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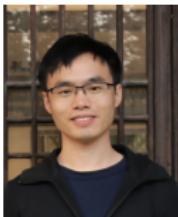


Spatiotemporal Traffic Data Imputation and Forecasting with Tensor Learning

Ph.D. Research Project

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Outline

- **Motivation**

- Multivariate traffic time series
- Multidimensional traffic time series
- Multiple data behaviors

- **Literature Review**

- Spatiotemporal traffic data imputation
- Spatiotemporal traffic forecasting
- Low-rank tensor learning

- **Objective A**

- Spatiotemporal traffic data imputation

- **Objective B**

- High-dimensional traffic forecasting
- Multidimensional traffic forecasting

- **Objective C**

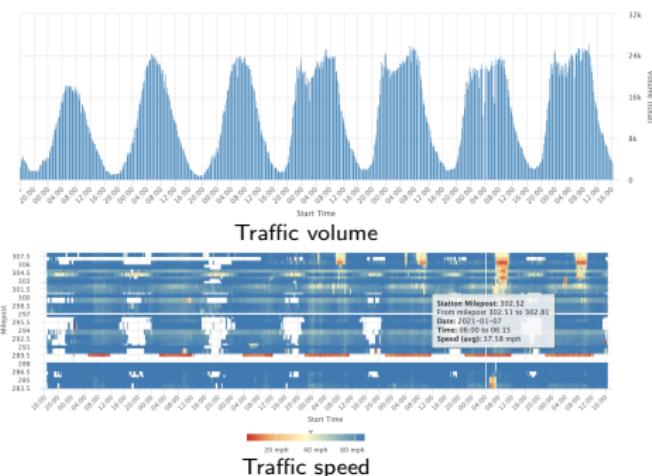
- Multivariate traffic forecasting on sparse data
- Multidimensional traffic forecasting on sparse data

- **Conclusion**

Multivariate Traffic Time Series

Many spatiotemporal traffic time series data are in the form of **matrix**.

- Example: Portland highway traffic data¹.



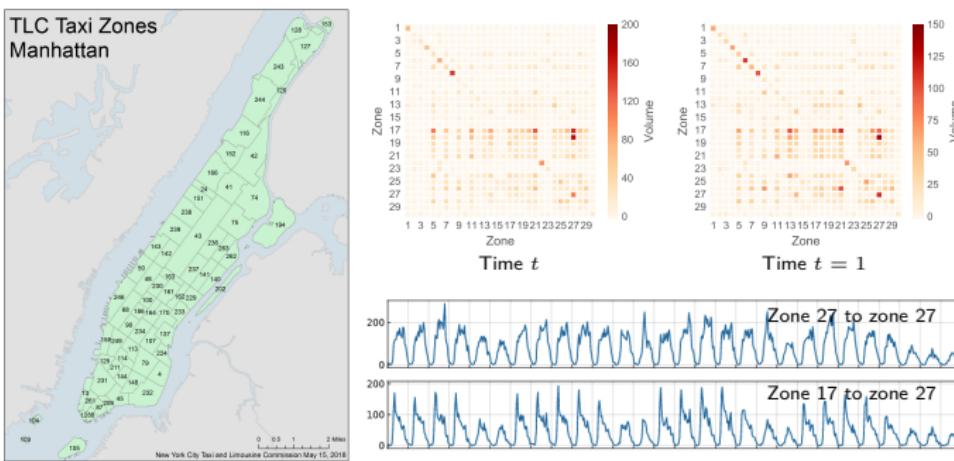
- $X \in \mathbb{R}^{N \times T}$ with N spatial locations $\times T$ time steps

¹<https://portal.its.pdx.edu/home>

Multidimensional Traffic Time Series

Many spatiotemporal traffic time series data are in the form of **tensor**.

- Example: NYC (hourly) taxi flow data².



- $\mathcal{X} \in \mathbb{R}^{M \times N \times T}$ with M zones \times N zones \times T time steps

²<https://www1.nyc.gov/site/tlc/about/tlc-trip-record-data.page>

Multiple Data Behaviors

Spatiotemporal traffic data are time series, but they involve multiple data behaviors.

- Incompleteness & sparsity
- High-dimensionality
- Multidimensionality
- Noises & outliers
- Time-varying behavior
- Nonstationarity
-

In addition, spatiotemporal correlations are also very important.

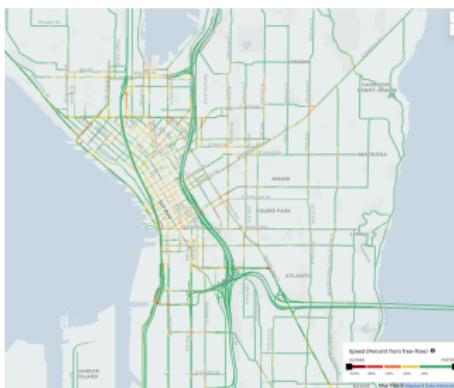
Multiple Data Behaviors

Sparsity & high-dimensionality

- Uber (hourly) movement speed data³



NYC movement



Seattle movement

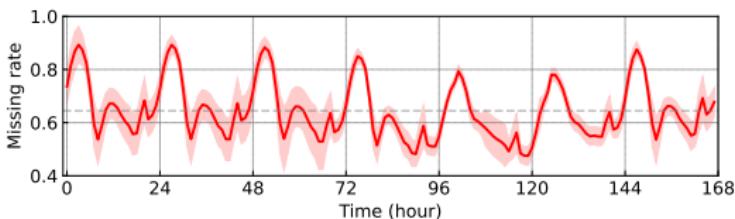
- The average speed on a given road segment for each hour of each day.
- Hourly speeds are computed when road segments have 5+ unique trips.
- **Issue:** insufficient sampling of ridesharing vehicles on the road network.

³<https://movement.uber.com/>

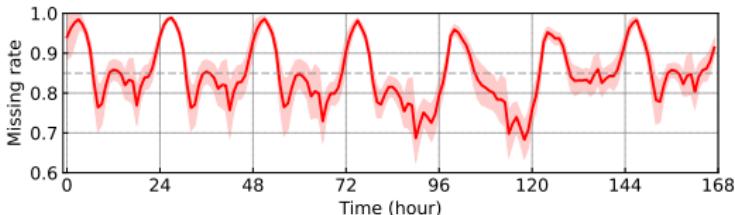
Multiple Data Behaviors

Sparsity & high-dimensionality

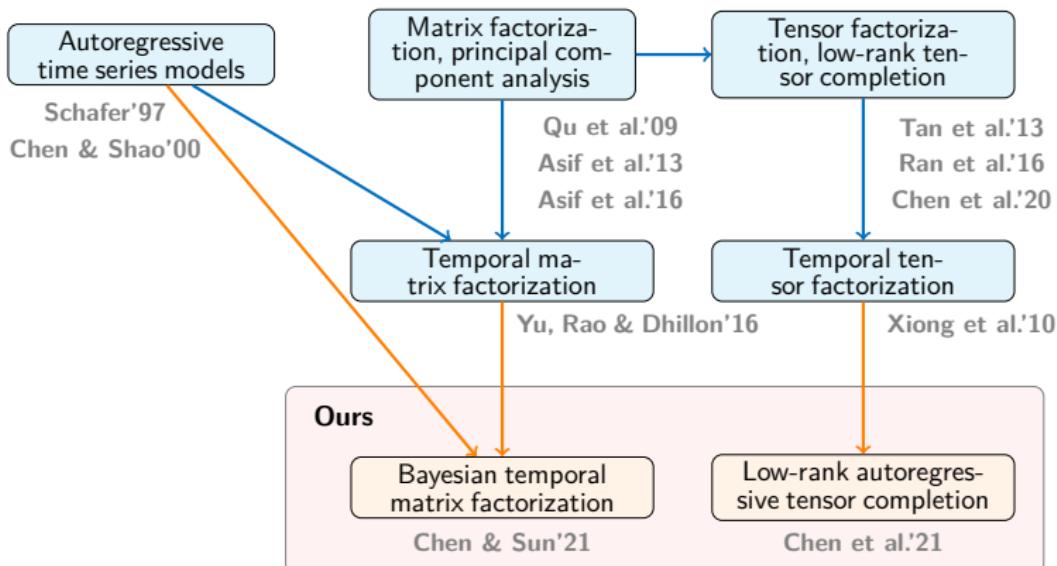
- **NYC** movement speed data (2019)
 - **98,210** road segments & 8,760 time steps (hours)
 - Overall missing rate: **64.43%**



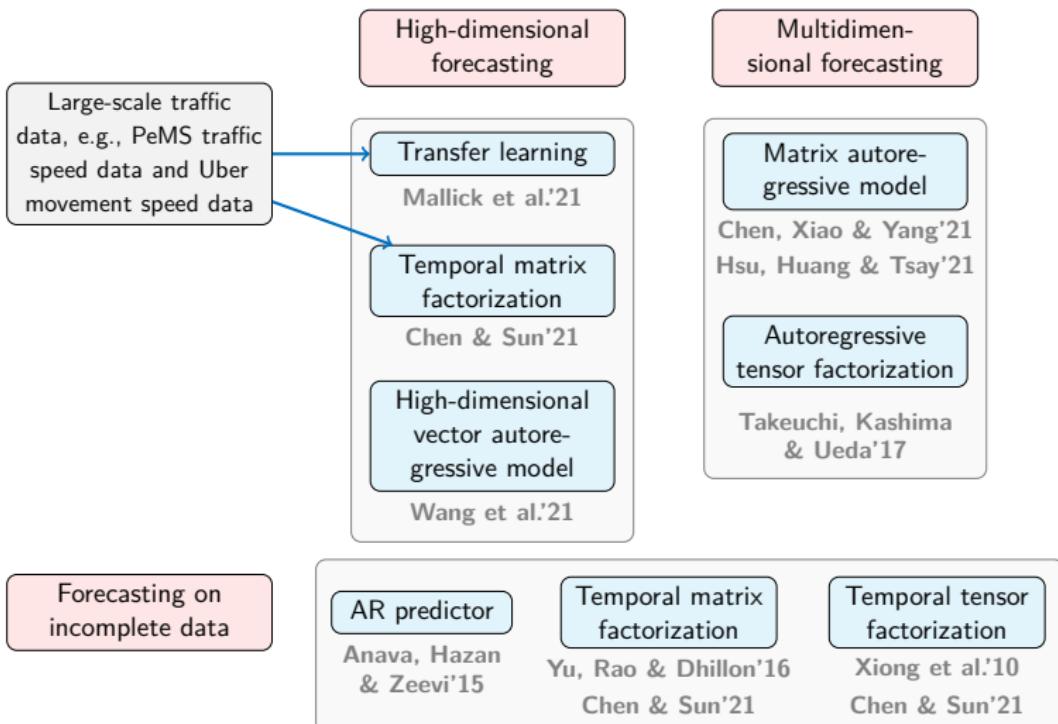
- **Seattle** movement speed data (2019)
 - **63,490** road segments & 8,760 time steps (hours)
 - Overall missing rate: **84.95%**



Spatiotemporal Traffic Data Imputation



Spatiotemporal Traffic Forecasting

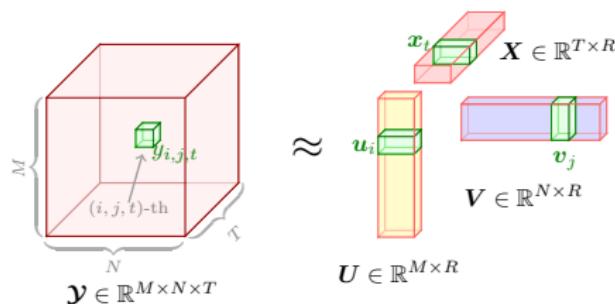


Low-Rank Tensor Learning

From data to model

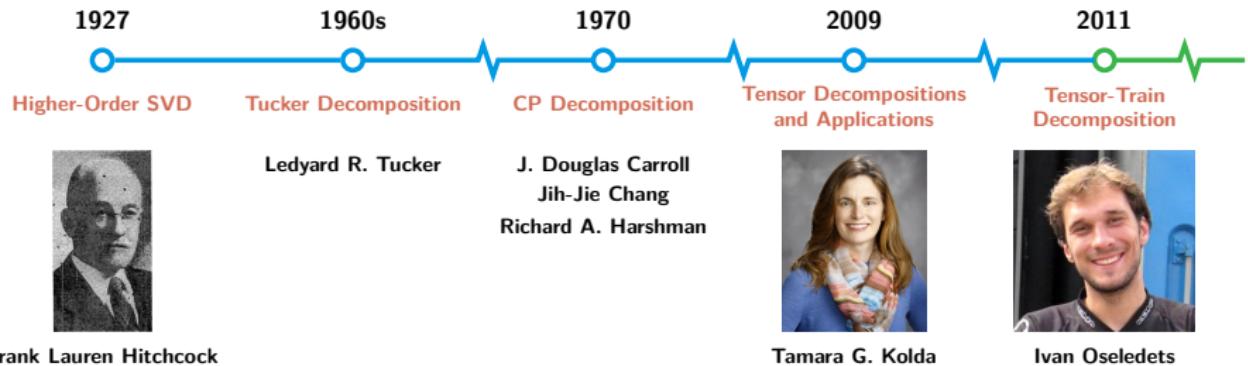
- Matrix factorization:

$$\min_{\mathbf{W}, \mathbf{X}} \frac{1}{2} \|\mathcal{P}_{\Omega}(\mathbf{Y} - \mathbf{W}^{\top} \mathbf{X})\|_F^2 + \frac{\lambda}{2} (\|\mathbf{W}\|_F^2 + \|\mathbf{X}\|_F^2)$$



- Tensor factorization:

$$\min_{\mathbf{U}, \mathbf{V}, \mathbf{X}} \frac{1}{2} \|\mathcal{P}_{\Omega}(\mathbf{Y} - \sum_{r=1}^R \mathbf{u}_r \circ \mathbf{v}_r \circ \mathbf{x}_r)\|_F^2 + \frac{\lambda}{2} (\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2 + \|\mathbf{X}\|_F^2)$$



Low-Rank Tensor Learning

- Low-rank matrix/tensor completion



Candès & Recht'09: Convex nuclear norm minimization for matrix completion.

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{X}\|_* \\ \text{s.t. } & \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{Y}) \end{aligned}$$



Cai, Candès & Shen'10: Singular value thresholding algorithm.

$$\begin{cases} \mathbf{X}^\ell = \mathcal{D}_\tau(\mathbf{Z}^{\ell-1}) \\ \mathbf{Z}^\ell = \mathbf{Z}^{\ell-1} + \delta_\ell \mathcal{P}_\Omega(\mathbf{Y} - \mathbf{X}^\ell) \end{cases}$$



Zhang et al.'12: Nonconvex truncated nuclear norm minimization.



Liu et al.'13: Convex nuclear norm minimization for tensor completion.

$$\begin{aligned} \min_{\mathcal{X}} \quad & \|\mathcal{X}\|_* \\ \text{s.t. } & \mathcal{P}_\Omega(\mathcal{X}) = \mathcal{P}_\Omega(\mathcal{Y}) \end{aligned}$$

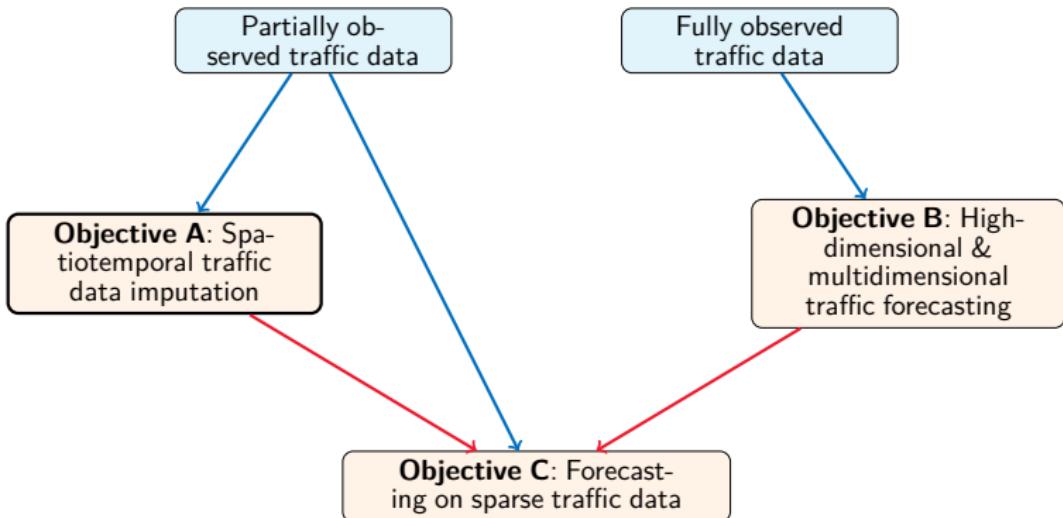


Lu, Peng & Wei'19: Tensor nuclear norm induced by linear transform.



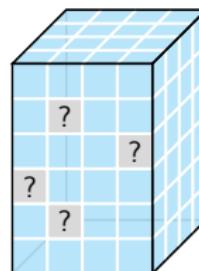
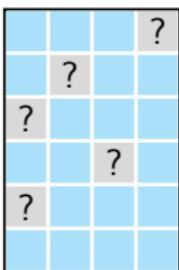
A Whole Picture of Objectives

We are working on **spatiotemporal traffic data modeling**.



Spatiotemporal Traffic Data Imputation

- **Objective A:** Given a multivariate time series data like $\mathbf{Y} \in \mathbb{R}^{N \times T}$ or a multidimensional time series data like $\mathcal{Y} \in \mathbb{R}^{M \times N \times T}$, impute the missing values of the data.



[Q]

- How to learn and reconstruct missing values from observed data?
- How to make use of spatiotemporal correlations?
- How to make use of traffic time series dynamics?

Spatiotemporal Traffic Data Imputation

Low-rank matrix completion (Candès & Recht'09)

For any partially observed data matrix $\mathbf{Y} \in \mathbb{R}^{N \times T}$ with observed index set Ω , then low-rank matrix completion takes the form of

$$\begin{aligned} & \min_{\mathbf{X}} \|\mathbf{X}\|_* \\ & \text{s.t. } \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{Y}). \end{aligned} \tag{1}$$

Low-rank tensor completion (Liu et al.'13)

For any partially observed data matrix $\mathcal{Y} \in \mathbb{R}^{M \times N \times T}$ with observed index set Ω , then low-rank matrix completion takes the form of

$$\begin{aligned} & \min_{\boldsymbol{\mathcal{X}}} \|\boldsymbol{\mathcal{X}}\|_* \\ & \text{s.t. } \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}}) = \mathcal{P}_\Omega(\mathcal{Y}). \end{aligned} \tag{2}$$

- **Limitation:** Only cover the global consistency.
- **Comment:** For modeling spatiotemporal traffic data, local consistency (e.g., temporal correlations) is also important.

Spatiotemporal Traffic Data Imputation

Low-rank autoregressive matrix completion

Given the multivariate traffic time series data $\mathbf{Y} \in \mathbb{R}^{N \times T}$ with the observed index set Ω , the low-rank matrix completion combines both nuclear norm and univariate autoregressive process:

$$\begin{aligned} & \min_{\mathbf{X}} \|\mathbf{X}\|_* + \frac{\lambda}{2} \sum_{n=1}^N \sum_{t=d+1}^T (z_{n,t} - \sum_{k=1}^d a_{n,k} z_{n,t-k})^2 \\ \text{s.t. } & \begin{cases} \mathbf{X} = \mathbf{Z}, \\ \mathcal{P}_\Omega(\mathbf{Z}) = \mathcal{P}_\Omega(\mathbf{Y}). \end{cases} \end{aligned} \tag{3}$$

Low-rank autoregressive tensor completion

If T time steps can be separated into I time steps per day and J days, i.e., $T = IJ$, then we can define an operator $\mathcal{Q}(\cdot)$ to convert multivariate data to third-order tensor:

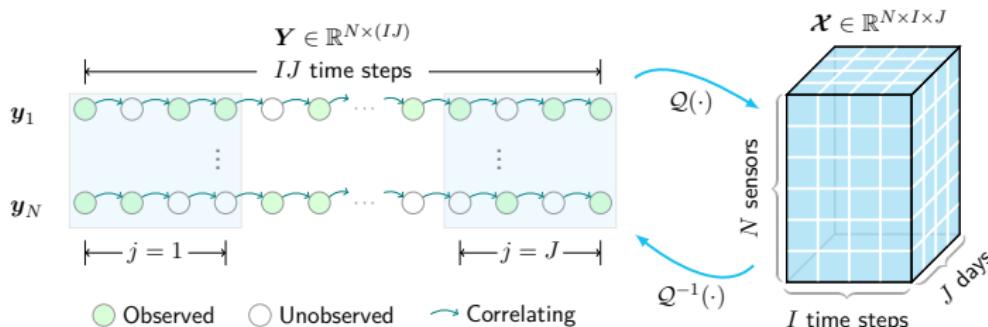
$$\begin{aligned} & \min_{\boldsymbol{\mathcal{X}}} \|\boldsymbol{\mathcal{X}}\|_* + \frac{\lambda}{2} \sum_{n=1}^N \sum_{t=d+1}^T (z_{n,t} - \sum_{k=1}^d a_{n,k} z_{n,t-k})^2 \\ \text{s.t. } & \begin{cases} \boldsymbol{\mathcal{X}} = \mathcal{Q}(\mathbf{Z}), \\ \mathcal{P}_\Omega(\mathbf{Z}) = \mathcal{P}_\Omega(\mathbf{Y}). \end{cases} \end{aligned} \tag{4}$$

Spatiotemporal Traffic Data Imputation

Low-rank autoregressive tensor completion

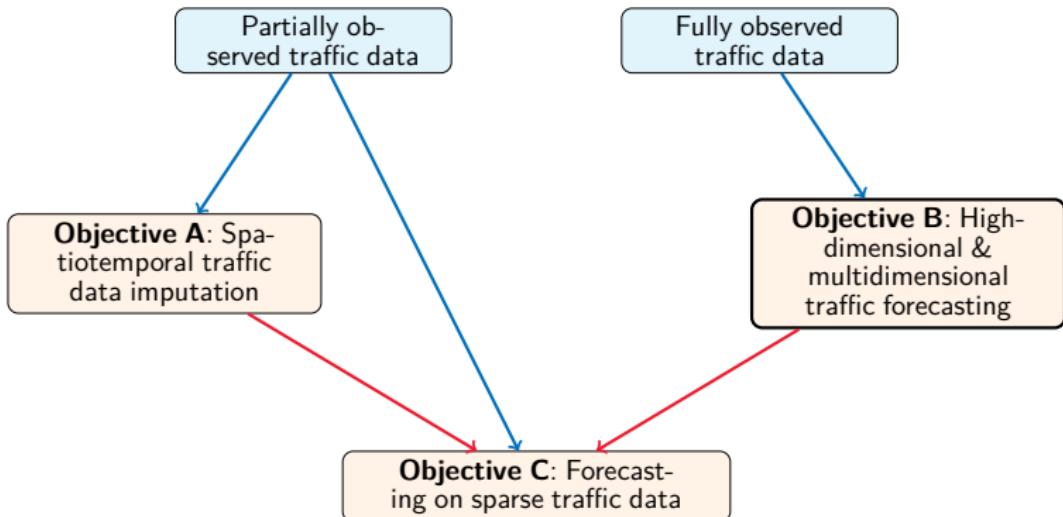
$$\begin{aligned}
 & \min_{\mathcal{X}} \|\mathcal{X}\|_* + \frac{\lambda}{2} \sum_{n=1}^N \sum_{t=d+1}^T (z_{n,t} - \sum_{k=1}^d a_{n,k} z_{n,t-k})^2 \\
 & \text{s.t. } \begin{cases} \mathcal{X} = \mathcal{Q}(\mathbf{Z}), \\ \mathcal{P}_{\Omega}(\mathbf{Z}) = \mathcal{P}_{\Omega}(\mathbf{Y}). \end{cases}
 \end{aligned} \tag{5}$$

- **Advantage:** Global consistency + local consistency.



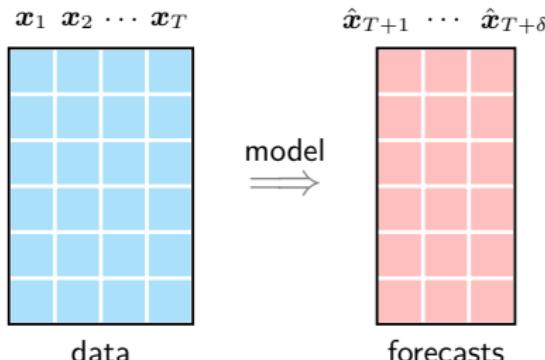
A Whole Picture of Objectives

We are working on **spatiotemporal traffic data modeling**.



High-Dimensional Traffic Forecasting

- **Objective B-1:** Given a multivariate traffic time series $\mathbf{x}_1, \dots, \mathbf{x}_T \in \mathbb{R}^N$ with $N \gg T$ ("tall-skinny"), forecast data points $\hat{\mathbf{x}}_{T+\delta}, \delta \in \mathbb{N}^+$.

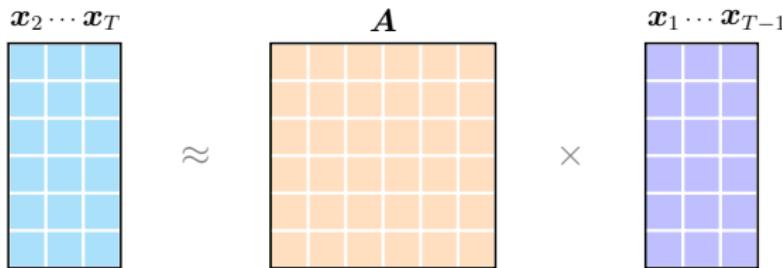


- **Solution:** For time series $\mathbf{x}_1, \dots, \mathbf{x}_T \in \mathbb{R}^N$, the d th-order vector autoregressive (VAR(d)) model: $\mathbf{x}_t = \sum_{k=1}^d \mathbf{A}_k \mathbf{x}_{t-k} + \boldsymbol{\epsilon}_t$.
- **Advantage:** Co-evolution patterns
- **Limitation:** Over-parameterization

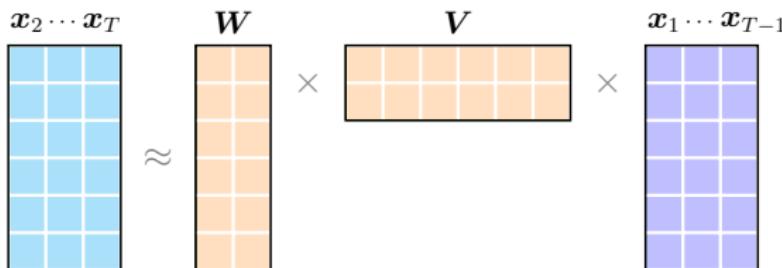
High-Dimensional Traffic Forecasting

VAR(1) model

- Over-parameterization in the case of $N \gg T$.



- Reduced-rank autoregression: $\mathbf{A} = \mathbf{WV}$ with $\mathbf{W} \in \mathbb{R}^{N \times R}, \mathbf{V} \in \mathbb{R}^{R \times N}$.



High-Dimensional Traffic Forecasting

Multivariate reduced-rank regression

- Data: $Z \in \mathbb{R}^{N \times T}$ (input) & $Y \in \mathbb{R}^{M \times T}$ (output)
- Component matrices $W \in \mathbb{R}^{M \times R}$, $V \in \mathbb{R}^{R \times N}$ such that $A = WV$
- Optimization problem:

$$W^*, V^* \triangleq \arg \min_{W, V} \frac{1}{2} \|Y - WVZ\|_F^2 \quad (6)$$

Theorem 1 (Izenman'75)

Suppose the optimization problem of multivariate reduced-rank regression trained on the centered data matrices Z and Y , then for any positive definite matrix Γ , the solutions are

$$W^* = \Gamma^{-1/2} \xi, \quad V^* = \xi^\top \Gamma^{1/2} \Sigma_{yz} \Sigma_{zz}^{-1}, \quad (7)$$

where $\xi \in \mathbb{R}^{N \times R}$ consists of the R eigenvectors corresponding to the R largest eigenvalues of the matrix $\Gamma^{1/2} \Sigma_{yz} \Sigma_{zz}^{-1} \Sigma_{zy} \Gamma^{1/2}$.

High-Dimensional Traffic Forecasting

VAR(d) model

- Recall that $\mathbf{x}_t = \sum_{k=1}^d \mathbf{A}_k \mathbf{x}_{t-k} + \boldsymbol{\epsilon}_t$.
- Coefficients $\mathbf{A}_k \in \mathbb{R}^{N \times N}$, $k = 1, \dots, d$ are tensor, e.g., $\mathcal{A} \in \mathbb{R}^{N \times N \times d}$.

VAR(d) with Tucker decomposition (Wang et al.'21)

For VAR(d) on the multivariate time series $\mathbf{x}_t \in \mathbb{R}^N$, $t = 1, \dots, T$, the reduced-rank VAR via Tucker decomposition is given by

$$\min_{\mathcal{G}, \mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3} \frac{1}{2} \sum_{t=d+1}^T \|\mathbf{x}_t - (\mathcal{G} \times_1 \mathbf{W} \times_2 \mathbf{U} \times_3 \mathbf{H})_{(1)} \mathbf{z}_t\|_2^2 \quad (8)$$

where $\mathbf{z}_t = (\mathbf{x}_{t-1}^\top, \dots, \mathbf{x}_{t-d}^\top)^\top \in \mathbb{R}^{dN}$. The multilinear rank is (R_1, R_2, R_3) .

$\mathcal{G} \in \mathbb{R}^{R_1 \times R_2 \times R_3}$ is the core tensor, while $\mathbf{W} \in \mathbb{R}^{N \times R_1}$, $\mathbf{U} \in \mathbb{R}^{N \times R_2}$, and $\mathbf{H} \in \mathbb{R}^{d \times R_3}$ are the component matrices.

Advantage: High compression rate.

Limitations: Nonconvex optimization; the model is failing in nonstationary time series.

High-Dimensional Traffic Forecasting

Reduced-rank time-varying VAR

Given traffic data samples $\{\mathbf{x}_{t-d}, \dots, \mathbf{x}_t\}$ at time t , let $\mathbf{y}_t \triangleq \mathbf{x}_t \in \mathbb{R}^N$ and $\mathbf{z}_t \triangleq (\mathbf{x}_{t-1}^\top, \dots, \mathbf{x}_{t-d}^\top)^\top \in \mathbb{R}^{dN}$, then the reduced-rank time-varying VAR takes

$$\begin{aligned} \min_{\mathbf{G}_t, \mathbf{W}_t, \mathbf{U}_t, \mathbf{H}_t} \quad & \frac{1}{2} (\|\mathbf{G}_t - \mathbf{G}_{t-1}\|_F^2 + \|\mathbf{W}_t - \mathbf{W}_{t-1}\|_F^2 \\ & + \|\mathbf{U}_t - \mathbf{U}_{t-1}\|_F^2 + \|\mathbf{H}_t - \mathbf{H}_{t-1}\|_F^2) \\ & + \frac{\lambda}{2} \|\mathbf{y}_t - \mathbf{W}_t \mathbf{G}_t (\mathbf{H}_t \otimes \mathbf{U}_t)^\top \mathbf{z}_t\|_2^2 \end{aligned} \quad (9)$$

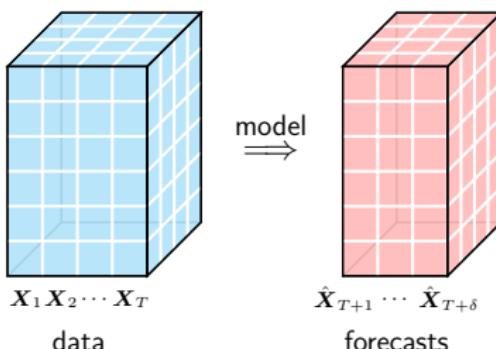
where $\mathbf{G}_t \in \mathbb{R}^{R \times (dR)}$, $\mathbf{W}_t \in \mathbb{R}^{N \times R}$, $\mathbf{U}_t \in \mathbb{R}^{N \times R}$, $\mathbf{H}_t \in \mathbb{R}^{d \times d}$.

Advantages:

- Produce time-varying parameters.
- Overcome nonstationarity.

Multidimensional Traffic Forecasting

- **Objective B-2:** Given a multidimensional traffic time series $\mathbf{X}_1, \dots, \mathbf{X}_T \in \mathbb{R}^{M \times N}$, forecast data points $\hat{\mathbf{X}}_{T+\delta}, \delta \in \mathbb{N}^+$.



[Q]

- How to perform forecasting on this kind of data?
- How to preserve the intrinsic tensor representation of data?

Multidimensional Traffic Forecasting

Matrix autoregressive model (Chen, Xiao & Yang'21)

Given matrix-variate time series $\mathbf{X}_t \in \mathbb{R}^{M \times N}$, $t = 1, \dots, T$, then the d th-order matrix autoregressive (MAR(d)) model takes the form of

$$\mathbf{X}_t = \sum_{k=1}^d \mathbf{A}_k \mathbf{X}_{t-k} \mathbf{B}_k^\top + \mathbf{E}_t \quad (10)$$

where $\mathbf{A}_k \in \mathbb{R}^{M \times M}$, $\mathbf{B}_k \in \mathbb{R}^{N \times N}$, $k = 1, \dots, d$ are the coefficient matrices.

Advantages:

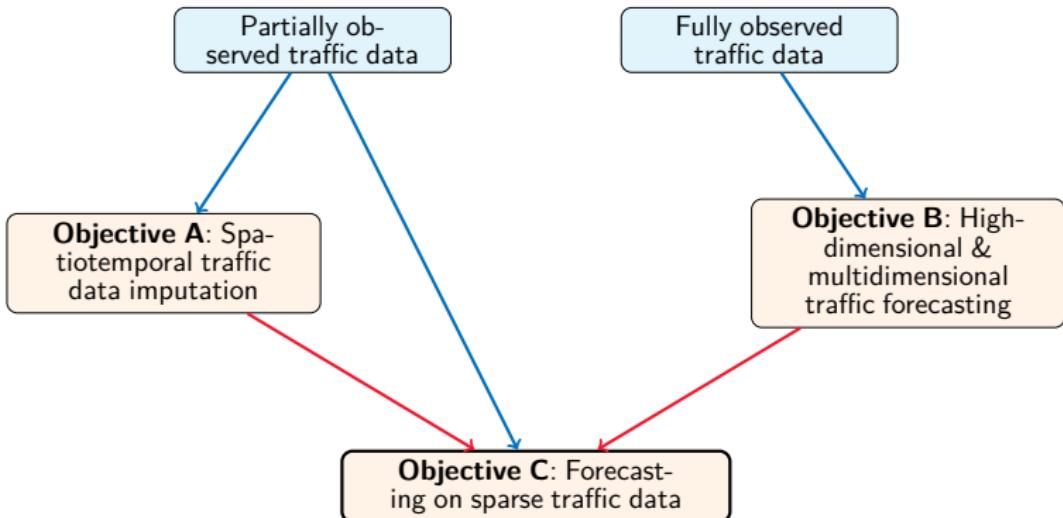
- Preserve the intrinsic tensor representation.
- Reduce parameters in autoregressive models (if $n = \max\{M, N\}$), e.g.,

$$\mathcal{O}(n^4) \text{ in VAR(1)} \quad \text{vs.} \quad \mathcal{O}(n^2) \text{ in MAR(1)}$$

Limitation: Failing in nonstationary time series.

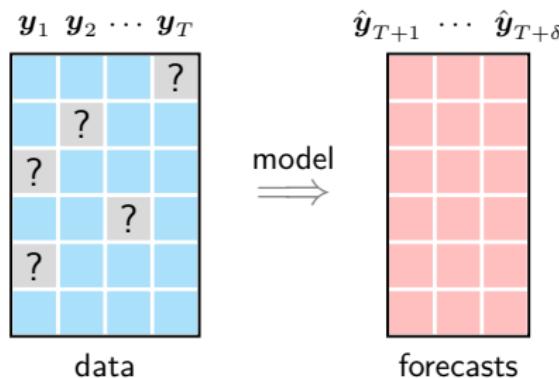
A Whole Picture of Objectives

We are working on **spatiotemporal traffic data modeling**.



Multivariate Traffic Forecasting on Sparse Data

- **Objective C-1:** Given a partially observed data $\mathbf{Y} \in \mathbb{R}^{N \times T}$ consisting of time series $y_1, \dots, y_T \in \mathbb{R}^N$, forecast data points $y_{T+\delta}, \delta \in \mathbb{N}^+$.



[Q]

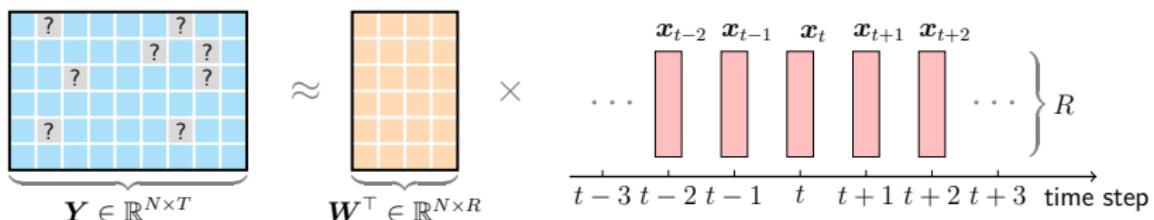
- How to learn from *high-dimensional* and *sparse* data?
- How to model *nonstationarity* in time series?
- How to perform forecasting on these time series?

Multivariate Traffic Forecasting on Sparse Data

Temporal matrix factorization (Yu et al.'16; Chen & Sun'21)

Given any partially observed time series data $\mathbf{Y} \in \mathbb{R}^{N \times T}$ with observed index set Ω , then temporal matrix factorization assumes a d th-order vector autoregressive (VAR) process on the temporal factor matrix:

$$\begin{aligned} \min_{\mathbf{W}, \mathbf{X}, \{\mathbf{A}_k\}_{k=1}^d} & \frac{1}{2} \|\mathcal{P}_\Omega(\mathbf{Y} - \mathbf{W}^\top \mathbf{X})\|_F^2 + \frac{\rho}{2} (\|\mathbf{W}\|_F^2 + \|\mathbf{X}\|_F^2) \\ & + \frac{\lambda}{2} \sum_{t=d+1}^T \|\mathbf{x}_t - \sum_{k=1}^d \mathbf{A}_k \mathbf{x}_{t-k}\|_2^2 \end{aligned} \quad (11)$$



⚠ VAR is usually built on stationary time series (temporal factors).

Multivariate Traffic Forecasting on Sparse Data

Nonstationary temporal matrix factorization (NoTMF)

Given any partially observed time series data $\mathbf{Y} \in \mathbb{R}^{N \times T}$ with observed index set Ω , then we assume a season- m differencing on the latent temporal factors:

$$\begin{aligned} \min_{\mathbf{W}, \mathbf{X}, \{\mathbf{A}_k\}_{k=1}^d} & \frac{1}{2} \|\mathcal{P}_\Omega(\mathbf{Y} - \mathbf{W}^\top \mathbf{X})\|_F^2 + \frac{\rho}{2} (\|\mathbf{W}\|_F^2 + \|\mathbf{X}\|_F^2) \\ & + \frac{\lambda}{2} \sum_{t=d+m+1}^T \|(\mathbf{x}_t - \mathbf{x}_{t-m}) - \sum_{k=1}^d \mathbf{A}_k (\mathbf{x}_{t-k} - \mathbf{x}_{t-m-k})\|_2^2 \end{aligned} \quad (12)$$

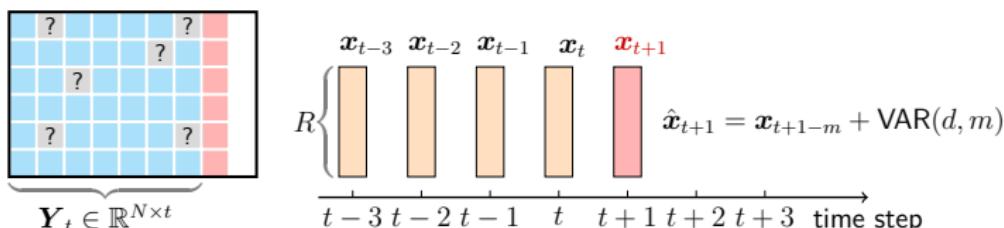
- First-order differencing $\mathbf{x}'_t = \mathbf{x}_t - \mathbf{x}_{t-1}$.
 - Second-order differencing $\mathbf{x}''_t = (\mathbf{x}_t - \mathbf{x}_{t-1}) - (\mathbf{x}_{t-1} - \mathbf{x}_{t-2})$.
 - Twice-differenced series $\mathbf{x}'''_t = (\mathbf{x}_t - \mathbf{x}_{t-m}) - (\mathbf{x}_{t-1} - \mathbf{x}_{t-m-1})$.
- 😊 Stationarizing a time series with differencing can improve the prediction.⁴

⁴ Stationarity and differencing: <https://otexts.com/fpp2/stationarity.html>

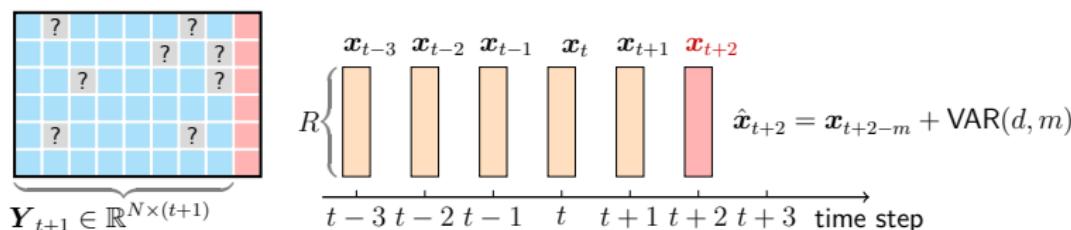
Multivariate Traffic Forecasting on Sparse Data

NoTMF forecasting on streaming data?

- NoTMF: Use \mathbf{Y}_t to estimate $\{\mathbf{W}, \mathbf{X}, \mathbf{A}\}$.

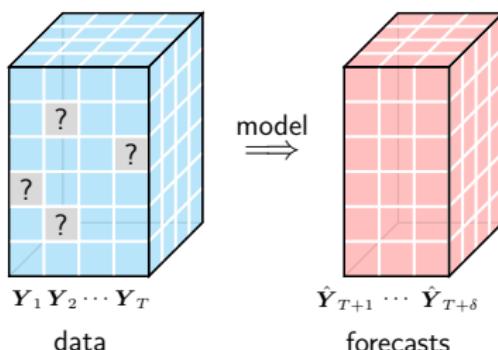


- Online forecasting (Gultekin & Paisley'18): Fix \mathbf{W} and use \mathbf{Y}_{t+1} to update $\{\mathbf{X}, \mathbf{A}\}$.



Multidimensional Traffic Forecasting on Sparse Data

- **Objective C-2:** Given a partially observed data $\mathcal{Y} \in \mathbb{R}^{M \times N \times T}$ consisting of time series $\mathbf{Y}_1, \dots, \mathbf{Y}_T \in \mathbb{R}^{M \times N}$, forecast data points $\hat{\mathbf{Y}}_{T+\delta}, \delta \in \mathbb{N}^+$.



- **Solution:** Temporal tensor factorization, e.g., tensor factorization + VAR (on latent temporal factors).

Conclusion

- Contributions

- *Objective A: Spatiotemporal traffic data imputation.* Develop a low-rank temporal modeling framework and improve the imputation accuracy, efficiency, and scalability.
- *Objective B: High-dimensional and multidimensional forecasting.* Fast and accurate forecasting approach for high-dimensional and large-scale data; tensor representation based autoregressive model for multidimensional data.
- *Objective C: Forecasting on sparse data.* Low-rank temporal modeling framework for traffic time series forecasting in the presence of missing values.

- Significance

- Improve traffic data quality.
- Support data-driven intelligent transportation applications.

Research Work during Ph.D. Research

- Publications
 - [J1] X. Chen, M. Lei, N. Saunier, L. Sun (2021). Low-rank autoregressive tensor completion for spatiotemporal traffic data imputation. *IEEE Transactions on Intelligent Transportation Systems*. (Early access)
 - [J2] X. Chen, Y. Chen, N. Saunier, L. Sun (2021). Scalable low-rank tensor learning for spatiotemporal traffic data imputation. *Transportation Research Part C: Emerging Technologies*, 129: 103226.
 - [C1] X. Chen, M. Lei, N. Saunier, L. Sun (2021). Low-rank autoregressive tensor completion for spatiotemporal traffic data imputation. *The 7th SIGKDD Workshop on Mining and Learning from Time Series (MiLeTS)* at KDD 2021.
- Preprint (under review)
 - [P1] X. Chen, C. Zhang, X.L. Zhao, N. Saunier, L. Sun (2022). Nonstationary temporal matrix factorization for multivariate time series forecasting. arXiv preprint arXiv:2203.10651.
- Open-source projects
 - **transdim**: Machine learning for spatiotemporal traffic data imputation and forecasting. (780 stars & 240 forks on GitHub)
<https://github.com/xinchen/transdim>
 - **tracebase**: Multivariate time series forecasting on high-dimensional and sparse Uber movement speed data. (19 stars & 5 forks on GitHub)
<https://github.com/xinchen/tracebase>



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Thanks for your attention!

Any Questions?

About me:

-  Homepage: <https://xinychen.github.io>
-  GitHub: <https://github.com/xinychen> (2.4K+ stars)
-  Blog: <https://medium.com/@xinyu.chen> (30K+ views)
-  How to reach me: chenxy346@gmail.com