrace{\frac{1}{2} \|\boldsymbol{\ell} * \mathbf{x}\|_2^2}_{\text{convolution}*}$$

Global Trend Modeling

Circulant matrix $\mathcal{C}(\mathbf{x})$ vs. convolution matrix $\mathcal{C}_{\tilde{\tau}}(\mathbf{x})$



- Circulant/Convolution nuclear norm minimization
 - A balance between global and local trends modeling?

CircNNM (Liu'22, Liu & Zhang'23)

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathcal{C}(\mathbf{x})\|_* \\ \text{s.t. } & \|\mathcal{P}_\Omega(\mathbf{x} - \mathbf{y})\|_2 \leq \epsilon \end{aligned}$$

on data \mathbf{y} w/ observed index set Ω .

ConvNNM (Liu'22, Liu & Zhang'23)

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathcal{C}_{\tilde{\tau}}(\mathbf{x})\|_* \\ \text{s.t. } & \|\mathcal{P}_\Omega(\mathbf{x} - \mathbf{y})\|_2 \leq \epsilon \end{aligned}$$

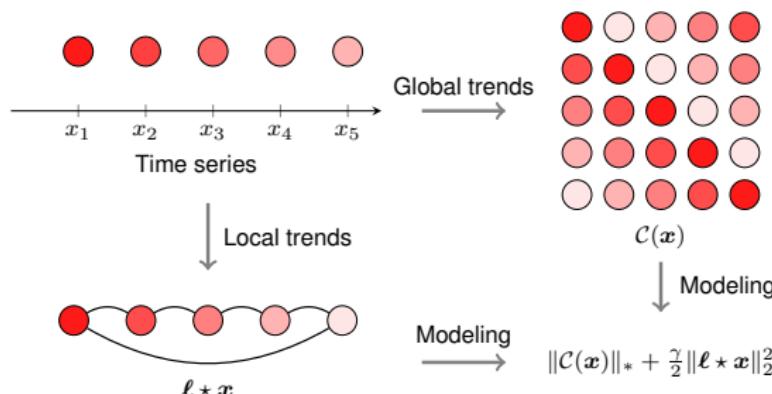
on data \mathbf{y} w/ observed index set Ω .

Global + Local Trends?

Laplacian Convolutional Representation (LCR)

For any partially observed time series $\mathbf{y} \in \mathbb{R}^T$ with observed index set Ω , LCR utilizes **circulant matrix** and **Laplacian kernel** to characterize global/local time series trends:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \underbrace{\|\mathcal{C}(\mathbf{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\ell * \mathbf{x}\|_2^2}_{\text{local}} \\ \text{s.t. } & \|\mathcal{P}_\Omega(\mathbf{x} - \mathbf{y})\|_2 \leq \epsilon \end{aligned}$$



Laplacian Convolutional Representation

- LCR model:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2} \|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 \\ \text{s.t. } & \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon \end{aligned}$$

$$\implies \min_{\boldsymbol{x}} \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2} \|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{global} + \text{local}} + \underbrace{\frac{\eta}{2} \|\mathcal{P}_\Omega(\boldsymbol{z} - \boldsymbol{y})\|_2^2}_{\text{regularization}}$$

s.t. $\boldsymbol{z} = \boldsymbol{x}$

“The alternating direction method of multipliers (ADMM) is an algorithm that solves convex optimization problems by breaking them into smaller pieces, each of which are then easier to handle.”

— Source: <https://stanford.edu/~boyd/admm.html>

Laplacian Convolutional Representation

- Augmented Lagrangian function:

$$\mathcal{L} = \underbrace{\|\mathcal{C}(\mathbf{x})\|_* + \frac{\gamma}{2}\|\ell * \mathbf{x}\|_2^2}_{\text{global + local}} + \underbrace{\frac{\lambda}{2}\|\mathbf{x} - \mathbf{z}\|_2^2 + \langle \mathbf{w}, \mathbf{x} - \mathbf{z} \rangle}_{\text{Lagrangian multiplier } \mathbf{w}} + \underbrace{\frac{\eta}{2}\|\mathcal{P}_\Omega(\mathbf{z} - \mathbf{y})\|_2^2}_{\text{observations } \mathbf{y}}$$

- Optimize \mathbf{x} w/ FFT in $\mathcal{O}(T \log T)$ time:

$$\begin{cases} \|\mathcal{C}(\mathbf{x})\|_* = \|\mathcal{F}(\mathbf{x})\|_1 = \|\hat{\mathbf{x}}\|_1 & (\text{circulant matrix}) \\ \frac{1}{2}\|\ell * \mathbf{x}\|_2^2 = \frac{1}{2T}\|\mathcal{F}(\ell) \circ \mathcal{F}(\mathbf{x})\|_2^2 = \frac{1}{2T}\|\hat{\ell} \circ \hat{\mathbf{x}}\|_2^2 & (\text{circular convolution}) \end{cases}$$

- Reformulate the optimization as ℓ_1 -norm minimization:

$$\begin{aligned} \mathbf{x} &:= \arg \min_{\mathbf{x}} \|\mathcal{C}(\mathbf{x})\|_* + \frac{\gamma}{2}\|\ell * \mathbf{x}\|_2^2 + \frac{\lambda}{2}\|\mathbf{x} - \mathbf{z} + \mathbf{w}/\lambda\|_2^2 \\ \implies \hat{\mathbf{x}} &:= \arg \min_{\hat{\mathbf{x}}} \underbrace{\|\hat{\mathbf{x}}\|_1}_{\ell_1\text{-norm}} + \frac{\gamma}{2T}\|\hat{\ell} \circ \hat{\mathbf{x}}\|_2^2 + \frac{\lambda}{2T}\|\hat{\mathbf{x}} - \hat{\mathbf{z}} + \hat{\mathbf{w}}/\lambda\|_2^2 \end{aligned}$$

Laplacian Convolutional Representation

ℓ_1 -norm Minimization (Liu & Zhang'23)

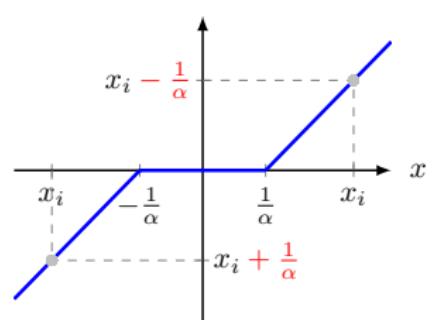
For any $\hat{\mathbf{h}} \in \mathbb{C}^T$ and $\delta \in \mathbb{R}$:

$$\min_{\hat{\mathbf{x}}} \|\hat{\mathbf{x}}\|_1 + \frac{\delta}{2} \|\hat{\mathbf{x}} - \hat{\mathbf{h}}\|_2^2$$

The solution to $\hat{\mathbf{x}}$:

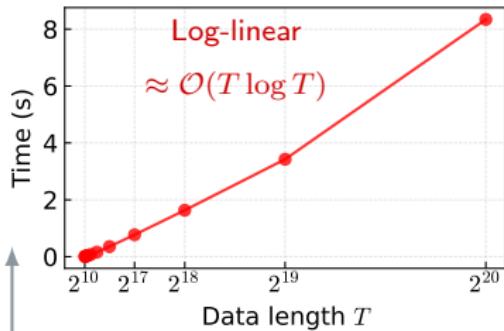
$$\hat{x}_t := \frac{\hat{h}_t}{|\hat{h}_t|} \cdot \underbrace{\max\{0, |\hat{h}_t| - 1/\delta\}}_{\text{shrinkage (e.g., ReLU)}}, t \in [T]$$

$$y_i = \frac{x_i}{|x_i|} \cdot \max\{|x_i| - 1/\alpha, 0\}$$

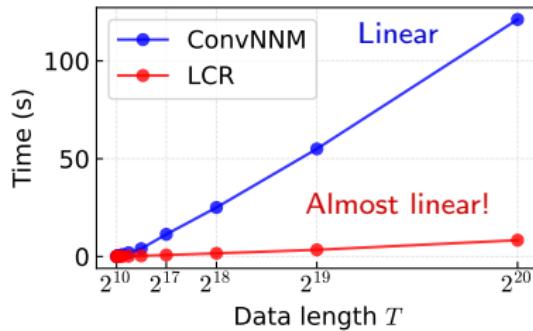


Laplacian Convolutional Representation

Time complexity & scalability & efficiency?



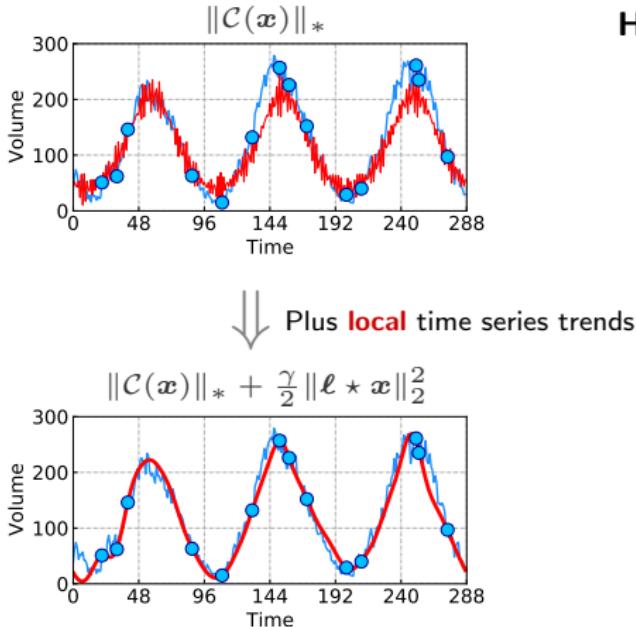
Empirical time complexity



On the synthetic data $y \in \mathbb{R}^T$ with
 $T \in \{2^{10}, 2^{11}, \dots, 2^{20}\}$

Experiments

- Traffic speed imputation² (95% missing rate)



Highlights:

- Rethink the importance of local trend modeling in traffic data imputation tasks.
- Find a unified global and local trend modeling framework whose optimization can be efficiently solved by **FFT**:

$$\min_{\mathbf{x}} \underbrace{\|\mathcal{C}(\mathbf{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\ell * \mathbf{x}\|_2^2}_{\text{local}}$$

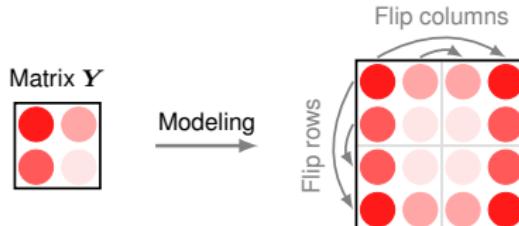
s. t. $\|\mathcal{P}_\Omega(\mathbf{x} - \mathbf{y})\|_2 \leq \epsilon$

²Blue dot: partial observation; red line: imputation.

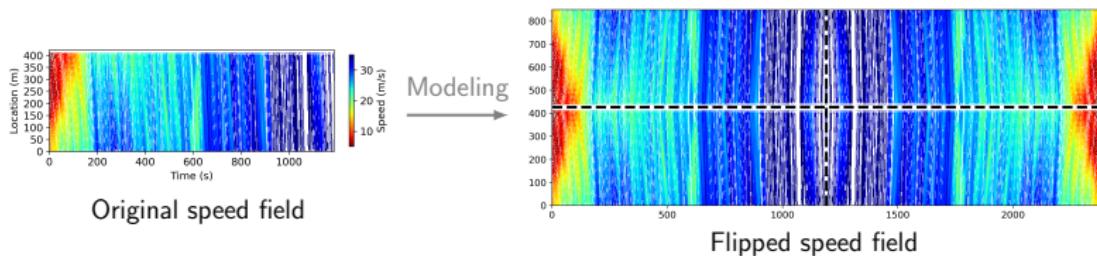
Experiments

Speed field reconstruction³

- Flipping operation on a matrix:



- Flipping operation on a speed field of vehicular traffic flow:



³Highway Drone (HighD) dataset at <https://www.hightd-dataset.com/>

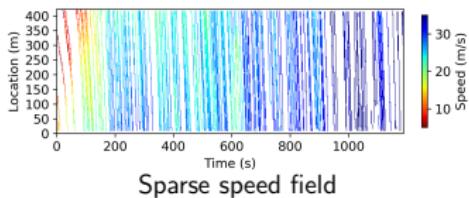
Experiments

Speed field reconstruction in German highways⁴

- Scenario: Mask trajectories of 70% vehicles
- LCR-2D on partially observed $\mathbf{Y} \in \mathbb{R}^{N \times T}$:

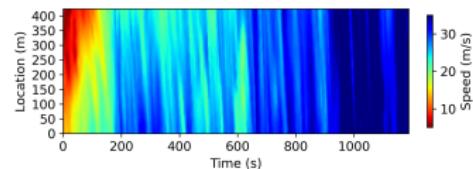
$$\min_{\mathbf{X}} \underbrace{\|\mathcal{C}(\mathbf{X})\|_*}_{\text{global trend}} + \frac{\gamma}{2} \underbrace{\|(\ell_s \ell^\top) \star \mathbf{X}\|_F^2}_{\text{local trend}}$$

s.t. $\|\mathcal{P}_\Omega(\mathbf{X} - \mathbf{Y})\|_F \leq \epsilon$

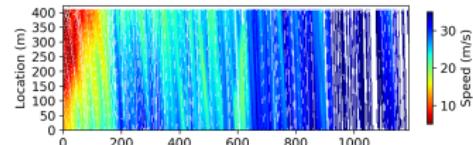


Sparse speed field

LCR-2D



Reconstructed speed field



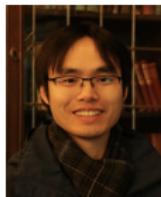
Ground-truth speed field

⁴Highway Drone (HighD) dataset at <https://www.hightd-dataset.com/>

Quantifying Time Series Periodicity

(Ongoing Research)

- Interpretable ML Optimization w/ ℓ_0 -norm Mixed-integer programming
- Human mobility regularity Climate system seasonality



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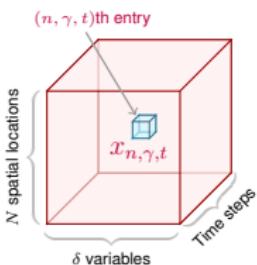


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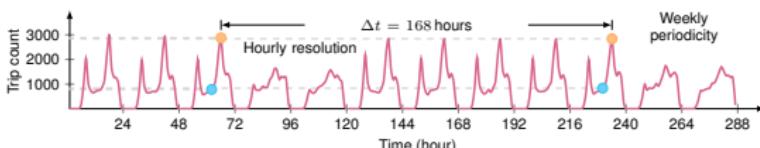
Motivation

Human mobility data show daily/weekly regularity and periodicity?

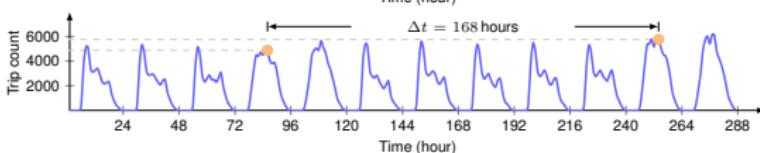
A



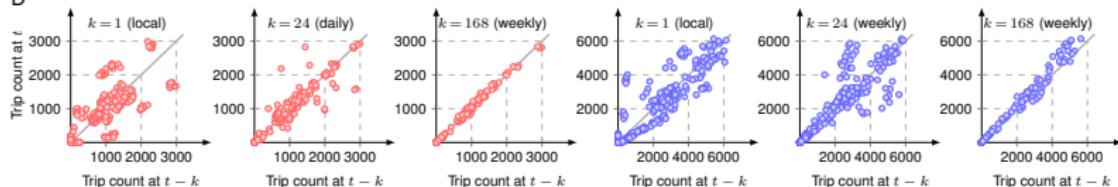
B



C



D

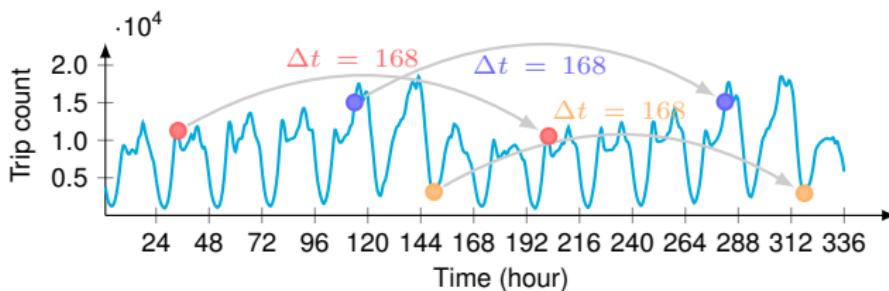


“Closeness” to the
anti-diagonal $y = x$

$x_t \approx x_{t-168}$ (weekly periodicity)

Motivation

Weekly periodicity of ridesharing trip time series in Chicago



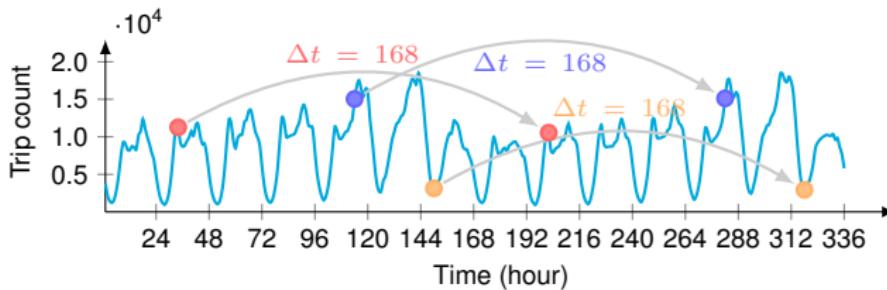
What motivate us most about periodicity?

- ① **Resilience and stability of systems:** Empirically measure the periodicity and predictability of urban systems.
- ② **Optimization of transport systems:** Optimize resources (e.g., public transit, taxi, rideshare, and micromobility) to meet transport demand efficiently.
- ③ **Design of sustainable transport & infrastructure:** Implement energy-efficient solutions tailored to peak hours.

Motivation

- Time series autoregression on $\mathbf{x} \in \mathbb{R}^T$

$$\mathbf{w} := \arg \min_{\{w_k\}_{k \in [d]}} \sum_{t \in [d+1, T]} \left(\mathbf{x}_t - \sum_{k \in [d]} w_k \mathbf{x}_{t-k} \right)^2$$



Periodicity of rideshare trip time series

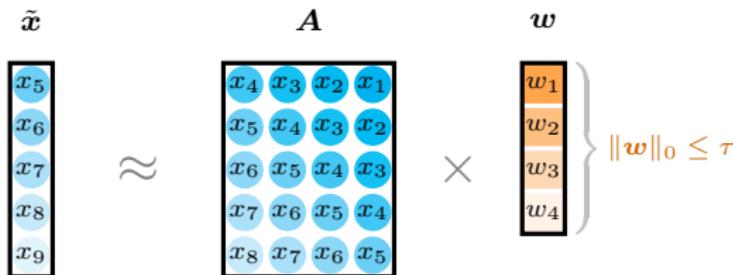
- Sparse coefficient vector \mapsto **Interpretability?**

$$\mathbf{w} = (\underbrace{0.33}_{k=1}, 0, \dots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

Valorizing Autoregression

- Time series autoregression

$$\begin{aligned} \mathbf{w} &:= \arg \min_{\{w_k\}_{k \in [d]}} \sum_{t \in [d+1, T]} \left(\mathbf{x}_t - \sum_{k \in [d]} w_k \mathbf{x}_{t-k} \right)^2 \\ &= \arg \min_{\mathbf{w}} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 \end{aligned}$$



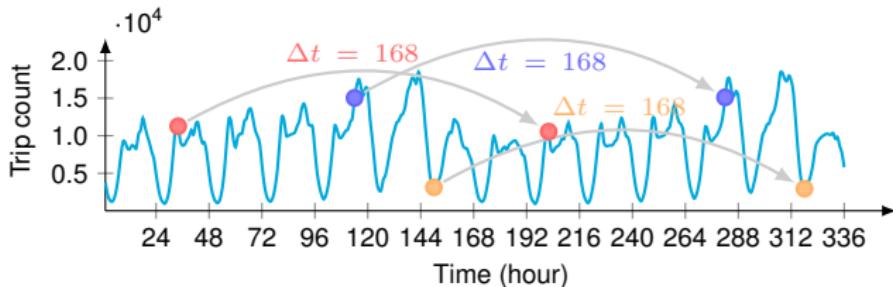
Autoregression on time series $\mathbf{x} = (x_1, x_2, \dots, x_9)^\top$ w/ sparsity $\tau \in \mathbb{Z}^+$

- Sparse autoregression

$$\begin{array}{ll} \min_{\mathbf{w} \geq 0} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 & \min_{\mathbf{w}, \boldsymbol{\beta}} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 \\ \text{s.t. } \underbrace{\|\mathbf{w}\|_0 \leq \tau}_{\text{sparsity w/ } \ell_0\text{-norm}} & \iff \text{s.t. } \begin{cases} 0 \leq \mathbf{w} \leq \boldsymbol{\beta}, \boldsymbol{\beta} \in \{0, 1\}^d \\ \|\boldsymbol{\beta}\|_1 \leq \tau \end{cases} \end{array}$$

Solution Quality

- Subspace pursuit (SP) sometimes fails

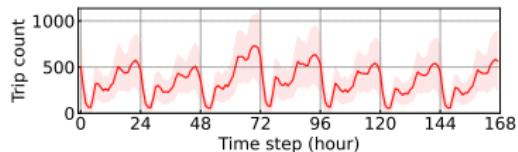
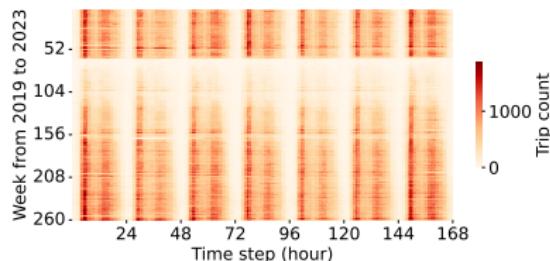
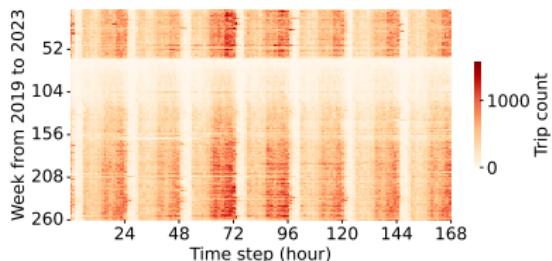


- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity $\tau = 2$):

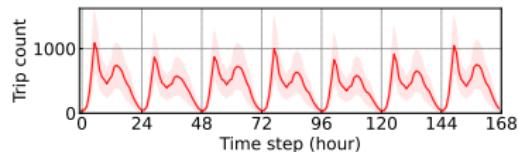
$$\underbrace{\boldsymbol{w} = (\dots, \underbrace{0.02}_{k=53}, \dots, \underbrace{0.96}_{k=168})^\top}_{\text{loss func. } = 8.32 \times 10^7 \text{ (SP)}} \quad \text{vs.} \quad \underbrace{\boldsymbol{w} = (\underbrace{0.22}_{k=1}, \dots, \underbrace{0.77}_{k=168})^\top}_{\text{loss func. } = 6.25 \times 10^7 \text{ (MIP)}}$$

John F. Kennedy International Airport

- Pickup/Dropoff trips in airport
 - Pickup trips are relevant to flight delay, baggage claim, and other factors.
 - Dropoff trips to airport are highly related to flight schedules.



Pickup trips from airport



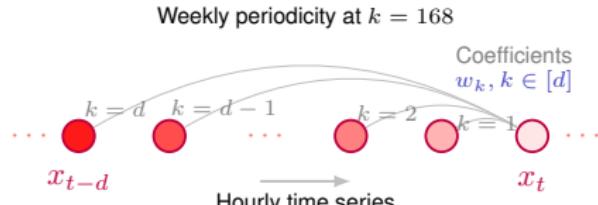
Dropoff trips to airport

- Sparse coefficient vectors (**sparsity $\tau = 3$**):

$$\mathbf{w} = (\underbrace{0.31}_{k=1}, \dots, \underbrace{0.28}_{k=24}, \dots, \underbrace{0.41}_{k=168})^\top \quad \text{vs.} \quad \mathbf{w} = (\underbrace{0.18}_{k=1}, \dots, \underbrace{0.35}_{k=24}, \dots, \underbrace{0.47}_{k=168})^\top$$

Spatially- and Time-Varying Autoregression

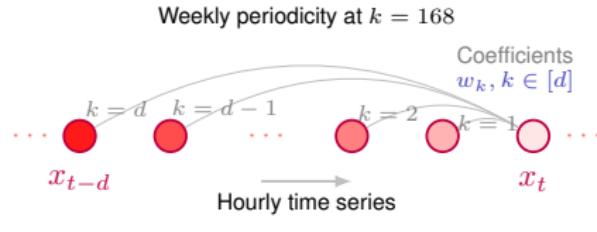
Univariate autoregression



$$\min_t \sum_{k \in [d]} \left(x_t - \sum_{k \in [d]} w_k x_{t-k} \right)^2$$

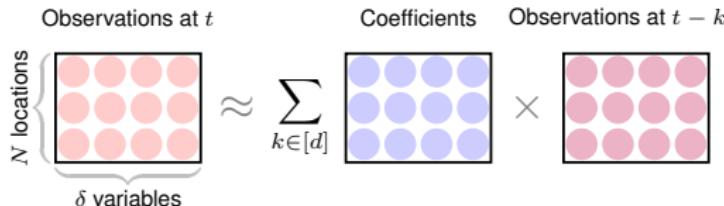
Spatially- and Time-Varying Autoregression

Univariate autoregression



$$\min_t \sum_{k \in [d]} \left(x_t - \sum_{k \in [d]} w_k x_{t-k} \right)^2$$

Multidimensional autoregression



$$\min_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_t \left(x_{n, \gamma, t} - \sum_{k \in [d]} w_{n, \gamma, k} x_{n, \gamma, t-k} \right)^2$$

Envisioning Human Mobility

- Ridesharing trip data $\{x_{n,\gamma}\}$ across $\gamma \in [\delta]$ years
- Reformulate sparse autoregression:

$$\begin{array}{c} \text{year} \\ \swarrow \quad \searrow \\ \text{spatial location} \quad \text{hourly time step} \\ \downarrow \quad \quad \quad \downarrow \\ \min_{\{\boldsymbol{w}_{n,\gamma}\}, \boldsymbol{\beta}} \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_{t \in [d+1, T_\gamma]} \left(x_{n,\gamma,t} - \sum_{k \in [d]} w_{n,\gamma,k} x_{n,\gamma,t-k} \right)^2 \\ \text{s.t. } \underbrace{\boldsymbol{\beta} \in \{0, 1\}^d}_{\text{binary var.}} \quad \underbrace{0 \leq \boldsymbol{w}_{n,\gamma} \leq \boldsymbol{\beta}}_{\text{upper bound in } \{0, 1\}} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\text{sum of binary var.}} \end{array}$$

- MIP problem w/ $(N\delta + 1)d$ variables!
- How to handle thousands or millions of (e.g., $N\delta = 10^6$) time series?

Quantify weekly periodicity by $\{w_{n,\gamma,k}\}$ at index $k = 168$

Spatially- and Time-Varying Systems

- Rideshare trip data $\{\mathbf{x}_{n,\gamma}\}_{n \in [N], \gamma \in [\delta]}$ across $\gamma \in [\delta]$ months/years
interpretable sparse autoregression:

$$\begin{aligned} & \min_{\{\mathbf{w}_\gamma\}_{\gamma \in [\delta]}} \sum_{\gamma \in [\delta]} \|\tilde{\mathbf{x}}_\gamma - \mathbf{A}_\gamma \mathbf{w}_\gamma\|_2^2 \\ \text{s.t. } & \begin{cases} \mathbf{w}_\gamma \geq 0 & \text{(non-negativity)} \\ \|\mathbf{w}_\gamma\|_0 \leq \tau & \text{(sparsity)} \\ \text{supp}(\mathbf{w}_\gamma) = \text{supp}(\mathbf{w}_{\gamma+1}) & \text{(no local difference)} \end{cases} \end{aligned}$$

making these coefficient vectors comparable across δ months/years.

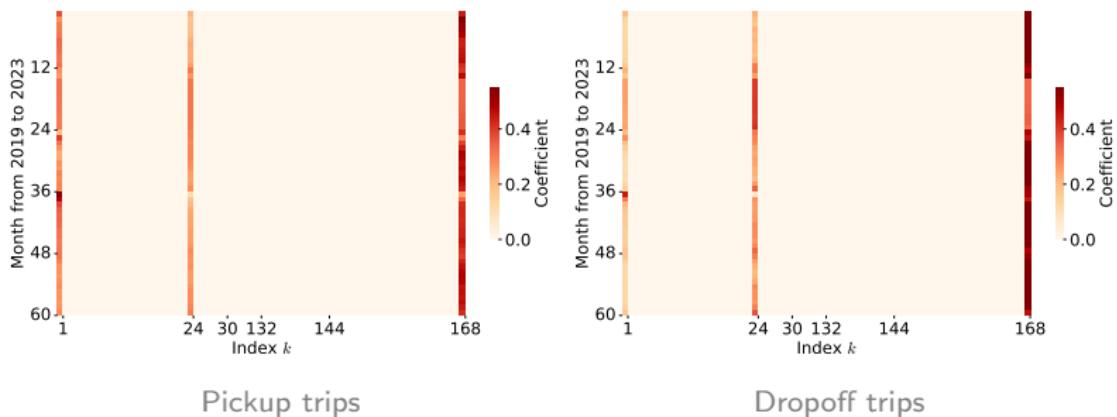
- Constraints w/ binary variables $\beta_\gamma \in \{0, 1\}^d$:

$$\underbrace{0 \leq \mathbf{w}_\gamma \leq \beta_\gamma}_{\text{upper bound } \{0, 1\}} \quad \underbrace{\sum_{k \in [d]} \beta_{\gamma,k} \leq \tau}_{\text{sum of binary var.}} \quad \underbrace{\beta_\gamma - \beta_{\gamma+1} = 0}_{\text{comparability across } \mathbf{w}_\gamma, \forall \gamma}$$

- MIP problem w/ $2d\delta$ decision variables!
- (Efficiency?) ML prunes the search space, e.g., $2\tau_0\delta$ decision variables ($\tau < \tau_0 \ll d$) instead.

John F. Kennedy International Airport

- Coefficients $\{w_\gamma\}_{\gamma \in [\delta]}$ at $S = \{\underbrace{1}_{\text{local}}, \underbrace{24}_{\text{daily}}, \underbrace{168}_{\text{weekly}}\}$ across $\delta = 60$ months
 - ① Stronger weekly periodicity of dropoff trips than pickup trips
 - ② Stronger daily periodicity in 2020
 - ③ Weaker weekly periodicity in 2020



- Identify system patterns that evolve over time for human mobility

Spatially- and Time-Varying Autoregression

$$\min_{\{\boldsymbol{w}_{m,n,\gamma}\}, \boldsymbol{\beta}} \sum_{m \in [M]} \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_{t \in [d+1, T_\gamma]} (x_{m,n,\gamma,t} - \sum_{k \in [d]} w_{m,n,\gamma,k} x_{m,n,\gamma,t-k})^2$$

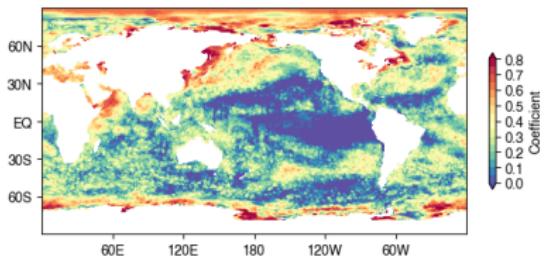
s.t.

$$\begin{cases} \boldsymbol{\beta} \in \{0, 1\}^d & \text{binary decision var.} \\ 0 \leq \boldsymbol{w}_{m,n,\gamma} \leq \boldsymbol{\beta}, \forall m, n, \gamma \\ \|\boldsymbol{\beta}\|_1 \leq \tau \\ \|\boldsymbol{w}_{m,n,\gamma}\|_1 = 1, \forall m, n, \gamma \end{cases}$$

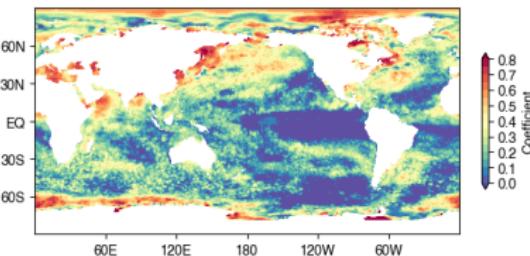
sparsity constraints

ℓ_1 -normalization

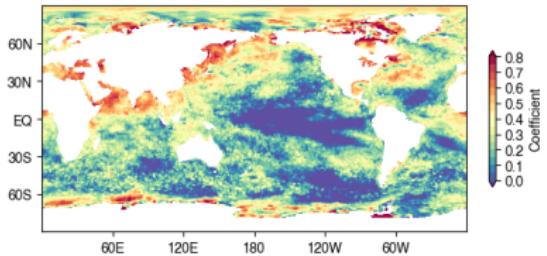
Sea Surface Temperature



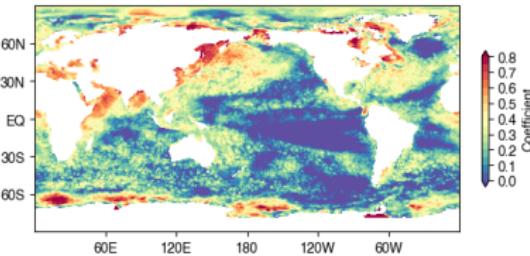
1980s



1990s



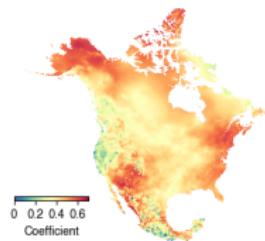
2000s



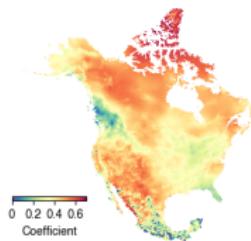
2010s

- Identify yearly periodicity at $k = 12$ from SST data ($\tau = 4$)
 - ❶ The areas of El Niño events are less seasonal/predictable
 - ❷ Arctic becomes less seasonal/predictable in the past 20 years
- Insights into climate change & global warming & sustainable development

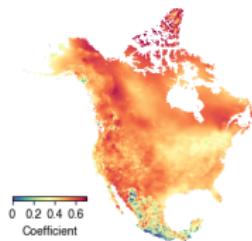
North America Temperature



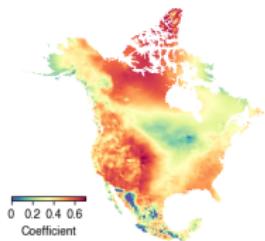
tmin, 1980s



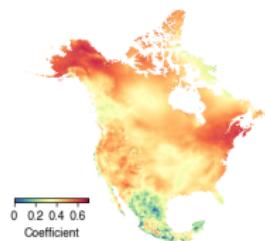
tmin, 1990s



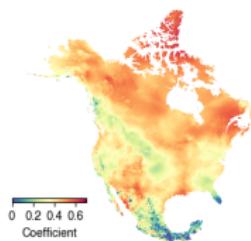
tmin, 2000s



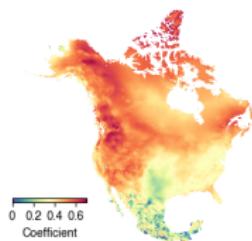
tmin, 2010s



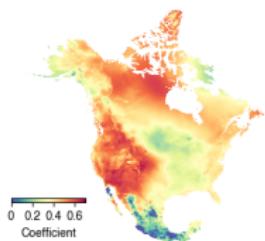
tmax, 1980s



tmax, 1990s



tmax, 2000s



tmax, 2010s

- Identify yearly periodicity at $k = 12$ from temperature data ($\tau = 4$)
 - ❶ Stronger yearly seasonality in high-latitude areas
 - ❷ Less seasonal temperature in south areas (e.g., Mexico)
 - ❸ Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s



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Thanks for your attention!

Any Questions?

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