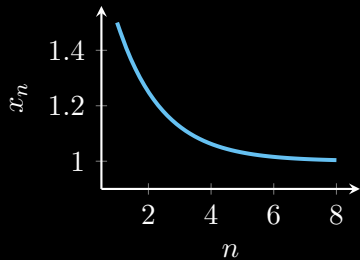


Three rates of convergence on a sequence $\{x_n\}_{n \in \mathbb{Z}^+}$

Linear convergence

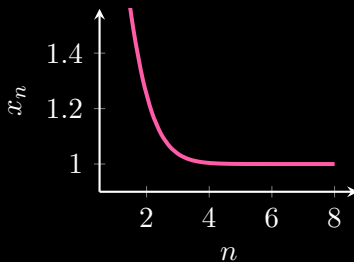
$$\frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} \leq r, r \in (0, 1)$$



Sequence $x_n = 1 + \left(\frac{1}{2}\right)^n$
converges linearly to $x_\infty = 1$
because $r = \frac{1}{2}$

Superlinear convergence

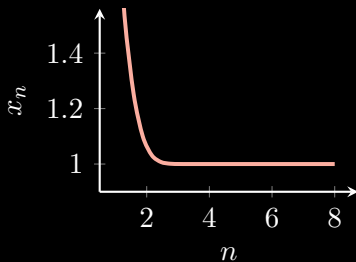
$$\lim_{n \rightarrow \infty} \frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} = 0$$



Sequence $x_n = 1 + \left(\frac{1}{n}\right)^n$
converges superlinearly to
 $x_\infty = 1$

Quadratic convergence

$$\frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|^2} \leq M$$

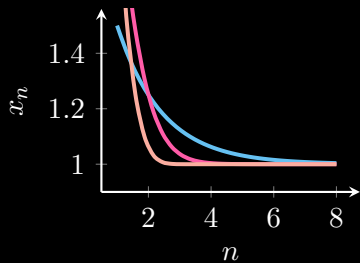


Sequence $x_n = 1 + \left(\frac{1}{n}\right)^{2^n}$
converges quadratically to
 $x_\infty = 1$ because $M = 1$

Three rates of convergence on a sequence $\{x_n\}_{n \in \mathbb{Z}^+}$

Linear convergence

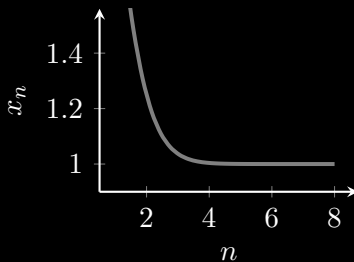
$$\frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} \leq r, r \in (0, 1)$$



Sequence $x_n = 1 + \left(\frac{1}{2}\right)^n$
converges linearly to $x_\infty = 1$
because $r = \frac{1}{2}$

Superlinear convergence

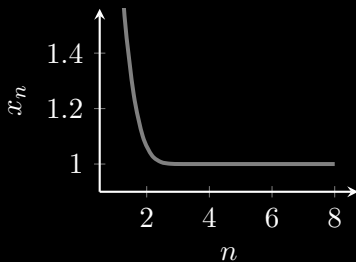
$$\lim_{n \rightarrow \infty} \frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} = 0$$



Sequence $x_n = 1 + \left(\frac{1}{n}\right)^n$
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Quadratic convergence

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Sequence $x_n = 1 + \left(\frac{1}{n}\right)^{2^n}$
converges quadratically to
 $x_\infty = 1$ because $M = 1$