



UNIVERSITY OF  
CENTRAL FLORIDA

# Machine Learning and Optimization for Understanding Spatiotemporal Systems

Time Series Imputation & Periodicity Quantification

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Orlando, USA

# Spatiotemporal Data

- Transport & mobility application scenarios



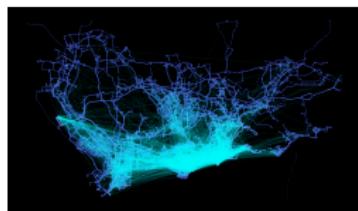
Highway (Portland)



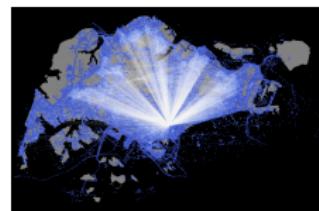
Uber movement (NYC)



Uber movement (Seattle)



Taxi trajectory (Shenzhen)



Human movement (Singapore)

- Challenges: Sparsity, high-dimensionality (network-scale), and multi-dimensionality (complicated data structure), time-varying systems

# Spatiotemporal Data Imputation

- Convolution     Fast Fourier transform     Optimization w/  $\ell_1$ -norm
- Time series imputation     Speed field reconstruction



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McGill → UF



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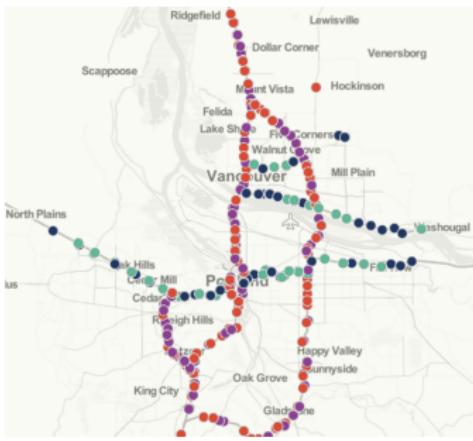


Lijun Sun  
McGill

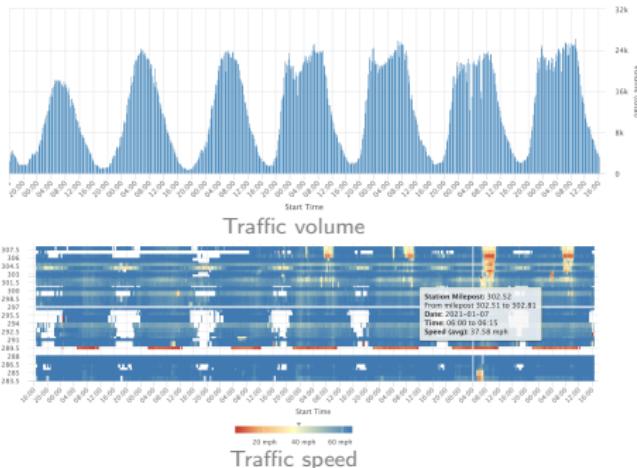
- **Xinyu Chen**, Zhanhong Chen, HanQin Cai, Nicolas Saunier, Lijun Sun (2024). “Laplacian Convolutional Representation for Traffic Time Series Imputation”. *IEEE Transactions on Knowledge and Data Engineering*, 36 (11): 6490–6502.
- Blog post: Understanding time series convolution.  
[https://spatiotemporal-data.github.io/posts/ts\\_conv](https://spatiotemporal-data.github.io/posts/ts_conv)

# Motivation

- Portland highway traffic data<sup>1</sup>



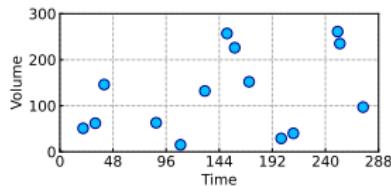
Highway network & sensor locations



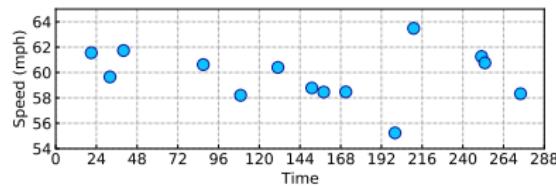
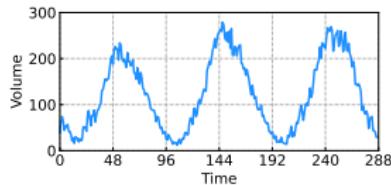
- $\mathbf{X} \in \mathbb{R}^{N \times T}$  with  $N$  spatial locations  $\times$   $T$  time steps
- Traffic volume/speed shows strong spatial/temporal dependencies
- Missing data are there, how to improve data quality?

<sup>1</sup><https://portal.its.pdx.edu/home>

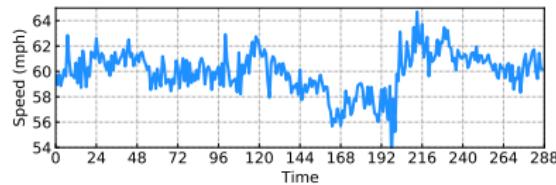
# Motivation



↓  
Reconstruct  
traffic volume?

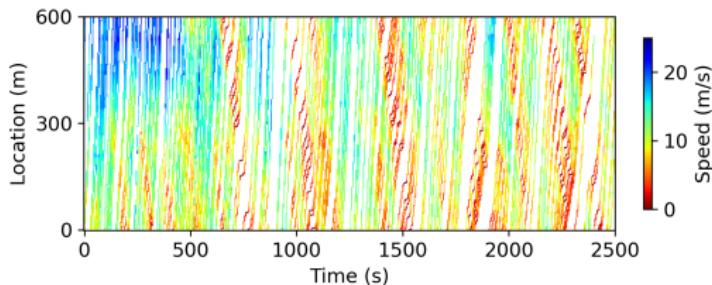


↓  
Reconstruct  
traffic speed?



- How to utilize the global trends of traffic time series?
- How to produce local consistency of traffic data?

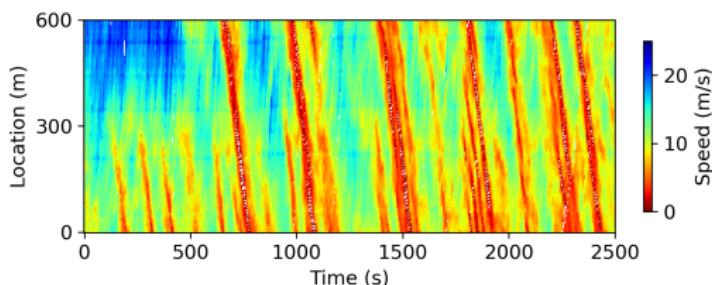
# Motivation



200-by-500 matrix  
(NGSIM)



Reconstruct speed field from  
20% sparse trajectories?

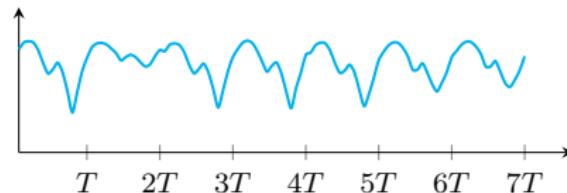


- How to learn from sparse spatiotemporal data?
- How to characterize spatial/temporal local dependencies?

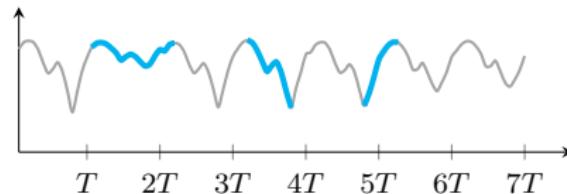
# Time Series Imputation

Global/local trends in sparse data?

- Global trends (e.g., daily/weekly periodicity):



- Local trends (e.g., short-term time series trends):



# Local Trend Modeling

- Intuition of Laplacian matrix

Undirected and circulant graph

Modeling  $\longrightarrow$

$$\mathbf{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Laplacian kernel:

$$\boldsymbol{\ell} \triangleq \underbrace{(2, -1, 0, 0, -1)}_{\text{first column of } \mathbf{L}}^\top$$

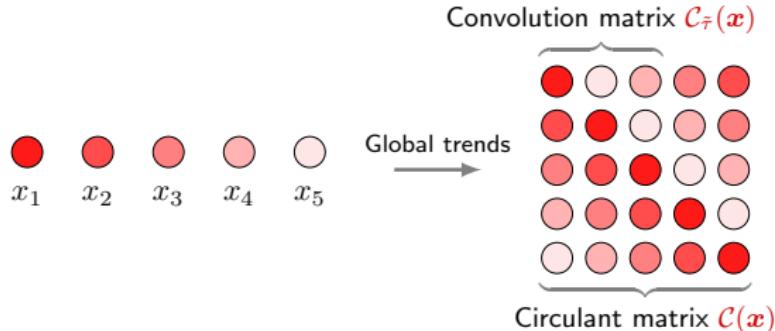
extending to the degree  $2\tau$  (i.e., graph connectivity) for  $\mathbf{x} \in \mathbb{R}^T$ .

- Temporal regularization:

$$\mathcal{R}(\mathbf{x}) = \underbrace{\frac{1}{2} \|\mathbf{L}\mathbf{x}\|_2^2}_{\text{mat-vec mul.}} = \underbrace{\frac{1}{2} \|\boldsymbol{\ell} * \mathbf{x}\|_2^2}_{\text{convolution}*}$$

# Global Trend Modeling

Circulant matrix  $\mathcal{C}(\mathbf{x})$  vs. convolution matrix  $\mathcal{C}_{\tilde{\tau}}(\mathbf{x})$



- Circulant/Convolution nuclear norm minimization
  - A balance between global and local trends modeling?

CircNNM (Liu'22, Liu & Zhang'23)

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathcal{C}(\mathbf{x})\|_* \\ \text{s.t. } & \|\mathcal{P}_\Omega(\mathbf{x} - \mathbf{y})\|_2 \leq \epsilon \end{aligned}$$

on data  $\mathbf{y}$  w/ observed index set  $\Omega$ .

ConvNNM (Liu'22, Liu & Zhang'23)

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathcal{C}_{\tilde{\tau}}(\mathbf{x})\|_* \\ \text{s.t. } & \|\mathcal{P}_\Omega(\mathbf{x} - \mathbf{y})\|_2 \leq \epsilon \end{aligned}$$

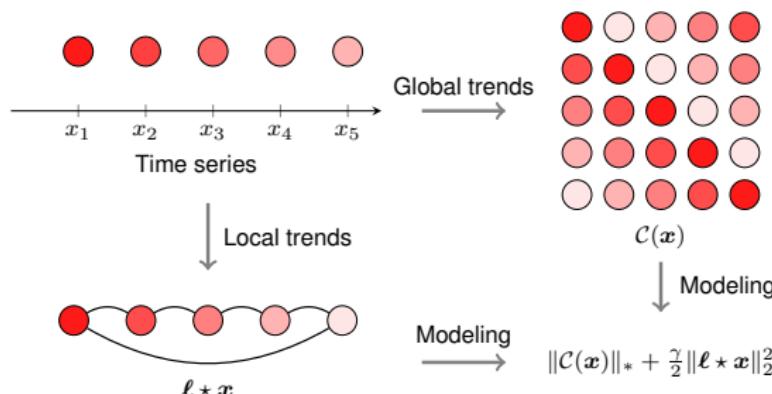
on data  $\mathbf{y}$  w/ observed index set  $\Omega$ .

# Global + Local Trends?

## Laplacian Convolutional Representation (LCR)

For any partially observed time series  $\mathbf{y} \in \mathbb{R}^T$  with observed index set  $\Omega$ , LCR utilizes **circulant matrix** and **Laplacian kernel** to characterize global/local time series trends:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \underbrace{\|\mathcal{C}(\mathbf{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\ell * \mathbf{x}\|_2^2}_{\text{local}} \\ \text{s.t. } & \|\mathcal{P}_\Omega(\mathbf{x} - \mathbf{y})\|_2 \leq \epsilon \end{aligned}$$



## Laplacian Convolutional Representation

- LCR model:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2} \|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 \\ \text{s.t. } & \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon \end{aligned}$$

$$\implies \min_{\boldsymbol{x}} \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2} \|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{global} + \text{local}} + \underbrace{\frac{\eta}{2} \|\mathcal{P}_\Omega(\boldsymbol{z} - \boldsymbol{y})\|_2^2}_{\text{regularization}}$$

s.t.  $\boldsymbol{z} = \boldsymbol{x}$

“The alternating direction method of multipliers (ADMM) is an algorithm that solves convex optimization problems by breaking them into smaller pieces, each of which are then easier to handle.”

— Source: <https://stanford.edu/~boyd/admm.html>

# Laplacian Convolutional Representation

- Augmented Lagrangian function:

$$\mathcal{L} = \underbrace{\|\mathcal{C}(\mathbf{x})\|_* + \frac{\gamma}{2}\|\ell * \mathbf{x}\|_2^2}_{\text{global + local}} + \underbrace{\frac{\lambda}{2}\|\mathbf{x} - \mathbf{z}\|_2^2 + \langle \mathbf{w}, \mathbf{x} - \mathbf{z} \rangle}_{\text{Lagrangian multiplier } \mathbf{w}} + \underbrace{\frac{\eta}{2}\|\mathcal{P}_\Omega(\mathbf{z} - \mathbf{y})\|_2^2}_{\text{observations } \mathbf{y}}$$

- Optimize  $\mathbf{x}$  w/ FFT in  $\mathcal{O}(T \log T)$  time:

$$\begin{cases} \|\mathcal{C}(\mathbf{x})\|_* = \|\mathcal{F}(\mathbf{x})\|_1 = \|\hat{\mathbf{x}}\|_1 & (\text{circulant matrix}) \\ \frac{1}{2}\|\ell * \mathbf{x}\|_2^2 = \frac{1}{2T}\|\mathcal{F}(\ell) \circ \mathcal{F}(\mathbf{x})\|_2^2 = \frac{1}{2T}\|\hat{\ell} \circ \hat{\mathbf{x}}\|_2^2 & (\text{circular convolution}) \end{cases}$$

- Reformulate the optimization as  $\ell_1$ -norm minimization:

$$\begin{aligned} \mathbf{x} &:= \arg \min_{\mathbf{x}} \|\mathcal{C}(\mathbf{x})\|_* + \frac{\gamma}{2}\|\ell * \mathbf{x}\|_2^2 + \frac{\lambda}{2}\|\mathbf{x} - \mathbf{z} + \mathbf{w}/\lambda\|_2^2 \\ \implies \hat{\mathbf{x}} &:= \arg \min_{\hat{\mathbf{x}}} \underbrace{\|\hat{\mathbf{x}}\|_1}_{\ell_1\text{-norm}} + \frac{\gamma}{2T}\|\hat{\ell} \circ \hat{\mathbf{x}}\|_2^2 + \frac{\lambda}{2T}\|\hat{\mathbf{x}} - \hat{\mathbf{z}} + \hat{\mathbf{w}}/\lambda\|_2^2 \end{aligned}$$

# Laplacian Convolutional Representation

$\ell_1$ -norm Minimization (Liu & Zhang'23)

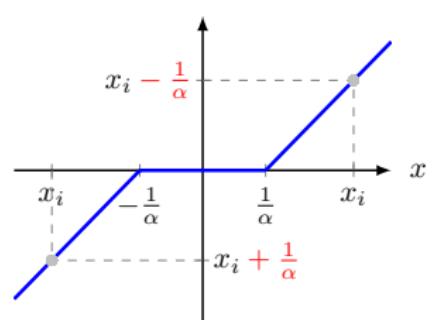
For any  $\hat{\mathbf{h}} \in \mathbb{C}^T$  and  $\delta \in \mathbb{R}$ :

$$\min_{\hat{\mathbf{x}}} \|\hat{\mathbf{x}}\|_1 + \frac{\delta}{2} \|\hat{\mathbf{x}} - \hat{\mathbf{h}}\|_2^2$$

The solution to  $\hat{\mathbf{x}}$ :

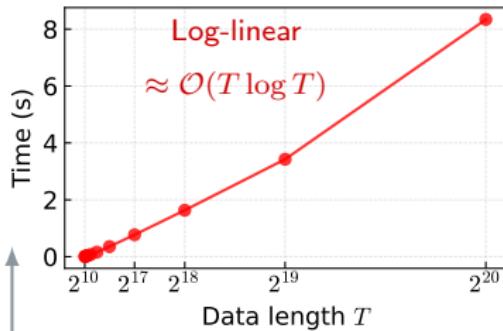
$$\hat{x}_t := \frac{\hat{h}_t}{|\hat{h}_t|} \cdot \underbrace{\max\{0, |\hat{h}_t| - 1/\delta\}}_{\text{shrinkage (e.g., ReLU)}}, t \in [T]$$

$$y_i = \frac{x_i}{|x_i|} \cdot \max\{|x_i| - 1/\alpha, 0\}$$

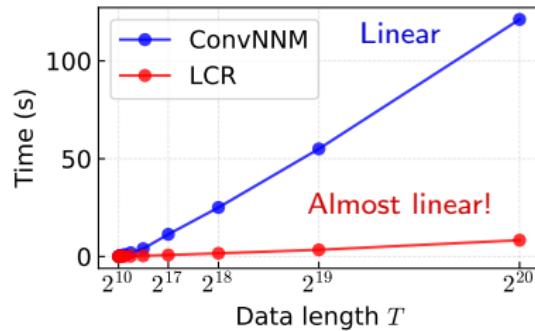


# Laplacian Convolutional Representation

Time complexity & scalability & efficiency?



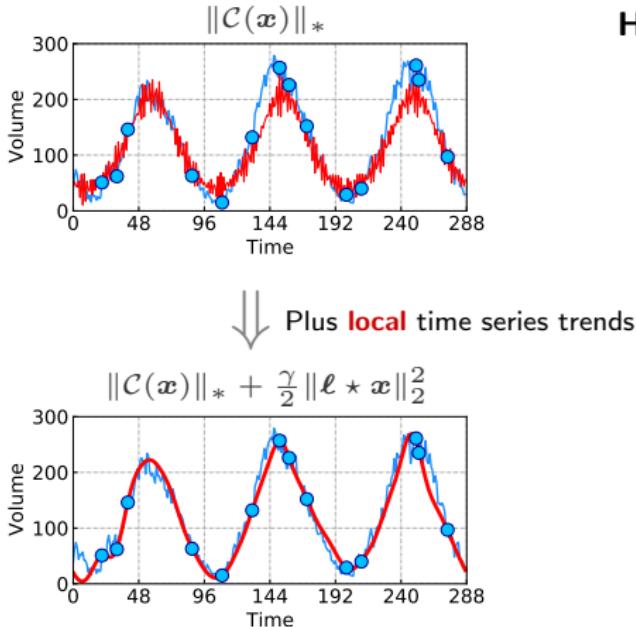
Empirical time complexity



On the synthetic data  $y \in \mathbb{R}^T$  with  
 $T \in \{2^{10}, 2^{11}, \dots, 2^{20}\}$

# Experiments

- Traffic speed imputation<sup>2</sup> (95% missing rate)



## Highlights:

- Rethink the importance of local trend modeling in traffic data imputation tasks.
- Find a unified global and local trend modeling framework whose optimization can be efficiently solved by FFT:

$$\min_{\mathbf{x}} \underbrace{\|\mathcal{C}(\mathbf{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\ell * \mathbf{x}\|_2^2}_{\text{local}}$$

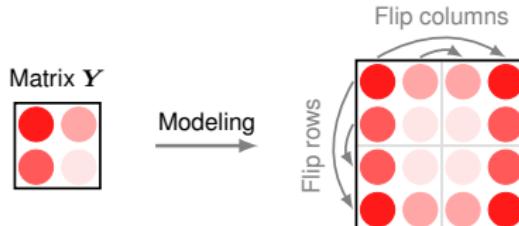
s. t.  $\|\mathcal{P}_\Omega(\mathbf{x} - \mathbf{y})\|_2 \leq \epsilon$

<sup>2</sup>Blue dot: partial observation; red line: imputation.

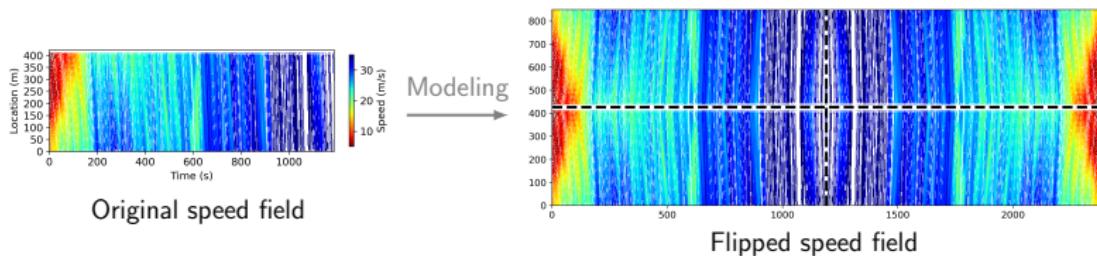
# Experiments

## Speed field reconstruction<sup>3</sup>

- Flipping operation on a matrix:



- Flipping operation on a speed field of vehicular traffic flow:



<sup>3</sup>Highway Drone (HighD) dataset at <https://www.hightd-dataset.com/>

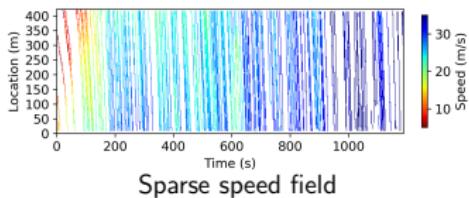
# Experiments

Speed field reconstruction in German highways<sup>4</sup>

- Scenario: Mask trajectories of 70% vehicles
- LCR-2D on partially observed  $\mathbf{Y} \in \mathbb{R}^{N \times T}$ :

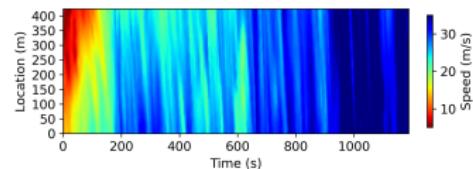
$$\min_{\mathbf{X}} \underbrace{\|\mathcal{C}(\mathbf{X})\|_*}_{\text{global trend}} + \frac{\gamma}{2} \underbrace{\|(\ell_s \ell^\top) \star \mathbf{X}\|_F^2}_{\text{local trend}}$$

s.t.  $\|\mathcal{P}_\Omega(\mathbf{X} - \mathbf{Y})\|_F \leq \epsilon$

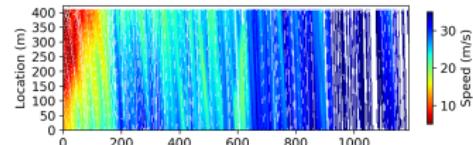


Sparse speed field

LCR-2D



Reconstructed speed field



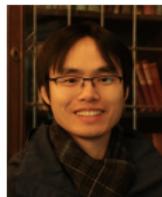
Ground-truth speed field

<sup>4</sup>Highway Drone (HighD) dataset at <https://www.hightd-dataset.com/>

# Quantifying Time Series Periodicity

(Ongoing Research)

- Interpretable ML     Optimization w/  $\ell_0$ -norm     Mixed-integer programming
- Human mobility regularity     Climate system seasonality



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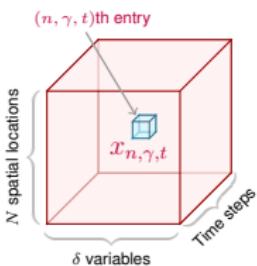


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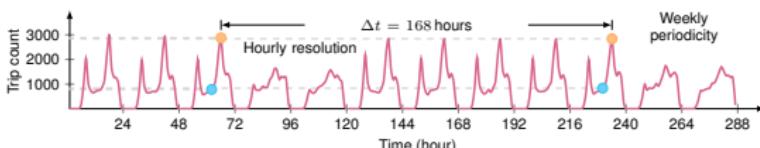
# Motivation

Human mobility data show daily/weekly regularity and periodicity?

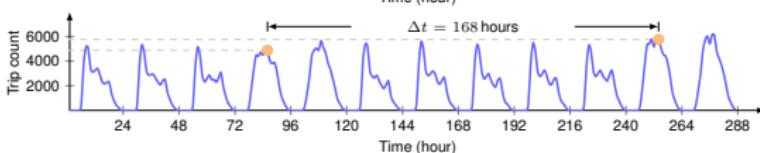
A



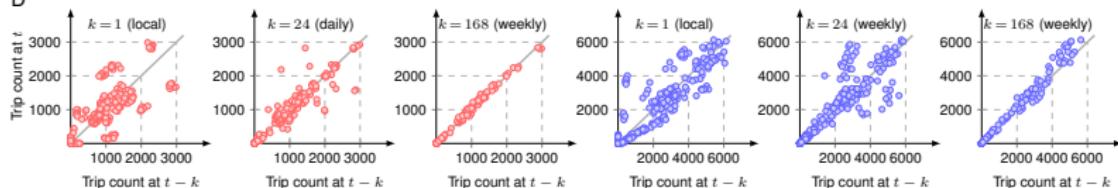
B



C



D

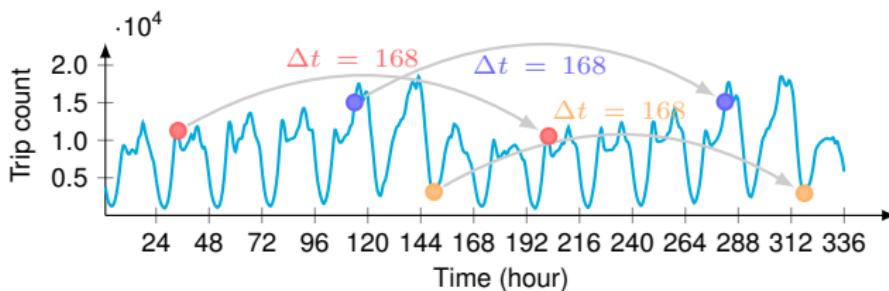


“Closeness” to the  
anti-diagonal  $y = x$

$x_t \approx x_{t-168}$  (weekly periodicity)

# Motivation

Weekly periodicity of ridesharing trip time series in Chicago



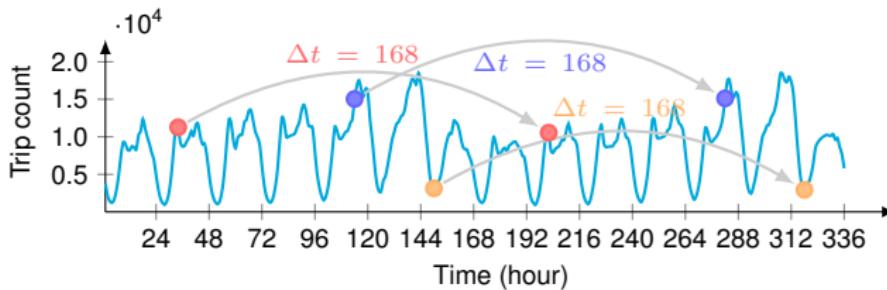
What motivate us most about periodicity?

- ① **Resilience and stability of systems:** Empirically measure the periodicity and predictability of urban systems.
- ② **Optimization of transport systems:** Optimize resources (e.g., public transit, taxi, rideshare, and micromobility) to meet transport demand efficiently.
- ③ **Design of sustainable transport & infrastructure:** Implement energy-efficient solutions tailored to peak hours.

## Motivation

- Time series autoregression on  $\mathbf{x} \in \mathbb{R}^T$

$$\mathbf{w} := \arg \min_{\{\mathbf{w}_k\}_{k \in [d]}} \sum_{t \in [d+1, T]} \left( \mathbf{x}_t - \sum_{k \in [d]} w_k \mathbf{x}_{t-k} \right)^2$$



Periodicity of rideshare trip time series

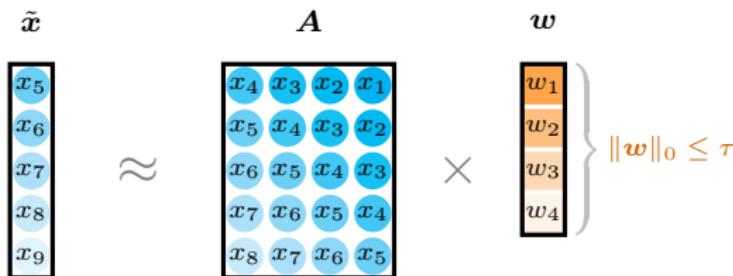
- Sparse coefficient vector  $\mapsto$  **Interpretability?**

$$\mathbf{w} = (\underbrace{0.33}_{k=1}, 0, \dots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

# Valorizing Autoregression

- Time series autoregression

$$\begin{aligned} \mathbf{w} &:= \arg \min_{\{w_k\}_{k \in [d]}} \sum_{t \in [d+1, T]} \left( \mathbf{x}_t - \sum_{k \in [d]} w_k \mathbf{x}_{t-k} \right)^2 \\ &= \arg \min_{\mathbf{w}} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 \end{aligned}$$



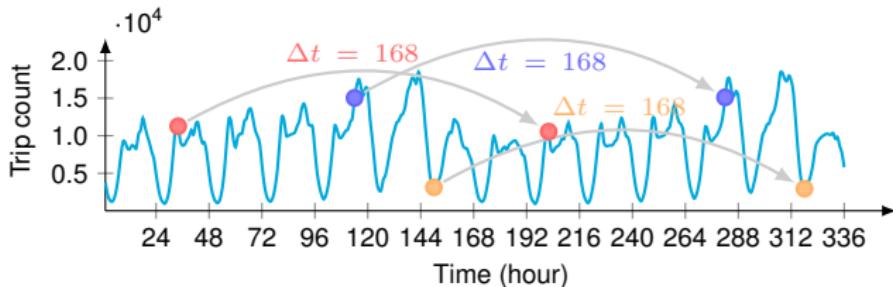
Autoregression on time series  $\mathbf{x} = (x_1, x_2, \dots, x_9)^\top$  w/ sparsity  $\tau \in \mathbb{Z}^+$

- Sparse autoregression

$$\begin{array}{ll} \min_{\mathbf{w} \geq 0} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 & \min_{\mathbf{w}, \boldsymbol{\beta}} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 \\ \text{s.t. } \underbrace{\|\mathbf{w}\|_0 \leq \tau}_{\text{sparsity w/ } \ell_0\text{-norm}} & \iff \text{s.t. } \begin{cases} 0 \leq \mathbf{w} \leq \boldsymbol{\beta}, \boldsymbol{\beta} \in \{0, 1\}^d \\ \|\boldsymbol{\beta}\|_1 \leq \tau \end{cases} \end{array}$$

## Solution Quality

- Subspace pursuit (SP) sometimes fails

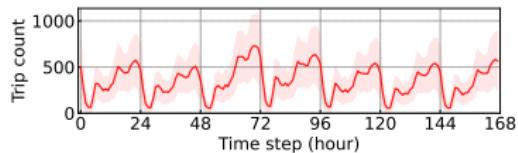
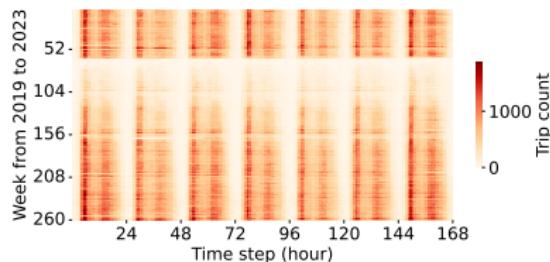
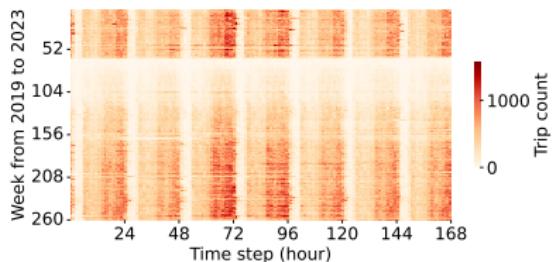


- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity  $\tau = 2$ ):

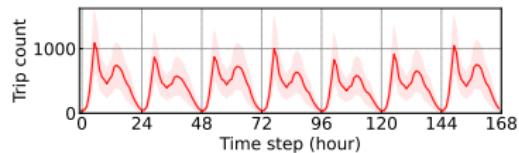
$$\underbrace{\boldsymbol{w} = (\dots, \underbrace{0.02}_{k=53}, \dots, \underbrace{0.96}_{k=168})^\top}_{\text{loss func. } = 8.32 \times 10^7 \text{ (SP)}} \quad \text{vs.} \quad \underbrace{\boldsymbol{w} = (\underbrace{0.22}_{k=1}, \dots, \underbrace{0.77}_{k=168})^\top}_{\text{loss func. } = 6.25 \times 10^7 \text{ (MIP)}}$$

# John F. Kennedy International Airport

- Pickup/Dropoff trips in airport
  - Pickup trips are relevant to flight delay, baggage claim, and other factors.
  - Dropoff trips to airport are highly related to flight schedules.



Pickup trips from airport



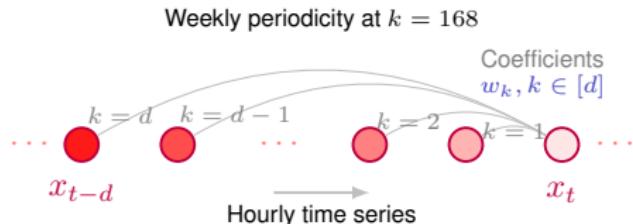
Dropoff trips to airport

- Sparse coefficient vectors (**sparsity  $\tau = 3$** ):

$$\mathbf{w} = (\underbrace{0.31}_{k=1}, \dots, \underbrace{0.28}_{k=24}, \dots, \underbrace{0.41}_{k=168})^\top \quad \text{vs.} \quad \mathbf{w} = (\underbrace{0.18}_{k=1}, \dots, \underbrace{0.35}_{k=24}, \dots, \underbrace{0.47}_{k=168})^\top$$

# Spatially- and Time-Varying Autoregression

## Univariate autoregression

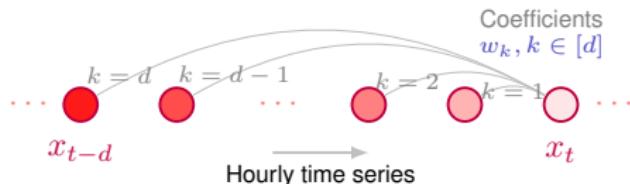


$$\min_t \sum_{k \in [d]} \left( x_t - \sum_{k \in [d]} w_k x_{t-k} \right)^2$$

# Spatially- and Time-Varying Autoregression

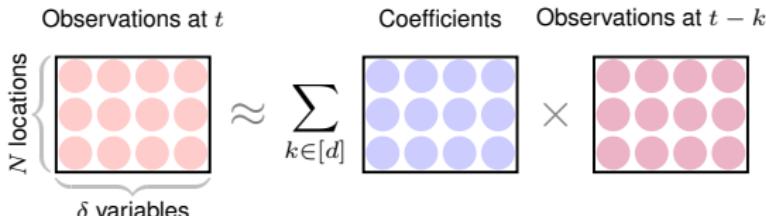
## Univariate autoregression

Weekly periodicity at  $k = 168$



$$\min \sum_t \left( x_t - \sum_{k \in [d]} w_k x_{t-k} \right)^2$$

## Multidimensional autoregression



$$\min \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_t \left( x_{n, \gamma, t} - \sum_{k \in [d]} w_{n, \gamma, k} x_{n, \gamma, t-k} \right)^2$$

# Envisioning Human Mobility

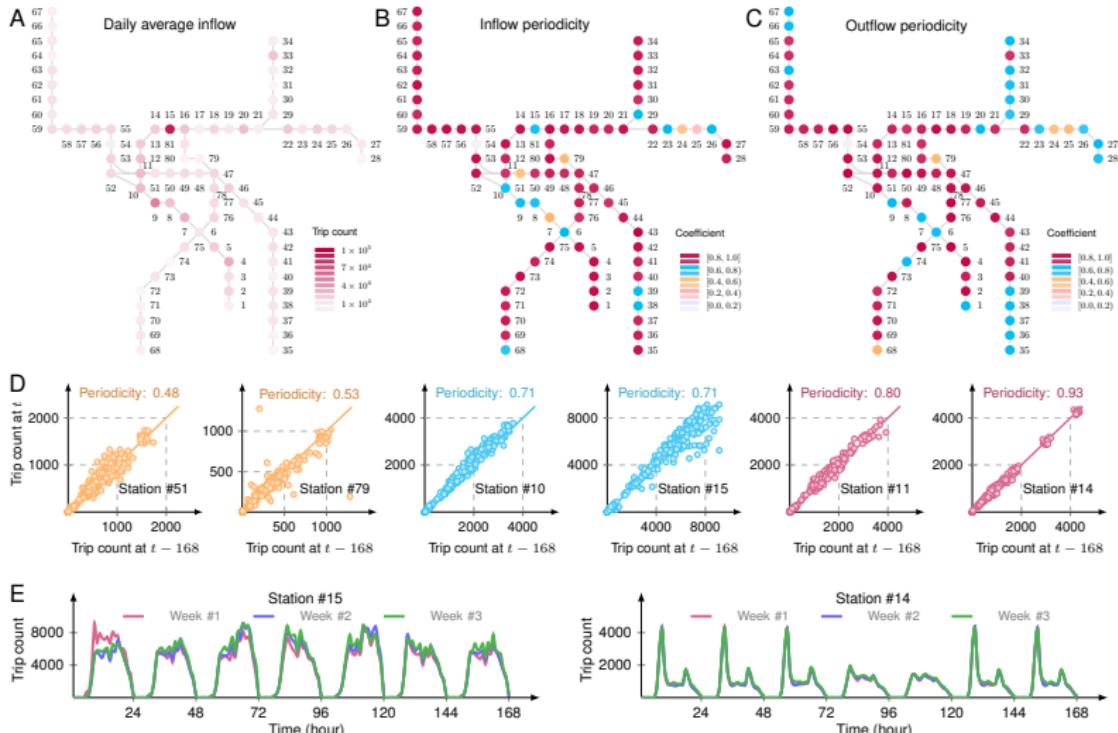
- Ridesharing trip data  $\{x_{n,\gamma}\}$  across  $\gamma \in [\delta]$  years
- Reformulate sparse autoregression:

$$\begin{array}{c} \text{year} \\ \swarrow \quad \searrow \\ \text{spatial location} \quad \text{hourly time step} \\ \downarrow \quad \quad \quad \downarrow \\ \min_{\{\boldsymbol{w}_{n,\gamma}\}, \boldsymbol{\beta}} \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_{t \in [d+1, T_\gamma]} \left( x_{n,\gamma,t} - \sum_{k \in [d]} w_{n,\gamma,k} x_{n,\gamma,t-k} \right)^2 \\ \text{s.t. } \underbrace{\boldsymbol{\beta} \in \{0, 1\}^d}_{\text{binary var.}} \quad \underbrace{0 \leq \boldsymbol{w}_{n,\gamma} \leq \boldsymbol{\beta}}_{\text{upper bound in } \{0, 1\}} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\text{sum of binary var.}} \end{array}$$

- MIP problem w/  $(N\delta + 1)d$  variables!
- How to handle thousands or millions of (e.g.,  $N\delta = 10^6$ ) time series?

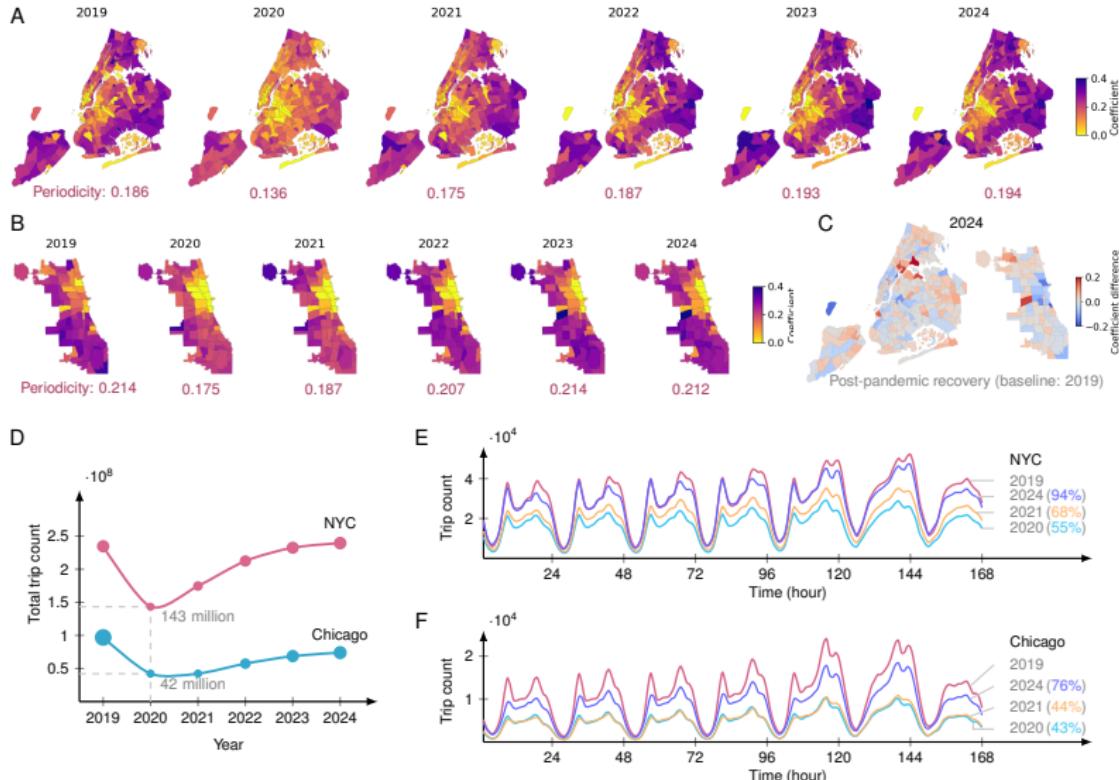
Quantify weekly periodicity by  $\{w_{n,\gamma,k}\}$  at index  $k = 168$

# Envisioning Human Mobility



Hangzhou metro passenger flow in January 2019

# Envisioning Human Mobility



Weekly periodicity reveals spatial patterns of ridesharing systems

# Understanding Climate Systems

Quantify yearly seasonality by  $\{w_{m,n,\gamma,k}\}$  at index  $k = 12$

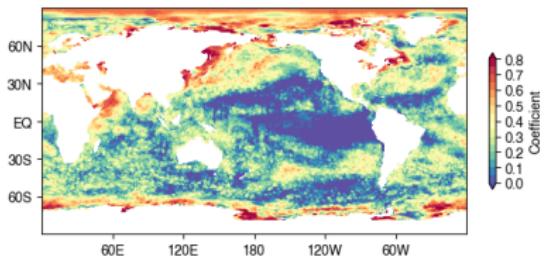
$$\min_{\{\boldsymbol{w}_{m,n,\gamma}\}, \boldsymbol{\beta}} \sum_{m \in [M]} \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_{t \in [d+1, T_\gamma]} \left( x_{m,n,\gamma,t} - \sum_{k \in [d]} w_{m,n,\gamma,k} x_{m,n,\gamma,t-k} \right)^2$$

s.t. 
$$\begin{cases} \boldsymbol{\beta} \in \{0, 1\}^d & \text{binary decision var.} \\ 0 \leq \boldsymbol{w}_{m,n,\gamma} \leq \boldsymbol{\beta}, \forall m, n, \gamma \\ \|\boldsymbol{\beta}\|_1 \leq \tau \\ \|\boldsymbol{w}_{m,n,\gamma}\|_1 = 1, \forall m, n, \gamma \end{cases}$$

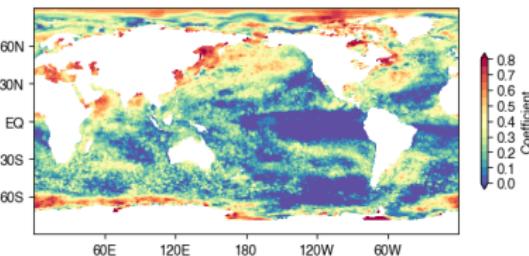
$\ell_1$ -normalization

longitude  
latitude      decade      monthly time step

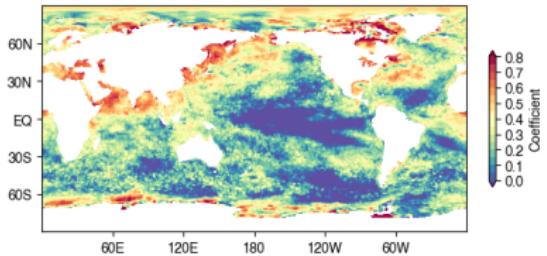
# Sea Surface Temperature



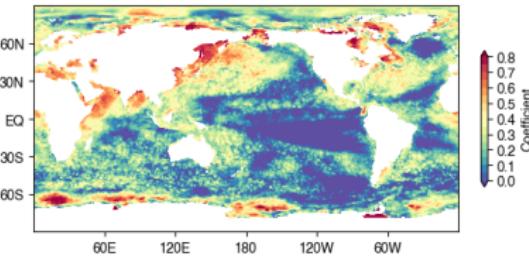
1980s



1990s



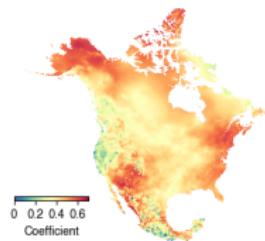
2000s



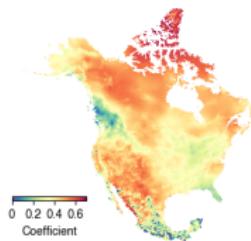
2010s

- Identify yearly periodicity at  $k = 12$  from SST data ( $\tau = 4$ )
  - ❶ The areas of El Niño events are less seasonal/predictable
  - ❷ Arctic becomes less seasonal/predictable in the past 20 years
- Insights into climate change & global warming & sustainable development

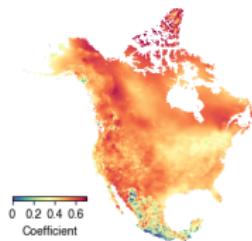
## North America Temperature



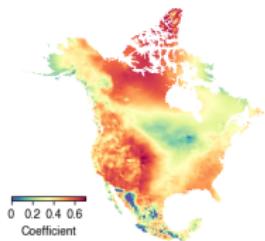
tmin, 1980s



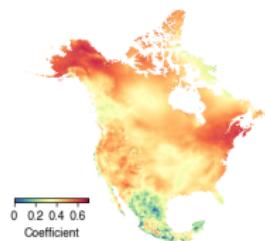
tmin, 1990s



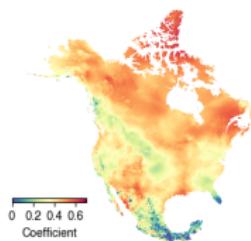
tmin, 2000s



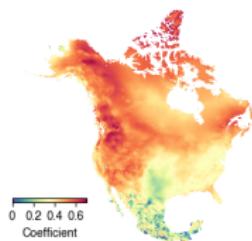
tmin, 2010s



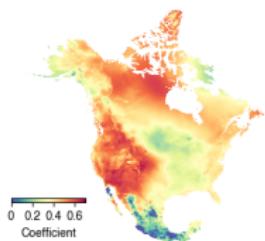
tmax, 1980s



tmax, 1990s



tmax, 2000s



tmax, 2010s

- Identify yearly periodicity at  $k = 12$  from temperature data ( $\tau = 4$ )
  - ❶ Stronger yearly seasonality in high-latitude areas
  - ❷ Less seasonal temperature in south areas (e.g., Mexico)
  - ❸ Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s



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# Thanks for your attention!

Any Questions?

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