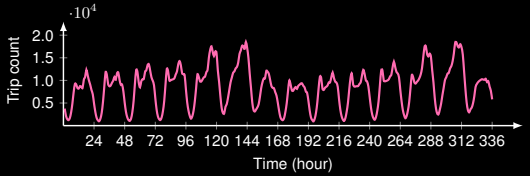
Essential Idea of Sparse Autoregression & Periodicity Quantification

① Hourly ridesharing trip time series

(336 data pints x_t)



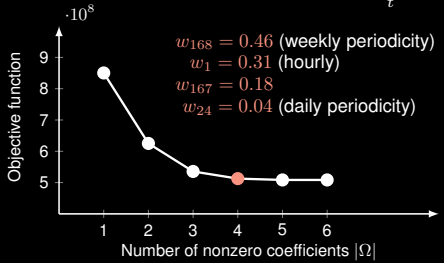
② Autoregression (order-168)

$$\min \sum_t \left(x_t - \sum_{k=1}^{168} w_k x_{t-k} \right)^2$$

Dense

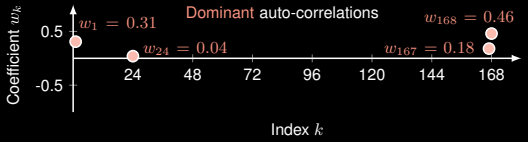
③ Sparse autoregression

$$\min \sum_t \left(x_t - \sum_{k \in \Omega} w_k x_{t-k} \right)^2 \quad \text{s.t.} \ \underbrace{|\Omega| \leq \tau}_{\text{sparsity}}$$



$w_{168} = 0.46$ (weekly periodicity)
$w_1 = 0.31$ (hourly)
$w_{167} = 0.18$
$w_{24} = 0.04$ (daily periodicity)

Sparse



Sparse Autoregression Done Right

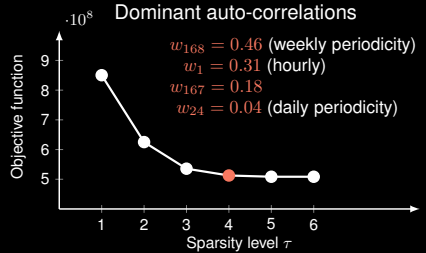
$$\min_{\boldsymbol{w},\,\boldsymbol{\beta}} \underbrace{\sum_{t=d+1}^{T}\left(x_t - \sum_{k=1}^{d} w_k x_{t-k}\right)^2}_{\text{Time series autoregression}} \quad \text{s.t.} \quad \underbrace{-\beta_k \le w_k \le \beta_k}_{\text{Lower and upper bounds}}, \quad \underbrace{\sum_{k=1}^{d}\beta_k \le \tau}_{\clubsuit\ \text{Sparsity}}, \quad \underbrace{\beta_k \in \{0,1\}}_{\text{Binary variable}}$$

- $\boldsymbol{w} \in \mathbb{R}^d$: Auto-correlations
- $\boldsymbol{\beta} \in \{0,1\}^d$: Sparsity pattern
- $d = 168$: Autoregression order

```python
import numpy as np
from docplex.mp.model import Model

def sparse_ar(x, d, tau):
    model = Model('Sparse Autoregression')
    T = x.shape[0]
    w = [model.continuous_var(name = f'w_{k}') for k in range(d)]
    beta = [model.binary_var(name = f'beta_{k}') for k in range(d)]
    model.minimize(model.sum((x[t] - model.sum(w[k] * x[t - k - 1]
                                               for k in range(d))) ** 2
                             for t in range(d, T)))
    model.add_constraint(model.sum(beta[k] for k in range(d)) <= tau)
    for k in range(d):
        model.add_constraint(w[k] <= beta[k])
        model.add_constraint(w[k] >= - beta[k])
    solution = model.solve()
    return np.array(solution.get_values(w))
```

https://github.com/xinychen/integers



Thanks for your attention!

Any Questions?

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