Quantifying Time Series Periodicity with Interpretable Machine Learning

Urban Human Mobility & Climate Variables

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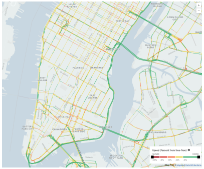
Cambridge, USA

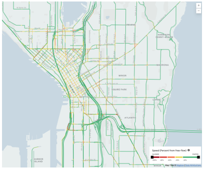
Spatiotemporal Data

- Transport & mobility & climate application scenarios

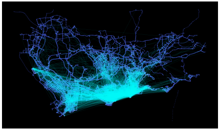
Highway (Portland)



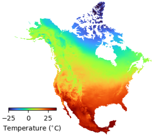
Uber movement (NYC)



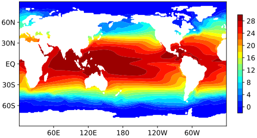
Uber movement (Seattle)



Taxi trajectory (Shenzhen)



Temperature (NA)

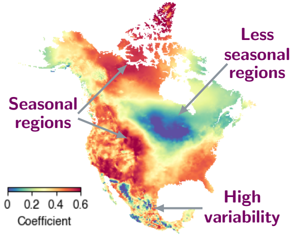


Temperature (sea surface)

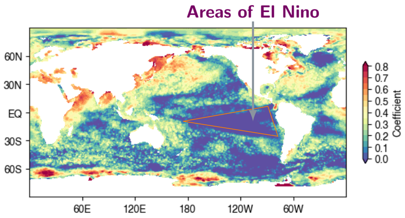
- Challenges: Sparsity, high-dimensionality, multi-dimensionality, heavy tails, irregular sampling, and time-varying systems

Motivation

Yearly temperature seasonality patterns in 2010s



North America



Global sea surface

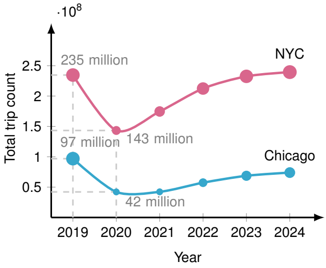
What motivate us most about periodicity?

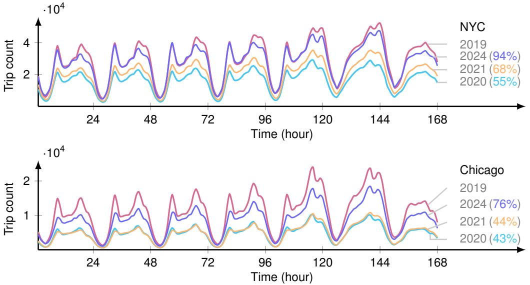
❶ **Monitoring climate systems**: Empirically measure the periodicity of climate variables (e.g., temperature, precipitation).

❷ **Discovering spatiotemporal patterns**: Identify periodicity pattern shift and special climate events.

Motivation

Ridesharing trip data



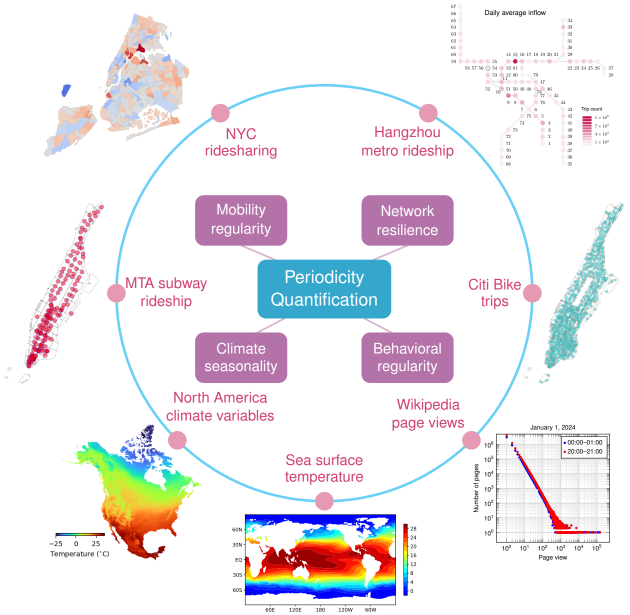


What motivate us most about **periodicity**?

❶ **Resilience and stability of systems**: Empirically measure the periodicity and predictability of urban systems.

❷ **Optimization of transport systems**: Optimize resources (e.g., public transit, taxi, ridesharing, micromobility) to meet transport demand efficiently.

❸ **Design of sustainable transport & infrastructure**: Implement energy-efficient solutions (e.g., congestion pricing) tailored to peak hours.



Periodicity Quantification
Mobility regularity
Network resilience
Climate seasonality
Behavioral regularity
NYC ridesharing
Hangzhou metro rideship
MTA subway rideship
Citi Bike trips
North America climate variables
Wikipedia page views
Sea surface temperature
Daily average inflow
Trip count
Temperature (°C)
January 1, 2024
00:00–01:00
20:00–21:00
Number of pages
Page view

Interpretable Time Series Autoregression

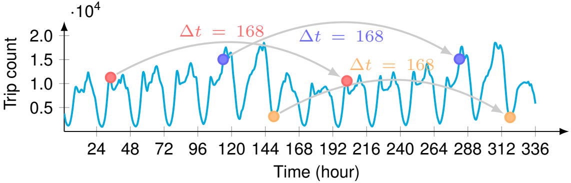
https://github.com/xinychen/integers

- Interpretable ML
- ℓ_0-norm optimization
- Climate system seasonality
- Sparse autoregression
- Mixed-integer optimization
- Human mobility regularity

Valorizing Autoregression

- Time series autoregression on $\boldsymbol{x} \in \mathbb{R}^T$

$$\boldsymbol{w} := \arg\min_{\boldsymbol{w}} \sum_{t=d+1}^{T} \Big(x_t - \sum_{k=1}^{d} w_k x_{t-k}\Big)^2$$



Periodicity of hourly rideshare trip time series

- **Sparse** coefficient vector $\mapsto$ **Interpretability**?

$$\underbrace{\boldsymbol{w}}_{\text{sparsity } \|\boldsymbol{w}\|_0 \triangleq 3} = (\underbrace{0.33}_{k=1}, 0, \cdots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

Sparse Autoregression

- Identify the dominant auto-correlations
  - $\tau \in \mathbb{Z}^+$: Upper bound of the number of nonzero entries in $\boldsymbol{w} \in \mathbb{R}^d$

$$\underbrace{\begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}}_{\tilde{\boldsymbol{x}}} \approx \underbrace{\begin{bmatrix} x_4 & x_3 & x_2 & x_1 \\ x_5 & x_4 & x_3 & x_2 \\ x_6 & x_5 & x_4 & x_3 \\ x_7 & x_6 & x_5 & x_4 \\ x_8 & x_7 & x_6 & x_5 \end{bmatrix}}_{\boldsymbol{A}} \times \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}}_{\boldsymbol{w}} \Big\} \|\boldsymbol{w}\|_0 \leq \tau$$

$$\begin{aligned} \boldsymbol{w} &:= \underset{\|\boldsymbol{w}\|_0 \leq \tau}{\arg\min} \sum_{t=d+1}^{T} \left(x_t - \sum_{k=1}^{d} w_k x_{t-k} \right)^2 \\ &= \underset{\|\boldsymbol{w}\|_0 \leq \tau}{\arg\min} \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \end{aligned}$$

- ℓ_0-norm optimization is NP-hard
- Formulate it as a mixed-integer programming
  - Introduce binary decision variables $\boldsymbol{\beta} \in \{0,1\}^d$

$$\begin{aligned} \min_{\boldsymbol{w}} \ & \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \\ \text{s.t.} \ & \underbrace{\|\boldsymbol{w}\|_0 \leq \tau}_{\clubsuit \text{ sparsity of } \boldsymbol{w}} \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} \min_{\boldsymbol{w},\,\boldsymbol{\beta}} \ & \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \\ \text{s.t.} \ & \underbrace{-\boldsymbol{\beta} \leq \boldsymbol{w} \leq \boldsymbol{\beta},}_{\text{bounds being either } 0 \text{ or } \pm 1} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\clubsuit \text{ sparsity of } \boldsymbol{\beta}} \end{aligned}$$

Sparse Autoregression Done Right

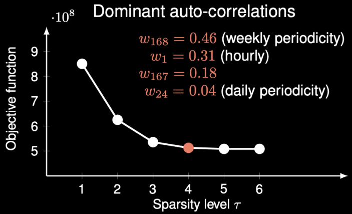
$$\min_{\boldsymbol{w},\,\boldsymbol{\beta}}\ \underbrace{\sum_{t=d+1}^{T}\left(x_t-\sum_{k=1}^{d}w_kx_{t-k}\right)^2}_{\text{Time series autoregression}}\quad \text{s.t.}\ \underbrace{-\beta_k\leq w_k\leq\beta_k}_{\text{Lower and upper bounds}},\ \underbrace{\sum_{k=1}^{d}\beta_k\leq\tau}_{\clubsuit\ \text{Sparsity}},\ \underbrace{\beta_k\in\{0,1\}}_{\text{Binary variable}}$$

- $\boldsymbol{w}\in\mathbb{R}^d$: Auto-correlations
- $\boldsymbol{\beta}\in\{0,1\}^d$: Sparsity pattern
- $d=168$: Autoregression order

```python
import numpy as np
from docplex.mp.model import Model

def sparse_ar(x, d, tau):
    model = Model('Sparse Autoregression')
    T = x.shape[0]
    w = [model.continuous_var(name = f'w_{k}') for k in range(d)]
    beta = [model.binary_var(name = f'beta_{k}') for k in range(d)]
    model.minimize(model.sum((x[t] - model.sum(w[k] * x[t - k - 1]
                                for k in range(d))) ** 2
                        for t in range(d, T)))
    model.add_constraint(model.sum(beta[k] for k in range(d)) <= tau)
    for k in range(d):
        model.add_constraint(w[k] <= beta[k])
        model.add_constraint(w[k] >= - beta[k])
    solution = model.solve()
    return np.array(solution.get_values(w))
```

https://github.com/xinychen/integers

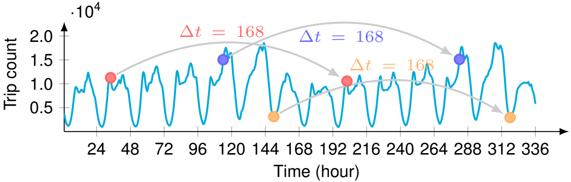


Solution Quality → Better Interpretability?

- Sparse autoregression

$$\min_{\boldsymbol{w}\geq 0} \;\|\tilde{\boldsymbol{x}}-\boldsymbol{A}\boldsymbol{w}\|_2^2 \quad \text{s.t. } \|\boldsymbol{w}\|_0\leq\tau$$

- Subspace pursuit (SP) sometimes fails



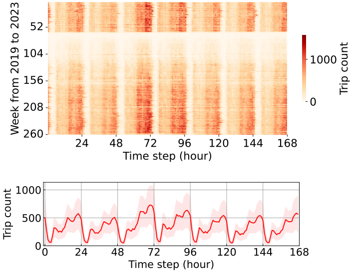
Periodicity of ridesharing trip time series

- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity $\tau = 2$):

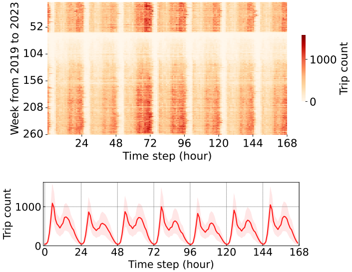
$$\underbrace{\boldsymbol{w}=(\cdots,\underbrace{0.02}_{k=53},\cdots,\underbrace{0.96}_{k=168})^\top}_{\text{obj. }=8.32\times 10^7\text{ (SP)}} \quad \text{vs.} \quad \underbrace{\boldsymbol{w}=(\underbrace{0.22}_{k=1},\cdots,\underbrace{0.77}_{k=168})^\top}_{\clubsuit\text{ obj. }=6.25\times 10^7\text{ (MIP)}}$$

John F. Kennedy International Airport

- Pickup/Dropoff trips in airport
  - Pickup trips are relevant to flight delay, baggage claim, and other factors.
  - Dropoff trips to airport are highly related to flight schedules.



Pickup trips from airport



Dropoff trips to airport

- Sparse coefficient vectors (sparsity $\tau = 3$):

$$\boldsymbol{w} = (\underbrace{0.31}_{k=1}, \cdots, \underbrace{0.28}_{k=24}, \cdots, \underbrace{0.41}_{k=168})^\top \quad \text{vs.} \quad \boldsymbol{w} = (\underbrace{0.18}_{k=1}, \cdots, \underbrace{0.35}_{k=24}, \cdots, \underbrace{0.47}_{k=168})^\top$$

High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large N:

$$\underbrace{\min_{\{\boldsymbol{w}_n\}_{n=1}^{N},\boldsymbol{\beta}}}_{(N+1)d\text{ decision var.}} \quad \underbrace{\sum_{n=1}^{N}\|\tilde{\boldsymbol{x}}_n-\boldsymbol{A}_n\boldsymbol{w}_n\|_2^2}_{\text{multivariate time series}}$$

$$\text{s.t.}\quad \underbrace{0\leq\boldsymbol{w}_n\leq\boldsymbol{\beta},}_{\text{bounds being either }0\text{ or }1}\quad \underbrace{\|\boldsymbol{\beta}\|_1\leq\tau,}_{\text{sparsity of }\boldsymbol{\beta}}\quad \boldsymbol{\beta}\in\{0,1\}^d$$

High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large N:

$$\underbrace{\min_{\{\boldsymbol{w}_n\}_{n=1}^N, \boldsymbol{\beta}}}_{(N+1)d \text{ decision var.}} \quad \underbrace{\sum_{n=1}^{N} \|\tilde{\boldsymbol{x}}_n - \boldsymbol{A}_n \boldsymbol{w}_n\|_2^2}_{\text{multivariate time series}}$$

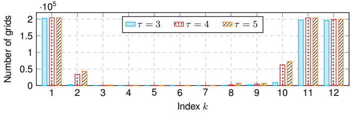
$$\text{s.t.} \quad \underbrace{0 \leq \boldsymbol{w}_n \leq \boldsymbol{\beta},}_{\text{bounds being either } 0 \text{ or } 1} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau,}_{\text{sparsity of } \boldsymbol{\beta}} \quad \boldsymbol{\beta} \in \{0,1\}^d$$

- Two-stage optimization (♣):
  - ❶ **Learn sparsity patterns in $\boldsymbol{\beta} \in \{0,1\}^d$**

$$\min_{\boldsymbol{w}, \boldsymbol{\beta}} \; \underbrace{\operatorname{tr}(\boldsymbol{w}\boldsymbol{w}^\top \boldsymbol{P})}_{\text{quadratic}} - \underbrace{2\boldsymbol{w}^\top \boldsymbol{q}}_{\text{linear}}$$

$$\text{s.t. } 0 \leq \boldsymbol{w} \leq \boldsymbol{\beta}, \; \|\boldsymbol{\beta}\|_1 \leq \tau$$

Global sparsity verification



204k monthly NA temperature time series

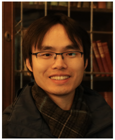
  - ❷ **Quadratic programming** with index set $\Omega = \operatorname{supp}(\boldsymbol{\beta})$

$$\boldsymbol{w}_n := \arg\min_{\boldsymbol{w} \geq 0} \; \|\tilde{\boldsymbol{x}}_n - \boldsymbol{A}_n \boldsymbol{w}\|_2^2 \quad \text{s.t. } w_k = 0, \; \forall k \notin \Omega$$

Climate System Seasonality Patterns

(arXiv:2506.22895)

❒ North America temperature/precipitation ❒ Sea surface temperature
❒ Climate variable seasonality ❒ Spatiotemporal patterns



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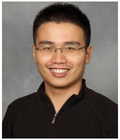

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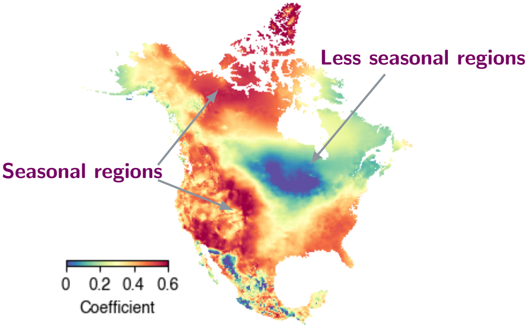
Motivation

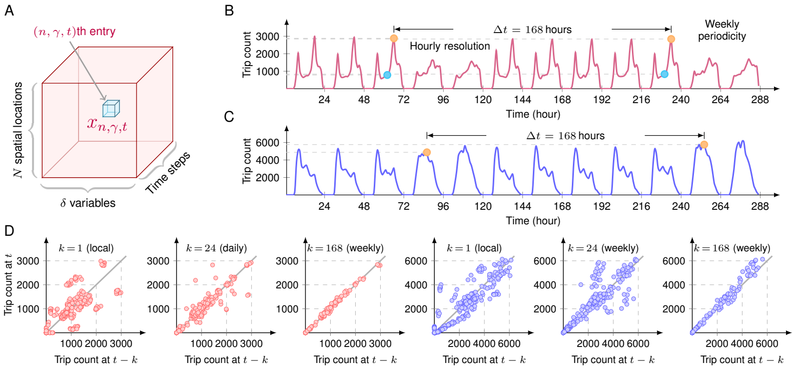


Yearly temperature **seasonality** pattern in 2010s

Motivation

Motivation

Human mobility data show daily/weekly regularity and periodicity?



"**Closeness**" to the anti-diagonal $y = x$

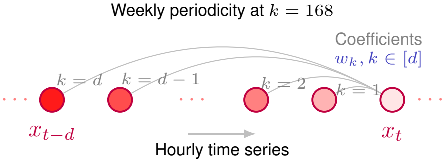
$x_t \approx x_{t-168}$ (weekly periodicity)

John F. Kennedy International Airport

No free lunch

Spatially- and Time-Varying Autoregression

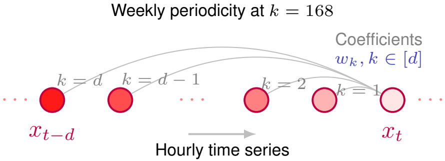
Univariate autoregression



$$\min \sum_t \Big(x_t - \sum_{k \in [d]} w_k x_{t-k} \Big)^2$$

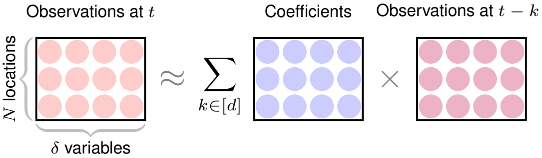
Spatially- and Time-Varying Autoregression

Univariate autoregression



$$\min \sum_t \Big(x_t - \sum_{k \in [d]} w_k x_{t-k} \Big)^2$$

Multidimensional autoregression



$$\min \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_t \Big(x_{n,\gamma,t} - \sum_{k \in [d]} w_{n,\gamma,k} x_{n,\gamma,t-k} \Big)^2$$

Envisioning Human Mobility

- Ridesharing trip data $\{\boldsymbol{x}_{n,\gamma}\}$ across $\gamma \in [\delta]$ years
- Reformulate sparse autoregression:

year

spatial location hourly time step

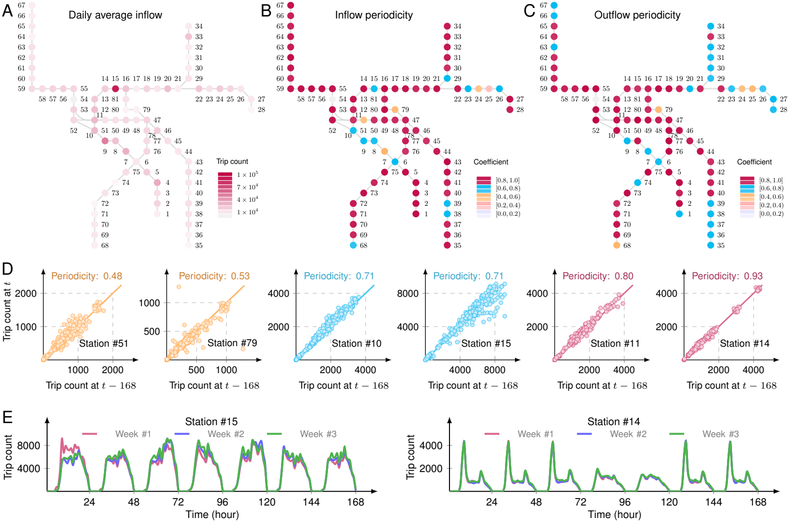
$$\min_{\{\boldsymbol{w}_{n,\gamma}\},\boldsymbol{\beta}} \sum_{n\in[N]} \sum_{\gamma\in[\delta]} \sum_{t\in[d+1,T_\gamma]} \Big(x_{n,\gamma,t} - \sum_{k\in[d]} w_{n,\gamma,k} x_{n,\gamma,t-k}\Big)^2$$

$$\text{s.t. } \underbrace{\boldsymbol{\beta} \in \{0,1\}^d}_{\text{binary var.}} \quad \underbrace{0 \leq \boldsymbol{w}_{n,\gamma} \leq \boldsymbol{\beta}}_{\text{upper bound in } \{0,1\}} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\text{sum of binary var.}}$$

- MIP problem w/ $(N\delta + 1)d$ variables!
- How to handle thousands or millions of (e.g., $N\delta = 10^6$) time series?

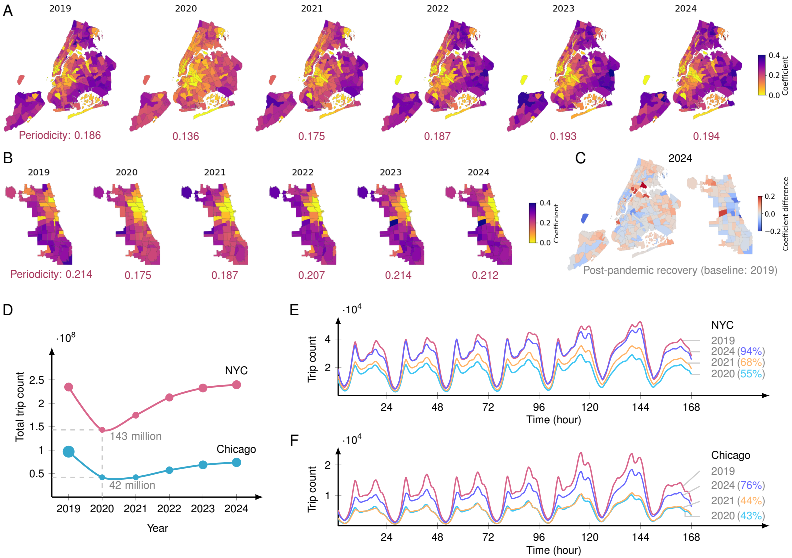
Quantify weekly periodicity by $\{w_{n,\gamma,k}\}$ at index $k = 168$

Envisioning Human Mobility



Hangzhou metro passenger flow in January 2019

Envisioning Human Mobility



Weekly periodicity reveals spatial patterns of ridesharing systems

Understanding Climate Systems

Quantify yearly seasonality by $\{w_{m,n,\gamma,k}\}$ at index $k = 12$

latitude

longitude

decade

monthly time step

temperature

$$\min_{\{\boldsymbol{w}_{m,n,\gamma}\},\boldsymbol{\beta}} \sum_{m\in[M]} \sum_{n\in[N]} \sum_{\gamma\in[\delta]} \sum_{t\in[d+1,T_\gamma]} \Big(x_{m,n,\gamma,t} - \sum_{k\in[d]} w_{m,n,\gamma,k} x_{m,n,\gamma,t-k} \Big)^2$$

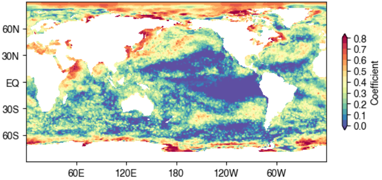
$$\text{s.t.} \begin{cases} \boldsymbol{\beta} \in \{0,1\}^d \\ 0 \leq \boldsymbol{w}_{m,n,\gamma} \leq \boldsymbol{\beta},\ \forall m,n,\gamma \\ \|\boldsymbol{\beta}\|_1 \leq \tau \\ \|\boldsymbol{w}_{m,n,\gamma}\|_1 = 1,\ \forall m,n,\gamma \end{cases}$$

binary decision var.

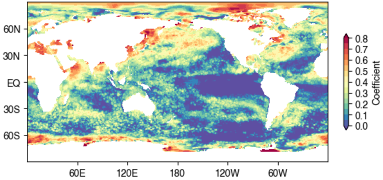
sparsity constraints

ℓ_1-normalization

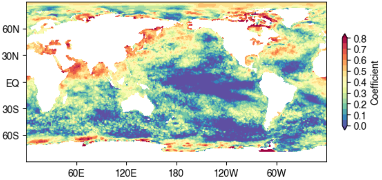
Sea Surface Temperature



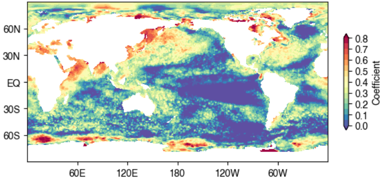
1980s



1990s



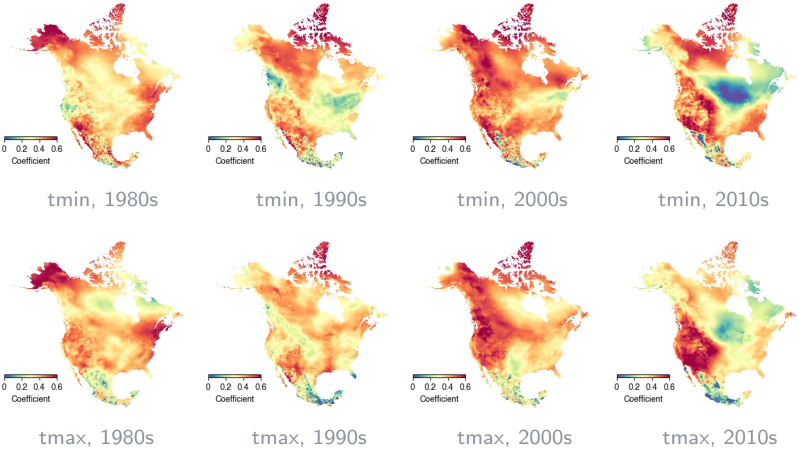
2000s



2010s

- Identify yearly periodicity at $k = 12$ from SST data ($\tau = 3$)
  1. The areas of El Nino events are less seasonal/predictable
  2. Arctic becomes less seasonal/predictable in the past 20 years
- Insights into climate system monitoring

North America Temperature



- Identify yearly periodicity at $k = 12$ from temperature data ($\tau = 3$)
  1. Stronger yearly seasonality in high-latitude areas
  2. Less seasonal temperature in south areas (e.g., Mexico)
  3. Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s

Future Work

- Quantifying Behavioral Regularity of Wikipedia

Thanks for your attention!

Any Questions?

Slides: https://xinychen.github.io/slides/ints.pdf

About me:

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