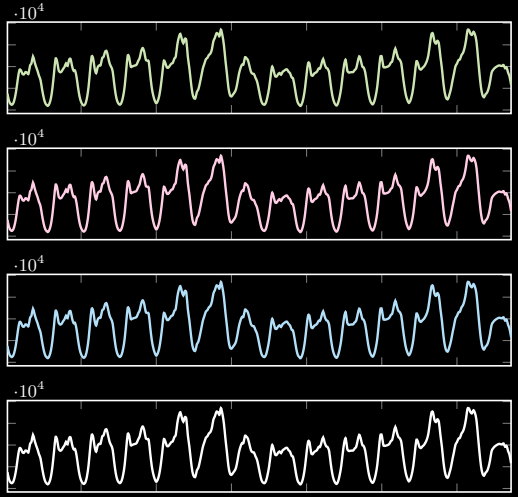



Annotate Ridesharing Trip Time Series

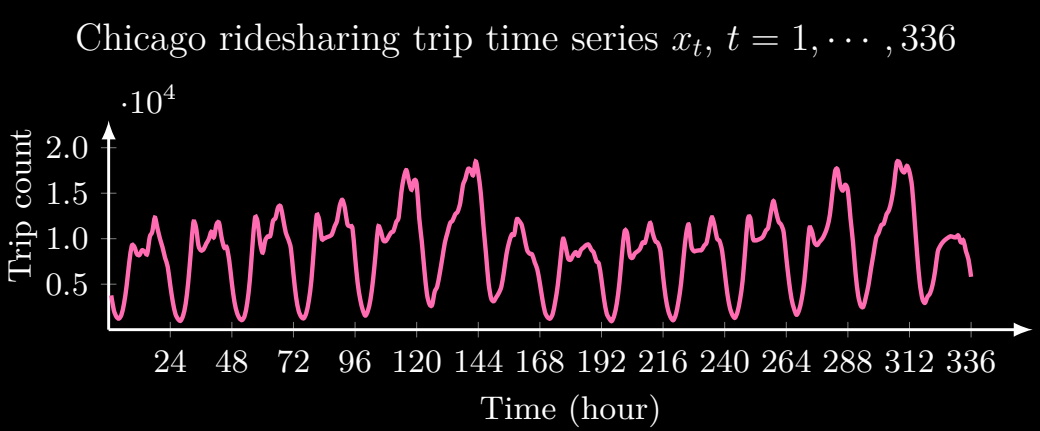
# Interpretable Time Series Autoregression

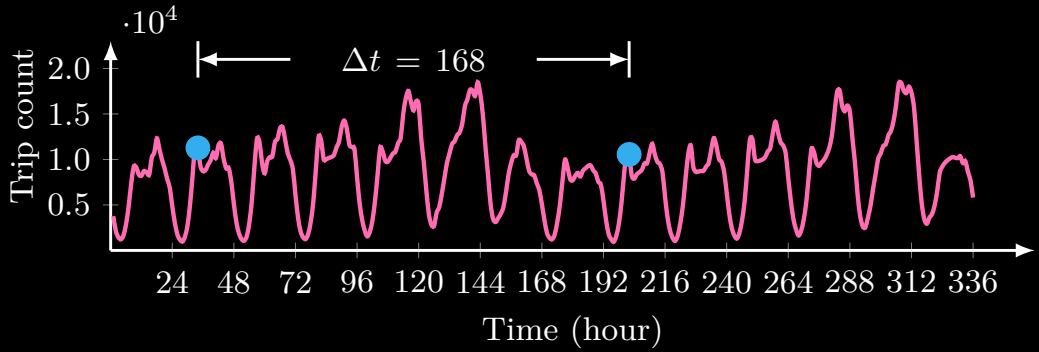
Xinyu Chen, MIT

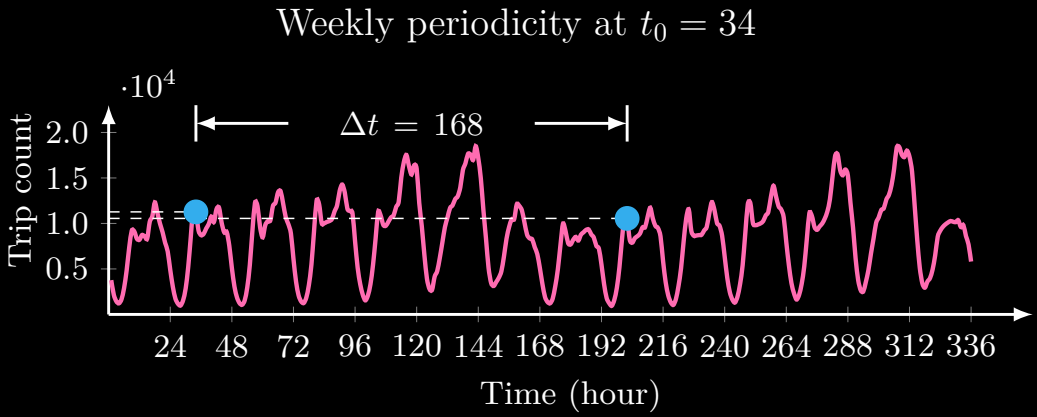
Vassilis Digalakis Jr, HEC Paris

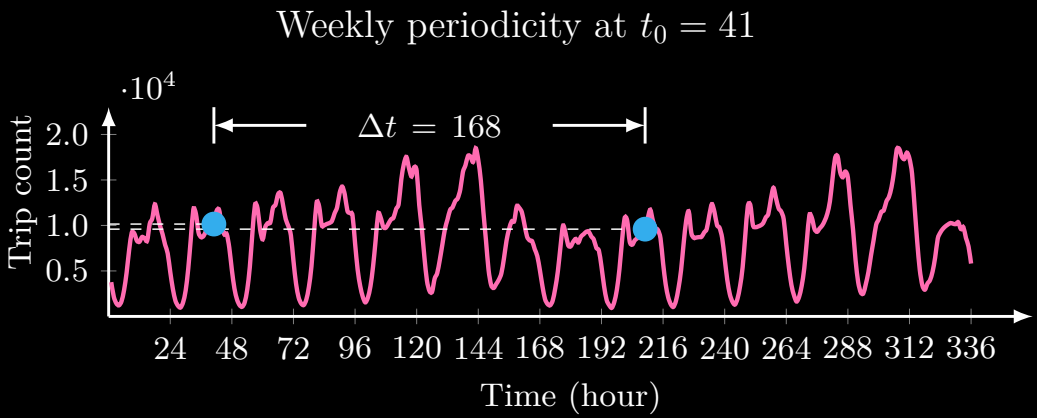
Lijun Ding, UCSD

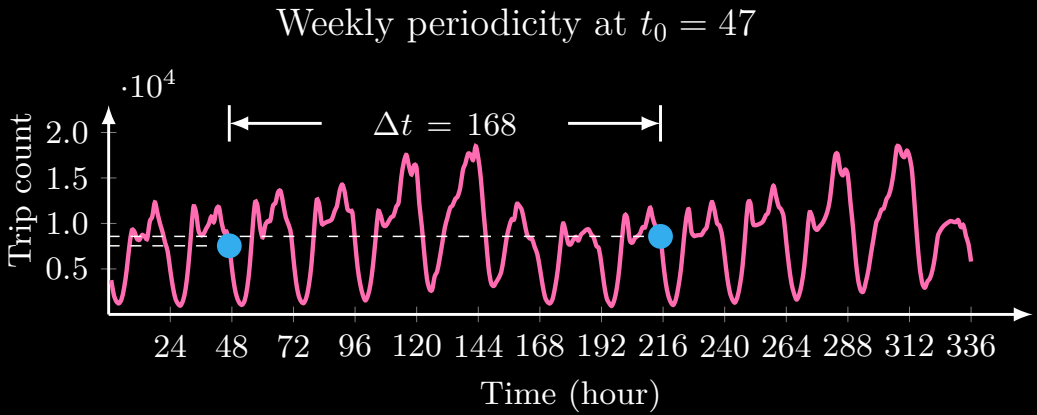
Jinhua Zhao, MIT

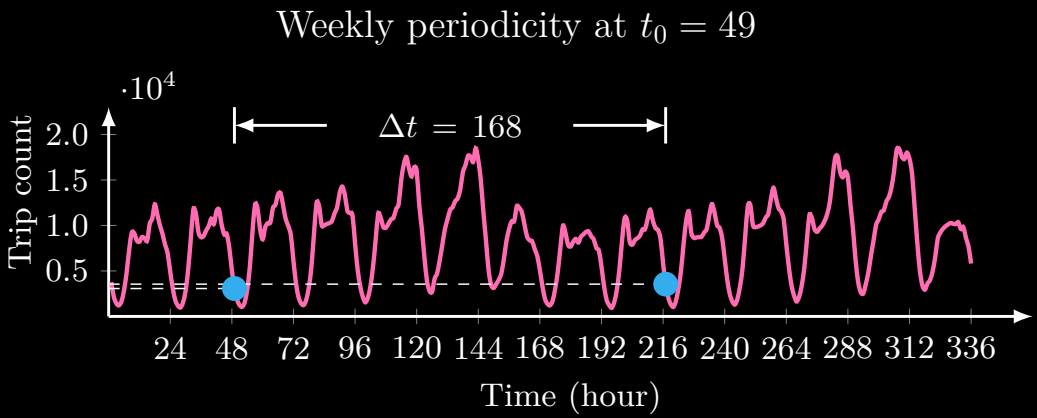

Chicago ridesharing trip time series $x_t$, $t = 1, \cdots, 336$
$\cdot 10^4$
Trip count
2.0
1.5
1.0
0.5
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)

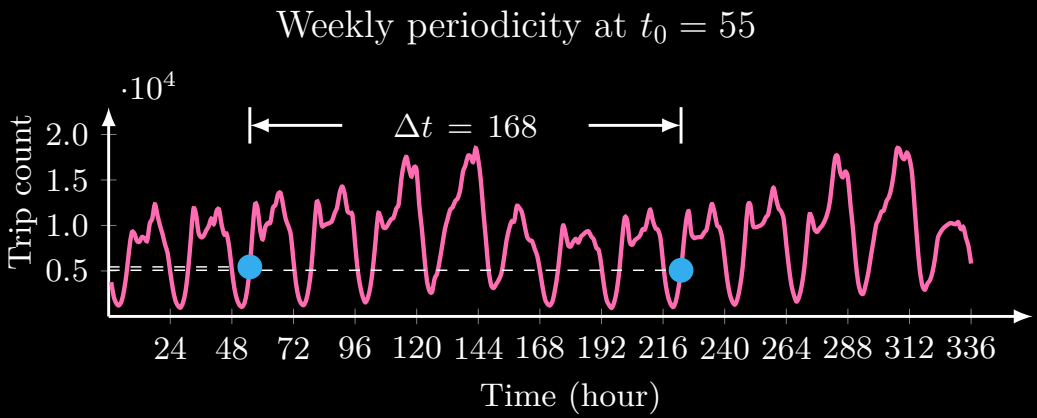

$\cdot 10^4$
$\Delta t = 168$
Trip count
2.0
1.5
1.0
0.5
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)


Weekly periodicity at $t_0 = 34$
$\cdot 10^4$
$\Delta t = 168$
Trip count
2.0
1.5
1.0
0.5
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)

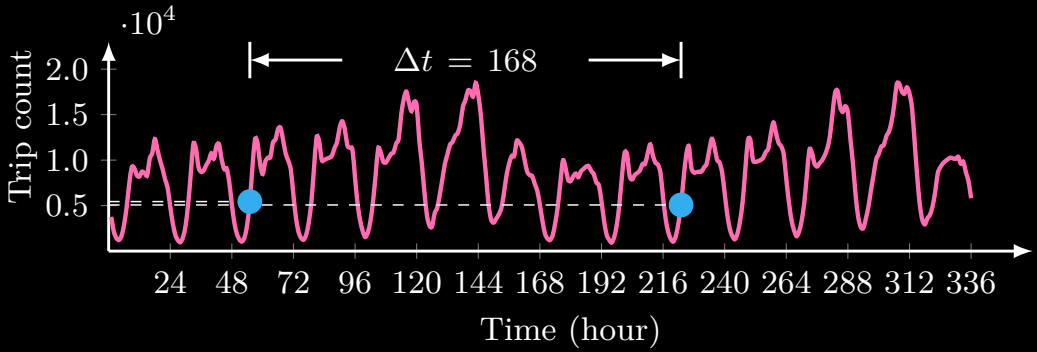

Weekly periodicity at $t_0 = 41$
$\cdot 10^4$
$\Delta t = 168$
Trip count
2.0
1.5
1.0
0.5
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)

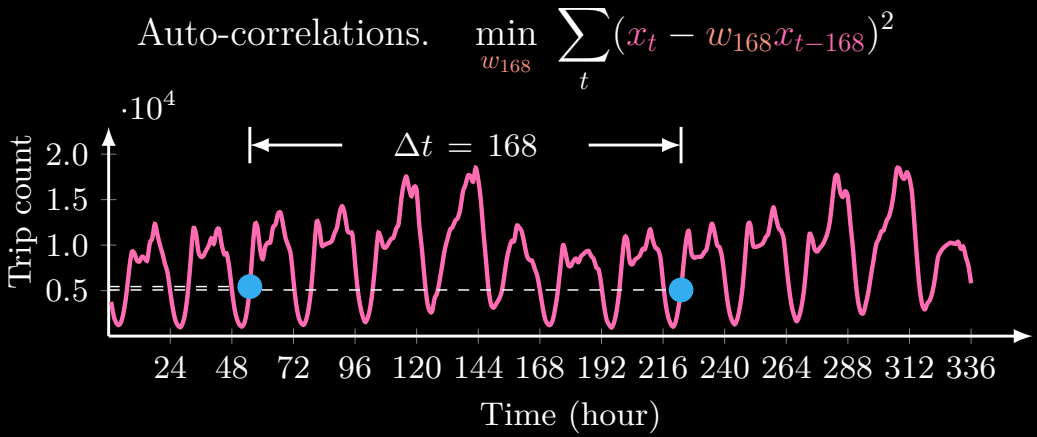

Weekly periodicity at $t_0 = 47$
$\cdot 10^4$
$\Delta t = 168$
Trip count
2.0
1.5
1.0
0.5
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)

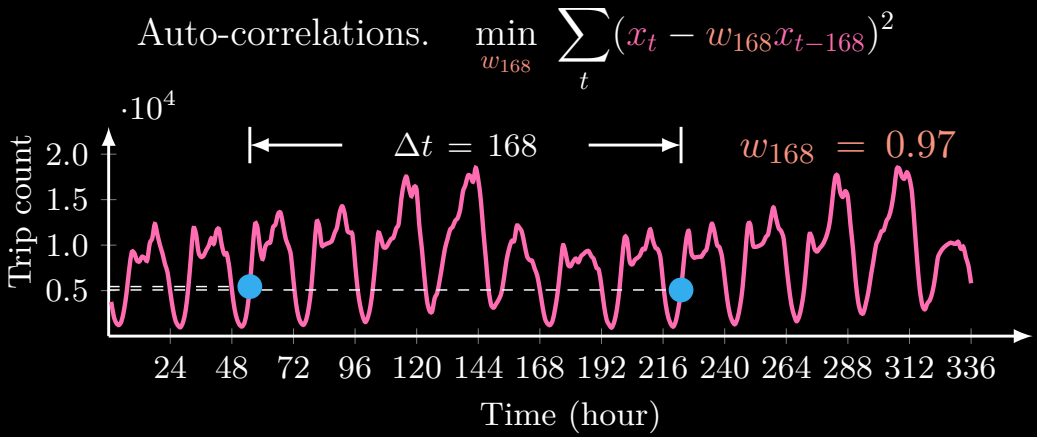

Weekly periodicity at $t_0 = 49$
$\cdot 10^4$
$\Delta t = 168$
Trip count
2.0
1.5
1.0
0.5
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)


Weekly periodicity at $t_0 = 55$
$\cdot 10^4$
$\Delta t = 168$
Trip count
2.0
1.5
1.0
0.5
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)

Auto-correlations. $(x_t - w_{168}x_{t-168})^2$




Auto-correlations. $\min_{w_{168}} \sum_t (x_t - w_{168} x_{t-168})^2$
$\cdot 10^4$
$\Delta t = 168$
Trip count
2.0
1.5
1.0
0.5
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)


Auto-correlations.
$\min_{w_{168}} \sum_t (x_t - w_{168} x_{t-168})^2$
$\cdot 10^4$
Trip count
2.0
1.5
1.0
0.5
$\Delta t = 168$
$w_{168} = 0.97$
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)

Time Series Autoregression

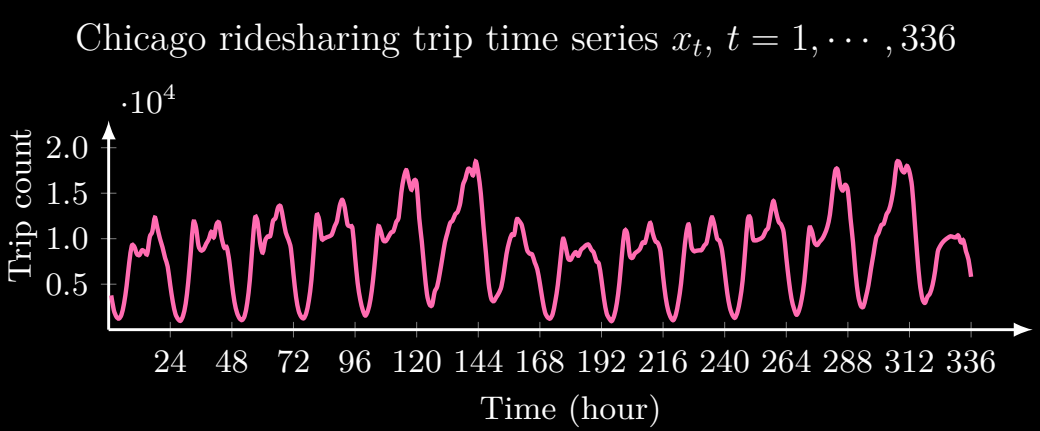
$$\left(x_t - \sum_{k=1}^{d} w_k x_{t-k}\right)^2$$

Time Series Autoregression

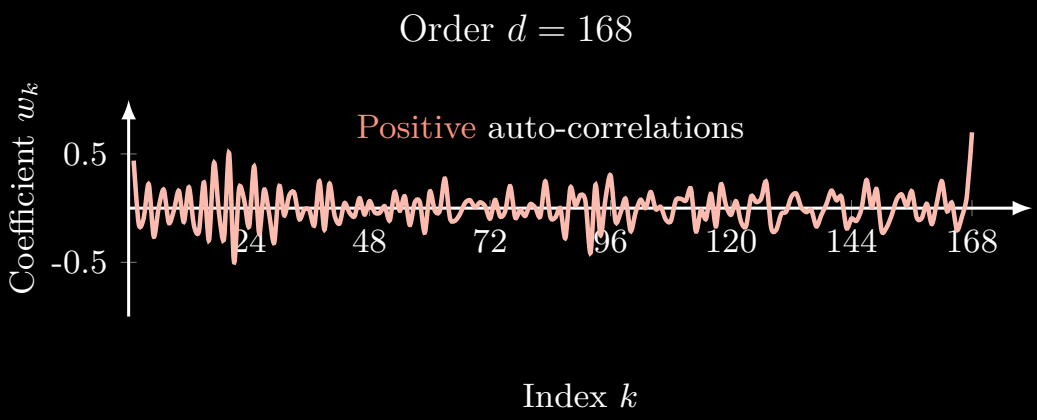
$$f \triangleq \sum_t \left( x_t - \sum_{k=1}^{d} w_k x_{t-k} \right)^2$$

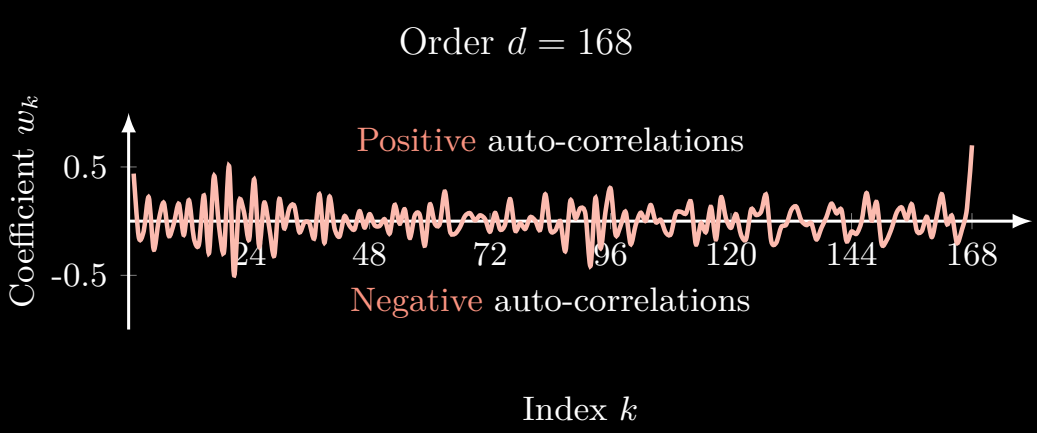
Time Series Autoregression

$$\min_{w_1,\cdots,w_d} \quad f \triangleq \sum_t \left( x_t - \sum_{k=1}^{d} w_k x_{t-k} \right)^2$$

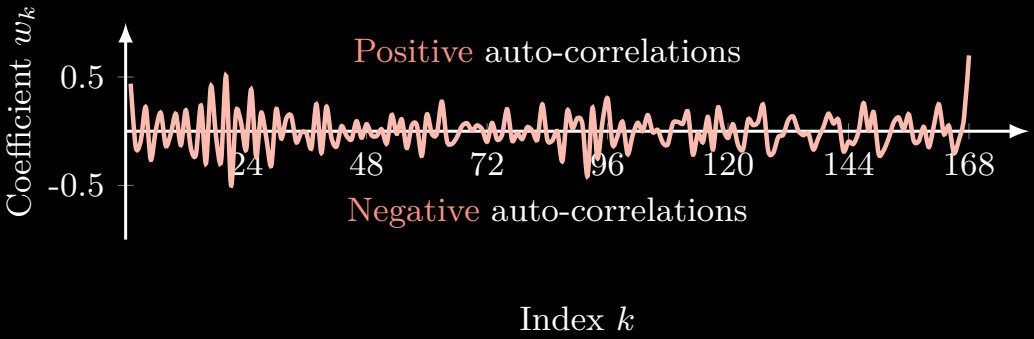

Chicago ridesharing trip time series $x_t$, $t = 1, \cdots, 336$
$\cdot 10^4$
Trip count
2.0
1.5
1.0
0.5
24 48 72 96 120 144 168 192 216 240 264 288 312 336
Time (hour)

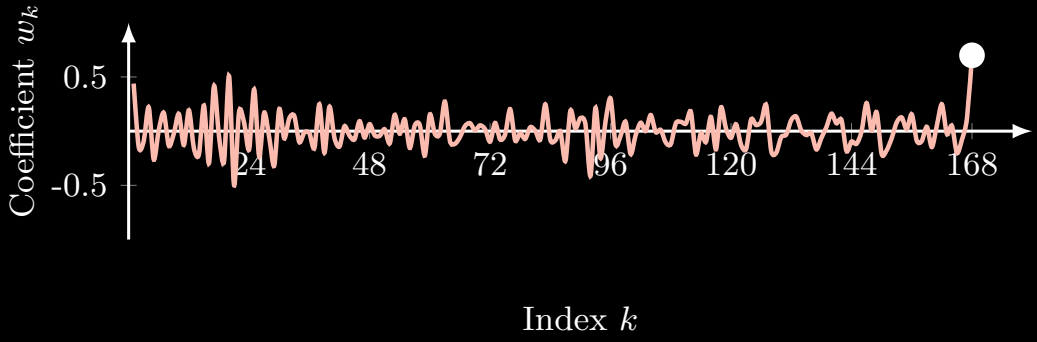

Order $d = 168$
Coefficient $w_k$
0.5
-0.5
24
48
72
96
120
144
168
Index $k$

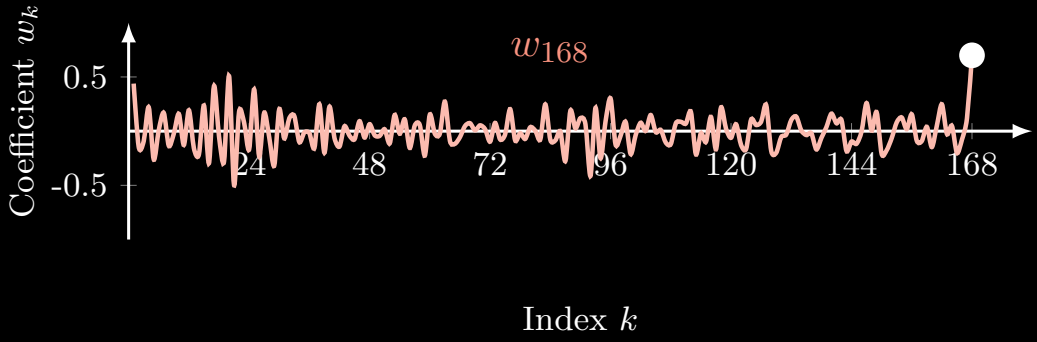

Order $d = 168$
Positive auto-correlations
Coefficient $w_k$
0.5
-0.5
24
48
72
96
120
144
168
Index $k$

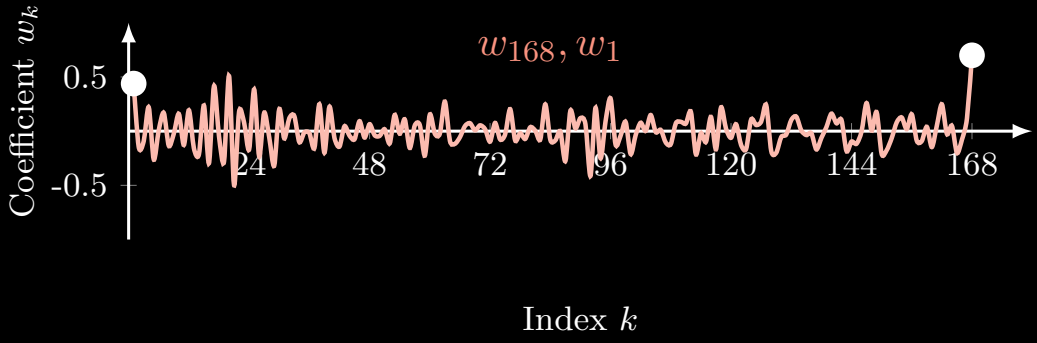

Order $d = 168$
Coefficient $w_k$
0.5
-0.5
Positive auto-correlations
Negative auto-correlations
24
48
72
96
120
144
168
Index $k$

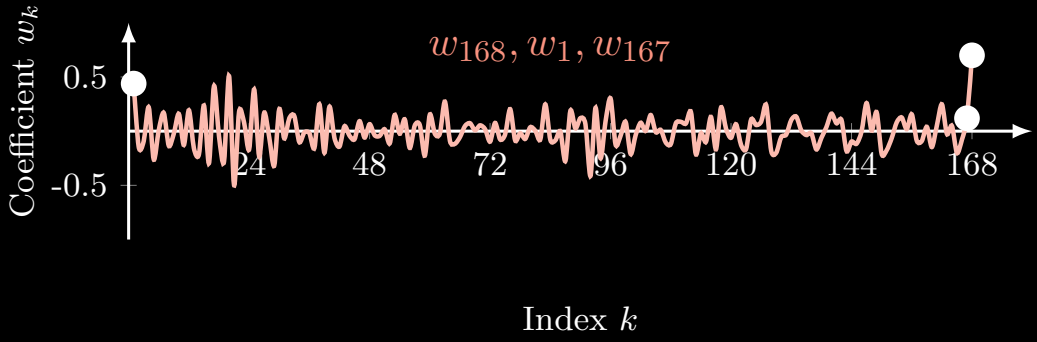
Which are the dominant coefficients in $w_1, \cdots, w_{168}$?




Coefficient $w_k$
0.5
-0.5
24
48
72
96
120
144
168
Index $k$

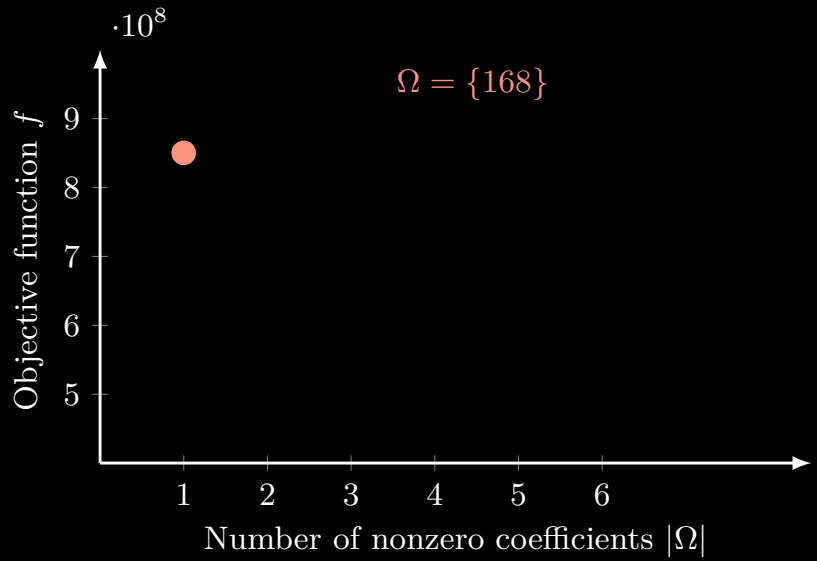

$w_{168}$
0.5
-0.5
Coefficient $w_k$
24
48
72
96
120
144
168
Index $k$

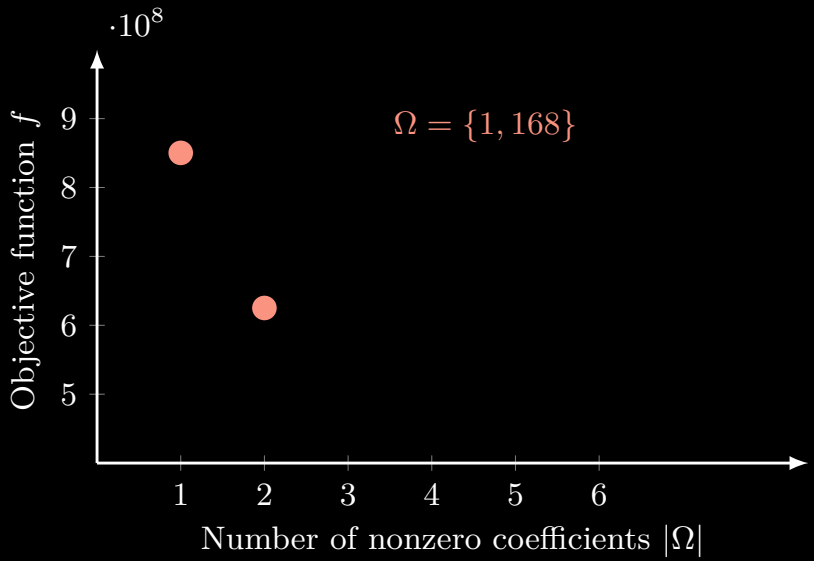

$w_{168}, w_1$
0.5
-0.5
Coefficient $w_k$
24
48
72
96
120
144
168
Index $k$

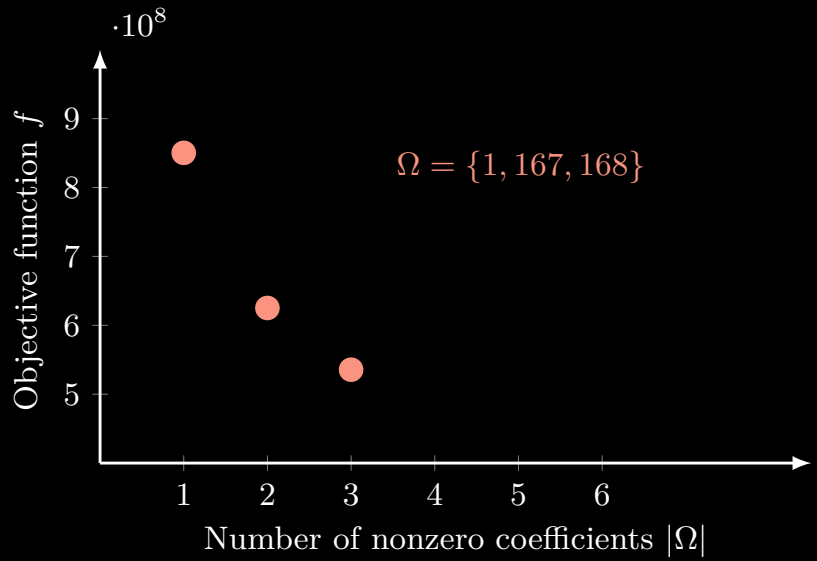

$w_{168}, w_1, w_{167}$
Coefficient $w_k$
0.5
-0.5
24
48
72
96
120
144
168
Index $k$

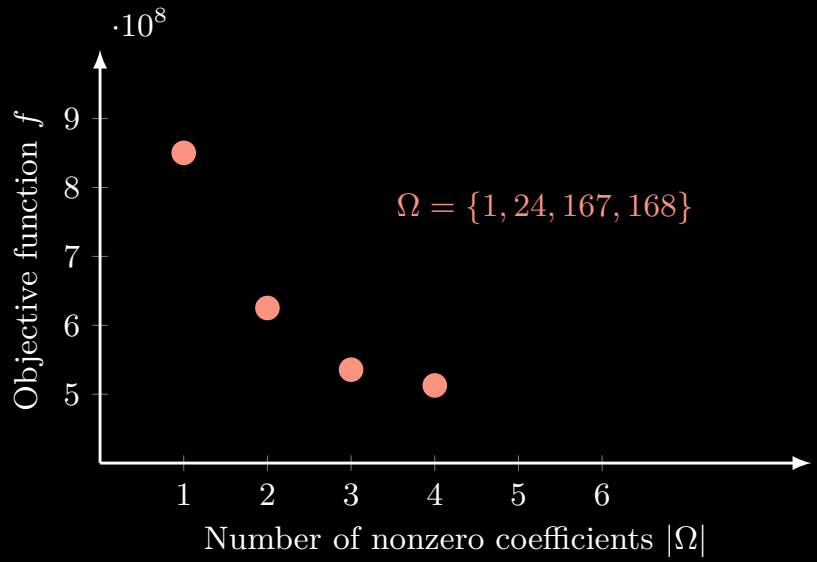
Select dominant coefficients $\boldsymbol{w}_k,\ k \in \Omega$

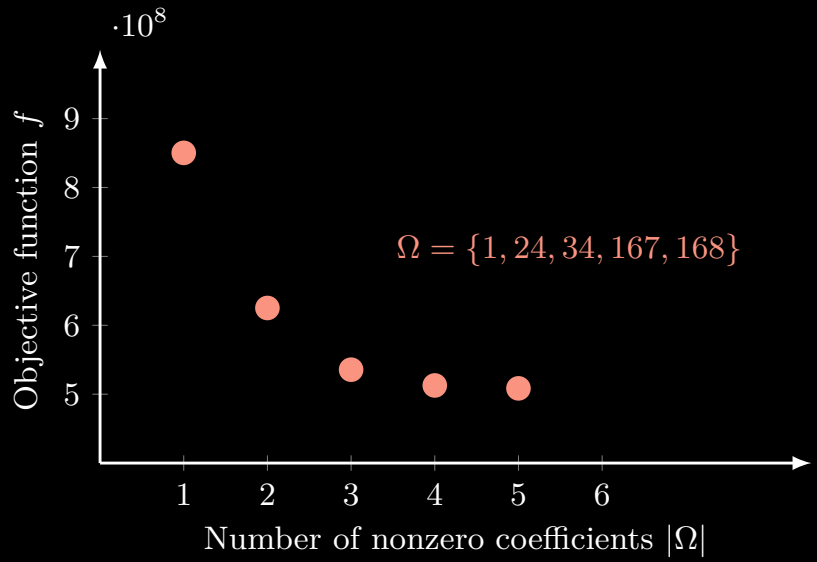
$$\min_{w_k,\, k\in\Omega} \ f \triangleq \sum_t \left( x_t - \sum_{k\in\Omega} w_k x_{t-k} \right)^2$$

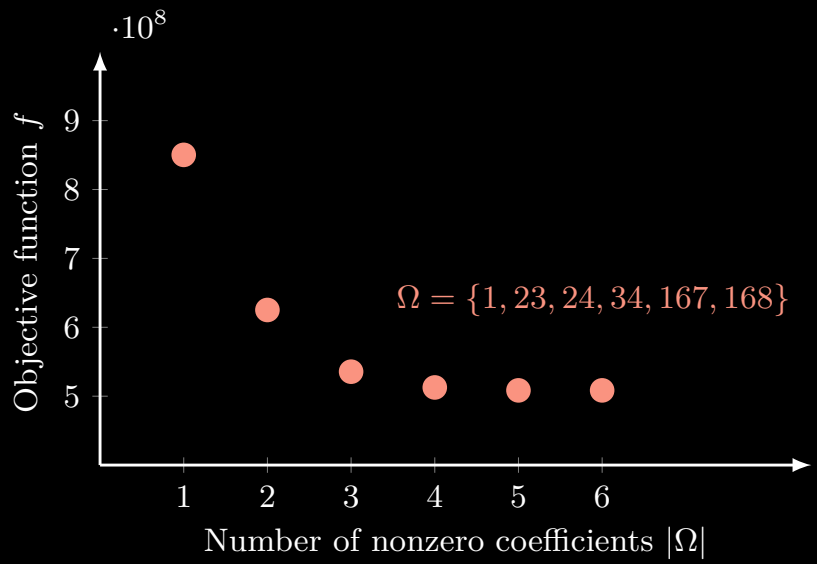

$\cdot 10^8$
$\Omega = \{168\}$
Objective function $f$
9
8
7
6
5
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$

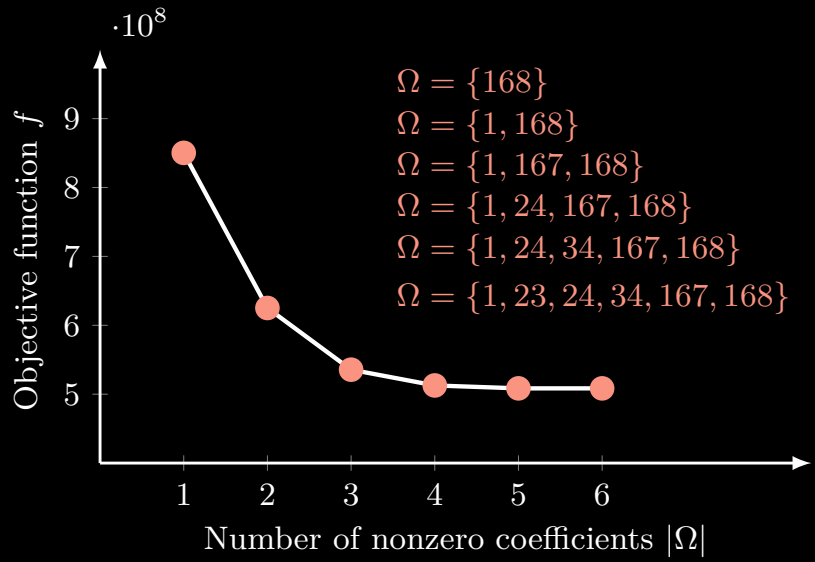

$\cdot 10^8$
9
8
7
6
5
Objective function $f$
$\Omega = \{1, 168\}$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$


$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$\Omega = \{1, 167, 168\}$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$


$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$\Omega = \{1, 24, 167, 168\}$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$

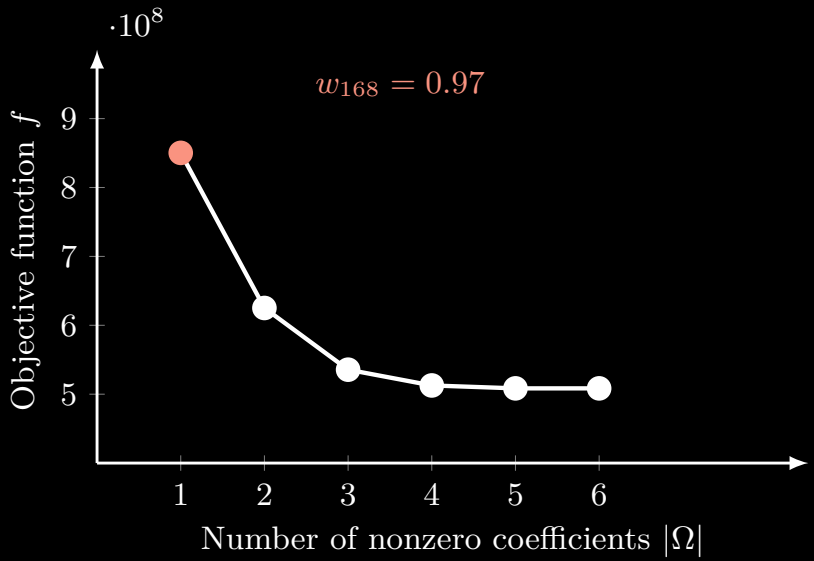

$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$\Omega = \{1, 24, 34, 167, 168\}$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$

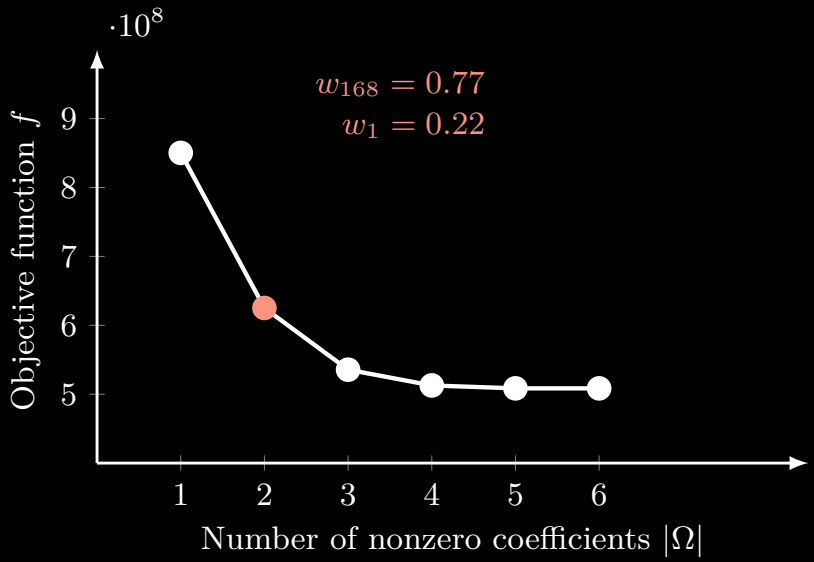

$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$\Omega = \{1, 23, 24, 34, 167, 168\}$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$

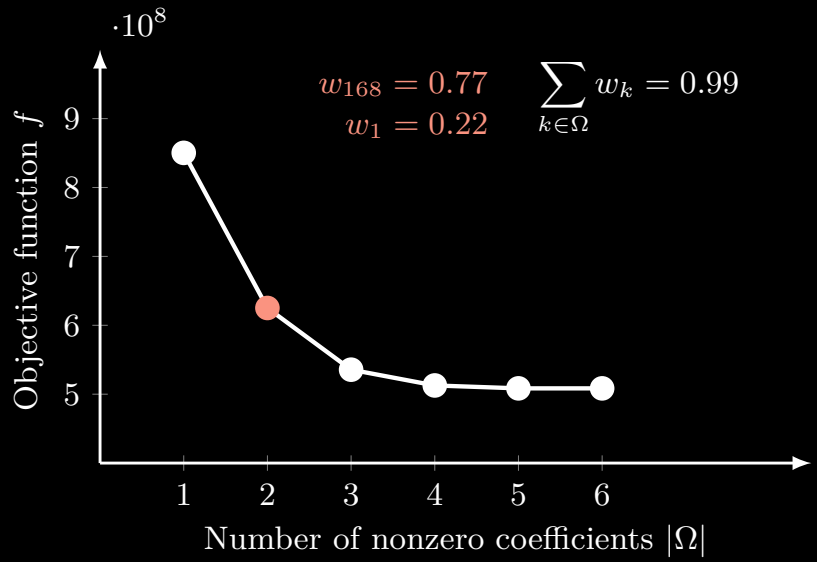

$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$\Omega = \{168\}$
$\Omega = \{1, 168\}$
$\Omega = \{1, 167, 168\}$
$\Omega = \{1, 24, 167, 168\}$
$\Omega = \{1, 24, 34, 167, 168\}$
$\Omega = \{1, 23, 24, 34, 167, 168\}$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$

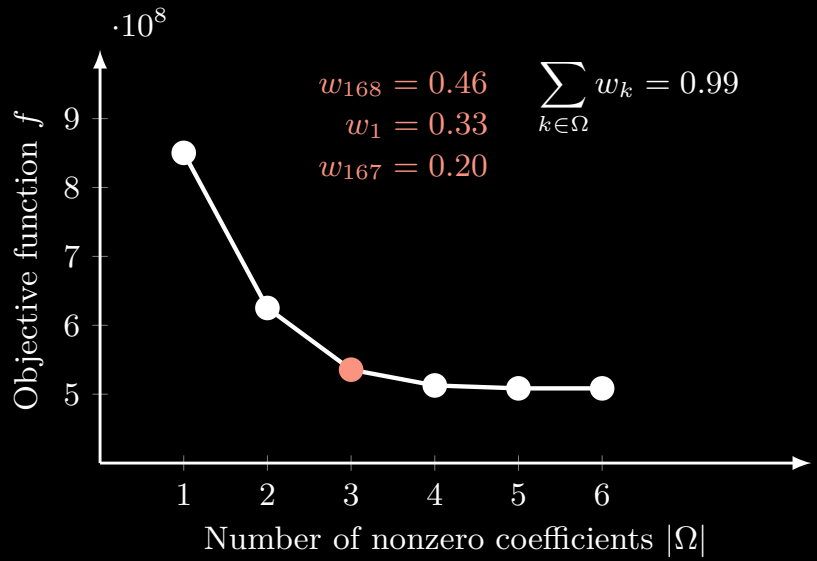
Select dominant coefficients via optimization

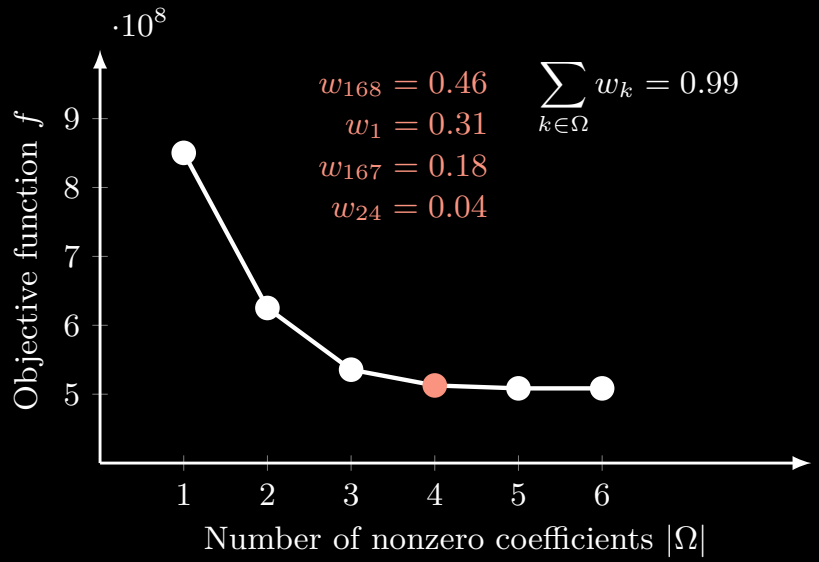
$$\min_{w_k, k\in\Omega} \sum_t \left( x_t - \sum_{k\in\Omega} w_k x_{t-k} \right)^2$$
$$\text{s.t. } |\Omega| \leq \tau,\ \tau = 1, 2, 3, \ldots$$

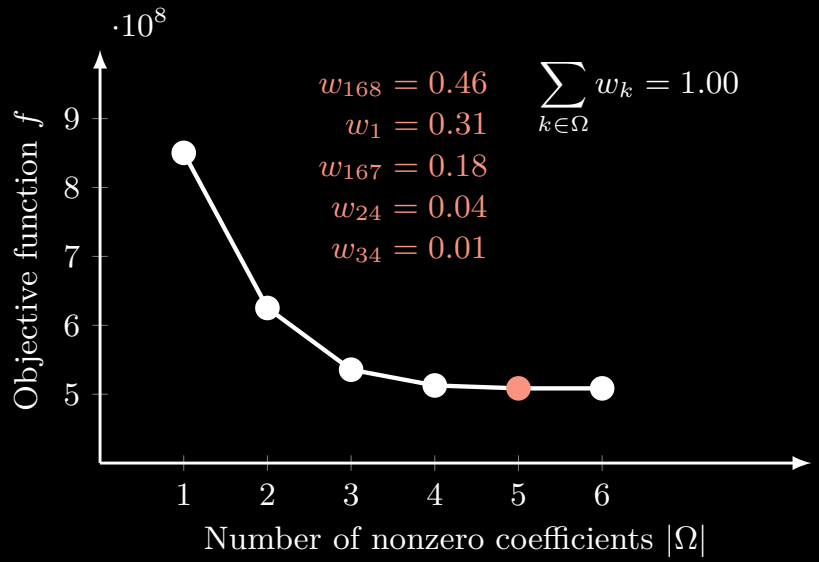

$\cdot 10^8$
$w_{168} = 0.97$
Objective function $f$
9
8
7
6
5
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$

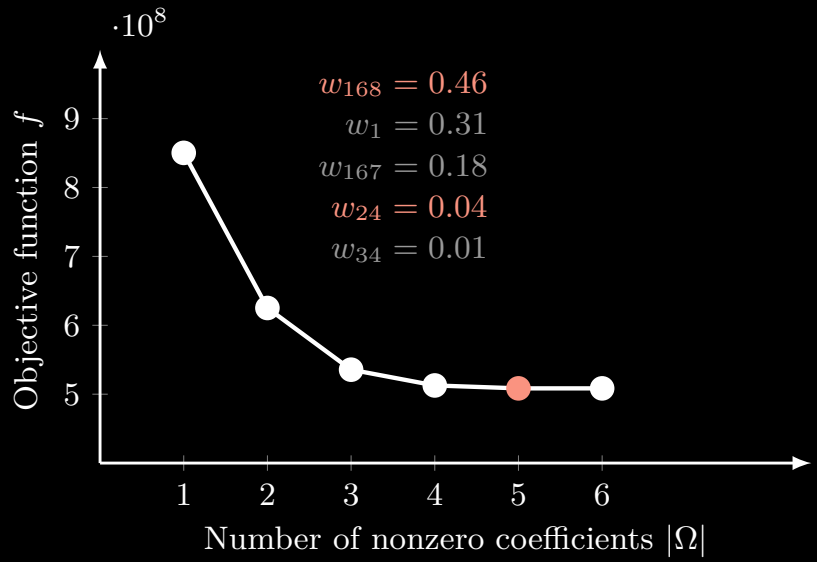

$\cdot 10^8$
$w_{168} = 0.77$
$w_1 = 0.22$
Objective function $f$
9
8
7
6
5
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$

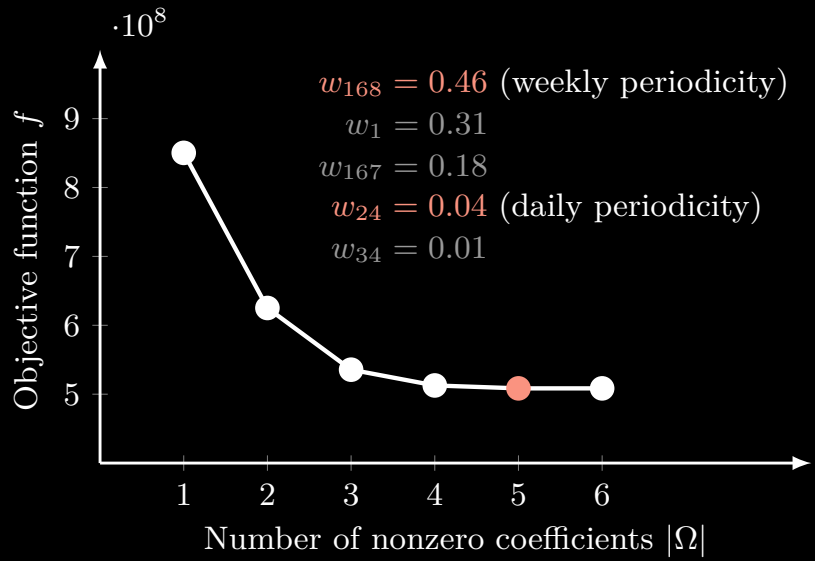

$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$w_{168} = 0.77$
$w_1 = 0.22$
$\sum_{k\in\Omega} w_k = 0.99$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$


$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$w_{168} = 0.46$
$w_1 = 0.33$
$w_{167} = 0.20$
$\sum_{k \in \Omega} w_k = 0.99$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$


$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$w_{168} = 0.46$
$w_1 = 0.31$
$w_{167} = 0.18$
$w_{24} = 0.04$
$\sum_{k \in \Omega} w_k = 0.99$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$


$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$w_{168} = 0.46$
$w_1 = 0.31$
$w_{167} = 0.18$
$w_{24} = 0.04$
$w_{34} = 0.01$
$\sum_{k \in \Omega} w_k = 1.00$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$


$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$w_{168} = 0.46$
$w_1 = 0.31$
$w_{167} = 0.18$
$w_{24} = 0.04$
$w_{34} = 0.01$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$


$\cdot 10^8$
Objective function $f$
9
8
7
6
5
$w_{168} = 0.46$ (weekly periodicity)
$w_1 = 0.31$
$w_{167} = 0.18$
$w_{24} = 0.04$ (daily periodicity)
$w_{34} = 0.01$
1
2
3
4
5
6
Number of nonzero coefficients $|\Omega|$