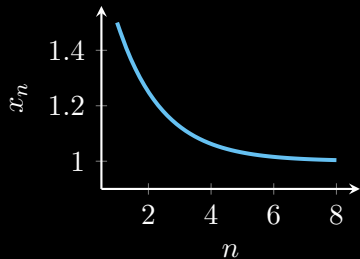# Three rates of convergence on a sequence $\{x_n\}_{n \in \mathbb{Z}^+}$

Linear convergence

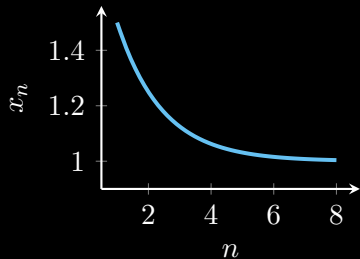$$\frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} \leq r, \, r \in (0, 1)$$

Sequence $x_n = 1 + \left(\frac{1}{2}\right)^n$
converges linearly to $x_\infty = 1$
because $r = \frac{1}{2}$

Three rates of convergence on a sequence $\{x_n\}_{n \in \mathbb{Z}^+}$
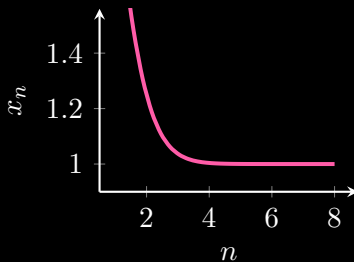
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Superlinear convergence

$$\lim_{n \to \infty} \frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} = 0$$
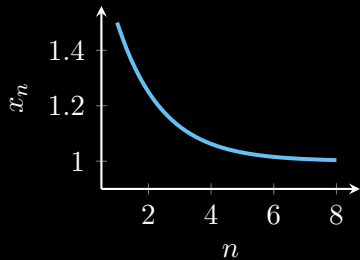

Sequence $x_n = 1 + \left(\frac{1}{n}\right)^n$
converges superlinearly to
$x_\infty = 1$

Three rates of convergence on a sequence $\{x_n\}_{n \in \mathbb{Z}^+}$
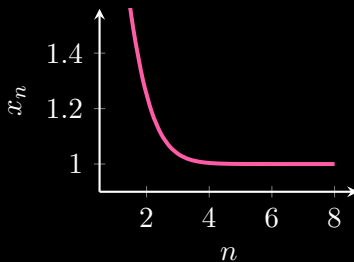
Linear convergence

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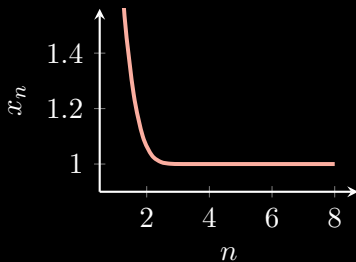
Superlinear convergence

$$\lim_{n \to \infty} \frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} = 0$$

Sequence $x_n = 1 + \left(\frac{1}{n}\right)^n$ converges superlinearly to $x_\infty = 1$

Quadratic convergence

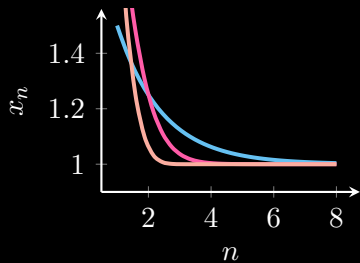$$\frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|^2} \leq M$$

Sequence $x_n = 1 + \left(\frac{1}{n}\right)^{2^n}$ converges quadratically to $x_\infty = 1$ because $M = 1$

Three rates of convergence on a sequence $\{x_n\}_{n\in\mathbb{Z}^+}$
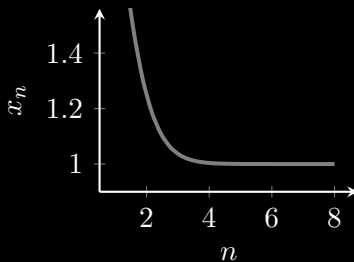
Linear convergence

$$\frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} \leq r, \ r \in (0,1)$$

Sequence $x_n = 1 + \left(\frac{1}{2}\right)^n$ converges linearly to $x_\infty = 1$ because $r = \frac{1}{2}$
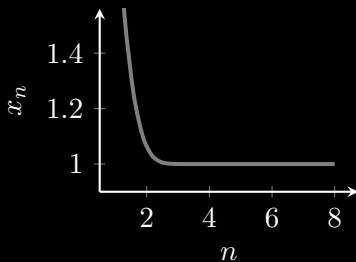
Superlinear convergence

$$\lim_{n\to\infty} \frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|} = 0$$

Sequence $x_n = 1 + \left(\frac{1}{n}\right)^n$ converges superlinearly to $x_\infty = 1$

Quadratic convergence

$$\frac{\|x_{n+1} - x_\infty\|}{\|x_n - x_\infty\|^2} \leq M$$

Sequence $x_n = 1 + \left(\frac{1}{n}\right)^{2^n}$ converges quadratically to $x_\infty = 1$ because $M = 1$

Thanks for your attention!

Any Questions?

About me:

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