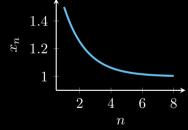
## Three rates of convergence on a sequence $\{x_n\}_{n\in\mathbb{Z}^+}$

## Linear convergence

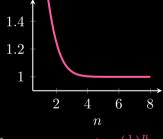
$$\frac{\|x_{n+1} - x_{\infty}\|}{\|x_n - x_{\infty}\|} \le r, \, r \in (0, 1)$$



Sequence  $x_n = 1 + \left(\frac{1}{2}\right)^n$  converges linearly to  $x_\infty = 1$  because  $r = \frac{1}{2}$ 

## Superlinear convergence

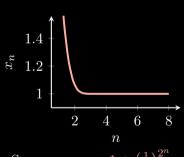
$$\lim_{n \to \infty} \frac{\|x_{n+1} - x_{\infty}\|}{\|x_n - x_{\infty}\|} = 0$$



Sequence  $x_n = 1 + \left(\frac{1}{n}\right)^n$  converges superlinearly to  $x_{\infty} = 1$ 

Quadratic convergence

$$\frac{\|x_{n+1} - x_{\infty}\|}{\|x_n - x_{\infty}\|^2} \le M$$



Sequence  $x_n = 1 + \left(\frac{1}{n}\right)^{2^n}$  converges quadratically to  $x_{\infty} = 1$  because M = 1