Model Settings in Imputation Experiments

"Matrix and Tensor Model for Spatiotemporal Traffic Data Imputation and Forecasting"

Training Set, Validation Set, & Testing Set

In terms of Table 4.1 and 4.3 in the thesis, we separate the data into training set (e.g., 25% data), validation set (e.g., 5% data), and testing set (e.g., 70% data) in which validation set and testing set are randomly selected from the whole data points. As shown in Table 1, we consider **the best hyperparameters** $\gamma = 10\lambda$ **and** $r = 25$ **tuned on the validation set (i.e., 5% data). The imputation performance on the testing set (i.e., 70% data) is:**

$$\text{MAPE} = 6.21\% \qquad \text{RMSE} = 3.84$$

Compared to Table 1 on the validation set, Table 2 shows that the imputation performance on the testing set is very consistent with the validation set. This is because the validation set ($\approx 5\%$ data points) and the testing set ($\approx 70\%$ data points) are randomly selected from the original data. There should be no difference between the validation set and the testing set (see e.g., at 90% missing rate).

Table 1: Imputation performance (in MAPE/RMSE) of LATC with different truncation values and different ratios γ/λ on the Seattle freeway traffic speed dataset (on the 5% validation set). Note that the best results are highlighted in bold fonts.

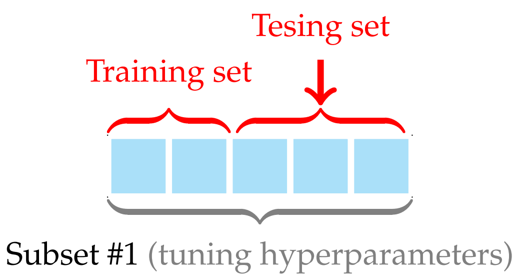
Missing rate	γ/λ	Truncation parameter				
		$r = 5$	$r = 10$	$r = 15$	$r = 20$	$r = 25$
70%, RM	1/10	7.84/4.52	7.20/4.25	6.82/4.08	6.60/3.98	6.41/3.92
	1/5	7.84/4.52	7.20/4.25	6.82/4.08	6.59/3.97	6.41/3.92
	1	7.81/4.51	7.18/4.24	6.80/4.07	6.58/3.97	6.39/3.91
	5	7.70/4.45	7.09/4.20	6.72/4.04	6.49/3.93	6.29/3.87
	10	7.59/4.39	7.00/4.16	6.64/4.00	6.41/3.89	**6.22**/**3.83**
90%, RM	1/10	10.39/5.77	9.45/5.41	9.32/5.36	9.38/5.39	9.50/5.48
	1/5	10.41/5.78	9.43/5.39	9.32/5.34	9.34/5.37	9.46/5.45
	1	10.35/5.75	9.36/5.36	9.18/5.29	9.21/5.31	9.32/5.37
	5	10.29/5.72	9.21/5.29	**8.91**/5.17	8.96/**5.16**	8.99/5.17
	10	-/-	-/-	-/-	-/-	-/-

Table 2: Imputation performance (in MAPE/RMSE) of LATC with different truncation values and different ratios γ/λ on the Seattle freeway traffic speed dataset (on the 70% testing set). Note that the best results are highlighted in bold fonts.

Missing rate	γ/λ	Truncation parameter				
		$r=5$	$r=10$	$r=15$	$r=20$	$r=25$
70%, RM	1/10	7.83/4.53	7.18/4.27	6.80/4.09	6.58/3.99	6.41/3.92
	1/5	7.83/4.53	7.18/4.26	6.80/4.09	6.57/3.98	6.40/3.92
	1	7.80/4.52	7.16/4.25	6.78/4.08	6.55/3.98	6.40/3.92
	5	7.70/4.47	7.08/4.21	6.70/4.04	6.46/3.94	6.29/3.87
	10	7.58/4.41	6.99/4.17	6.62/4.01	6.39/3.90	**6.21**/**3.84**
90%, RM	1/10	10.30/5.73	9.33/5.37	9.22/5.32	9.30/5.36	9.45/5.45
	1/5	10.32/5.74	9.32/5.35	9.19/5.30	9.26/5.35	9.41/5.42
	1	10.26/5.71	9.26/5.32	9.12/5.27	9.14/5.29	9.27/5.35
	5	10.16/5.68	9.12/5.26	**8.89**/5.15	8.91/**5.14**	8.93/5.15
	10	-/-	-/-	-/-	-/-	-/-

Another Way to Tune Hyperparameters

We consider a new mechanism for our missing traffic data imputation tasks (see the illustration below): 1) Suppose the traffic data can be separated into 10 fractions (e.g., 4-week data); 2) Separate the first 5 fractions (**subset #1**, e.g., the data in the first two weeks) into the training set and the validation set, helping tune hyperparameters; 3) Separate the last 5 fractions (**subset #2**, e.g., the data in the last two weeks) into the training set and the testing set, allowing one to report imputation results on this subset. In the experiment setting, both subsets preserve the same missing rate level.



How to Select Hyperparameters in LATC and Baseline Imputation Methods?

Table 3 shows that the best hyperparameters for LATC on the Seattle freeway traffic speed dataset are:

- 30%, RM: $\gamma = 10\lambda$ and $r = 25$;
- 70%, RM: $\gamma = 10\lambda$ and $r = 25$;
- 90%, RM: $\gamma = 10\lambda$ and $r = 20$;
- 30%, NM: $\gamma = 5\lambda$ and $r = 15$;
- 70%, RM: $\gamma = 10\lambda$ and $r = 10$;
- 30%, BM-12: $\gamma = 10\lambda$ and $r = 25$.

Table 4 shows that the best hyperparameters for LAMC on the Seattle freeway traffic speed dataset are:

- 30%, RM: $\gamma = 10\lambda$ and $r = 20$;
- 70%, RM: $\gamma = \lambda$ and $r = 5$;
- 90%, RM: $\gamma = 10\lambda$ and $r = 10$;
- 30%, NM: $\gamma = 10\lambda$ and $r = 10$;
- 70%, NM: $\gamma = 10\lambda$ and $r = 5$;
- 30%, BM-12: $\gamma = 10\lambda$ and $r = 25$.

Table 5 shows that the best hyperparameters for LRTC-TNN on the Seattle freeway traffic speed dataset are:

- 30%/70%, RM: $r = 25$;
- 90%, RM: $r = 10$;

Table 3: Imputation performance (in MAPE/RMSE) of LATC with different truncation values and different ratios γ/λ on the Seattle freeway traffic speed dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts. The number next to the BM denotes the window length.

Missing rate	γ/λ	Truncation parameter				
		$r=5$	$r=10$	$r=15$	$r=20$	$r=25$
30%, RM	1/10	5.19/3.33	4.93/3.22	4.77/3.16	4.69/3.13	4.63/3.11
	1/5	5.19/3.32	4.93/3.22	4.77/3.16	4.69/3.13	4.63/3.10
	1	5.18/3.32	4.92/3.22	4.77/3.16	4.68/3.12	4.62/3.10
	5	5.13/3.30	4.88/3.20	4.74/3.14	4.65/3.11	4.59/3.09
	10	5.09/3.27	4.85/3.19	4.71/3.13	4.63/3.10	**4.57**/**3.08**
70%, RM	1/10	6.64/4.11	6.14/3.88	5.87/3.77	5.77/3.73	5.73/3.72
	1/5	6.64/4.11	6.13/3.88	5.87/3.77	5.77/3.73	5.73/3.72
	1	6.62/4.10	6.12/3.87	5.86/3.76	5.75/3.72	5.71/3.71
	5	6.55/4.06	6.07/3.84	5.80/3.73	5.69/3.69	5.63/3.67
	10	6.48/4.02	6.01/3.81	5.75/3.71	5.63/3.66	**5.56**/**3.64**
90%, RM	1/10	8.33/4.99	7.60/4.69	7.45/4.65	7.54/4.72	7.68/4.81
	1/5	8.33/4.99	7.60/4.69	7.44/4.64	7.54/4.71	7.67/4.81
	1	8.34/4.99	7.57/4.67	7.39/4.62	7.46/4.67	7.57/4.76
	5	8.26/4.95	7.47/4.62	7.22/4.53	7.22/4.55	7.29/4.61
	10	8.22/4.92	7.41/4.58	7.12/**4.48**	**7.08**/**4.48**	7.10/4.51
30%, NM	1/10	7.69/4.65	7.30/4.49	7.16/4.46	7.11/4.51	7.09/4.55
	1/5	7.69/4.65	7.30/4.49	7.16/4.46	7.10/4.50	7.09/4.57
	1	7.69/4.65	7.29/4.49	7.17/4.46	7.10/4.50	7.07/4.53
	5	7.71/4.65	7.32/4.49	7.15/**4.45**	7.09/4.48	7.05/4.50
	10	7.73/4.66	7.32/4.50	7.18/**4.45**	7.09/4.46	**7.04**/4.48
70%, NM	1/10	12.54/**11.35**	12.49/11.38	12.55/11.41	12.64/11.44	12.71/11.48
	1/5	12.54/**11.35**	12.52/11.40	12.56/11.41	12.65/11.44	12.68/11.47
	1	12.54/**11.35**	12.48/11.38	12.55/11.41	12.63/11.44	12.66/11.46
	5	12.55/**11.35**	12.50/11.38	12.53/11.38	12.59/11.40	12.63/11.43
	10	12.56/**11.35**	**12.41**/**11.35**	12.50/11.37	12.56/11.40	12.58/11.41
30%, BM-12	1/10	8.35/5.08	8.29/5.05	8.32/5.09	8.36/5.11	8.41/5.15
	1/5	8.35/5.08	8.28/5.05	8.28/5.07	8.33/5.10	8.37/5.13
	1	8.34/5.07	8.27/5.04	8.25/5.05	8.28/5.08	8.34/5.12
	5	8.28/5.03	8.22/5.01	8.20/5.00	8.18/5.00	8.20/5.01
	10	8.23/5.00	8.17/4.97	8.15/**4.96**	8.16/4.97	**8.14**/**4.96**

- 30%/70%, NM: $r=5$;
- 30%, BM-12: $r=5$.

Table 6 summarizes the imputation performance of LATC and baseline models on the Seattle freeway traffic speed dataset (i.e., on the last two-week data). Of these results, LATC outperforms LRTC-TNN, highlighting the importance of temporal autoregression. In most cases, LATC performs better than LAMC, demonstrating the significance of utilizing the tensor data structure.

Table 4: Imputation performance (in MAPE/RMSE) of LAMC with different truncation values and different ratios γ/λ on the Seattle freeway traffic speed dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts. The number next to the BM denotes the window length.

Missing rate	γ/λ	Truncation parameter				
		$r=5$	$r=10$	$r=15$	$r=20$	$r=25$
30%, RM	1/10	5.60/3.59	5.51/3.56	5.46/3.55	5.42/3.55	5.41/3.57
	1/5	5.60/3.59	5.51/3.56	5.46/3.55	5.42/3.55	5.41/3.57
	1	5.59/3.59	5.51/3.56	5.45/3.55	5.41/3.55	5.41/3.56
	5	5.58/3.58	5.49/3.55	5.43/3.53	5.38/3.53	5.37/3.54
	10	5.57/3.57	5.46/3.53	5.40/3.51	5.35/**3.50**	**5.33**/3.51
70%, RM	1/10	7.34/4.54	7.31/4.58	7.55/4.83	7.89/5.13	8.39/5.59
	1/5	7.34/4.54	7.31/4.58	7.54/4.82	7.89/5.14	8.37/5.54
	1	7.33/**4.53**	**7.29**/4.57	7.48/4.77	7.89/5.12	8.28/5.49
	5	-/-	-/-	-/-	-/-	-/-
	10	-/-	-/-	-/-	-/-	-/-
90%, RM	1/10	9.90/6.02	11.77/7.61	13.71/8.94	15.48/9.98	16.69/10.56
	1/5	9.81/5.97	11.54/7.30	13.08/8.34	14.47/9.21	15.55/9.79
	1	9.34/5.66	9.80/6.05	10.81/6.72	11.47/7.12	12.14/7.51
	5	8.65/5.24	8.36/5.13	8.53/5.27	8.79/5.44	8.96/5.57
	10	8.41/5.06	7.94/**4.88**	**7.90**/4.90	7.99/4.95	8.07/5.04
30%, NM	1/10	7.04/4.37	7.01/4.37	7.08/4.43	7.34/4.91	7.55/5.19
	1/5	7.04/4.37	7.00/4.37	7.07/4.44	7.36/4.89	7.51/5.21
	1	7.04/**4.36**	7.01/4.37	7.07/4.43	7.37/4.94	7.50/5.18
	5	7.04/**4.36**	7.00/**4.36**	7.04/4.42	7.31/4.85	7.47/5.09
	10	7.03/**4.36**	**6.99**/**4.36**	7.06/4.42	7.26/4.74	7.41/5.02
70%, NM	1/10	13.04/11.84	13.89/12.16	14.53/12.49	15.28/12.79	15.51/12.90
	1/5	13.23/11.84	14.11/12.19	14.62/12.55	15.33/12.81	15.33/12.84
	1	12.95/11.70	13.85/12.15	14.76/12.61	15.34/12.81	15.40/12.86
	5	12.90/11.71	13.52/11.98	14.46/12.38	14.90/12.62	15.12/12.69
	10	**12.76**/**11.58**	13.60/12.00	14.40/12.39	14.66/12.44	14.87/12.52
30%, BM-12	1/10	29.96/15.63	29.92/15.62	29.84/15.60	29.82/15.60	29.80/15.59
	1/5	38.64/22.22	38.59/22.20	38.55/22.20	38.46/22.18	38.43/22.18
	1	59.28/38.65	59.12/38.63	59.07/38.64	59.03/38.64	58.99/38.63
	5	33.72/25.90	33.37/35.89	33.23/25.91	33.08/25.90	32.99/25.89
	10	18.56/13.98	18.17/13.98	17.99/13.98	17.83/**13.94**	**17.74**/**13.94**

Table 5: Imputation performance (in MAPE/RMSE) of LRTC-TNN with different truncation values on the Seattle freeway traffic speed dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts. The number next to the BM denotes the window length.

Missing rate	Truncation parameter				
	$r=5$	$r=10$	$r=15$	$r=20$	$r=25$
30%, RM	5.24/3.35	4.97/3.25	4.80/3.18	4.71/3.14	**4.63**/**3.11**
70%, RM	6.70/4.14	6.16/3.90	5.86/3.77	5.73/3.71	**5.65**/**3.68**
90%, RM	8.41/5.04	7.66/**4.73**	**7.60**/4.74	7.83/4.91	8.12/5.09
30%, NM	**7.70**/**4.64**	15.40/17.04	19.29/20.32	-/-	-/-
70%, NM	**24.44**/**21.76**	79.33/49.93	-/-	-/-	-/-
30%, BM-12	**8.38**/**5.08**	46.05/35.61	79.33/50.27	-/-	-/-

Table 6: Imputation performance comparison (in MAPE/RMSE) of LATC and baseline models on the Seattle freeway traffic speed dataset (on the last two-week data for reporting results). Note that the best results are highlighted in bold fonts.

Missing rate	LATC	LAMC	LRTC-TNN	BTMF	SPC
30%, RM	**5.18**/**3.23**	6.20/3.74	5.27/3.27	-/-	-/-
70%, RM	**6.40**/**3.83**	8.77/4.93	6.54/3.89	-/-	-/-
90%, RM	**8.32**/**4.80**	9.63/5.34	9.16/5.11	-/-	-/-
30%, NM	8.29/4.81	**8.09**/**4.70**	8.97/5.04	-/-	-/-
70%, NM	**12.52**/**8.04**	13.15/8.46	23.58/17.78	-/-	-/-
30%, BM-12	**11.77**/**6.20**	22.10/14.67	12.07/6.23	-/-	-/-

Table 7 shows that the best hyperparameters for LATC on the Portland traffic volume dataset are:

- 30%, RM: $\gamma = \lambda$ and $r = 10$;
- 70%, RM: $\gamma = \lambda$ and $r = 25$;
- 90%, RM: $\gamma = \lambda$ and $r = 10$;
- 30%, NM: $\gamma = \lambda/5$ and $r = 10$;
- 70%, NM: $\gamma = \lambda/10$ and $r = 15$;
- 30%, BM-4: $\gamma = \lambda$ and $r = 5$.

Table 8 shows that the best hyperparameters for LAMC on the Portland traffic volume dataset are:

- 30%, RM: $\gamma = 10\lambda$ and $r = 25$;
- 70%, RM: $\gamma = 10\lambda$ and $r = 20$;
- 90%, RM: $\gamma = 10\lambda$ and $r = 10$;
- 30%, NM: $\gamma = 10\lambda$ and $r = 10$;
- 70%, NM: $\gamma = \lambda/5$ and $r = 5$;
- 30%, BM-4: $\gamma = 10\lambda$ and $r = 25$.

Table 9 shows that the best hyperparameters for LRTC-TNN on the Portland traffic volume dataset are:

- 30%, RM: $r = 25$;
- 70%, RM: $r = 15$;
- 90%, RM: $r = 5$;
- 30%, NM: $r = 10$;
- 70%, NM: $r = 5$;
- 30%, BM-4: $r = 5$.

Table 10 summarizes the imputation performance of LATC and baseline models on the Portland highway traffic volume dataset (i.e., on the last 17-day data). As can be seen, the proposed LATC model outperforms LAMC and LRTC-TNN in most cases.

Table 7: Imputation performance (in MAPE/RMSE) of LATC with different truncation values and different ratios γ/λ on the Portland traffic volume dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts. The number next to the BM denotes the window length.

Missing rate	γ/λ	Truncation parameter				
		$r=5$	$r=10$	$r=15$	$r=20$	$r=25$
30%, RM	1/10	19.50/17.43	19.13/16.97	19.04/17.11	18.97/17.23	18.85/17.41
	1/5	19.49/17.41	19.12/16.96	19.03/17.05	18.95/17.16	18.83/17.26
	1	19.40/17.28	19.03/**16.88**	18.93/16.97	18.84/16.97	18.73/16.99
	5	19.25/17.56	18.96/17.41	18.83/17.44	18.76/17.40	**18.65**/17.35
	10	19.28/18.23	19.06/18.22	18.94/18.26	18.88/18.23	18.79/18.20
70%, RM	1/10	21.79/20.34	21.15/20.08	20.99/19.75	20.89/19.59	20.91/19.58
	1/5	21.77/20.31	21.10/19.87	20.95/19.65	20.85/19.51	20.84/19.51
	1	21.66/20.21	20.99/19.76	20.84/19.49	20.71/19.38	20.68/**19.36**
	5	21.48/20.43	20.97/20.28	20.83/20.36	20.76/20.28	**20.67**/20.35
	10	21.40/20.83	21.01/20.92	20.93/21.09	20.87/21.07	20.81/21.15
90%, RM	1/10	25.19/24.49	24.71/24.39	24.83/24.50	25.03/24.74	25.41/24.95
	1/5	25.16/24.46	24.62/24.17	24.65/24.12	24.89/24.35	25.25/24.53
	1	25.17/24.58	24.35/**24.00**	24.48/24.07	24.56/24.20	24.81/24.35
	5	25.01/24.86	24.22/24.90	24.26/25.11	24.31/25.26	24.46/25.42
	10	24.91/25.50	24.29/26.80	**24.12**/27.53	24.18/27.84	24.44/28.24
30%, NM	1/10	22.01/22.61	21.62/22.65	21.67/23.52	21.81/23.75	21.82/23.87
	1/5	22.01/22.65	**21.55**/**22.60**	21.69/23.70	21.84/23.87	21.82/23.72
	1	22.05/22.75	21.64/23.45	25.24/52.86	25.46/52.94	25.39/52.78
	5	22.37/23.56	26.75/55.29	51.87/124.84	52.52/125.87	52.89/125.99
	10	22.88/24.44	38.10/89.49	64.03/148.00	64.31/148.53	64.86/148.72
70%, NM	1/10	52.17/108.21	57.34/112.18	**48.04**/**97.43**	48.29/97.47	48.51/97.46
	1/5	53.33/110.79	59.89/116.05	54.61/109.94	54.75/109.95	55.02/109.99
	1	51.04/105.28	72.48/138.43	73.73/143.10	73.78/143.10	74.08/143.09
	5	55.95/114.62	82.36/157.25	86.18/161.35	85.83/161.71	85.39/161.38
	10	53.63/109.04	80.04/156.88	90.15/168.07	89.45/168.49	89.16/168.30
30%, BM-4	1/10	31.18/57.15	33.34/75.29	34.16/79.52	34.39/79.76	34.49/79.87
	1/5	26.96/40.67	31.16/65.41	31.74/69.04	31.99/69.27	31.98/69.40
	1	24.31/**23.07**	26.25/42.89	26.19/44.60	26.37/44.79	26.33/44.87
	5	23.55/23.08	23.44/23.73	23.52/25.98	23.53/26.10	23.70/26.28
	10	23.29/23.35	**23.15**/24.31	23.41/25.77	23.50/25.89	23.76/26.05

Table 8: Imputation performance (in MAPE/RMSE) of LAMC with different truncation values and different ratios γ/λ on the Portland traffic volume dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts. The number next to the BM denotes the window length.

Missing rate	γ/λ	Truncation parameter				
		$r = 5$	$r = 10$	$r = 15$	$r = 20$	$r = 25$
30%, RM	1/10	21.37/20.35	21.44/20.95	21.31/20.44	21.18/20.43	21.06/20.16
	1/5	21.35/20.14	21.35/20.30	21.28/19.75	21.15/19.80	21.03/19.66
	1	21.42/19.80	21.24/19.52	21.10/18.87	20.95/18.62	20.86/18.42
	5	20.97/18.20	20.86/17.72	20.80/17.53	20.63/17.33	20.50/17.31
	10	20.77/17.73	20.62/17.33	20.56/17.26	20.43/17.11	**20.32**/**17.04**
70%, RM	1/10	24.17/23.37	24.32/23.26	24.46/23.78	24.45/24.37	24.79/25.40
	1/5	24.14/22.54	24.15/23.46	24.25/23.58	24.56/23.95	24.80/24.12
	1	23.92/21.82	23.90/21.57	23.91/21.70	23.92/21.47	24.17/21.93
	5	23.43/20.66	23.15/20.45	23.18/20.31	23.17/20.26	23.13/20.23
	10	23.04/20.31	22.76/20.29	22.73/20.10	**22.60**/**19.79**	**22.60**/19.82
90%, RM	1/10	29.90/34.26	30.59/33.76	32.69/35.83	35.29/38.35	38.13/40.53
	1/5	29.47/33.36	29.96/30.14	31.74/32.04	34.29/34.75	36.50/36.87
	1	28.61/26.96	29.06/26.93	30.54/28.12	31.97/29.44	34.16/31.03
	5	-/-	-/-	-/-	-/-	-/-
	10	26.83/24.97	**26.38**/**24.51**	26.81/25.17	27.52/26.05	28.36/26.63
30%, NM	1/10	23.69/28.48	23.66/28.80	24.18/30.46	24.57/31.01	24.80/32.23
	1/5	23.64/27.54	23.59/27.79	23.93/27.42	24.35/28.30	24.82/29.29
	1	23.40/30.17	23.29/27.75	23.77/27.47	24.01/28.63	-/-
	5	23.17/29.11	23.18/26.16	23.60/25.70	23.81/26.35	24.52/27.79
	10	23.06/28.04	**23.04**/**25.30**	23.49/25.45	23.47/25.52	23.73/25.55
70%, NM	1/10	31.20/46.90	31.80/43.49	32.41/43.04	33.04/43.02	33.52/43.22
	1/5	31.88/**41.70**	31.30/47.18	32.42/47.11	32.82/47.06	33.29/46.97
	1	30.80/44.07	30.69/51.23	31.41/51.07	31.73/51.14	32.62/51.20
	5	31.33/48.77	30.27/52.19	30.80/52.02	31.40/51.92	31.94/51.87
	10	**29.53**/49.50	29.81/52.22	-/-	31.30/50.92	31.49/50.59
30%, BM-4	1/10	-/-	-/-	-/-	-/-	-/-
	1/5	-/-	-/-	-/-	-/-	-/-
	1	44.91/58.60	42.21/57.89	41.39/57.50	40.69/57.30	40.34/57.13
	5	36.02/30.81	32.68/29.24	31.78/28.85	31.31/28.60	30.85/28.43
	10	34.08/28.80	30.93/27.39	30.02/27.04	29.57/26.85	**29.28**/**26.72**

Table 9: Imputation performance (in MAPE/RMSE) of LRTC-TNN with different truncation values on the Portland traffic volume dataset (on the first two-week subset for tuning hyper-parameters). Note that the best results are highlighted in bold fonts. The number next to the BM denotes the window length.

Missing rate	Truncation parameter				
	$r = 5$	$r = 10$	$r = 15$	$r = 20$	$r = 25$
30%, RM	19.90/17.42	19.56/16.97	19.42/16.93	19.34/16.80	**19.22**/**16.75**
70%, RM	21.89/20.19	21.24/19.70	21.08/**19.41**	**21.02**/19.52	**21.02**/19.88
90%, RM	24.97/**24.38**	**24.09**/24.78	24.42/26.80	25.08/29.36	26.29/32.37
30%, NM	**22.26**/**22.40**	22.39/39.77	56.22/136.73	80.00/170.10	92.23/180.96
70%, NM	**37.49**/**75.24**	73.64/144.96	95.50/173.41	-/-	-/-
30%, BM-4	**26.88**/**39.32**	50.35/118.94	83.45/160.64	91.43/173.29	92.70/179.45

Table 10: Imputation performance comparison (in MAPE/RMSE) of LATC and baseline models on the Portland highway traffic volume dataset (on the last 17-day data for reporting results). Note that the best results are highlighted in bold fonts.

Missing rate	LATC	LAMC	LRTC-TNN	BTMF	SPC
30%, RM	**17.57**/**15.81**	18.60/16.00	17.79/16.92	-/-	-/-
70%, RM	**19.60**/**19.20**	21.48/19.76	20.17/19.52	-/-	-/-
90%, RM	**23.84**/**23.78**	25.60/25.55	24.81/24.67	-/-	-/-
30%, NM	**20.77**/21.42	22.00/28.48	20.94/**19.81**	-/-	-/-
70%, NM	50.98/108.83	**29.94**/**57.84**	32.47/64.49	-/-	-/-
30%, BM-4	**24.80**/**24.51**	33.40/29.26	29.71/69.57	-/-	-/-

How to Select Hyperparameters in LCR and Baseline Imputation Methods?

Table 11 shows that the best hyperparameters for LCR-2D on the PeMS traffic speed dataset are with $\lambda = 10^{-5}NT$:

- 30%/50%, RM: $\gamma = 10\lambda$ and $\tau = 1$;
- 70%, RM: $\gamma = 5\lambda$ and $\tau = 2$;
- 90%, RM: $\gamma = 5\lambda$ and $\tau = 3$.

Table 11: Imputation performance (in MAPE/RMSE) of LCR-2D with different hyperparameters on the PeMS traffic speed dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts.

Missing rate	γ/λ	$\lambda = 10^{-5}NT$			$\lambda = 10^{-6}NT$		
		$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 1$	$\tau = 2$	$\tau = 3$
30%, RM	1/10	2.12/1.77	1.99/1.70	1.89/1.67	2.15/1.85	2.15/1.85	2.14/1.85
	1/5	2.06/1.73	1.87/1.64	1.78/1.63	2.15/1.85	2.14/1.84	2.12/1.84
	1	1.80/1.60	1.61/1.54	1.64/1.63	2.14/1.84	2.11/1.83	2.10/1.86
	5	1.56/1.50	1.53/1.55	1.70/1.72	2.12/1.83	2.10/1.85	2.14/1.93
	10	**1.50**/**1.49**	1.56/1.58	1.77/1.79	2.11/1.83	2.10/1.87	2.18/1.99
50%, RM	1/10	2.49/2.04	2.30/1.93	2.15/1.87	2.52/2.13	2.51/2.12	2.50/2.12
	1/5	2.40/1.99	2.15/1.86	2.00/1.81	2.52/2.13	2.50/2.12	2.47/2.11
	1	2.12/1.84	1.83/1.72	1.80/1.77	2.50/2.11	2.46/2.09	2.43/2.10
	5	1.85/1.72	1.70/**1.71**	1.83/1.85	2.47/2.10	2.43/2.09	2.44/2.16
	10	1.76/**1.69**	1.71/1.73	1.90/1.92	2.45/2.09	2.42/2.11	2.47/2.20
70%, RM	1/10	3.11/2.49	2.83/2.33	2.61/2.22	3.18/2.61	3.16/2.60	3.12/2.58
	1/5	2.99/2.43	2.65/2.24	2.43/2.14	3.17/2.60	3.13/2.58	3.09/2.56
	1	2.69/2.27	2.28/2.07	2.14/2.06	3.13/2.58	3.06/2.54	3.02/2.53
	5	2.40/2.13	2.08/**2.02**	2.11/2.12	3.08/2.55	3.02/2.53	3.01/2.56
	10	2.29/2.08	**2.05**/2.04	2.16/2.18	3.06/2.54	3.01/2.53	3.02/2.59
30%, RM	1/10	4.73/3.63	4.28/3.37	3.96/3.20	4.87/3.79	4.79/3.74	4.72/3.70
	1/5	4.59/3.55	4.10/3.27	3.76/3.11	4.84/3.77	4.74/3.71	4.68/3.68
	1	4.27/3.38	3.73/3.10	3.41/2.98	4.76/3.72	4.67/3.67	4.62/3.65
	5	3.97/3.23	3.43/2.99	3.20/**2.96**	4.70/3.69	4.63/3.65	4.60/3.65
	10	3.84/3.17	3.32/**2.96**	**3.16**/2.98	4.68/3.68	4.61/3.65	4.60/3.66

Table 12 shows that the best hyperparameters for LCR on the PeMS traffic speed dataset are with $\lambda = 10^{-3}T$:

- 30%/50%, RM: $\gamma = 10\lambda$ and $\tau = 1$;
- 70%, RM: $\gamma = 5\lambda$ and $\tau = 2$;
- 90%, RM: $\gamma = 5\lambda$ and $\tau = 3$.

Table 13 shows that the best hyperparameters for LCR on the PeMS traffic speed dataset are with $\lambda = 10^{-5}NT$:

- 30%/50%, RM: $\gamma = 10\lambda$ and $\tau = 1$;

Table 12: Imputation performance (in MAPE/RMSE) of LCR$_N$ with different hyperparameters on the PeMS traffic speed dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts.

Missing rate	γ/λ	$\lambda=10^{-3}T$			$\lambda=10^{-4}T$		
		$\tau=1$	$\tau=2$	$\tau=3$	$\tau=1$	$\tau=2$	$\tau=3$
30%, RM	1/10	1.97/1.80	1.85/1.71	1.79/1.68	2.16/1.93	2.15/1.92	2.14/1.92
	1/5	1.91/1.75	1.76/1.65	1.71/1.65	2.15/1.93	2.14/1.91	2.12/1.90
	1	1.70/1.62	1.57/1.56	1.62/1.64	2.13/1.91	2.09/1.88	2.08/1.89
	5	1.52/1.53	1.52/1.56	1.70/1.72	2.09/1.88	2.06/1.87	2.11/1.95
	10	**1.48/1.51**	1.55/1.59	1.77/1.79	2.08/1.87	2.06/1.89	2.15/2.01
50%, RM	1/10	2.36/2.11	2.18/1.98	2.05/1.90	2.57/2.26	2.55/2.25	2.53/2.23
	1/5	2.28/2.05	2.05/1.90	1.93/1.84	2.56/2.26	2.53/2.23	2.49/2.21
	1	2.04/1.90	1.79/1.75	1.78/1.79	2.52/2.23	2.46/2.18	2.42/2.17
	5	1.81/1.77	**1.70/1.73**	1.83/1.86	2.47/2.18	2.40/2.15	2.42/2.20
	10	1.74/1.74	1.71/1.75	1.90/1.93	2.45/2.17	2.39/2.15	2.44/2.24
70%, RM	1/10	3.07/2.68	2.78/2.48	2.56/2.34	3.34/2.87	3.30/2.84	3.25/2.80
	1/5	2.95/2.61	2.61/2.37	2.39/2.24	3.33/2.86	3.26/2.81	3.19/2.76
	1	2.66/2.43	2.26/2.18	2.13/2.13	3.25/2.80	3.13/2.72	3.06/2.67
	5	2.38/2.27	2.08/**2.10**	2.11/2.16	3.16/2.74	3.05/2.66	3.01/2.66
	10	2.28/2.21	**2.05/2.10**	2.16/2.21	3.12/2.71	3.02/2.65	3.02/2.68
90%, RM	1/10	5.08/4.21	4.53/3.86	4.15/3.63	5.62/4.51	5.47/4.39	5.33/4.29
	1/5	4.91/4.11	4.32/3.75	3.93/3.52	5.56/4.46	5.36/4.32	5.21/4.21
	1	4.53/3.90	3.91/3.53	3.54/3.32	5.39/4.34	5.16/4.18	5.03/4.09
	5	4.18/3.70	3.57/3.35	3.29/3.21	5.23/4.23	5.03/4.09	4.93/4.03
	10	4.04/3.62	3.45/3.29	**3.24/3.20**	5.18/4.19	4.99/4.06	4.91/4.02

- 70%, RM: $\gamma=5\lambda$ and $\tau=2$;
- 90%, RM: $\gamma=5\lambda$ and $\tau=3$.

Table 14 shows that the best hyperparameter for CTNNM and CircNNM on the PeMS traffic speed dataset is:

- 30%/50%, RM: $\lambda=10^{-5}NT$;
- 70%/90%, RM: $\lambda=10^{-6}NT$.

Table 15 shows that the best hyperparameter for HaLRTC on the PeMS traffic speed dataset is $\lambda=10^{-5}$, while the best hyperparameters for LRTC-TNN are:

- 30%, RM: $\lambda=10^{-5}$ and $r=25$;
- 50%, RM: $\lambda=10^{-5}$ and $r=25$;
- 70%, RM: $\lambda=10^{-5}$ and $r=25$;
- 90%, RM: $\lambda=10^{-5}$ and $r=25$.

Table 16 summarizes the imputation performance of LCR models and baseline models on the PeMS-4W traffic speed dataset (i.e., on the last two-week data). As can be seen, LCR models (e.g., LCR-2D and LCR) perform better than the circulant matrix/tensor nuclear norm minimization models (e.g., CTNNM and CircNNM), highlighting the importance of temporal regularization.

Table 13: Imputation performance (in MAPE/RMSE) of LCR with different hyperparameters on the PeMS traffic speed dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts.

Missing rate	γ/λ	$\lambda = 10^{-5}NT$			$\lambda = 10^{-6}NT$		
		$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 1$	$\tau = 2$	$\tau = 3$
30%, RM	1/10	2.13/1.77	1.99/1.70	1.89/1.67	2.16/1.85	2.15/1.85	2.14/1.85
	1/5	2.06/1.74	1.87/1.64	1.78/1.63	2.16/1.85	2.15/1.85	2.13/1.85
	1	1.81/1.60	1.61/1.54	1.64/1.63	2.15/1.85	2.12/1.84	2.11/11.86
	5	1.56/1.50	1.54/1.55	1.70/1.72	2.13/1.84	2.10/1.85	2.14/1.93
	10	**1.50**/**1.49**	1.56/1.58	1.78/1.79	2.12/1.83	2.10/1.87	2.18/1.99
50%, RM	1/10	2.49/2.04	2.30/1.94	2.15/1.87	2.54/2.13	2.52/2.13	2.51/2.12
	1/5	2.41/1.99	2.15/1.86	2.01/1.81	2.53/2.13	2.51/2.12	2.48/2.11
	1	2.13/1.85	1.83/1.72	1.80/1.77	2.51/2.12	2.47/2.10	2.44/2.11
	5	1.85/1.72	**1.71**/1.71	1.84/1.85	2.48/2.10	2.44/2.10	2.45/2.16
	10	1.77/**1.69**	1.72/1.73	1.90/1.92	2.46/2.10	2.43/2.11	2.48/2.21
70%, RM	1/10	3.13/2.51	2.84/2.34	2.62/2.23	3.21/2.62	3.18/2.61	3.15/2.59
	1/5	3.01/2.44	2.66/2.25	2.44/2.15	3.20/2.62	3.15/2.59	3.11/22.57
	1	2.70/2.28	2.29/2.08	2.15/2.06	3.15/2.59	3.08/2.55	3.04/2.54
	5	2.41/2.13	2.08/**2.03**	2.11/2.12	3.10/2.56	3.04/2.54	3.03/2.57
	10	2.30/2.09	**2.06**/2.04	2.16/2.18	3.08/2.55	3.03/2.54	3.04/2.60
90%, RM	1/10	4.80/3.67	4.34/3.41	4.01/3.23	4.97/3.84	4.88/3.79	4.81/3.75
	1/5	4.66/3.59	4.15/3.31	3.81/3.14	4.93/3.82	4.83/3.76	4.76/3.72
	1	4.33/3.41	3.77/3.13	3.44/3.00	4.85/3.77	4.75/3.71	4.70/3.69
	5	4.02/3.26	3.46/3.01	3.22/**2.98**	4.78/3.73	4.70/3.69	4.67/3.69
	10	3.89/3.20	3.35/2.99	**3.18**/3.00	4.76/3.72	4.69/3.68	4.67/3.69

Table 14: Imputation performance (in MAPE/RMSE) of CTNNM and CircNNM on the PeMS traffic speed dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts.

Missing rate	Model	$\lambda = 10^{-5}NT$	$\lambda = 10^{-6}NT$
30%, RM	CTNNM	2.21/**1.82**	**2.16**/1.85
	CircNNM	2.21/**1.83**	**2.16**/1.85
50%, RM	CTNNM	2.62/**2.12**	**2.53**/2.13
	CircNNM	2.62/**2.12**	**2.54**/2.14
70%, RM	CTNNM	3.34/2.63	**3.20**/**2.62**
	CircNNM	3.36/2.64	**3.22**/**2.63**
90%, RM	CTNNM	5.22/3.91	**4.93**/**3.82**
	CircNNM	5.30/3.96	**5.03**/**3.88**

Table 15: Imputation performance (in MAPE/RMSE) of LRMC, HaLRTC ($r=0$), and LRTC-TNN ($r>0$) on the PeMS traffic speed dataset (on the first two-week subset for tuning hyperparameters). Note that the best results are highlighted in bold fonts.

Missing rate	Truncation parameter	$\lambda=10^{-4}$	$\lambda=10^{-5}$
30%, RM	LRMC	1.82/1.65	2.74/2.45
	$r=0$ (i.e., HaLRTC)	1.76/1.62	1.76/1.62
	$r=5$ (i.e., LRTC-TNN)	1.71/1.60	1.71/1.60
	$r=10$ (i.e., LRTC-TNN)	1.68/1.57	1.67/1.57
	$r=15$ (i.e., LRTC-TNN)	1.65/1.55	1.65/1.55
	$r=20$ (i.e., LRTC-TNN)	1.63/1.55	1.63/1.55
	$r=25$ (i.e., LRTC-TNN)	**1.62/1.54**	**1.62/1.54**
50%, RM	LRMC	2.19/1.96	3.18/2.85
	$r=0$ (i.e., HaLRTC)	2.10/1.90	2.10/1.90
	$r=5$ (i.e., LRTC-TNN)	2.01/1.86	2.01/1.86
	$r=10$ (i.e., LRTC-TNN)	1.96/1.82	1.95/1.82
	$r=15$ (i.e., LRTC-TNN)	1.91/1.79	1.91/1.79
	$r=20$ (i.e., LRTC-TNN)	1.89/1.78	1.89/1.78
	$r=25$ (i.e., LRTC-TNN)	**1.88/1.76**	**1.88/1.76**
70%, RM	LRMC	2.80/2.47	3.85/3.42
	$r=0$ (i.e., HaLRTC)	2.67/2.38	2.67/2.38
	$r=5$ (i.e., LRTC-TNN)	2.49/2.30	2.49/2.30
	$r=10$ (i.e., LRTC-TNN)	2.38/2.22	2.38/2.22
	$r=15$ (i.e., LRTC-TNN)	2.31/2.17	2.31/2.17
	$r=20$ (i.e., LRTC-TNN)	2.28/2.14	2.28/2.14
	$r=25$ (i.e., LRTC-TNN)	2.26/**2.12**	**2.25/2.12**
90%, RM	LRMC	4.26/3.63	5.47/4.53
	$r=0$ (i.e., HaLRTC)	4.90/3.91	4.18/3.53
	$r=5$ (i.e., LRTC-TNN)	3.64/3.30	3.59/3.26
	$r=10$ (i.e., LRTC-TNN)	3.40/3.14	3.33/3.09
	$r=15$ (i.e., LRTC-TNN)	3.36/3.10	3.20/3.00
	$r=20$ (i.e., LRTC-TNN)	3.49/3.17	**3.16/2.97**
	$r=25$ (i.e., LRTC-TNN)	3.91/3.40	**3.16**/2.98

Table 16: Imputation performance (MAPE/RMSE) on the PeMS-4W traffic speed dataset (on the last two-week subset for reporting results). Note that the best results are highlighted in bold fonts.

Model	Missing rate			
	30%	50%	70%	90%
LCR-2D	1.56/1.50	1.84/**1.71**	2.18/**2.07**	**3.39**/**3.06**
LCR_N	**1.54**/1.52	**1.81**/1.76	**2.16**/2.15	3.41/3.29
LCR	1.57/**1.50**	1.85/1.72	2.19/2.08	3.43/3.09
CTNNM	2.25/1.89	2.65/2.18	3.36/2.68	5.19/3.92
CircNNM	2.31/1.86	2.75/2.17	3.40/2.71	5.35/4.01
LRMC	1.92/1.70	2.30/2.02	2.97/2.56	4.49/3.74
HaLRTC	1.85/1.66	2.20/1.96	2.81/2.45	4.40/3.62
LRTC-TNN	1.70/1.58	1.97/1.82	2.38/2.19	3.33/3.06
NoTMF	-/-	-/-	-/-	-/-