POLYTECHNIQUE MONTRÉAL
UNIVERSITÉ D'INGÉNIERIE

IVADO

Matrix and Tensor Models for Spatiotemporal Traffic Data Imputation and Forecasting

Ph.D. Defense

Xinyu Chen

Polytechnique Montreal, Canada

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President
Prof. Francesco Ciari
Polytechnique Montréal

Supervisor
Prof. Nicolas Saunier
Polytechnique Montréal

Co-supervisor
Prof. Lijun Sun
McGill University

Member
Prof. James Goulet
Polytechnique Montréal

External member
Prof. Guillaume Rabusseau
Université de Montréal

Outline

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- **Nonstationary Temporal Matrix Factorization**
- **Low-Rank Autoregressive Tensor Completion**
- **Laplacian Convolutional Representation**
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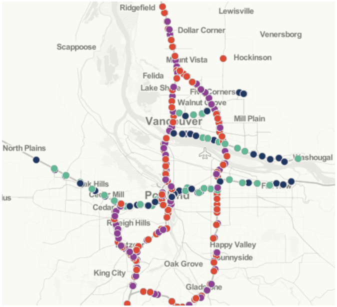
  - Reformulate Laplacian Regularization
  - Traffic Time Series Imputation
- **Hankel Tensor Factorization**
  - Motivation
  - Hankel Structure
- **Experiments**
- **Conclusion**

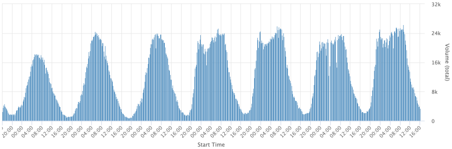
Multivariate Traffic Time Series

Many spatiotemporal traffic time series data are in the form of **matrix**.

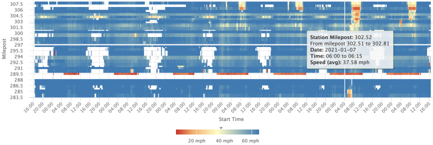
- **Example**: Portland highway traffic data[1].



Highway network & sensor locations



Traffic volume



Traffic speed

- $\boldsymbol{X} \in \mathbb{R}^{N \times T}$ with N spatial locations $\times$ T time steps
- Traffic volume/speed shows strong spatial/temporal dependencies

[1] https://portal.its.pdx.edu/home

Multiple Data Behaviors

Spatiotemporal traffic data are time series, but they involve multiple data behaviors.

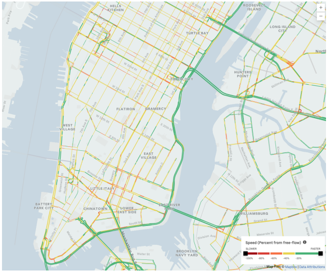
- Incompleteness & sparsity
- High-dimensionality
- Multidimensionality
- Noises & outliers
- Time-varying behavior
- Nonstationarity
- ......

In addition, spatiotemporal correlations are also very important.

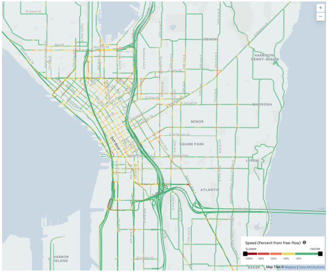
Multiple Data Behaviors

Sparsity & high-dimensionality

- Uber (hourly) movement speed data[2]



NYC movement



Seattle movement

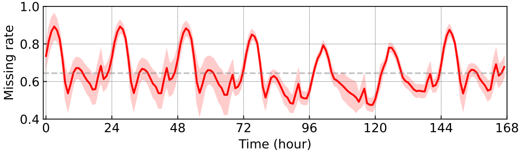
- The average speed on a given road segment for each hour of each day.
- Hourly speeds are computed when road segments have 5+ unique trips.
- **Issue**: insufficient sampling of ridesharing vehicles on the road network.

[2] https://movement.uber.com/

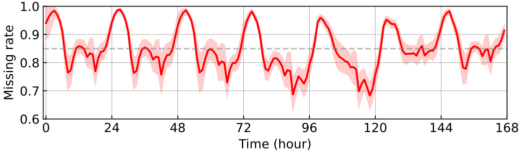
Multiple Data Behaviors

Sparsity & high-dimensionality

- **NYC** movement speed data (2019)
  - **98,210** road segments & 8,760 time steps (hours)
  - Overall missing rate: **64.43%**

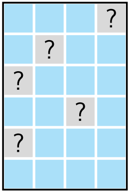


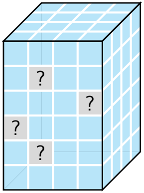
- **Seattle** movement speed data (2019)
  - **63,490** road segments & 8,760 time steps (hours)
  - Overall missing rate: **84.95%**



Problem Formulation

- **Objective A:** Given a multivariate time series data like $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$ or a multidimensional time series data like $\boldsymbol{\mathcal{Y}} \in \mathbb{R}^{M \times N \times T}$ with the observed index set Ω, impute the missing values of the data.



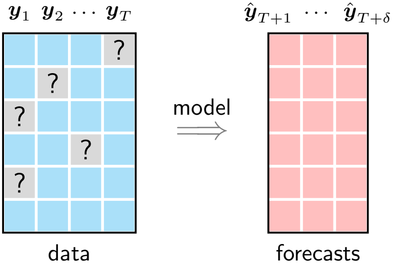


[Q]

- How to reconstruct missing values from observed data?
  - Matrix completion: From $\mathcal{P}_{\Omega}(\boldsymbol{Y})$ (observed) to $\mathcal{P}_{\Omega}^{\perp}(\boldsymbol{Y})$ (unobserved)
  - Tensor completion: From $\mathcal{P}_{\Omega}(\boldsymbol{\mathcal{Y}})$ (observed) to $\mathcal{P}_{\Omega}^{\perp}(\boldsymbol{\mathcal{Y}})$ (unobserved)
- How to make use of spatiotemporal correlations?
- How to make use of traffic time series dynamics?

Problem Formulation

- **Objective B:** Given a partially observed data $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$ consisting of time series $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T \in \mathbb{R}^N$, forecast data points $\boldsymbol{y}_{T+\delta}, \delta \in \mathbb{N}^+$.

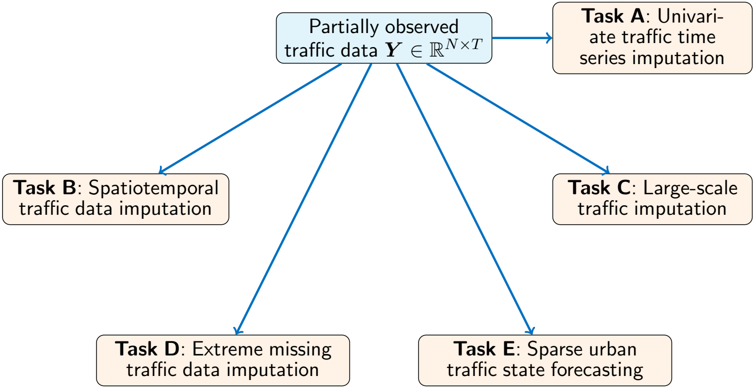


[Q]

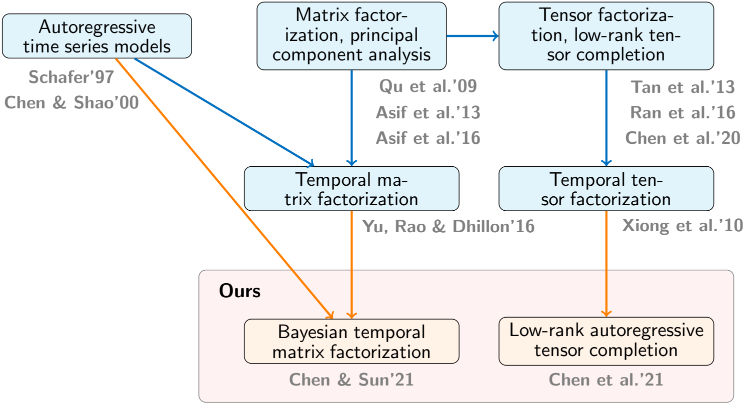
- How to learn from *high-dimensional* and *sparse* data?
- How to model *nonstationarity* in time series?
- How to perform forecasting on these time series?

Whole Picture

We are working on **spatiotemporal traffic data modeling**.



Spatiotemporal Traffic Data Imputation



Spatiotemporal Traffic Forecasting

Large-scale traffic data, e.g., PeMS traffic speed data and Uber movement speed data

High-dimensional forecasting

- Transfer learning — Mallick et al.'21
- Temporal matrix factorization — Chen & Sun'21
- High-dimensional vector autoregressive model — Wang et al.'21

Multidimensional forecasting

- Matrix autoregressive model — Chen, Xiao & Yang'21; Hsu, Huang & Tsay'21
- Autoregressive tensor factorization — Takeuchi, Kashima & Ueda'17

Forecasting on incomplete data

- AR predictor — Anava, Hazan & Zeevi'15
- Temporal matrix factorization — Yu, Rao & Dhillon'16; Chen & Sun'21
- Temporal tensor factorization — Xiong et al.'10; Chen & Sun'21

Tensor Factorization

- Revisit tensor factorization

1927 — Higher-Order SVD

Frank Lauren Hitchcock

1960s — Tucker Decomposition

Ledyard R. Tucker

1970 — CP Decomposition

J. Douglas Carroll
Jih-Jie Chang
Richard A. Harshman

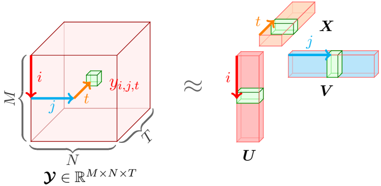
2009 — Tensor Decompositions and Applications

Tamara G. Kolda

2011 — Tensor-Train Decomposition

Ivan Oseledets

- **CP tensor factorization**: Factorize $\boldsymbol{\mathcal{Y}}$ into the combination of three rank-R factor matrices (i.e., low-dimensional latent factors).



$$\begin{cases} y_{i,j,t} \approx \sum_{r=1}^{R} u_{i,r} v_{j,r} x_{t,r} \\ \boldsymbol{\mathcal{Y}} \approx \sum_{r=1}^{R} \boldsymbol{u}_r \otimes \boldsymbol{v}_r \otimes \boldsymbol{x}_r \end{cases}$$

Matrix/Tensor Completion

Matrix nuclear norm ($\|X\|_*$) minimization
Candès & Recht'09

Singular value thresholding
Cai et al.'10

Truncated nuclear norm ($\|X\|_{r,*}, r \in \mathbb{N}^+$) minimization
Zhang et al.'12
Hu et al.'12

Tensor nuclear norm ($\|\boldsymbol{\mathcal{X}}\|_*$) minimization
Liu et al.'13

Generalized nonconvex nonsmooth low-rank minimization
Lu et al.'14

Tensor nuclear norm minimization with linear transform
Lu et al.'19

Low-rank Hankel matrix/tensor ($\mathcal{H}_\tau(\cdot)$) completion
Yokota et al.'18
Cai et al.'21

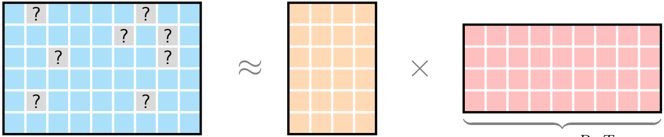
Circulant/convolutional nuclear norm ($\mathcal{C}(\cdot), \mathcal{C}_\tau(\cdot)$) minimization
Liu'22
Liu & Zhang'23

This research

- Integrate temporal modeling techniques (e.g., temporal smoothing and time series autoregression) into low-rank matrix and tensor methods
- Implement spatiotemporal traffic data imputation and forecasting on partially observed data

Matrix Factorization

A simple approach to reconstruct missing values.



$$\underbrace{\boldsymbol{Y}}_{\boldsymbol{Y}\in\mathbb{R}^{N\times T}} \approx \underbrace{\boldsymbol{W}^\top}_{\boldsymbol{W}^\top\in\mathbb{R}^{N\times R}} \times \underbrace{\boldsymbol{X}}_{\boldsymbol{X}\in\mathbb{R}^{R\times T}}$$

MF (Koren et al.'09)

Estimating low-dimensional $\boldsymbol{W},\boldsymbol{X}$:

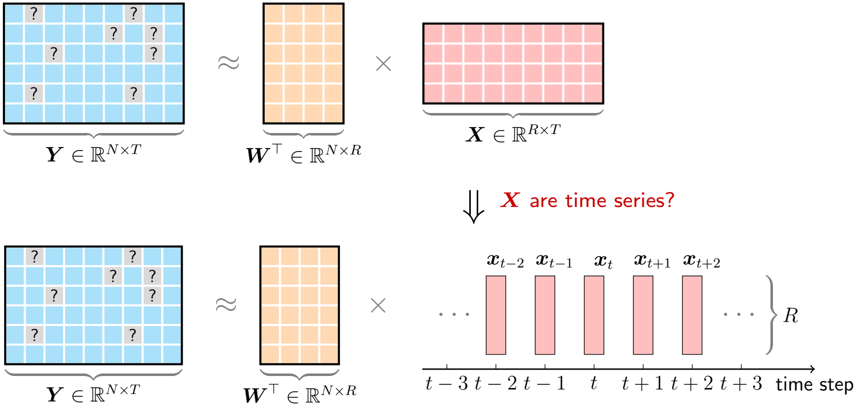
$$\min_{\boldsymbol{W},\boldsymbol{X}}\ \frac{1}{2}\|\mathcal{P}_\Omega(\boldsymbol{Y}-\boldsymbol{W}^\top\boldsymbol{X})\|_F^2$$

on data $\boldsymbol{Y}$ w/ observed index set Ω.

- ✓ Learn from sparse data
- ✗ Temporal correlations
- ✗ Time series forecasting

Temporal Matrix Factorization

Vector autoregression (VAR) on the temporal factor matrix.



Why? $\boldsymbol{X} \in \mathbb{R}^{R \times T}$ is the low-dimensional representation of time series dynamics of $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$.

Temporal Matrix Factorization

MF (Koren et al.'09)

Estimating low-dimensional $\boldsymbol{W}, \boldsymbol{X}$:

$$\min_{\boldsymbol{W},\boldsymbol{X}} \frac{1}{2}\|\mathcal{P}_{\Omega}(\boldsymbol{Y}-\boldsymbol{W}^{\top}\boldsymbol{X})\|_F^2$$

on data $\boldsymbol{Y}$ w/ observed index set Ω.

$+$

dth-order VAR

$$\boldsymbol{x}_t=\sum_{k=1}^{d}\boldsymbol{A}_k\boldsymbol{x}_{t-k}+\boldsymbol{\epsilon}_t$$

w/ coefficients $\{\boldsymbol{A}_k\}$.

$\Downarrow$ Yu et al.'16, Chen & Sun'21

$$\min_{\boldsymbol{W},\boldsymbol{X},\{\boldsymbol{A}_k\}_{k=1}^{d}} \frac{1}{2}\underbrace{\|\mathcal{P}_{\Omega}(\boldsymbol{Y}-\boldsymbol{W}^{\top}\boldsymbol{X})\|_F^2}_{\text{MF on data } \boldsymbol{Y}}+\frac{\gamma}{2}\underbrace{\sum_{t=d+1}^{T}\left\|\boldsymbol{x}_t-\sum_{k=1}^{d}\boldsymbol{A}_k\boldsymbol{x}_{t-k}\right\|_2^2}_{\text{VAR on temporal factors } \boldsymbol{X}}$$

Nonstationary Temporal Matrix Factorization

Nonstationary temporal matrix factorization (NoTMF)

Given any partially observed time series data $\boldsymbol{Y}\in\mathbb{R}^{N\times T}$ with observed index set Ω, then we assume a season-m differencing on the latent temporal factors:

$$\min_{\boldsymbol{W},\boldsymbol{X},\{\boldsymbol{A}_k\}_{k=1}^{d}} \frac{1}{2}\|\mathcal{P}_{\Omega}(\boldsymbol{Y}-\boldsymbol{W}^{\top}\boldsymbol{X})\|_F^2+\frac{\rho}{2}(\|\boldsymbol{W}\|_F^2+\|\boldsymbol{X}\|_F^2) + \frac{\gamma}{2}\sum_{t=d+m+1}^{T}\left\|(\boldsymbol{x}_t-\boldsymbol{x}_{t-m})-\sum_{k=1}^{d}\boldsymbol{A}_k(\boldsymbol{x}_{t-k}-\boldsymbol{x}_{t-m-k})\right\|_2^2$$

- First-order differencing $\boldsymbol{x}_t'=\boldsymbol{x}_t-\boldsymbol{x}_{t-1}$.
- Second-order differencing $\boldsymbol{x}_t''=(\boldsymbol{x}_t-\boldsymbol{x}_{t-1})-(\boldsymbol{x}_{t-1}-\boldsymbol{x}_{t-2})$.
- Twice-differenced series $\boldsymbol{x}_t''=(\boldsymbol{x}_t-\boldsymbol{x}_{t-m})-(\boldsymbol{x}_{t-1}-\boldsymbol{x}_{t-m-1})$.

✓ Stationarizing a time series with differencing can improve the prediction.[3]

[3] Stationarity and differencing: https://otexts.com/fpp2/stationarity.html

Nonstationary Temporal Matrix Factorization

Rewrite VAR in the form of matrix

Temporal operators

For any multivariate time series $\boldsymbol{X}\in\mathbb{R}^{R\times T}$ with $m,d\in\mathbb{N}^{+}$, if we define temporal operators as

$$\begin{aligned}\boldsymbol{\Psi}_k \triangleq & \begin{bmatrix}\boldsymbol{0}_{(T-d-m)\times(d-k)} & -\boldsymbol{I}_{T-d-m} & \boldsymbol{0}_{(T-d-m)\times(k+m)}\end{bmatrix} \\ & +\begin{bmatrix}\boldsymbol{0}_{(T-d-m)\times(d+m-k)} & \boldsymbol{I}_{T-d-m} & \boldsymbol{0}_{(T-d-m)\times k}\end{bmatrix} \\ & \in\mathbb{R}^{(T-d-m)\times T},\, k=0,1,\ldots,d\end{aligned}$$

then

$$\sum_{t=d+m+1}^{T}\|(\boldsymbol{x}_t-\boldsymbol{x}_{t-m})-\sum_{k=1}^{d}\boldsymbol{A}_k(\boldsymbol{x}_{t-k}-\boldsymbol{x}_{t-m-k})\|_2^2$$

$$\equiv\|\boldsymbol{X}\boldsymbol{\Psi}_0^\top-\sum_{k=1}^{d}\boldsymbol{A}_k\boldsymbol{X}\boldsymbol{\Psi}_k^\top\|_F^2\triangleq\|\boldsymbol{X}\boldsymbol{\Psi}_0^\top-\boldsymbol{A}(\boldsymbol{I}_d\otimes\boldsymbol{X})\boldsymbol{\Psi}^\top\|_F^2$$

where $\boldsymbol{A}\triangleq\begin{bmatrix}\boldsymbol{A}_1 & \cdots & \boldsymbol{A}_d\end{bmatrix}$ and $\boldsymbol{\Psi}\triangleq\begin{bmatrix}\boldsymbol{\Psi}_1 & \cdots & \boldsymbol{\Psi}_d\end{bmatrix}$.

Nonstationary Temporal Matrix Factorization

Rewrite NoTMF:

$$\begin{aligned}\min_{\boldsymbol{W},\boldsymbol{X},\boldsymbol{A}}\ & \frac{1}{2}\|\mathcal{P}_{\Omega}(\boldsymbol{Y}-\boldsymbol{W}^\top\boldsymbol{X})\|_F^2+\frac{\rho}{2}(\|\boldsymbol{W}\|_F^2+\|\boldsymbol{X}\|_F^2) \\ & +\frac{\gamma}{2}\|\boldsymbol{X}\boldsymbol{\Psi}_0^\top-\boldsymbol{A}(\boldsymbol{I}_d\otimes\boldsymbol{X})\boldsymbol{\Psi}^\top\|_F^2\end{aligned}$$

Alternating minimization method:

- w.r.t. $\boldsymbol{W}$:

$$\frac{\partial f}{\partial\boldsymbol{W}}=-\boldsymbol{X}\mathcal{P}_{\Omega}^\top(\boldsymbol{Y}-\boldsymbol{W}^\top\boldsymbol{X})+\rho\boldsymbol{W}=\boldsymbol{0}\qquad\text{(Least squares)}$$

- w.r.t. $\boldsymbol{X}$:

$$\frac{\partial f}{\partial\boldsymbol{X}}=-\boldsymbol{W}\mathcal{P}_{\Omega}(\boldsymbol{Y}-\boldsymbol{W}^\top\boldsymbol{X})+\rho\boldsymbol{X}+\gamma\sum_{k=0}^{d}\boldsymbol{A}_k^\top\left(\sum_{h=0}^{d}\boldsymbol{A}_h\boldsymbol{X}\boldsymbol{\Psi}_h^\top\right)\boldsymbol{\Psi}_k=\boldsymbol{0}$$

  This generalized Sylvester equation can be solved by **conjugate gradient**.

- w.r.t. $\boldsymbol{A}$:

$$\boldsymbol{A}=\boldsymbol{X}\boldsymbol{\Psi}_0^\top[(\boldsymbol{I}_d\otimes\boldsymbol{X})\boldsymbol{\Psi}^\top]^\dagger\qquad\text{(Least squares)}$$

Nonstationary Temporal Matrix Factorization

NoTMF forecasting on streaming data?

- NoTMF: Use $\boldsymbol{Y}_t$ to estimate $\boldsymbol{W}, \boldsymbol{X}, \boldsymbol{A}$.

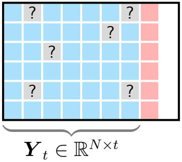
Implementation

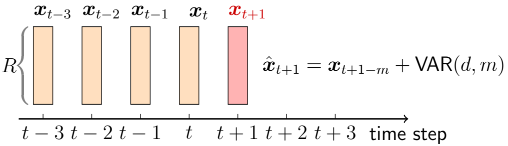
- Estimate $\boldsymbol{W}, \boldsymbol{X}, \boldsymbol{A}$
- Forecast $\hat{\boldsymbol{x}}_{t+1}$ with VAR
- Return $\hat{\boldsymbol{y}}_{t+1} = \boldsymbol{W}^\top \hat{\boldsymbol{x}}_{t+1}$

✓ Sparse input $\boldsymbol{Y}_t$

✓ Low-dimensional temporal factors

✓ Forecast in latent spaces





Nonstationary Temporal Matrix Factorization

NoTMF forecasting on streaming data?

- Online forecasting (Gultekin & Paisley'18): Fix $\boldsymbol{W}$ and use $\boldsymbol{Y}_{t+1}$ to update $\boldsymbol{X}, \boldsymbol{A}$.

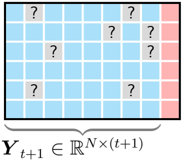
Implementation

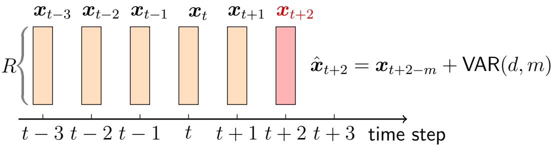
- Estimate $\boldsymbol{X}, \boldsymbol{A}$
- Forecast $\hat{\boldsymbol{x}}_{t+2}$ with VAR
- Return $\hat{\boldsymbol{y}}_{t+2} = \boldsymbol{W}^\top \hat{\boldsymbol{x}}_{t+2}$

✓ Sparse input $\boldsymbol{Y}_{t+1}$

✓ Fixed spatial factors $\boldsymbol{W}$

✓ Forecast in latent spaces





Matrix/Tensor Completion

Cornerstone: Nuclear norm minimization in matrix/tensor completion

LRMC (Candès & Recht'09)

Estimating the matrix $\boldsymbol{X}$:

$$\min_{\boldsymbol{X}} \ \|\boldsymbol{X}\|_*$$
$$\text{s.t. } \mathcal{P}_\Omega(\boldsymbol{X}) = \mathcal{P}_\Omega(\boldsymbol{Y})$$

on data $\boldsymbol{Y}$ w/ observed index set Ω.

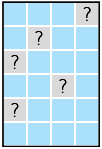
vs.

LRTC (Liu et al.'13)

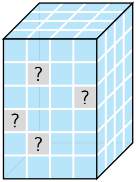
Estimating the tensor $\boldsymbol{\mathcal{X}}$:

$$\min_{\boldsymbol{\mathcal{X}}} \ \|\boldsymbol{\mathcal{X}}\|_*$$
$$\text{s.t. } \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}}) = \mathcal{P}_\Omega(\boldsymbol{\mathcal{Y}})$$

on data $\boldsymbol{\mathcal{Y}}$ w/ observed index set Ω.



$\mathcal{P}_\Omega(\boldsymbol{Y}) \in \mathbb{R}^{N\times T}$

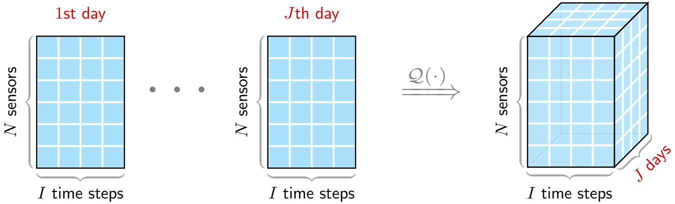


$\mathcal{P}_\Omega(\boldsymbol{\mathcal{Y}}) \in \mathbb{R}^{N\times I\times J}$

- **Limitation**: Only cover global consistency

Low-Rank Autoregressive Tensor Completion

- Introduce traffic tensors with day dimension[4] (Tan et al.'13, Chen et al.'19, ...)



[4] There are $T = IJ$ time steps in total.

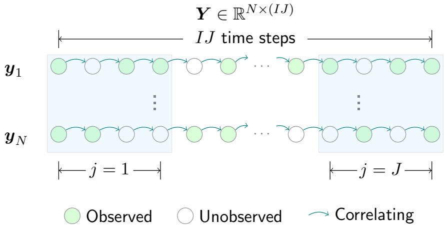
Low-Rank Autoregressive Tensor Completion

- Build temporal correlations with autoregression

On the time series $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

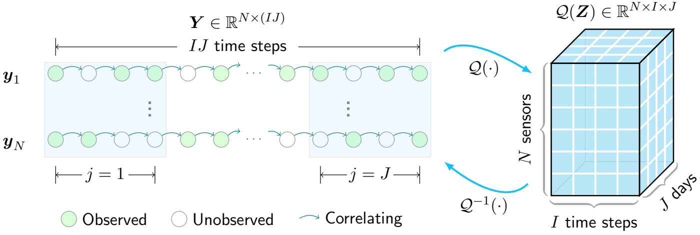
$$\boxed{\|\boldsymbol{Y}\|_{\boldsymbol{A},\mathcal{H}} \triangleq \sum_{n,t}\Big(y_{n,t} - \sum_{k} a_{n,k} y_{n,t-h_k}\Big)^2}$$

with the time lag set $\mathcal{H} = \{h_1, \ldots, h_d\}$ and the coefficient matrix $\boldsymbol{A} \in \mathbb{R}^{N \times d}$.



Low-Rank Autoregressive Tensor Completion

Local consistency w/ autoregression **Global consistency** w/ tensor structure



LATC

Optimization problem:

$$\min_{\boldsymbol{Z},\boldsymbol{A}} \ \|\mathcal{Q}(\boldsymbol{Z})\|_{r,*} + \frac{\gamma}{2}\|\boldsymbol{Z}\|_{\boldsymbol{A},\mathcal{H}}$$

$$\text{s.t. } \mathcal{P}_\Omega(\boldsymbol{Z}) = \mathcal{P}_\Omega(\boldsymbol{Y})$$

on data $\boldsymbol{Y}$ w/ observed index set Ω.

$\Rightarrow$

Two subproblems

$$\begin{cases} \boldsymbol{Z} := \underset{\mathcal{P}_\Omega(\boldsymbol{Z})=\mathcal{P}_\Omega(\boldsymbol{Y})}{\arg\min} \ \|\mathcal{Q}(\boldsymbol{Z})\|_{r,*} + \frac{\gamma}{2}\|\boldsymbol{Z}\|_{\boldsymbol{A},\mathcal{H}} \\ \boldsymbol{A} := \frac{1}{2}\|\boldsymbol{Z}\|_{\boldsymbol{A},\mathcal{H}} \qquad \text{(Least squares)} \end{cases}$$

Low-Rank Autoregressive Tensor Completion

$\boldsymbol{Z}$-subproblem:

$$\boldsymbol{Z} := \underset{\mathcal{P}_{\Omega}(\boldsymbol{Z})=\mathcal{P}_{\Omega}(\boldsymbol{Y})}{\arg\min} \quad \|\mathcal{Q}(\boldsymbol{Z})\|_{r,*} + \frac{\gamma}{2}\|\boldsymbol{Z}\|_{\boldsymbol{A},\mathcal{H}}$$

- Augmented Lagrangian function:[5]

$$\mathcal{L}(\boldsymbol{\mathcal{X}},\boldsymbol{Z},\boldsymbol{\mathcal{W}}) = \|\boldsymbol{\mathcal{X}}\|_{r,*}+\frac{\gamma}{2}\|\boldsymbol{Z}\|_{\boldsymbol{A},\mathcal{H}}+\frac{\lambda}{2}\|\boldsymbol{\mathcal{X}}-\mathcal{Q}(\boldsymbol{Z})\|_F^2+\langle\boldsymbol{\mathcal{W}},\boldsymbol{\mathcal{X}}-\mathcal{Q}(\boldsymbol{Z})\rangle+\pi(\boldsymbol{Z})$$

Implementation

Repeat

- Compute $\boldsymbol{Z}$
- Compute $\boldsymbol{A}$

$\Longrightarrow$

Implementation

Repeat

- Repeat
  - Compute $\boldsymbol{\mathcal{X}}$
  - Compute $\boldsymbol{Z}$
  - Compute $\boldsymbol{\mathcal{W}}$
- Compute $\boldsymbol{A}$

[5] $\boldsymbol{\mathcal{W}} \in \mathbb{R}^{N\times I\times J}$ (Lagrange multiplier); The indicator function:

$$\pi(\boldsymbol{Z}) = \begin{cases} 0, & \text{if } \mathcal{P}_{\Omega}(\boldsymbol{Z}) = \mathcal{P}_{\Omega}(\boldsymbol{Y}), \\ +\infty, & \text{otherwise.} \end{cases}$$

Low-Rank Autoregressive Tensor Completion

- Augmented Lagrangian function:

$$\mathcal{L}(\boldsymbol{\mathcal{X}},\boldsymbol{Z},\boldsymbol{\mathcal{W}})=\|\boldsymbol{\mathcal{X}}\|_{r,*}+\frac{\gamma}{2}\|\boldsymbol{Z}\|_{\boldsymbol{A},\mathcal{H}}+\frac{\lambda}{2}\|\boldsymbol{\mathcal{X}}-\mathcal{Q}(\boldsymbol{Z})\|_F^2+\langle\boldsymbol{\mathcal{W}},\boldsymbol{\mathcal{X}}-\mathcal{Q}(\boldsymbol{Z})\rangle+\pi(\boldsymbol{Z})$$

- The ADMM[6] scheme:

$$\begin{cases}\boldsymbol{\mathcal{X}}:=\arg\min\limits_{\boldsymbol{\mathcal{X}}}\ \mathcal{L}(\boldsymbol{\mathcal{X}},\boldsymbol{Z},\boldsymbol{\mathcal{W}}) & \text{(Truncated nuclear norm minimization)}\\ \boldsymbol{Z}:=\arg\min\limits_{\boldsymbol{Z}}\ \mathcal{L}(\boldsymbol{\mathcal{X}},\boldsymbol{Z},\boldsymbol{\mathcal{W}}) & \text{(Generalized Sylvester equation)}\\ \boldsymbol{\mathcal{W}}:=\boldsymbol{\mathcal{W}}+\lambda(\boldsymbol{\mathcal{X}}-\mathcal{Q}(\boldsymbol{Z})) & \text{(Standard update)}\end{cases}$$

✓ Solution to $\boldsymbol{\mathcal{X}}$: singular value thresholding

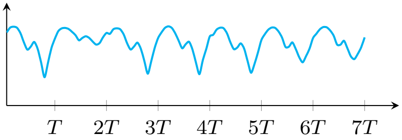
✓ Solution to $\boldsymbol{Z}$: conjugate gradient

[6] Alternating Direction Method of Multipliers.

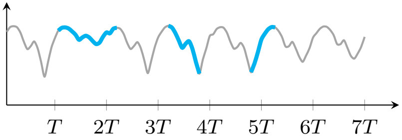
Laplacian Convolutional Representation

Motivation: Time series imputation

- Global trends (e.g., long-term quasi-seasonality & daily/weekly rhythm):



- Local trends (e.g., short-term time series trends):

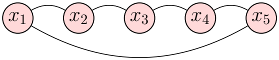


- **[Question]** How to characterize both global and local trends in sparse time series data?

Laplacian Convolutional Representation

Local trend modeling: Reformulate temporal regularization with circular convolution.

- Intuition of (circulant) Laplacian matrix.



Undirected and circulant graph

$\xrightarrow{\text{Modeling}}$

$$\boldsymbol{L}=\begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Define Laplacian kernel:

$$\boldsymbol{\ell} \triangleq (2,-1,0,0,-1)^\top$$

$$\Downarrow$$

$$\boldsymbol{\ell} \triangleq (\underbrace{2\tau}_{\text{degree}}, \underbrace{-1,\cdots,-1}_{\tau}, 0, \cdots, 0, \underbrace{-1,\cdots,-1}_{\tau})^\top \in \mathbb{R}^T$$

for any time series $\boldsymbol{x}=(x_1,\ldots,x_T)^\top \in \mathbb{R}^T$.

- (Laplacian) Temporal regularization:

$$\mathcal{R}_\tau(\boldsymbol{x})=\frac{1}{2}\|\boldsymbol{L}\boldsymbol{x}\|_2^2=\frac{1}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2$$

Laplacian Convolutional Representation

Global trend modeling
Circulant matrix definition
Literature review of circulant/convolution matrix

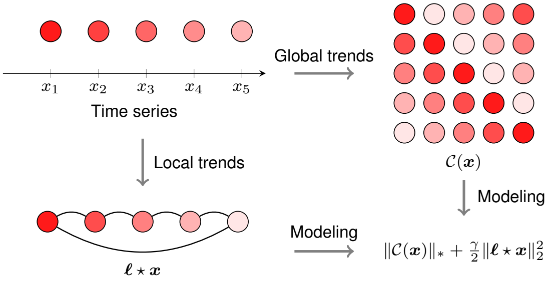
Laplacian Convolutional Representation

Laplacian Convolutional Representation (LCR)

For any partially observed time series $\boldsymbol{y} \in \mathbb{R}^T$ with observed index set Ω, LCR utilizes circulant matrix and Laplacian kernel to characterize **global and local trends** in time series, respectively, i.e.,

$$\min_{\boldsymbol{x}} \; \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2$$
$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon$$

where $\mathcal{C} : \mathbb{R}^T \to \mathbb{R}^{T \times T}$ denotes the circulant operator.



Laplacian Convolutional Representation

- Augmented Lagrangian function:

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) = \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{z}\|_2^2 + \langle\boldsymbol{w},\boldsymbol{x}-\boldsymbol{z}\rangle + \frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{z}-\boldsymbol{y})\|_2^2$$

  where $\boldsymbol{w}\in\mathbb{R}^T$ is the Lagrange multiplier, and $\langle\cdot,\cdot\rangle$ denotes the inner product.
- The ADMM scheme:

$$\begin{cases} \boldsymbol{x} := \arg\min\limits_{\boldsymbol{x}}\ \mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) & \text{(Nuclear norm minimization)} \\ \boldsymbol{z} := \arg\min\limits_{\boldsymbol{z}}\ \mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) & \text{(Closed-form solution)} \\ \quad = \frac{1}{\lambda+\eta}\mathcal{P}_\Omega(\lambda\boldsymbol{x}+\boldsymbol{w}+\eta\boldsymbol{y}) + \frac{1}{\lambda}\mathcal{P}_\Omega^\perp(\lambda\boldsymbol{x}+\boldsymbol{w}) & \\ \boldsymbol{w} := \boldsymbol{w}+\lambda(\boldsymbol{x}-\boldsymbol{z}) & \text{(Standard update)} \end{cases}$$

- Optimize $\boldsymbol{x}$?

$$\|\mathcal{C}(\boldsymbol{x})\|_* = \|\mathcal{F}(\boldsymbol{x})\|_1 \qquad \& \qquad \frac{1}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 = \frac{1}{2T}\|\mathcal{F}(\boldsymbol{\ell})\circ\mathcal{F}(\boldsymbol{x})\|_2^2$$

  Nuclear norm minimization $\Rightarrow$ ℓ_1**-norm minimization with FFT** (in $\mathcal{O}(T\log T)$ time).

Laplacian Convolutional Representation

- Optimize $\boldsymbol{x}$ via FFT (in $\mathcal{O}(T\log T)$ time):

$$\begin{aligned} \boldsymbol{x} &:= \arg\min_{\boldsymbol{x}}\ \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{z}+\boldsymbol{w}/\lambda\|_2^2 \\ \Longrightarrow \hat{\boldsymbol{x}} &:= \arg\min_{\hat{\boldsymbol{x}}}\ \|\hat{\boldsymbol{x}}\|_1 + \frac{\gamma}{2T}\|\hat{\boldsymbol{\ell}}\circ\hat{\boldsymbol{x}}\|_2^2 + \frac{\lambda}{2T}\|\hat{\boldsymbol{x}}-\hat{\boldsymbol{z}}+\hat{\boldsymbol{w}}/\lambda\|_2^2 \end{aligned}$$

where we introduce $\{\hat{\boldsymbol{\ell}},\hat{\boldsymbol{x}},\hat{\boldsymbol{z}},\hat{\boldsymbol{w}}\}\triangleq\mathcal{F}\{\boldsymbol{\ell},\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}\}$ (i.e., FFT).

ℓ_1-norm Minimization in Complex Space (Liu & Zhang'23)

For any optimization problem in the form of ℓ_1-norm minimization in complex space:

$$\min_{\hat{\boldsymbol{x}}}\ \|\hat{\boldsymbol{x}}\|_1 + \frac{\omega}{2}\|\hat{\boldsymbol{x}}-\hat{\boldsymbol{h}}\|_2^2$$

with complex-valued vectors $\hat{\boldsymbol{x}},\hat{\boldsymbol{h}}\in\mathbb{C}^T$ and weight parameter ω, element-wise, the solution is given by

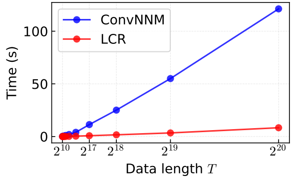
$$\hat{x}_t := \frac{\hat{h}_t}{|\hat{h}_t|}\cdot\max\{0,|\hat{h}_t|-1/\omega\}, t=1,\dots,T.$$

Laplacian Convolutional Representation

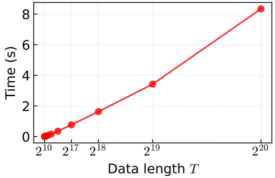
Empirical time complexity

On the synthetic data $\boldsymbol{y} \in \mathbb{R}^T$ with $T \in \{2^{10}, 2^{11}, \ldots, 2^{20}\}$

- Ours: **LCR**
  - An FFT implementation in $\mathcal{O}(T \log T)$
  - The logarithmic factor $\log T$ makes the FFT highly efficient
- Baseline: **ConvNNM**[7] (Liu'22, Liu & Zhang'23)
  - Convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{y}) \in \mathbb{R}^{T \times \tilde{\tau}}$ with kernel size $\tilde{\tau} \in \mathbb{N}^+$
  - Singular value thresholding in $\mathcal{O}(\tilde{\tau}^2 T)$



ConvNNM vs. LCR



LCR

[7] Convolution nuclear norm minimization.

Two-Dimensional LCR (LCR-2D)

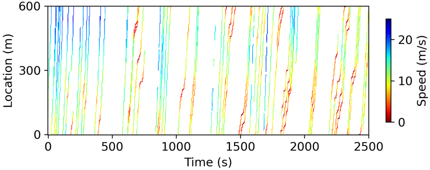
For any partially observed time series $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$ with observed index set Ω, LCR can be formulated as follows,

$$\begin{aligned} \min_{\boldsymbol{X}} \; & \|\mathcal{C}(\boldsymbol{X})\|_* + \frac{\gamma}{2}\|(\boldsymbol{\ell}_s\boldsymbol{\ell}^\top) \star \boldsymbol{X}\|_F^2 \\ \text{s.t. } & \|\mathcal{P}_\Omega(\boldsymbol{X} - \boldsymbol{Y})\|_F \leq \epsilon \end{aligned}$$

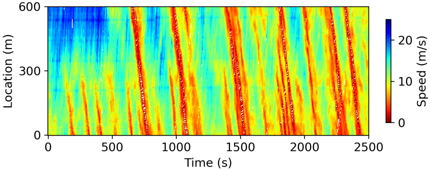
where $\mathcal{C} : \mathbb{R}^{N \times T} \to \mathbb{R}^{N \times N \times T \times T}$ denotes the circulant operator.

Motivation: Spatiotemporal data reconstruction

- Speed field reconstruction problem in vehicular traffic flow.



200-by-500 matrix (NGSIM) $\Downarrow$ Reconstruct speed field from 5% sparse trajectories?



- How to learn from sparse spatiotemporal data?
- How to characterize spatial/temporal dependencies?

Hankel Tensor Factorization

- Hankel matrix
  - Given $\boldsymbol{x}=(1,2,3,4,5)^\top$ and window length $\tau=2$, we have

$$\mathcal{H}_\tau(\boldsymbol{x})=\begin{bmatrix}1 & 2\\ 2 & 3\\ 3 & 4\\ 4 & 5\end{bmatrix}\in\mathbb{R}^{4\times 2}$$

Hankel matrix (Source: Twitter)

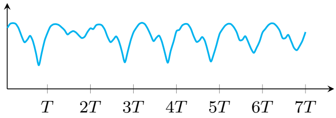
Hankel Tensor Factorization

- Hankel matrix
  - On time series $\boldsymbol{y}=(y_1,y_2,\ldots,y_5)^\top$ with $\tau=2$:

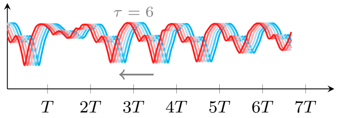
$$\mathcal{H}_\tau(\boldsymbol{y})=\begin{bmatrix} y_1 & y_2 \\ y_2 & y_3 \\ y_3 & y_4 \\ y_4 & y_5 \end{bmatrix}\approx\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}\otimes\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Longrightarrow\quad \hat{\boldsymbol{y}}=\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \\ \hat{y}_5 \end{bmatrix}=\mathcal{H}_\tau^{-1}\left(\begin{bmatrix} v_1x_1 & v_1x_2 \\ v_2x_1 & v_2x_2 \\ v_3x_1 & v_3x_2 \\ v_4x_1 & v_4x_2 \end{bmatrix}\right)=\begin{bmatrix} v_1x_1 \\ (v_1x_2+v_2x_1)/2 \\ (v_2x_2+v_3x_1)/2 \\ (v_3x_2+v_4x_1)/2 \\ v_4x_2 \end{bmatrix}$$

  - Automatic temporal modeling.



Traffic time series



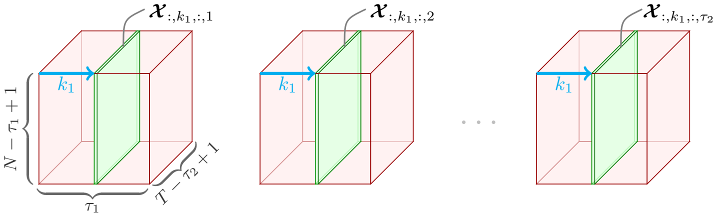
Hankel matrix

Hankel Tensor Factorization

- Hankel tensor: Given any matrix $\boldsymbol{X} \in \mathbb{R}^{N \times T}$, we have

$$\boldsymbol{\mathcal{X}} \triangleq \mathcal{H}_{\tau_1,\tau_2}(\boldsymbol{X})$$

  - Window lengths: $\tau_1, \tau_2 \in \mathbb{N}^{+}$;
  - Tensor size: $(N-\tau_1+1) \times \tau_1 \times (T-\tau_2+1) \times \tau_2$;



(Figure) 4th order Hankel tensor: A sequence of third-order tensors.

  - Slice: $\boldsymbol{\mathcal{X}}_{:,k_1,:,k_2}, \forall k_1, k_2$;
  - Slice size: $(N-\tau_1+1) \times (T-\tau_2+1)$.

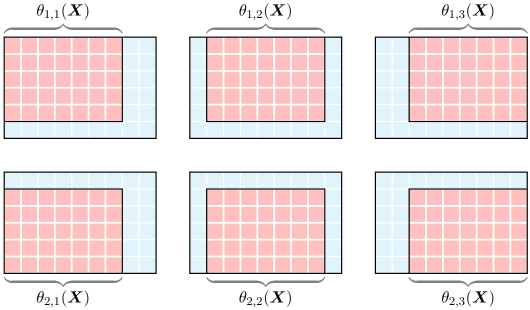
Hankel Tensor Factorization

Hankel indexing:

- Sampling function for the Hankelization:

$$\theta_{k_1,k_2}(\boldsymbol{X}) \triangleq [\mathcal{H}_{\tau_1,\tau_2}(\boldsymbol{X})]_{:,k_1,:,k_2},$$

  referring to the tensor slice with $k_1 \in \{1,\ldots,\tau_1\}$, $k_2 \in \{1,\ldots,\tau_2\}$.
- **[Importance]** Developing memory-efficient algorithms.



- Tensor slices $\theta_{k_1,k_2}(\boldsymbol{X})$ vs. data matrix $\boldsymbol{X}$

Hankel Tensor Factorization

Ours:

- Convolutional tensor decomposition (circular convolution $\star_{\text{row}}$):

$$\theta_{k_1,k_2}(\boldsymbol{Y}) \approx (\boldsymbol{Q} \star_{\text{row}} \boldsymbol{s}_{k_1}^\top)(\boldsymbol{U} \star_{\text{row}} \boldsymbol{v}_{k_2}^\top)^\top$$

Baselines:

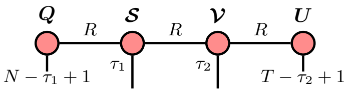
- CP tensor decomposition (Khatri-Rao product $\odot$):

$$\theta_{k_1,k_2}(\boldsymbol{Y}) \approx (\boldsymbol{Q} \odot \boldsymbol{s}_{k_1}^\top)(\boldsymbol{U} \odot \boldsymbol{v}_{k_2}^\top)^\top$$

- Tensor-train decomposition:

$$\theta_{k_1,k_2}(\boldsymbol{Y}) \approx (\boldsymbol{Q}\boldsymbol{S}_{k_1})(\boldsymbol{U}\boldsymbol{V}_{k_2})^\top$$

  - $\{\boldsymbol{S}_{k_1}, \boldsymbol{V}_{k_2}\}$ are **circulant matrices** $\Rightarrow$ convolutional decomposition
  - $\{\boldsymbol{S}_{k_1}, \boldsymbol{V}_{k_2}\}$ are **diagonal matrices** $\Rightarrow$ CP decomposition



Hankel Tensor Factorization

HTF (convolutional decomposition)

- Optimization problem:

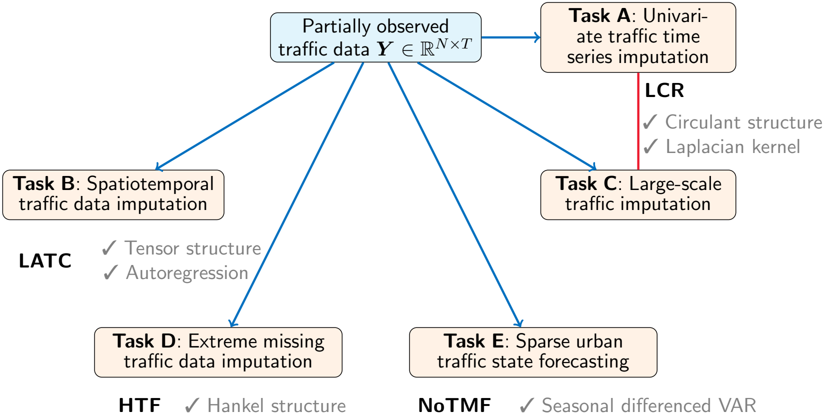
$$\min_{\boldsymbol{Q},\boldsymbol{S},\boldsymbol{U},\boldsymbol{V}} \ \frac{1}{2}\sum_{k_1,k_2}\left\|\mathcal{P}_{\Omega_{k_1,k_2}}\left(\theta_{k_1,k_2}(\boldsymbol{Y})-(\boldsymbol{Q}\star_{\text{row}}\boldsymbol{s}_{k_1})(\boldsymbol{U}\star_{\text{row}}\boldsymbol{v}_{k_2})^\top\right)\right\|_F^2 + \frac{\rho}{2}\left(\|\boldsymbol{Q}\|_F^2+\|\boldsymbol{S}\|_F^2+\|\boldsymbol{U}\|_F^2+\|\boldsymbol{V}\|_F^2\right)$$

- Alternating minimization:

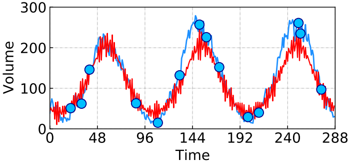
$$\begin{cases} \boldsymbol{Q}:=\{\boldsymbol{Q}\mid \frac{\partial f}{\partial \boldsymbol{Q}}=\boldsymbol{0}\} & \text{(conjugate gradient)} \\ \boldsymbol{s}_{k_1}:=\{\boldsymbol{s}_{k_1}\mid \frac{\partial f}{\partial \boldsymbol{s}_{k_1}}=\boldsymbol{0}\},\ \forall k_1 & \text{(conjugate gradient)} \\ \boldsymbol{U}:=\{\boldsymbol{U}\mid \frac{\partial f}{\partial \boldsymbol{U}}=\boldsymbol{0}\} & \text{(conjugate gradient)} \\ \boldsymbol{v}_{k_2}:=\{\boldsymbol{v}_{k_2}\mid \frac{\partial f}{\partial \boldsymbol{v}_{k_2}}=\boldsymbol{0}\},\ \forall k_2 & \text{(conjugate gradient)} \end{cases}$$

Whole Picture

We are working on **spatiotemporal traffic data modeling**.



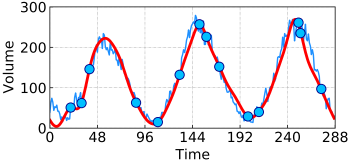
Univariate Traffic Time Series Imputation



CircNNM:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_*$$
$$\text{s. t.} \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

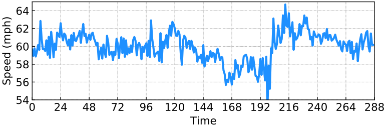
$\Downarrow$ Plus temporal regularization (TR)



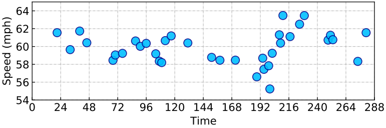
LCR:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2$$
$$\text{s. t.} \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

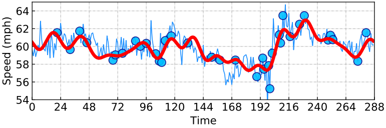
Univariate Traffic Time Series Imputation



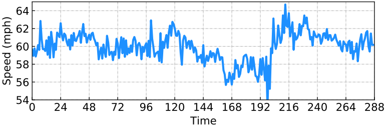
Mask **90%** observations



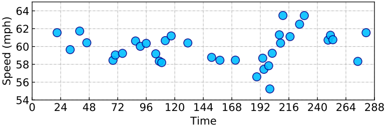
Reconstruct time series



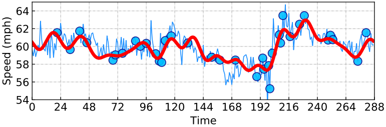
Univariate Traffic Time Series Imputation

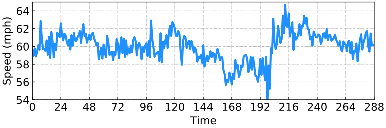


Mask **90%** observations

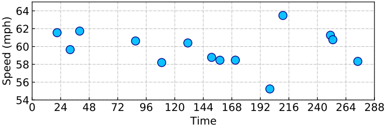


Reconstruct time series

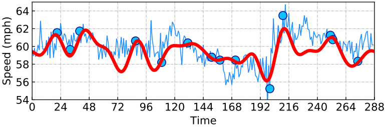




Mask **95%** observations



Reconstruct time series



Spatiotemporal Traffic Data Imputation

LATC vs. baseline models (in MAPE/RMSE)

- On the Seattle freeway traffic speed dataset ($\boldsymbol{Y} \in \mathbb{R}^{323 \times 8064}$)

Missing rate	LATC	LAMC	LRTC-TNN	BTMF	SPC
30%, Random Missing	**4.90**/**3.16**	5.98/3.73	4.99/3.20	5.91/3.72	5.92/3.62
70%, Random Missing	**5.96**/**3.71**	8.02/4.70	6.10/3.77	6.47/3.98	7.38/4.30
90%, Random Missing	**7.46**/**4.50**	10.56/5.91	8.08/4.80	8.17/4.81	9.75/5.31
30%, Nonrandom Missing	7.10/4.33	6.99/4.25	**6.85**/**4.21**	9.26/5.36	8.87/4.99
70%, Nonrandom Missing	9.40/5.40	9.75/5.60	**9.23**/**5.35**	10.47/6.15	11.32/5.92
30%, Block-out Missing	**9.43**/**5.36**	27.05/13.66	9.52/5.41	14.33/13.60	11.30/5.84

- On the Portland highway traffic volume dataset ($\boldsymbol{Y} \in \mathbb{R}^{1156 \times 2976}$)

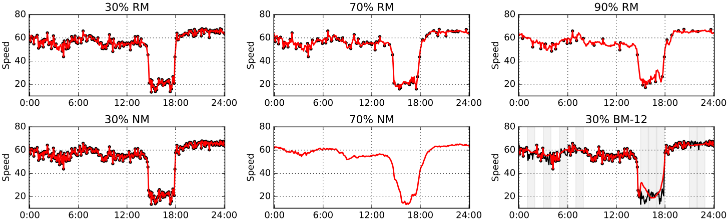
Missing rate	LATC	LAMC	LRTC-TNN	BTMF	SPC
30%, Random Missing	**16.95**/**15.99**	17.93/16.03	17.27/16.08	18.22/19.14	21.29/56.73
70%, Random Missing	**19.59**/**18.70**	21.26/19.37	19.99/18.73	19.96/22.21	24.35/43.32
90%, Random Missing	23.15/22.83	25.64/23.75	**22.90**/**22.68**	23.90/25.71	28.45/39.65
30%, Nonrandom Missing	**19.48**/19.14	19.93/19.69	19.59/**18.91**	19.55/20.38	26.96/60.33
70%, Nonrandom Missing	27.67/45.03	25.75/28.25	30.26/60.85	**23.86**/**26.74**	33.42/47.34
30%, Block-out Missing	**24.01**/**23.50**	29.21/27.60	31.74/74.42	27.85/25.68	31.01/60.33

- LATC vs. LAMC: The significance of tensor representation
- LATC vs. LRTC-TNN: The significance of temporal autoregression

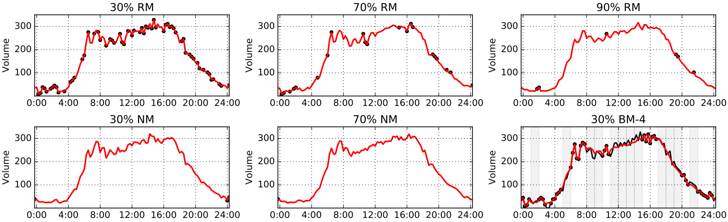
Spatiotemporal Traffic Data Imputation

LATC imputation

- Seattle freeway traffic speed data



- Portland highway traffic volume data



Large-Scale Traffic Data Imputation

LCR vs. baseline models (in MAPE/RMSE)

- PeMS-4W: California freeway traffic speed dataset ($\boldsymbol{Y} \in \mathbb{R}^{11160 \times 8064}$)

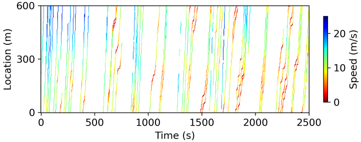
Model	Missing rate			
	30%	50%	70%	90%
LCR-2D	1.50/**1.49**	1.76/**1.69**	**2.07**/**2.06**	**3.19**/**3.05**
LCR$_N$	**1.48**/1.50	**1.73**/1.73	**2.07**/2.12	3.24/3.22
LCR	1.50/**1.49**	1.76/**1.69**	2.08/2.07	3.21/3.06
CTNNM	2.26/1.84	2.67/2.14	3.40/2.66	5.22/3.90
CircNNM	2.26/1.84	2.69/2.15	3.43/2.67	5.34/3.96
LRMC	2.04/1.80	2.43/2.12	3.08/2.66	6.05/4.43
HaLRTC	1.98/1.73	2.22/1.98	2.84/2.49	4.39/3.66
LRTC-TNN	1.68/1.55	1.93/1.77	2.33/2.14	3.40/3.10
NoTMF	2.95/2.65	3.05/2.73	3.33/2.97	5.22/4.71

Results

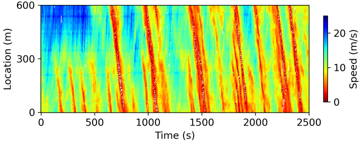
- LCR-2D $>$ CTNNM: The importance of temporal regularization
- CTNNM $\geq$ CircNNM: Ciculant tensor is superior to circulant matrix
- LCR $>$ LRMC/LRTC: The importance of global/local modeling

$$\mathcal{O}(NT\log(NT)) \quad \text{(FFT)} \quad \text{vs.} \quad \mathcal{O}(\min\{N^2T, NT^2\}) \quad \text{(SVD)}$$

Extreme Missing Traffic Data Imputation

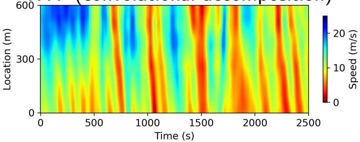


Sparse speed field



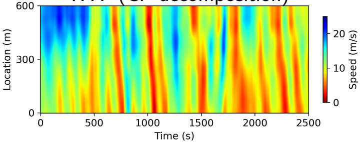
Ground truth speed field

HTF (convolutional decomposition)



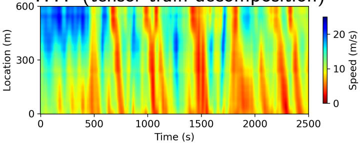
MAPE = **51.92%**

HTF (CP decomposition)



MAPE = 53.93%

HTF (tensor-train decomposition)



MAPE = 56.48%

Sparse Urban Traffic State Forecasting

NoTMF

Conclusion

- Data (large-scale, high-dimensional, city-wide, sparse)
- Modeling (meaningfulness and importance of temporal correlations)

Conclusion

Low-Rank Autoregressive Tensor Completion (LATC):

- (Highlight) Global & local consistency
  - ✓ Tensor structure $\|\mathcal{Q}(\boldsymbol{\mathcal{Z}})\|_{r,*}$ ✓ Autoregression $\|\boldsymbol{Z}\|_{\boldsymbol{A},\mathcal{H}}$

Laplacian Convolutional Representation (LCR):

- (Solution) Time series trend modeling in the low-rank framework?
  - Global time series trend modeling: $\|\mathcal{C}(\boldsymbol{x})\|_*$
  - Local time series trend modeling: $\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2$
- (Highlight) A unified framework with the FFT implementation.

Hankel Tensor Factorization:

- (Highlight) Memory-efficient Hankel indexing & convolutional parameterization.

Collaborators

Dr. HanQin Cai

Xiaoxu Chen

Dr. Zhanhong Cheng

Chengyuan Zhang

Dr. Xi-Le Zhao

References

A short list:

- **[Candès & Recht'09]** Exact matrix completion via convex optimization. Foundations of Computational Mathematics, 9 (6), 717-772.
- **[Cai et al.'10]** A singular value thresholding algorithm for matrix completion
- **[Zhang et al.'12]** Matrix completion by truncated nuclear norm regularization
- **[Hu et al.'12]** Fast and accurate matrix completion via truncated nuclear norm regularization
- **[Lu et al.'14]** Generalized Nonconvex Nonsmooth Low-Rank Minimization
- **[Lu et al.'19]** Tensor Robust Principal Component Analysis with A New Tensor Nuclear Norm
- **[Yokota et al.'18]** Missing Slice Recovery for Tensors Using a Low-rank Model in Embedded Space
- **[Cai et al.'21]** Accelerated Structured Alternating Projections for Robust Spectrally Sparse Signal Recovery
- **[Liu'22]**
- **[Liu & Zhang'23]**

Thanks for your attention!

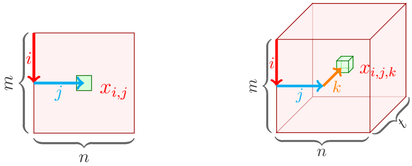
Any Questions?

About me:

- Homepage: https://xinychen.github.io
- GitHub: https://github.com/xinychen
- How to reach me: chenxy346@gmail.com

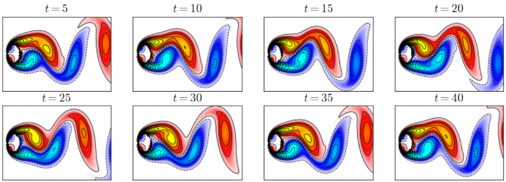
What Is Tensors?

- What is tensor? $\boldsymbol{X} \in \mathbb{R}^{m\times n}$ vs. $\boldsymbol{\mathcal{X}} \in \mathbb{R}^{m\times n\times t}$



- Tensors are everywhere!

Color image with RGB channels



Dynamical system (fluid flow)

Convolution & FFT

Flipping Operation in LCR

Results on speed fields

Training, Validation & Testing

Tuning Hyperparameters