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# Quantifying Time Series Periodicity with Interpretable Machine Learning

Urban Human Mobility & Climate Variables

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Cambridge, USA

# Spatiotemporal Data

- Transport & mobility & climate application scenarios



Highway (Portland)



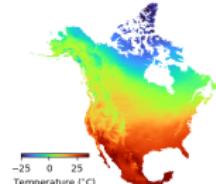
Uber movement (NYC)



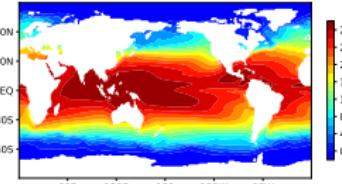
Uber movement (Seattle)



Taxi trajectory (Shenzhen)



Temperature (NA)

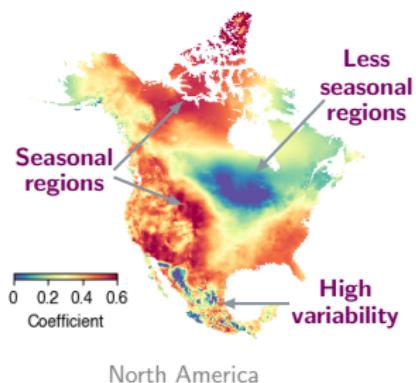


Temperature (sea surface)

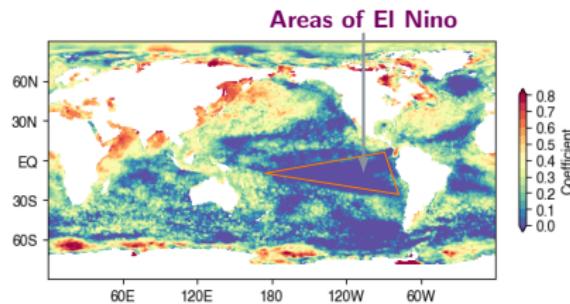
- Challenges: Sparsity, high-dimensionality, multi-dimensionality, heavy tails, irregular sampling, and time-varying systems

# Motivation

Yearly temperature **seasonality** patterns in 2010s



North America



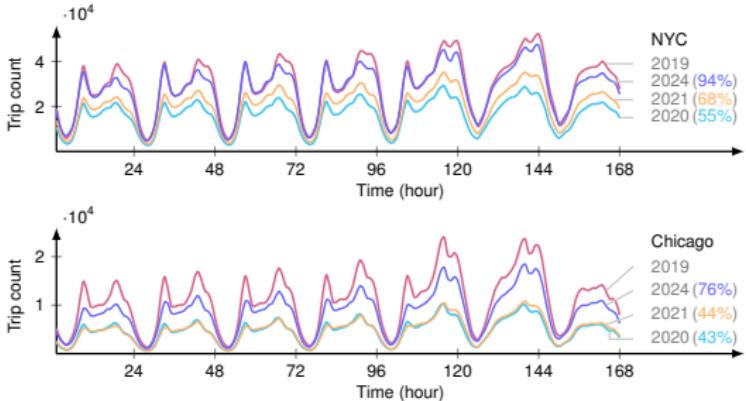
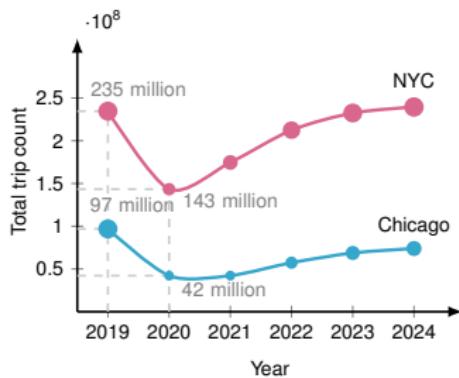
Global sea surface

What motivate us most about **periodicity**?

- ① **Monitoring climate systems:** Empirically measure the periodicity of climate variables (e.g., temperature, precipitation).
- ② **Discovering spatiotemporal patterns:** Identify periodicity pattern shift and special climate events.

# Motivation

## Ridesharing trip data



## What motivate us most about periodicity?

- ① Resilience and stability of systems:** Empirically measure the periodicity and predictability of urban systems.
- ② Optimization of transport systems:** Optimize resources (e.g., public transit, taxi, ridesharing, micromobility) to meet transport demand efficiently.
- ③ Design of sustainable transport & infrastructure:** Implement energy-efficient solutions (e.g., congestion pricing) tailored to peak hours.



## Interpretable Time Series Autoregression



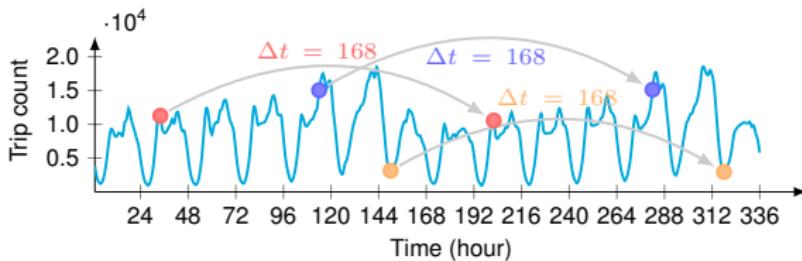
<https://github.com/xinychen/integers>

- Interpretable ML
- $\ell_0$ -norm optimization
- Climate system seasonality
- Sparse autoregression
- Mixed-integer optimization
- Human mobility regularity

# Valorizing Autoregression

- Time series autoregression on  $\mathbf{x} \in \mathbb{R}^T$

$$\mathbf{w} := \arg \min_{\mathbf{w}} \sum_{t=d+1}^T \left( \mathbf{x}_t - \sum_{k=1}^d w_k \mathbf{x}_{t-k} \right)^2$$



Periodicity of hourly rideshare trip time series

- Sparse** coefficient vector  $\mapsto$  **Interpretability?**

$$\underbrace{\mathbf{w}}_{\text{sparsity } \|\mathbf{w}\|_0 \triangleq 3} = (\underbrace{0.33}_{k=1}, 0, \dots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

# Sparse Autoregression

- Identify the dominant auto-correlations
  - $\tau \in \mathbb{Z}^+$ : Upper bound of the number of nonzero entries in  $w \in \mathbb{R}^d$

$$\tilde{x} \approx A \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} \quad \left\| w \right\|_0 \leq \tau$$

$$w := \arg \min_{\|w\|_0 \leq \tau} \sum_{t=d+1}^T \left( x_t - \sum_{k=1}^d w_k x_{t-k} \right)^2$$

$$= \arg \min_{\|w\|_0 \leq \tau} \|\tilde{x} - Aw\|_2^2$$

Diagram illustrating the sparse autoregression model:

- $\tilde{x}$  is a vector of length 9:  $(x_5, x_6, x_7, x_8, x_9, x_4, x_3, x_2, x_1)$ .
- $A$  is a 4x9 matrix representing lags of  $x$ :

$x_4$	$x_3$	$x_2$	$x_1$					
$x_5$	$x_4$	$x_3$	$x_2$					
$x_6$	$x_5$	$x_4$	$x_3$	$x_2$				
$x_7$	$x_6$	$x_5$	$x_4$	$x_3$				
$x_8$	$x_7$	$x_6$	$x_5$	$x_4$				
$x_9$	$x_8$	$x_7$	$x_6$	$x_5$				

- $w$  is a vector of length 4:  $(w_1, w_2, w_3, w_4)$ . Only  $w_1, w_2, w_3$  are non-zero.

- $\ell_0$ -norm optimization is NP-hard
- Formulate it as a mixed-integer programming
  - Introduce binary decision variables  $\beta \in \{0, 1\}^d$

$$\min_w \|\tilde{x} - Aw\|_2^2 \quad \iff \quad \min_{w, \beta} \|\tilde{x} - Aw\|_2^2$$

$$\text{s.t. } \underbrace{\|w\|_0 \leq \tau}_{\clubsuit \text{ sparsity of } w} \quad \iff \quad \text{s.t. } \underbrace{-\beta \leq w \leq \beta}_{\text{bounds being either 0 or } \pm 1}, \quad \underbrace{\|\beta\|_1 \leq \tau}_{\clubsuit \text{ sparsity of } \beta}$$

# Sparse Autoregression Done Right

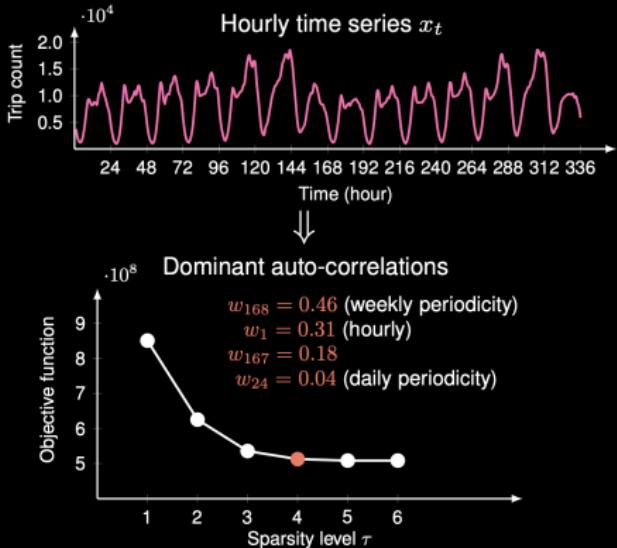
$$\min_{\mathbf{w}, \boldsymbol{\beta}} \underbrace{\sum_{t=d+1}^T \left( x_t - \sum_{k=1}^d w_k x_{t-k} \right)^2}_{\text{Time series autoregression}} \quad \text{s.t. } \underbrace{-\beta_k \leq w_k \leq \beta_k}_{\text{Lower and upper bounds}}, \quad \underbrace{\sum_{k=1}^d \beta_k \leq \tau}_{\clubsuit \text{ Sparsity}} , \quad \underbrace{\beta_k \in \{0, 1\}}_{\text{Binary variable}}$$

- $\mathbf{w} \in \mathbb{R}^d$ : Auto-correlations
- $\boldsymbol{\beta} \in \{0, 1\}^d$ : Sparsity pattern
- $d = 168$ : Autoregression order

```

1 import numpy as np
2 from docplex.mp.model import Model
3
4 def sparse_ar(x, d, tau):
5     model = Model('Sparse Autoregression')
6     T = x.shape[0]
7     w = [model.continuous_var(name = f'w_{k}') for k in range(d)]
8     beta = [model.binary_var(name = f'beta_{k}') for k in range(d)]
9     model.minimize(model.sum((x[t] - model.sum(w[k] * x[t - k - 1]
10                                for k in range(d))) ** 2
11                                for t in range(d, T)))
12     model.add_constraint(model.sum(beta[k] for k in range(d)) <= tau)
13     for k in range(d):
14         model.add_constraint(w[k] <= beta[k])
15         model.add_constraint(w[k] >= -beta[k])
16     solution = model.solve()
17     return np.array(solution.get_values(w))

```



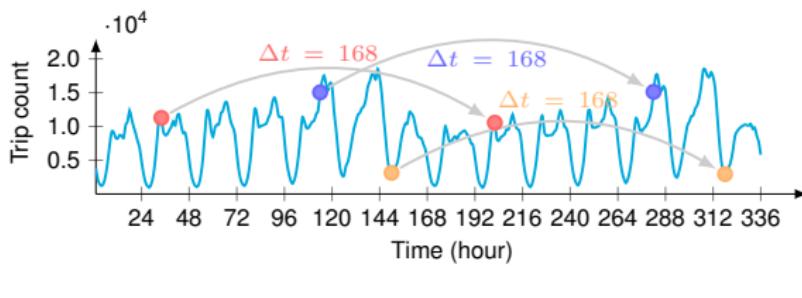
<https://github.com/xinychen/integers>

## Solution Quality → Better Interpretability?

- Sparse autoregression

$$\min_{\mathbf{w} \geq 0} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 \quad \text{s.t. } \|\mathbf{w}\|_0 \leq \tau$$

- Subspace pursuit (SP) sometimes fails



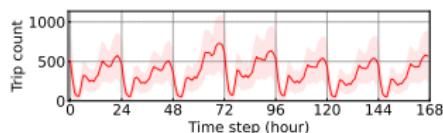
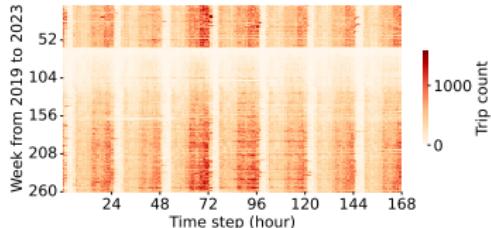
Periodicity of ridesharing trip time series

- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity  $\tau = 2$ ):

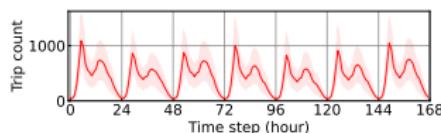
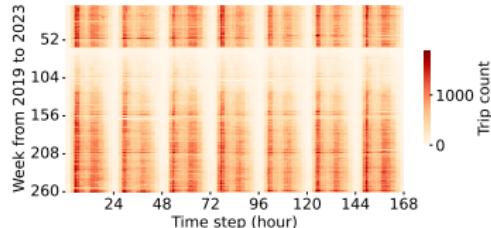
$$\underbrace{\mathbf{w} = (\dots, \underbrace{0.02}_{k=53}, \dots, \underbrace{0.96}_{k=168})^\top}_{\text{obj. } = 8.32 \times 10^7 \text{ (SP)}} \quad \text{vs.} \quad \underbrace{\mathbf{w} = (\underbrace{0.22}_{k=1}, \dots, \underbrace{0.77}_{k=168})^\top}_{\clubsuit \text{ obj. } = 6.25 \times 10^7 \text{ (MIP)}}$$

# John F. Kennedy International Airport

- Pickup/Dropoff trips in airport
  - Pickup trips are relevant to flight delay, baggage claim, and other factors.
  - Dropoff trips to airport are highly related to flight schedules.



Pickup trips from airport



Dropoff trips to airport

- Sparse coefficient vectors (**sparsity  $\tau = 3$** ):

$$\mathbf{w} = (\underbrace{0.31}_{k=1}, \dots, \underbrace{0.28}_{k=24}, \dots, \underbrace{0.41}_{k=168})^\top \quad \text{vs.} \quad \mathbf{w} = (\underbrace{0.18}_{k=1}, \dots, \underbrace{0.35}_{k=24}, \dots, \underbrace{0.47}_{k=168})^\top$$

## High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large  $N$ :

$$\min_{\underbrace{\{\boldsymbol{w}_n\}_{n=1}^N, \boldsymbol{\beta}}_{(N+1)d \text{ decision var.}}} \underbrace{\sum_{n=1}^N \|\tilde{\boldsymbol{x}}_n - \boldsymbol{A}_n \boldsymbol{w}_n\|_2^2}_{\text{multivariate time series}}$$

s.t.

$$\underbrace{0 \leq \boldsymbol{w}_n \leq \boldsymbol{\beta},}_{\text{bounds being either 0 or 1}} \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau,}_{\text{sparsity of } \boldsymbol{\beta}} \boldsymbol{\beta} \in \{0, 1\}^d$$

# High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large  $N$ :

$$\begin{aligned}
 & \underbrace{\min_{\{\mathbf{w}_n\}_{n=1}^N, \boldsymbol{\beta}}}_{(N+1)d \text{ decision var.}} \quad \underbrace{\sum_{n=1}^N \|\tilde{\mathbf{x}}_n - \mathbf{A}_n \mathbf{w}_n\|_2^2}_{\text{multivariate time series}} \\
 & \text{s.t.} \quad \underbrace{0 \leq \mathbf{w}_n \leq \boldsymbol{\beta},}_{\text{bounds being either 0 or 1}} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau,}_{\text{sparsity of } \boldsymbol{\beta}} \quad \boldsymbol{\beta} \in \{0, 1\}^d
 \end{aligned}$$

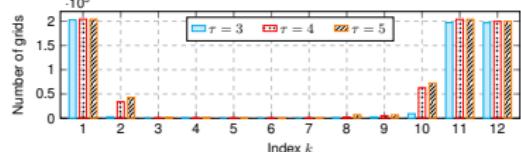
- Two-stage optimization (♣):

① Learn sparsity patterns in  $\boldsymbol{\beta} \in \{0, 1\}^d$

$$\min_{\mathbf{w}, \boldsymbol{\beta}} \underbrace{\text{tr}(\mathbf{w} \mathbf{w}^\top \mathbf{P})}_{\text{quadratic}} - \underbrace{2 \mathbf{w}^\top \mathbf{q}}_{\text{linear}}$$

$$\text{s.t. } 0 \leq \mathbf{w} \leq \boldsymbol{\beta}, \|\boldsymbol{\beta}\|_1 \leq \tau$$

Global sparsity verification



204k monthly NA temperature time series

② Quadratic programming with index set  $\Omega = \text{supp}(\boldsymbol{\beta})$

$$\mathbf{w}_n := \arg \min_{\mathbf{w} \geq 0} \|\tilde{\mathbf{x}}_n - \mathbf{A}_n \mathbf{w}\|_2^2 \quad \text{s.t. } w_k = 0, \forall k \notin \Omega$$

- Decision variable pruning strategy (SP → MIP ♣)

# Climate System Seasonality Patterns

(arXiv:2506.22895)

- North America temperature/precipitation     Sea surface temperature
- Climate variable seasonality     Spatiotemporal patterns



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UCSD

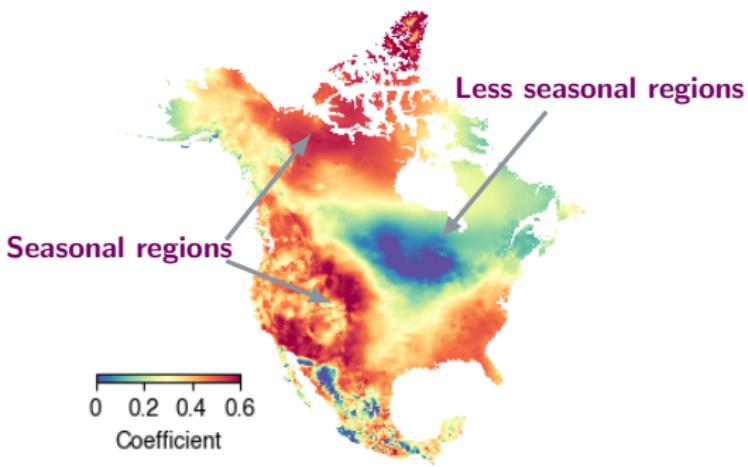


Dingyi Zhuang  
MIT



Jinhua Zhao  
MIT

## Motivation



Yearly temperature **seasonality** pattern in 2010s

# Understanding Climate Systems

Quantify yearly seasonality by  $\{w_{m,n,\gamma,k}\}$  at index  $k = 12$

$$\min_{\{\mathbf{w}_{m,n,\gamma}\}, \boldsymbol{\beta}} \sum_{m=1}^M \sum_{n=1}^N \sum_{\gamma=1}^{\delta} \|\tilde{\mathbf{x}}_{m,n,\gamma} - \mathbf{A}_{m,n,\gamma} \mathbf{w}_{m,n,\gamma}\|_2^2$$

longitude  
latitude | decade      monthly  
          ↓      ↓      ↓  
          M      N      δ      temperature

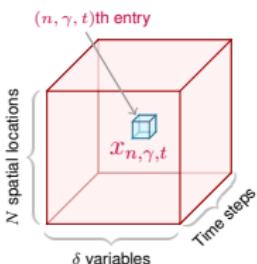
s.t.  $0 \leq \mathbf{w}_{m,n,\gamma} \leq \boldsymbol{\beta}$       sparsity constraint  
 $\|\boldsymbol{\beta}\|_1 \leq \tau$   
 $\boldsymbol{\beta} \in \{0, 1\}^d$       binary decision var.



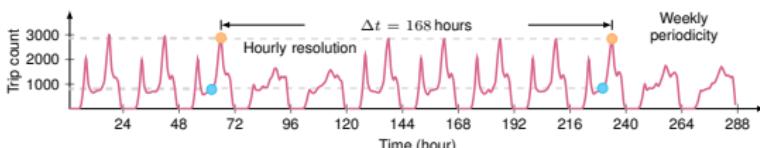
# Motivation

Human mobility data show daily/weekly regularity and periodicity?

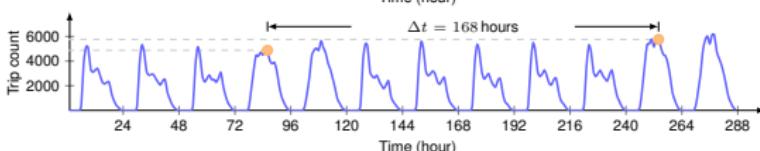
A



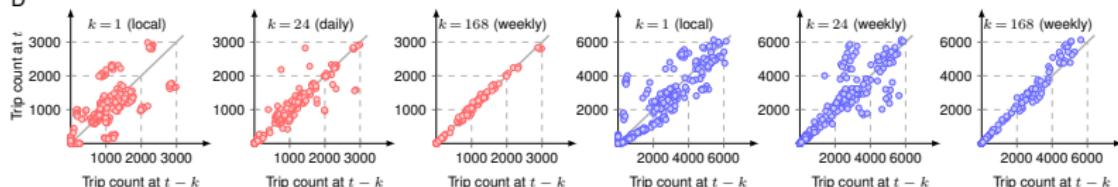
B



C



D



“Closeness” to the  
anti-diagonal  $y = x$

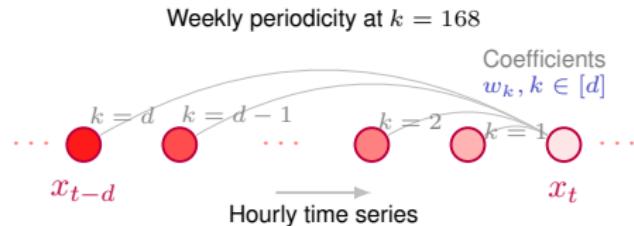
$x_t \approx x_{t-168}$  (weekly periodicity)

## John F. Kennedy International Airport

No free lunch

# Spatially- and Time-Varying Autoregression

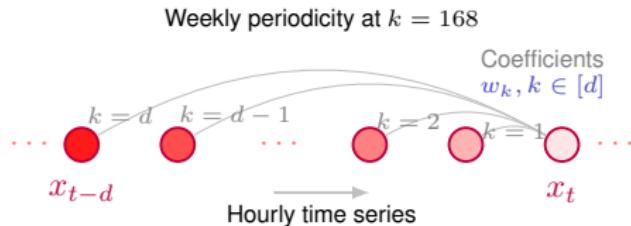
## Univariate autoregression



$$\min_t \sum_{k \in [d]} \left( x_t - \sum_{k \in [d]} w_k x_{t-k} \right)^2$$

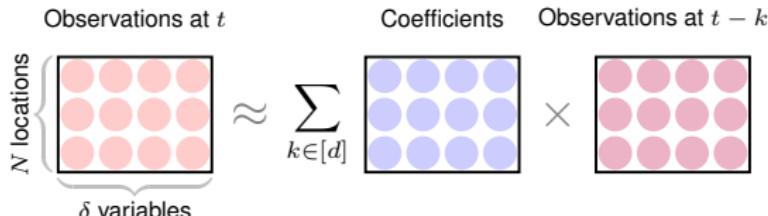
# Spatially- and Time-Varying Autoregression

## Univariate autoregression



$$\min \sum_t \left( x_t - \sum_{k \in [d]} w_k x_{t-k} \right)^2$$

## Multidimensional autoregression



$$\min \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_t \left( x_{n, \gamma, t} - \sum_{k \in [d]} w_{n, \gamma, k} x_{n, \gamma, t-k} \right)^2$$

# Envisioning Human Mobility

- Ridesharing trip data  $\{x_{n,\gamma}\}$  across  $\gamma \in [\delta]$  years
- Reformulate sparse autoregression:

$$\min_{\{\mathbf{w}_{n,\gamma}\}, \boldsymbol{\beta}} \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_{t \in [d+1, T_\gamma]} \left( x_{n,\gamma,t} - \sum_{k \in [d]} w_{n,\gamma,k} x_{n,\gamma,t-k} \right)^2$$

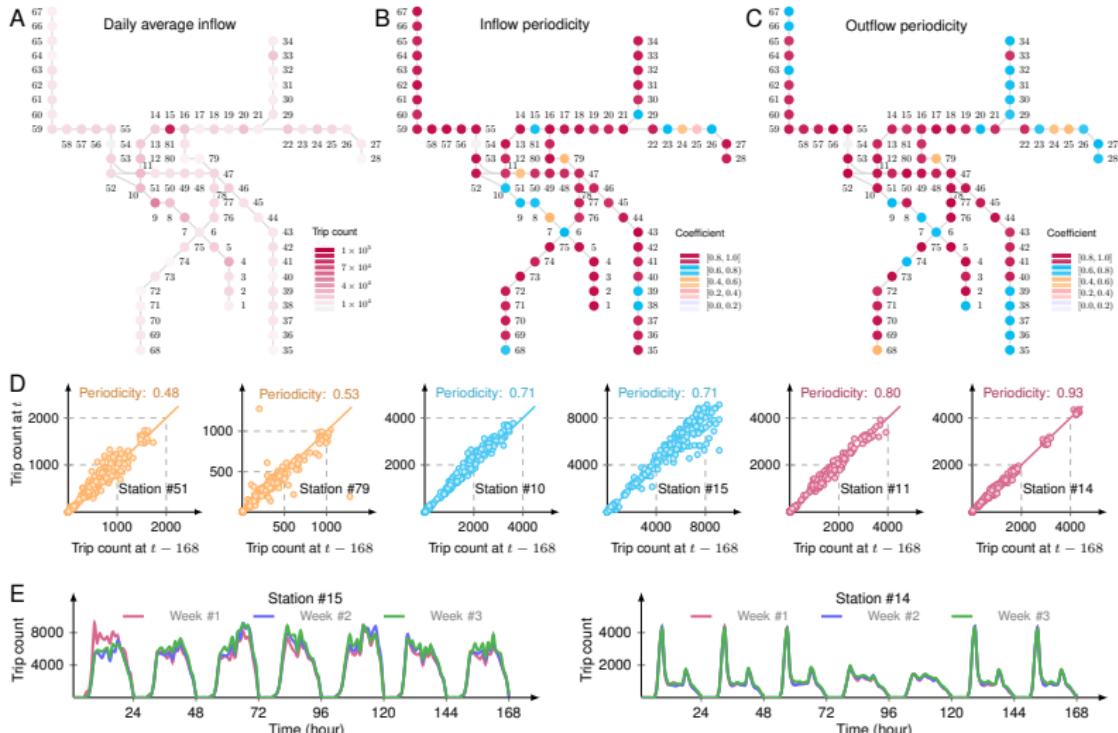
year  
spatial location      hourly time step

s.t.  $\underbrace{\boldsymbol{\beta} \in \{0, 1\}^d}_{\text{binary var.}}$      $\underbrace{0 \leq \mathbf{w}_{n,\gamma} \leq \boldsymbol{\beta}}_{\text{upper bound in } \{0, 1\}}$      $\underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\text{sum of binary var.}}$

- MIP problem w/  $(N\delta + 1)d$  variables!
- How to handle thousands or millions of (e.g.,  $N\delta = 10^6$ ) time series?

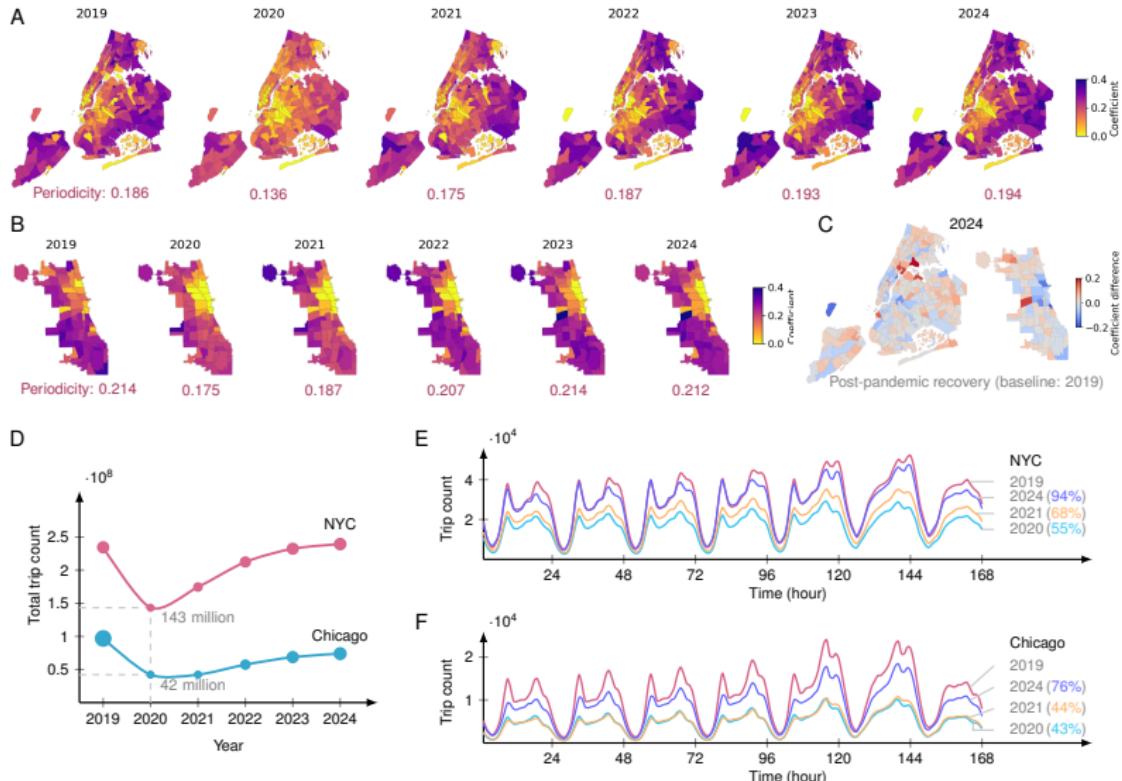
Quantify weekly periodicity by  $\{w_{n,\gamma,k}\}$  at index  $k = 168$

# Envisioning Human Mobility



Hangzhou metro passenger flow in January 2019

# Envisioning Human Mobility



Weekly periodicity reveals spatial patterns of ridesharing systems

# Understanding Climate Systems

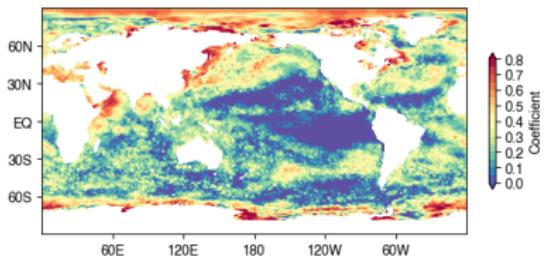
Quantify yearly seasonality by  $\{w_{m,n,\gamma,k}\}$  at index  $k = 12$

$$\min_{\{\boldsymbol{w}_{m,n,\gamma}\}, \boldsymbol{\beta}} \sum_{m \in [M]} \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_{t \in [d+1, T_\gamma]} \left( x_{m,n,\gamma,t} - \sum_{k \in [d]} w_{m,n,\gamma,k} x_{m,n,\gamma,t-k} \right)^2$$

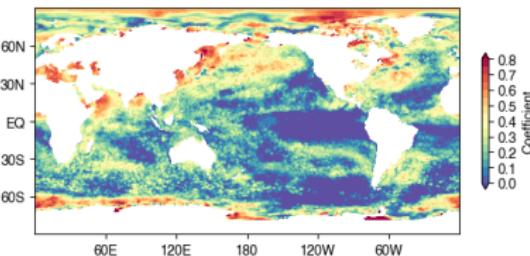
s.t. 
$$\begin{cases} \boldsymbol{\beta} \in \{0, 1\}^d & \text{binary decision var.} \\ 0 \leq \boldsymbol{w}_{m,n,\gamma} \leq \boldsymbol{\beta}, \forall m, n, \gamma \\ \|\boldsymbol{\beta}\|_1 \leq \tau \\ \|\boldsymbol{w}_{m,n,\gamma}\|_1 = 1, \forall m, n, \gamma \end{cases}$$

$\ell_1$ -normalization

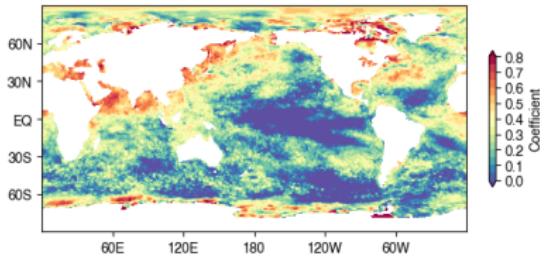
# Sea Surface Temperature



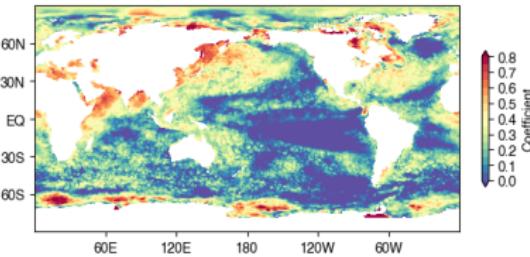
1980s



1990s



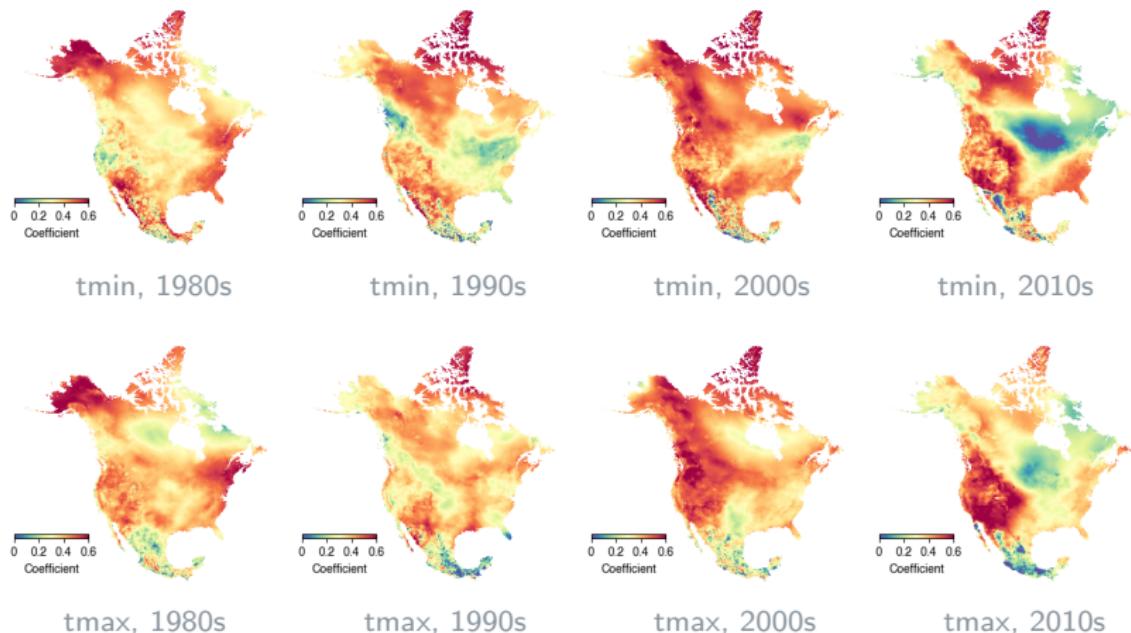
2000s



2010s

- Identify yearly periodicity at  $k = 12$  from SST data ( $\tau = 3$ )
  - ❶ The areas of El Niño events are less seasonal/predictable
  - ❷ Arctic becomes less seasonal/predictable in the past 20 years
- Insights into climate system monitoring

## North America Temperature



- Identify yearly periodicity at  $k = 12$  from temperature data ( $\tau = 3$ )
  - ❶ Stronger yearly seasonality in high-latitude areas
  - ❷ Less seasonal temperature in south areas (e.g., Mexico)
  - ❸ Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s

## Future Work

- Quantifying Behavioral Regularity of Wikipedia



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# Thanks for your attention!

Any Questions?

Slides: <https://xinychen.github.io/slides/intro.pdf>

## About me:

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