# Quantifying Time Series Periodicity with Interpretable Machine Learning

Climate Systems & Urban Human Mobility

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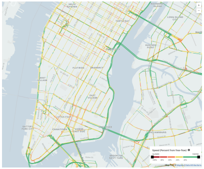
Atlanta, USA

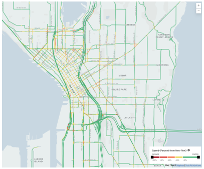
# Spatiotemporal Time Series Data

- Transport & mobility & climate application scenarios

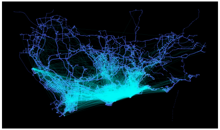

Highway (Portland)



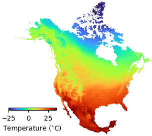
Uber movement (NYC)



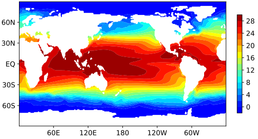
Uber movement (Seattle)



Taxi trajectory (Shenzhen)




Temperature (NA)

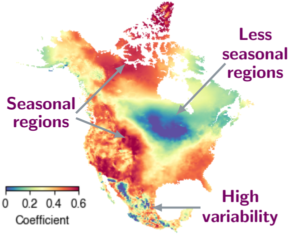



Temperature (sea surface)

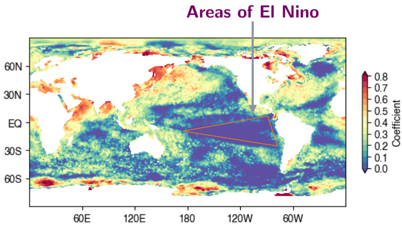
- Challenges: Sparsity, high-dimensionality, multi-dimensionality, heavy tails, irregular sampling, and time-varying systems

# Motivation

**Yearly temperature seasonality patterns in 2010s**




North America




Global sea surface

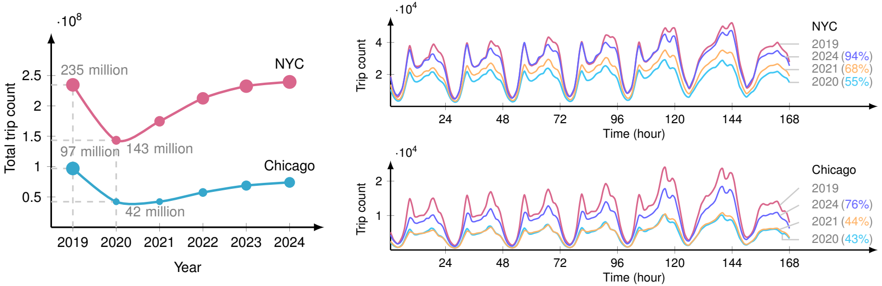
**What motivate us most about periodicity?**

❶ **Monitoring climate systems**: Empirically measure the periodicity of climate variables (e.g., temperature, precipitation).

❷ **Discovering spatiotemporal patterns**: Identify periodicity pattern shift and special climate events.

# Motivation

Ridesharing trip data

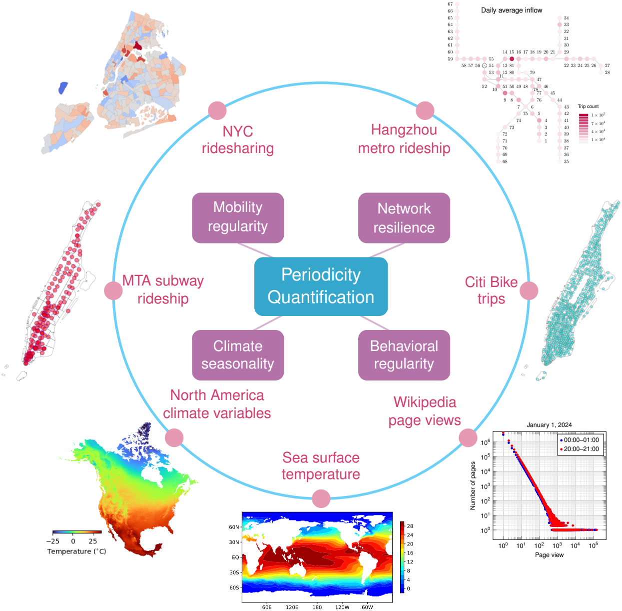



What motivate us most about **periodicity**?

❶ **Resilience and stability of systems**: Empirically measure the periodicity and predictability of urban systems.

❷ **Optimization of transport systems**: Optimize resources (e.g., public transit, taxi, ridesharing, micromobility) to meet transport demand efficiently.

❸ **Design of sustainable transport & infrastructure**: Implement energy-efficient solutions (e.g., congestion pricing) tailored to peak hours.



Periodicity Quantification
Mobility regularity
Network resilience
Climate seasonality
Behavioral regularity
NYC ridesharing
Hangzhou metro rideship
MTA subway rideship
Citi Bike trips
North America climate variables
Wikipedia page views
Sea surface temperature
Daily average inflow
Trip count
−25 0 25
Temperature (°C)
January 1, 2024
00:00–01:00
20:00–21:00
Number of pages
Page view

# Interpretable Time Series Autoregression

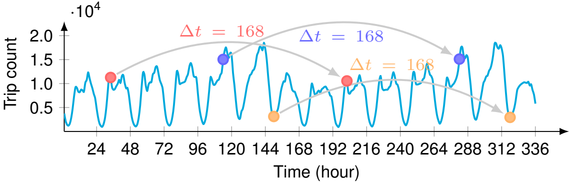
https://github.com/xinychen/integers

- Interpretable ML
- $\ell_0$-norm optimization
- Climate system seasonality
- Sparse autoregression
- Mixed-integer programming
- Human mobility regularity

# Valorizing Autoregression

- Time series autoregression on $\boldsymbol{x} \in \mathbb{R}^T$ with order $d \in \mathbb{Z}^+$

$$\boldsymbol{w} := \arg\min_{\boldsymbol{w}} \sum_{t=d+1}^{T} \left( x_t - \sum_{k=1}^{d} w_k x_{t-k} \right)^2$$




Periodicity of hourly rideshare trip time series

- **Sparse** coefficient vector $\mapsto$ **Interpretability**?

$$\underbrace{\boldsymbol{w}}_{\text{sparsity } \|\boldsymbol{w}\|_0 \triangleq 3} = (\underbrace{0.33}_{k=1}, 0, \cdots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

# Sparse Autoregression

- Identify the dominant auto-correlations
  - $\tau \in \mathbb{Z}^+$: Upper bound of the number of nonzero entries in $\boldsymbol{w} \in \mathbb{R}^d$

$$\underbrace{\begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}}_{\tilde{\boldsymbol{x}}} \approx \underbrace{\begin{bmatrix} x_4 & x_3 & x_2 & x_1 \\ x_5 & x_4 & x_3 & x_2 \\ x_6 & x_5 & x_4 & x_3 \\ x_7 & x_6 & x_5 & x_4 \\ x_8 & x_7 & x_6 & x_5 \end{bmatrix}}_{\boldsymbol{A}} \times \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}}_{\boldsymbol{w}} \Big\} \, \|\boldsymbol{w}\|_0 \leq \tau$$

$$\begin{aligned} \boldsymbol{w} := & \mathop{\arg\min}_{\|\boldsymbol{w}\|_0 \leq \tau} \sum_{t=d+1}^{T} \Big( x_t - \sum_{k=1}^{d} w_k x_{t-k} \Big)^2 \\ = & \mathop{\arg\min}_{\|\boldsymbol{w}\|_0 \leq \tau} \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \end{aligned}$$

- $\ell_0$-norm optimization is NP-hard
- Formulate it as a mixed-integer programming (MIP)
  - Introduce binary decision variables $\boldsymbol{\beta} \in \{0,1\}^d$

$$\begin{aligned} \min_{\boldsymbol{w}} \; & \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \\ \text{s.t.} \; & \underbrace{\|\boldsymbol{w}\|_0 \leq \tau}_{\clubsuit \text{ sparsity of } \boldsymbol{w}} \end{aligned} \iff \begin{aligned} \min_{\boldsymbol{w},\,\boldsymbol{\beta}} \; & \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \\ \text{s.t.} \; & \underbrace{-\boldsymbol{\beta} \leq \boldsymbol{w} \leq \boldsymbol{\beta},}_{\text{bounds being either } 0 \text{ or } \pm 1} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\clubsuit \text{ sparsity of } \boldsymbol{\beta}} \end{aligned}$$

# $\ell_0$-Norm Optimization

$$\min_{\boldsymbol{w}} \ \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \quad \text{s. t.} \ \|\boldsymbol{w}\|_0 \leq \tau$$

Greedy

**1993**

**Orthogonal Matching Pursuit (OMP)**

Y. C. Pati
R. Rezaiifar
P. S. Krishnaprasad

**2008**

**Subspace Pursuit (SP)**

Wei Dai
Olgica Milenkovic

**Compressive Sampling Matching Pursuit (CoSaMP)**

Deanna Needell
Joel A. Tropp

**2015**

**Modern Optimization (i.e., MIP)**

Dimitris Bertsimas
Angela King
Rahul Mazumder

# Sparse Autoregression Done Right

$$\underbrace{\min_{\boldsymbol{w},\,\boldsymbol{\beta}} \sum_{t=d+1}^{T}\left(x_t-\sum_{k=1}^{d} w_k x_{t-k}\right)^2}_{\text{Time series autoregression}} \quad \text{s.t.} \quad \underbrace{-\beta_k \leq w_k \leq \beta_k}_{\text{Lower and upper bounds}}, \quad \underbrace{\sum_{k=1}^{d}\beta_k \leq \tau}_{\clubsuit\ \text{Sparsity}}, \quad \underbrace{\beta_k \in \{0,1\}}_{\text{Binary variable}}$$

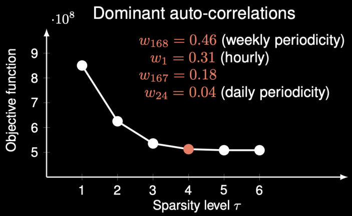
- $\boldsymbol{w} \in \mathbb{R}^d$: Auto-correlations
- $\boldsymbol{\beta} \in \{0,1\}^d$: Sparsity pattern
- $d = 168$: Autoregression order

```python
import numpy as np
from docplex.mp.model import Model

def sparse_ar(x, d, tau):
    model = Model('Sparse Autoregression')
    T = x.shape[0]
    w = [model.continuous_var(name = f'w_{k}') for k in range(d)]
    beta = [model.binary_var(name = f'beta_{k}') for k in range(d)]
    model.minimize(model.sum((x[t] - model.sum(w[k] * x[t - k - 1]
                                 for k in range(d))) ** 2
                    for t in range(d, T)))
    model.add_constraint(model.sum(beta[k] for k in range(d)) <= tau)
    for k in range(d):
        model.add_constraint(w[k] <= beta[k])
        model.add_constraint(w[k] >= - beta[k])
    solution = model.solve()
    return np.array(solution.get_values(w))
```

https://github.com/xinychen/integers


$\Downarrow$

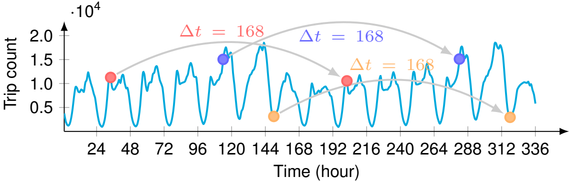


# Solution Quality → Better Interpretability?

- Sparse autoregression

$$\min_{\boldsymbol{w}\geq 0}\ \|\tilde{\boldsymbol{x}}-\boldsymbol{A}\boldsymbol{w}\|_2^2 \quad \text{s.t. } \|\boldsymbol{w}\|_0\leq\tau$$

- Subspace pursuit (SP) sometimes fails



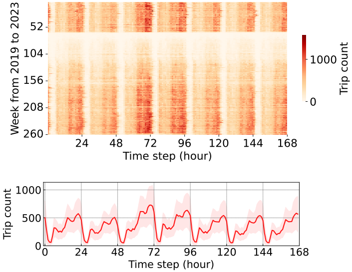

Periodicity of ridesharing trip time series

- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity $\tau=2$):

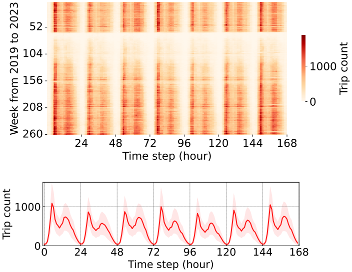
$$\underbrace{\boldsymbol{w}=(\cdots,\underbrace{0.02}_{k=53},\cdots,\underbrace{0.96}_{k=168})^\top}_{\text{obj.}\ =8.32\times 10^7\ \text{(SP)}} \quad \text{vs.} \quad \underbrace{\boldsymbol{w}=(\underbrace{0.22}_{k=1},\cdots,\underbrace{0.77}_{k=168})^\top}_{\clubsuit\ \text{obj.}\ =6.25\times 10^7\ \text{(MIP)}}$$

# John F. Kennedy International Airport

- Daily & weekly periodicity: **dropoff** $>$ **pickup** trips at JFK airport
  - Pickup trips are relevant to flight delay, baggage claim, and other factors.
  - Dropoff trips to airport are highly related to flight schedules.




**Pickup** trips from airport




**Dropoff** trips to airport

- Sparse coefficient vectors (sparsity $\tau = 3$):

$$\boldsymbol{w} = (\underbrace{0.31}_{k=1}, \cdots, \underbrace{0.28}_{k=24}, \cdots, \underbrace{0.41}_{k=168})^\top \quad \text{vs.} \quad \boldsymbol{w} = (\underbrace{0.18}_{k=1}, \cdots, \underbrace{0.35}_{k=24}, \cdots, \underbrace{0.47}_{k=168})^\top$$

# High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large $N$:

$$\underbrace{\min_{\{\boldsymbol{w}_n\}_{n=1}^{N},\,\boldsymbol{\beta}}}_{(N+1)d\text{ decision var.}} \quad \underbrace{\sum_{n=1}^{N}\|\tilde{\boldsymbol{x}}_n-\boldsymbol{A}_n\boldsymbol{w}_n\|_2^2}_{\text{multivariate time series}}$$

$$\text{s.t.}\quad \underbrace{0\leq\boldsymbol{w}_n\leq\boldsymbol{\beta},}_{\text{bounds being either }0\text{ or }1}\qquad \underbrace{\|\boldsymbol{\beta}\|_1\leq\tau,}_{\text{sparsity of }\boldsymbol{\beta}}\quad \boldsymbol{\beta}\in\{0,1\}^d$$

- How to handle millions of time series (e.g., $N\geq 10^6$)?

# High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large $N$:

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- How to handle millions of time series (e.g., $N\geq 10^6$)?

- Two-stage optimization (♣):

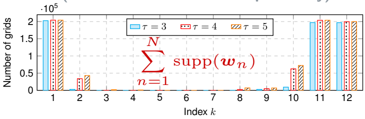
❶ **Learn sparsity patterns in $\boldsymbol{\beta}\in\{0,1\}^d$**

$$\min_{\boldsymbol{w},\boldsymbol{\beta}}\ \underbrace{\operatorname{tr}(\boldsymbol{w}\boldsymbol{w}^\top\boldsymbol{P})}_{\text{quadratic}}-\underbrace{2\boldsymbol{w}^\top\boldsymbol{q}}_{\text{linear}}$$

$$\text{s.t. } 0\leq\boldsymbol{w}\leq\boldsymbol{\beta},\quad \|\boldsymbol{\beta}\|_1\leq\tau$$

Global sparsity verification

(Tested $N$ time series independently)




204k monthly NA temperature time series

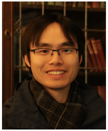
❷ **Quadratic programming** with index set $\Omega=\operatorname{supp}(\boldsymbol{\beta})$

$$\boldsymbol{w}_n:=\arg\min_{\boldsymbol{w}\geq 0}\ \|\tilde{\boldsymbol{x}}_n-\boldsymbol{A}_n\boldsymbol{w}\|_2^2\quad\text{s.t. } w_k=0,\ \forall k\notin\Omega$$

# Climate Seasonality Patterns

(arXiv:2506.22895)

- ❒ North America temperature
- ❒ Climate variable seasonality
- ❒ Sea surface temperature
- ❒ Spatiotemporal patterns



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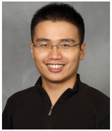

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MIT

## Understanding Climate Systems

Quantify yearly seasonality by $\{w_{m,n,\gamma,k}\}$ at index $k = 12$

$$\min_{\{\boldsymbol{w}_{m,n,\gamma}\},\,\boldsymbol{\beta}} \; \sum_{\substack{m=1 \\ \text{latitude}}}^{M} \sum_{\substack{n=1 \\ \text{longitude}}}^{N} \sum_{\substack{\gamma=1 \\ \text{decade}}}^{\delta} \|\underbrace{\tilde{\boldsymbol{x}}_{m,n,\gamma} - \boldsymbol{A}_{m,n,\gamma}}_{\text{monthly temperature}}\boldsymbol{w}_{m,n,\gamma}\|_2^2$$

$$\text{s.t. } 0 \leq \boldsymbol{w}_{m,n,\gamma} \leq \boldsymbol{\beta}$$

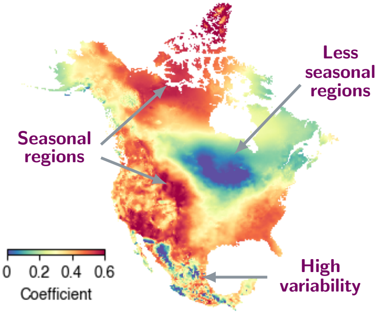
$$\|\boldsymbol{\beta}\|_1 \leq \tau$$

sparsity constraint

$$\boldsymbol{\beta} \in \{0,1\}^d$$

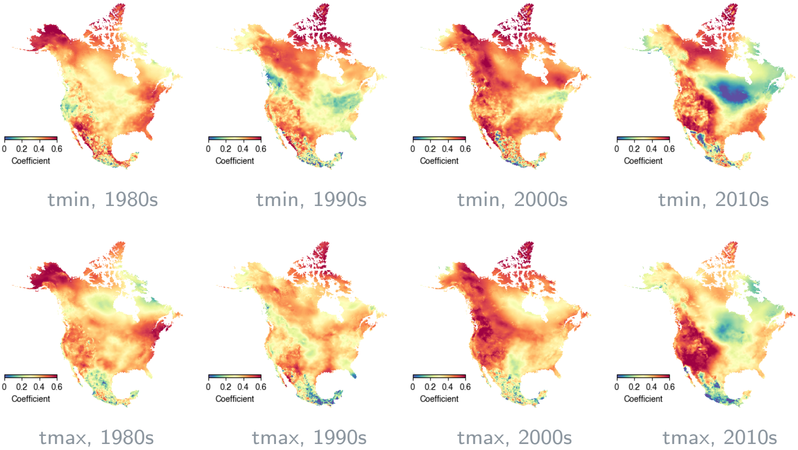
binary decision var.

# North America Temperature



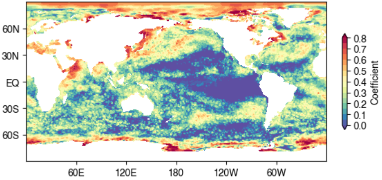

Yearly temperature **seasonality** pattern in 2010s

# North America Temperature

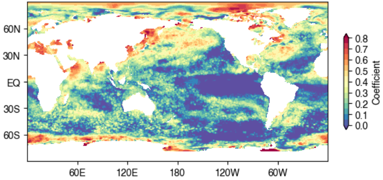



- Identify yearly periodicity at $k = 12$ from temperature data ($\tau = 3$)
  1. Stronger yearly seasonality in high-latitude areas
  2. Less seasonal temperature in south areas (e.g., Mexico)
  3. Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s

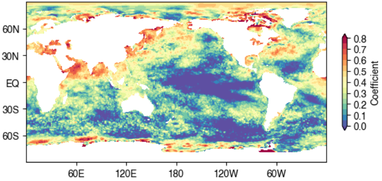
# Sea Surface Temperature



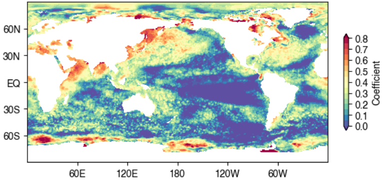

1980s




1990s




2000s




2010s

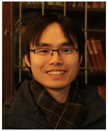
- Identify yearly periodicity at $k = 12$ from SST data ($\tau = 3$)
  1. The areas of **El Nino** events are less seasonal/predictable
  2. **Arctic** becomes less seasonal/predictable in the past 20 years

# Human Mobility Regularity

Applications and Case Studies

(arXiv:2508.03747)

- ❐ Daily & weekly periodicity
- ❐ NYC & Chicago ridesharing
- ❐ Multi-modal mobility
- ❐ Network resilience
- ❐ Post-pandemic recovery
- ❐ Metro passenger flow

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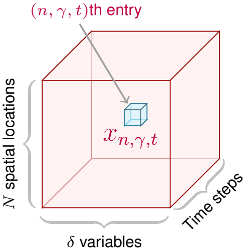
# Envisioning Human Mobility

- Human mobility data




spatial location — variable — hourly trip data

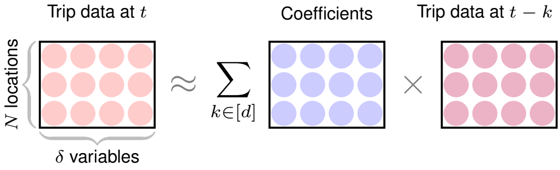
$$\min_{\{\boldsymbol{w}_{n,\gamma}\},\,\boldsymbol{\beta}} \sum_{n=1}^{N}\sum_{\gamma=1}^{\delta}\|\tilde{\boldsymbol{x}}_{n,\gamma}-\boldsymbol{A}_{n,\gamma}\boldsymbol{w}_{n,\gamma}\|_2^2$$

$$\text{s.t. } 0\leq\boldsymbol{w}_{n,\gamma}\leq\boldsymbol{\beta}$$

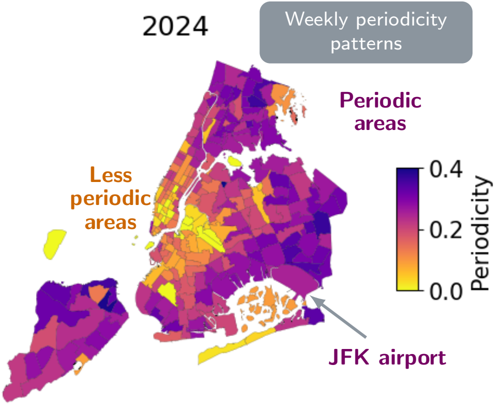
$$\|\boldsymbol{\beta}\|_1\leq\tau \quad \text{sparse}$$

$$\boldsymbol{\beta}\in\{0,1\}^d \quad \text{binary}$$

- Quantify **weekly periodicity** by $\{w_{n,\gamma,k}\}$ at index $k=168$



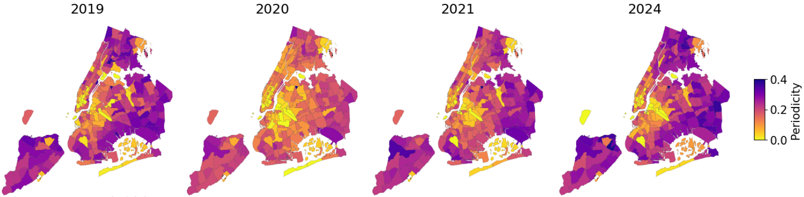
# NYC Ridesharing




Hourly trip time series of 265 pickup areas

# NYC Ridesharing

**Weekly periodicity patterns**




Periodicity: **0.186** | **0.136** | **0.175** | **0.194**

⇑ Pandemic disruption

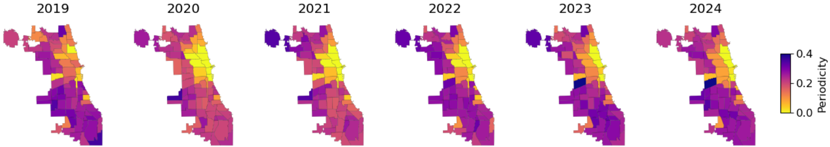
⇑ Recovering

Weekly periodicity ↓

Less periodic areas ↑

# Chicago Ridesharing

**Weekly periodicity patterns**




Periodicity: 0.214 0.175 0.187 0.207 0.214 0.212

Pandemic disruption

Weekly periodicity ↓

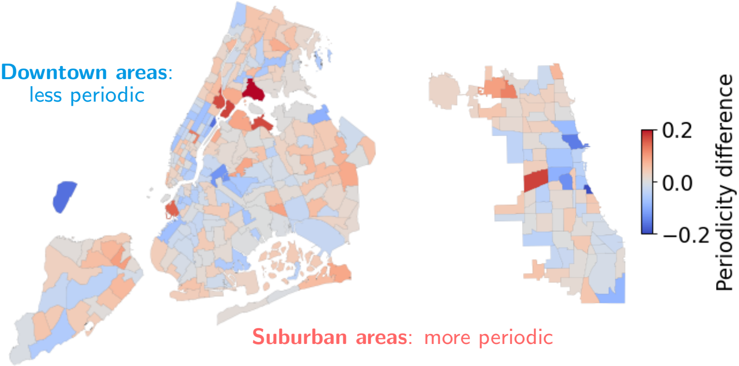
Less periodic areas ↑

Recovering

Hourly trip time series of 77 pickup areas

# Post-Pandemic Recovery

2024's periodicity **minus** 2019's periodicity




**Downtown areas**: less periodic

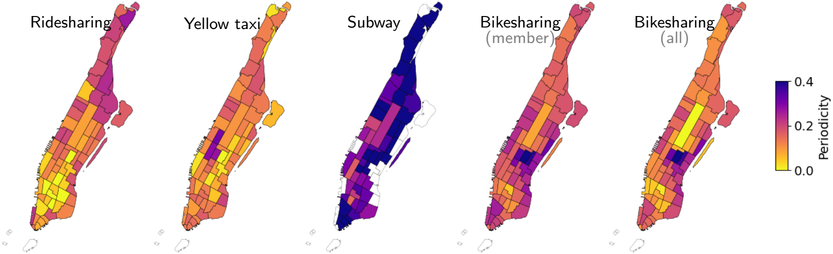
**Suburban areas**: more periodic

# Weekly Periodicity of Manhattan Mobility

Mobility data of 2024

**North areas**:
Ridesharing > yellow taxi

**Overall**:
Subway > others




Periodicity: **0.122** **0.120** **0.363** **0.191** **0.151**

**South areas**:
Yellow taxi > Ridesharing

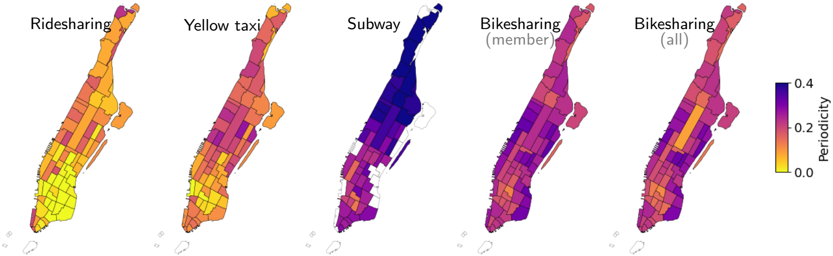
**Remarkable** weekly periodicity difference between membership and all travels

# Daily Periodicity of Manhattan Mobility (Weekdays)

Mobility data of 2024

**Overall**:
Yellow taxi > ridesharing

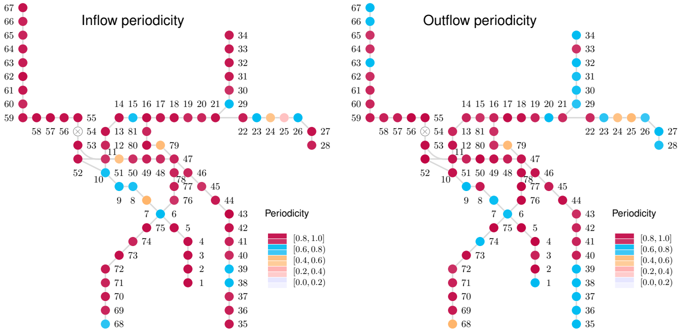
**Subway**:
North areas > south areas



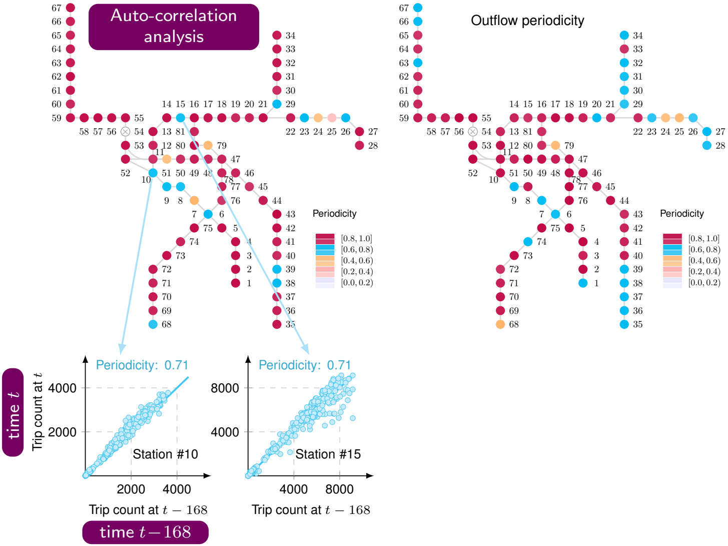

Periodicity: **0.072** **0.127** **0.305** **0.232** **0.214**

**Marginal** daily periodicity difference between membership and all travels

# Hangzhou Metro Passenger Flow

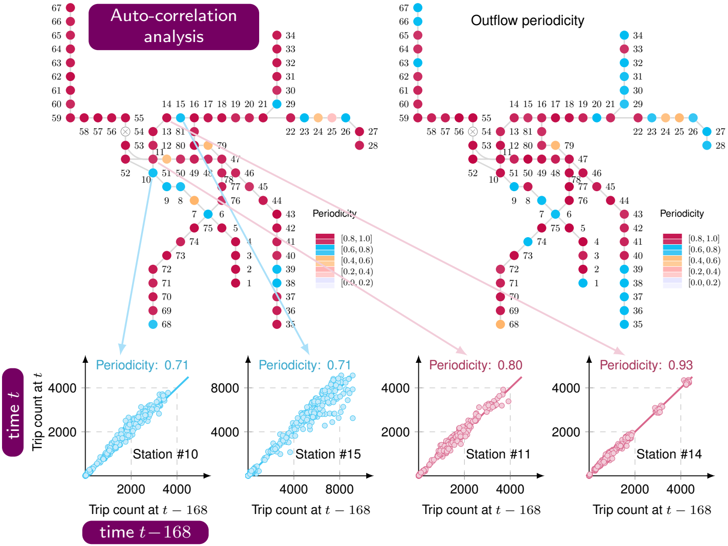


# Hangzhou Metro Passenger Flow




“Closeness” to the anti-diagonal curve $y = x$

# Hangzhou Metro Passenger Flow




"Closeness" to the anti-diagonal curve $y = x$

# Thanks for your attention!

## Any Questions?

Slides: https://xinychen.github.io/slides/informs25.pdf

**About me**:

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