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MANUS AND
MACHINA

Quantifying Time Series Periodicity with Interpretable Machine Learning

Climate Variables & Urban Human Mobility

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Spatiotemporal Data

- Transport & mobility & climate application scenarios



Highway (Portland)



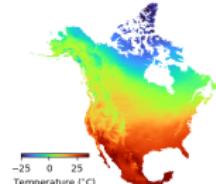
Uber movement (NYC)



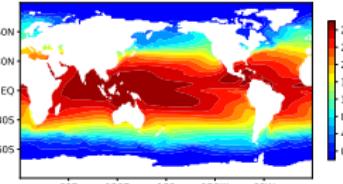
Uber movement (Seattle)



Taxi trajectory (Shenzhen)



Temperature (NA)

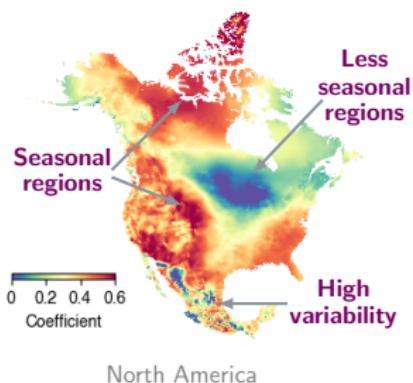


Temperature (sea surface)

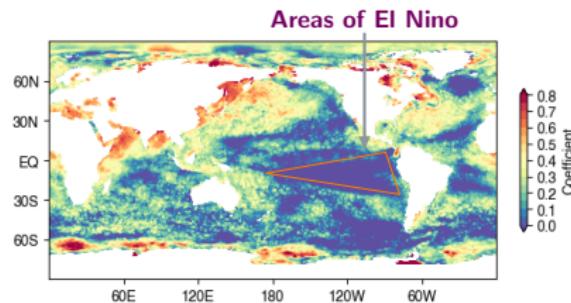
- Challenges: Sparsity, high-dimensionality, multi-dimensionality, heavy tails, irregular sampling, and time-varying systems

Motivation

Yearly temperature **seasonality** patterns in 2010s



North America



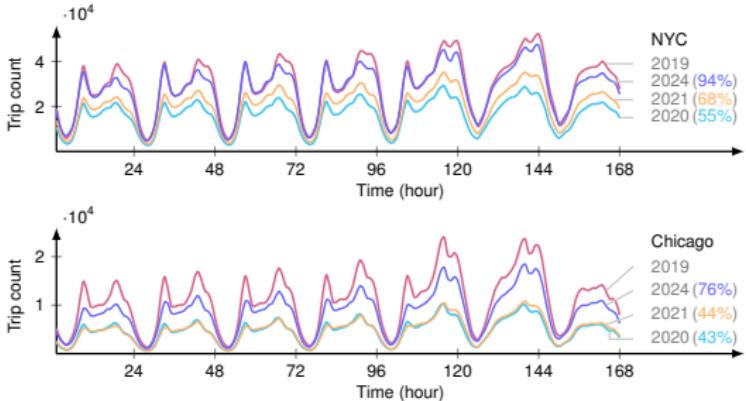
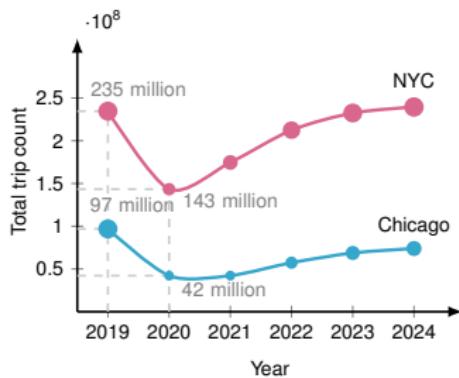
Global sea surface

What motivate us most about **periodicity**?

- ① **Monitoring climate systems:** Empirically measure the periodicity of climate variables (e.g., temperature, precipitation).
- ② **Discovering spatiotemporal patterns:** Identify periodicity pattern shift and special climate events.

Motivation

Ridesharing trip data



What motivate us most about periodicity?

- ① Resilience and stability of systems:** Empirically measure the periodicity and predictability of urban systems.
- ② Optimization of transport systems:** Optimize resources (e.g., public transit, taxi, ridesharing, micromobility) to meet transport demand efficiently.
- ③ Design of sustainable transport & infrastructure:** Implement energy-efficient solutions (e.g., congestion pricing) tailored to peak hours.



Interpretable Time Series Autoregression



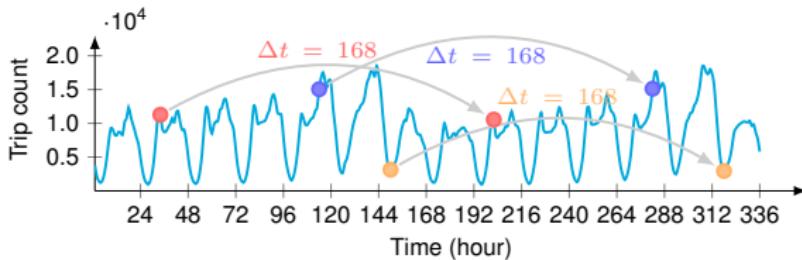
<https://github.com/xinychen/integers>

- Interpretable ML
- ℓ_0 -norm optimization
- Climate system seasonality
- Sparse autoregression
- Mixed-integer optimization
- Human mobility regularity

Valorizing Autoregression

- Time series autoregression on $\mathbf{x} \in \mathbb{R}^T$ with order $d \in \mathbb{Z}^+$

$$\mathbf{w} := \arg \min_{\mathbf{w}} \sum_{t=d+1}^T \left(\mathbf{x}_t - \sum_{k=1}^d w_k \mathbf{x}_{t-k} \right)^2$$



Periodicity of hourly rideshare trip time series

- Sparse** coefficient vector \mapsto **Interpretability?**

$$\underbrace{\mathbf{w}}_{\text{sparsity } \|\mathbf{w}\|_0 \triangleq 3} = (\underbrace{0.33}_{k=1}, 0, \dots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

Sparse Autoregression

- Identify the dominant auto-correlations
 - $\tau \in \mathbb{Z}^+$: Upper bound of the number of nonzero entries in $w \in \mathbb{R}^d$

$$\tilde{x} \approx A \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} \quad \left\{ \begin{array}{l} w := \arg \min_{\|w\|_0 \leq \tau} \sum_{t=d+1}^T \left(x_t - \sum_{k=1}^d w_k x_{t-k} \right)^2 \\ = \arg \min_{\|w\|_0 \leq \tau} \|\tilde{x} - Aw\|_2^2 \end{array} \right.$$

- ℓ_0 -norm optimization is NP-hard
- Formulate it as a mixed-integer programming
 - Introduce binary decision variables $\beta \in \{0, 1\}^d$

$$\begin{aligned} \min_w \|\tilde{x} - Aw\|_2^2 & \iff \min_{w, \beta} \|\tilde{x} - Aw\|_2^2 \\ \text{s.t. } \underbrace{\|w\|_0 \leq \tau}_{\clubsuit \text{ sparsity of } w} & \iff \text{s.t. } \underbrace{-\beta \leq w \leq \beta}_{\text{bounds being either } 0 \text{ or } \pm 1}, \underbrace{\|\beta\|_1 \leq \tau}_{\clubsuit \text{ sparsity of } \beta} \end{aligned}$$

Sparse Autoregression Done Right

$$\min_{\mathbf{w}, \boldsymbol{\beta}} \underbrace{\sum_{t=d+1}^T \left(x_t - \sum_{k=1}^d w_k x_{t-k} \right)^2}_{\text{Time series autoregression}} \quad \text{s.t. } \underbrace{-\beta_k \leq w_k \leq \beta_k}_{\text{Lower and upper bounds}}, \quad \underbrace{\sum_{k=1}^d \beta_k \leq \tau}_{\clubsuit \text{ Sparsity}} , \quad \underbrace{\beta_k \in \{0, 1\}}_{\text{Binary variable}}$$

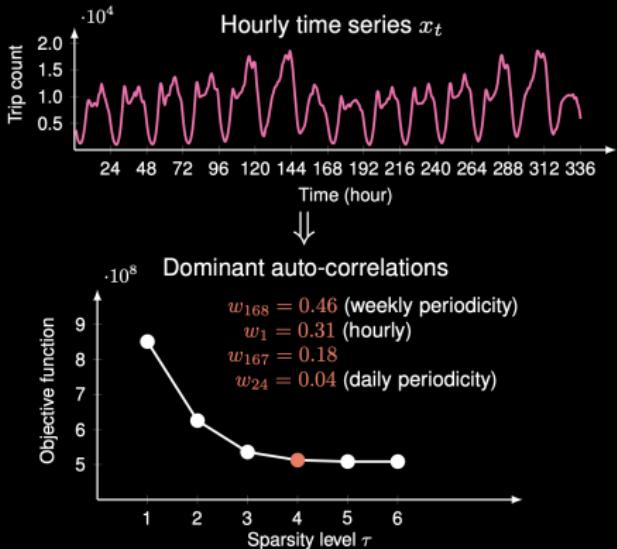
- $\mathbf{w} \in \mathbb{R}^d$: Auto-correlations
- $\boldsymbol{\beta} \in \{0, 1\}^d$: Sparsity pattern
- $d = 168$: Autoregression order

```

1 import numpy as np
2 from docplex.mp.model import Model
3
4 def sparse_ar(x, d, tau):
5     model = Model('Sparse Autoregression')
6     T = x.shape[0]
7     w = [model.continuous_var(name = f'w_{k}') for k in range(d)]
8     beta = [model.binary_var(name = f'beta_{k}') for k in range(d)]
9     model.minimize(model.sum((x[t] - model.sum(w[k] * x[t - k - 1]
10                                for k in range(d))) ** 2
11                                for t in range(d, T)))
12     model.add_constraint(model.sum(beta[k] for k in range(d)) <= tau)
13     for k in range(d):
14         model.add_constraint(w[k] <= beta[k])
15         model.add_constraint(w[k] >= -beta[k])
16     solution = model.solve()
17     return np.array(solution.get_values(w))

```

<https://github.com/xinychen/integers>

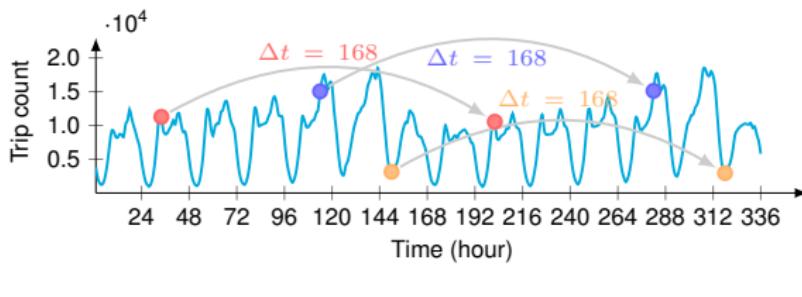


Solution Quality → Better Interpretability?

- Sparse autoregression

$$\min_{\mathbf{w} \geq 0} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 \quad \text{s.t. } \|\mathbf{w}\|_0 \leq \tau$$

- Subspace pursuit (SP) sometimes fails

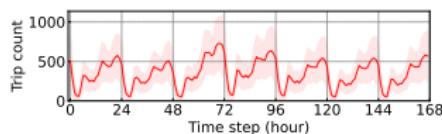
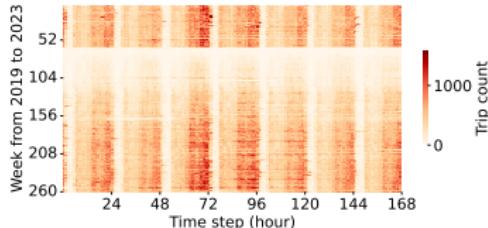


- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity $\tau = 2$):

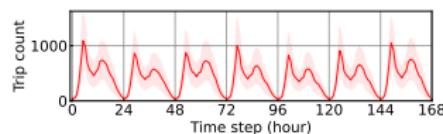
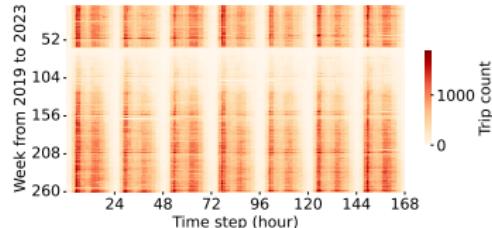
$$\underbrace{\mathbf{w} = (\dots, \underbrace{0.02}_{k=53}, \dots, \underbrace{0.96}_{k=168})^\top}_{\text{obj. } = 8.32 \times 10^7 \text{ (SP)}} \quad \text{vs.} \quad \underbrace{\mathbf{w} = (\underbrace{0.22}_{k=1}, \dots, \underbrace{0.77}_{k=168})^\top}_{\clubsuit \text{ obj. } = 6.25 \times 10^7 \text{ (MIP)}}$$

John F. Kennedy International Airport

- Daily & weekly periodicity: **dropoff > pickup** trips at JFK airport
 - Pickup trips are relevant to flight delay, baggage claim, and other factors.
 - Dropoff trips to airport are highly related to flight schedules.



Pickup trips from airport



Dropoff trips to airport

- Sparse coefficient vectors (**sparsity $\tau = 3$**):

$$\mathbf{w} = (\underbrace{0.31}_{k=1}, \dots, \underbrace{0.28}_{k=24}, \dots, \underbrace{0.41}_{k=168})^\top \quad \text{vs.} \quad \mathbf{w} = (\underbrace{0.18}_{k=1}, \dots, \underbrace{0.35}_{k=24}, \dots, \underbrace{0.47}_{k=168})^\top$$

High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large N :

$$\begin{aligned} & \underbrace{\min_{\{\boldsymbol{w}_n\}_{n=1}^N, \boldsymbol{\beta}}}_{(N+1)d \text{ decision var.}} \quad \underbrace{\sum_{n=1}^N \|\tilde{\boldsymbol{x}}_n - \mathbf{A}_n \boldsymbol{w}_n\|_2^2}_{\text{multivariate time series}} \\ \text{s.t.} \quad & \underbrace{0 \leq \boldsymbol{w}_n \leq \boldsymbol{\beta},}_{\text{bounds being either 0 or 1}} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau,}_{\text{sparsity of } \boldsymbol{\beta}} \quad \boldsymbol{\beta} \in \{0, 1\}^d \end{aligned}$$

High-Dimensional Sparse Autoregression

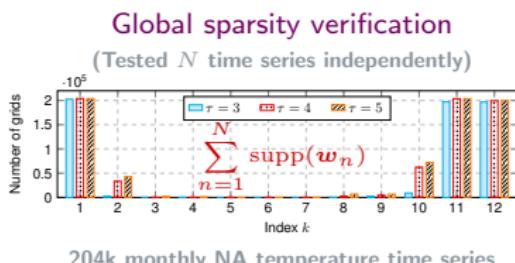
- On high-dimensional time series with a large N :

$$\begin{aligned}
 & \underbrace{\min_{\{\mathbf{w}_n\}_{n=1}^N, \boldsymbol{\beta}}}_{(N+1)d \text{ decision var.}} \underbrace{\sum_{n=1}^N \|\tilde{\mathbf{x}}_n - \mathbf{A}_n \mathbf{w}_n\|_2^2}_{\text{multivariate time series}} \\
 & \text{s.t. } \underbrace{0 \leq \mathbf{w}_n \leq \boldsymbol{\beta},}_{\text{bounds being either 0 or 1}} \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau,}_{\text{sparsity of } \boldsymbol{\beta}} \boldsymbol{\beta} \in \{0, 1\}^d
 \end{aligned}$$

- Two-stage optimization (♣):

- Learn sparsity patterns in $\boldsymbol{\beta} \in \{0, 1\}^d$

$$\begin{aligned}
 & \min_{\mathbf{w}, \boldsymbol{\beta}} \underbrace{\text{tr}(\mathbf{w} \mathbf{w}^\top \mathbf{P})}_{\text{quadratic}} - \underbrace{2 \mathbf{w}^\top \mathbf{q}}_{\text{linear}} \\
 & \text{s.t. } 0 \leq \mathbf{w} \leq \boldsymbol{\beta}, \quad \|\boldsymbol{\beta}\|_1 \leq \tau
 \end{aligned}$$



- Quadratic programming with index set $\Omega = \text{supp}(\boldsymbol{\beta})$

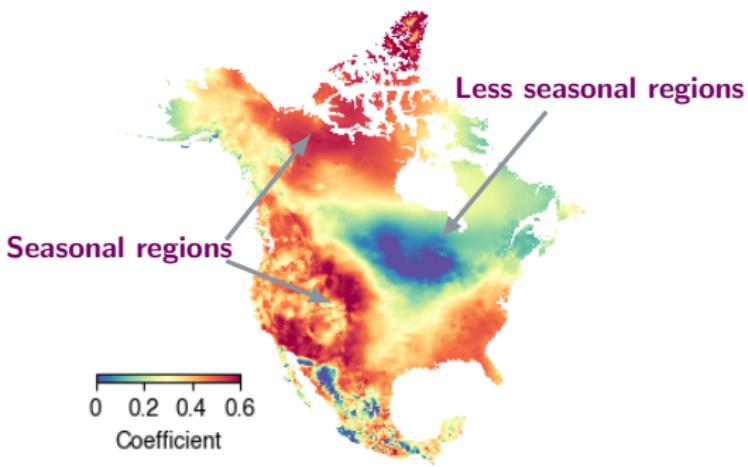
$$\mathbf{w}_n := \arg \min_{\mathbf{w} \geq 0} \|\tilde{\mathbf{x}}_n - \mathbf{A}_n \mathbf{w}\|_2^2 \quad \text{s.t. } w_k = 0, \forall k \notin \Omega$$

Climate System Seasonality Patterns

(arXiv:2506.22895)

- North America temperature/precipitation Sea surface temperature
- Climate variable seasonality Spatiotemporal patterns

Motivation



Yearly temperature **seasonality** pattern in 2010s

Understanding Climate Systems

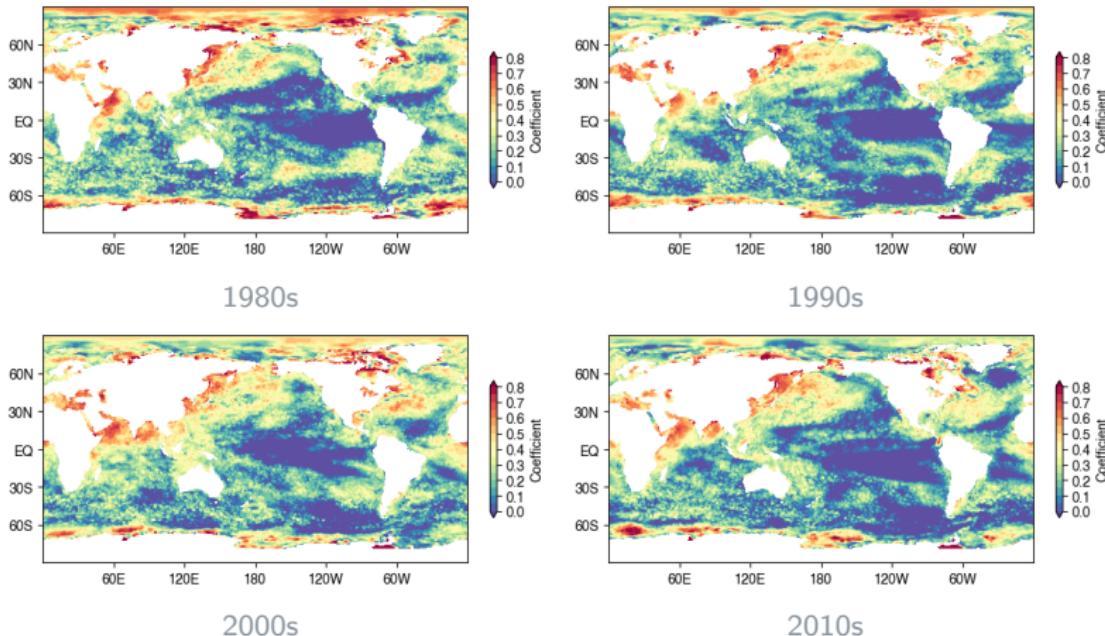
Quantify yearly seasonality by $\{w_{m,n,\gamma,k}\}$ at index $k = 12$

$$\min_{\{\mathbf{w}_{m,n,\gamma}\}, \boldsymbol{\beta}} \sum_{m=1}^M \sum_{n=1}^N \sum_{\gamma=1}^{\delta} \|\tilde{\mathbf{x}}_{m,n,\gamma} - \mathbf{A}_{m,n,\gamma} \mathbf{w}_{m,n,\gamma}\|_2^2$$

longitude
latitude | decade monthly
 ↓ ↓ ↓
 M N δ

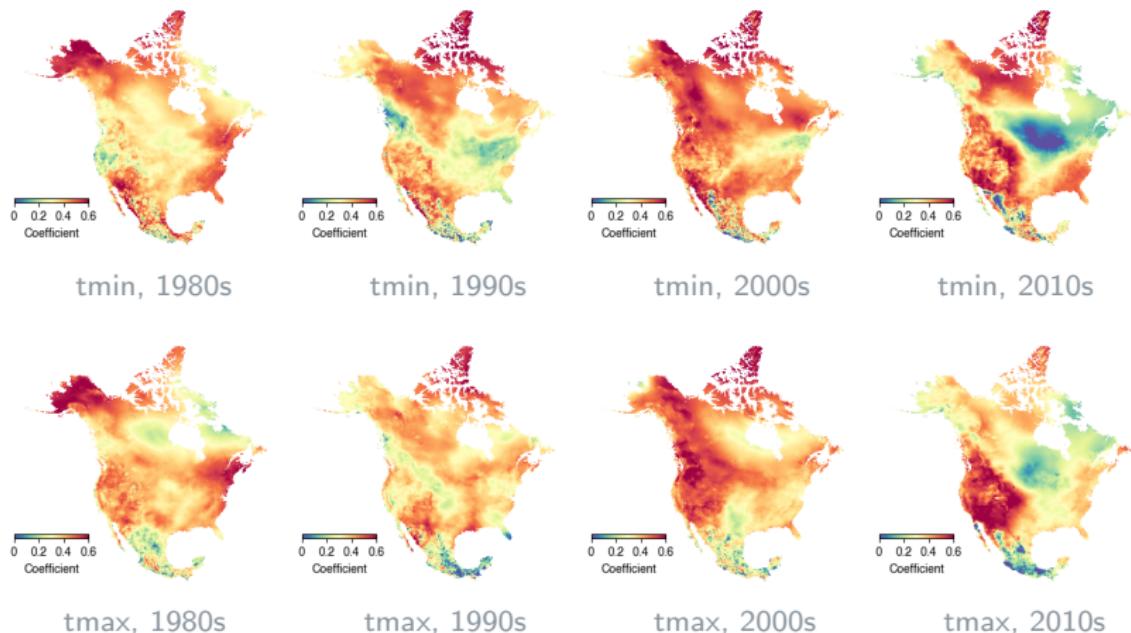
s.t. $0 \leq \mathbf{w}_{m,n,\gamma} \leq \boldsymbol{\beta}$ sparsity constraint
 $\|\boldsymbol{\beta}\|_1 \leq \tau$
 $\boldsymbol{\beta} \in \{0, 1\}^d$ binary decision var.

Sea Surface Temperature



- Identify yearly periodicity at $k = 12$ from SST data ($\tau = 3$)
 - ❶ The areas of El Nino events are less seasonal/predictable
 - ❷ Arctic becomes less seasonal/predictable in the past 20 years

North America Temperature



- Identify yearly periodicity at $k = 12$ from temperature data ($\tau = 3$)
 - ❶ Stronger yearly seasonality in high-latitude areas
 - ❷ Less seasonal temperature in south areas (e.g., Mexico)
 - ❸ Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s

Mexico

Human Mobility Regularity

Applications and Case Studies

(arXiv:2508.03747)

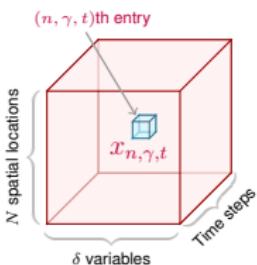
- NYC, Chicago Sea surface temperature
- Climate variable seasonality Spatiotemporal patterns

Envisioning Human Mobility

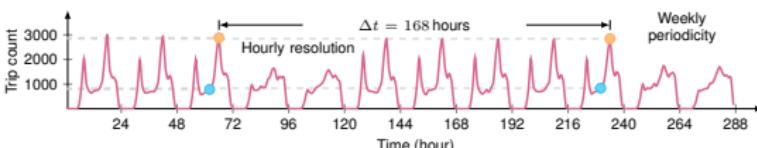
Motivation

Human mobility data show daily/weekly regularity and periodicity?

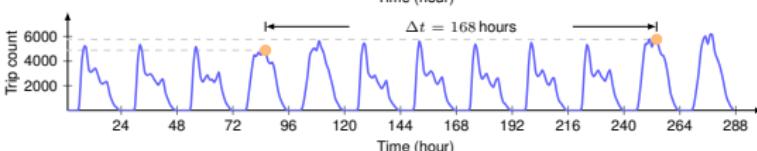
A



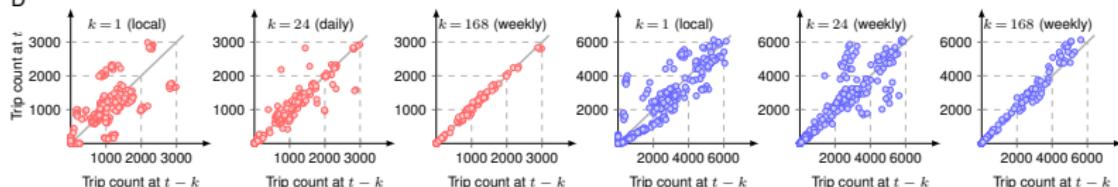
B



C



D

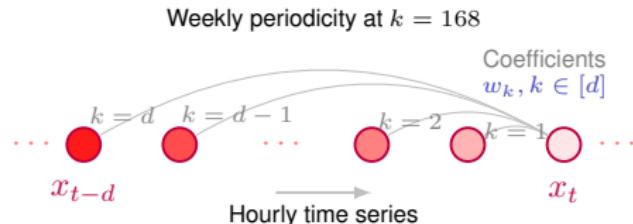


“Closeness” to the
anti-diagonal $y = x$

$x_t \approx x_{t-168}$ (weekly periodicity)

Spatially- and Time-Varying Autoregression

Univariate autoregression

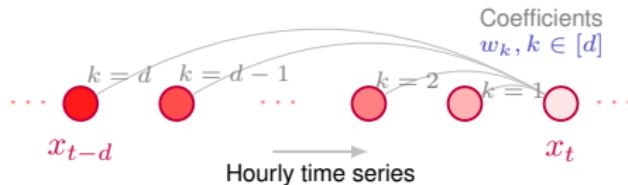


$$\min_t \sum_{k \in [d]} \left(x_t - \sum_{k \in [d]} w_k x_{t-k} \right)^2$$

Spatially- and Time-Varying Autoregression

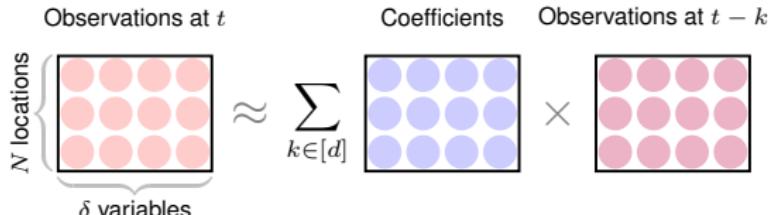
Univariate autoregression

Weekly periodicity at $k = 168$



$$\min \sum_t \left(x_t - \sum_{k \in [d]} w_k x_{t-k} \right)^2$$

Multidimensional autoregression



$$\min \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_t \left(x_{n, \gamma, t} - \sum_{k \in [d]} w_{n, \gamma, k} x_{n, \gamma, t-k} \right)^2$$

Envisioning Human Mobility

- Ridesharing trip data $\{x_{n,\gamma}\}$ across $\gamma \in [\delta]$ years
- Reformulate sparse autoregression:

$$\min_{\{\mathbf{w}_{n,\gamma}\}, \boldsymbol{\beta}} \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_{t \in [d+1, T_\gamma]} \left(x_{n,\gamma,t} - \sum_{k \in [d]} w_{n,\gamma,k} x_{n,\gamma,t-k} \right)^2$$

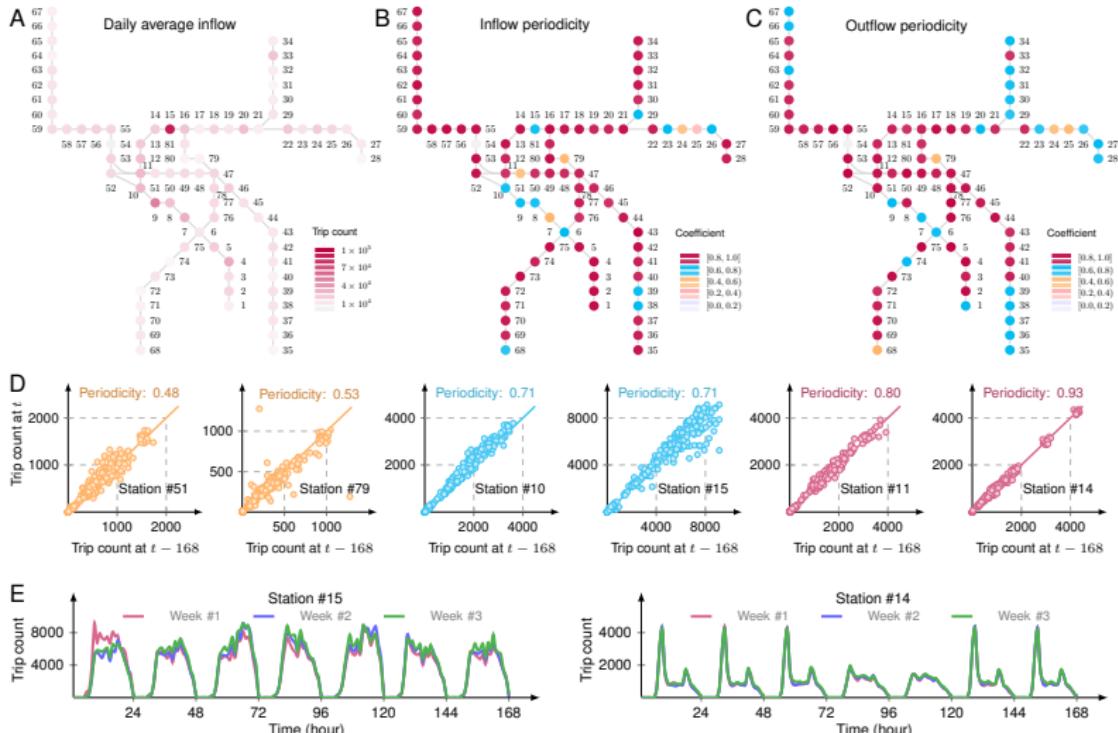
year
spatial location hourly time step

s.t. $\underbrace{\boldsymbol{\beta} \in \{0, 1\}^d}_{\text{binary var.}}$ $\underbrace{0 \leq \mathbf{w}_{n,\gamma} \leq \boldsymbol{\beta}}_{\text{upper bound in } \{0, 1\}}$ $\underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\text{sum of binary var.}}$

- MIP problem w/ $(N\delta + 1)d$ variables!
- How to handle thousands or millions of (e.g., $N\delta = 10^6$) time series?

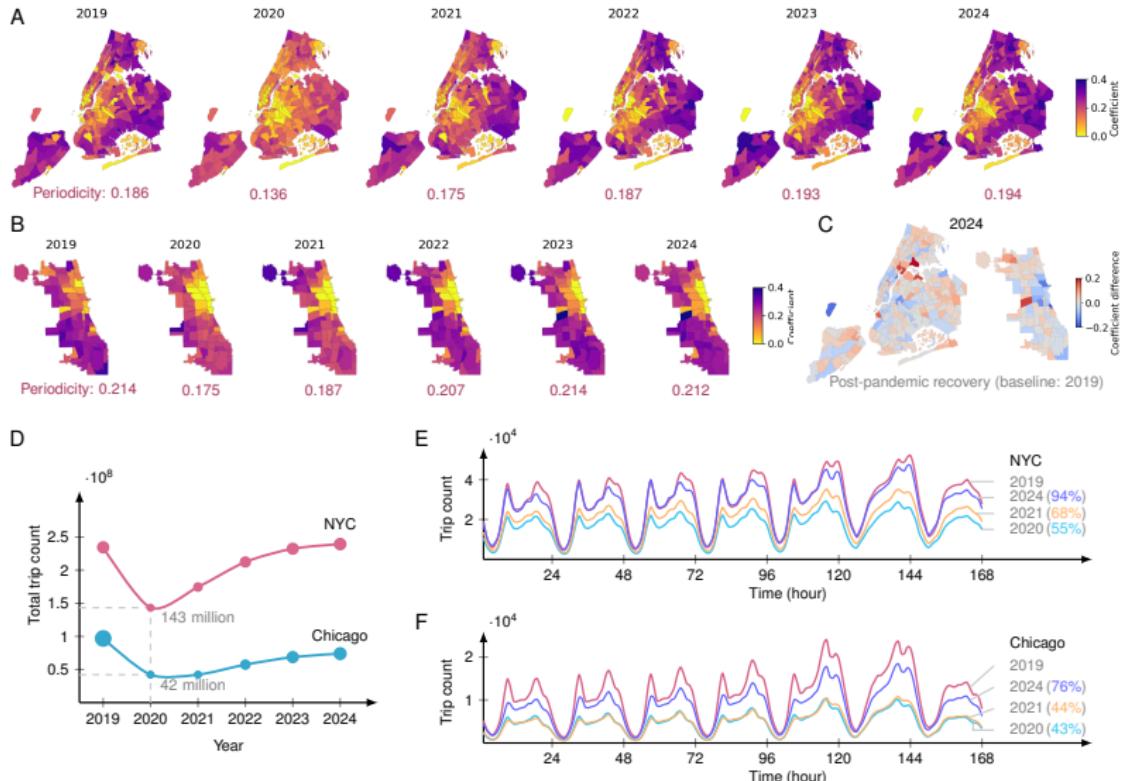
Quantify weekly periodicity by $\{w_{n,\gamma,k}\}$ at index $k = 168$

Envisioning Human Mobility



Hangzhou metro passenger flow in January 2019

Envisioning Human Mobility



Weekly periodicity reveals spatial patterns of ridesharing systems

Future Work

- Quantifying Behavioral Regularity of Wikipedia



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Thanks for your attention!

Any Questions?

Slides: <https://xinychen.github.io/slides/intro.pdf>

About me:

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- /github/ GitHub: <https://github.com/xinychen>
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