数字信号处理

周治国

2023.11

第五章 数字滤波器

FIR数字滤波器

频率取样设计法

设计原理

$$H(k) = H_d(k) = H_d(z)\Big|_{z=e^{j(\frac{2\pi}{N})k}}$$

$$= H_d(e^{j\omega})\Big|_{\omega=(\frac{2\pi}{N})k}$$

内插公式

$$H(z) = \frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-W_N^{-k}z^{-1}}$$

$$\begin{cases} H(z) = \sum\limits_{k=0}^{N-1} H(k)\Phi_k(z) \\ \Phi_k(z) = \dfrac{1}{N}\dfrac{1-z^{-N}}{1-W_N^{-k}z^{-1}} \end{cases}$$

设计原理

$$z = e^{j\omega}$$

$$\begin{cases} H\left(e^{j\omega}\right) = \sum_{k=0}^{N-1} H(k)\Phi_k\left(e^{j\omega}\right) \\ \Phi_k\left(e^{j\omega}\right) = \frac{1}{N}\frac{1-e^{-jN\omega}}{1-e^{-j(\omega-k\frac{2\pi}{N})}} \end{cases}$$

$$\Phi_k\left(e^{j\omega}\right) = \frac{1}{N}\frac{\sin\left|\frac{N\omega}{2}\right|}{\sin\left|\frac{\omega}{2}-\frac{\pi k}{N}\right|}e^{-j(\frac{N\omega}{2}-\frac{\omega}{2}+\frac{\pi k}{N})}$$

$$\Phi_k\left(e^{j\omega}\right) = \Phi\left|\omega - k\frac{2\pi}{N}\right|$$

$$\Rightarrow \Phi\left(\omega\right) = \frac{1}{N}\frac{\sin\left|\frac{N\omega}{2}\right|}{\sin\left|\frac{\omega}{2}\right|}e^{-j\left|\frac{N-1}{2}\right|\omega}$$

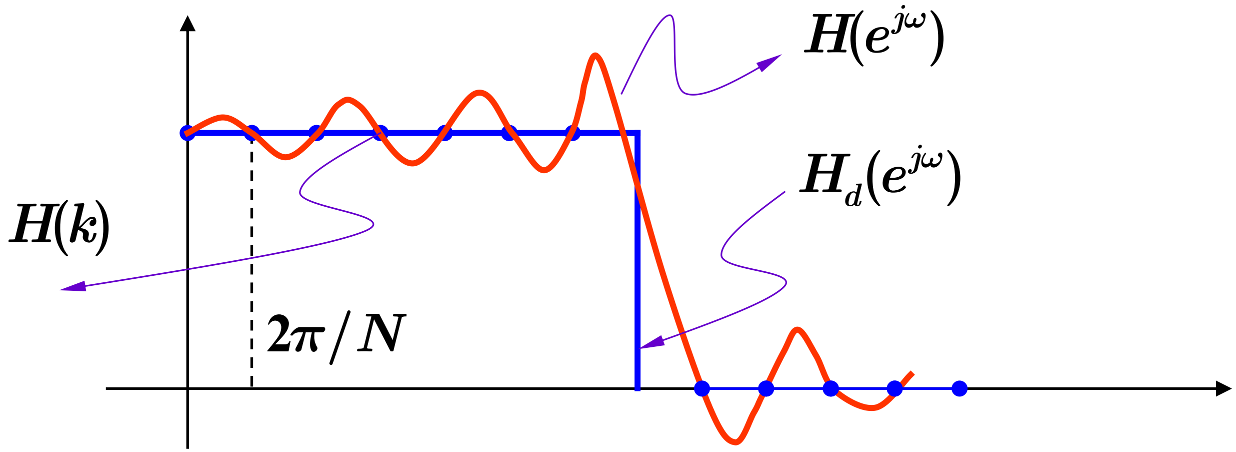
$$\Rightarrow H\left(e^{j\omega}\right) = \sum_{k=0}^{N-1} H\left(k\right)\Phi\left|\omega - k\frac{2\pi}{N}\right|$$

$$= \frac{e^{-j\left|\frac{N-1}{2}\right|\omega}}{N}\sum_{k=0}^{N-1} H\left(k\right)\frac{\sin\left|\frac{N\omega}{2}\right|}{\sin\left|\frac{\omega}{2}-\frac{\pi k}{N}\right|}e^{-j\frac{\pi k}{N}}$$

$$H\left(e^{j\omega}\right)=\sum_{k=0}^{N-1}H\left(k\right)\Phi\left|\omega-k\frac{2\pi}{N}\right|=\frac{e^{-j\left|\frac{N-1}{2}\right|\omega}}{N}\sum_{k=0}^{N-1}H\left(k\right)\frac{\sin\left|\frac{N\omega}{2}\right|}{\sin\left|\frac{\omega}{2}-\frac{\pi k}{N}\right|}e^{-j\frac{\pi k}{N}}$$

$$s\left(\omega,k\right)=\frac{1}{N}e^{-j\frac{\pi k}{N}}\frac{\sin\left|\frac{N\omega}{2}\right|}{\sin\left|\frac{\omega}{2}-\frac{\pi k}{N}\right|}\Rightarrow\boxed{H\left(e^{j\omega}\right)=e^{-j\left|\frac{N-1}{2}\right|\omega}\sum_{k=0}^{N-1}H\left(k\right)s\left(\omega,k\right)}$$

内插函数



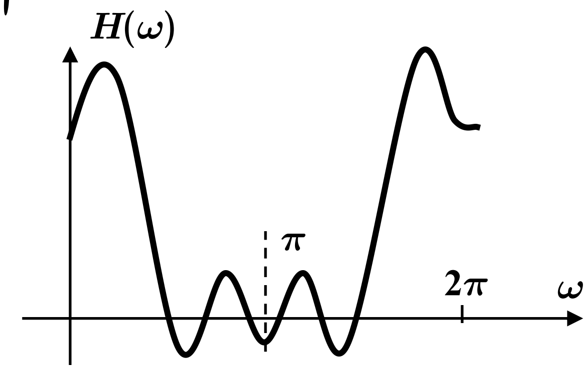
- 抽样点上，频率响应严格相等
- 抽样点之间，加权内插函数的延伸叠加
- 变化越平缓，内插越接近理想值，逼近误差较小

线性相位约束条件

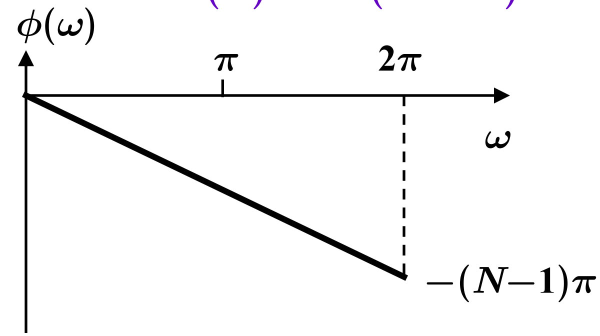
对第一类线性相位滤波器，h(n)为偶对称，N为奇数

$$H(e^{j\omega}) = H(\omega)e^{j\theta(\omega)}$$

$$\begin{cases} H(\omega) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos[\omega n] \\ \theta(\omega) = -\left|\dfrac{N-1}{2}\right|\omega \end{cases}$$



$H(\omega) = H(2\pi - \omega)$



$$H(k) = H(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k} \quad k = 0, 1, 2, \cdots, N-1$$

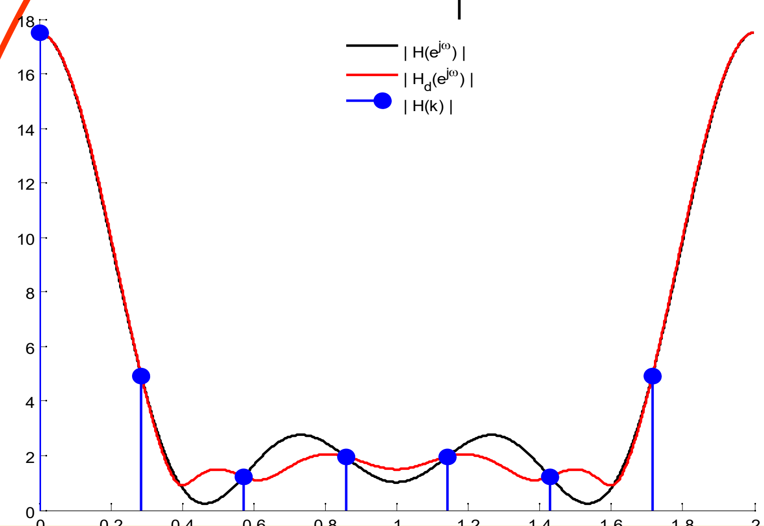
$$\begin{cases} H(k) = \boxed{H_k} e^{j\theta_k} \\ \theta_k = \theta(\omega)\Big|_{\omega=\frac{2\pi}{N}k} = -\left|\dfrac{N-1}{2}\right|\dfrac{2\pi}{N}k \end{cases}$$

$$H(\omega) = H(2\pi - \omega)$$

$$\Rightarrow \boxed{H_k = H_{N-k}}$$

N为偶数:

$$H_k = -H_{N-k}$$



A summary:

① **时域：**

$$h(n) = \pm h(N-n-1)$$

② **频域：**

$$H(e^{j\omega}) = H(\omega)e^{j\left[\frac{L}{2}\pi - \frac{N-1}{2}\omega\right]}$$

$$H(z) = (-1)^L z^{-(N-1)} H(z^{-1})$$

点线性相位FIR数字滤波器特

— $H(\omega)$ 为实函数

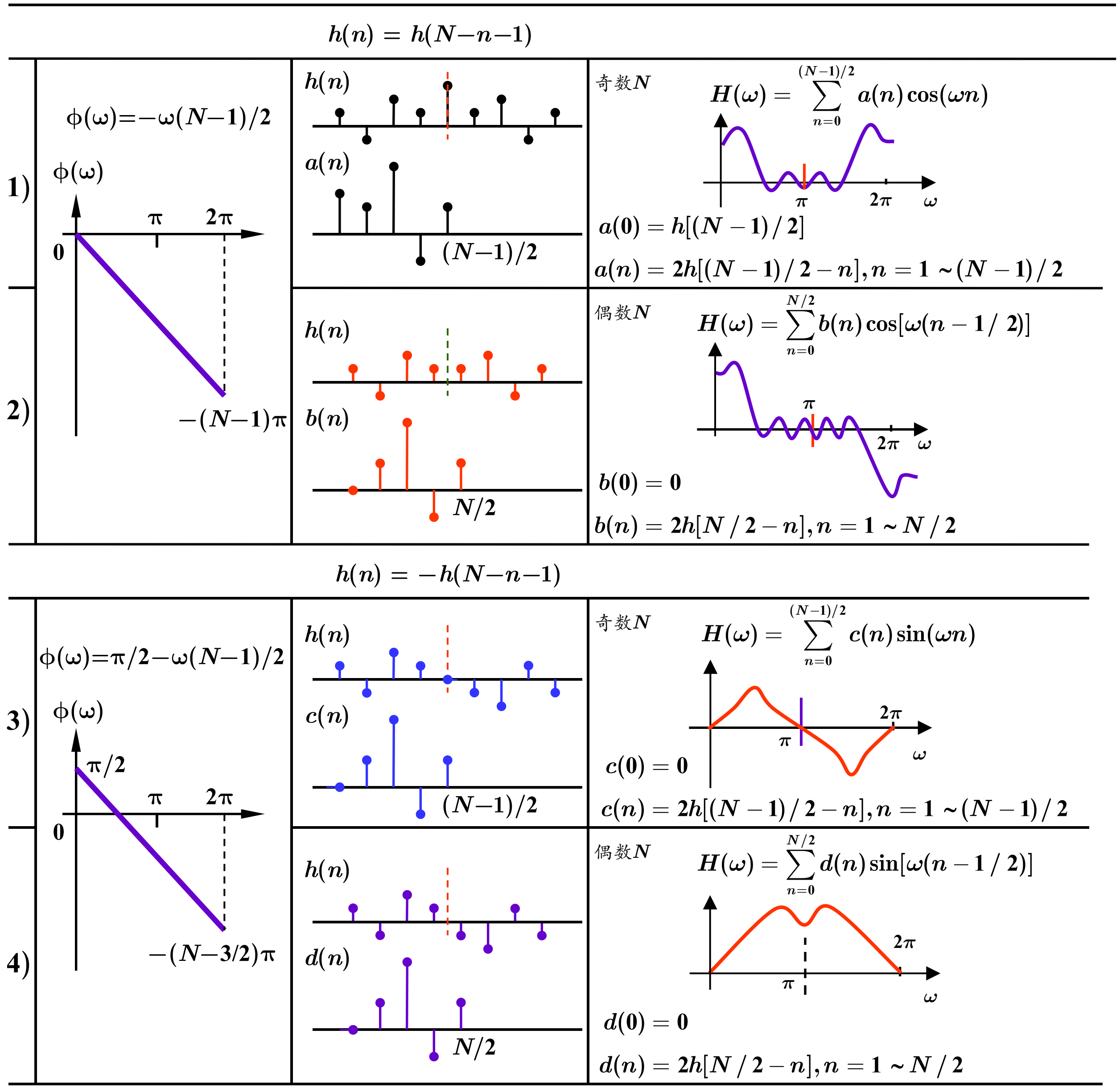
— $h(n)$ 偶对称： $L = 0$

— $h(n)$ 奇对称： $L = 1$

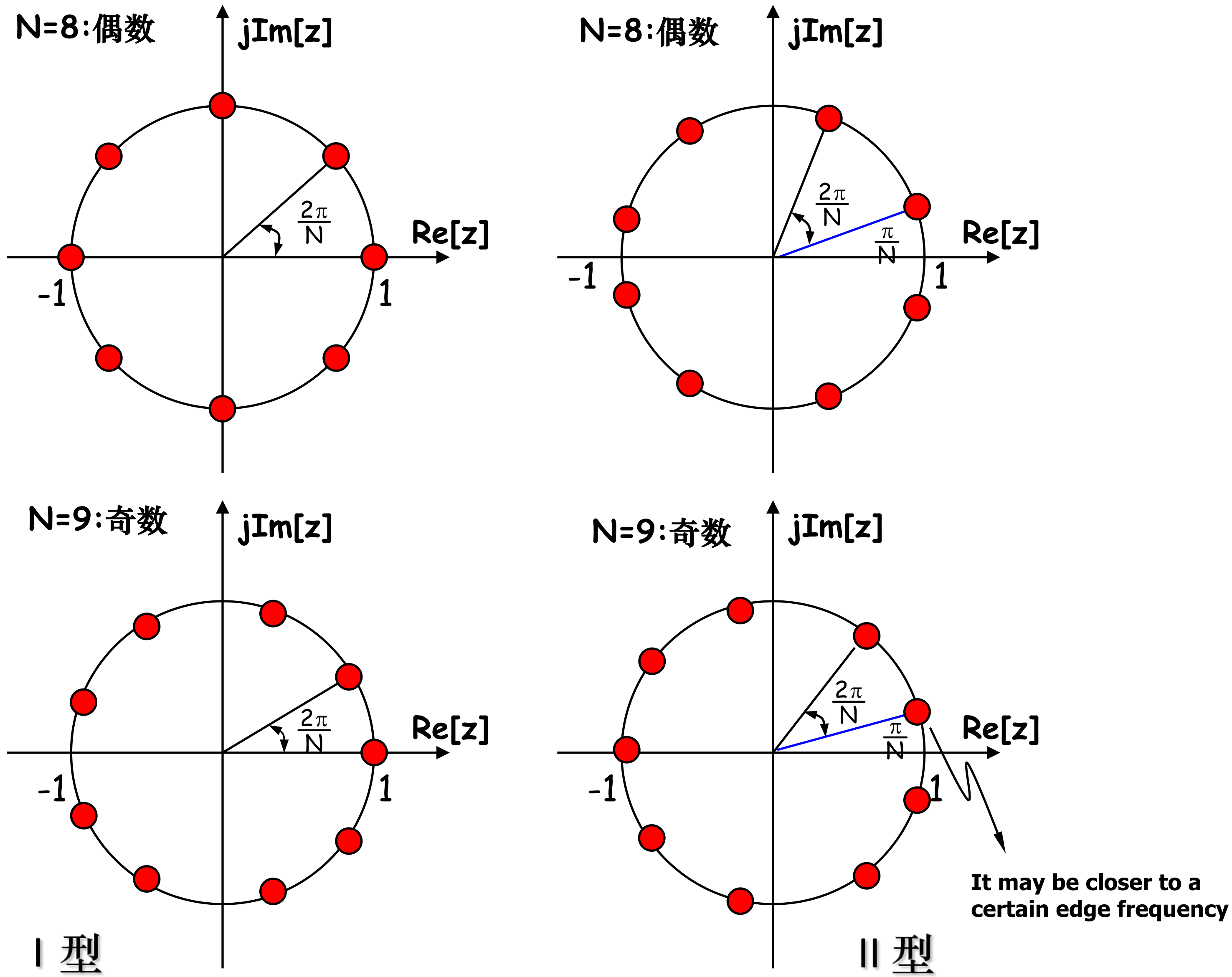
③ **零点：**

成倒易对出现

线性相位FIR滤波器的四种情况



频率抽样两种方法

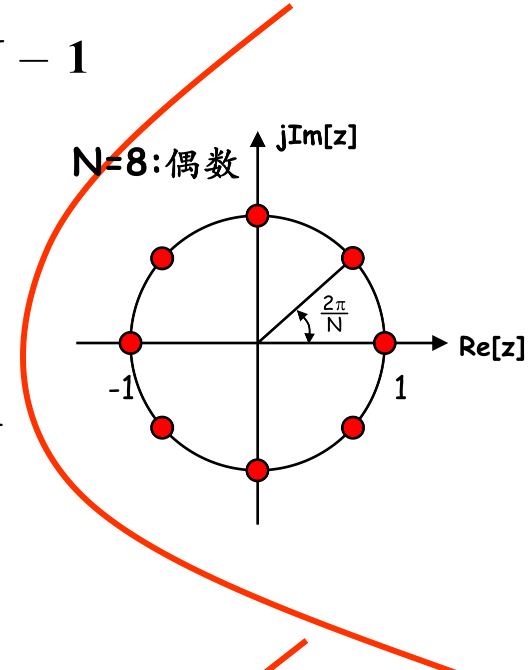

N=8:偶数
jIm[z]
Re[z]
-1
1
$\frac{2\pi}{N}$
N=8:偶数
jIm[z]
Re[z]
-1
1
$\frac{2\pi}{N}$
$\frac{\pi}{N}$
N=9:奇数
jIm[z]
Re[z]
-1
1
$\frac{2\pi}{N}$
N=9:奇数
jIm[z]
Re[z]
-1
1
$\frac{2\pi}{N}$
$\frac{\pi}{N}$
It may be closer to a
certain edge frequency
I 型
II 型

1）第一种频率抽样

$$H(k)=H_d(k)=H_d(z)\Big|_{z=e^{j\frac{2\pi}{N}k}}=H_d\left(e^{j\omega}\right)\Big|_{\omega=\frac{2\pi}{N}k}\qquad k=0,1,\ldots,N-1$$

系统函数：$$H(z)=\frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-W_N^{-k}z^{-1}}$$

频率响应：$$H\left(e^{j\omega}\right)=\frac{1}{N}e^{-j\left(\frac{N-1}{2}\right)\omega}\sum_{k=0}^{N-1}H(k)e^{-j\frac{\pi k}{N}}\frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}-\frac{\pi k}{N}\right)}$$



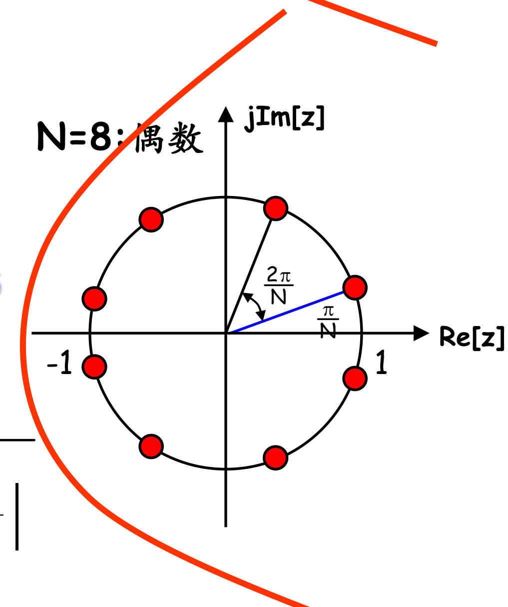
2）第二种频率抽样

$$H(k)=H_d(z)\Big|_{z=e^{j\left(\frac{2\pi}{N}k+\frac{\pi}{N}\right)}}=H_d\left(e^{j\omega}\right)\Big|_{\omega=\frac{2\pi}{N}k+\frac{\pi}{N}}\qquad k=0,1,\ldots,N-1$$

系统函数：$$H(z)=\frac{1+z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-e^{j\frac{2\pi}{N}\left(k+\frac{1}{2}\right)}z^{-1}}$$

pp. 245

频率响应：$$H\left(e^{j\omega}\right)=\frac{\cos\left(\frac{\omega N}{2}\right)}{N}e^{-j\left(\frac{N-1}{2}\right)\omega}\sum_{k=0}^{N-1}\frac{H(k)e^{-j\frac{\pi}{N}\left(k+\frac{1}{2}\right)}}{j\sin\left(\frac{\omega}{2}-\frac{\pi}{N}\left(k+\frac{1}{2}\right)\right)}$$



线性相位约束条件

第一种抽样方法				
	$h(n)$ 中心偶对称 N 为奇数	$h(n)$ 中心偶对称 N 为偶数	$h(n)$ 中心奇对称 N 为奇数	$h(n)$ 中心奇对称 N 为偶数
幅度约束	$\lvert H(k)\rvert = \lvert H(N-k)\rvert$ $k = 0 \sim (N-1)/2$	$\lvert H(k)\rvert = \lvert H(N-k)\rvert$ $k = 0 \sim (N/2-1)$ $\lvert H(N/2)\rvert = 0$	$\lvert H(k)\rvert = \lvert H(N-k)\rvert$ $k = 0 \sim (N-1)/2$ $\lvert H(0)\rvert = 0$	$\lvert H(k)\rvert = \lvert H(N-k)\rvert$ $k = 0 \sim (N/2-1)$ $\lvert H(0)\rvert = 0$
相位约束	$\varphi(k) = -\varphi(N-k)$ $= -k(1-N^{-1})\pi$ $k = 0 \sim (N-1)/2$	$\varphi(k) = -\varphi(N-k)$ $= -k(1-N^{-1})\pi$ $k = 0 \sim (N/2-1)$	$\varphi(k) = -\varphi(N-k)$ $= \frac{\pi}{2} - k(1-N^{-1})\pi$ $k = 0 \sim (N-1)/2$	$\varphi(k) = -\varphi(N-k)$ $= \frac{\pi}{2} - k(1-N^{-1})\pi$ $k = 0 \sim (N/2-1)$ $\varphi(N/2) = 0$

对于第一种抽样方式，当**h(n)**为实数时

$$h(n) = h^*(n)$$

$$H(k) = DFT[h(n)]$$

根据**P91（3-79）**

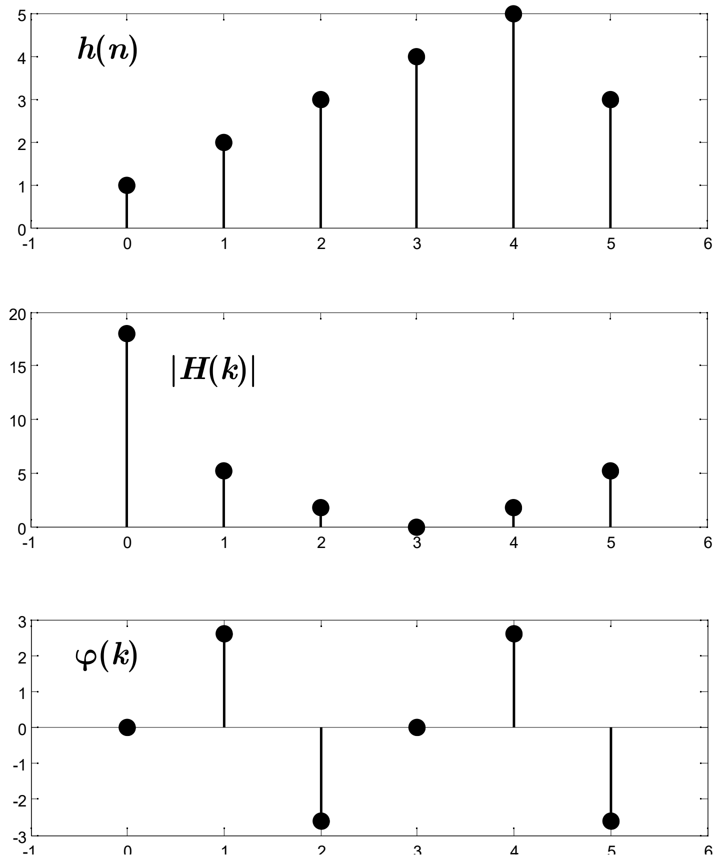
$$H^*(N-k) = DFT[h^*(n)]$$

于是

$$H(k) = H^*(N-k)$$

$$\begin{cases} |H(k)| = |H(N-k)| \\ \theta(k) = \arg[H(k)] = -\theta(N-k) \end{cases}$$

以$k = \dfrac{N}{2}$中心



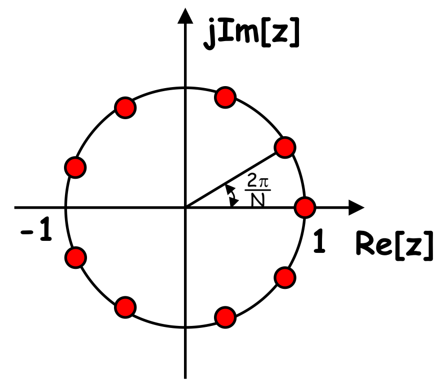
线性相位系统传函和频响

第一种频率抽样方法：

由： $\theta(\omega) = -\left|\frac{N-1}{2}\right|\omega$

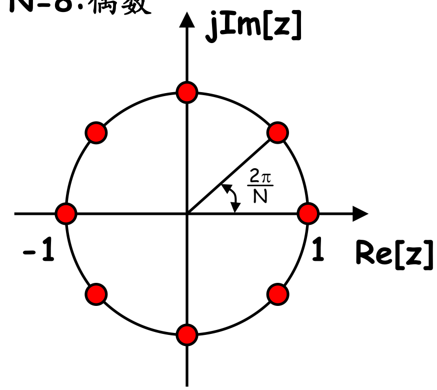
N为奇数： $$\theta(k) = \begin{cases} -\frac{2\pi}{N}k\left|\frac{N-1}{2}\right| & k = 0,\ldots,\frac{N-1}{2} \\ \frac{2\pi}{N}(N-k)\left|\frac{N-1}{2}\right| & k = \frac{N+1}{2},\ldots,N-1 \end{cases}$$

N=9:奇数



N为偶数： $$\theta(k) = \begin{cases} -\frac{2\pi}{N}k\ \frac{N-1}{2} & k = 0,\ldots,\frac{N}{2}-1 \\ 0 & k = \frac{N}{2} \\ \frac{2\pi}{N}(N-k)\ \frac{N-1}{2} & k = \frac{N}{2}+1,\ldots,N-1 \end{cases}$$

N=8:偶数



当N为奇数时：

$$H(k)=\begin{cases}\left|H(k)\right|e^{-j\frac{2\pi}{N}k\left(\frac{N-1}{2}\right)} & k=0,\ldots,\dfrac{N-1}{2}\\[2ex] \left|H(k)\right|e^{j\frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right)} & k=\dfrac{N+1}{2},\ldots,N-1\end{cases}$$

当N为偶数时：

$$H(k)=\begin{cases}\left|H(k)\right|e^{-j\frac{2\pi}{N}k\left(\frac{N-1}{2}\right)} & k=0,\ldots,\left(\dfrac{N}{2}-1\right)\\[2ex] 0 & k=\dfrac{N}{2}\\[2ex] \left|H(k)\right|e^{j\frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right)} & k=\left(\dfrac{N}{2}+1\right),\ldots,N-1\end{cases}$$

频率响应：

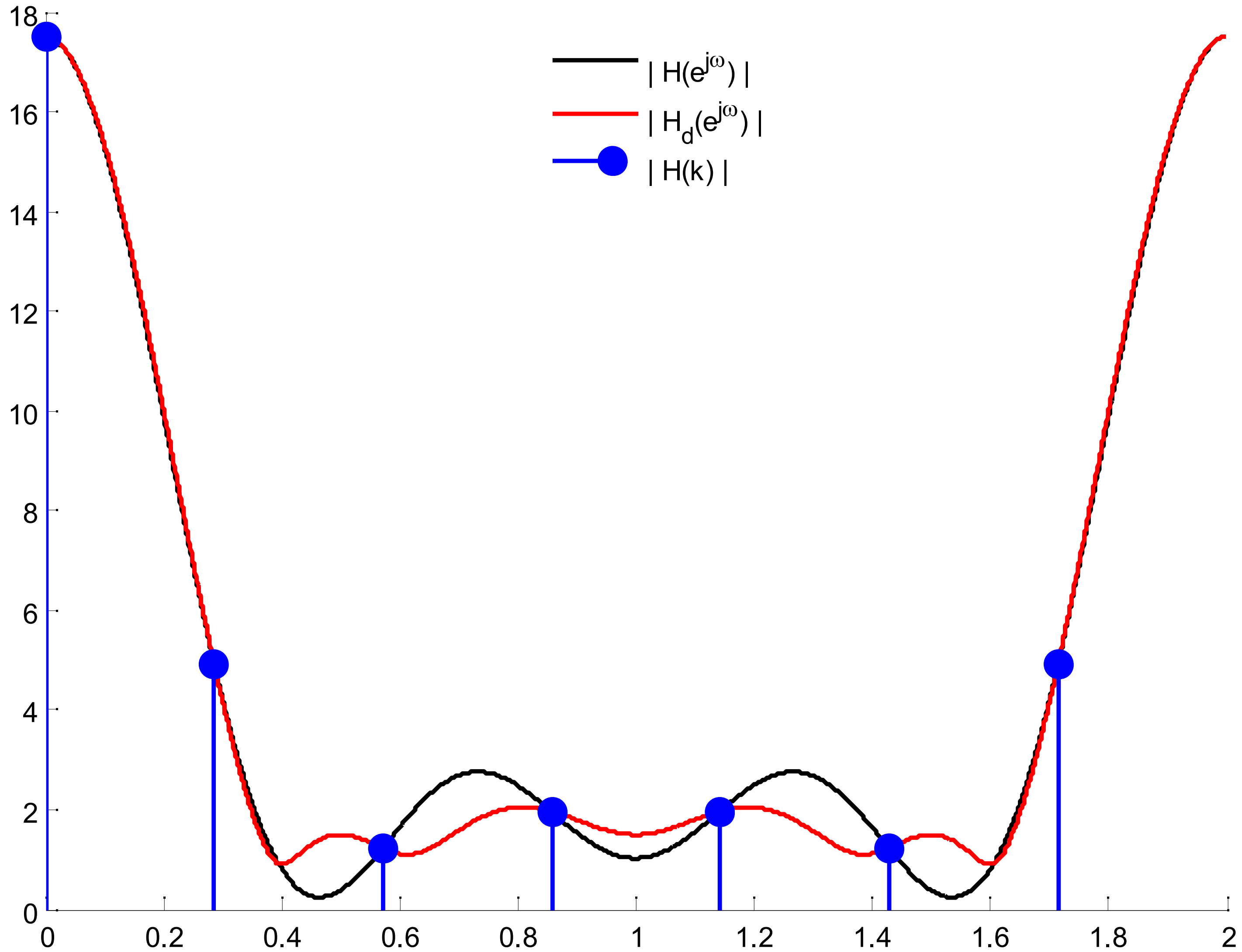
$$H(e^{j\omega})=e^{-j\left(\frac{N-1}{2}\right)\omega}\left\{\frac{|H(0)|\sin\left(\frac{\omega N}{2}\right)}{N\sin\left(\frac{\omega}{2}\right)}+\sum_{k=1}^{M}\frac{|H(k)|}{N}\left[\frac{\sin\left[N\left(\frac{\omega}{2}-\frac{k\pi}{N}\right)\right]}{\sin\left(\frac{\omega}{2}-\frac{k\pi}{N}\right)}+\frac{\sin\left[N\left(\frac{\omega}{2}+\frac{k\pi}{N}\right)\right]}{\sin\left(\frac{\omega}{2}+\frac{k\pi}{N}\right)}\right]\right\}$$

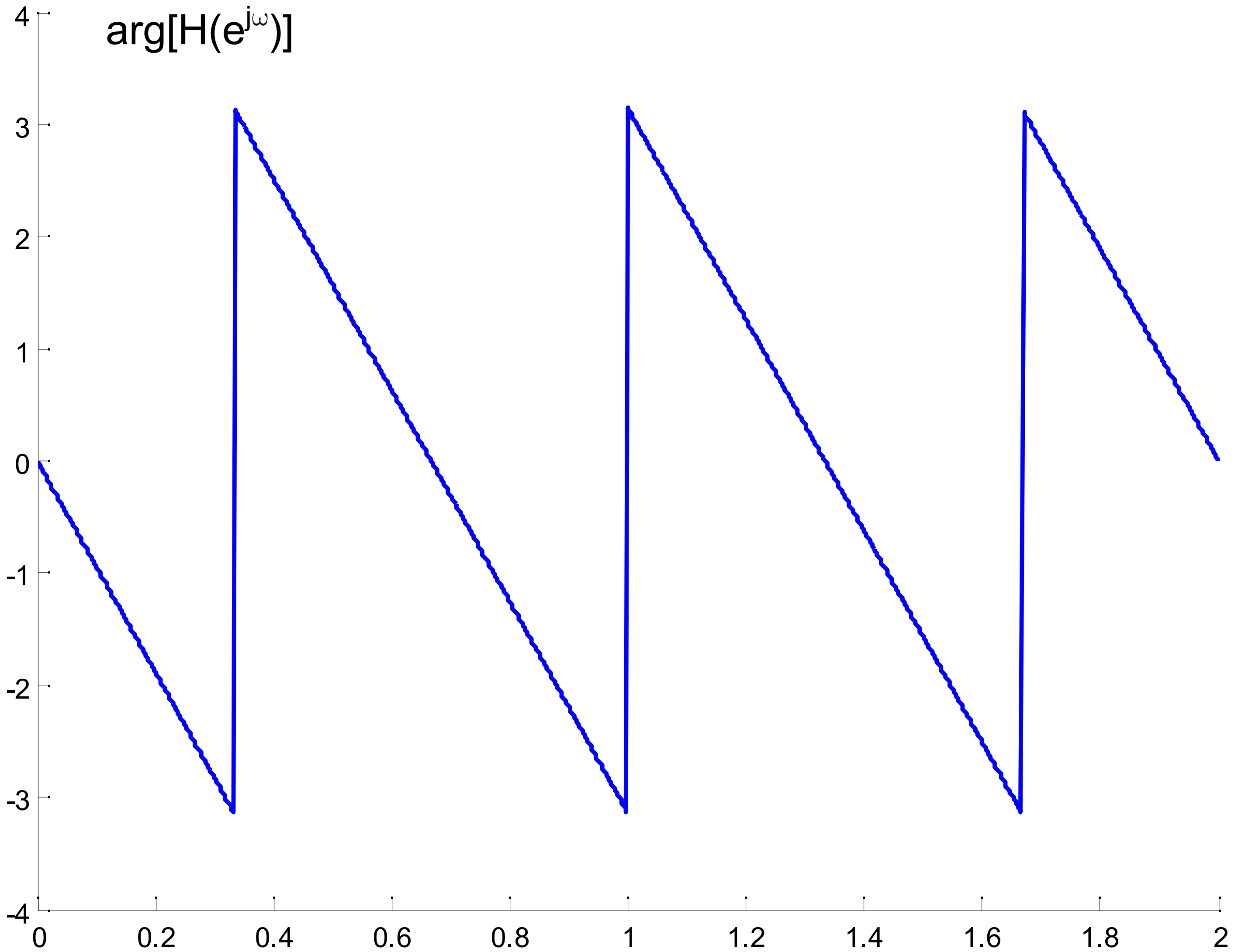
N为奇数$M=(N-1)/2$，　N为偶数$M=N/2-1$

第一种抽样

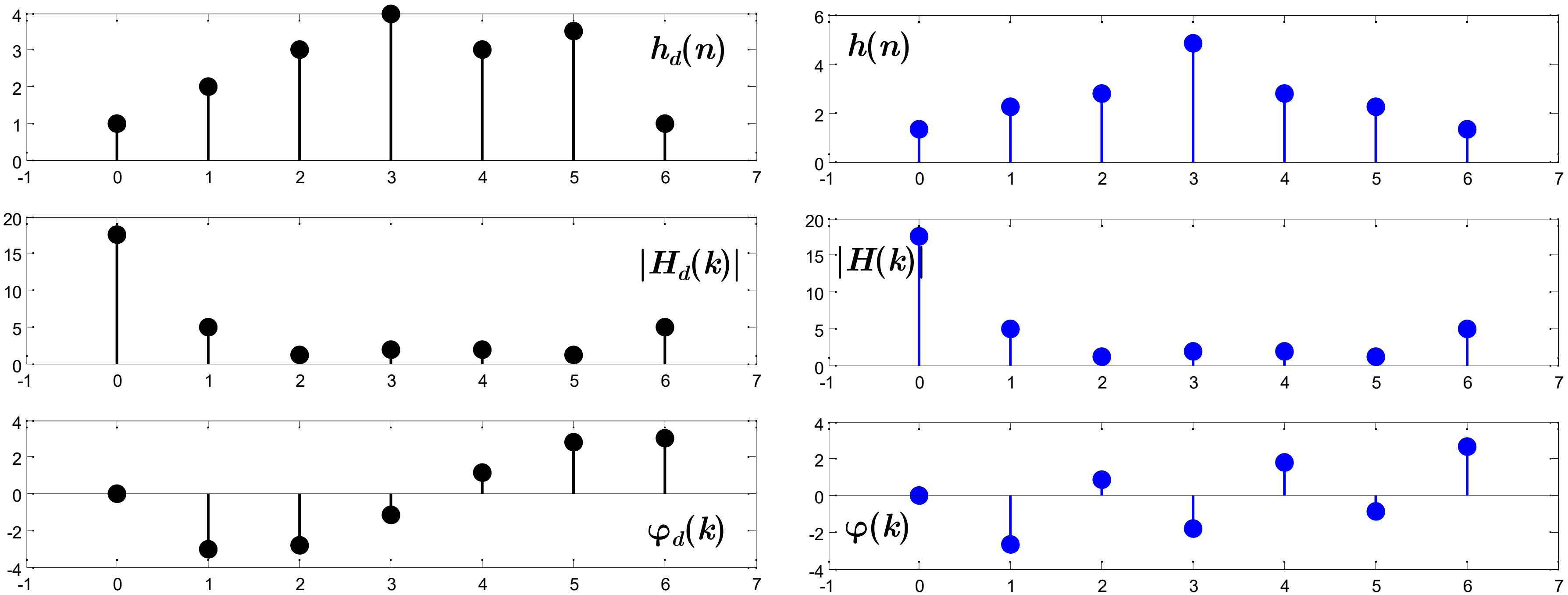
偶对称　$N = 7$

奇数点



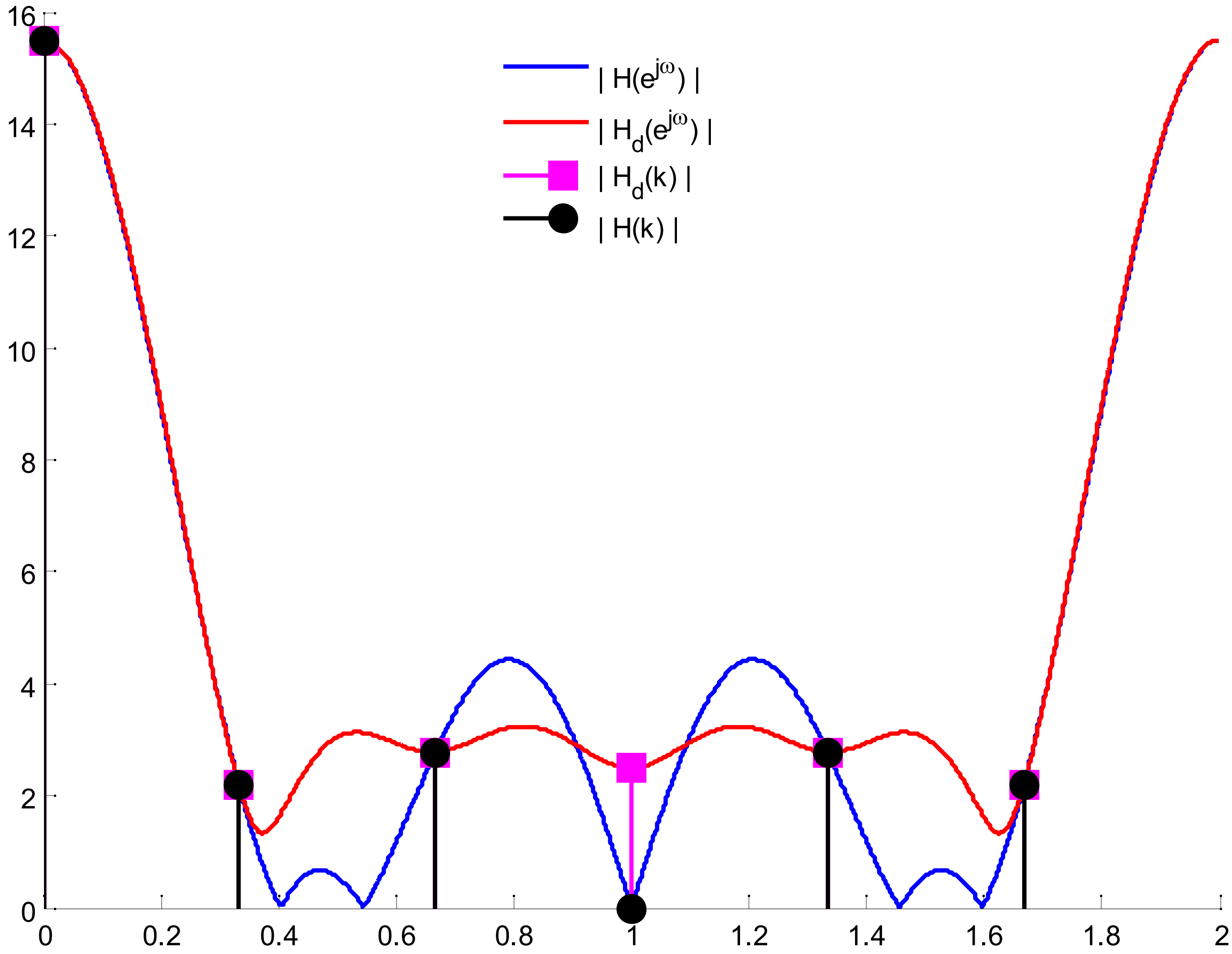


$\arg[H(e^{j\omega})]$
4
3
2
1
0
-1
-2
-3
-4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

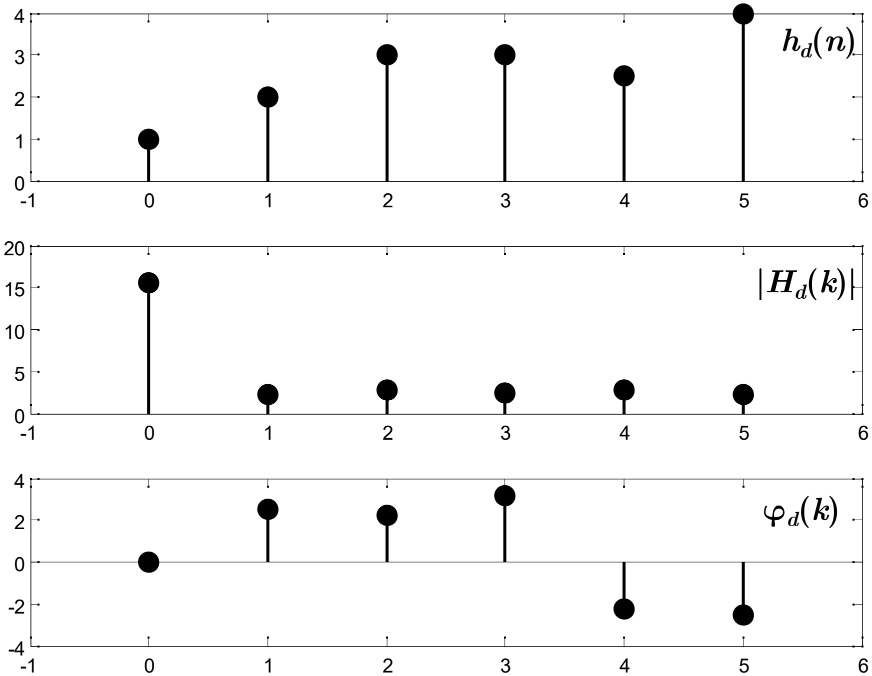


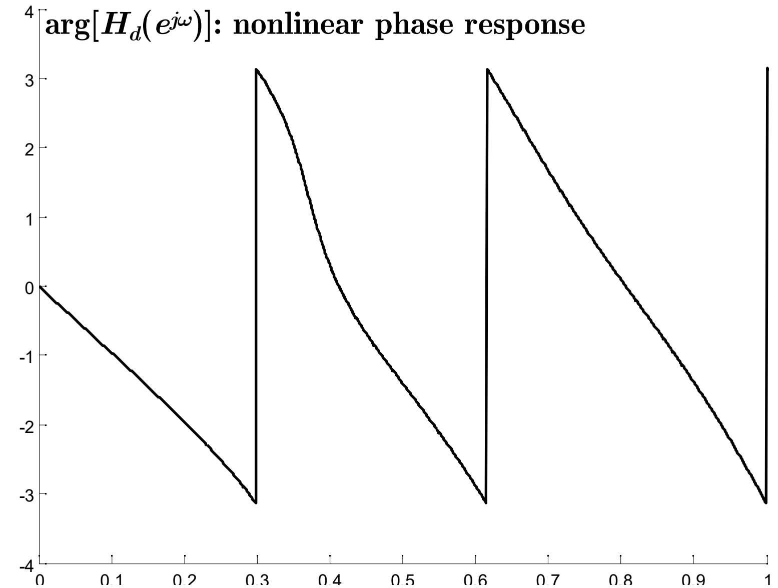
$h_d(n)$
$h(n)$
$|H_d(k)|$
$|H(k)|$
$\varphi_d(k)$
$\varphi(k)$

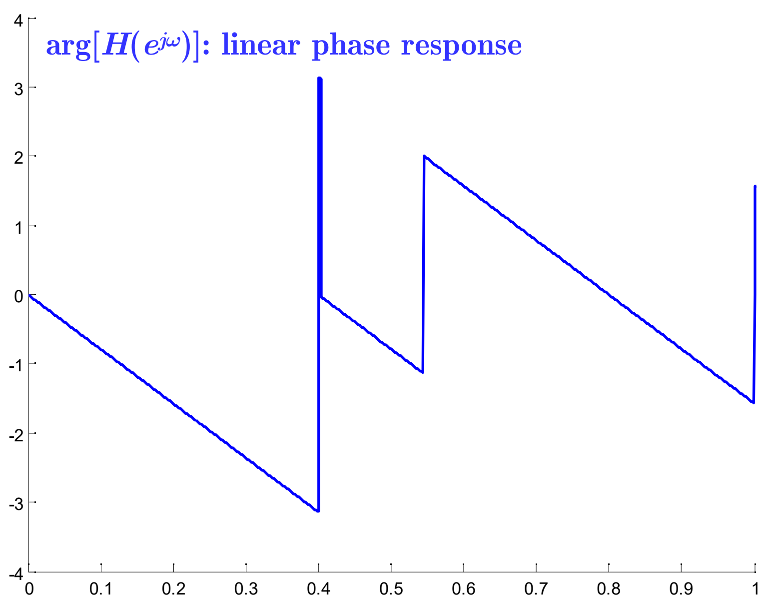
第一种抽样
偶对称　$N = 6$
偶数点

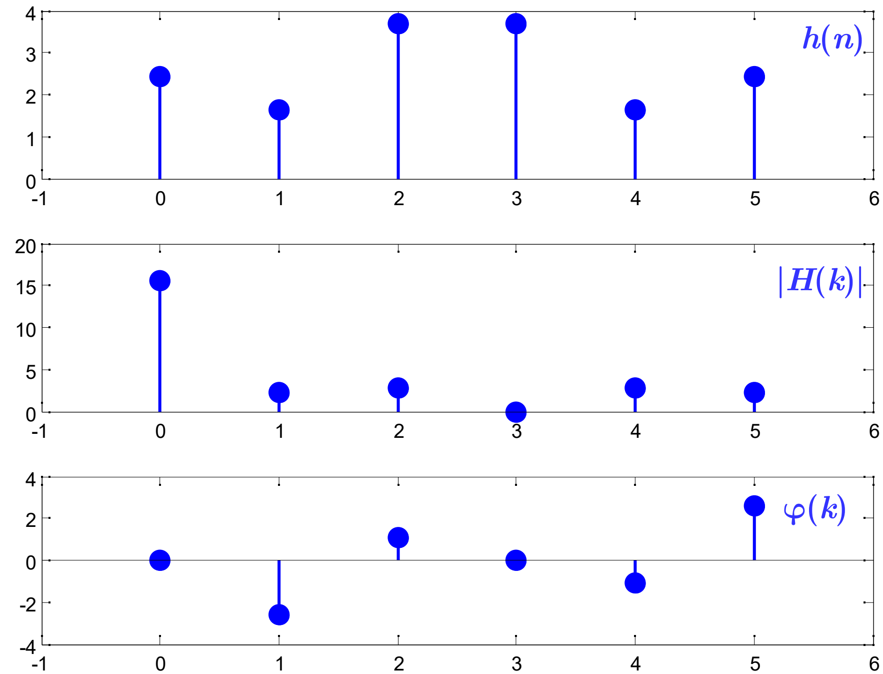


强行置零

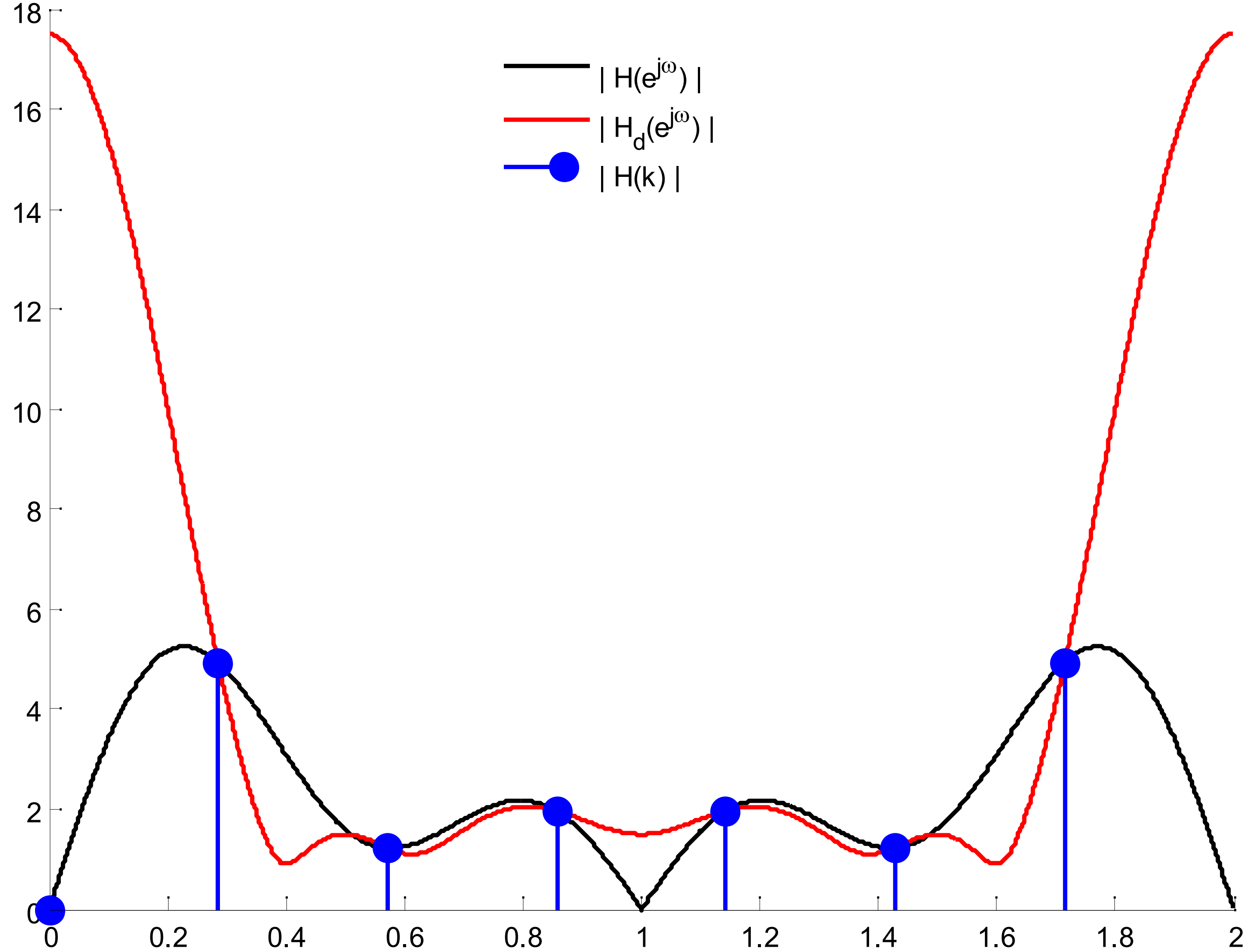

$h_d(n)$
$|H_d(k)|$
$\varphi_d(k)$

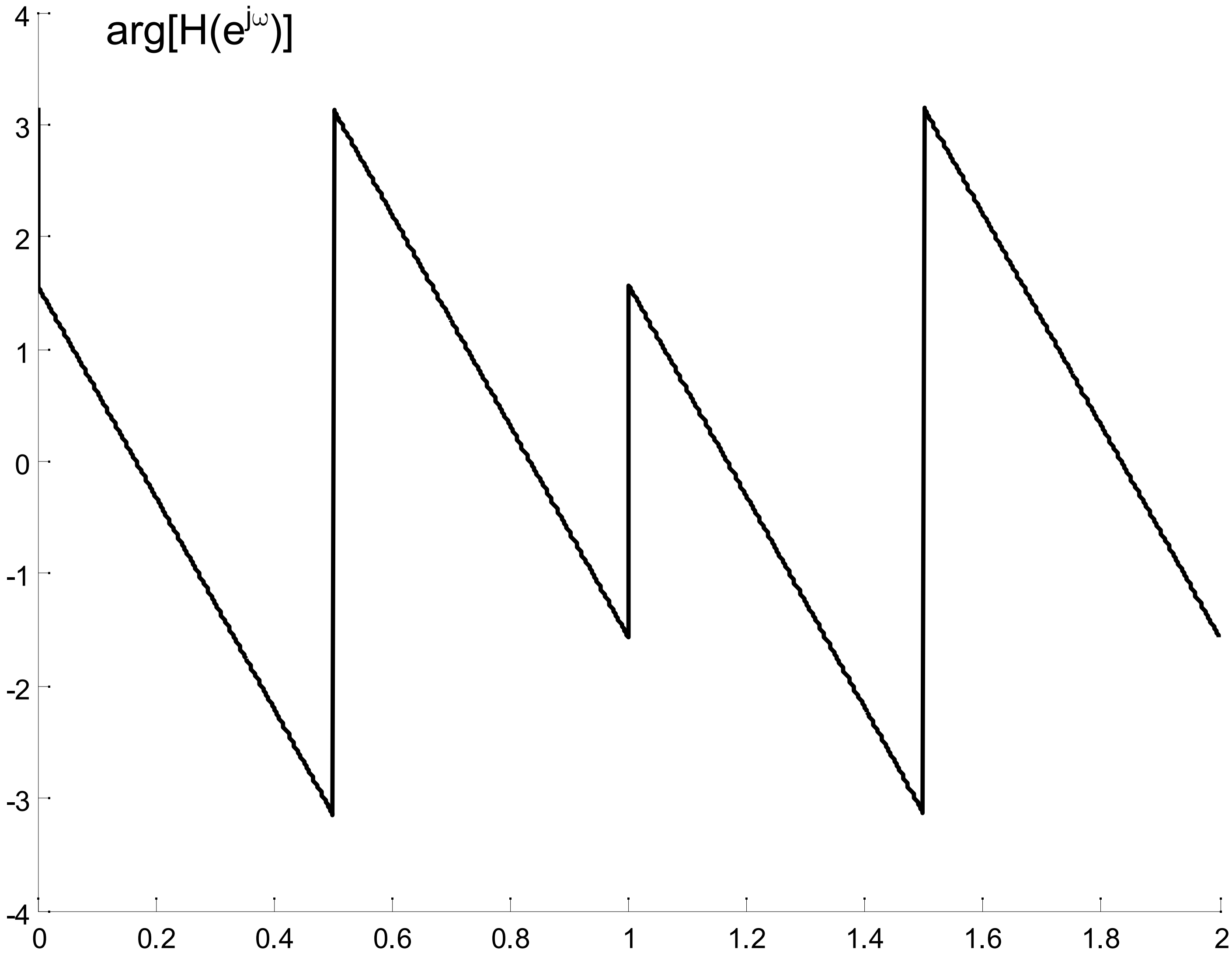

$\arg[H_d(e^{j\omega})]$: nonlinear phase response

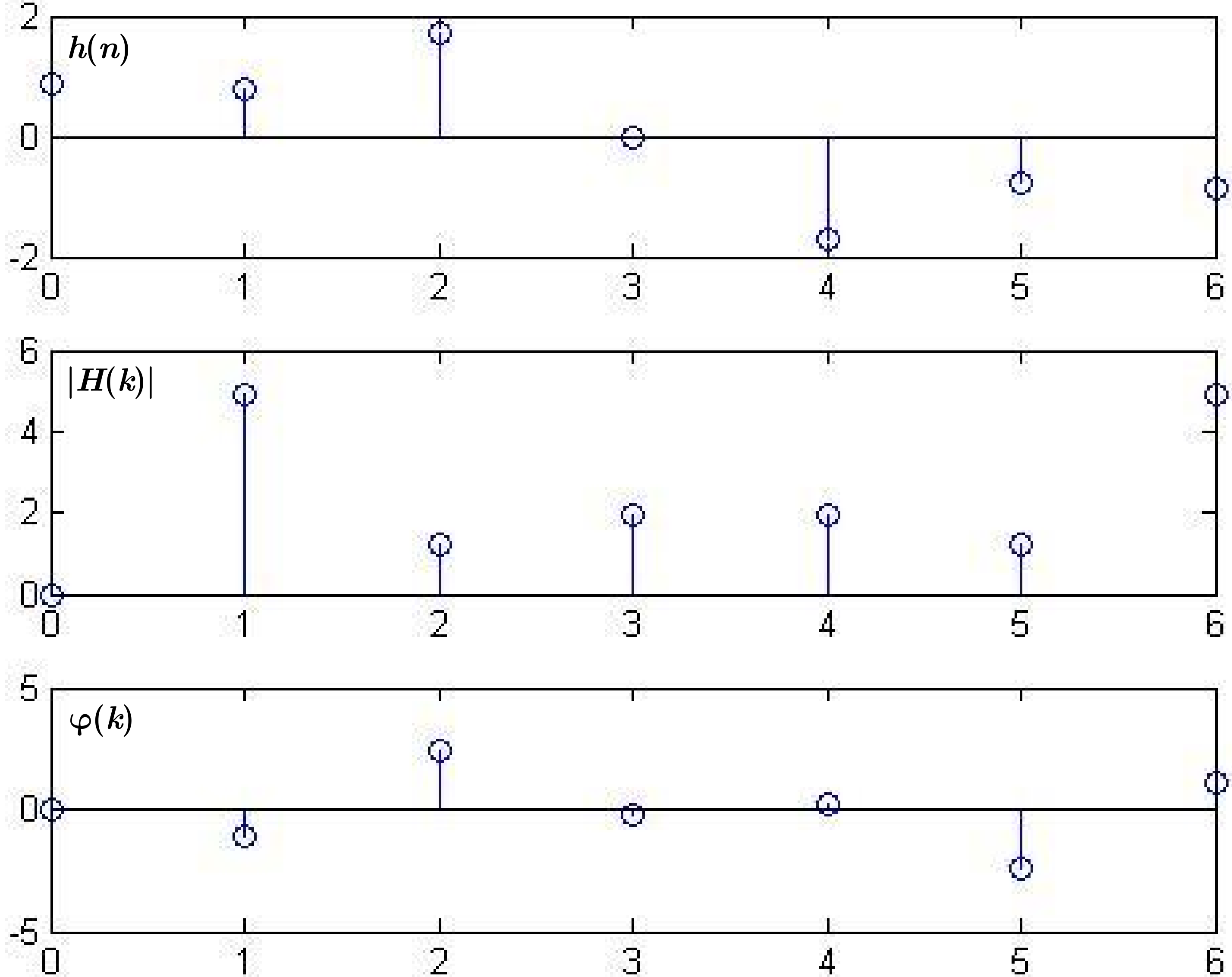

$\arg[H(e^{j\omega})]$: linear phase response


$h(n)$
$|H(k)|$
$\varphi(k)$

第一种抽样
奇对称 $N=7$
奇数点




arg[H(e^jω)]
4
3
2
1
0
-1
-2
-3
-4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

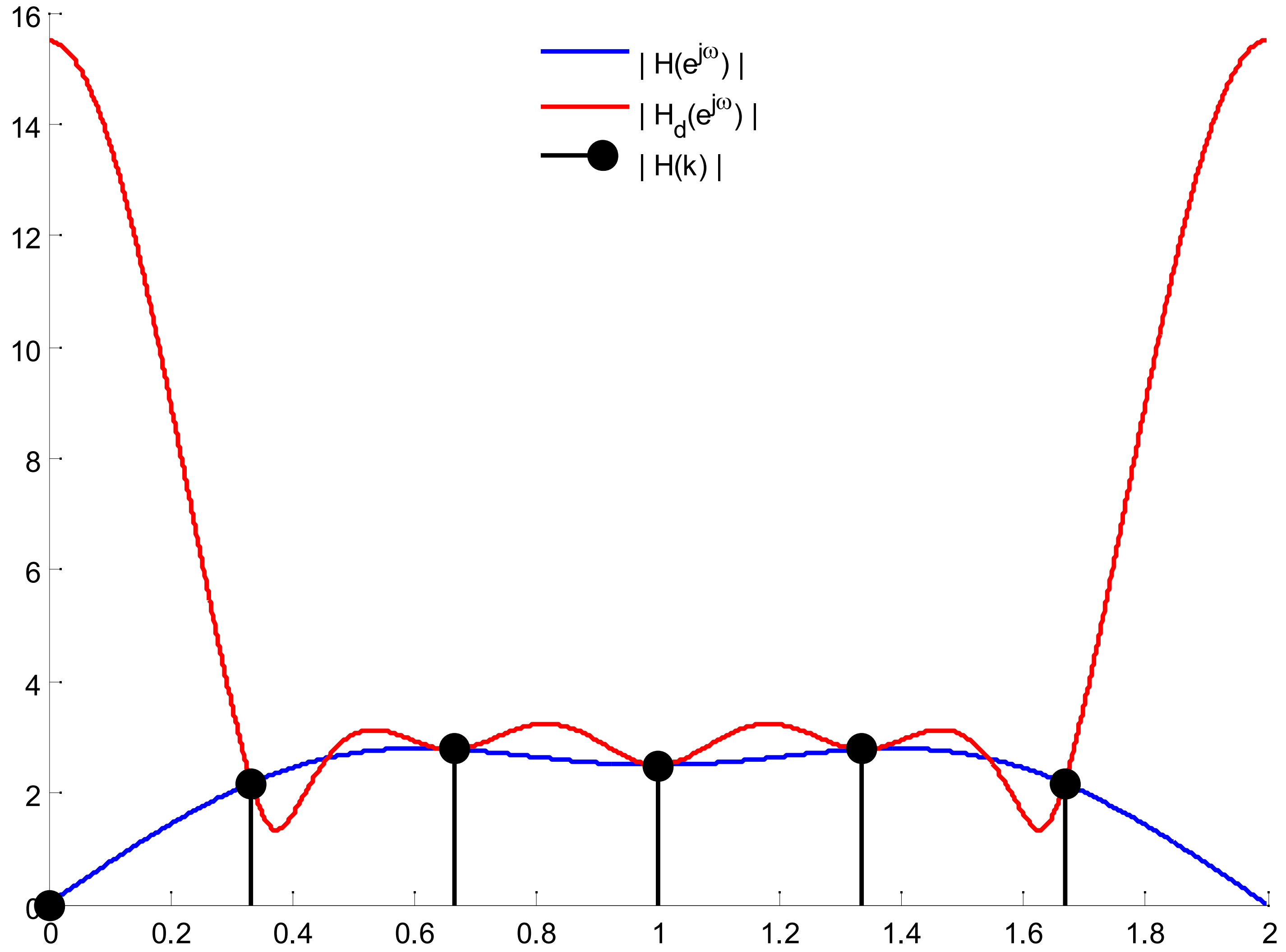


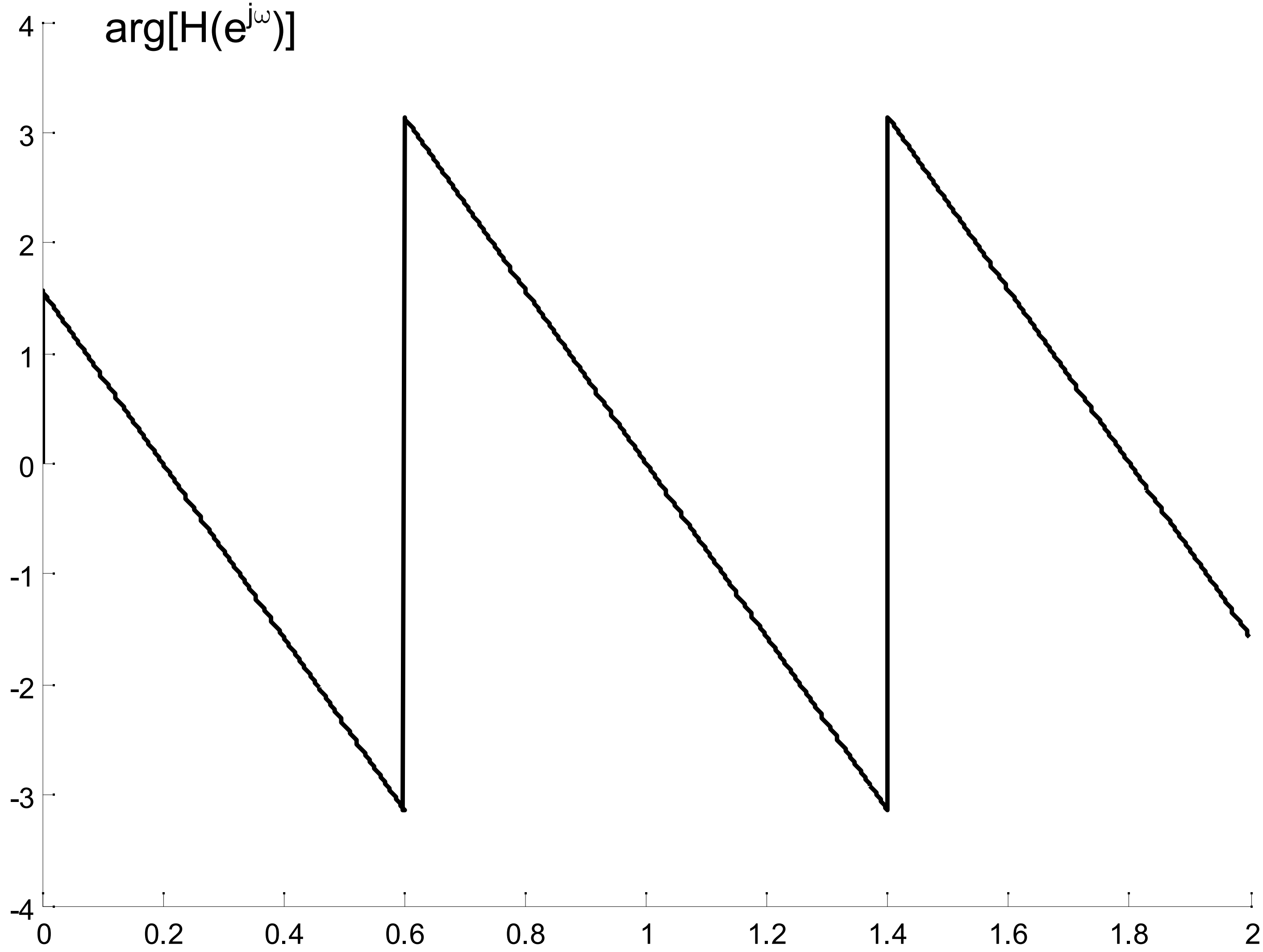
$h(n)$
$|H(k)|$
$\varphi(k)$
2
0
-2
6
4
5
-5
0 1 2 3 4 5 6

第一种抽样

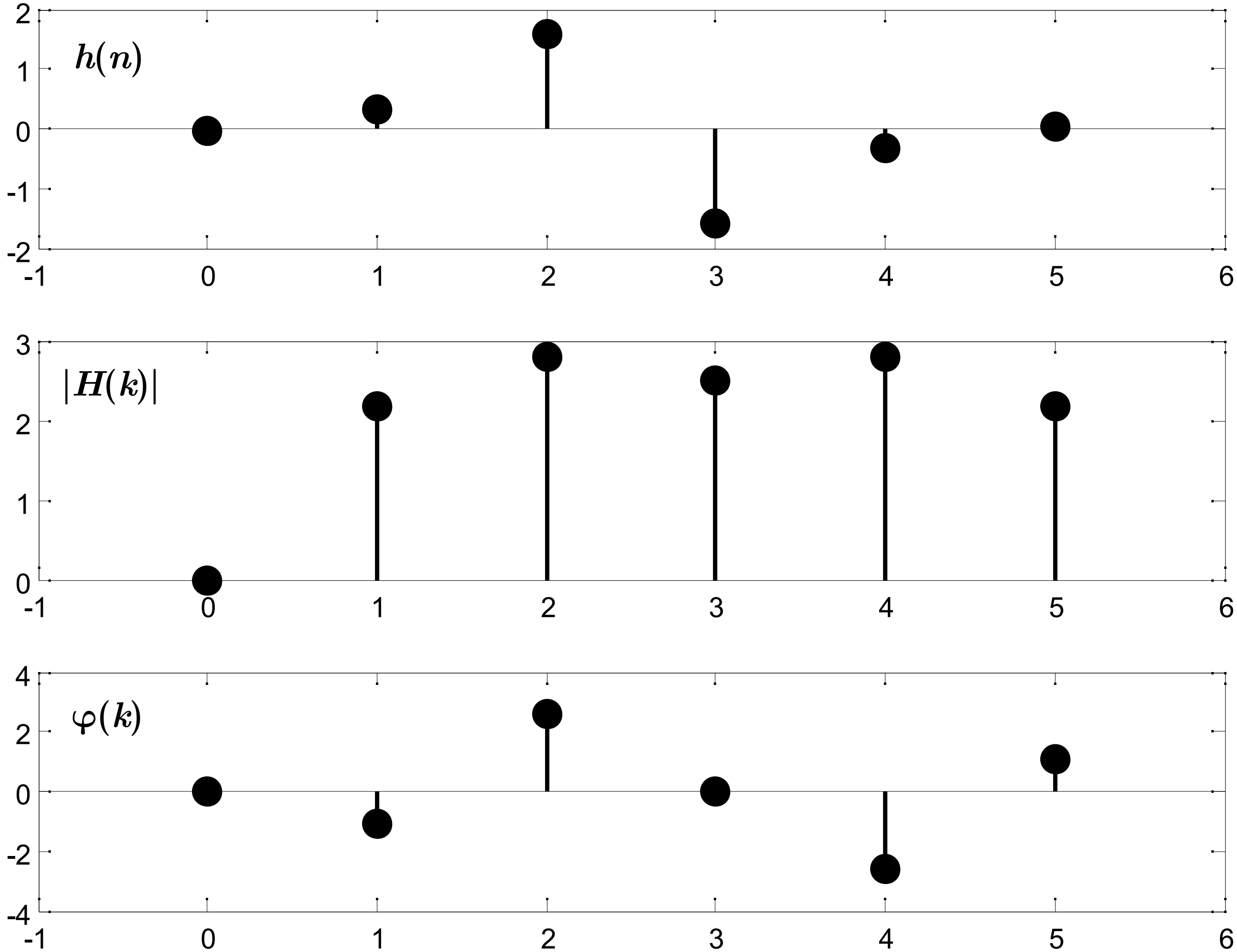
奇对称 $N=6$

偶数点





arg[H(e^jω)]
4
3
2
1
0
-1
-2
-3
-4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



$h(n)$
$|H(k)|$
$\varphi(k)$

线性相位约束条件

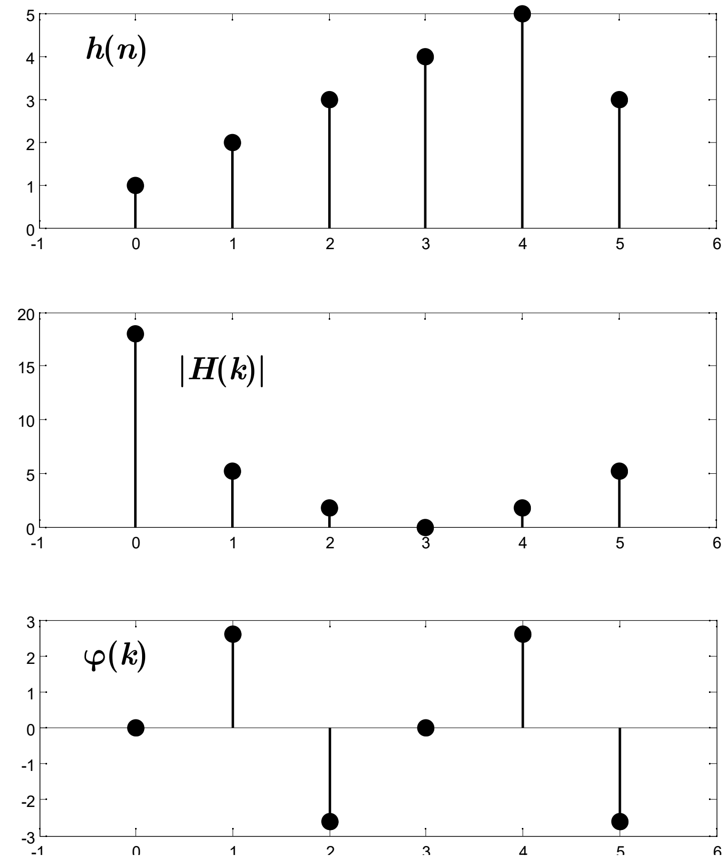
第二种抽样方法				
	$h(n)$中心偶对称 N为奇数	$h(n)$中心偶对称 N为偶数	$h(n)$中心奇对称 N为奇数	$h(n)$中心奇对称 N为偶数
幅度约束	$\|H(k)\|=\|H(N-k-1)\|$ $k=0\sim(N-1)/2$	$\|H(k)\|=\|H(N-k-1)\|$ $k=0\sim(N/2-1)$	$\|H(k)\|=\|H(N-k-1)\|$ $k=0\sim(N-1)/2$ $\|H(\frac{N-1}{2})\|=0$	$\|H(k)\|=\|H(N-k-1)\|$ $k=0\sim(N/2-1)$
相位约束	$\varphi(k)=-\varphi(N-k-1)$ $=-(k+\frac{1}{2})(1-\frac{1}{N})\pi$ $k=0\sim(N-3)/2$ $\varphi(\frac{N-1}{2})=0$	$\varphi(k)=-\varphi(N-k-1)$ $=-(k+\frac{1}{2})(1-\frac{1}{N})\pi$ $k=0\sim(N/2-1)$	$\varphi(k)=-\varphi(N-k-1)$ $=\frac{\pi}{2}-(k+\frac{1}{2})(1-\frac{1}{N})\pi$ $k=0\sim(N-3)/2$	$\varphi(k)=-\varphi(N-k-1)$ $=\frac{\pi}{2}-(k+\frac{1}{2})(1-\frac{1}{N})\pi$ $k=0\sim(N/2-1)$

对于第二种抽样方式，当h(n)为实数时

$$H(k) = H^*(N-1-k)$$

$$\begin{cases} |H(k)| = |H(N-1-k)| \\ \theta(k) = \arg[H(k)] = -\theta(N-1-k) \end{cases}$$

以$k = \dfrac{N-1}{2}$中心

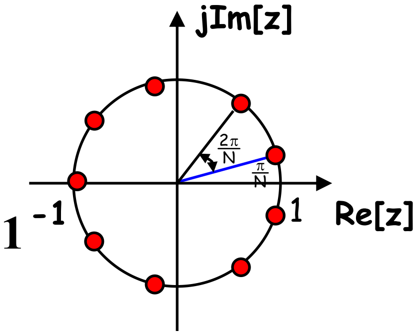


线性相位系统传函和频响

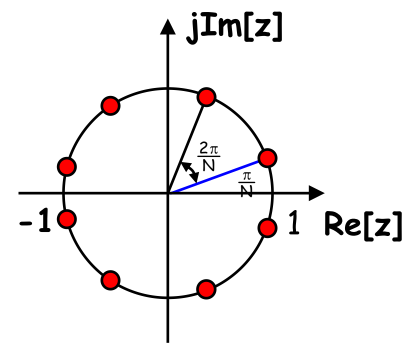
第二种频率抽样方法：

$$N为奇数：\theta(k)=\begin{cases} -\dfrac{2\pi}{N}\ k+\dfrac{1}{2}\ \ \dfrac{N-1}{2} & k=0,\dots,\dfrac{N-3}{2} \\ 0 & k=\dfrac{N-1}{2} \\ \dfrac{2\pi}{N}\ N-k-\dfrac{1}{2}\ \ \dfrac{N-1}{2} & k=\dfrac{N+1}{2},\dots,N-1 \end{cases}$$

N=9:奇数



N=8:偶数



$$N为偶数：\theta(k)=\begin{cases} -\dfrac{2\pi}{N}\ k+\dfrac{1}{2}\ \ \dfrac{N-1}{2} & k=0,\dots,\ \dfrac{N}{2}-1 \\ \dfrac{2\pi}{N}\ N-k-\dfrac{1}{2}\ \ \dfrac{N-1}{2} & k=\dfrac{N}{2},\dots,N-1 \end{cases}$$

当N为奇数时：

$$H(k)=\begin{cases} |H(k)|e^{-j\frac{2\pi}{N}\ k+\frac{1}{2}\ \frac{N-1}{2}} & k=0,\ldots,\frac{N-3}{2} \\ \left|H\left|\frac{N-1}{2}\right|\right| & k=\frac{N-1}{2} \\ |H(k)|e^{j\frac{2\pi}{N}\ N-k-\frac{1}{2}\ \frac{N-1}{2}} & k=\frac{N+1}{2},\ldots,N-1 \end{cases}$$

当N为偶数时：

$$H(k)=\begin{cases} |H(k)|e^{-j\frac{2\pi}{N}\ k+\frac{1}{2}\ \frac{N-1}{2}} & k=0,\ldots,\left|\frac{N}{2}-1\right| \\ |H(k)|e^{j\frac{2\pi}{N}\ N-k-\frac{1}{2}\ \frac{N-1}{2}} & k=\frac{N}{2},\ldots,N-1 \end{cases}$$

频率响应：

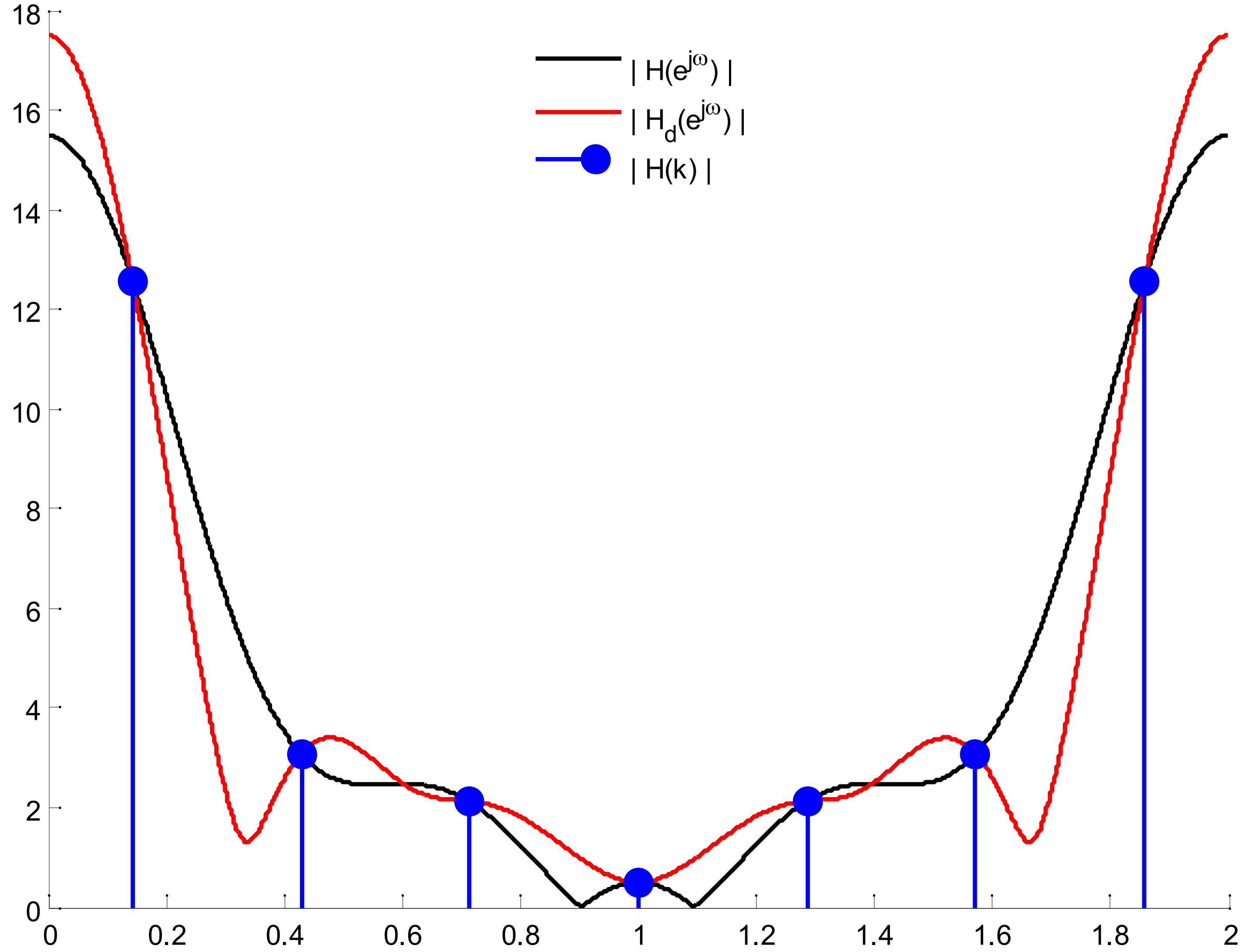
$$H\left(e^{j\omega}\right)=e^{-j\left|\frac{N-1}{2}\right|\omega}\left\{H_{\frac{N-1}{2}}(\omega)+\sum_{k=0}^{M}\frac{|H(k)|}{N}\left[\frac{\sin\ N\ \frac{\omega}{2}-\frac{\pi}{N}\ k+\frac{1}{2}}{\sin\ \frac{\omega}{2}-\frac{\pi}{N}\left|k+\frac{1}{2}\right|}+\frac{\sin\ N\ \frac{\omega}{2}+\frac{\pi}{N}\ k+\frac{1}{2}}{\sin\ \frac{\omega}{2}+\frac{\pi}{N}\left|k+\frac{1}{2}\right|}\right]\right\}$$

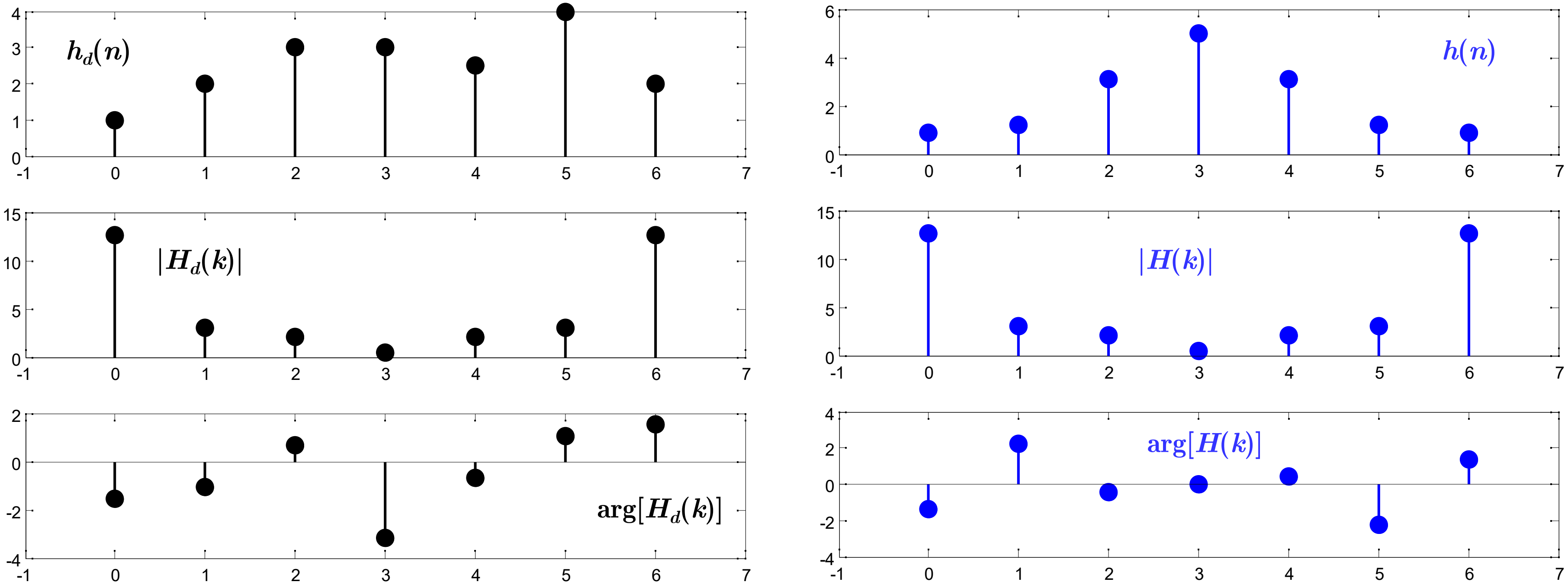
$$\begin{cases} H_{\frac{N-1}{2}}(\omega)=\frac{\left|H\left(\frac{N-1}{2}\right)\right|}{N}\cdot\frac{\cos\left(\frac{\omega N}{2}\right)}{\cos\left(\frac{\omega}{2}\right)},M=\frac{N-3}{2} & n:odd \\ H_{\frac{N-1}{2}}(\omega)=0\qquad ,M=\frac{N}{2}-1 & n:even \end{cases}$$

第二种抽样

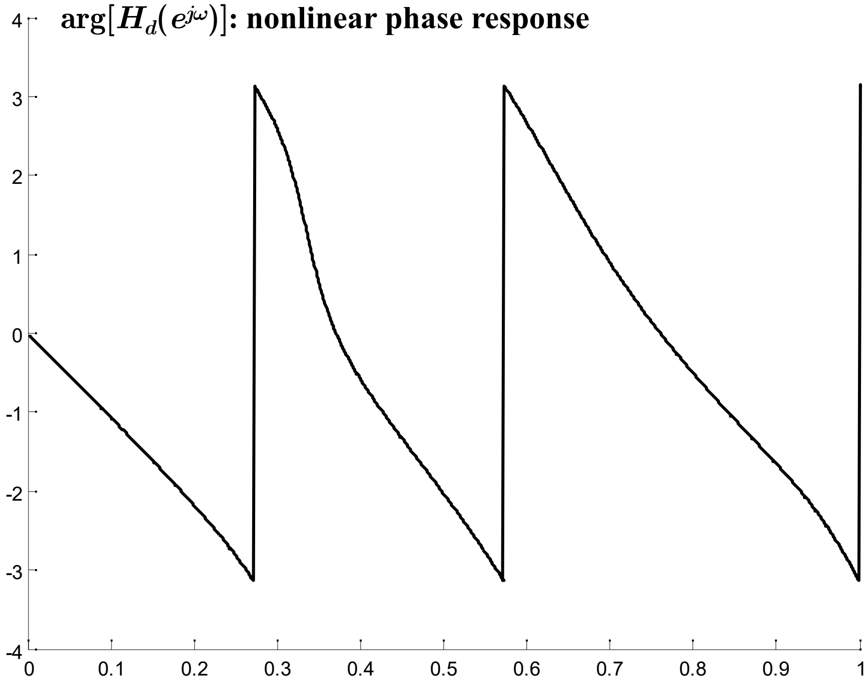
偶对称　$N = 7$

奇数点

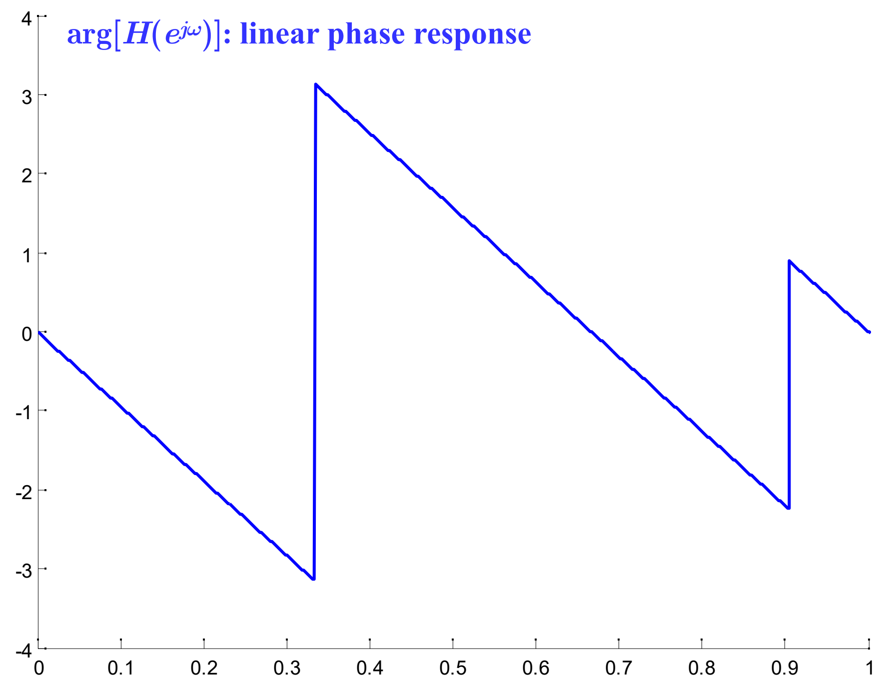




$h_d(n)$
$|H_d(k)|$
$\arg[H_d(k)]$
$h(n)$
$|H(k)|$
$\arg[H(k)]$



$\arg[H_d(e^{j\omega})]$: nonlinear phase response
4
3
2
1
0
-1
-2
-3
-4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

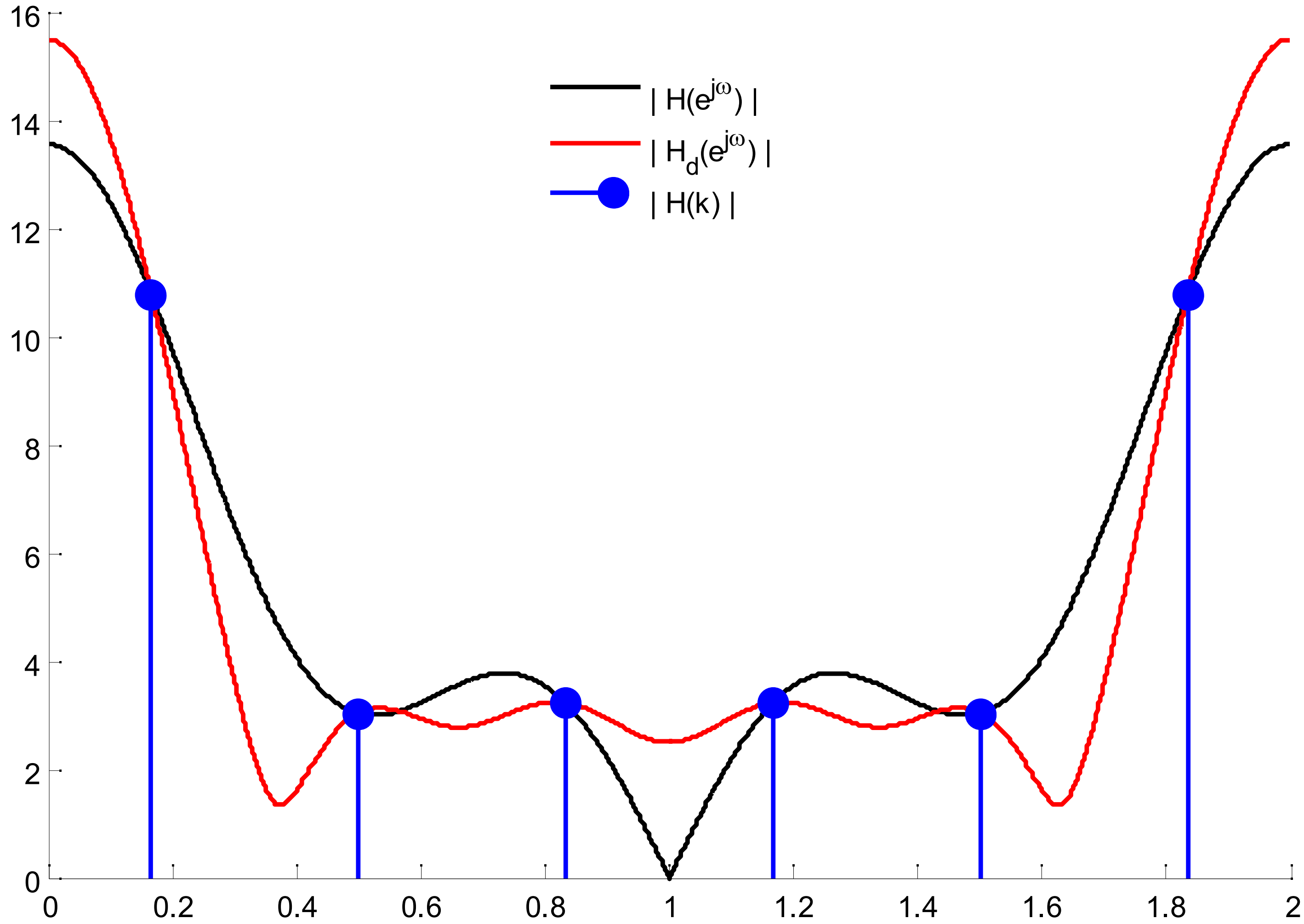


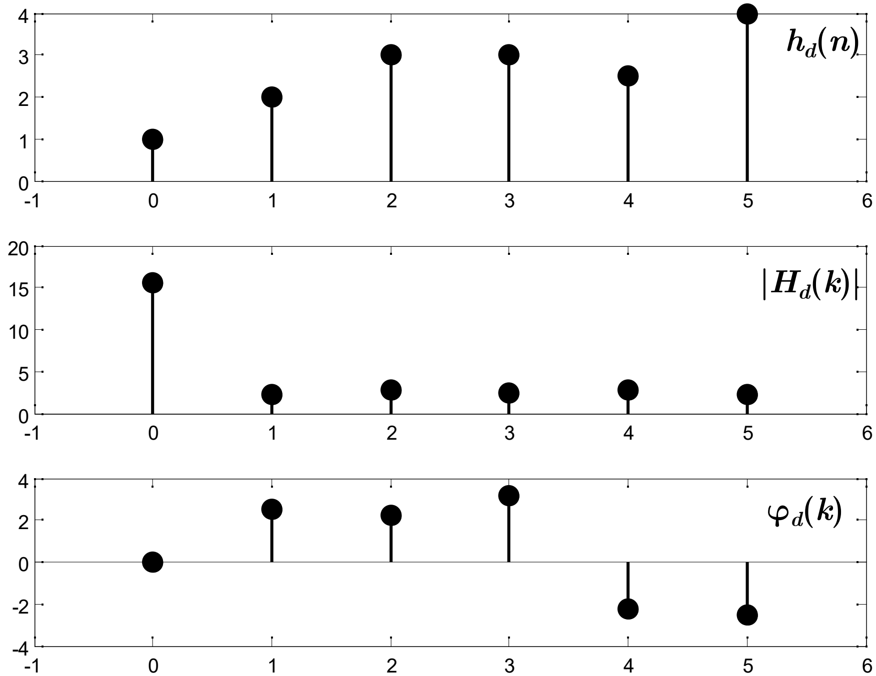
$\arg[H(e^{j\omega})]$: linear phase response
4
3
2
1
0
-1
-2
-3
-4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

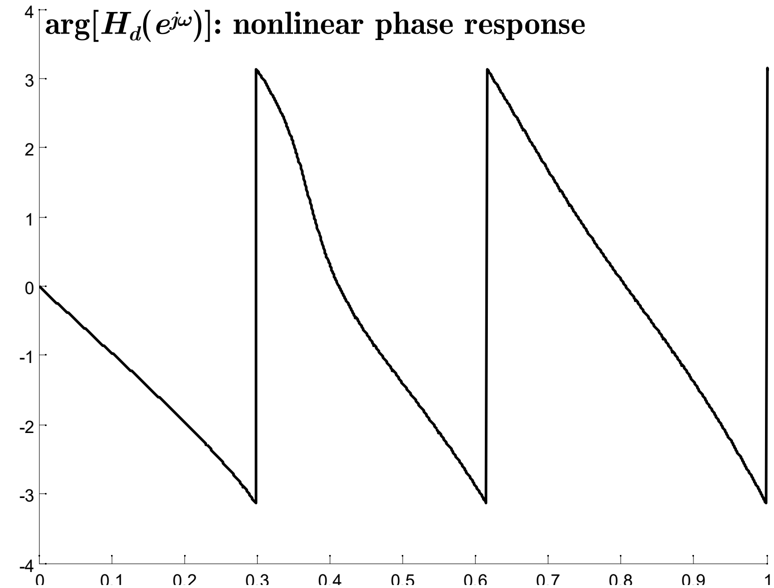
第二种抽样

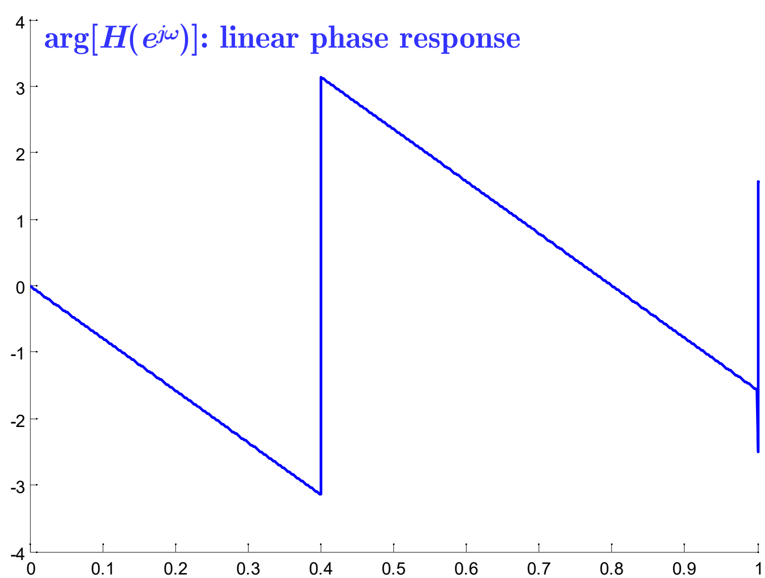
偶对称　$\boldsymbol{N=6}$

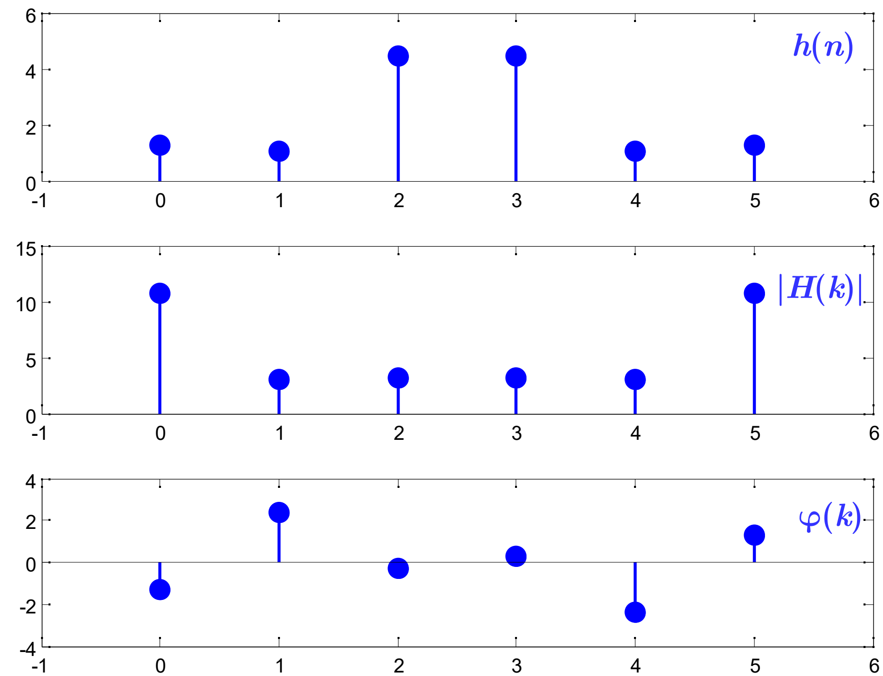
偶数点



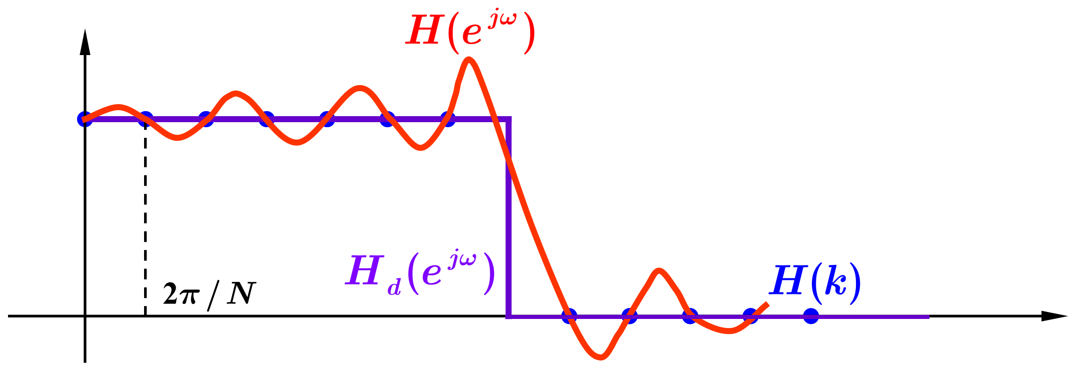

$h_d(n)$
$|H_d(k)|$
$\varphi_d(k)$


$\arg[H_d(e^{j\omega})]$: nonlinear phase response


$\arg[H(e^{j\omega})]$: linear phase response


$h(n)$
$|H(k)|$
$\varphi(k)$

过渡带的优化设计（增加自由度）



增加过渡带抽样点，可加大阻带衰减 ξ

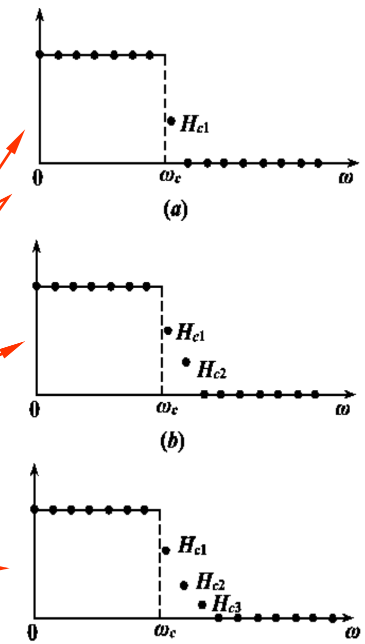
$$H\left(e^{j\omega}\right)=\sum_{k=0}^{N-1}H(k)\Phi\left(\omega-\frac{2\pi}{N}k\right)$$

不加过渡抽样点：　$\xi=-20\text{dB}$

加一点：　$\xi=-40\sim-54\text{dB}$

加两点：　$\xi=-60\sim-75\text{dB}$

加三点：　$\xi=-80\sim-95\text{dB}$



NOTE

- 增加过渡带抽样点，可加大阻带衰减，但导致过渡带变宽
- 增加N，使抽样点变密，减小过渡带宽度，但增加了计算量

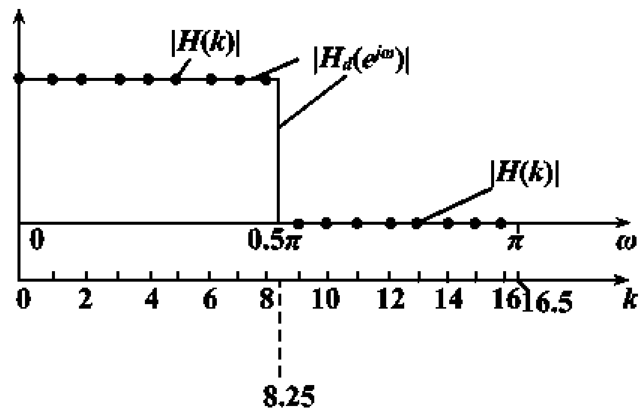
优点： 频域直接设计；窄带

缺点： 抽样频率只能是 2π/N 或者π/N 的整数倍，且截止频率 ω_c 不能任意取值(采样点可能无法触及)

例：利用频率抽样法设计一个频率特性为矩形的理想低通滤波器，截止频率为0.5π，抽样点数为$N=33$，要求滤波器具有线性相位。

解：理想低通频率特性：

$$|H_d(e^{j\omega})| = \begin{cases} 1 & 0 \le \omega \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$

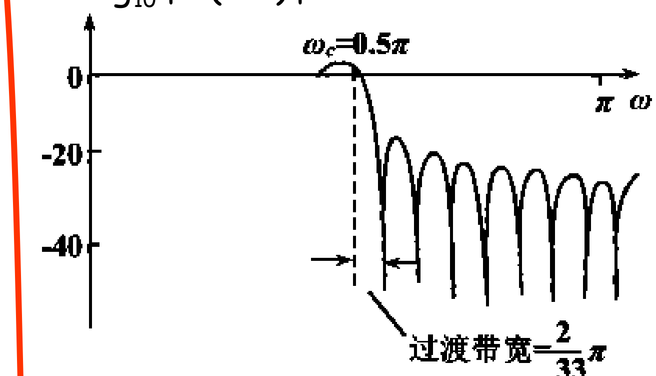


按第一种频率抽样方式，N=33，得抽样点

$$|H(k)| = \begin{cases} 1 & 0 \le k \le \mathrm{Int}\left[\dfrac{N\omega_c}{2\pi}\right] = \dfrac{N-1}{4} = 8 \\ 0 & \mathrm{Int}\left[\dfrac{N\omega_c}{2\pi}\right] + 1 = 9 \le k \le \dfrac{N-1}{2} = 16 \end{cases}$$

得线性相位FIR滤波器的频率响应：

$$H(e^{j\omega}) = e^{-j16\omega}\left\{\frac{\sin\left(\frac{33\omega}{2}\right)}{33\sin\left(\frac{\omega}{2}\right)} + \sum_{k=1}^{8}\left[\frac{\sin\left[33\left(\frac{\omega}{2}-\frac{k\pi}{33}\right)\right]}{33\sin\left(\frac{\omega}{2}-\frac{k\pi}{33}\right)} + \frac{\sin\left[33\left(\frac{\omega}{2}+\frac{k\pi}{33}\right)\right]}{33\sin\left(\frac{\omega}{2}+\frac{k\pi}{33}\right)}\right]\right\}$$

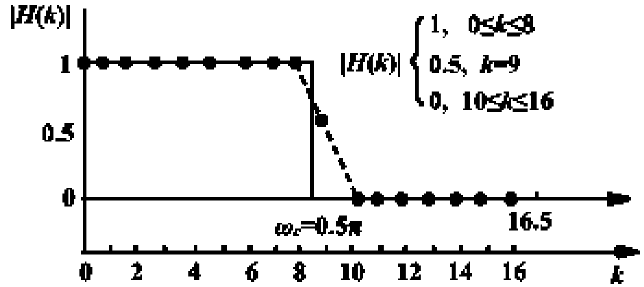


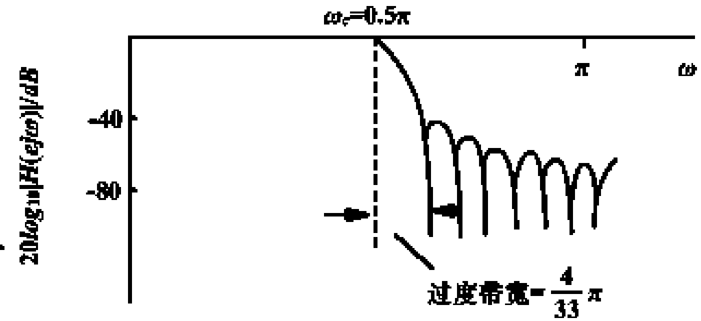
过渡带宽：$2\pi/33$　　阻带衰减：−20dB

AN EXAMPLE

* 增加一点过渡带抽样点

令H(9)=0.5（注意：原本H(9)=0）

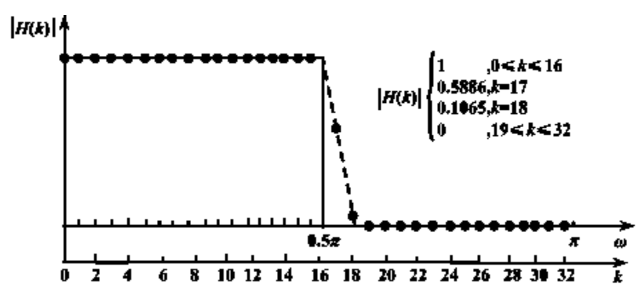


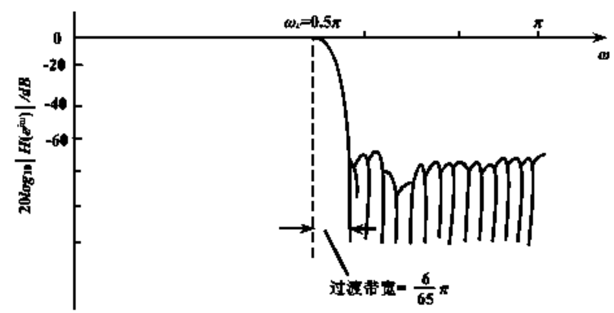


过渡带宽：4π/33　　阻带衰减：-40dB

* 增加两点过渡带抽样点，且增加抽样点数为N=65

令H(17)=0.5886，H(18)=0.1065





过渡带宽：6π/65　　阻带衰减：-60dB

FIR滤波器设计1--往年真题

设理想数字带通滤波器的幅频响应为

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/4 \le |\omega| \le \pi/2 \\ 0 & |\omega| < \pi/4,\ \pi/2 \le |\omega| \le \pi \end{cases}$$

要求用频率取样法设计相应的$N = 15$ 时FIR 线性相位数字带通滤波器，

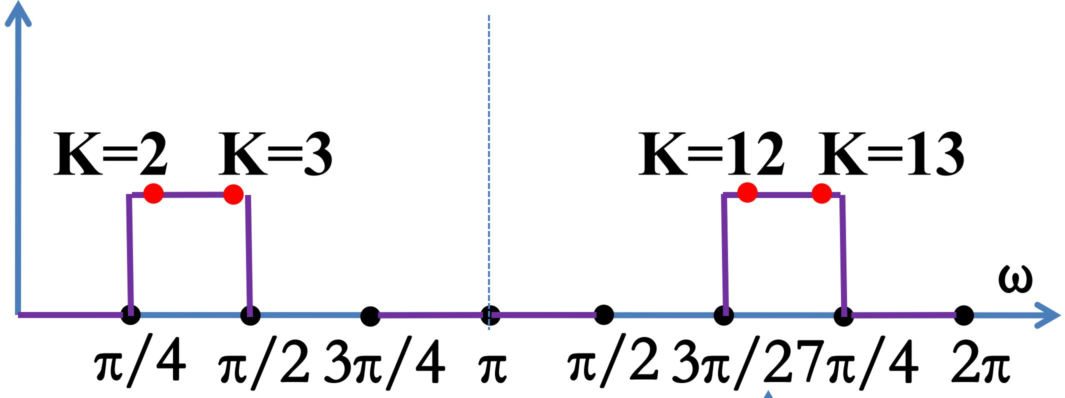
(1) 确定频率抽样序列$H(k), k = 0,1,\cdots,N-1$

(2) 确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

解：

(1) 理想数字带通滤波器的幅频响应为

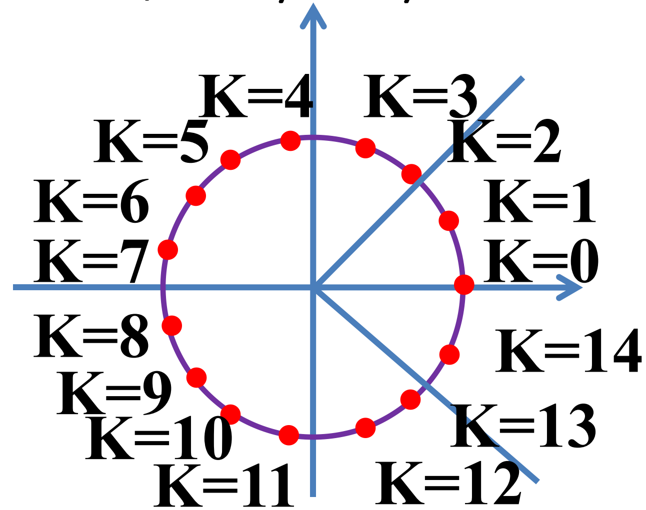


$$H(k) = H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k}$$

$$\Delta\omega = \frac{2\pi}{N} = \frac{2\pi}{15}$$

$$\begin{cases} 1 < {\pi/4}\big/{\Delta\omega} = {\pi/4}\big/{2\pi/15} = \dfrac{15}{8} < 2 \\ 3 < {\pi/2}\big/{\Delta\omega} = {\pi/2}\big/{2\pi/15} = \dfrac{15}{4} < 4 \\ 11 < {3\pi/2}\big/{\Delta\omega} = {3\pi/2}\big/{2\pi/15} = \dfrac{45}{4} < 12 \\ 13 < {7\pi/4}\big/{\Delta\omega} = {7\pi/4}\big/{2\pi/15} = \dfrac{105}{8} < 14 \end{cases}$$

$$\Rightarrow |H_d(k)| = \begin{cases} 1, & k = 2,3,12,13 \\ 0, & \text{其他} \end{cases}$$



$$H(k) = |H_d(k)| e^{-j\frac{N-1}{2}\cdot\frac{2\pi}{N}k} = e^{-j\frac{14}{15}\pi k}$$

$$k = 2,3,12,13$$

$$H(k) = \begin{cases} e^{-j\frac{14}{15}\pi k}, & k = 2,3,12,13 \\ 0, & \text{其他} \end{cases}$$

$$H(k)=\left|H_d(k)\right|e^{-j\frac{N-1}{2}\bullet\frac{2\pi}{N}k}=e^{-j\frac{14}{15}\pi k}$$

$$k=2,3,12,13$$

$$H(k)=\begin{cases} e^{-j\frac{14}{15}\pi k}, & k=2,3,12,13 \\ 0, & \text{其他} \end{cases}$$

$$H(0)=H(1)=H(4)=H(5)=H(6)=H(7)$$

$$=H(8)=H(9)=H(10)=H(11)=H(14)=0$$

$$H(2)=e^{-j\frac{28}{15}\pi}=e^{j\frac{2}{15}\pi}=0.91+j0.41,$$

$$H(13)=e^{-j\frac{182}{15}\pi}=e^{-j\frac{2}{15}\pi}=0.91-j0.41$$

$$H(3)=e^{-j\frac{42}{15}\pi}=e^{-j\frac{12}{15}\pi}=-0.81+j0.59,$$

$$H(12)=e^{-j\frac{168}{15}\pi}=e^{j\frac{12}{15}\pi}=-0.81-j0.59$$

$$H(k)=H^*(N-k)$$

或采用P243（5-238）求解

$$H(k)=\begin{cases}|H(k)|e^{-j\frac{2\pi}{N}k\left|\frac{N-1}{2}\right|} & k=0,\ldots,\frac{N-1}{2}\\ |H(k)|e^{j\frac{2\pi}{N}(N-k)\frac{N-1}{2}} \quad or\ |H(N-k)|e^{j\frac{2\pi}{N}(N-k)\frac{N-1}{2}} & k=\frac{N+1}{2},\ldots,N-1\end{cases}$$

$$\Rightarrow H(k)=\begin{cases}e^{-j\frac{2\pi}{N}k\left|\frac{N-1}{2}\right|}=e^{-j\frac{14}{15}\pi k}, & k=2,3\\ e^{j\frac{2\pi}{N}(N-k)\left|\frac{N-1}{2}\right|}=e^{j\frac{14}{15}\pi(15-k)}=e^{-j\frac{14}{15}\pi k}, & k=12,13\\ 0,\ \text{其他}\end{cases}$$

$$H(0)=H(1)=H(4)=H(5)=H(6)=H(7)$$
$$=H(8)=H(9)=H(10)=H(11)=H(14)=0$$

$$H(2)=e^{-j\frac{28}{15}\pi}=e^{j\frac{2}{15}\pi}=0.91+j0.41,$$

$$H(13)=e^{-j\frac{182}{15}\pi}=e^{-j\frac{2}{15}\pi}=0.91-j0.41$$

$$H(3)=e^{-j\frac{42}{15}\pi}=e^{-j\frac{12}{15}\pi}=-0.81+j0.59,$$

$$H(12)=e^{-j\frac{168}{15}\pi}=e^{j\frac{12}{15}\pi}=-0.81-j0.59$$

$$(2)H(z)=\frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-W_N^{-k}z^{-1}}$$

$$H(z)=\frac{1-z^{-15}}{15}\sum_{k=0}^{14}\frac{e^{-j\frac{14}{15}\pi k}}{1-W_{15}^{-k}z^{-1}}=\frac{1-z^{-15}}{15}\sum_{k=2,3,12,13}\frac{e^{-j\frac{14}{15}\pi k}}{1-e^{-j\frac{2\pi}{15}k}z^{-1}}$$

$$=\frac{1-z^{-15}}{15}\left(\frac{e^{-j\frac{14}{15}\pi 2}}{1-e^{-j\frac{2\pi}{15}2}z^{-1}}+\frac{e^{-j\frac{14}{15}\pi 3}}{1-e^{-j\frac{2\pi}{15}3}z^{-1}}+\frac{e^{-j\frac{14}{15}\pi 12}}{1-e^{-j\frac{2\pi}{15}12}z^{-1}}+\frac{e^{-j\frac{14}{15}\pi 13}}{1-e^{-j\frac{2\pi}{15}13}z^{-1}}\right)$$

$$=\frac{1-z^{-15}}{15}\left(\left(\frac{e^{-j\frac{14}{15}\pi 2}}{1-e^{-j\frac{2\pi}{15}2}z^{-1}}+\frac{e^{-j\frac{14}{15}\pi 13}}{1-e^{-j\frac{2\pi}{15}13}z^{-1}}\right)+\left(\frac{e^{-j\frac{14}{15}\pi 3}}{1-e^{-j\frac{2\pi}{15}3}z^{-1}}+\frac{e^{-j\frac{14}{15}\pi 12}}{1-e^{-j\frac{2\pi}{15}12}z^{-1}}\right)\right)$$

$$=\frac{1-z^{-15}}{15}\left(\left(\frac{e^{-j\frac{2}{15}\pi}}{1-e^{-j\frac{4\pi}{15}}z^{-1}}+\frac{e^{j\frac{2}{15}\pi}}{1-e^{j\frac{4\pi}{15}}z^{-1}}\right)+\left(\frac{e^{-j\frac{4}{5}\pi}}{1-e^{-j\frac{2\pi}{5}}z^{-1}}+\frac{e^{j\frac{4}{5}\pi}}{1-e^{j\frac{2\pi}{5}}z^{-1}}\right)\right)$$

$$=\frac{1-z^{-15}}{15}\left(\left(\frac{e^{-j\frac{2}{15}\pi}}{1-e^{-j\frac{4\pi}{15}}z^{-1}}+\frac{e^{j\frac{2}{15}\pi}}{1-e^{j\frac{4\pi}{15}}z^{-1}}\right)+\left(\frac{e^{-j\frac{4}{5}\pi}}{1-e^{-j\frac{2\pi}{5}}z^{-1}}+\frac{e^{j\frac{4}{5}\pi}}{1-e^{j\frac{2\pi}{5}}z^{-1}}\right)\right)$$

$$=\frac{1-z^{-15}}{15}\left(\frac{e^{-j\frac{2}{15}\pi}(1-e^{j\frac{4\pi}{15}}z^{-1})+e^{j\frac{2}{15}\pi}(1-e^{-j\frac{4\pi}{15}}z^{-1})}{(1-e^{-j\frac{4\pi}{15}}z^{-1})(1-e^{j\frac{4\pi}{15}}z^{-1})}+\frac{e^{-j\frac{4}{5}\pi}(1-e^{j\frac{2\pi}{5}}z^{-1})+e^{j\frac{4}{5}\pi}(1-e^{-j\frac{2\pi}{5}}z^{-1})}{(1-e^{-j\frac{2\pi}{5}}z^{-1})(1-e^{j\frac{2\pi}{5}}z^{-1})}\right)$$

$$=\frac{1-z^{-15}}{15}\left(\frac{(e^{j\frac{2}{15}\pi}+e^{-j\frac{2}{15}\pi})-(e^{j\frac{2}{15}\pi}+e^{-j\frac{2}{15}\pi})z^{-1}}{1-2\cos\frac{4\pi}{15}z^{-1}+z^{-2}}+\frac{(e^{j\frac{4}{5}\pi}+e^{-j\frac{4}{5}\pi})-(e^{j\frac{2}{5}\pi}+e^{-j\frac{2}{5}\pi})z^{-1}}{1-2\cos\frac{2\pi}{5}z^{-1}+z^{-2}}\right)$$

$$=\frac{1-z^{-15}}{15}\left(\frac{2\cos\frac{2\pi}{15}-2\cos\frac{2\pi}{15}z^{-1}}{1-2\cos\frac{4\pi}{15}z^{-1}+z^{-2}}+\frac{2\cos\frac{4\pi}{5}-2\cos\frac{2\pi}{5}z^{-1}}{1-2\cos\frac{2\pi}{5}z^{-1}+z^{-2}}\right)$$

$$=\frac{1-z^{-15}}{15}\left(\frac{1.83-1.83z^{-1}}{1-1.34z^{-1}+z^{-2}}+\frac{-1.62-0.62z^{-1}}{1-1.83z^{-1}+z^{-2}}\right)$$

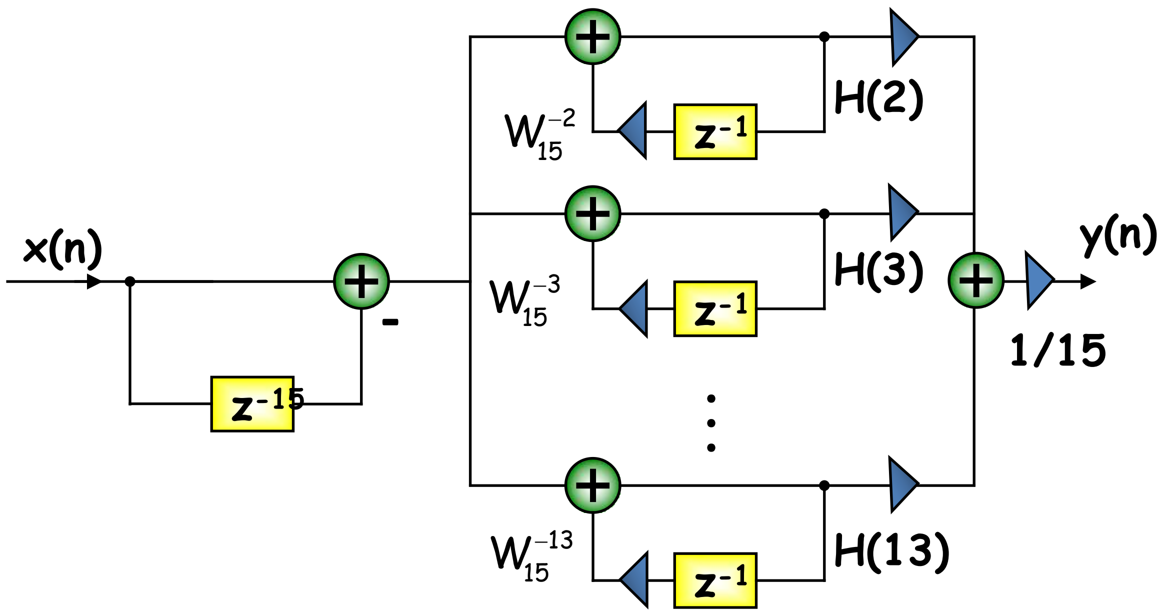
(3)

$$H(2)=e^{-j\frac{28}{15}\pi}=e^{j\frac{2}{15}\pi}=0.91+j0.41,$$

$$H(13)=e^{-j\frac{182}{15}\pi}=e^{-j\frac{2}{15}\pi}=0.91-j0.41$$

$$H(3)=e^{-j\frac{42}{15}\pi}=e^{-j\frac{12}{15}\pi}=-0.81+j0.59,$$

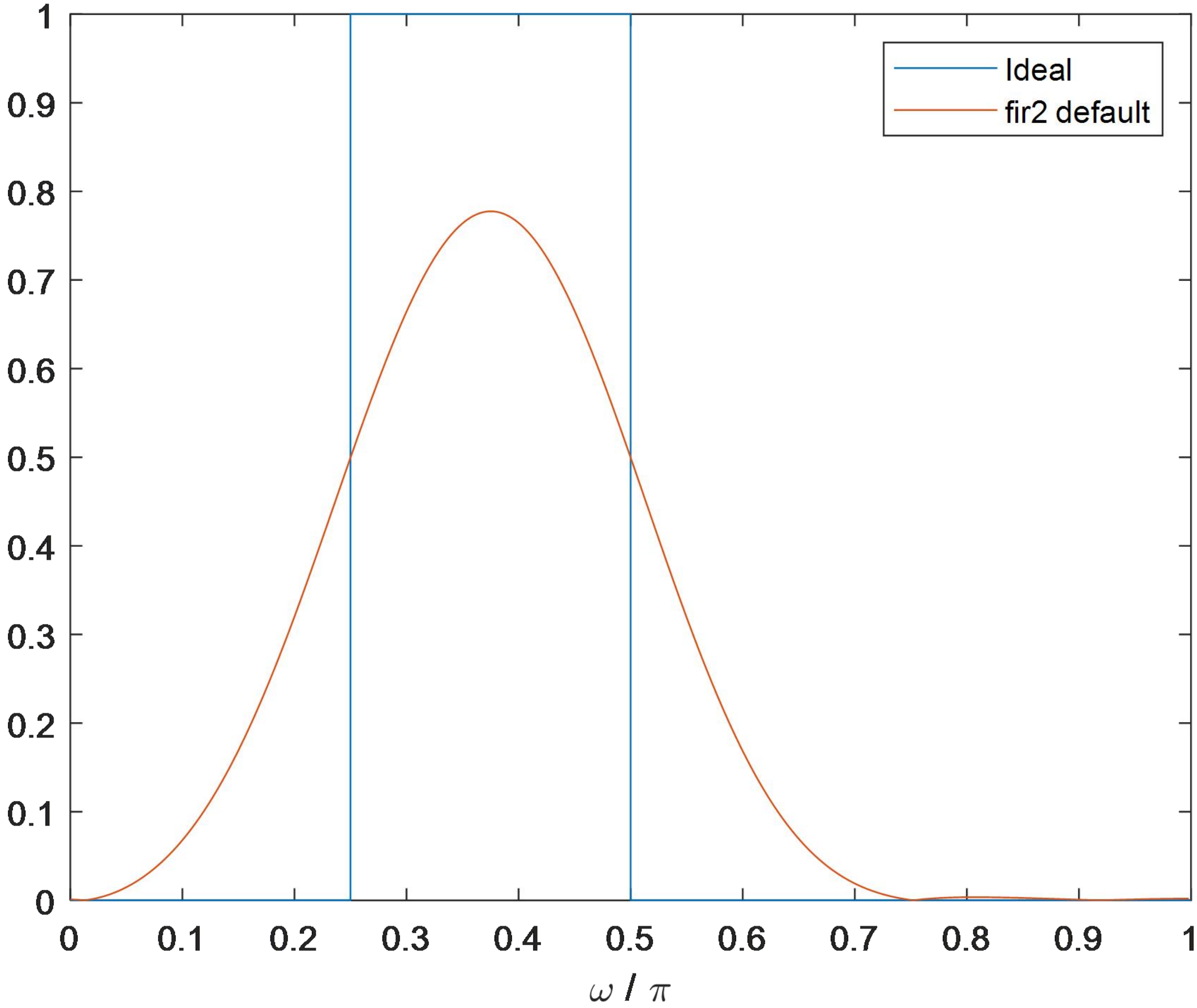
$$H(12)=e^{-j\frac{168}{15}\pi}=e^{j\frac{12}{15}\pi}=-0.81-j0.59$$



```
ord = 14;
f = [0 0.25 0.25 0.5 0.5 1];
m = [0 0 1 1 0 0];
b1 = fir2(ord,f,m);
fvtool(b1,1);
Hz = filt(bi,1);
Hk = fft(b1);
```

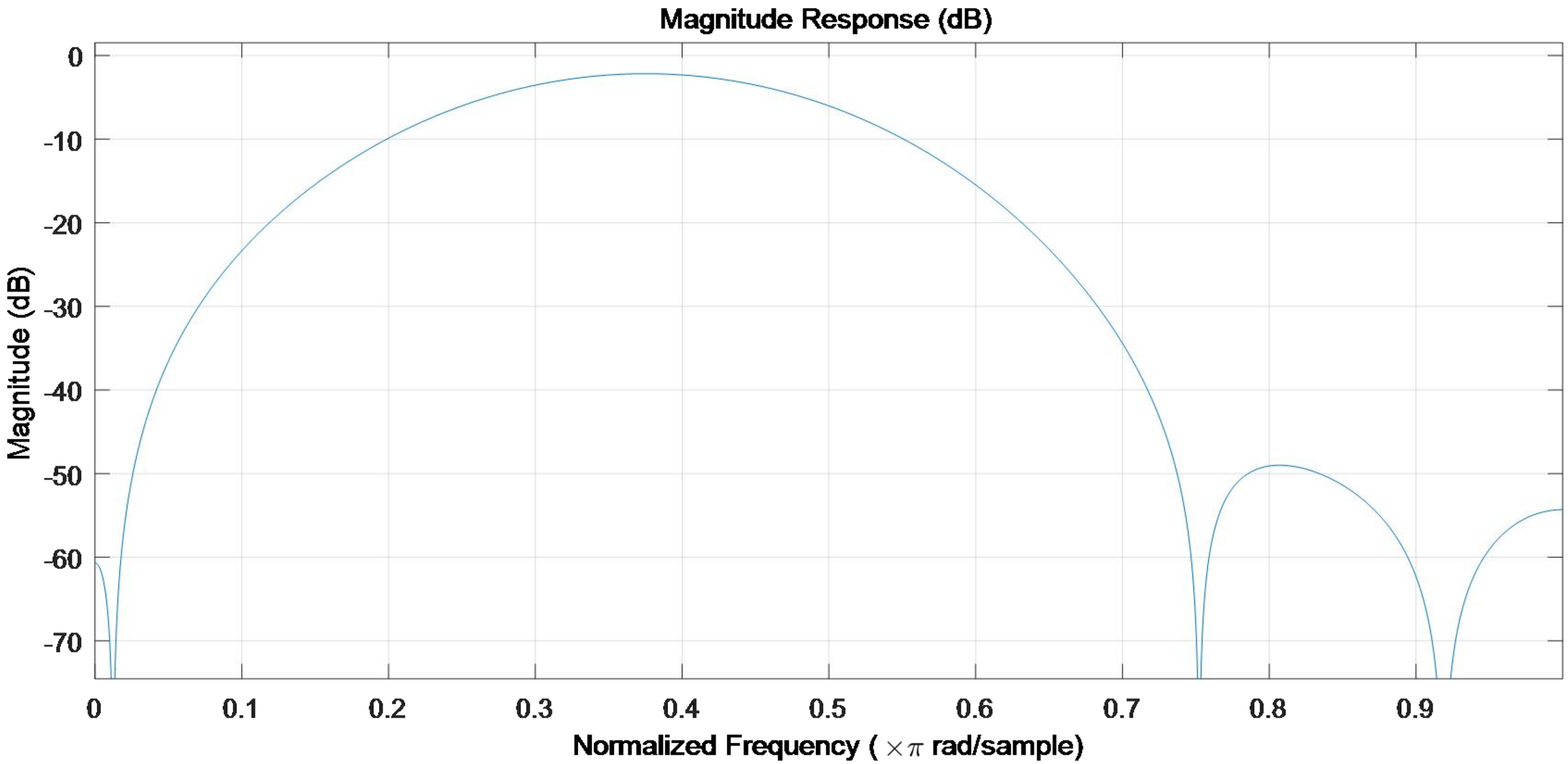
```
b =

  -0.0010    0.0065    0.0271   -0.0000   -0.1157   -0.1313    0.0889    0.2500    0.0889   -0.1313   -0.1157    0.0000    0.0271    0.0065   -0.0010

Hk =

  -0.0009 + 0.0000i  -0.1269 - 0.0270i   0.5104 + 0.2272i  -0.6184 - 0.4493i   0.2527 + 0.2807i  -0.0240 - 0.0415i

  -0.0011 - 0.0033i  -0.0001 - 0.0006i  -0.0001 + 0.0006i  -0.0011 + 0.0033i  -0.0240 + 0.0415i   0.2527 - 0.2807i

  -0.6184 + 0.4493i   0.5104 - 0.2272i  -0.1269 + 0.0270i

Hz =

  -0.001033 + 0.006512 z^-1 + 0.02709 z^-2 - 2.606e-17 z^-3 - 0.1157 z^-4 - 0.1313 z^-5 + 0.08893 z^-6

        + 0.25 z^-7 + 0.08893 z^-8 - 0.1313 z^-9 - 0.1157 z^-10 + 2.259e-17 z^-11 + 0.02709 z^

                                               -12 + 0.006512 z^-13 - 0.001033 z^-14
```

N=15



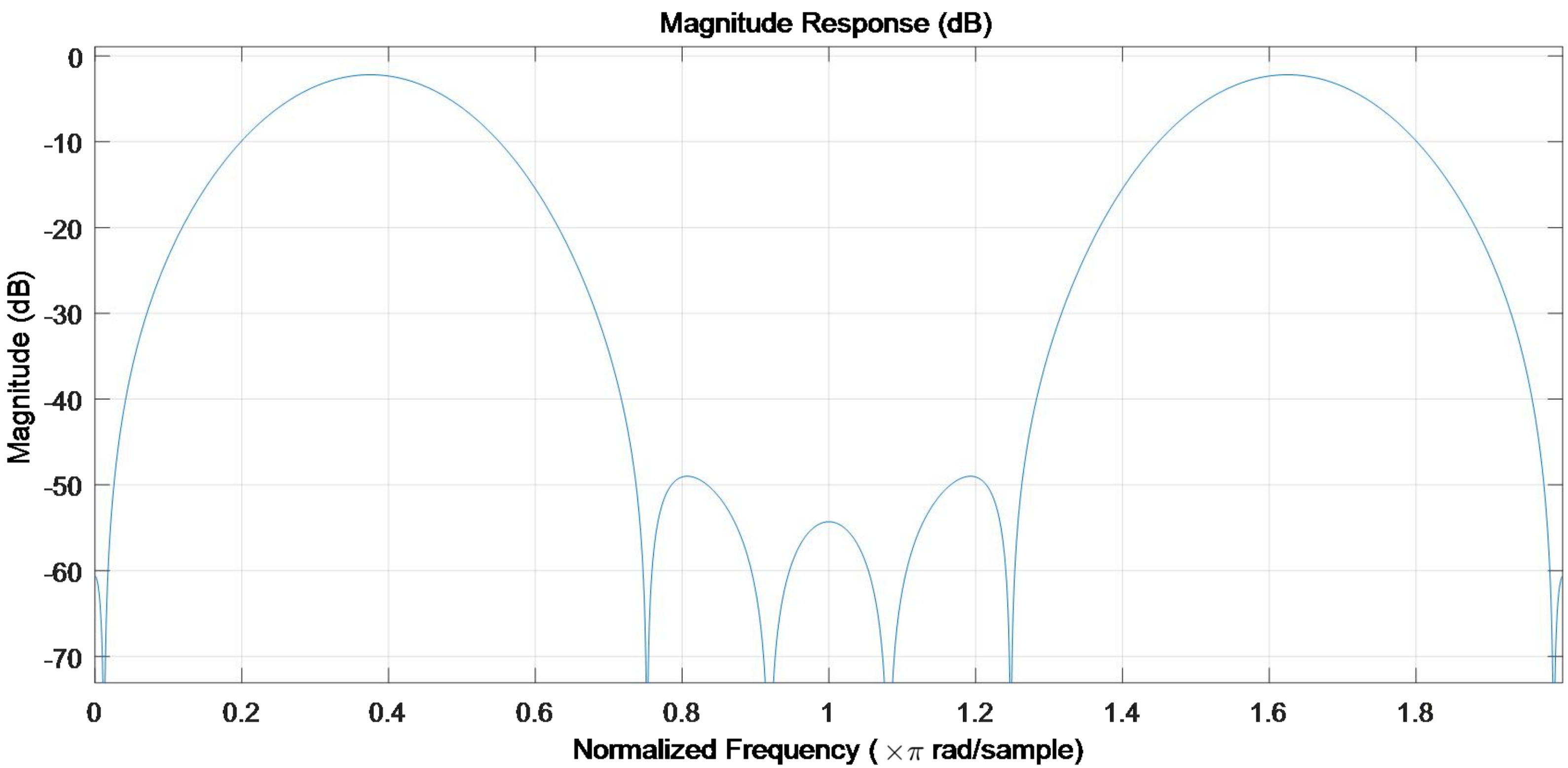
Ideal
fir2 default
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ω / π

N=15



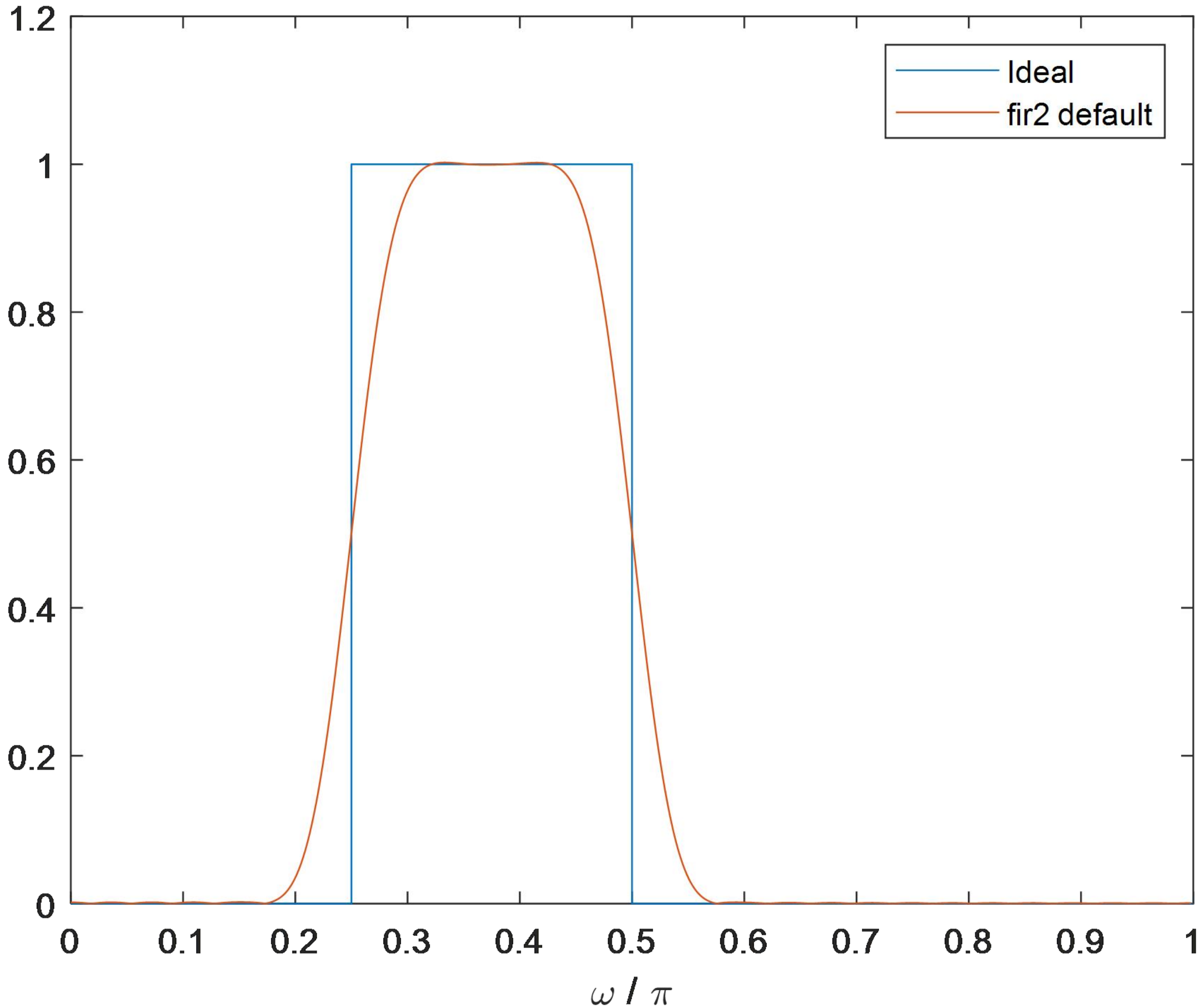
Magnitude Response (dB)
0
-10
-20
-30
-40
-50
-60
-70
Magnitude (dB)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)

N=15

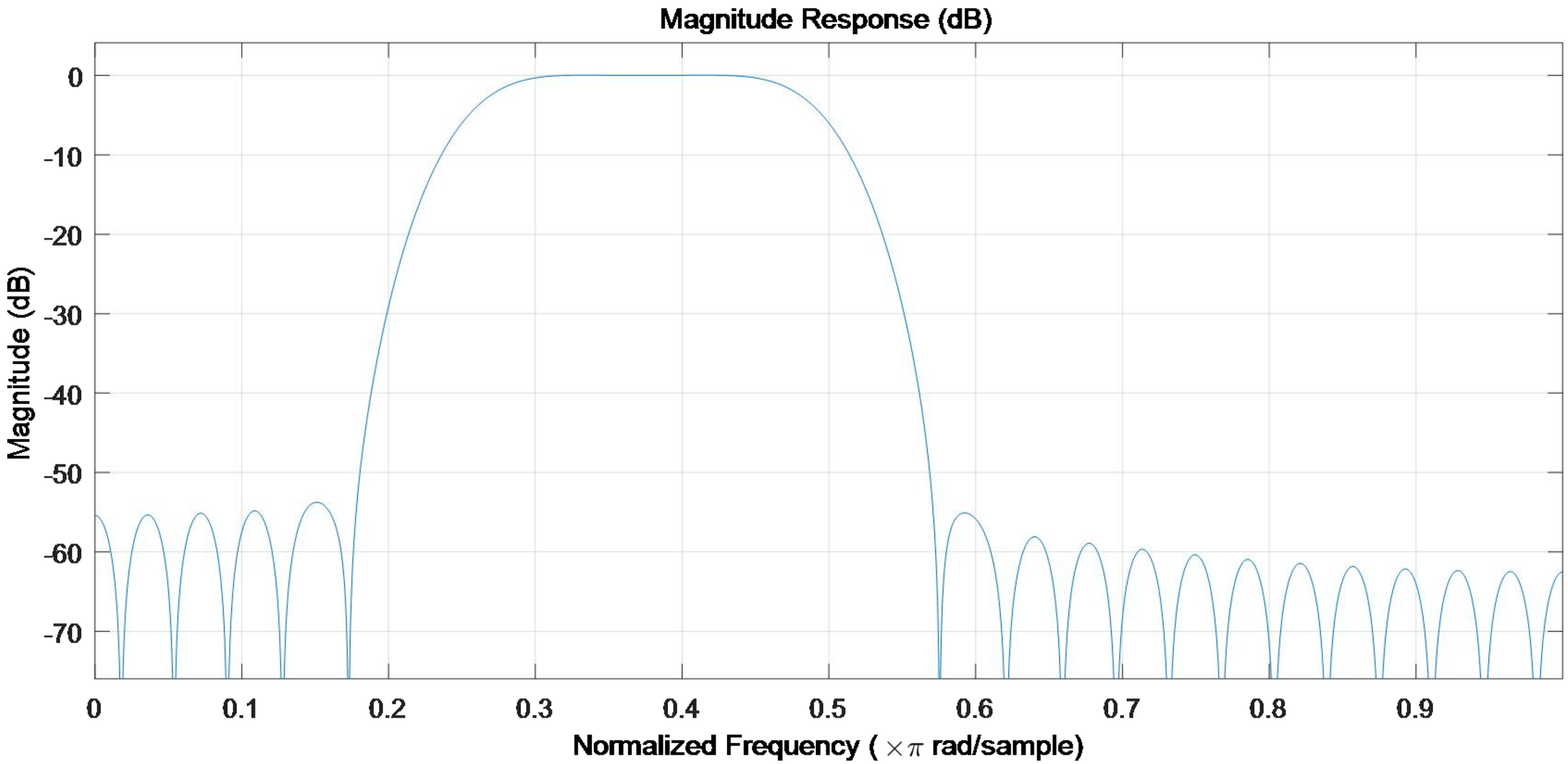


Magnitude Response (dB)
0
-10
-20
-30
-40
-50
-60
-70
Magnitude (dB)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Normalized Frequency (×π rad/sample)

N=55

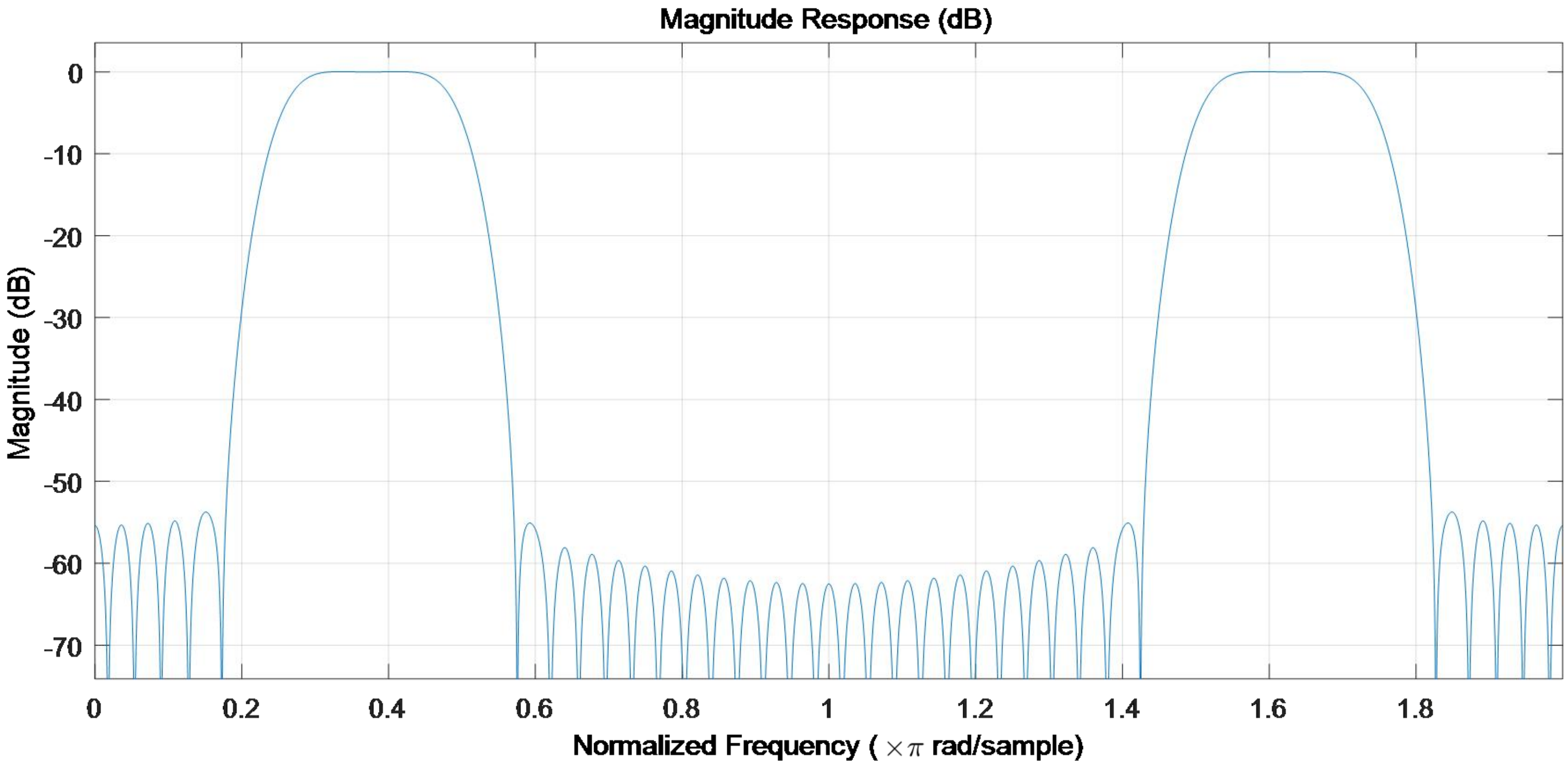

Ideal
fir2 default
1.2
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ω / π

N=55



Magnitude Response (dB)
0
-10
-20
-30
-40
-50
-60
-70
Magnitude (dB)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)

N=55


Magnitude Response (dB)
0
-10
-20
-30
-40
-50
-60
-70
Magnitude (dB)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Normalized Frequency (×π rad/sample)

FIR滤波器设计2--往年真题

设理想数字高通滤波器的幅频响应为

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/2 \le |\omega| \le \pi \\ 0 & |\omega| < \pi/2 \end{cases}$$

用频率取样法设计相应的$N=11$时FIR线性相位数字高通滤波器，

(1) 确定频域取样序列$H(k), k=0,1,\cdots,N-1$

(2) 确定滤波器的系统函数$H(z)$

(3) 确定滤波器的频率响应$H(e^{j\omega})$

(4)确定滤波器的单位脉冲响应$h(n)$

(5)给出滤波器的任意一种结构实现形式

√

注：四舍五入到小数点后2位

03B(05-06)

解：(1)理想数字高通滤波器的幅频响为

$$H(k)=H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k}$$

$$\Delta\omega=\frac{2\pi}{N}=\frac{2\pi}{11}$$

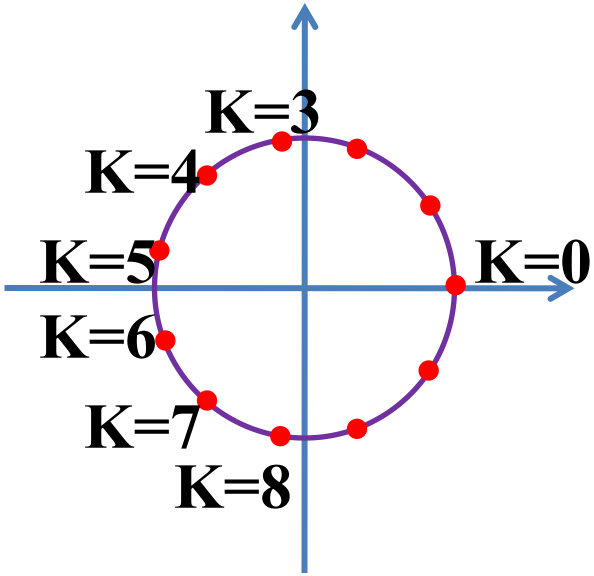
$$\begin{cases}\pi/2\Big/\Delta\omega=\pi/2\Big/2\pi/11=\dfrac{11}{4}<3\\ 3\pi/2\Big/\Delta\omega=3\pi/2\Big/2\pi/11=\dfrac{33}{4}>8\end{cases}$$

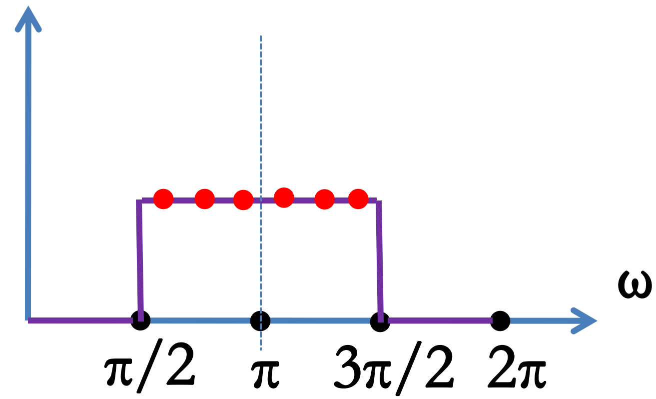
$$\Rightarrow |H_d(k)|=\begin{cases}1, & 3\le k\le 8\\ 0, & \text{其他}\end{cases}$$

$$H(k)=|H_d(k)|e^{-j\frac{N-1}{2}\frac{2\pi}{N}k}=e^{-j\frac{10}{11}\pi k}$$

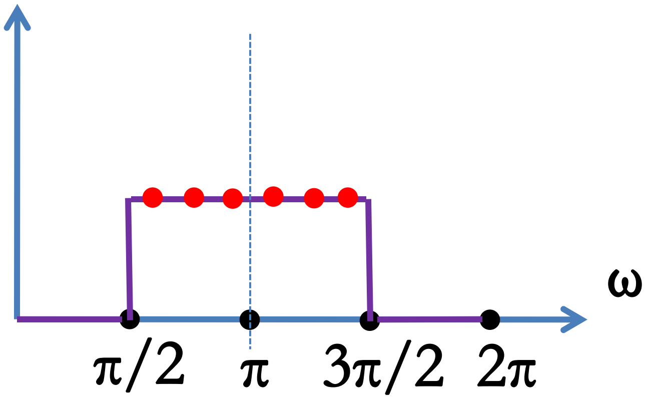
$$k=3,4,5,6,7,8$$

$$H(k)=\begin{cases}e^{-j\frac{10}{11}\pi k}, & 3\le k\le 8\\ 0, & \text{其他}\end{cases}$$





$$H(k)=\begin{cases} e^{-j\frac{10}{11}\pi k}, & 3\le k\le 8 \\ 0, & \text{其他} \end{cases}$$



$$H(0)=H(1)=H(2)=H(9)=H(10)=0$$

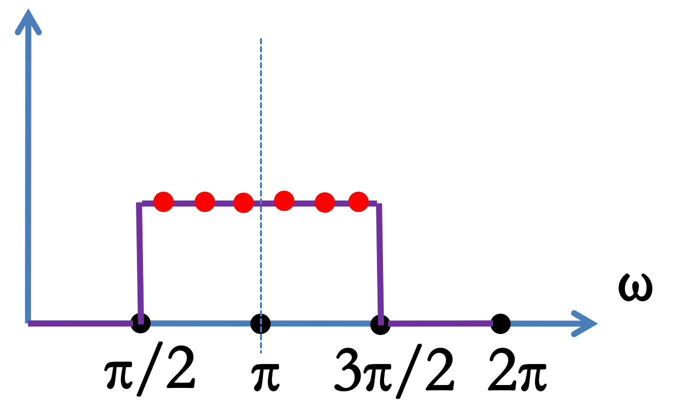
$$H(3)=e^{-j\frac{30}{11}\pi}=e^{-j\frac{8}{11}\pi}=-0.65-j0.76,\ H(8)=e^{-j\frac{80}{11}\pi}=e^{j\frac{8}{11}\pi}=-0.65+j0.76$$

$$H(4)=e^{-j\frac{40}{11}\pi}=e^{j\frac{4}{11}\pi}=0.42+j0.91,\quad H(7)=e^{-j\frac{70}{11}\pi}=e^{-j\frac{4}{11}\pi}=0.42-j0.91$$

$$H(5)=e^{-j\frac{50}{11}\pi}=e^{-j\frac{6}{11}\pi}=-0.14-j0.99,\ H(6)=e^{-j\frac{60}{11}\pi}=e^{j\frac{6}{11}\pi}=-0.14+j0.99$$

$$H(k)=H^*(N-k)$$

$$(2)H(z)=\frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-W_N^{-k}z^{-1}}$$



$$H(z)=\frac{1-z^{-11}}{11}\sum_{k=0}^{10}\frac{e^{-j\frac{10}{11}\pi k}}{1-W_{11}^{-k}z^{-1}}=\frac{1-z^{-11}}{11}\sum_{k=0}^{10}\frac{e^{-j\frac{10}{11}\pi k}}{1-e^{-j\frac{2\pi k}{11}}z^{-1}}$$

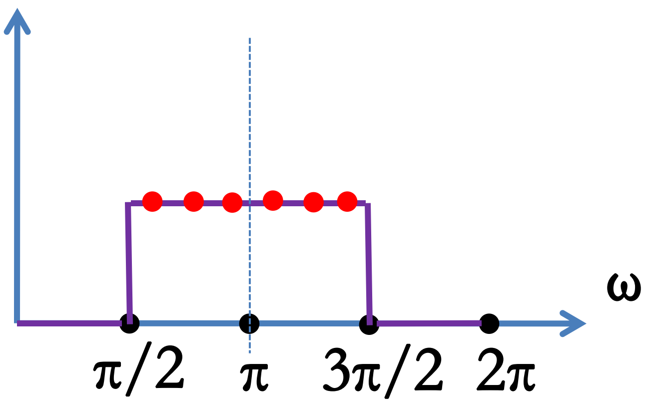
$$=\frac{1-z^{-11}}{11}\left(\frac{e^{-j\frac{10}{11}\pi 3}}{1-e^{-j\frac{2\pi 3}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}\pi 4}}{1-e^{-j\frac{2\pi 4}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}\pi 5}}{1-e^{-j\frac{2\pi 5}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}\pi 6}}{1-e^{-j\frac{2\pi 6}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}\pi 7}}{1-e^{-j\frac{2\pi 7}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}\pi 8}}{1-e^{-j\frac{2\pi 8}{11}}z^{-1}}\right)$$

$$=\frac{1-z^{-11}}{11}\left(\left(\frac{e^{-j\frac{10}{11}\pi 3}}{1-e^{-j\frac{2\pi 3}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}\pi 8}}{1-e^{-j\frac{2\pi 8}{11}}z^{-1}}\right)+\left(\frac{e^{-j\frac{10}{11}\pi 4}}{1-e^{-j\frac{2\pi 4}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}\pi 7}}{1-e^{-j\frac{2\pi 7}{11}}z^{-1}}\right)+\left(\frac{e^{-j\frac{10}{11}\pi 5}}{1-e^{-j\frac{2\pi 5}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}\pi 6}}{1-e^{-j\frac{2\pi 6}{11}}z^{-1}}\right)\right)$$

$$=\frac{1-z^{-11}}{11}\left(\frac{-1.31+1.31z^{-1}}{1-0.28z^{-1}+z^{-2}}+\frac{0.83-0.83z^{-1}}{1-1.31z^{-1}+z^{-2}}+\frac{-0.28+0.28z^{-1}}{1+1.92z^{-1}+z^{-2}}\right)$$

$$(3)H(e^{j\omega}) = H(z)\big|_{z=e^{j\omega}} = \frac{1-e^{-j\omega 11}}{11}\sum_{k=0}^{10}\frac{H(k)}{1-W_{11}^{-k}e^{-j\omega}}$$

$$(4)h(n) = \frac{1}{N}\sum_{k=0}^{10}H(k)e^{j\frac{2\pi}{11}kn}$$



$$h(0) = h(10) = -0.0694$$

$$h(1) = h(9) = 0.0540$$

$$h(2) = h(8) = 0.1094$$

$$h(3) = h(7) = -0.0474$$

$$h(4) = h(6) = -0.3194$$

$$h(5) = 0.5455$$

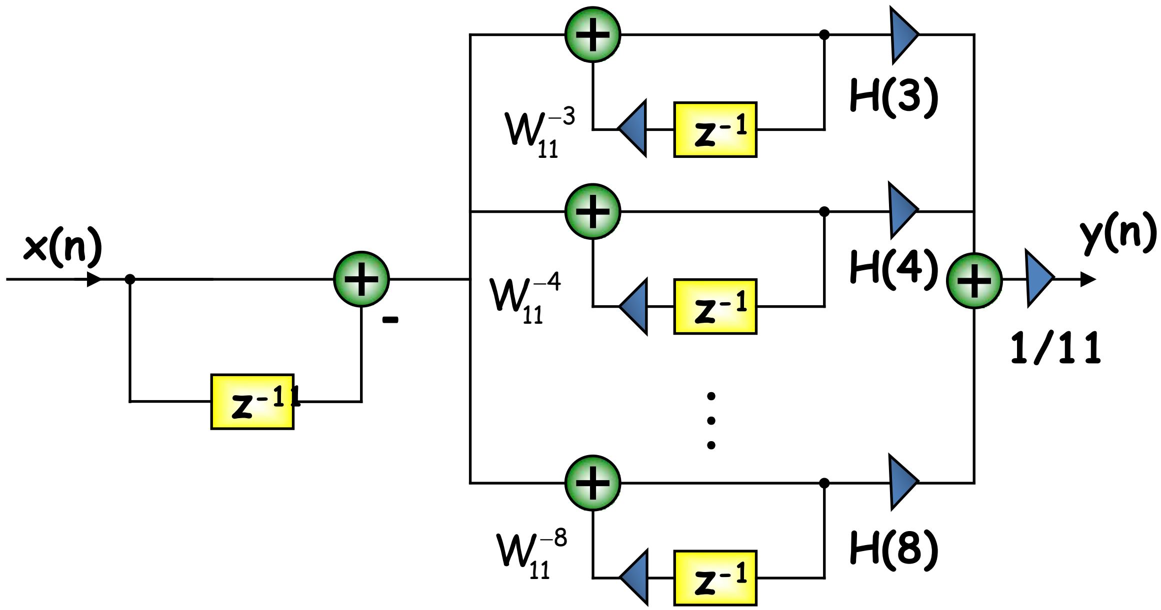
(5)给出滤波器的任意一种结构实现形式

频率取样型

$$H(3)=e^{-j\frac{30}{11}\pi}=e^{-j\frac{8}{11}\pi}=-0.65-j0.76,\ H(8)=e^{-j\frac{80}{11}\pi}=e^{j\frac{8}{11}\pi}=-0.65+j0.76$$

$$H(4)=e^{-j\frac{40}{11}\pi}=e^{j\frac{4}{11}\pi}=0.42+j0.91,\quad H(7)=e^{-j\frac{70}{11}\pi}=e^{-j\frac{4}{11}\pi}=0.42-j0.91$$

$$H(5)=e^{-j\frac{50}{11}\pi}=e^{-j\frac{6}{11}\pi}=-0.14-j0.99,\ H(6)=e^{-j\frac{60}{11}\pi}=e^{j\frac{6}{11}\pi}=-0.14+j0.99$$



FIR滤波器设计3

设理想数字带通滤波器的幅频响应为

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/4 \le |\omega| \le \pi/2 \\ 0 & \pi/2 \le |\omega| \le \pi, |\omega| \le \pi/4 \end{cases}$$

用频率取样法设计一个$N=9$时FIR线性相位数字带通滤波器，

(1)确定滤波器频域取样序列$H(k)\,n = 0,1,\cdots,N-1$

(2)确定滤波器的系统函数$H(z)$

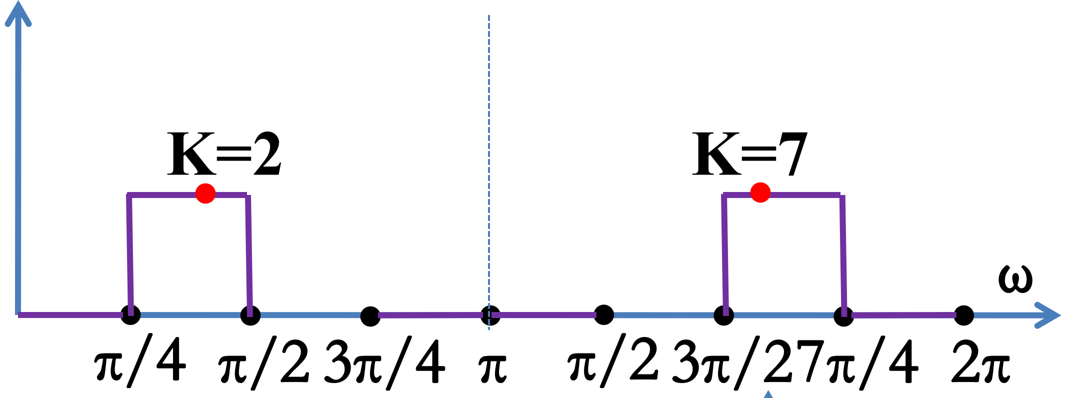
(3)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

04(06-07修改)

解：

(1) 理想数字带通滤波器的幅频响应为

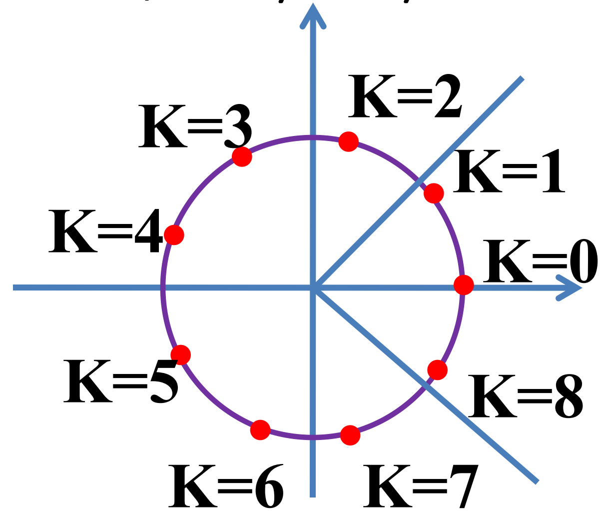


$$H(k)=H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k}$$

$$\Delta\omega=\frac{2\pi}{N}=\frac{2\pi}{9}$$

$$\begin{cases}1<\dfrac{\pi/4}{\Delta\omega}=\dfrac{\pi/4}{2\pi/9}=\dfrac{9}{8}<2\\[2ex] 2<\dfrac{\pi/2}{\Delta\omega}=\dfrac{\pi/2}{2\pi/9}=\dfrac{9}{4}<3\\[2ex] 6<\dfrac{3\pi/2}{\Delta\omega}=\dfrac{3\pi/2}{2\pi/9}=\dfrac{27}{4}<7\\[2ex] 7<\dfrac{7\pi/4}{\Delta\omega}=\dfrac{7\pi/4}{2\pi/9}=\dfrac{63}{8}<8\end{cases}$$

$$\Rightarrow |H_d(k)|=\begin{cases}1, & k=2,7\\ 0, & \text{其他}\end{cases}$$



$$H(k)=|H_d(k)|e^{-j\frac{N-1}{2}\cdot\frac{2\pi}{N}k}=e^{-j\frac{8}{9}\pi k}$$

$$k=2,7$$

$$H(k)=\begin{cases}e^{-j\frac{8}{9}\pi k}, & k=2,7\\ 0, & \text{其他}\end{cases}$$

$$H(k)=\left|H_d(k)\right|e^{-j\frac{N-1}{2}\bullet\frac{2\pi}{N}k}=e^{-j\frac{8}{9}\pi k}$$

$$k=2,7$$

$$H(k)=\begin{cases} e^{-j\frac{8}{9}\pi k}, & k=2,7 \\ 0, & \text{其他} \end{cases}$$

$$H(0)=H(1)=H(3)=H(4)=H(5)$$

$$=H(6)=H(8)=H(9)=0$$

$$H(2)=e^{-j\frac{16}{9}\pi}=e^{j\frac{2}{9}\pi}=0.77+j0.64,$$

$$H(7)=e^{-j\frac{56}{9}\pi}=e^{-j\frac{2}{9}\pi}=0.77-j0.64$$

$$H(k)=H^*(N-k)$$

$$(2)H(z)=\frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-W_N^{-k}z^{-1}}$$

$$H(z)=\frac{1-z^{-9}}{9}\sum_{k=0}^{8}\frac{e^{-j\frac{8}{9}\pi k}}{1-W_9^{-k}z^{-1}}=\frac{1-z^{-9}}{9}\sum_{k=2,7}\frac{e^{-j\frac{8}{9}\pi k}}{1-e^{-j\frac{2\pi}{9}k}z^{-1}}$$

$$=\frac{1-z^{-9}}{9}\left(\frac{e^{-j\frac{16}{9}\pi}}{1-e^{-j\frac{4\pi}{9}}z^{-1}}+\frac{e^{-j\frac{56}{9}\pi}}{1-e^{-j\frac{14\pi}{9}}z^{-1}}\right)$$

$$=\frac{1-z^{-9}}{9}\left(\frac{e^{j\frac{2}{9}\pi}}{1-e^{-j\frac{4\pi}{9}}z^{-1}}+\frac{e^{-j\frac{2}{9}\pi}}{1-e^{j\frac{4\pi}{9}}z^{-1}}\right)$$

$$=\frac{1-z^{-9}}{9}\frac{e^{j\frac{2}{9}\pi}(1-e^{j\frac{4\pi}{9}}z^{-1})+e^{-j\frac{2}{9}\pi}(1-e^{-j\frac{4\pi}{9}}z^{-1})}{(1-e^{-j\frac{4\pi}{9}}z^{-1})(1-e^{j\frac{4\pi}{9}}z^{-1})}$$

$$=\frac{1-z^{-9}}{9}\frac{(e^{j\frac{2}{9}\pi}+e^{-j\frac{2}{9}\pi})-(e^{j\frac{4\pi}{9}}+e^{-j\frac{4\pi}{9}})z^{-1}}{1-2\cos\frac{4\pi}{9}z^{-1}-z^{-2})}$$

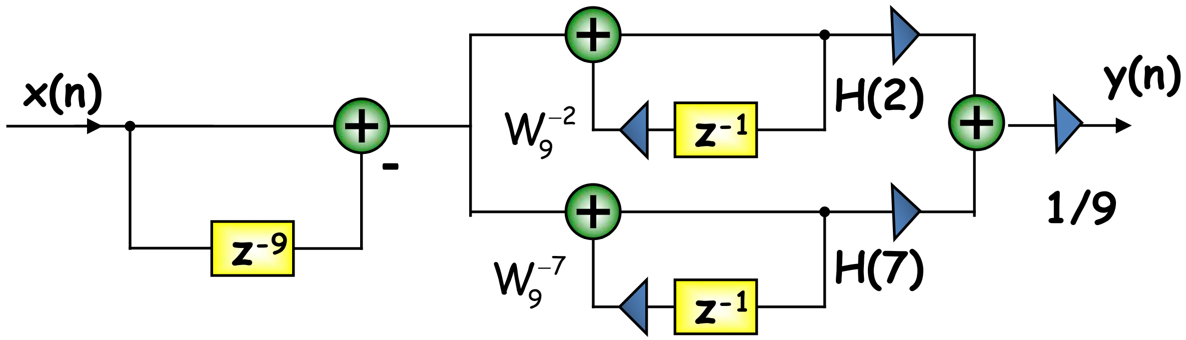
$$=\frac{1-z^{-9}}{9}\frac{2\cos\frac{2\pi}{9}-2\cos\frac{4\pi}{9}z^{-1}}{1-2\cos\frac{4\pi}{9}z^{-1}-z^{-2})}$$

$$=\frac{1-z^{-9}}{9}\frac{1.53-0.35z^{-1}}{1-0.35z^{-1}-z^{-2}}$$

(3)

$$H(2) = e^{-j\frac{16}{9}\pi} = e^{j\frac{2}{9}\pi} = 0.77 + j0.64,$$

$$H(7) = e^{-j\frac{56}{9}\pi} = e^{-j\frac{2}{9}\pi} = 0.77 - j0.64$$



FIR滤波器设计4--往年真题

若要求FIR数字滤波器具有线性相位特性，试分别给出其在时域、频域和变换域，即$h(n)$，$H(e^{j\omega})$，$H(z)$，应满足的条件。

04(06-07)