Machine Learning and Optimization for Data-Driven Transportation Analytics and Beyond

Tensor Decomposition, Interpretable ML, Mathematical Programming

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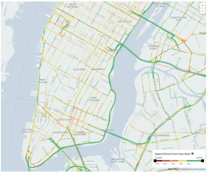
February 7, 2025

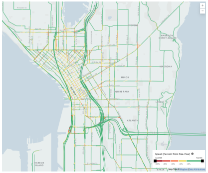
Transport Data

- Transport & mobility application scenarios

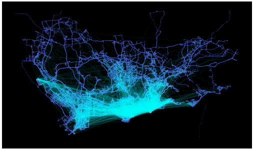
Highway (Portland)



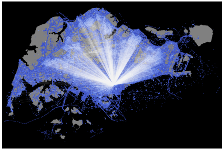
Uber movement (NYC)



Uber movement (Seattle)



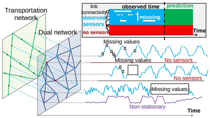
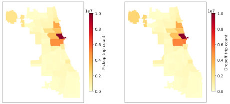
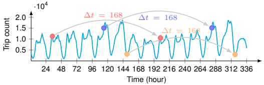
Taxi trajectory (Shenzhen)



Human movement (Singapore)

- Challenges: Sparsity, high-dimensionality (network-scale), and multi-dimensionality (complicated data structure), time-varying systems

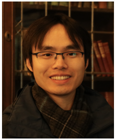
Data-Driven Transportation Analytics

Part I	Part II	Part III
Traffic Imputation	**Pattern Discovery**	**Periodicity Qualification**
		
Chen et al.'24	Chen et al.'24	Working paper
Matrix/tensor completion	Time series autoregression	Time series autoregression
Circulant matrix	Tensor decomposition	Sparse learning
Circular convolution	Conjugate gradient	Greedy methods
Fast Fourier transform	Procrustes problems	Mixed-integer programming

ML + Optimization for Transportation

Spatiotemporal Traffic Data Imputation

(Matrix/Tensor Factorization)



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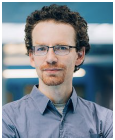


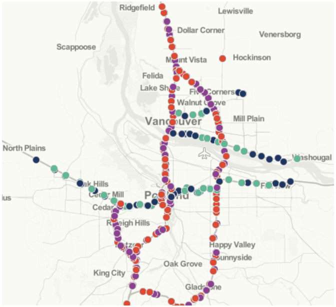
Nicolas Saunier
PolyMtl

Lijun Sun
McGill

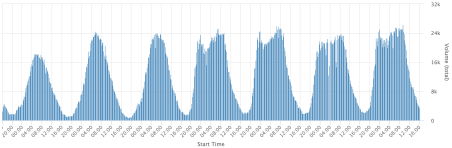
- **Chen** & Sun (2022). "Bayesian Temporal Factorization for Multidimensional Time Series Prediction". *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 44 (9): 4659–4673.
- **Chen** et al. (2024). "Laplacian Convolutional Representation for Traffic Time Series Imputation". *IEEE Transactions on Knowledge and Data Engineering*, 36 (11): 6490–6502.
- **Chen** et al. (2024). "Forecasting Urban Traffic States with Sparse Data Using Hankel Temporal Matrix Factorization". *INFORMS Journal on Computing*.

Traffic Flow Data

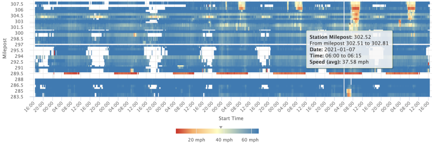
- Portland highway traffic data[1]



Highway network & sensor locations



Traffic volume



Traffic speed

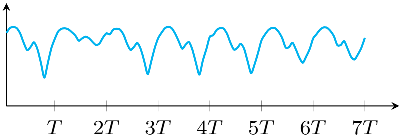
- $\boldsymbol{X} \in \mathbb{R}^{N \times T}$ with N spatial locations $\times$ T time steps
- Traffic volume/speed shows strong spatial/temporal dependencies
- Missing data are there, how to improve data quality?

[1] https://portal.its.pdx.edu/home

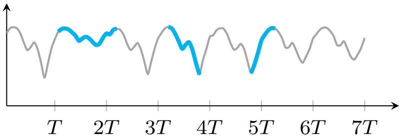
Time Series Imputation

Global/local trends in sparse data?

- Global trends (e.g., daily/weekly periodicity):

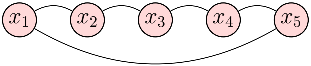


- Local trends (e.g., short-term time series trends):



Local Trend Modeling

- Intuition of Laplacian matrix



Undirected and circulant graph

$$\xrightarrow{\text{Modeling}} \quad \boldsymbol{L}=\begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Laplacian kernel:

$$\boldsymbol{\ell} \triangleq \underbrace{(2,-1,0,0,-1)^\top}_{\text{first column of } \boldsymbol{L}}$$

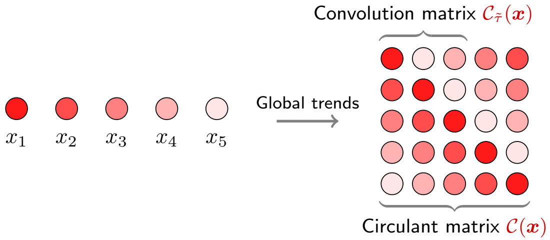
extending to the degree 2τ (i.e., graph connectivity) for $\boldsymbol{x} \in \mathbb{R}^T$.

- Temporal regularization:

$$\mathcal{R}(\boldsymbol{x}) = \underbrace{\frac{1}{2}\|\boldsymbol{L}\boldsymbol{x}\|_2^2}_{\text{mat-vec mul.}} = \underbrace{\frac{1}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{convolution } \star}$$

Global Trend Modeling

Circulant matrix $\mathcal{C}(\boldsymbol{x})$ vs. convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})$



- Circulant/Convolution nuclear norm minimization
  - A balance between global and local trends modeling?

CircNNM (Liu'22, Liu & Zhang'23)

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_*$$
$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

ConvNNM (Liu'22, Liu & Zhang'23)

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})\|_*$$
$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

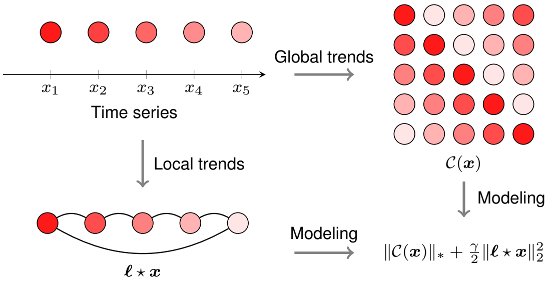
Global + Local Trends?

Laplacian Convolutional Representation (LCR)

For any partially observed time series $\boldsymbol{y} \in \mathbb{R}^T$ with observed index set Ω, LCR utilizes circulant matrix and Laplacian kernel to characterize global/local time series trends:

$$\min_{\boldsymbol{x}} \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{local}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon$$



Laplacian Convolutional Representation

- Augmented Lagrangian function:[2]

$$\mathcal{L} = \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{global + local}} + \underbrace{\frac{\lambda}{2}\|\boldsymbol{x} - \boldsymbol{z}\|_2^2 + \langle \boldsymbol{w}, \boldsymbol{x} - \boldsymbol{z}\rangle}_{\text{Lagrangian multiplier } \boldsymbol{w}} + \underbrace{\frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{z} - \boldsymbol{y})\|_2^2}_{\text{observations } \boldsymbol{y}}$$

- Optimize $\boldsymbol{x}$ w/ FFT (Properties of circulant matrix & circular convolution)

$$\begin{cases} \|\mathcal{C}(\boldsymbol{x})\|_* = \|\mathcal{F}(\boldsymbol{x})\|_1 = \|\hat{\boldsymbol{x}}\|_1 \\ \frac{1}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 = \frac{1}{2T}\|\mathcal{F}(\boldsymbol{\ell}) \circ \mathcal{F}(\boldsymbol{x})\|_2^2 = \frac{1}{2T}\|\hat{\boldsymbol{\ell}} \circ \hat{\boldsymbol{x}}\|_2^2 \end{cases}$$

- Reformulate the optimization as ℓ_1-norm minimization:

$$\begin{aligned} \boldsymbol{x} &:= \arg\min_{\boldsymbol{x}} \; \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x} - \boldsymbol{z} + \boldsymbol{w}/\lambda\|_2^2 \\ \Longrightarrow \hat{\boldsymbol{x}} &:= \arg\min_{\hat{\boldsymbol{x}}} \; \underbrace{\|\hat{\boldsymbol{x}}\|_1}_{\ell_1\text{-norm}} + \frac{\gamma}{2T}\|\hat{\boldsymbol{\ell}} \circ \hat{\boldsymbol{x}}\|_2^2 + \frac{\lambda}{2T}\|\hat{\boldsymbol{x}} - \hat{\boldsymbol{z}} + \hat{\boldsymbol{w}}/\lambda\|_2^2 \end{aligned}$$

in $\mathcal{O}(T \log T)$ time.

[2] $\boldsymbol{w} \in \mathbb{R}^T$ (Lagrange multiplier); $\langle \cdot, \cdot \rangle$ (inner product).

Laplacian Convolutional Representation

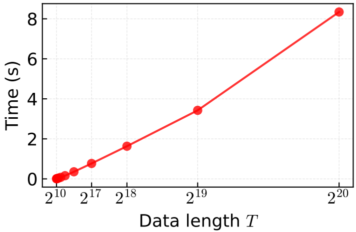
ℓ_1-norm Minimization (Liu & Zhang'23)

$$\min_{\hat{\boldsymbol{x}}} \ \|\hat{\boldsymbol{x}}\|_1 + \frac{\delta}{2}\|\hat{\boldsymbol{x}} - \hat{\boldsymbol{h}}\|_2^2$$

with complex-valued $\hat{\boldsymbol{x}}, \hat{\boldsymbol{h}} \in \mathbb{C}^T$ and weight parameter δ, the solution is

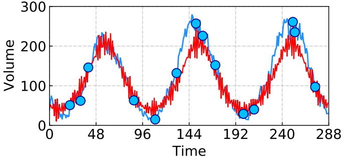
$$\hat{x}_t := \frac{\hat{h}_t}{|\hat{h}_t|} \cdot \underbrace{\max\{0, |\hat{h}_t| - 1/\delta\}}_{\text{shrinkage (e.g., ReLU)}}, t \in [T].$$

- **Empirical time complexity of LCR**: On the synthetic data $\boldsymbol{y} \in \mathbb{R}^T$ with $T \in \{2^{10}, 2^{11}, \ldots, 2^{20}\}$ (w.r.t. FFT in $\mathcal{O}(T \log T)$ time)



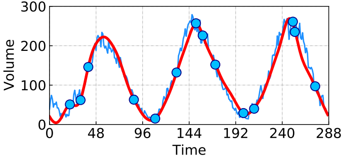
- Traffic speed imputation[3] (95% missing rate)

$\|\mathcal{C}(\boldsymbol{x})\|_*$



$\Downarrow$ Plus **local** time series trends

$\|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2$



Highlights:

- Rethink the importance of local trend modeling in traffic data imputation tasks.
- Find a unified global and local trend modeling framework whose optimization can be efficiently solved by FFT:

$$\min_{\boldsymbol{x}}\ \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2}\underbrace{\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2}_{\text{local}}$$
$$\text{s.t.}\ \|\mathcal{P}_{\Omega}(\boldsymbol{x}-\boldsymbol{y})\|_2\leq\epsilon$$

[3] Blue dot: partial observation; red line: imputation.

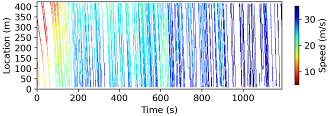
Experiments

Speed field reconstruction in German highways[4]

- Scenario: Mask trajectories of 70% vehicles
- LCR-2D on partially observed $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

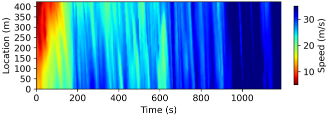
$$\min_{\boldsymbol{X}} \quad \underbrace{\|\mathcal{C}(\boldsymbol{X})\|_*}_{\text{global trend}} + \frac{\gamma}{2} \underbrace{\|(\boldsymbol{\ell}_s \boldsymbol{\ell}^\top) \star \boldsymbol{X}\|_F^2}_{\text{local trend}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{X} - \boldsymbol{Y})\|_F \leq \epsilon$$

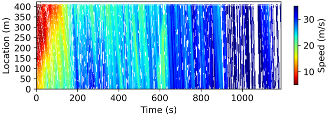


Sparse speed field

LCR-2D $\longrightarrow$



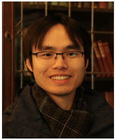
Reconstructed speed field



Ground-truth speed field

[4] Highway Drone (HighD) dataset at https://www.highd-dataset.com/

Discovering Dynamic Patterns from Spatiotemporal Systems



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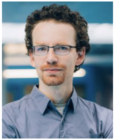

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Dingyi Zhuang
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HanQin Cai
UCF

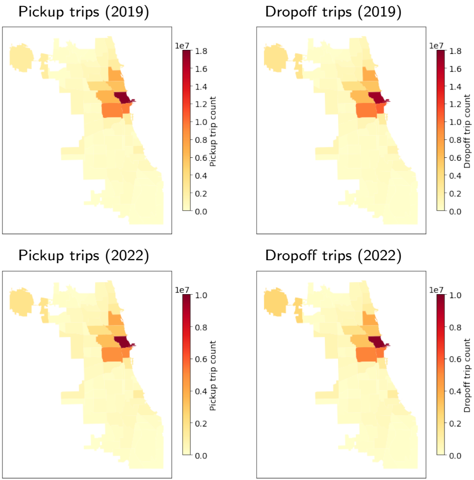
Shenhao Wang
UF

Jinhua Zhao
MIT

- **Chen** et al. (2024). "Discovering Dynamic Patterns from Spatiotemporal Data with Time-Varying Low-Rank Autoregression". *IEEE Transactions on Knowledge and Data Engineering*, 36 (2): 504–517.
- **Chen** et al. (2025). "Dynamic autoregressive tensor factorization for pattern discovery of spatiotemporal systems". Major revision at *IEEE Transactions on Pattern Analysis and Machine Intelligence*.

Motivation

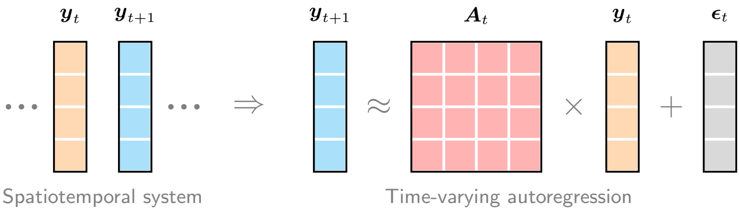
- Chicago rideshare: 96.6M trips (2019) vs. 57.3M trips (2022)



- **Contradict**: No difference in aggregation. How about latent patterns?

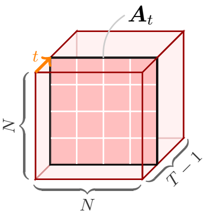
Autoregression

- How to characterize dynamical systems?



- On spatiotemporal data $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

$$\underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-invariant (e.g., DMD)}} \quad \text{v.s.} \quad \underbrace{\boldsymbol{y}_{t+1} = \boldsymbol{A}_t\boldsymbol{y}_t + \boldsymbol{\epsilon}_t}_{\text{time-varying (ours)}}$$



- How to discover spatial/temporal modes (patterns) from the tensor $\boldsymbol{\mathcal{A}} \triangleq \{\boldsymbol{A}_t\}_{t \in [T-1]}$?

1927 — Higher-Order SVD

Frank Lauren Hitchcock

1960s — Tucker Decomposition

Ledyard R. Tucker

1970 — CP Decomposition

J. Douglas Carroll
Jih-Jie Chang
Richard A. Harshman

2009 — Tensor Decompositions and Applications

Tamara G. Kolda

2011 — Tensor-Train Decomposition

Ivan Oseledets

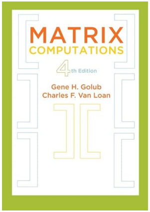
Dynamic Autoregressive Tensor Factorization

- Tensor factorization:

$$\mathcal{A} = \underbrace{\mathcal{G} \times_1 \boldsymbol{W} \times_2 \boldsymbol{V} \times_3 \boldsymbol{X}}_{\text{Tucker decomposition}}$$

$$\Updownarrow$$

$$\boldsymbol{A}_t = \mathcal{G} \times_1 \underbrace{\boldsymbol{W}}_{\text{spatial modes}} \times_2 \boldsymbol{V} \times_3 \underbrace{\boldsymbol{x}_t^\top}_{\text{temporal modes}}$$



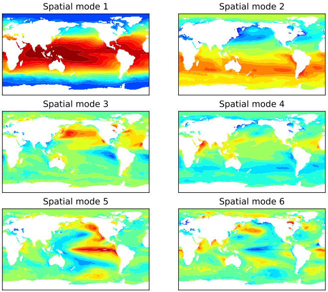
- Optimization problem:

$$\begin{aligned} \min_{\mathcal{G},\boldsymbol{W},\boldsymbol{V},\boldsymbol{X}} \; & \frac{1}{2}\sum_{t\in[T-1]} \left\|\boldsymbol{y}_{t+1} - (\mathcal{G} \times_1 \boldsymbol{W} \times_2 \boldsymbol{V} \times_3 \boldsymbol{x}_t^\top)\boldsymbol{y}_t\right\|_2^2 \\ \text{s.t.} \; & \underbrace{\boldsymbol{W}^\top \boldsymbol{W} = \boldsymbol{I}_R}_{\text{orthogonal spatial modes}} \end{aligned}$$

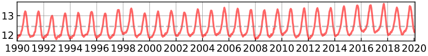
on spatiotemporal data $\boldsymbol{Y} \in \mathbb{R}^{N\times T}$.

Beyond Transport: Sea Surface Temperature

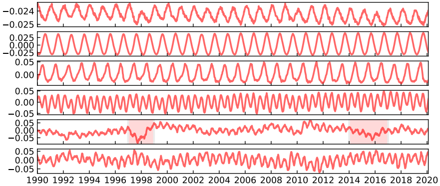
- Sea surface temperature (**SST**) dataset



Spatial modes in $\boldsymbol{W}$



Time series of mean temperature

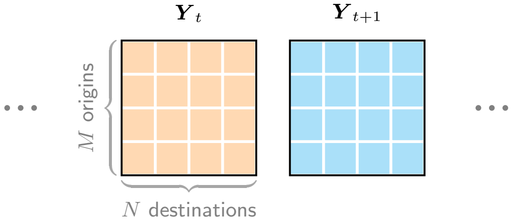


Temporal modes in $\boldsymbol{X}$

- Identify two strongest El Nino events (on 1997-98 & 2014-16)
- Insights into climate change

Dynamic Autoregressive Tensor Factorization

- Origin-Destination (OD) matrices



- On spatiotemporal systems $\boldsymbol{\mathcal{Y}} \in \mathbb{R}^{M \times N \times T}$:

$$\underbrace{\boldsymbol{y}_{n,t+1} = \boldsymbol{A}_{n,t} \boldsymbol{y}_{n,t} + \boldsymbol{\epsilon}_{n,t}}_{\text{time-varying \& destination-varying}}$$

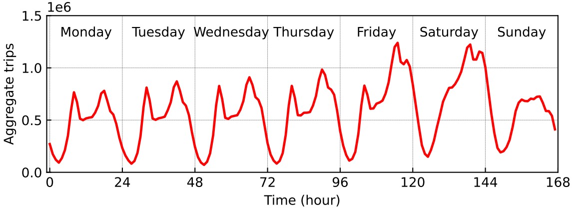
- Optimization problem:

$$\min_{\boldsymbol{\mathcal{G}}, \boldsymbol{W}, \boldsymbol{U}, \boldsymbol{V}, \boldsymbol{X}} \frac{1}{2} \sum_{n \in [N]} \sum_{t \in [T-1]} \left\| \boldsymbol{y}_{n,t+1} - (\underbrace{\boldsymbol{\mathcal{G}} \times_1 \boldsymbol{W} \times_2 \boldsymbol{U} \times_3 \boldsymbol{V} \times_4 \boldsymbol{x}_t^\top}_{\text{Tucker decomposition}}) \boldsymbol{y}_{n,t} \right\|_2^2$$

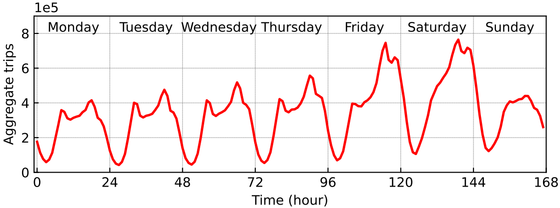
$$\text{s.t.} \quad \underbrace{\boldsymbol{W}^\top \boldsymbol{W} = \boldsymbol{I}_R}_{\text{orthogonal origin patterns}}$$

- Chicago rideshare: 96.6M trips (2019) vs. 57.3M trips (2022)

Pickup trips aggregated over 52 weeks in 2019

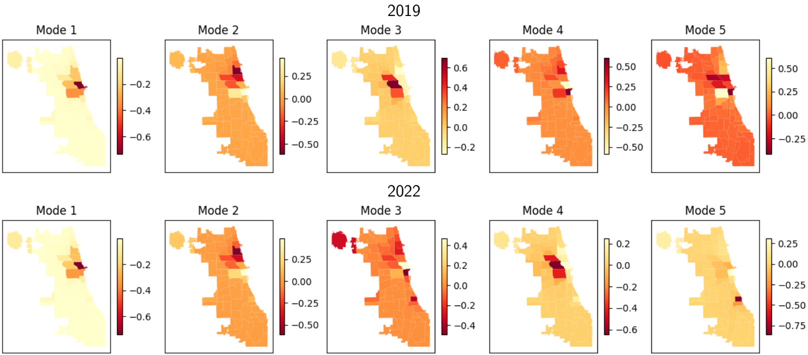


Pickup trips aggregated over 52 weeks in 2022



Human Mobility

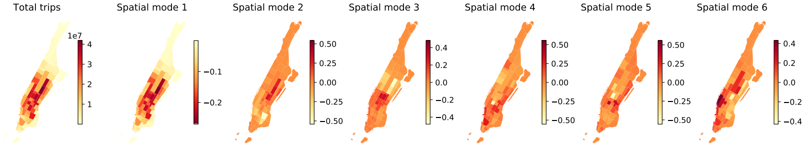
- Rideshare trip data: 77 origins $\times$ 77 destinations $\times$ 168 hours



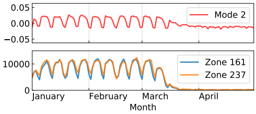
- Identify the changes in pickup zones before/after COVID-19

Human Mobility

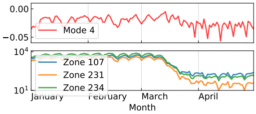
- NYC taxi dataset (pickup)
  - Remarkable decrease of trips due to COVID-19



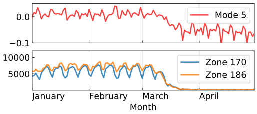
Spatial patterns in $\boldsymbol{W}$



Pattern #2 & taxi trips (2020)



Pattern #4 & taxi trips (2020)

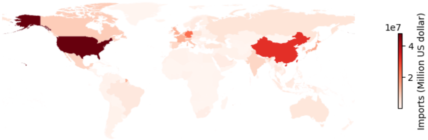


Pattern #5 & taxi trips (2020)

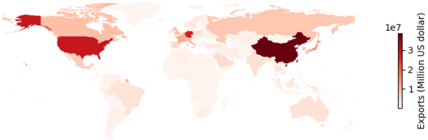
- Identify the changes of temporal patterns due to COVID-19

Beyond Transport: International Trade

- **Import/Export merchandise trade values (annual)**[5] (215 countries/regions & period of 2000-2022)
  - Total merchandise trade values
  - Represent import/export trade data as a 215-by-23 matrix

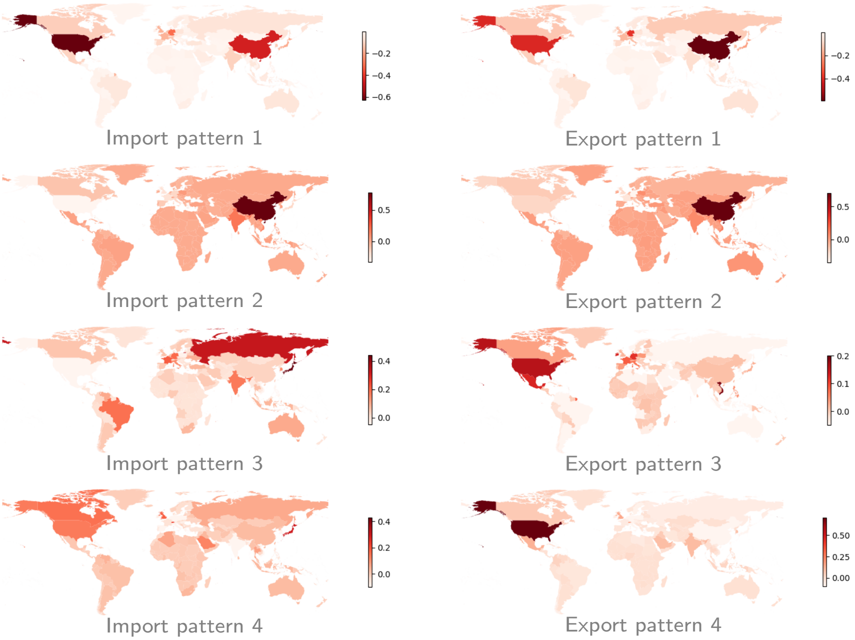


Imports from 2000 to 2022



Exports from 2000 to 2022

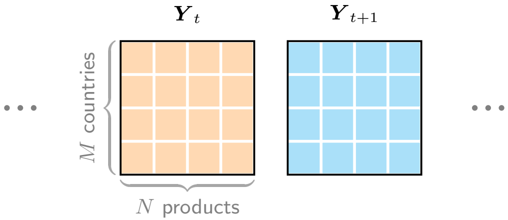
[5] The dataset is available at https://stats.wto.org.



Import pattern 1
−0.2
−0.4
−0.6
Export pattern 1
−0.2
−0.4
Import pattern 2
0.5
0.0
Export pattern 2
0.5
0.0
Import pattern 3
0.4
0.2
0.0
Export pattern 3
0.2
0.1
0.0
Import pattern 4
0.4
0.2
0.0
Export pattern 4
0.50
0.25
0.00

Beyond Transport: International Trade

- Three-dimensional trade (Economy, Product, Year)



- On spatiotemporal systems $\boldsymbol{\mathcal{Y}} \in \mathbb{R}^{M \times N \times T}$:

$$\underbrace{\boldsymbol{y}_{n,t+1} = \boldsymbol{A}_{n,t}\boldsymbol{y}_{n,t} + \boldsymbol{\epsilon}_{n,t}}_{\text{time-varying \& product-varying}}$$

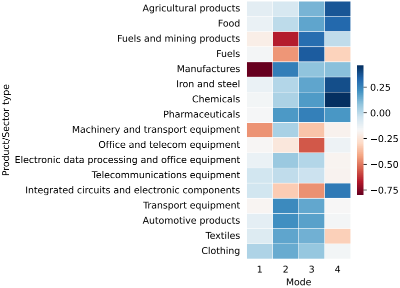
- Optimization problem of DATF:

$$\min_{\boldsymbol{\mathcal{G}},\boldsymbol{W},\boldsymbol{U},\boldsymbol{V},\boldsymbol{X}} \ \frac{1}{2}\sum_{n\in[N]}\sum_{t\in[T-1]}\left\|\boldsymbol{y}_{n,t+1} - (\boldsymbol{\mathcal{G}} \times_1 \boldsymbol{W} \times_2 \boldsymbol{U} \times_3 \boldsymbol{V} \times_4 \boldsymbol{x}_t^\top)\boldsymbol{y}_{n,t}\right\|_2^2$$

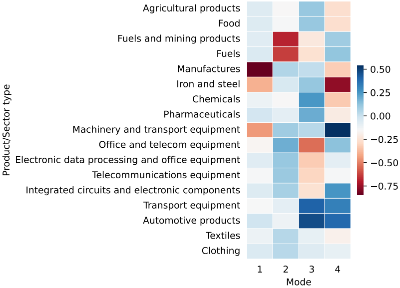
$$\text{s.t.} \quad \underbrace{\boldsymbol{W}^\top\boldsymbol{W} = \boldsymbol{I}_R}_{\text{orthogonal country patterns}}$$

Product Patterns

- On 17 merchandise types



Imports



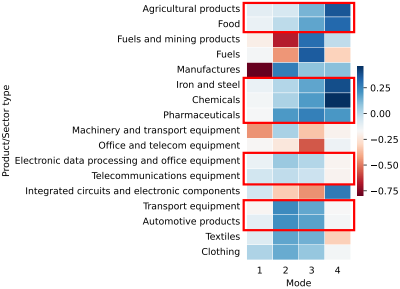
Exports

- Classify import/export merchandise according to product patterns
- Basic principle:

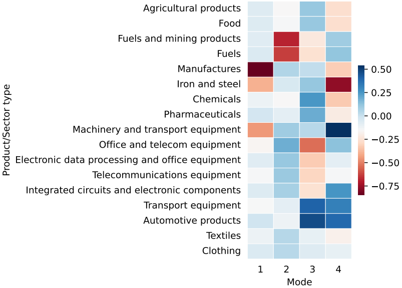
Import: What we buy? (demand) vs. Export: What we sell? (supply)

Product Patterns

- On 17 merchandise types



Imports



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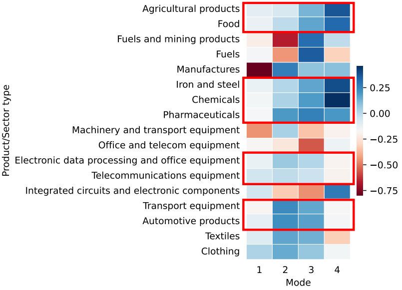
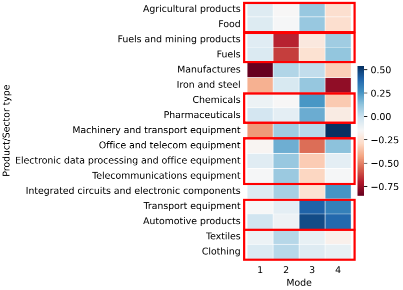


Imports



Exports

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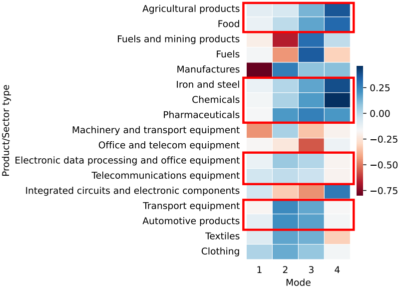
- Basic principle:

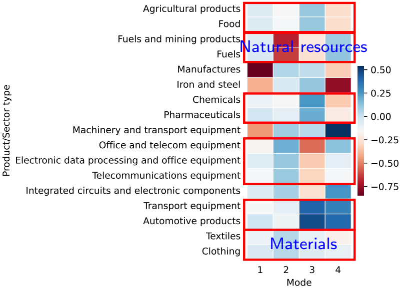
Import: What we buy? (demand) vs. Export: What we sell? (supply)

Product Patterns

- On 17 merchandise types



Imports



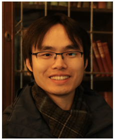
Exports

- Classify import/export merchandise according to product patterns
- Basic principle:

Import: What we buy? (demand) vs. Export: What we sell? (supply)

Quantifying Periodicity of Human Mobility

(Working Ideas on Interpretable ML & Causality & Nonlinear Programming)



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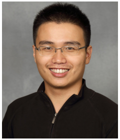

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Vassilis Digalakis Jr
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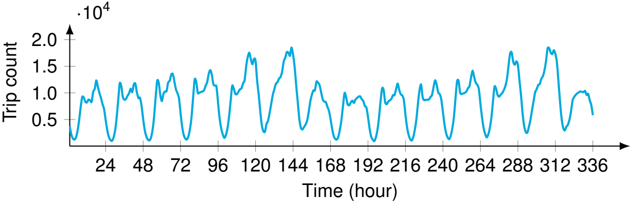
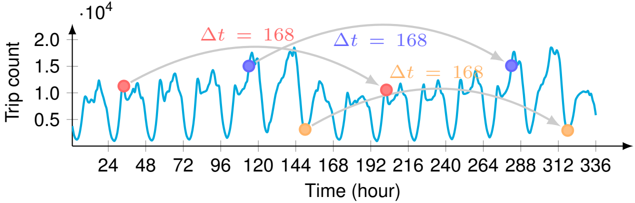
- **Xinyu Chen**, Vassilis Digalakis Jr, Lijun Ding, Jinhua Zhao (2025). "Interpretable Time Series Autoregression". Ready for submission.

Motivation



Two-week rideshare trip time series in Chicago since April 1, 2024

Motivation



Weekly periodicity of rideshare trip time series

What motivate us most about **periodicity**?

❶ **Resilience and stability of systems**: Empirically measure the periodicity and predictability of urban systems.

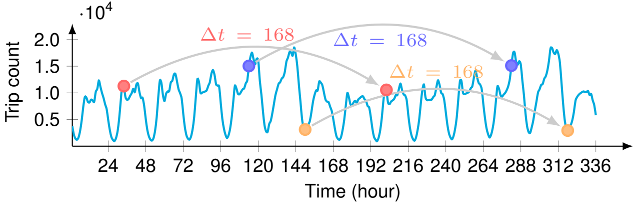
❷ **Optimization of transport systems**: Optimize resources (e.g., public transit, taxi, rideshare, and micromobility) to meet transport demand efficiently.

❸ **Design of sustainable transport & infrastructure**: Implement energy-efficient solutions tailored to peak hours.

Motivation

- Time series autoregression on $\boldsymbol{x} \in \mathbb{R}^T$

$$\boldsymbol{w} := \arg\min_{\{w_k\}_{k\in[d]}} \sum_{t\in[d+1,T]} \Big(x_t - \sum_{k\in[d]} w_k x_{t-k}\Big)^2$$



Periodicity of rideshare trip time series

- Sparse coefficient vector $\mapsto$ **Interpretability**?

$$\boldsymbol{w} = (\underbrace{0.33}_{k=1}, 0, \cdots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

Valorizing Autoregression

- Time series autoregression

$$\begin{aligned}\boldsymbol{w} &:= \arg\min_{\{w_k\}_{k\in[d]}} \sum_{t\in[d+1,T]}\Big(x_t-\sum_{k\in[d]} w_k x_{t-k}\Big)^2 \\ &= \arg\min_{\boldsymbol{w}}\ \|\tilde{\boldsymbol{x}}-\boldsymbol{A}\boldsymbol{w}\|_2^2\end{aligned}$$

$$\overset{\tilde{\boldsymbol{x}}}{\begin{bmatrix}x_5\\x_6\\x_7\\x_8\\x_9\end{bmatrix}} \approx \overset{\boldsymbol{A}}{\begin{bmatrix}x_4 & x_3 & x_2 & x_1\\ x_5 & x_4 & x_3 & x_2\\ x_6 & x_5 & x_4 & x_3\\ x_7 & x_6 & x_5 & x_4\\ x_8 & x_7 & x_6 & x_5\end{bmatrix}} \times \overset{\boldsymbol{w}}{\left.\begin{bmatrix}w_1\\w_2\\w_3\\w_4\end{bmatrix}\right\}} \|\boldsymbol{w}\|_0\leq\tau$$

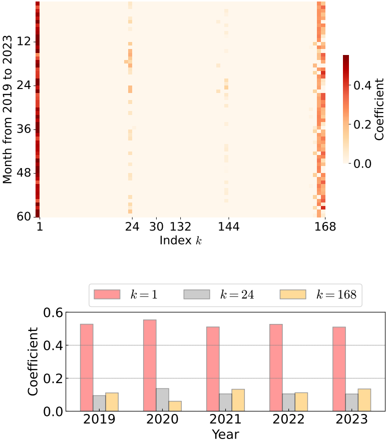
dth-order autoregression on time series $\boldsymbol{x}=(x_1,x_2,\ldots,x_9)^\top$ w/ integer $\tau\in\mathbb{Z}^+$

- Sparse autoregression (Subspace Pursuit Algorithm (Dai & Milenkovic'09))

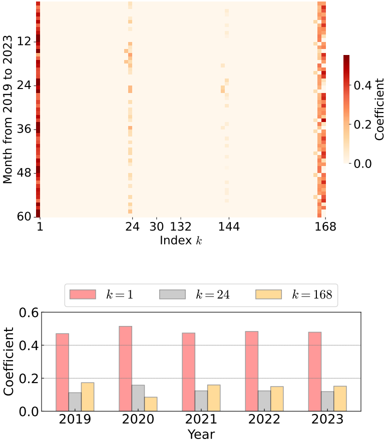
$$\begin{aligned}&\min_{\boldsymbol{w}\geq 0}\ \|\tilde{\boldsymbol{x}}-\boldsymbol{A}\boldsymbol{w}\|_2^2 \\ &\text{s.t.}\quad \underbrace{\|\boldsymbol{w}\|_0\leq\tau}_{\text{Sparsity w/ }\ell_0\text{-norm}}\end{aligned}$$

Envisioning Human Mobility

- NYC rideshare: Sparse coefficient vectors across different months



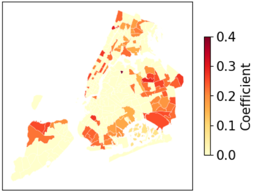
NYC pickup trips



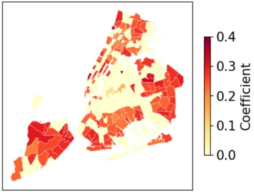
NYC dropoff trips

- Stronger daily periodicity vs. weaker weekly periodicity in 2020

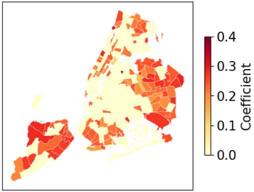
Stronger Daily Periodicity in 2020



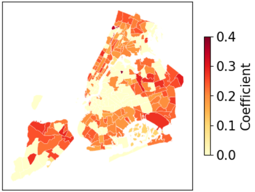
2019



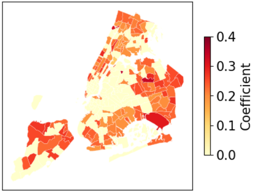
2020



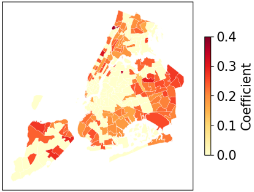
2021



2022



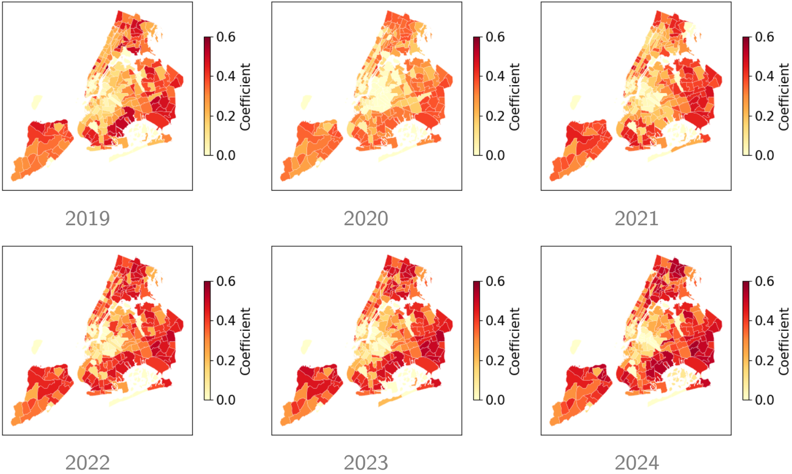
2023



2024

- High-demand areas are less periodic
- More areas show remarkable daily periodicity in 2020

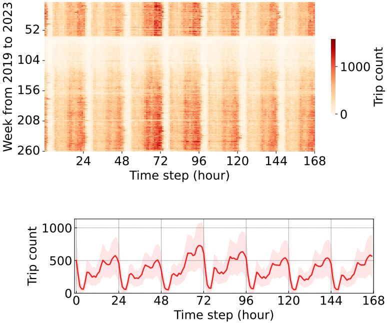
Weaker Weekly Periodicity in 2020



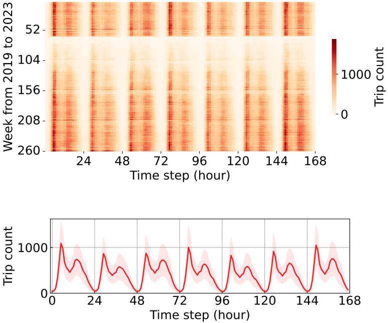
- High-demand areas are less periodic
- Less areas show remarkable weekly periodicity in 2020

John F. Kennedy International Airport

- Pickup/Dropoff trips in airport
  - Pickup trips are relevant to flight delay, baggage claim, and other factors.
  - Dropoff trips to airport are highly related to flight schedules.



Pickup trips



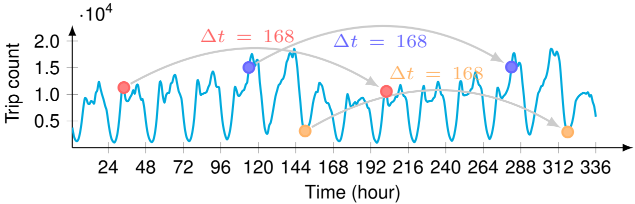
Dropoff trips

- Sparse coefficient vectors:

$$\boldsymbol{w} = (\underbrace{0.31}_{k=1}, \cdots, \underbrace{\color{red}{0.28}}_{k=24}, \cdots, \underbrace{\color{red}{0.41}}_{k=168})^\top \quad \text{vs.} \quad \boldsymbol{w} = (\underbrace{0.18}_{k=1}, \cdots, \underbrace{\color{red}{0.35}}_{k=24}, \cdots, \underbrace{\color{red}{0.47}}_{k=168})^\top$$

Solution Quality

- Subspace pursuit (SP) sometimes fails



Periodicity of rideshare trip time series

- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity $\tau = 2$):

$$\underbrace{\boldsymbol{w} = (\cdots, \underbrace{0.02}_{k=53}, \cdots, \underbrace{0.96}_{k=168})^\top}_{\text{loss func. } = 8.32 \times 10^7 \text{ (SP)}} \quad \text{vs.} \quad \underbrace{\boldsymbol{w} = (\underbrace{0.22}_{k=1}, \cdots, \underbrace{0.77}_{k=168})^\top}_{\text{loss func. } = 6.25 \times 10^7 \text{ (MIP)}}$$

Time-Varying Transport Systems

- Rideshare trip data $\{\boldsymbol{x}_\gamma\}_{\gamma\in[\delta]}$ across $\gamma \in [\delta]$ months/years
- **(Ours)** Reformulate interpretable sparse autoregression:

$$\min_{\{\boldsymbol{w}_\gamma\}_{\gamma\in[\delta]}} \sum_{\gamma\in[\delta]} \|\tilde{\boldsymbol{x}}_\gamma - \boldsymbol{A}_\gamma \boldsymbol{w}_\gamma\|_2^2$$

$$\text{s.t.} \begin{cases} \boldsymbol{w}_\gamma \geq 0 & \text{(non-negativity)} \\ \|\boldsymbol{w}_\gamma\|_0 \leq \tau & \text{(sparsity)} \\ \operatorname{supp}(\boldsymbol{w}_\gamma) = \operatorname{supp}(\boldsymbol{w}_{\gamma+1}) & \text{(no local difference)} \end{cases}$$

making these coefficient vectors comparable across δ months/years.

- Constraints w/ binary variables $\boldsymbol{\beta}_\gamma \in \{0,1\}^d$:

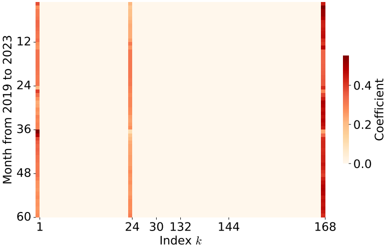
$$\underbrace{0 \leq \boldsymbol{w}_\gamma \leq \boldsymbol{\beta}_\gamma}_{\text{upper bound } \{0,1\}} \qquad \underbrace{\sum_{k\in[d]} \beta_{\gamma,k} \leq \tau}_{\text{sum of binary var.}} \qquad \underbrace{\boldsymbol{\beta}_\gamma - \boldsymbol{\beta}_{\gamma+1} = 0}_{\text{comparability across } \boldsymbol{w}_\gamma, \forall\gamma}$$

- MIP problem w/ $2d\delta$ decision variables!
- (**Efficiency?**) ML prunes the search space, e.g., $2\tau_0\delta$ decision variables ($\tau < \tau_0 \ll d$) instead.

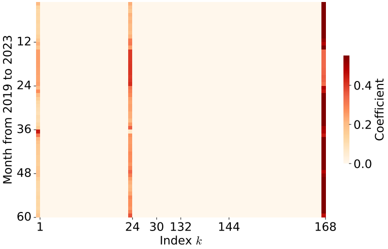
John F. Kennedy International Airport

- Coefficients $\{\boldsymbol{w}_\gamma\}_{\gamma\in[\delta]}$ at $S=\{\underbrace{1}_{\text{local}},\underbrace{24}_{\text{daily}},\underbrace{168}_{\text{weekly}}\}$ across $\delta=60$ months

  ❶ Stronger weekly periodicity of dropoff trips than pickup trips
  ❷ Stronger daily periodicity in 2020
  ❸ Weaker weekly periodicity in 2020



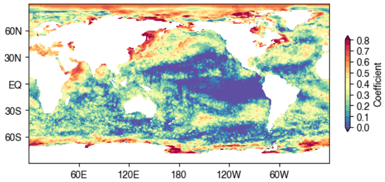
Pickup trips



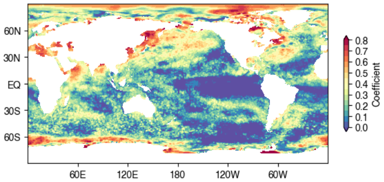
Dropoff trips

- Identify system patterns that evolve over time for human mobility

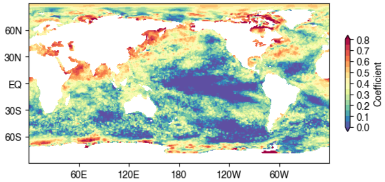
Beyond Transport: Sea Surface Temperature



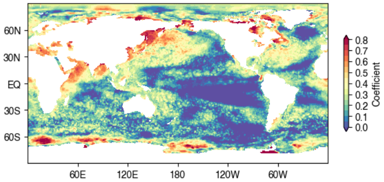
1980s



1990s



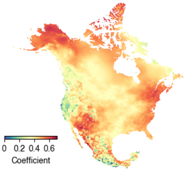
2000s



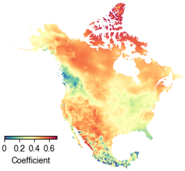
2010s

- Identify yearly periodicity at $k = 12$ from SST data ($\tau = 4$)
  1. The areas of El Nino events are less seasonal/predictable
  2. Arctic becomes less seasonal/predictable in the past 20 years
- Insights into climate change & global warming & sustainable development

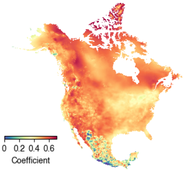
Beyond Transport: North America Temperature



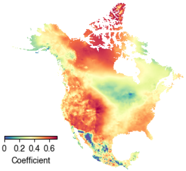
tmin, 1980s



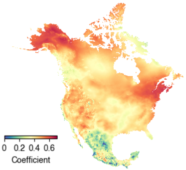
tmin, 1990s



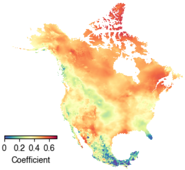
tmin, 2000s



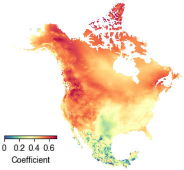
tmin, 2010s



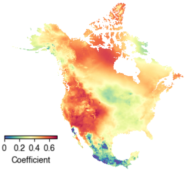
tmax, 1980s



tmax, 1990s



tmax, 2000s



tmax, 2010s

- Identify yearly periodicity at $k = 12$ from temperature data ($\tau = 4$)
  - ❶ Stronger yearly seasonality in high-latitude areas
  - ❷ Less seasonal temperature in south areas (e.g., Mexico)
  - ❸ Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s

Thanks for your attention!

Any Questions?

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