Modeling Urban Traffic Data with Matrix and Tensor Approaches

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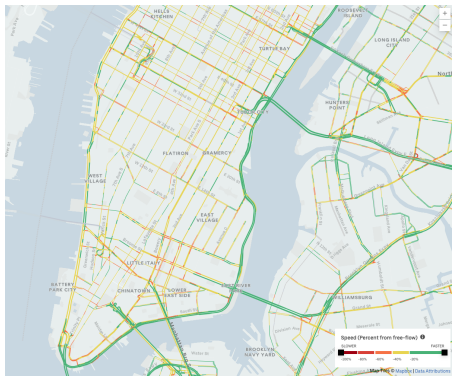
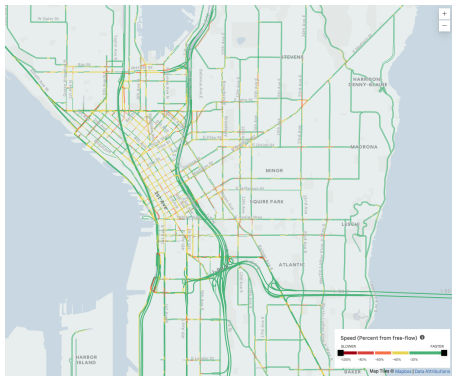
Urban Traffic Data

- Transport & mobility application scenarios

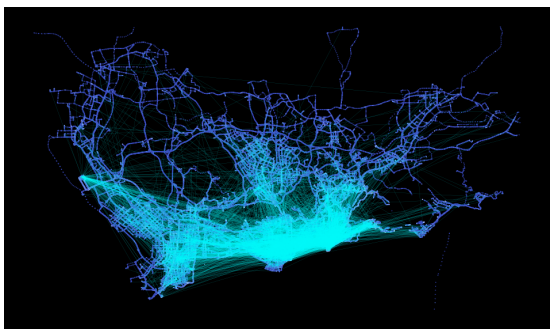
Highway (Portland)



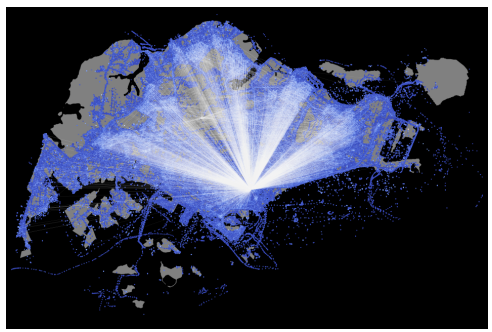
Uber movement (NYC)



Uber movement (Seattle)



Taxi trajectory (Shenzhen)



Passenger flow (Singapore)

- Challenges: Sparsity, time-varying system, high-dimensionality, and multi-dimensionality

Papers:

- X. Chen, Z. Cheng, H.Q. Cai, N. Saunier, L. Sun (2024). "Laplacian Convolutional Representation for Traffic Time Series Imputation". IEEE Transactions on Knowledge and Data Engineering, 36 (11): 6490–6502.

- X. Chen, L. Sun (2022). "Bayesian Temporal Factorization for Multidimensional Time Series Prediction". IEEE Transactions on Pattern Analysis and Machine Intelligence, 44 (9): 4659–4673.

- X. Chen, X.L. Zhao, C. Cheng (2024). "Forecasting Urban Traffic States with Sparse Data Using Hankel Temporal Matrix Factorization". INFORMS Journal on Computing. Early access.

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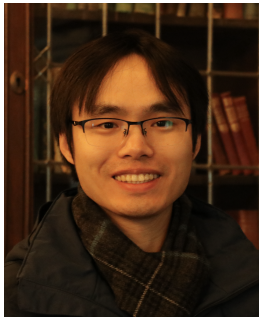
ML $\Rightarrow$ Imputation & Prediction & Pattern Discovery

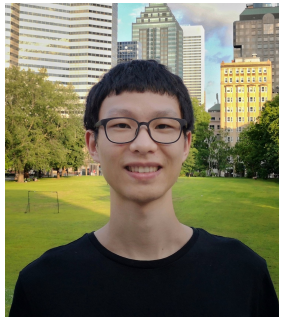
Laplacian Convolutional Representation for Traffic Time Series Imputation

IEEE Transactions on Knowledge and Data Engineering, 2024

https://doi.org/10.1109/TKDE.2024.3419698



Xinyu Chen



Zhanhong Cheng

HanQin Cai

Nicolas Saunier

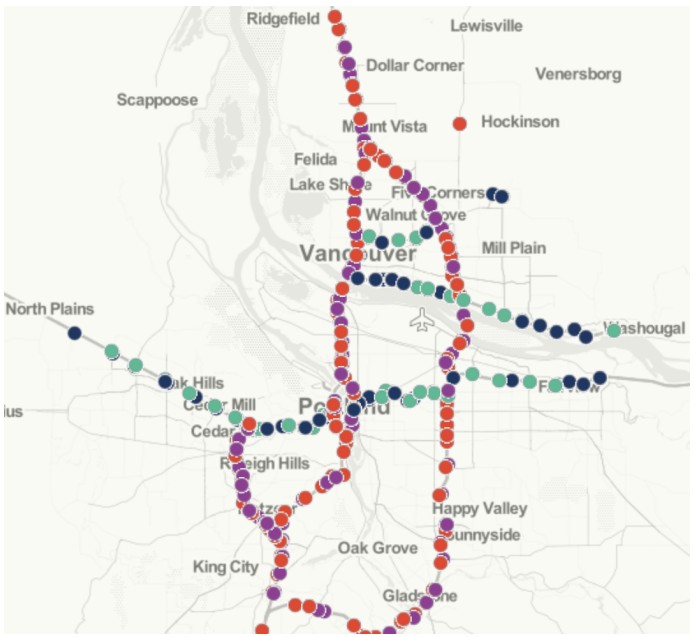
Lijun Sun

Materials:

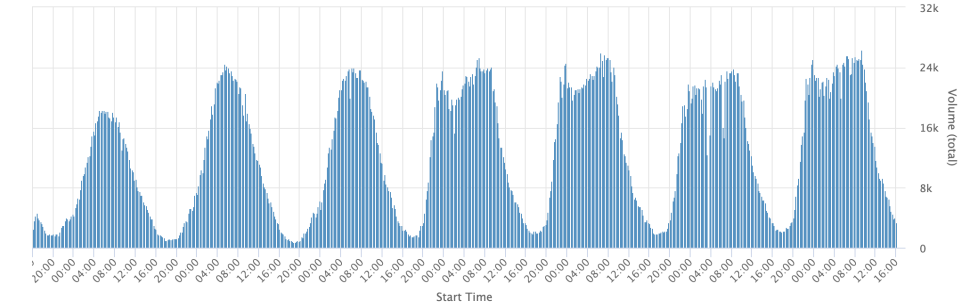
- GitHub: https://github.com/xinychen/transdim
- Blog: https://spatiotemporal-data.github.io/posts/ts_conv

Traffic Flow Data

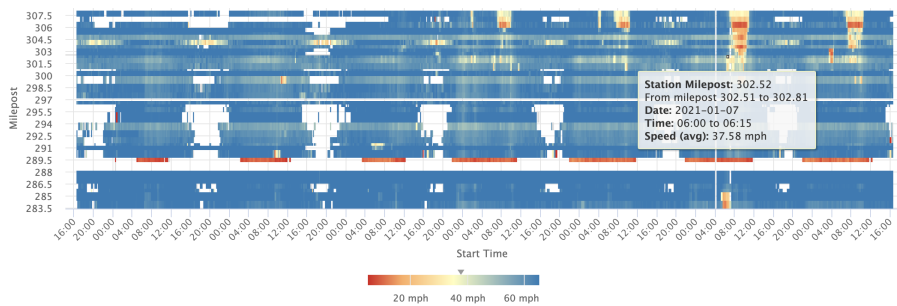
- Portland highway traffic data[1]



Highway network & sensor locations



Traffic volume



Traffic speed

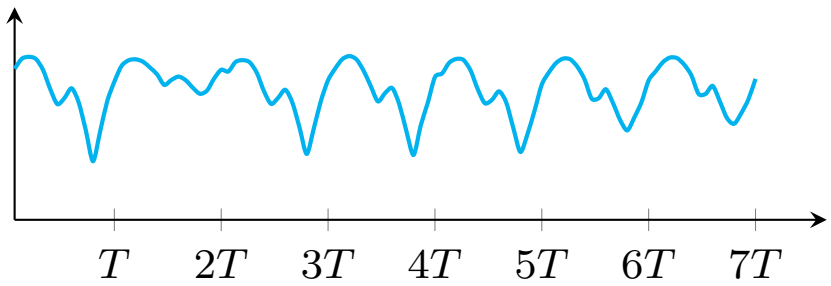
- $\boldsymbol{X} \in \mathbb{R}^{N \times T}$ with N spatial locations $\times$ T time steps
- Traffic volume/speed shows strong spatial/temporal dependencies

[1] https://portal.its.pdx.edu/home

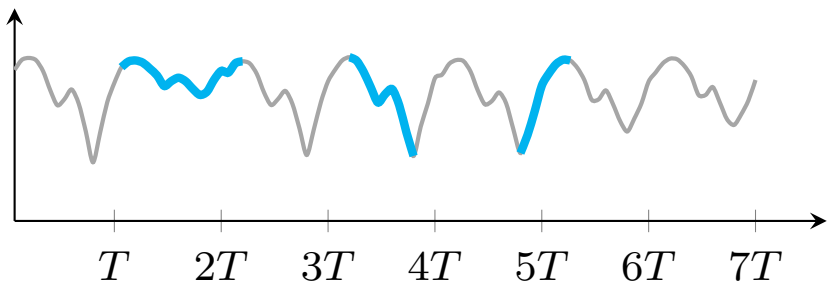
Time Series Imputation

Motivation: Traffic imputation

- Global trends (e.g., long-term quasi-seasonality & daily/weekly rhythm):



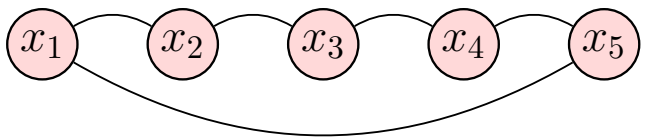
- Local trends (e.g., short-term time series trends):



How to characterize both global and local trends in sparse time series?

Local Trend Modeling

- Intuition of (circulant) Laplacian matrix



Undirected and circulant graph

Modeling $\longrightarrow$

$$\boldsymbol{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Define Laplacian kernel:

$$\boldsymbol{\ell} \triangleq (2, -1, 0, 0, -1)^\top$$

$$\Downarrow$$

$$\boldsymbol{\ell} \triangleq (\underbrace{2\tau}_{\text{degree}}, \underbrace{-1, \cdots, -1}_{\tau}, 0, \cdots, 0, \underbrace{-1, \cdots, -1}_{\tau})^\top \in \mathbb{R}^T$$

for any time series $\boldsymbol{x} = (x_1, \ldots, x_T)^\top \in \mathbb{R}^T$.

- Temporal regularization (w/ circular convolution $\star$):

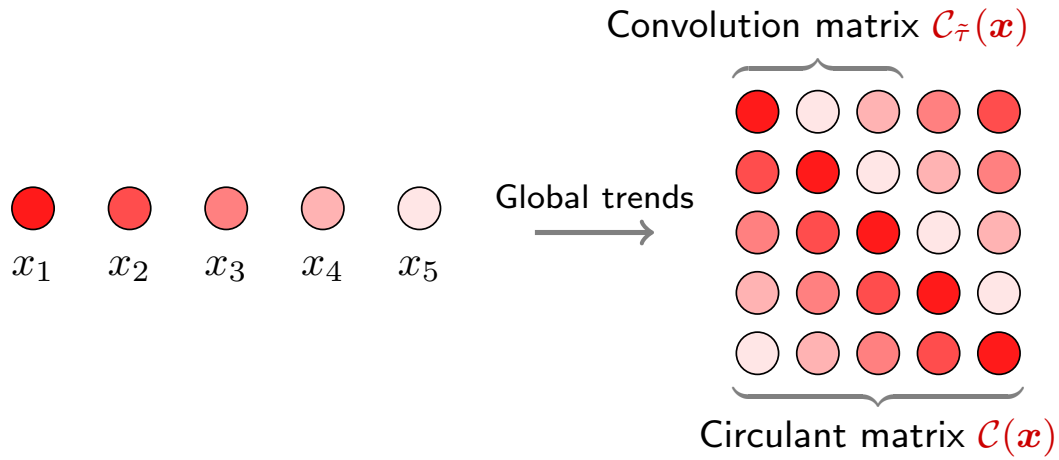
$$\mathcal{R}_\tau(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{L}\boldsymbol{x}\|_2^2 = \frac{1}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2$$

"... The circulant graph has an adjacency matrix that is a circulant matrix."

— Circulant graph on Wikipedia

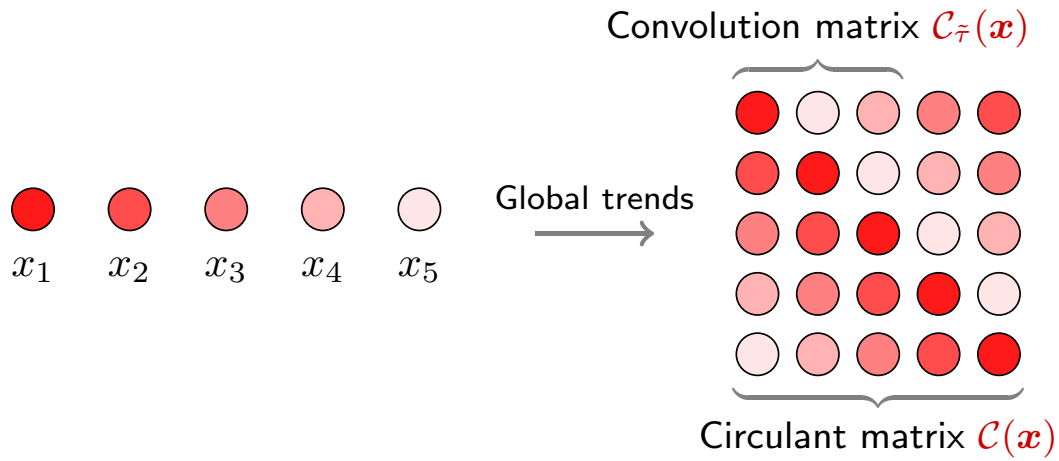
Global Trend Modeling

Circulant matrix $\mathcal{C}(\boldsymbol{x})$ vs. convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})$



Global Trend Modeling

Circulant matrix $\mathcal{C}(\boldsymbol{x})$ vs. convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})$



- Circulant/Convolution nuclear norm minimization
  - A balance between global and local trends modeling?

CircNNM (Liu'22, Liu & Zhang'23)

Estimating $\boldsymbol{x}$:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* \quad \text{s.t.} \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \le \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

ConvNNM (Liu'22, Liu & Zhang'23)

Estimating $\boldsymbol{x}$:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})\|_* \quad \text{s.t.} \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \le \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

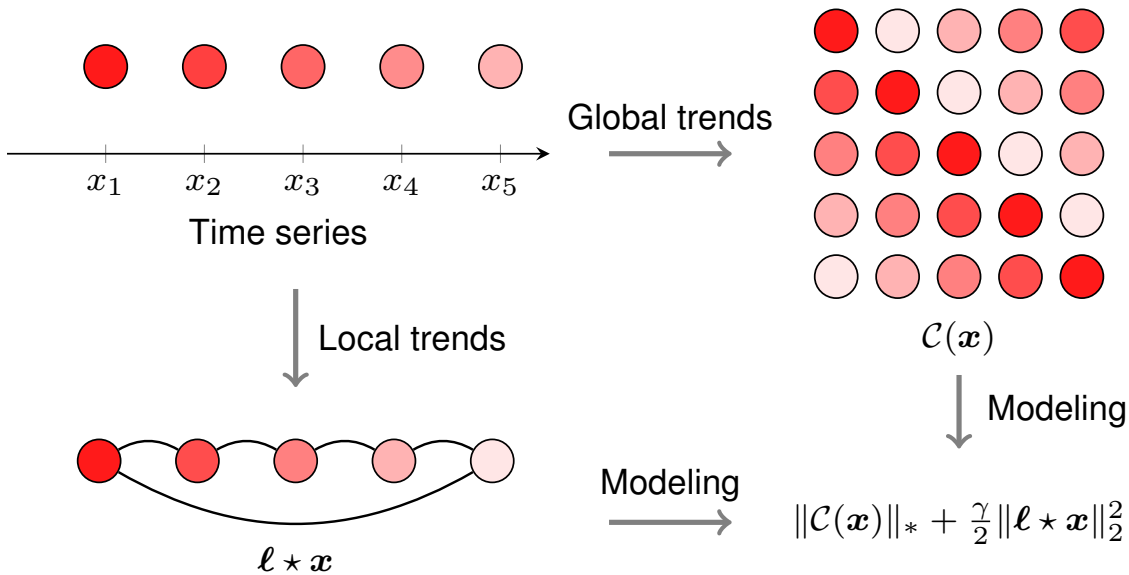
Global + Local Trends?

Laplacian Convolutional Representation (LCR)

For any partially observed time series $\boldsymbol{y} \in \mathbb{R}^T$ with observed index set Ω, LCR utilizes circulant matrix and Laplacian kernel to characterize global and local trends in time series, respectively, i.e.,

$$\min_{\boldsymbol{x}} \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{local}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon$$



Laplacian Convolutional Representation

- Augmented Lagrangian function:[2]

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) = \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{z}\|_2^2 + \langle\boldsymbol{w},\boldsymbol{x}-\boldsymbol{z}\rangle + \frac{\eta}{2}\|\mathcal{P}_{\Omega}(\boldsymbol{z}-\boldsymbol{y})\|_2^2$$

- The ADMM scheme:

$$\begin{cases} \boldsymbol{x} := \arg\min\limits_{\boldsymbol{x}} \ \mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) & \text{(Nuclear norm minimization)} \\ \boldsymbol{z} := \arg\min\limits_{\boldsymbol{z}} \ \mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) & \text{(Closed-form solution)} \\ \boldsymbol{w} := \boldsymbol{w} + \lambda(\boldsymbol{x}-\boldsymbol{z}) & \text{(Standard update)} \end{cases}$$

- Optimize $\boldsymbol{x}$?

$$\underbrace{\|\mathcal{C}(\boldsymbol{x})\|_* = \|\mathcal{F}(\boldsymbol{x})\|_1}_{\text{property of circulant matrix}} \quad \& \quad \underbrace{\frac{1}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 = \frac{1}{2T}\|\mathcal{F}(\boldsymbol{\ell})\circ\mathcal{F}(\boldsymbol{x})\|_2^2}_{\text{property of circular convolution}}$$

Nuclear norm minimization $\Rightarrow$ ℓ_1-**norm minimization with FFT** in $\mathcal{O}(T\log T)$ time.

[2] $\boldsymbol{w}\in\mathbb{R}^T$ (Lagrange multiplier); $\langle\cdot,\cdot\rangle$ (inner product).

Laplacian Convolutional Representation

- Optimize $\boldsymbol{x}$ via FFT (in $\mathcal{O}(T \log T)$ time):

$$\boldsymbol{x} := \arg\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x} - \boldsymbol{z} + \boldsymbol{w}/\lambda\|_2^2$$

$$\Longrightarrow \hat{\boldsymbol{x}} := \arg\min_{\hat{\boldsymbol{x}}} \ \|\hat{\boldsymbol{x}}\|_1 + \frac{\gamma}{2T}\|\hat{\boldsymbol{\ell}} \circ \hat{\boldsymbol{x}}\|_2^2 + \frac{\lambda}{2T}\|\hat{\boldsymbol{x}} - \hat{\boldsymbol{z}} + \hat{\boldsymbol{w}}/\lambda\|_2^2$$

where we introduce $\{\hat{\boldsymbol{\ell}}, \hat{\boldsymbol{x}}, \hat{\boldsymbol{z}}, \hat{\boldsymbol{w}}\} \triangleq \mathcal{F}\{\boldsymbol{\ell}, \boldsymbol{x}, \boldsymbol{z}, \boldsymbol{w}\}$ (i.e., FFT).

ℓ_1-norm Minimization in Complex Space (Liu & Zhang'23)

For any optimization problem in the form of ℓ_1-norm minimization in complex space:

$$\min_{\hat{\boldsymbol{x}}} \ \|\hat{\boldsymbol{x}}\|_1 + \frac{\delta}{2}\|\hat{\boldsymbol{x}} - \hat{\boldsymbol{h}}\|_2^2$$

with complex-valued $\hat{\boldsymbol{x}}, \hat{\boldsymbol{h}} \in \mathbb{C}^T$ and weight parameter δ, element-wise, the solution is given by

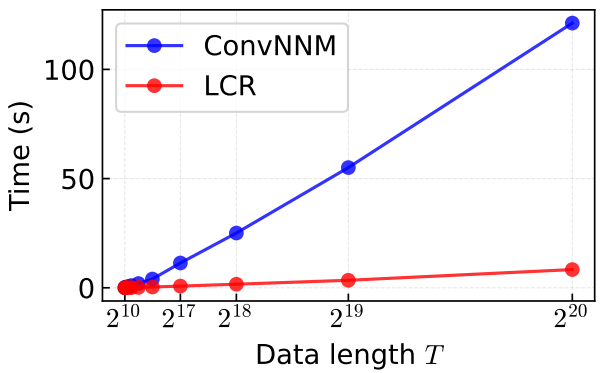
$$\hat{x}_t := \frac{\hat{h}_t}{|\hat{h}_t|} \cdot \max\{0, |\hat{h}_t| - 1/\delta\}, t \in [T].$$

Laplacian Convolutional Representation

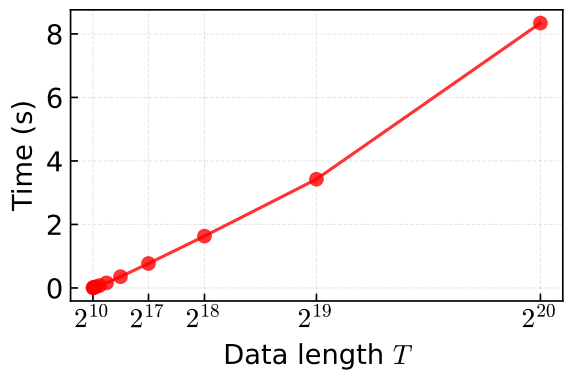
Empirical time complexity

On the synthetic data $\boldsymbol{y} \in \mathbb{R}^T$ with $T \in \{2^{10}, 2^{11}, \ldots, 2^{20}\}$

- Ours: **LCR**
  - An FFT implementation in $\mathcal{O}(T \log T)$
  - The logarithmic factor $\log T$ makes the FFT highly efficient
- Baseline: **ConvNNM** (Liu'22, Liu & Zhang'23)
  - Convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{y}) \in \mathbb{R}^{T \times \tilde{\tau}}$ with kernel size $\tilde{\tau} = 2^4$
  - Singular value thresholding in $\mathcal{O}(\tilde{\tau}^2 T)$

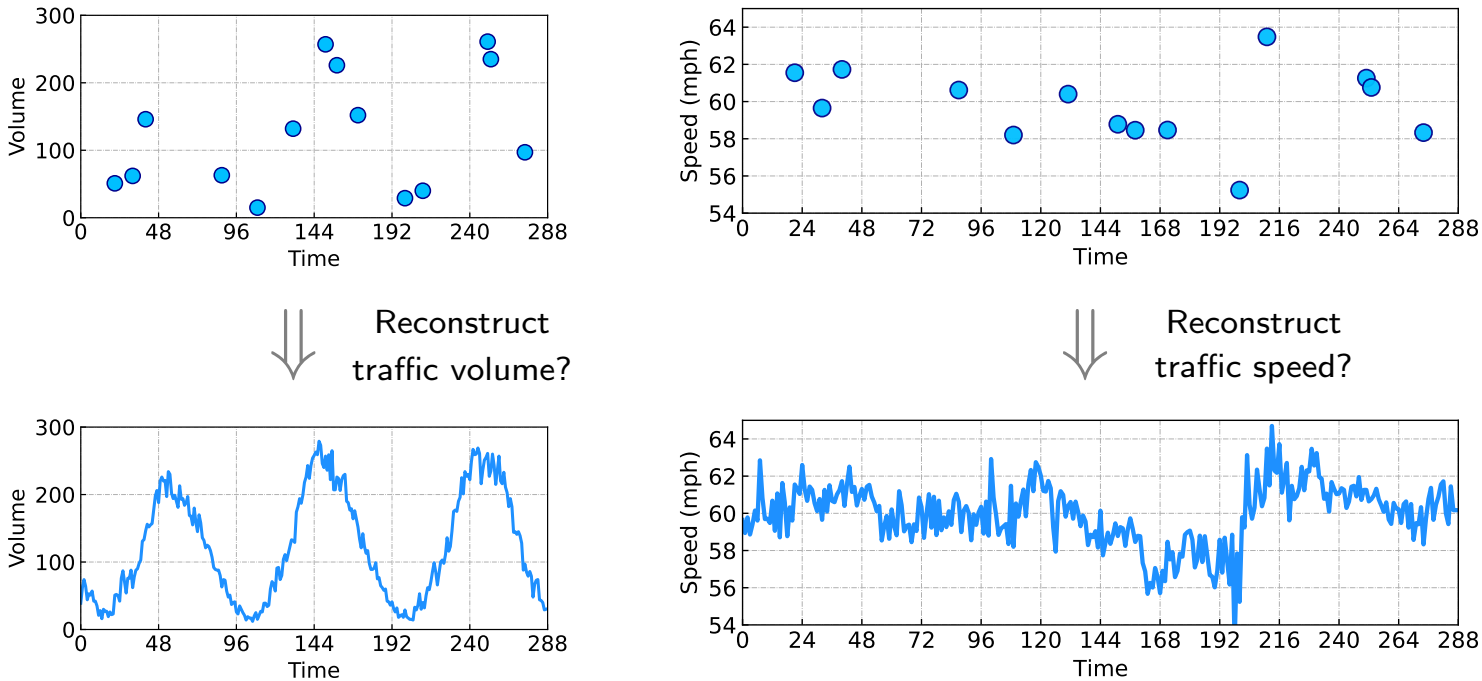


ConvNNM vs. LCR



LCR

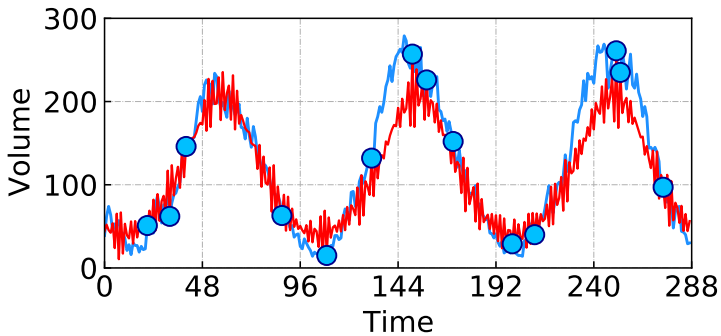
Experiments



- How to utilize the global trends of traffic time series?
- How to produce local consistency of traffic data?

Experiments

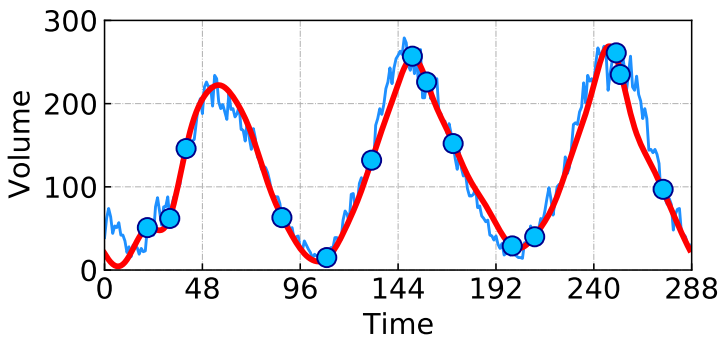
- Substantial performance gains?



CircNNM:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* \quad \text{s.t.} \ \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \le \epsilon$$

⇓ Plus **local** time series trends

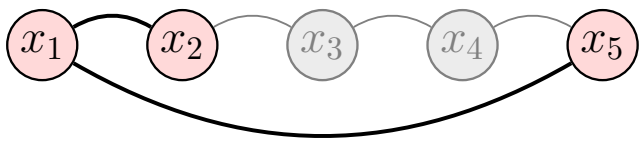


LCR:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 \quad \text{s.t.} \ \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \le \epsilon$$

Experiments

- The start data points and end data points are connected?



Undirected and circulant graph

Modeling $\longrightarrow$

$$\boldsymbol{L} = \begin{bmatrix} \mathbf{2} & -1 & 0 & 0 & -1 \\ \mathbf{-1} & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ \mathbf{-1} & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Flipping operation on $\boldsymbol{x} \in \mathbb{R}^5$:

$$\boldsymbol{x}_{\text{new}} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{J}\boldsymbol{x} \end{bmatrix} = (\underbrace{x_1, x_2, x_3, x_4, x_5}_{\text{original time series}}, \underbrace{x_5, x_4, x_3, x_2, x_1}_{\text{flipped time series}})^\top \in \mathbb{R}^{10}$$

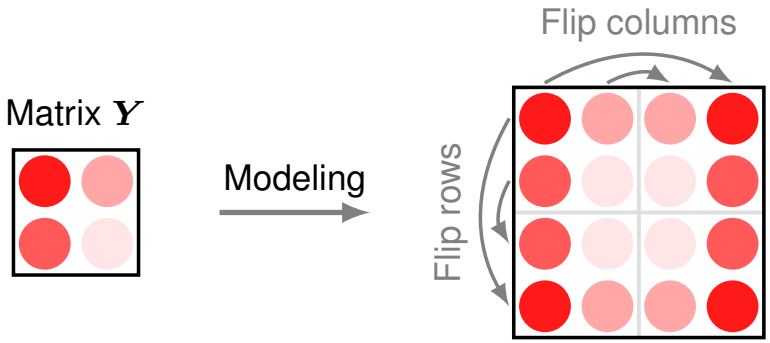
  where $\boldsymbol{J} \in \mathbb{R}^{5\times 5}$ is the exchange matrix.

- Potential applications: Passenger flow prediction with strong global/local trends

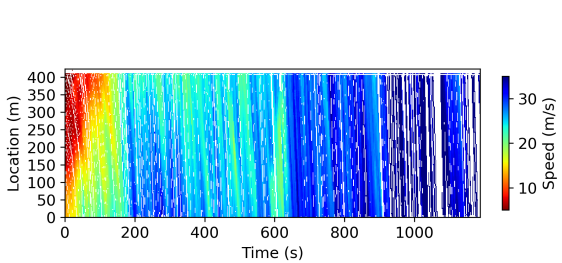
Experiments

Speed field reconstruction[3]

- Flipping operation on a matrix:

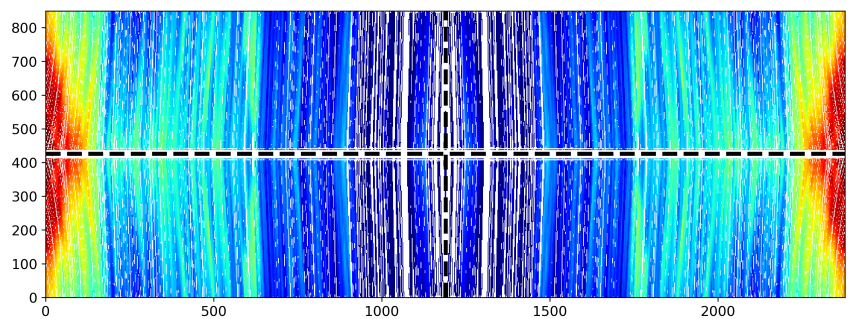


- Flipping operation on a speed field of vehicular traffic flow:



Original speed field

Modeling



Flipped speed field

[3]Highway Drone (HighD) dataset at https://www.highd-dataset.com/

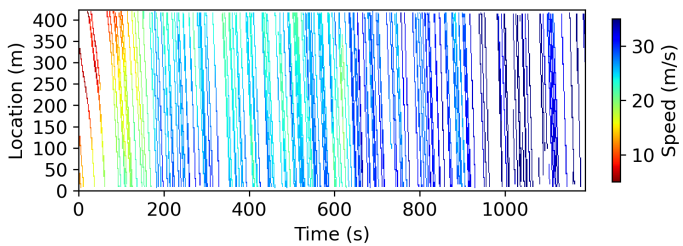
Experiments

Speed field reconstruction[4]

- Scenario: Mask trajectories of 70% vehicles
- LCR-2D on partially observed $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

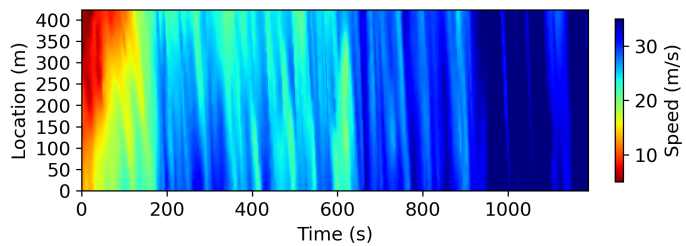
$$\min_{\boldsymbol{X}} \quad \underbrace{\|\mathcal{C}(\boldsymbol{X})\|_*}_{\text{global trend}} + \frac{\gamma}{2} \underbrace{\|(\boldsymbol{\ell}_s \boldsymbol{\ell}^\top) \star \boldsymbol{X}\|_F^2}_{\text{local trend}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{X} - \boldsymbol{Y})\|_F \leq \epsilon$$

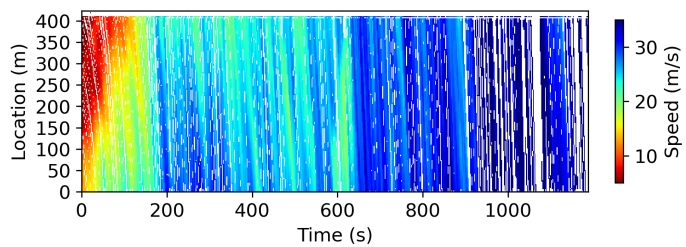


Sparse speed field

LCR-2D



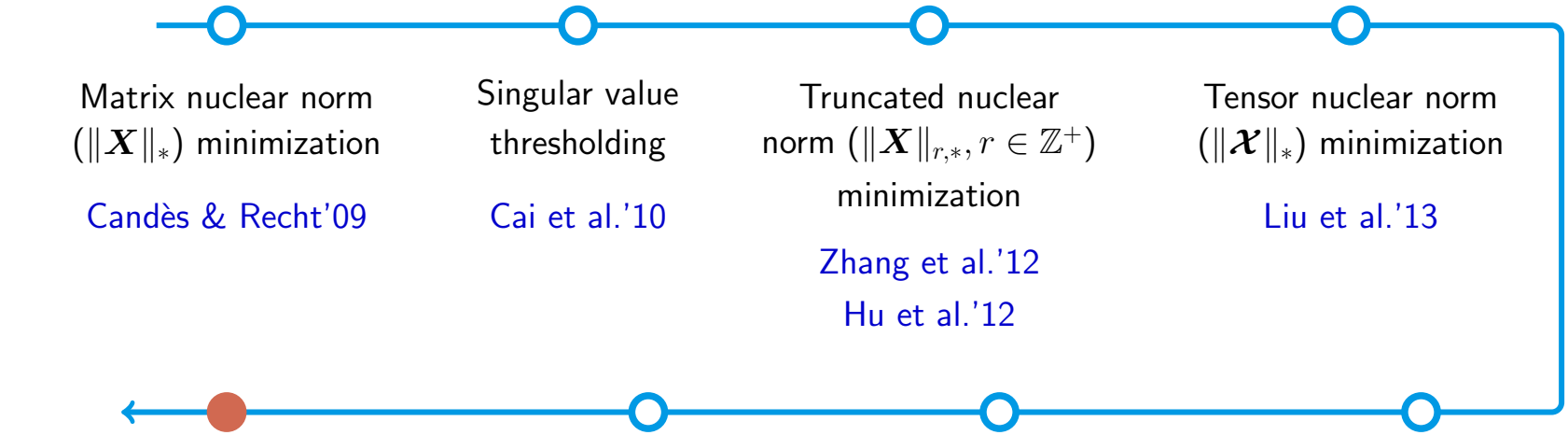
Reconstructed speed field



Ground-truth speed field

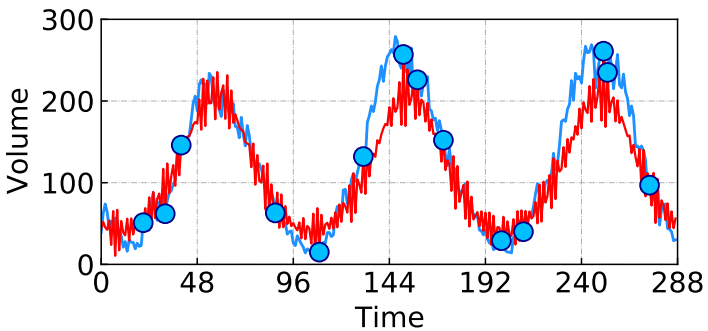
[4] Highway Drone (HighD) dataset at https://www.highd-dataset.com/

Contributions

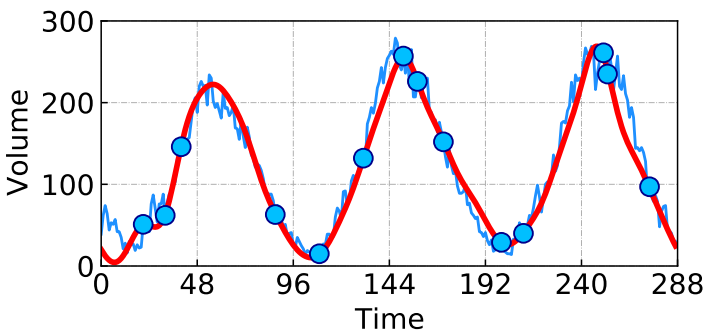


(Ours) **LCR**:
- ✓ Local trend modeling
- ✓ An FFT implementation

Vision & Insight



⇓ Plus **local** time series trends



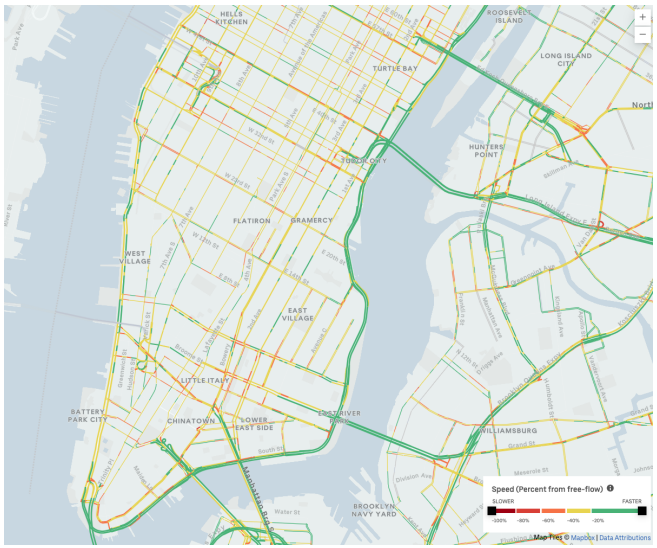
Highlights:

- Rethinking the importance of local trend modeling in traffic data imputation tasks.
- Finding a unified global and local trend modeling framework whose optimization can be efficiently solved by FFT:

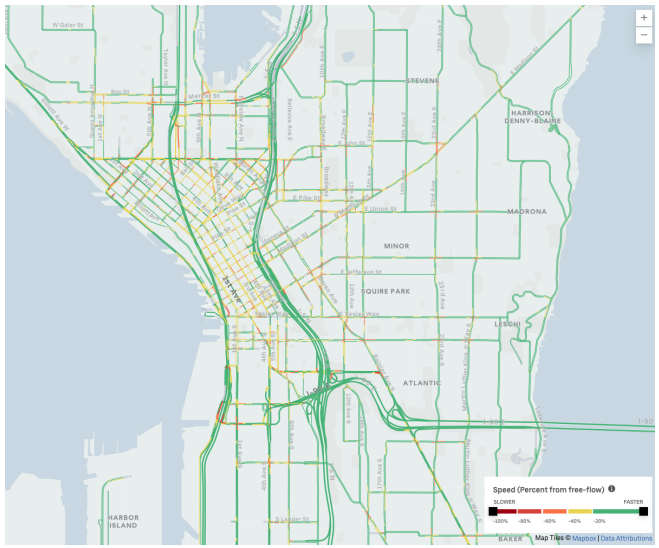
$$\min_{\boldsymbol{x}} \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{local}}$$
$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon$$

Vision & Insight

- Uber (hourly) movement speed data[5]



NYC movement



Seattle movement

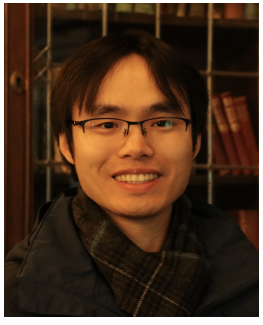
- {road segment, time slot (hour), average speed}
- Computing hourly speed: Road segments have 5+ unique trips.
- Estimating network-wide traffic states for traffic planning & management.
  (Data/model biases/fairness concerns for imputation, interpolation, and prediction.)

[5] https://movement.uber.com/ (not available now)

Discovering Dynamic Patterns from Spatiotemporal Data with Time-Varying Low-Rank Autoregression

IEEE Transactions on Knowledge and Data Engineering, 2024

https://doi.org/10.1109/TKDE.2023.3294440



Xinyu Chen

Chengyuan Zhang*

Xiaoxu Chen

Nicolas Saunier

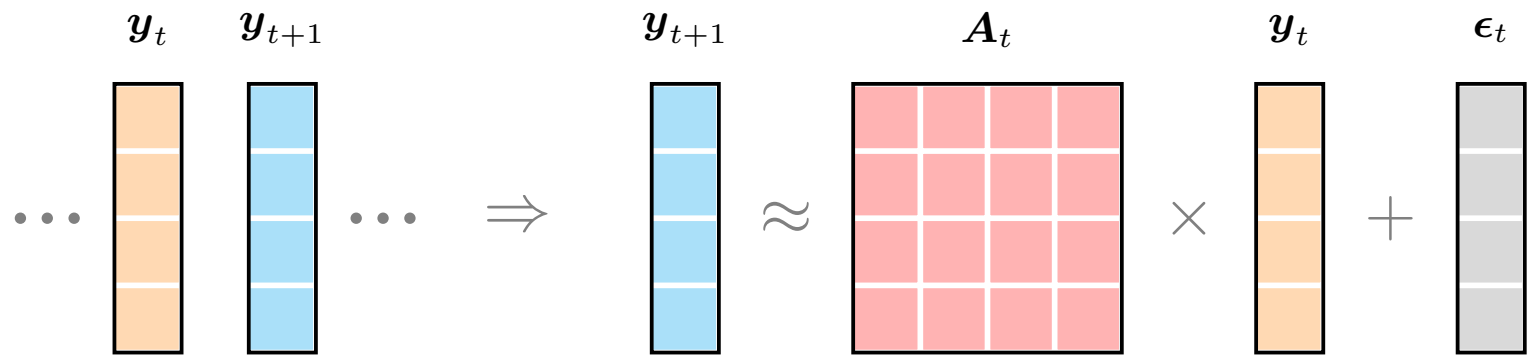
Lijun Sun

Materials:

- GitHub: https://github.com/xinychen/vars
- Blog: https://spatiotemporal-data.github.io/posts/time_varying_model

Autoregression

- How to characterize dynamical systems?

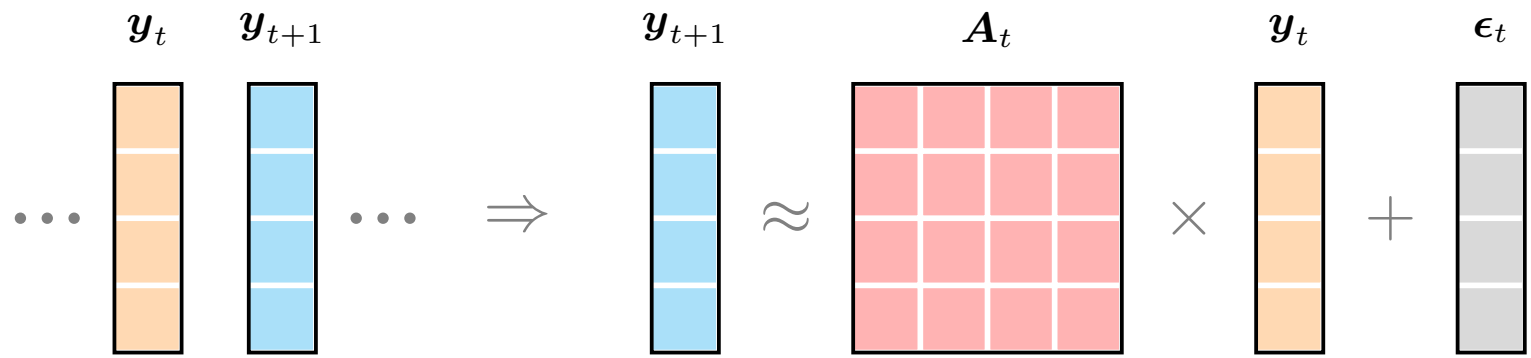


Spatiotemporal system

Time-varying autoregression

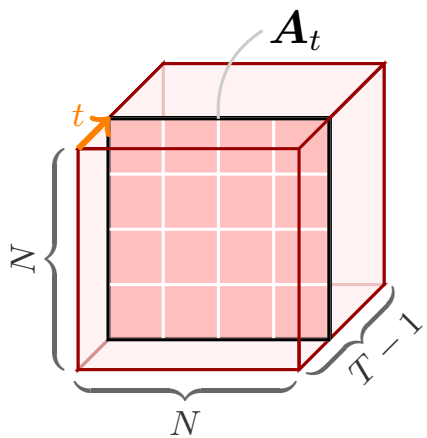
Autoregression

- How to characterize dynamical systems?

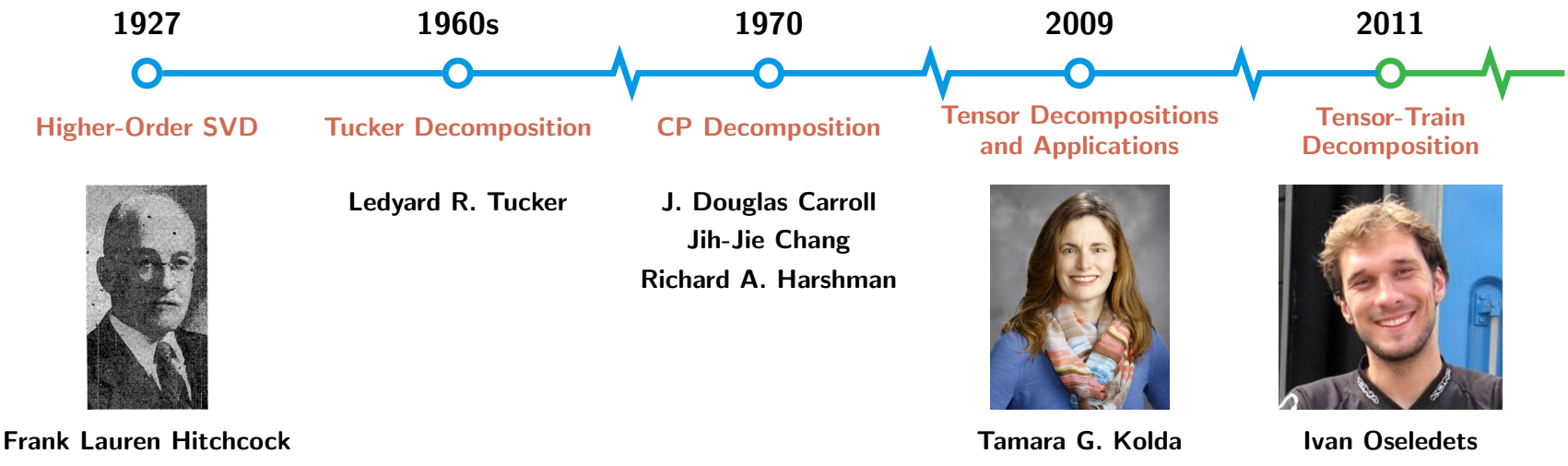


- On spatiotemporal systems $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

$$\underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-invariant (e.g., DMD)}} \quad \text{v.s.} \quad \underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}_t\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-varying}}$$



- How to discover spatial/temporal modes (patterns) from the tensor $\boldsymbol{\mathcal{A}} \triangleq \{\boldsymbol{A}_t\}_{t \in [T-1]}$?


1927
1960s
1970
2009
2011
Higher-Order SVD
Tucker Decomposition
CP Decomposition
Tensor Decompositions and Applications
Tensor-Train Decomposition
Ledyard R. Tucker
J. Douglas Carroll
Jih-Jie Chang
Richard A. Harshman
Frank Lauren Hitchcock
Tamara G. Kolda
Ivan Oseledets

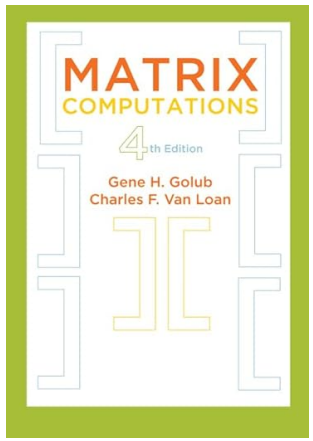
Time-Varying Autoregression

- Tensor factorization[6]:

$$\boldsymbol{\mathcal{A}} = \underbrace{\boldsymbol{\mathcal{G}} \times_1 \boldsymbol{W} \times_2 \boldsymbol{V} \times_3 \boldsymbol{X}}_{\text{Tucker decomposition}}$$

$$\Updownarrow$$

$$\boldsymbol{A}_t = \boldsymbol{\mathcal{G}} \times_1 \underbrace{\boldsymbol{W}}_{\text{spatial modes}} \times_2 \boldsymbol{V} \times_3 \underbrace{\boldsymbol{x}_t^\top}_{\text{temporal modes}}$$



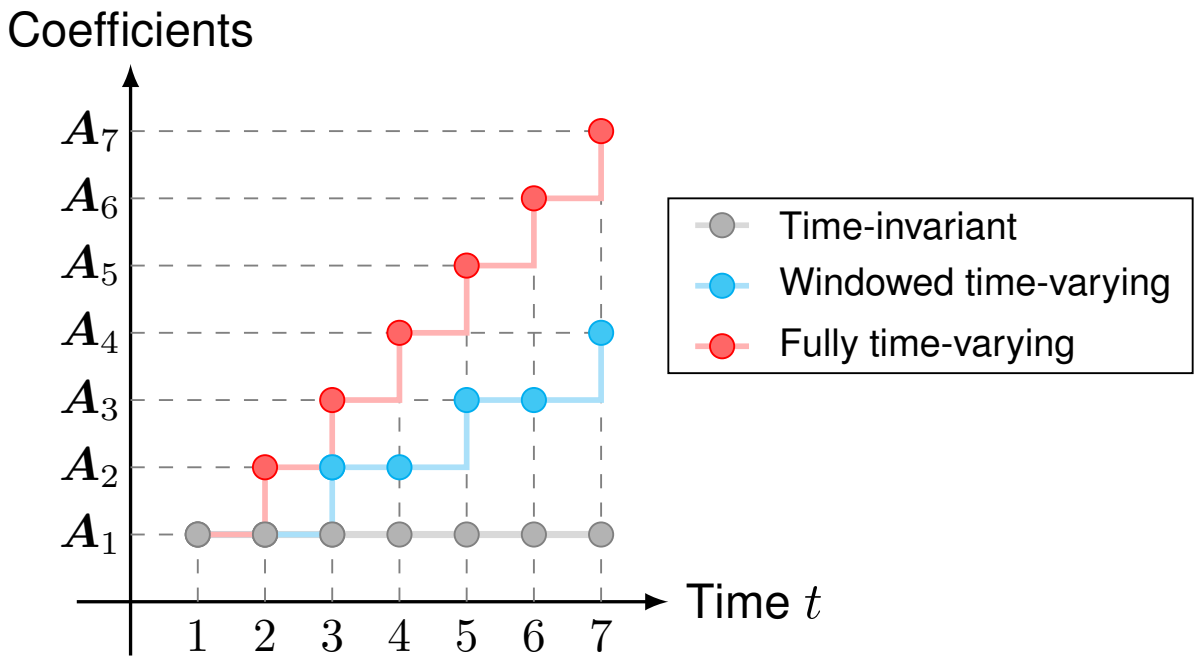
- **(Ours)** Time-varying low-rank autoregression:

$$\min_{\boldsymbol{\mathcal{G}}, \boldsymbol{W}, \boldsymbol{V}, \boldsymbol{X}} \frac{1}{2} \sum_{t \in [T-1]} \left\| \boldsymbol{y}_{t+1} - \underbrace{(\boldsymbol{\mathcal{G}} \times_1 \boldsymbol{W} \times_2 \boldsymbol{V} \times_3 \boldsymbol{x}_t^\top)}_{\text{tensor factorization}} \boldsymbol{y}_t \right\|_2^2$$

[6] $\times_k$, $\forall k$ is the mode-k product between tensor and matrix/vector.

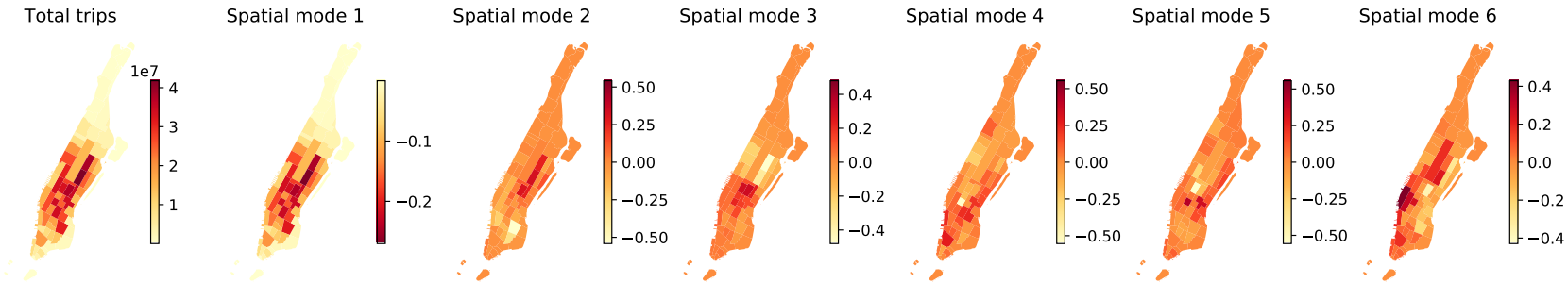
- On the data $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

$$\underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-invariant (e.g., DMD)}} \quad \text{v.s.} \quad \underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}_t\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{fully time-varying (ours)}}$$

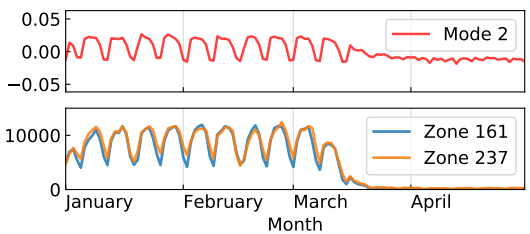


NYC Taxi Data

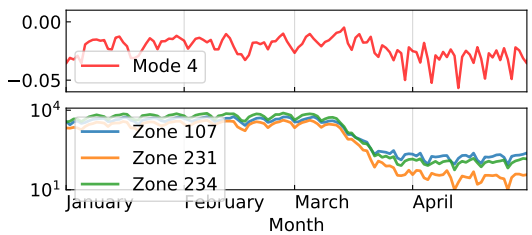
- **NYC taxi dataset** (pickup)



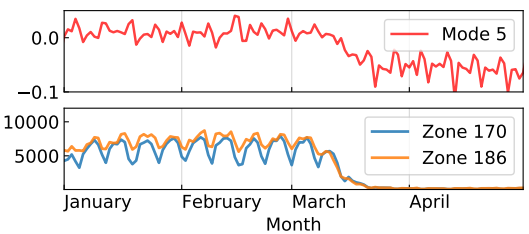
Spatial patterns in $\boldsymbol{W}$



Pattern #2 & taxi trips (2020)



Pattern #4 & taxi trips (2020)



Pattern #5 & taxi trips (2020)

Human Mobility

- Chicago taxi/ridesharing data

Matching Taxi Trips with Community Areas

There are three basic steps to follow for processing taxi trip data:

- Download taxi trips in 2022 in the `.csv` format, e.g., `Taxi_Trips_-_2022.csv`.
- Use the `pandas` package in Python to process the raw trip data.
- Match trip pickup/dropoff locations with boundaries of the community area.

```
import pandas as pd

data = pd.read_csv('Taxi_Trips_-_2022.csv')
data.head()
```

For each taxi trip, one can select some important information:

- `Trip Start Timestamp`: When the trip started, rounded to the nearest 15 minutes.
- `Trip Seconds`: Time of the trip in seconds.
- `Trip Miles`: Distance of the trip in miles.
- `Pickup Community Area`: The Community Area where the trip began. This column will be blank for locations ourside Chicago.
- `Dropoff Community Area`: The Community Area where the trip ended. This column will be blank for locations outside Chicago.

```
df = pd.DataFrame()
df['Trip Start Timestamp'] = data['Trip Start Timestamp']
df['Trip Seconds'] = data['Trip Seconds']
df['Trip Miles'] = data['Trip Miles']
df['Pickup Community Area'] = data['Pickup Community Area']
df['Dropoff Community Area'] = data['Dropoff Community Area']
del data
df
```

Figure 2 shows taxi pickup and dropoff trips (2022) on 77 community areas in the City of Chicago. Note that the average trip duration is **1207.75 seconds** and the average trip distance is **6.16 miles**.

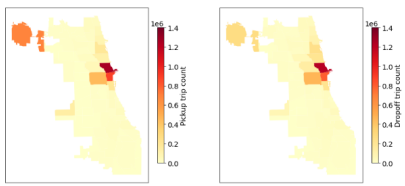


Figure 2. Taxi pickup and dropoff trips (2022) in the City of Chicago, USA. There are 4,763,961 remaining trips after the data processing.

For comparison, Figure 3 shows taxi pickup and dropoff trips (2019) on 77 community areas in the City of Chicago. Note that the average trip duration is **915.62 seconds** and the average trip distance is **3.93 miles**.

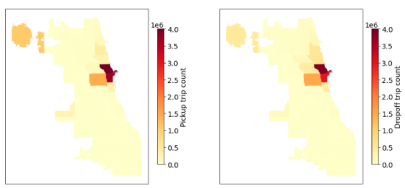


Figure 3. Taxi pickup and dropoff trips (2019) in the City of Chicago, USA. There are 12,484,572 remaining trips after the data processing. See the data processing codes.

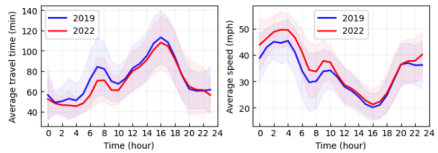


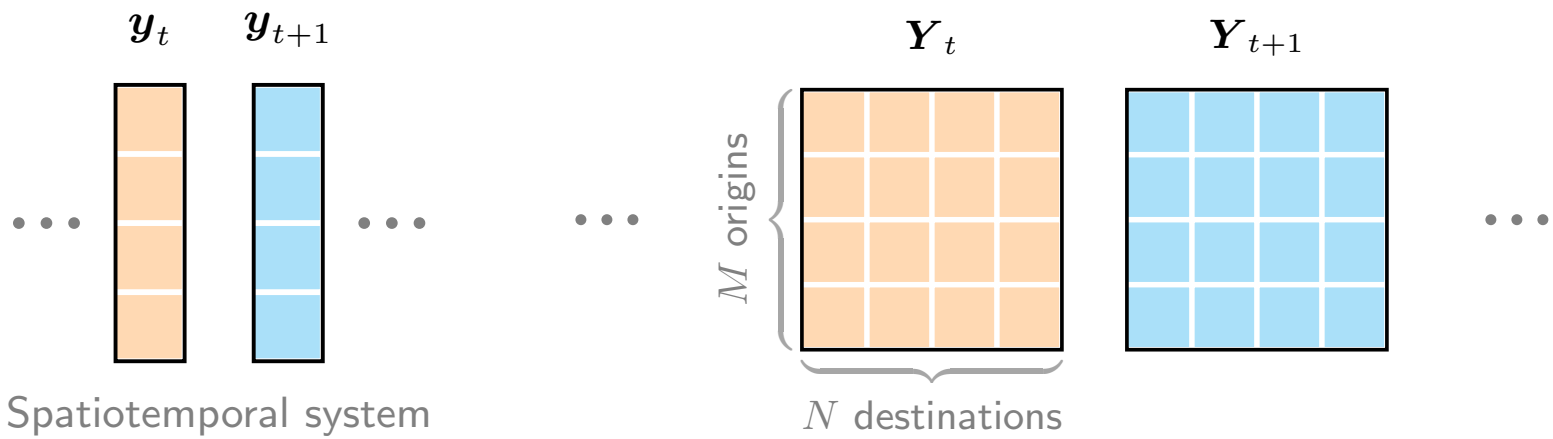
Figure 6. Average travel time and speed from area 32 (i.e., Downtown) to area 76 (i.e., Airport) in both 2019 and 2022.

```
import numpy as np
import matplotlib.pyplot as plt

fig = plt.figure(figsize = (8, 2.5))
ax = fig.add_subplot(1, 2, 1)
# Average travel time in 2019
m1 = df1.groupby(['hour'])['Trip Seconds'].mean().values / 30
s1 = df1.groupby(['hour'])['Trip Seconds'].std().values / 30
plt.plot(m1, color = 'blue', linewidth = 1.8, label = '2019')
upper = m1 + s1
lower = m1 - s1
x_bound = np.append(np.append(np.append(np.array([0, 0]), np.arange(0, 24)),
                              np.array([24 - 1, 24 - 1])), np.arange(24 - 1, -1,
y_bound = np.append(np.append(np.append(np.array([upper[0], lower[0]]), lower),
                              np.array([lower[-1], upper[-1]])), np.flip(upper))
plt.fill(x_bound, y_bound, color = 'blue', alpha = 0.05)

# Average travel time in 2022
m1 = df2.groupby(['hour'])['Trip Seconds'].mean().values / 30
s1 = df2.groupby(['hour'])['Trip Seconds'].std().values / 30
plt.plot(m1, color = 'red', linewidth = 1.8, label = '2022')
upper = m1 + s1
lower = m1 - s1
```

Source: https://spatiotemporal-data.github.io/Chicago-mobility/taxi-data

Concluding Remark

- Discovering spatial/temporal patterns from 2D and 3D spatiotemporal systems with unsupervised learning:
  - Time-varying autoregression on the data
  - Tensor factorization on the coefficients



Thanks for your attention!

Any Questions?

Slides: https://xinychen.github.io/slides/informs24.pdf

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