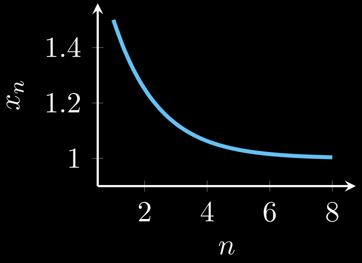
Three rates of convergence on a sequence $\{x_n\}_{n \in \mathbb{Z}^+}$

Linear convergence

$$\frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} \le r,\ r \in (0,1)$$



Sequence $x_n = 1 + \left(\frac{1}{2}\right)^n$
converges linearly to $x_\infty = 1$
because $r = \frac{1}{2}$

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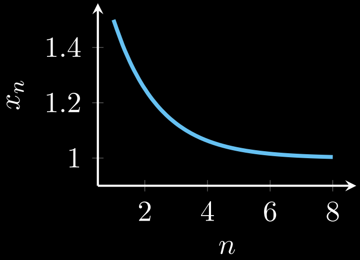
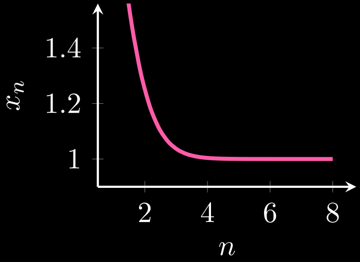


Sequence $x_n = 1 + \left(\frac{1}{2}\right)^n$ converges linearly to $x_\infty = 1$ because $r = \frac{1}{2}$

Superlinear convergence

$$\lim_{n\to\infty} \frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} = 0$$

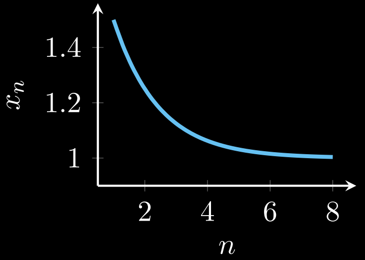


Sequence $x_n = 1 + \left(\frac{1}{n}\right)^n$ converges superlinearly to $x_\infty = 1$

Three rates of convergence on a sequence $\{x_n\}_{n\in\mathbb{Z}^+}$

Linear convergence

$$\frac{\|x_{n+1}-x_\infty\|}{\|x_n-x_\infty\|}\leq r,\; r\in(0,1)$$



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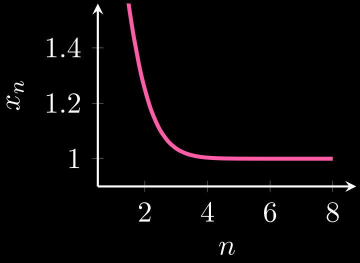
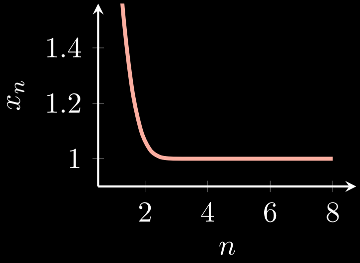


Sequence $x_n = 1+\left(\frac{1}{n}\right)^n$ converges superlinearly to $x_\infty = 1$

Quadratic convergence

$$\frac{\|x_{n+1}-x_\infty\|}{\|x_n-x_\infty\|^2}\leq M$$

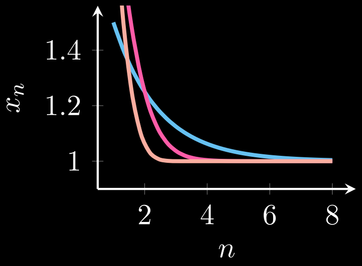


Sequence $x_n = 1+\left(\frac{1}{n}\right)^{2^n}$ converges quadratically to $x_\infty = 1$ because $M=1$

Three rates of convergence on a sequence $\{x_n\}_{n\in\mathbb{Z}^+}$

Linear convergence

$$\frac{\|x_{n+1}-x_\infty\|}{\|x_n-x_\infty\|} \le r,\ r\in(0,1)$$



Sequence $x_n = 1 + \left(\frac{1}{2}\right)^n$ converges linearly to $x_\infty = 1$ because $r = \frac{1}{2}$

Superlinear convergence

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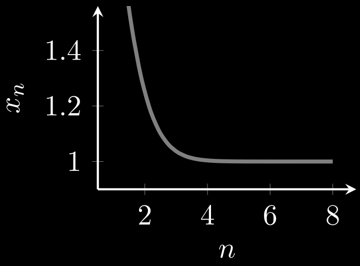

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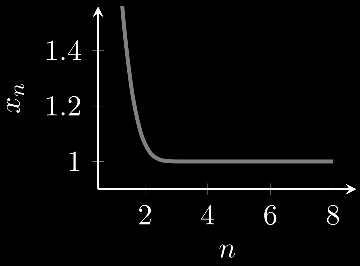

Sequence $x_n = 1 + \left(\frac{1}{n}\right)^{2^n}$ converges quadratically to $x_\infty = 1$ because $M = 1$

Thanks for your attention!

Any Questions?

About me:

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