POLYTECHNIQUE MONTRÉAL
UNIVERSITÉ D'INGÉNIERIE

IVADO

Low-Rank Matrix and Tensor Methods for Spatiotemporal Traffic Data Modeling

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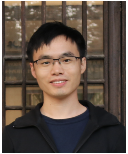
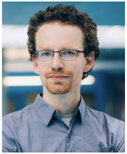


Ph.D. Candidate
Xinyu Chen



Supervisor
Prof. Nicolas Saunier

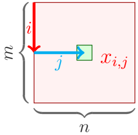
Co-supervisor
Prof. Lijun Sun

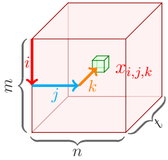
Outline

- **Spatiotemporal Traffic Data**
  Preliminaries: What are matrix & tensor?
  Traffic data & representation & problems
- **Spatiotemporal Traffic Data Imputation**
  Speed field reconstruction
  Hankel matrix & tensor
  Revisit tensor factorization
  Hankel tensor factorization
- **Sparse Traffic Flow Forecasting**
  Uber movement speed data
  Temporal matrix factorization
- **Dynamic Pattern Discovery**
  NYC taxi trip changes over COVID-19
  Time-varying low-rank autoregression
  London bike trip pattern discovery
- **Conclusion**

What are Matrix & Tensor?

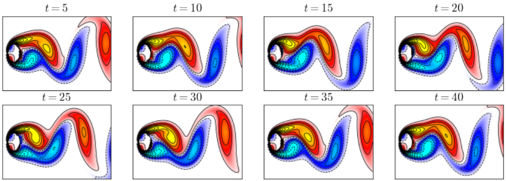
- What is tensor? $\boldsymbol{X} \in \mathbb{R}^{m\times n}$ vs. $\boldsymbol{\mathcal{X}} \in \mathbb{R}^{m\times n\times t}$





- Tensors are everywhere!

Color image with RGB channels

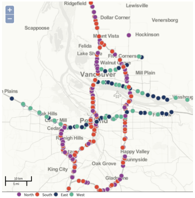


Dynamical system (fluid flow)

Spatiotemporal Traffic Data

- Spatiotemporal traffic data are indeed matrices or tensors.

Network

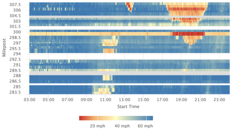


Highway network

Urban road network

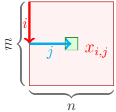
$\Longrightarrow$

Data & Representation

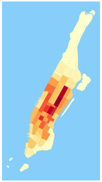


Speed field

$\Longrightarrow$

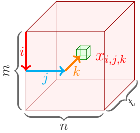


Matrix



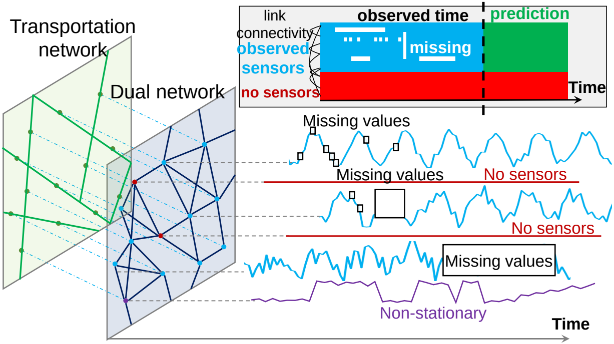
OD matrices over time

$\Longrightarrow$



Tensor

Spatiotemporal Traffic Data



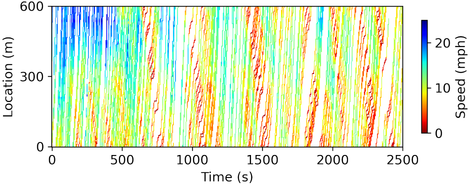
Spatiotemporal Traffic Data Imputation

1. X. Chen, Z. Cheng, N. Saunier, L. Sun (2022). Laplacian convolutional representation for traffic time series imputation. arXiv preprint arXiv:2212.01529.

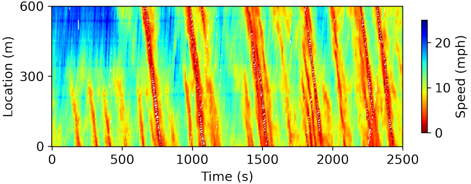
   (Under 1st review at IEEE Transactions on Signal Processing)

2. X. Chen Z. Cheng, L. Sun, N. Saunier (2023). Memory-efficient Hankel tensor factorization for extreme missing traffic data imputation. (coming soon)

Spatiotemporal Traffic Data Imputation



200-by-500 matrix (NGSIM) $\Downarrow$ Reconstruct speed field from 20% sparse trajectories?



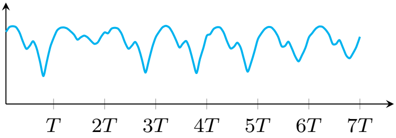
Motivation:

- How to learn from sparse spatiotemporal data?
- How to characterize spatial/temporal local dependencies?

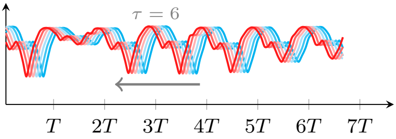
- Hankel matrix
  - Given $\boldsymbol{y}=(1,2,3,4,5)^\top$ and window length $\tau=2$, we have

$$\mathcal{H}_\tau(\boldsymbol{y})=\begin{bmatrix}1 & 2\\ 2 & 3\\ 3 & 4\\ 4 & 5\end{bmatrix}\in\mathbb{R}^{4\times 2}$$

  - Hankel matrix $\mathcal{H}_\tau(\boldsymbol{y})$ on time series $\boldsymbol{y}$:



$\Downarrow$ Construct Hankel matrix



- Hankel matrix
  - Given $\boldsymbol{y}=(1,2,3,4,5)^\top$ and window length $\tau=2$, we have

$$\mathcal{H}_\tau(\boldsymbol{y})=\begin{bmatrix}1 & 2\\ 2 & 3\\ 3 & 4\\ 4 & 5\end{bmatrix}\in\mathbb{R}^{4\times 2}$$

  - On time series $\boldsymbol{y}=(y_1,y_2,\ldots,y_5)^\top$ with $\tau=2$:

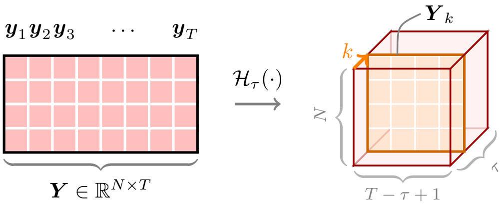
$$\mathcal{H}_\tau(\boldsymbol{y})=\begin{bmatrix}y_1 & y_2\\ y_2 & y_3\\ y_3 & y_4\\ y_4 & y_5\end{bmatrix}\approx\begin{bmatrix}v_1\\ v_2\\ v_3\\ v_4\end{bmatrix}\otimes\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$\Longrightarrow\quad\hat{\boldsymbol{y}}=\begin{bmatrix}\hat{y}_1\\ \hat{y}_2\\ \hat{y}_3\\ \hat{y}_4\\ \hat{y}_5\end{bmatrix}=\mathcal{H}_\tau^{-1}\left(\begin{bmatrix}v_1x_1 & v_1x_2\\ v_2x_1 & v_2x_2\\ v_3x_1 & v_3x_2\\ v_4x_1 & v_4x_2\end{bmatrix}\right)=\begin{bmatrix}v_1x_1\\ (v_1x_2+v_2x_1)/2\\ (v_2x_2+v_3x_1)/2\\ (v_3x_2+v_4x_1)/2\\ v_4x_2\end{bmatrix}$$

  - Automatic **temporal** modeling

Spatiotemporal Traffic Data Imputation

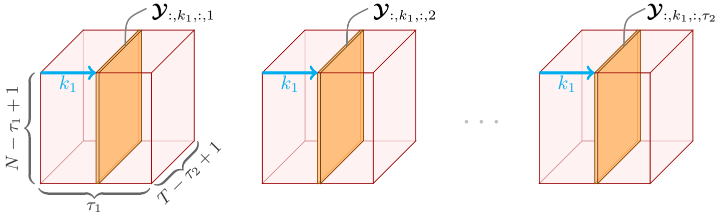
- Hankelization from $\boldsymbol{Y}\in\mathbb{R}^{N\times T}$ to Hankel tensor $\mathcal{H}_\tau(\boldsymbol{Y})$.
  - Tensor size: $N\times(T-\tau+1)\times\tau$;
  - Slices: $\boldsymbol{Y}_k=\begin{bmatrix} | & | & & | \\ \boldsymbol{y}_k & \boldsymbol{y}_{k+1} & \cdots & \boldsymbol{y}_{T-\tau+k} \\ | & | & & | \end{bmatrix}, k=1,2,\ldots,\tau;$
  - Slice size: $N\times(T-\tau+1)$.



  - Automatic **temporal** modeling

Spatiotemporal Traffic Data Imputation

- Hankelization from $\boldsymbol{Y}\in\mathbb{R}^{N\times T}$ to $\boldsymbol{\mathcal{Y}}\triangleq\mathcal{H}_{\tau_1,\tau_2}(\boldsymbol{Y})$ (Hankel tensor).
  - Tensor size: $(N-\tau_1+1)\times\tau_1\times(T-\tau_2+1)\times\tau_2$;
  - Slice: $\boldsymbol{\mathcal{Y}}_{:,k_1,:,k_2},\forall k_1,k_2$;
  - Slice size: $(N-\tau_1+1)\times(T-\tau_2+1)$.



  - Automatic **spatial** and **temporal** modeling

- Revisit tensor factorization (TF)

1927
Higher-Order SVD

Frank Lauren Hitchcock

1960s
Tucker Decomposition
Ledyard R. Tucker

1970
CP Decomposition
J. Douglas Carroll
Jih-Jie Chang
Richard A. Harshman

2009
Tensor Decompositions and Applications

Tamara G. Kolda

2011
Tensor-Train Decomposition

Ivan Oseledets

- Revisit tensor factorization (TF)

1927 — Higher-Order SVD

Frank Lauren Hitchcock

1960s — Tucker Decomposition

Ledyard R. Tucker

1970 — CP Decomposition

J. Douglas Carroll
Jih-Jie Chang
Richard A. Harshman

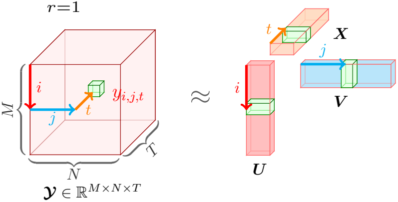
2009 — Tensor Decompositions and Applications

Tamara G. Kolda

2011 — Tensor-Train Decomposition

Ivan Oseledets

- **CP decomposition**: Factorize $\mathcal{Y}$ into the combination of rank-R factor matrices, i.e., $\mathcal{Y} \approx \sum_{r=1}^{R} \boldsymbol{u}_r \otimes \boldsymbol{v}_r \otimes \boldsymbol{x}_r$.



Spatiotemporal Traffic Data Imputation

- Hankel tensor factorization:

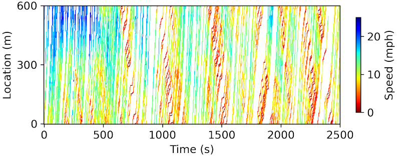
$$\min_{\boldsymbol{Q},\boldsymbol{S},\boldsymbol{U},\boldsymbol{V}}\ \frac{1}{2}\Big\|\mathcal{P}_{\dot{\Omega}}\Big(\mathcal{H}_{\tau_1,\tau_2}(\boldsymbol{Y})-\sum_{r=1}^{R}\boldsymbol{q}_r\otimes\boldsymbol{s}_r\otimes\boldsymbol{u}_r\otimes\boldsymbol{v}_r\Big)\Big\|_F^2$$

- Advantage/disadvantage:
  - ✓ Automatic spatial and temporal modeling
  - ✗ High memory consumption
- Space complexity:

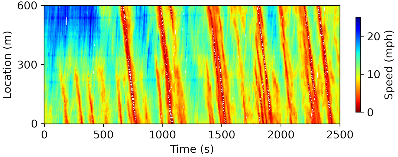
$$\mathcal{O}(\tau_1\tau_2(N-\tau_1+1)(T-\tau_2+1))$$

- **(Contribution)** Reduce the space complexity to $\mathcal{O}(NT)$.

Which Model Is Better?

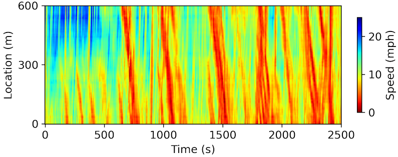


Sparse speed field

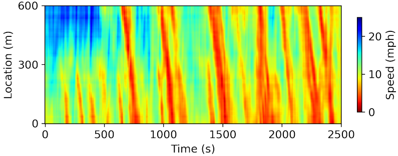


Ground truth speed field

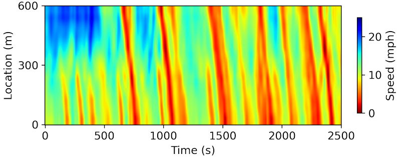
Matrix factorization



Smoothing matrix factorization

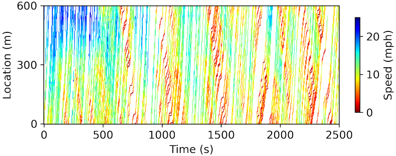


Hankel tensor factorization

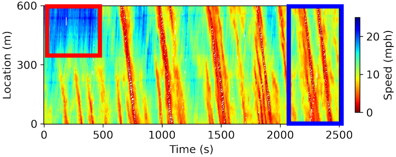


Spatiotemporal Traffic Data Imputation

Which Model Is Better?

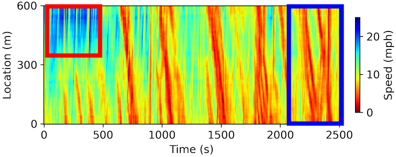


Sparse speed field

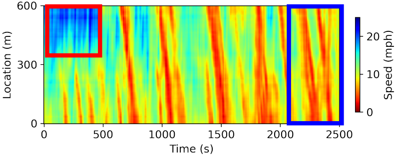


Ground truth speed field

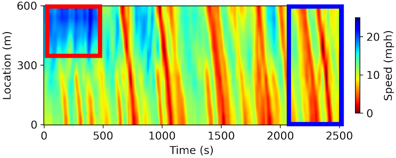
Matrix factorization



Smoothing matrix factorization



Hankel tensor factorization



Sparse Traffic Flow Forecasting

❸ X. Chen, L. Sun (2022). Bayesian temporal factorization for multidimensional time series prediction. IEEE Transactions on Pattern Analysis and Machine Intelligence, 44 (9): 4659–4673.

- 100+ citations on Google Scholar
- ESI highly cited paper (top 1%)
- ESI hot paper (top 0.1%)

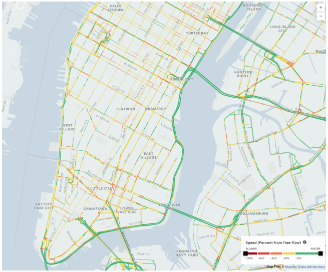
❹ X. Chen, C. Zhang, X.-L. Zhao, N. Saunier, L. Sun (2022). Nonstationary temporal matrix factorization for sparse traffic time series forecasting. arXiv preprint arXiv:2203.10651.

(Under 2nd review at Transportation Research Part C: Emerging Technologies)

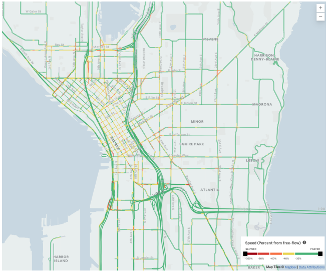
Sparse Traffic Flow Forecasting

Motivation:

- Uber (hourly) movement speed data[1]



NYC movement



Seattle movement

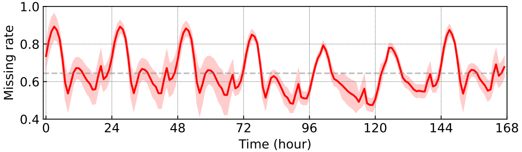
- The average speed on a given road segment for each hour of each day.
- Hourly speeds are computed when road segments have 5+ unique trips.
- **Issue**: insufficient sampling of ridesharing vehicles on the road network.

[1] https://movement.uber.com/

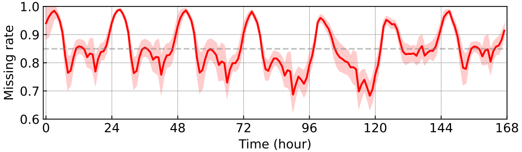
Sparse Traffic Flow Forecasting

High-dimensionality & Sparsity

- **NYC** movement speed data (2019)
  - **98,210** road segments & 8,760 time steps (hours)
  - Whole missing rate: **64.43%**

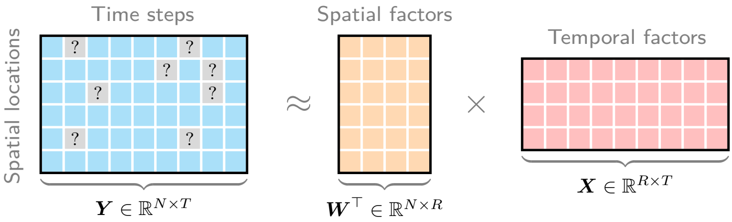


- **Seattle** movement speed data (2019)
  - **63,490** road segments & 8,760 time steps (hours)
  - Whole missing rate: **84.95%**



Sparse Traffic Flow Forecasting

- Spatiotemporal data can be reconstructed by low-dimensional latent factors!



- MF optimization problem

$$\min_{\boldsymbol{W},\boldsymbol{X}} \frac{1}{2}\left\|\mathcal{P}_\Omega(\boldsymbol{Y}-\boldsymbol{W}^\top\boldsymbol{X})\right\|_F^2+\frac{\rho}{2}\left(\|\boldsymbol{W}\|_F^2+\|\boldsymbol{X}\|_F^2\right)$$

with factor matrices $\boldsymbol{W}$ and $\boldsymbol{X}$.

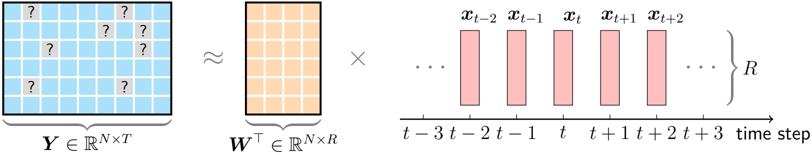
- Disadvantages:
  - Cannot capture temporal correlations.
  - Cannot perform time series forecasting.

Sparse Traffic Flow Forecasting

Temporal matrix factorization (Yu et al.'16; Chen & Sun'22)

Given any partially observed time series data $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$ with observed index set Ω, then temporal matrix factorization assumes a dth-order vector autoregressive (VAR) process on the temporal factor matrix:

$$\begin{aligned}\min_{\boldsymbol{W}, \boldsymbol{X}, \{\boldsymbol{A}_k\}_{k=1}^{d}} \quad & \frac{1}{2}\left\|\mathcal{P}_{\Omega}(\boldsymbol{Y} - \boldsymbol{W}^\top \boldsymbol{X})\right\|_F^2 + \frac{\rho}{2}(\|\boldsymbol{W}\|_F^2 + \|\boldsymbol{X}\|_F^2) \\ & + \frac{\lambda}{2}\sum_{t=d+1}^{T}\left\|\boldsymbol{x}_t - \sum_{k=1}^{d}\boldsymbol{A}_k \boldsymbol{x}_{t-k}\right\|_2^2\end{aligned}$$



GitHub repositories:

- **transdim**: Machine learning for spatiotemporal traffic data imputation and forecasting. (970+ stars & 270+ forks)

  https://github.com/xinychen/transdim

- **tracebase**: Multivariate time series forecasting on high-dimensional and sparse Uber movement speed data. (30+ stars)

  https://github.com/xinychen/tracebase

- **awesome-latex-drawing**: Academic drawing examples in LaTeX. (1,000+ stars & 140+ forks)

  https://github.com/xinychen/awesome-latex-drawing

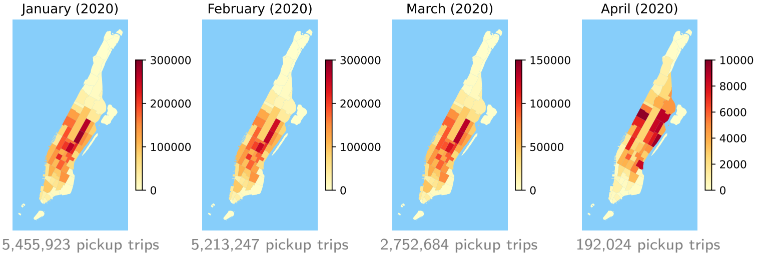
Dynamic Pattern Discovery

❺ X. Chen, C. Zhang†, X. Chen, N. Saunier, L. Sun (2022). Discovering dynamic patterns from spatiotemporal data with time-varying low-rank autoregression. arXiv preprint arXiv:2211.15482.

(Under 2nd review at IEEE Transactions on Knowledge and Data Engineering)

Motivation:

- NYC (yellow) taxi data[2]
  - Special event: COVID-19
  - Changed human mobility behavior (e.g., trip volume)
  - Changed human mobility patterns (e.g., frequent areas)



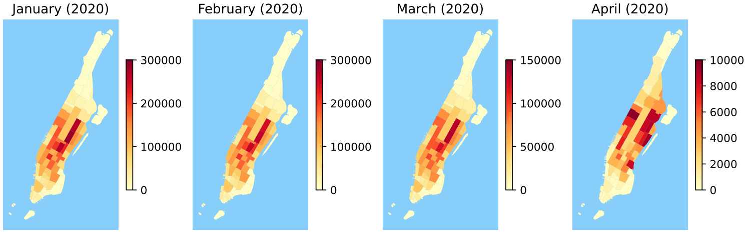
Geographical distribution of aggregated (pickup) taxi trips

[2] https://www1.nyc.gov/site/tlc/about/tlc-trip-record-data.page

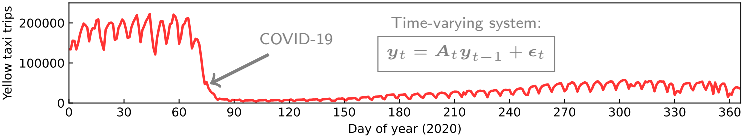
Dynamic Pattern Discovery

Motivation:

- NYC (yellow taxi data)



5,455,923 pickup trips | 5,213,247 pickup trips | 2,752,684 pickup trips | 192,024 pickup trips



- How to characterize the dynamic pattern?

Dynamic Pattern Discovery

- Given a sequence of spatiotemporal measurements $\boldsymbol{y}_t \in \mathbb{R}^N$, $t = 1, 2, \ldots, T$

$$\min_{\{\boldsymbol{A}_t\}} \quad \underbrace{\frac{1}{2}\sum_{t}\left\|\boldsymbol{y}_t - \boldsymbol{A}_t\boldsymbol{y}_{t-1}\right\|_2^2}_{\text{Time-varying autoregression}}$$

[Over-parameterization] $\mathcal{O}(N^2(T-1))$ parameters vs. $\mathcal{O}(NT)$ data.

Dynamic Pattern Discovery

- Given a sequence of spatiotemporal measurements $\boldsymbol{y}_t \in \mathbb{R}^N$, $t = 1, 2, \ldots, T$

$$\min_{\{\boldsymbol{A}_t\}} \ \underbrace{\frac{1}{2}\sum_t \left\|\boldsymbol{y}_t - \boldsymbol{A}_t\boldsymbol{y}_{t-1}\right\|_2^2}_{\text{Time-varying autoregression}}$$

[**Over-parameterization**] $\mathcal{O}(N^2(T-1))$ parameters vs. $\mathcal{O}(NT)$ data.

- **(Ours)** Parameterize coefficients via tensor factorization (TF):

$$\min_{\boldsymbol{W},\boldsymbol{G},\boldsymbol{V},\boldsymbol{X}} \ \underbrace{\frac{1}{2}\sum_t \left\|\boldsymbol{y}_t - \boldsymbol{W}\boldsymbol{G}(\boldsymbol{x}_t^\top \otimes \boldsymbol{V})^\top \boldsymbol{y}_{t-1}\right\|_2^2}_{\text{Let } \boldsymbol{A}_t = \boldsymbol{\mathcal{G}} \times_1 \boldsymbol{W} \times_2 \boldsymbol{V} \times_3 \boldsymbol{x}_t^\top \text{ be the TF}}$$

- Alternating minimization (Let f be the obj.)

$$\boldsymbol{W} := \{\boldsymbol{W} \mid \frac{\partial f}{\partial \boldsymbol{W}} = \boldsymbol{0}\} \qquad \boldsymbol{G} := \{\boldsymbol{G} \mid \frac{\partial f}{\partial \boldsymbol{G}} = \boldsymbol{0}\}$$

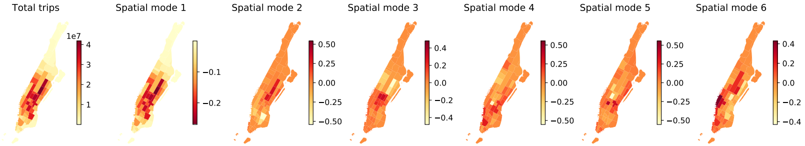
$$\boldsymbol{V} := \{\boldsymbol{V} \mid \frac{\partial f}{\partial \boldsymbol{V}} = \boldsymbol{0}\} \qquad \boldsymbol{x}_t := \{\boldsymbol{x}_t \mid \frac{\partial f}{\partial \boldsymbol{x}_t} = \boldsymbol{0}\}$$

- Solve each subproblem by **conjugate gradient** and **least squares**.

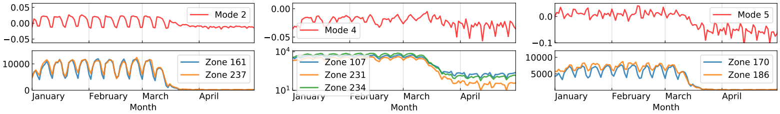
- Time-varying autoregression with TF

$$\min_{\boldsymbol{W},\boldsymbol{G},\boldsymbol{V},\boldsymbol{X}} \ \frac{1}{2}\sum_{t}\left\|\boldsymbol{y}_t-\boldsymbol{W}\boldsymbol{G}(\boldsymbol{x}_t^\top\otimes\boldsymbol{V})^\top\boldsymbol{y}_{t-1}\right\|_2^2$$

- **NYC taxi dataset** (pickup)



Spatial modes in $\boldsymbol{W}$



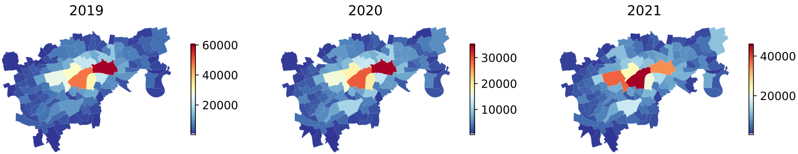
Mode #2 & taxi trips (2020) Mode #4 & taxi trips (2020) Mode #5 & taxi trips (2020)

Temporal modes in $\boldsymbol{X}$

- Produce interpretable patterns and identify the changing point of system (mainly due to COVID-19).

Dynamic Pattern Discovery

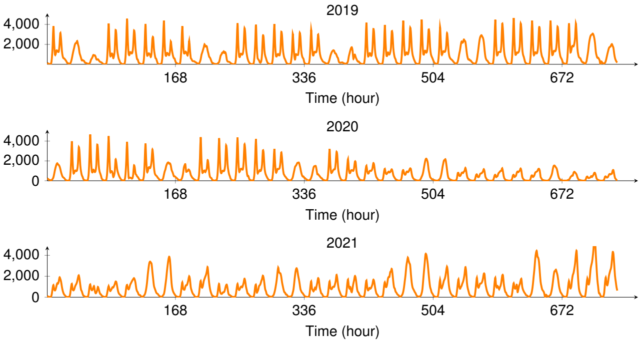
- **London bikeshare trips** (March, 2019-2021)
  - Significant decline of bike trips in 2020, but no pattern changes.
  - The most frequent areas change in 2021, showing new patterns.



Geographical distribution of aggregated bike trips

Dynamic Pattern Discovery

- Hourly aggregated trips (31 days in March)
    - Significant trip decline in late March 2020
    - (Before COVID-19) Daily trips at weekdays > weekends (see 2019)
    - (During COVID-19) Daily trips at weekdays < weekends (see 2021)
    - Bike trips shifted from weekdays to weekends $\Longrightarrow$ preference over active travel mode



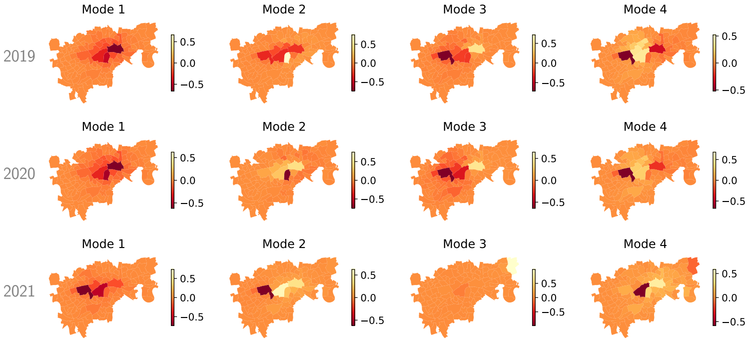
- Time-varying system with pattern transition!

Dynamic Pattern Discovery

- Time-varying autoregression with TF

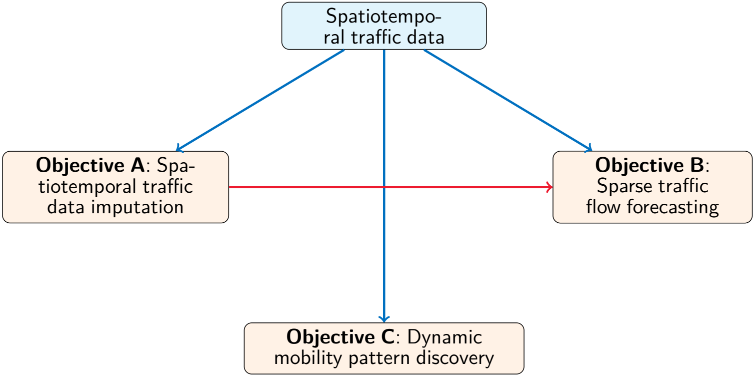
$$\min_{\boldsymbol{W},\boldsymbol{G},\boldsymbol{V},\boldsymbol{X}} \ \frac{1}{2}\sum_{t}\left\|\boldsymbol{y}_t-{\color{red}\boldsymbol{W}}\boldsymbol{G}(\boldsymbol{x}_t^\top\otimes\boldsymbol{V})^\top\boldsymbol{y}_{t-1}\right\|_2^2$$

  - (2019 vs. 2020) Consistent spatial modes #1, #3, and #4
  - (2021) Quite different spatial modes & mobility pattern changes



Conclusion

We give some studies on **spatiotemporal traffic data modeling**.



Future Works

Prior Works

Other studies about spatiotemporal data modeling:

- ❻ X. Chen, M. Lei, N. Saunier, L. Sun (2022). Low-rank autoregressive tensor completion for spatiotemporal traffic data imputation. IEEE Transactions on Intelligent Transportation Systems, 23 (8): 12301–12310.
- ❼ X. Chen, Y. Chen, N. Saunier, L. Sun (2021). Scalable low-rank tensor learning for spatiotemporal traffic data imputation. Transportation Research Part C: Emerging Technologies, 129: 103226.
- ❽ X. Chen, J. Yang, L. Sun (2020). A nonconvex low-rank tensor completion model for spatiotemporal traffic data imputation. Transportation Research Part C: Emerging Technologies, 117: 102673.
- ❾ X. Chen, Z. He, Y. Chen, Y. Lu, J. Wang (2019). Missing traffic data imputation and pattern discovery with a Bayesian augmented tensor factorization model. Transportation Research Part C: Emerging Technologies, 104: 66-77.
- ❿ X. Chen, Z. He, L. Sun (2019). A Bayesian tensor decomposition approach for spatiotemporal traffic data imputation. Transportation Research Part C: Emerging Technologies, 98: 73-84.

References

A short list:

- **[Liu & Zhang'22]** G. Liu and W. Zhang (2022). Recovery of future data via convolution nuclear norm minimization. IEEE Transactions on Information Theory, 69(1), 650–665.
- **[Yu et al.'16]** H.-F. Yu, N. Rao, and I. S. Dhillon (2016). Temporal regularized matrix factorization for high-dimensional time series prediction. Advances in neural information processing systems (NIPS).

Thanks for your attention!

Any Questions?

Slides: https://xinychen.github.io/slides/traffic_data_modeling_v2.pdf

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