Machine Learning and Optimization for Understanding Spatiotemporal Systems

Time Series Imputation & Periodicity Quantification

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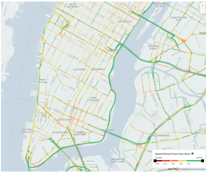
May 22, 2025

Orlando, USA

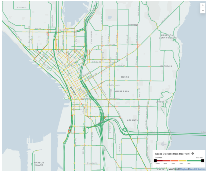
Spatiotemporal Data

- Transport & mobility application scenarios

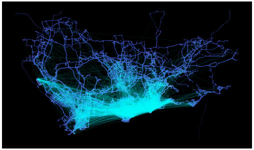
Highway (Portland)



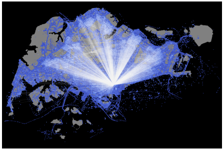
Uber movement (NYC)



Uber movement (Seattle)



Taxi trajectory (Shenzhen)

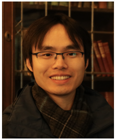


Human movement (Singapore)

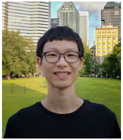
- Challenges: Sparsity, high-dimensionality (network-scale), and multi-dimensionality (complicated data structure), time-varying systems

Spatiotemporal Data Imputation

❒ Convolution ❒ Fast Fourier transform ❒ Optimization w/ ℓ_1-norm
❒ Time series imputation ❒ Speed field reconstruction



Xinyu Chen
UdeM $\rightarrow$ MIT



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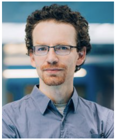
HanQin Cai
UCF



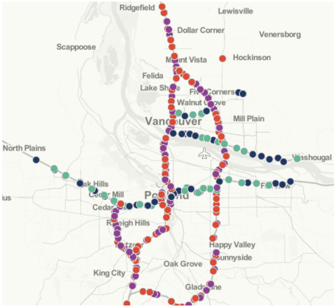
Nicolas Saunier
PolyMtl

Lijun Sun
McGill

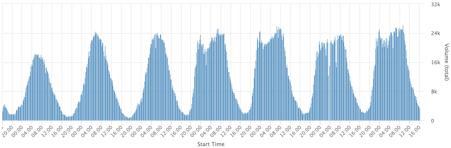
- **Xinyu Chen**, Zhanhong Chen, HanQin Cai, Nicolas Saunier, Lijun Sun (2024). "Laplacian Convolutional Representation for Traffic Time Series Imputation". *IEEE Transactions on Knowledge and Data Engineering*, 36 (11): 6490–6502.
- Blog post: Understanding time series convolution. https://spatiotemporal-data.github.io/posts/ts_conv

Motivation

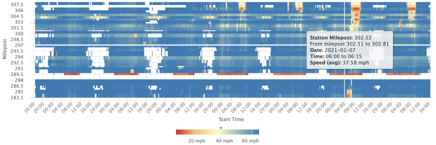
- Portland highway traffic data[1]



Highway network & sensor locations



Traffic volume

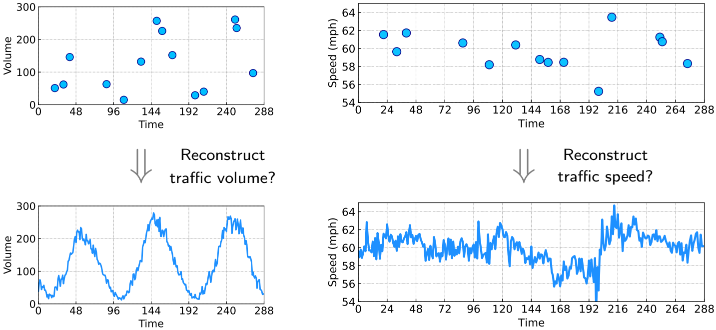


Traffic speed

- $\boldsymbol{X} \in \mathbb{R}^{N \times T}$ with N spatial locations $\times$ T time steps
- Traffic volume/speed shows strong spatial/temporal dependencies
- Missing data are there, how to improve data quality?

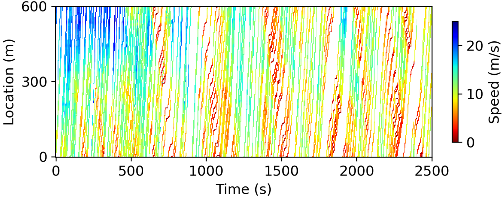
[1] https://portal.its.pdx.edu/home

Motivation

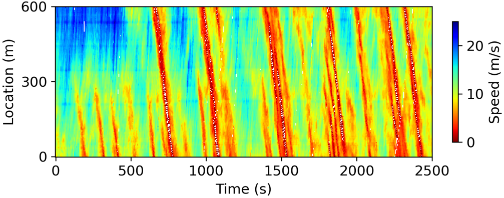


- How to utilize the global trends of traffic time series?
- How to produce local consistency of traffic data?

Motivation



200-by-500 matrix (NGSIM) $\Downarrow$ Reconstruct speed field from 20% sparse trajectories?

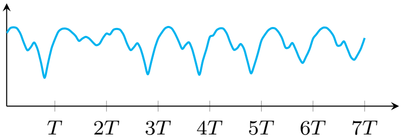


- How to learn from sparse spatiotemporal data?
- How to characterize spatial/temporal local dependencies?

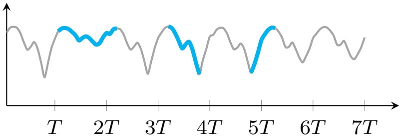
Time Series Imputation

Global/local trends in sparse data?

- Global trends (e.g., daily/weekly periodicity):

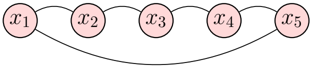


- Local trends (e.g., short-term time series trends):



Local Trend Modeling

- Intuition of Laplacian matrix



Undirected and circulant graph

$\xrightarrow{\text{Modeling}}$

$$\boldsymbol{L}=\begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Laplacian kernel:

$$\boldsymbol{\ell} \triangleq \underbrace{(2,-1,0,0,-1)^\top}_{\text{first column of } \boldsymbol{L}}$$

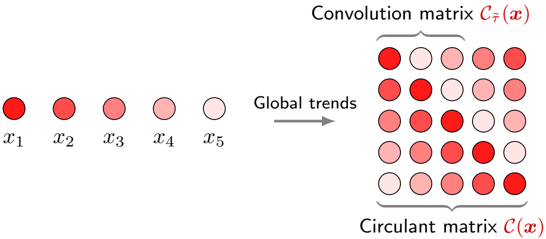
extending to the degree 2τ (i.e., graph connectivity) for $\boldsymbol{x} \in \mathbb{R}^T$.

- Temporal regularization:

$$\mathcal{R}(\boldsymbol{x}) = \underbrace{\frac{1}{2}\|\boldsymbol{L}\boldsymbol{x}\|_2^2}_{\text{mat-vec mul.}} = \underbrace{\frac{1}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{convolution } \star}$$

Global Trend Modeling

Circulant matrix $\mathcal{C}(\boldsymbol{x})$ vs. convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})$



- Circulant/Convolution nuclear norm minimization
  - A balance between global and local trends modeling?

CircNNM (Liu'22, Liu & Zhang'23)

$$\min_{\boldsymbol{x}} \; \|\mathcal{C}(\boldsymbol{x})\|_*$$
$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

ConvNNM (Liu'22, Liu & Zhang'23)

$$\min_{\boldsymbol{x}} \; \|\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})\|_*$$
$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

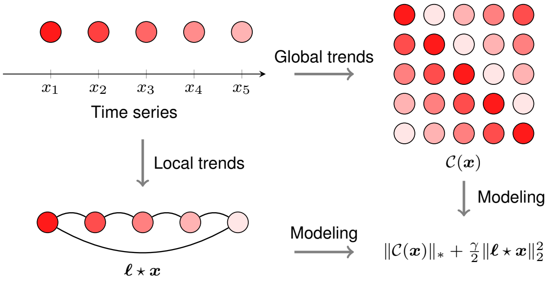
Global + Local Trends?

Laplacian Convolutional Representation (LCR)

For any partially observed time series $\boldsymbol{y} \in \mathbb{R}^T$ with observed index set Ω, LCR utilizes circulant matrix and Laplacian kernel to characterize global/local time series trends:

$$\min_{\boldsymbol{x}} \; \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{local}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon$$



Laplacian Convolutional Representation

- LCR model:

$$\begin{aligned} \min_{\boldsymbol{x}} \ & \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 \\ \text{s.t. } & \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon \end{aligned}$$

$$\Longrightarrow \quad \begin{aligned} \min_{\boldsymbol{x}} \ & \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{global + local}} + \underbrace{\frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{z} - \boldsymbol{y})\|_2^2}_{\text{regularization}} \\ \text{s.t. } & \boldsymbol{z} = \boldsymbol{x} \end{aligned}$$

"The alternating direction method of multipliers (ADMM) is an algorithm that solves convex optimization problems by breaking them into smaller pieces, each of which are then easier to handle."

— Source: https://stanford.edu/~boyd/admm.html

Laplacian Convolutional Representation

- Augmented Lagrangian function:

$$\mathcal{L} = \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{global + local}} + \underbrace{\frac{\lambda}{2}\|\boldsymbol{x} - \boldsymbol{z}\|_2^2 + \langle \boldsymbol{w}, \boldsymbol{x} - \boldsymbol{z} \rangle}_{\text{Lagrangian multiplier } \boldsymbol{w}} + \underbrace{\frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{z} - \boldsymbol{y})\|_2^2}_{\text{observations } \boldsymbol{y}}$$

- Optimize $\boldsymbol{x}$ w/ FFT in $\mathcal{O}(T \log T)$ time: :

$$\begin{cases} \|\mathcal{C}(\boldsymbol{x})\|_* = \|\mathcal{F}(\boldsymbol{x})\|_1 = \|\hat{\boldsymbol{x}}\|_1 & \text{(circulant matrix)} \\ \dfrac{1}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 = \dfrac{1}{2T}\|\mathcal{F}(\boldsymbol{\ell}) \circ \mathcal{F}(\boldsymbol{x})\|_2^2 = \dfrac{1}{2T}\|\hat{\boldsymbol{\ell}} \circ \hat{\boldsymbol{x}}\|_2^2 & \text{(circular convolution)} \end{cases}$$

- Reformulate the optimization as ℓ_1-norm minimization:

$$\boldsymbol{x} := \arg\min_{\boldsymbol{x}} \; \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x} - \boldsymbol{z} + \boldsymbol{w}/\lambda\|_2^2$$

$$\Longrightarrow \hat{\boldsymbol{x}} := \arg\min_{\hat{\boldsymbol{x}}} \; \underbrace{\|\hat{\boldsymbol{x}}\|_1}_{\ell_1\text{-norm}} + \frac{\gamma}{2T}\|\hat{\boldsymbol{\ell}} \circ \hat{\boldsymbol{x}}\|_2^2 + \frac{\lambda}{2T}\|\hat{\boldsymbol{x}} - \hat{\boldsymbol{z}} + \hat{\boldsymbol{w}}/\lambda\|_2^2$$

Laplacian Convolutional Representation

ℓ_1-norm Minimization (Liu & Zhang'23)

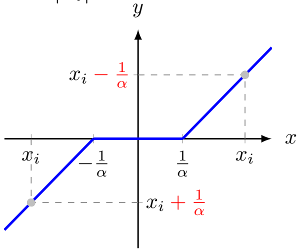
For any $\hat{\boldsymbol{h}} \in \mathbb{C}^T$ and $\delta \in \mathbb{R}$:

$$\min_{\hat{\boldsymbol{x}}} \ \|\hat{\boldsymbol{x}}\|_1 + \frac{\delta}{2}\|\hat{\boldsymbol{x}} - \hat{\boldsymbol{h}}\|_2^2$$

The solution to $\hat{\boldsymbol{x}}$:

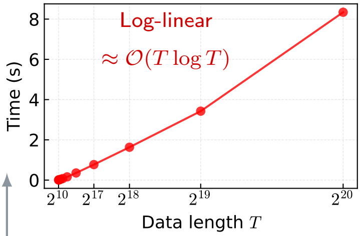
$$\hat{x}_t := \frac{\hat{h}_t}{|\hat{h}_t|} \cdot \underbrace{\max\{0, |\hat{h}_t| - 1/\delta\}}_{\text{shrinkage (e.g., ReLU)}}, t \in [T]$$

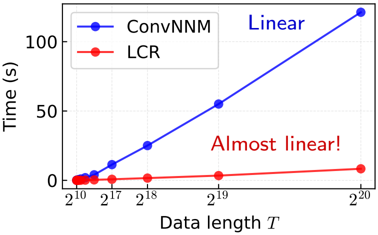
$$y_i = \frac{x_i}{|x_i|} \cdot \max\{|x_i| - 1/\alpha, 0\}$$



Laplacian Convolutional Representation

Time complexity & scalability & efficiency?





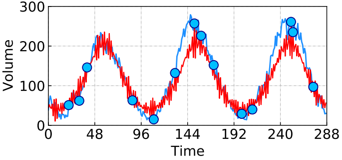
Empirical time complexity

On the synthetic data $\boldsymbol{y} \in \mathbb{R}^T$ with $T \in \{2^{10}, 2^{11}, \dots, 2^{20}\}$

Experiments

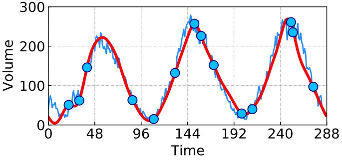
- Traffic speed imputation[2] (95% missing rate)

$\|\mathcal{C}(\boldsymbol{x})\|_*$



$\Downarrow$ Plus **local** time series trends

$\|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2$



Highlights:

- Rethink the importance of local trend modeling in traffic data imputation tasks.
- Find a unified global and local trend modeling framework whose optimization can be efficiently solved by FFT:

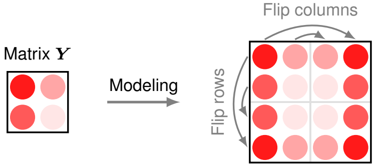
$$\min_{\boldsymbol{x}} \; \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{local}}$$

$$\text{s.t.} \;\; \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon$$

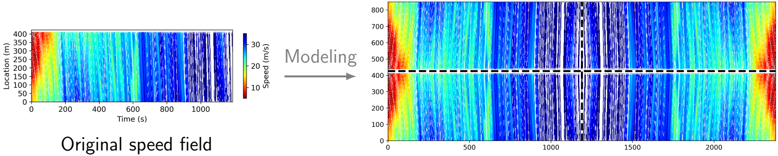
[2]Blue dot: partial observation; red line: imputation.

Speed field reconstruction[3]

- Flipping operation on a matrix:



- Flipping operation on a speed field of vehicular traffic flow:



Original speed field

Flipped speed field

[3]Highway Drone (HighD) dataset at https://www.highd-dataset.com/

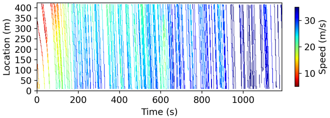
Experiments

Speed field reconstruction in German highways[4]

- Scenario: Mask trajectories of 70% vehicles
- LCR-2D on partially observed $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

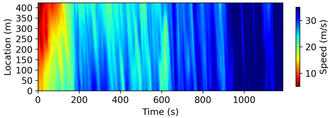
$$\min_{\boldsymbol{X}} \quad \underbrace{\|\mathcal{C}(\boldsymbol{X})\|_*}_{\text{global trend}} + \frac{\gamma}{2} \underbrace{\|(\boldsymbol{\ell}_s \boldsymbol{\ell}^\top) \star \boldsymbol{X}\|_F^2}_{\text{local trend}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{X} - \boldsymbol{Y})\|_F \leq \epsilon$$

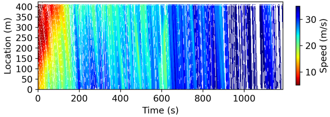


Sparse speed field

LCR-2D



Reconstructed speed field



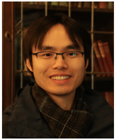
Ground-truth speed field

[4] Highway Drone (HighD) dataset at https://www.highd-dataset.com/

Quantifying Time Series Periodicity

(Ongoing Research)

❒ Interpretable ML ❒ Optimization w/ ℓ_0-norm ❒ Mixed-integer programming
❒ Human mobility regularity ❒ Climate system seasonality



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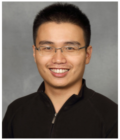

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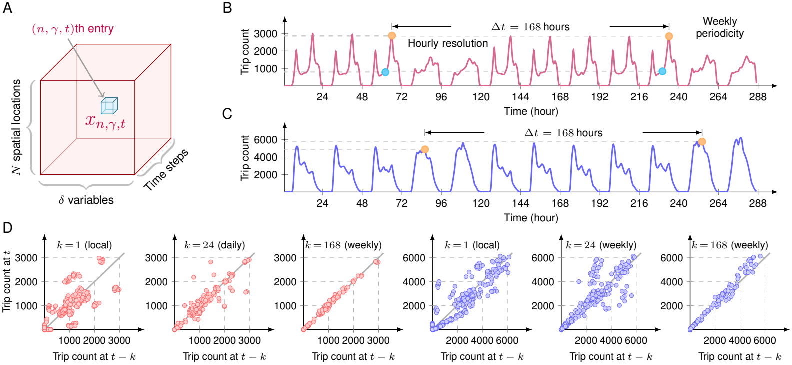
Motivation

Human mobility data show daily/weekly regularity and periodicity?

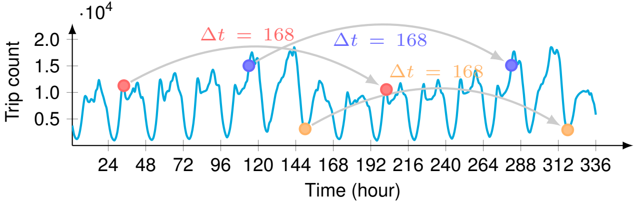


"**Closeness**" to the anti-diagonal $y = x$

$x_t \approx x_{t-168}$ (weekly periodicity)

Motivation

Weekly periodicity of ridesharing trip time series in Chicago



What motivate us most about **periodicity**?

❶ **Resilience and stability of systems**: Empirically measure the periodicity and predictability of urban systems.

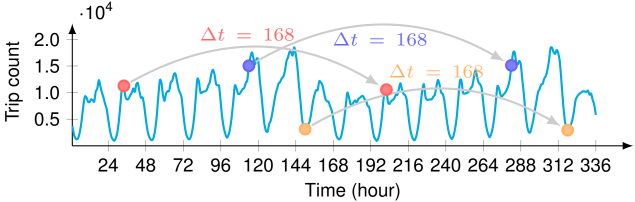
❷ **Optimization of transport systems**: Optimize resources (e.g., public transit, taxi, rideshare, and micromobility) to meet transport demand efficiently.

❸ **Design of sustainable transport & infrastructure**: Implement energy-efficient solutions tailored to peak hours.

Motivation

- Time series autoregression on $\boldsymbol{x} \in \mathbb{R}^T$

$$\boldsymbol{w} := \arg \min_{\{w_k\}_{k\in[d]}} \sum_{t\in[d+1,T]} \left(x_t - \sum_{k\in[d]} w_k x_{t-k} \right)^2$$



Periodicity of rideshare trip time series

- Sparse coefficient vector $\mapsto$ **Interpretability**?

$$\boldsymbol{w} = (\underbrace{0.33}_{k=1}, 0, \cdots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

Valorizing Autoregression

- Time series autoregression

$$\begin{aligned}\boldsymbol{w} &:= \arg\min_{\{w_k\}_{k\in[d]}} \sum_{t\in[d+1,T]} \Big(x_t - \sum_{k\in[d]} w_k x_{t-k} \Big)^2 \\ &= \arg\min_{\boldsymbol{w}} \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2\end{aligned}$$

$$\underbrace{\begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}}_{\tilde{\boldsymbol{x}}} \approx \underbrace{\begin{bmatrix} x_4 & x_3 & x_2 & x_1 \\ x_5 & x_4 & x_3 & x_2 \\ x_6 & x_5 & x_4 & x_3 \\ x_7 & x_6 & x_5 & x_4 \\ x_8 & x_7 & x_6 & x_5 \end{bmatrix}}_{\boldsymbol{A}} \times \underbrace{\left.\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}\right\} \|\boldsymbol{w}\|_0 \leq \tau}_{\boldsymbol{w}}$$

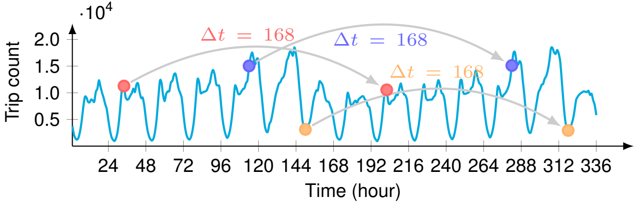
Autoregression on time series $\boldsymbol{x} = (x_1, x_2, \ldots, x_9)^\top$ w/ sparsity $\tau \in \mathbb{Z}^+$

- Sparse autoregression

$$\begin{aligned}&\min_{\boldsymbol{w}\geq 0} \ \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \\ &\text{s.t.} \ \underbrace{\|\boldsymbol{w}\|_0 \leq \tau}_{\text{sparsity w/ } \ell_0\text{-norm}}\end{aligned} \iff \begin{aligned}&\min_{\boldsymbol{w},\boldsymbol{\beta}} \ \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \\ &\text{s.t.} \ \begin{cases} 0 \leq \boldsymbol{w} \leq \boldsymbol{\beta}, \ \boldsymbol{\beta} \in \{0,1\}^d \\ \|\boldsymbol{\beta}\|_1 \leq \tau \end{cases}\end{aligned}$$

Solution Quality

- Subspace pursuit (SP) sometimes fails



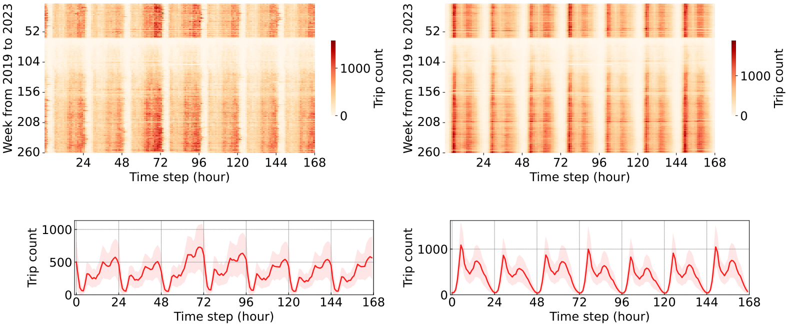
Periodicity of ridesharing trip time series

- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity $\tau = 2$):

$$\underbrace{\boldsymbol{w} = (\cdots, \underbrace{0.02}_{k=53}, \cdots, \underbrace{0.96}_{k=168})^\top}_{\text{loss func.} = 8.32 \times 10^7 \text{ (SP)}} \quad \text{vs.} \quad \underbrace{\boldsymbol{w} = (\underbrace{0.22}_{k=1}, \cdots, \underbrace{0.77}_{k=168})^\top}_{\text{loss func.} = 6.25 \times 10^7 \text{ (MIP)}}$$

John F. Kennedy International Airport

- Pickup/Dropoff trips in airport
  - Pickup trips are relevant to flight delay, baggage claim, and other factors.
  - Dropoff trips to airport are highly related to flight schedules.



Pickup trips from airport

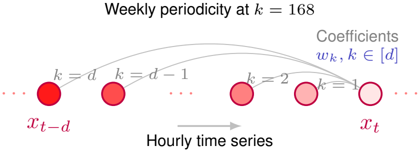
Dropoff trips to airport

- Sparse coefficient vectors (sparsity $\tau = 3$):

$$\boldsymbol{w} = (\underbrace{0.31}_{k=1}, \cdots, \underbrace{0.28}_{k=24}, \cdots, \underbrace{0.41}_{k=168})^\top \quad \text{vs.} \quad \boldsymbol{w} = (\underbrace{0.18}_{k=1}, \cdots, \underbrace{0.35}_{k=24}, \cdots, \underbrace{0.47}_{k=168})^\top$$

Spatially- and Time-Varying Autoregression

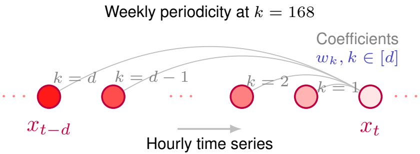
Univariate autoregression



$$\min \sum_t \left(x_t - \sum_{k \in [d]} w_k x_{t-k} \right)^2$$

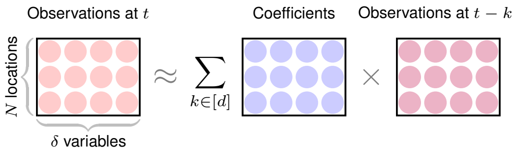
Spatially- and Time-Varying Autoregression

Univariate autoregression



$$\min \sum_{t} \Big(x_t - \sum_{k \in [d]} w_k x_{t-k} \Big)^2$$

Multidimensional autoregression



$$\min \sum_{n \in [N]} \sum_{\gamma \in [\delta]} \sum_{t} \Big(x_{n,\gamma,t} - \sum_{k \in [d]} w_{n,\gamma,k} x_{n,\gamma,t-k} \Big)^2$$

Envisioning Human Mobility

- Ridesharing trip data $\{\boldsymbol{x}_{n,\gamma}\}$ across $\gamma \in [\delta]$ years
- Reformulate sparse autoregression:

$$\min_{\{\boldsymbol{w}_{n,\gamma}\},\boldsymbol{\beta}} \sum_{n\in[N]} \sum_{\gamma\in[\delta]} \sum_{t\in[d+1,T_\gamma]} \Big(x_{n,\gamma,t} - \sum_{k\in[d]} w_{n,\gamma,k} x_{n,\gamma,t-k}\Big)^2$$

$$\text{s.t.} \quad \underbrace{\boldsymbol{\beta} \in \{0,1\}^d}_{\text{binary var.}} \quad \underbrace{0 \leq \boldsymbol{w}_{n,\gamma} \leq \boldsymbol{\beta}}_{\text{upper bound in } \{0,1\}} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\text{sum of binary var.}}$$

- MIP problem w/ $(N\delta + 1)d$ variables!
- How to handle thousands or millions of (e.g., $N\delta = 10^6$) time series?

Spatially- and Time-Varying Systems

- Rideshare trip data $\{\boldsymbol{x}_{n,\gamma}\}_{n\in[N],\gamma\in[\delta]}$ across $\gamma\in[\delta]$ months/years interpretable sparse autoregression:

$$\min_{\{\boldsymbol{w}_\gamma\}_{\gamma\in[\delta]}} \sum_{\gamma\in[\delta]} \|\tilde{\boldsymbol{x}}_\gamma - \boldsymbol{A}_\gamma \boldsymbol{w}_\gamma\|_2^2$$

$$\text{s.t. } \begin{cases} \boldsymbol{w}_\gamma \geq 0 & \text{(non-negativity)} \\ \|\boldsymbol{w}_\gamma\|_0 \leq \tau & \text{(sparsity)} \\ \operatorname{supp}(\boldsymbol{w}_\gamma) = \operatorname{supp}(\boldsymbol{w}_{\gamma+1}) & \text{(no local difference)} \end{cases}$$

  making these coefficient vectors comparable across δ months/years.

- Constraints w/ binary variables $\boldsymbol{\beta}_\gamma \in \{0,1\}^d$:

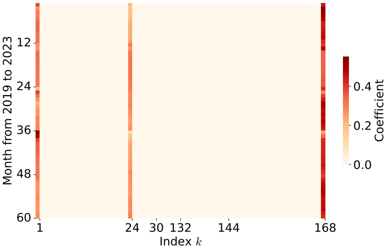
$$\underbrace{0 \leq \boldsymbol{w}_\gamma \leq \boldsymbol{\beta}_\gamma}_{\text{upper bound } \{0,1\}} \qquad \underbrace{\sum_{k\in[d]} \beta_{\gamma,k} \leq \tau}_{\text{sum of binary var.}} \qquad \underbrace{\boldsymbol{\beta}_\gamma - \boldsymbol{\beta}_{\gamma+1} = 0}_{\text{comparability across } \boldsymbol{w}_\gamma, \forall\gamma}$$

- MIP problem w/ $2d\delta$ decision variables!
- (**Efficiency?**) ML prunes the search space, e.g., $2\tau_0\delta$ decision variables ($\tau < \tau_0 \ll d$) instead.

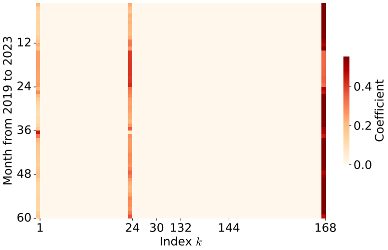
John F. Kennedy International Airport

- Coefficients $\{\boldsymbol{w}_\gamma\}_{\gamma\in[\delta]}$ at $S=\{\underbrace{1}_{\text{local}},\underbrace{24}_{\text{daily}},\underbrace{168}_{\text{weekly}}\}$ across $\delta=60$ months

  ❶ Stronger weekly periodicity of dropoff trips than pickup trips
  ❷ Stronger daily periodicity in 2020
  ❸ Weaker weekly periodicity in 2020



Pickup trips



Dropoff trips

- Identify system patterns that evolve over time for human mobility

Spatially- and Time-Varying Autoregression

longitude

latitude decade monthly time step

$$\min_{\{\boldsymbol{w}_{m,n,\gamma}\},\boldsymbol{\beta}} \sum_{m\in[M]}\sum_{n\in[N]}\sum_{\gamma\in[\delta]}\sum_{t\in[d+1,T_\gamma]}\Big(x_{m,n,\gamma,t} - \sum_{k\in[d]} w_{m,n,\gamma,k}x_{m,n,\gamma,t-k}\Big)^2$$

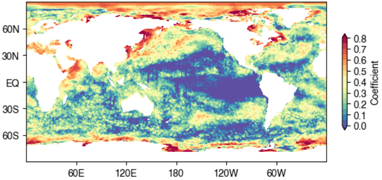
$$\text{s.t.} \begin{cases} \boldsymbol{\beta}\in\{0,1\}^d \\ 0\leq \boldsymbol{w}_{m,n,\gamma}\leq\boldsymbol{\beta},\ \forall m,n,\gamma \\ \|\boldsymbol{\beta}\|_1\leq\tau \\ \|\boldsymbol{w}_{m,n,\gamma}\|_1 = 1,\ \forall m,n,\gamma \end{cases}$$

binary decision var.

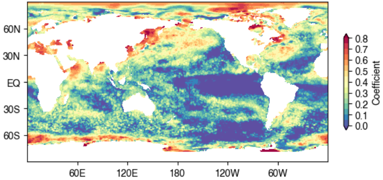
sparsity constraints

ℓ_1-normalization

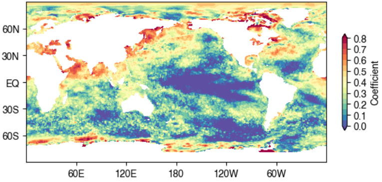
Sea Surface Temperature



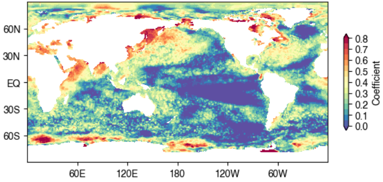
1980s



1990s



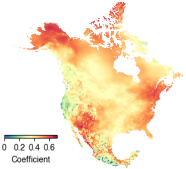
2000s



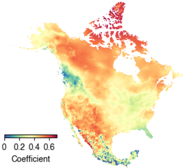
2010s

- Identify yearly periodicity at $k = 12$ from SST data ($\tau = 4$)
  1. The areas of El Nino events are less seasonal/predictable
  2. Arctic becomes less seasonal/predictable in the past 20 years
- Insights into climate change & global warming & sustainable development

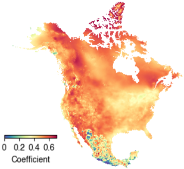
North America Temperature



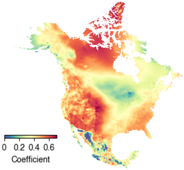
tmin, 1980s



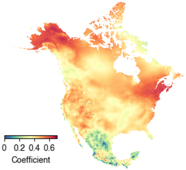
tmin, 1990s



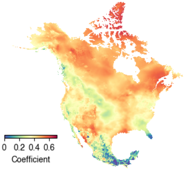
tmin, 2000s



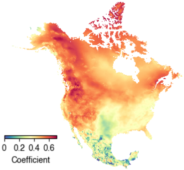
tmin, 2010s



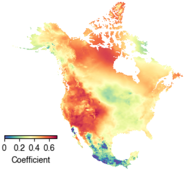
tmax, 1980s



tmax, 1990s



tmax, 2000s



tmax, 2010s

- Identify yearly periodicity at $k = 12$ from temperature data ($\tau = 4$)
  1. Stronger yearly seasonality in high-latitude areas
  2. Less seasonal temperature in south areas (e.g., Mexico)
  3. Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s

Thanks for your attention!

Any Questions?

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