数字信号处理

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第五章 数字滤波器

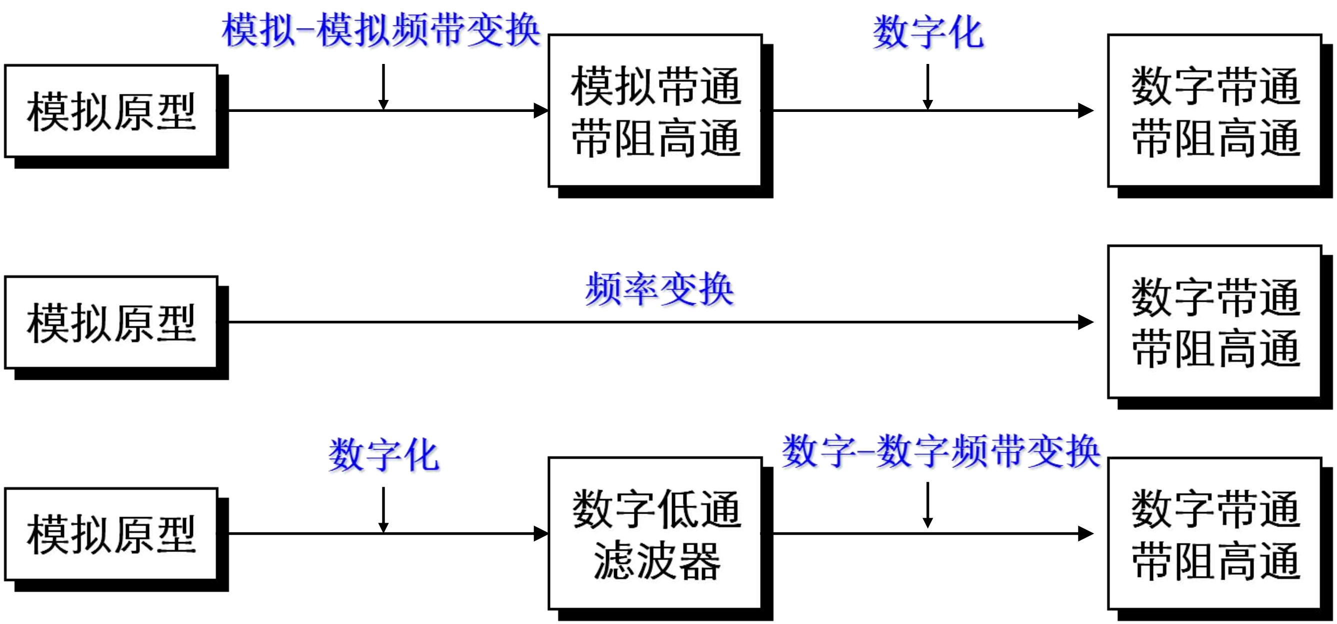
IIR数字滤波器的频率变换

数字带通、带阻、高通滤波器的设计

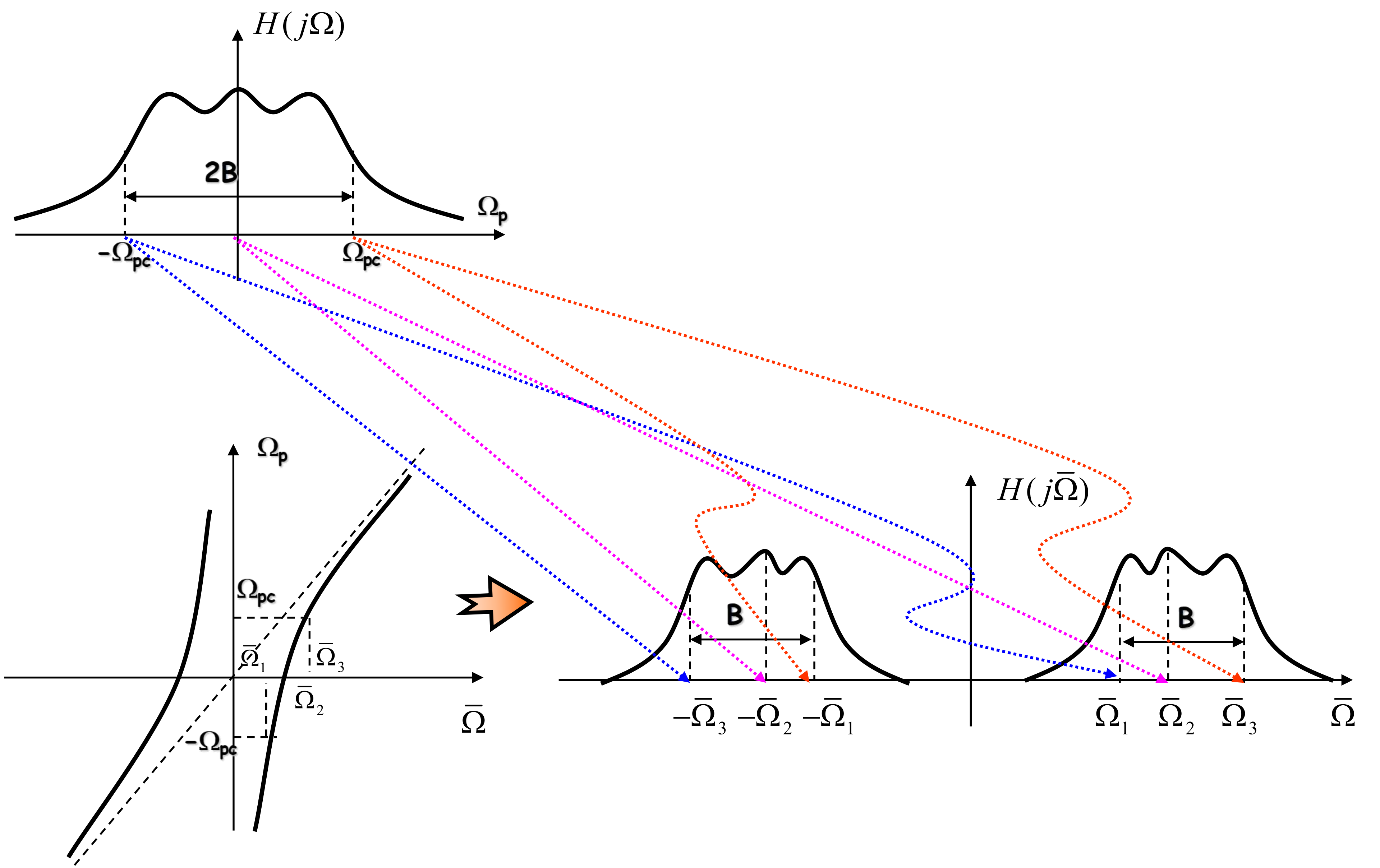
把一个归一化原型模拟低通滤波器变换成另一个所需类型的模拟滤波器，再将其数字化

直接从模拟滤波器通过一定的频率变换关系完成所需类型数字滤波器的设计

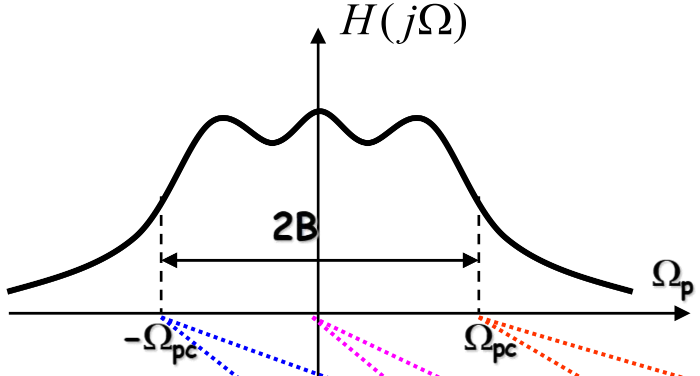
先设计低通型的数字滤波器，再用数字频率变化方法将其转换成所需类型数字滤波器



1 模拟原型方法：模拟低通-〉模拟带通



1 模拟原型方法：模拟低通-〉模拟带通



模拟低通(p平面)到模拟带通($\bar{s}$平面)的变换是

$$p=\bar{s}+\frac{\bar{\Omega}_2^2}{\bar{s}},\begin{cases}p=\sigma_p+j\Omega_p\\ \bar{s}=\bar{\sigma}+j\bar{\Omega}\end{cases}\Rightarrow\Omega_p=\frac{\bar{\Omega}^2-\bar{\Omega}_2^2}{\bar{\Omega}}$$

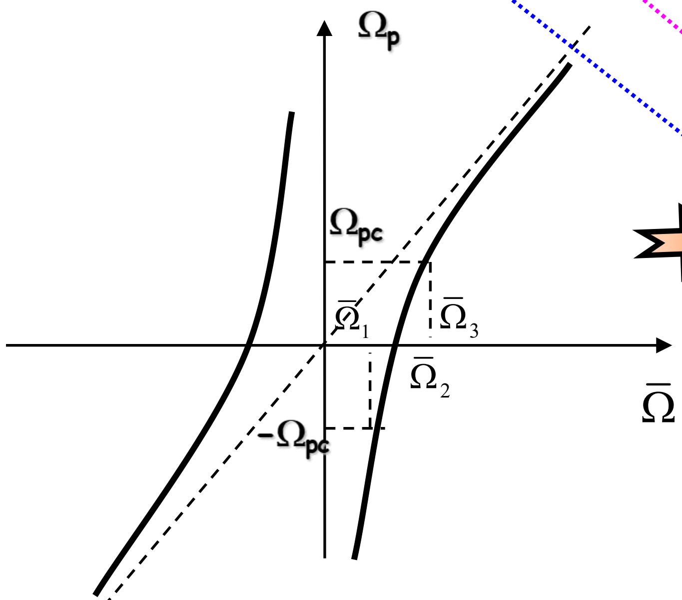
$$\begin{cases}\Omega_{pc}=\dfrac{\bar{\Omega}_3{}^2-\bar{\Omega}_2{}^2}{\bar{\Omega}_3}\\ -\Omega_{pc}=\dfrac{\bar{\Omega}_1{}^2-\bar{\Omega}_2{}^2}{\bar{\Omega}_1}\end{cases}\Rightarrow\begin{cases}\bar{\Omega}_2=\sqrt{\bar{\Omega}_1\bar{\Omega}_3}\\ B=\bar{\Omega}_3-\bar{\Omega}_1=\Omega_{pc}\end{cases}$$

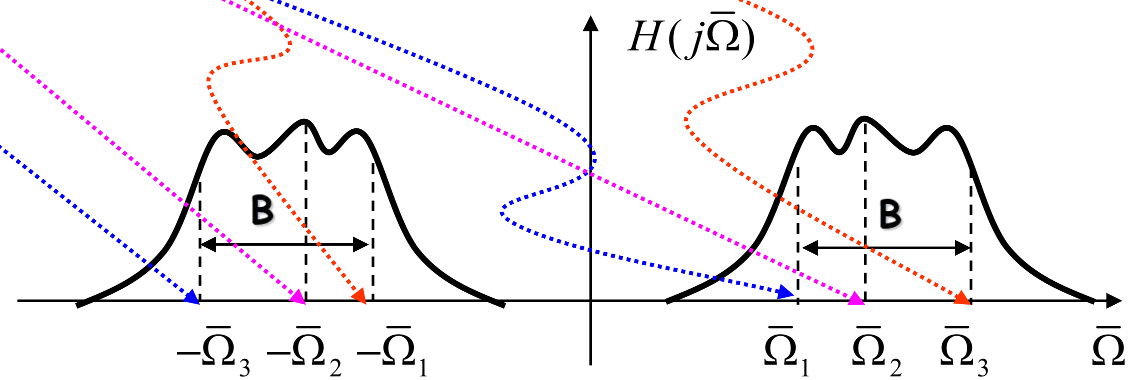
由模拟低通滤波器到模拟带通滤波器变换：

$$H_{bp}(\bar{s})=H_{lp}(p)\Big|_{p=\bar{s}+\frac{\bar{\Omega}_2^2}{\bar{s}}}$$

$\bar{\Omega}_2$为带通模拟滤波器的几何中心

B为带通模拟滤波器的带宽





数字带通滤波器设计

利用双线性变换将模拟带通滤波器转换为数字带通滤波器

$$H(z)=H_{bp}(\bar{s})\Big|_{\bar{s}=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

推导：利用双线性变换将模拟低通滤波器转换为数字带通滤波器

$$\bar{s}=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\Rightarrow p=\bar{s}+\frac{\bar{\Omega}_2^2}{\bar{s}}=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}+\frac{\bar{\Omega}_2^2}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

$$\text{由}\begin{cases} D=\Omega_{pc}ctg\left(\dfrac{\Omega_3-\Omega_1}{2}\right)T \\ E=2\dfrac{\left(\dfrac{2}{T}\right)^2-\bar{\Omega}_2^2}{\left(\dfrac{2}{T}\right)^2+\bar{\Omega}_2^2}=2\cos(\Omega_2 T) \end{cases}\Rightarrow p=D\left[\frac{1-Ez^{-1}+z^{-2}}{1-z^{-2}}\right]$$

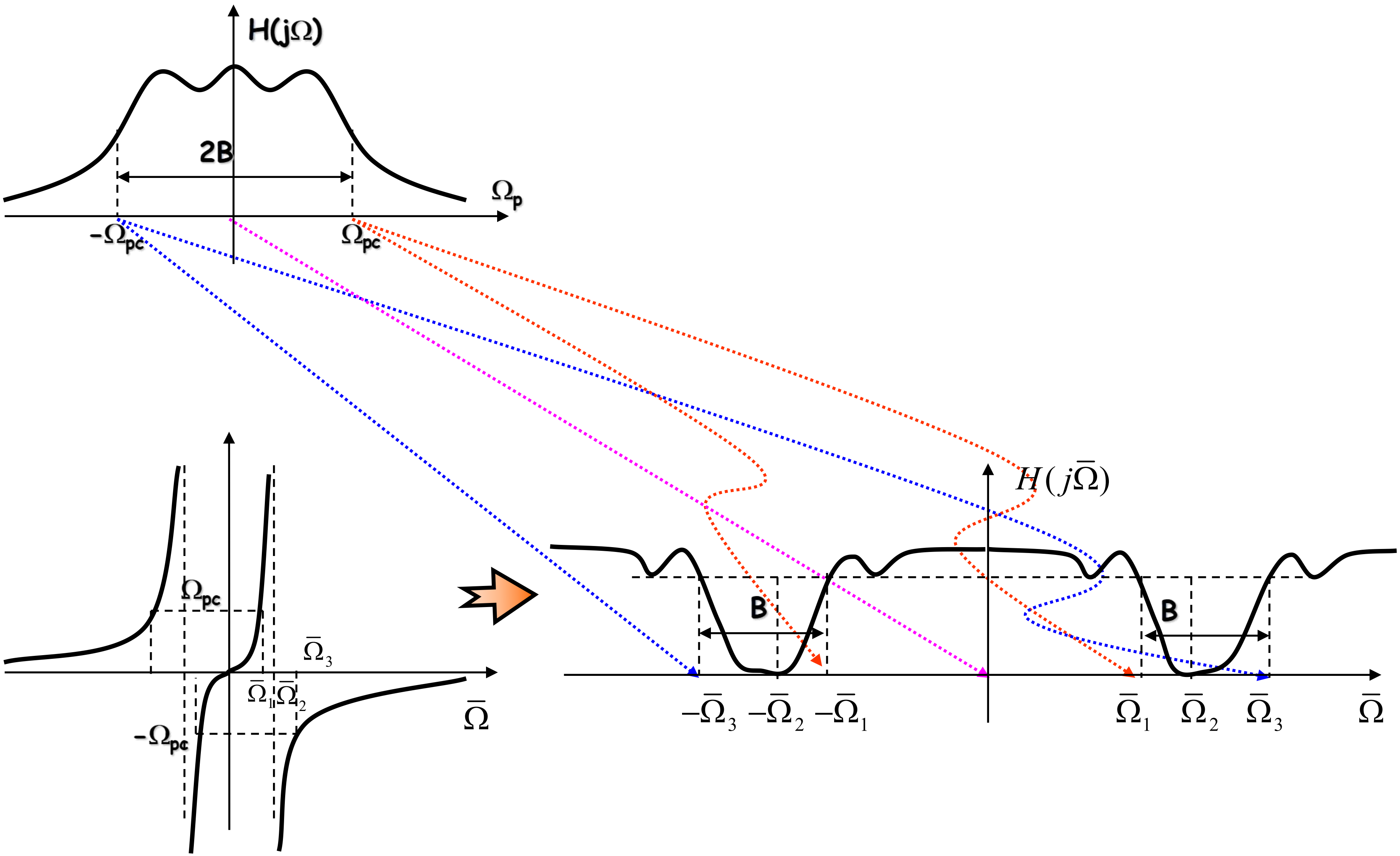
$$\Rightarrow H(z)=H_{lp}(p)\Big|_{p=D\left[\frac{1-Ez^{-1}+z^{-2}}{1-z^{-2}}\right]}$$

$$p=\bar{s}+\frac{\bar{\Omega}_2^2}{\bar{s}},\begin{cases} p=\sigma_p+j\Omega_p \\ \bar{s}=\bar{\sigma}+j\bar{\Omega} \end{cases}$$

$$\Rightarrow\Omega_p=D\frac{E/2-\cos\Omega T}{\sin\Omega T}$$

(用来确定低通原型滤波器截止频率Ω_{pc})

2 模拟原型方法：模拟低通-〉模拟带阻

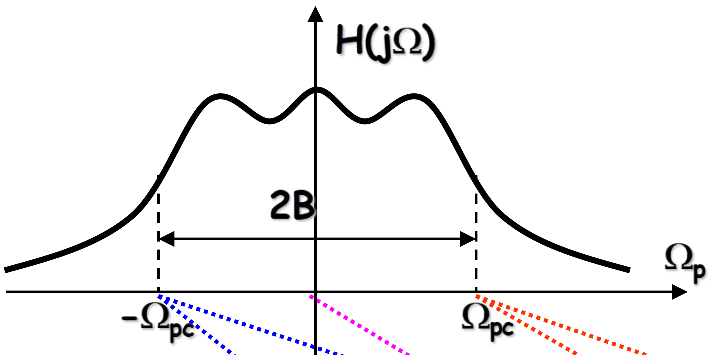


2 模拟原型方法：模拟低通-〉模拟带阻

低通(p平面)到带阻($\bar{s}$平面)的变换是

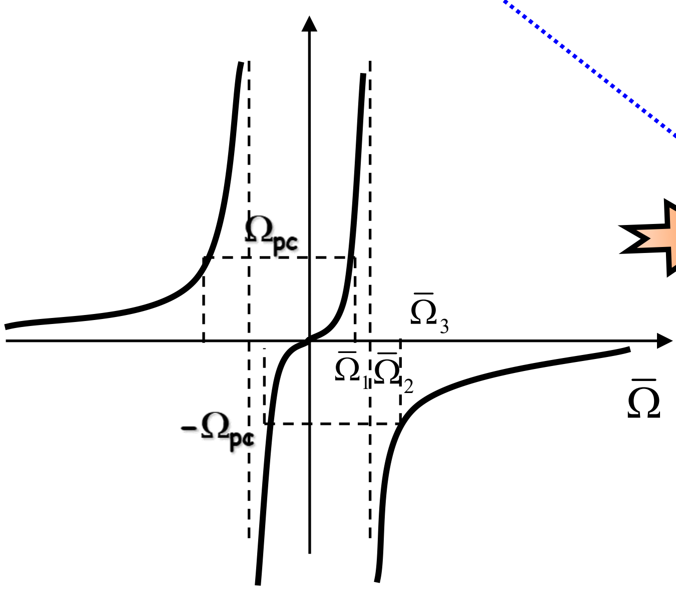
$$p=\frac{\bar{\Omega}_2^2\bar{s}}{\bar{s}^2+\bar{\Omega}_2^2},\begin{cases}p=\sigma_p+j\Omega_p\\ \bar{s}=\bar{\sigma}+j\bar{\Omega}\end{cases}\Rightarrow \Omega_p=\frac{\bar{\Omega}_2^2\bar{\Omega}}{\bar{\Omega}_2^2-\bar{\Omega}^2}$$

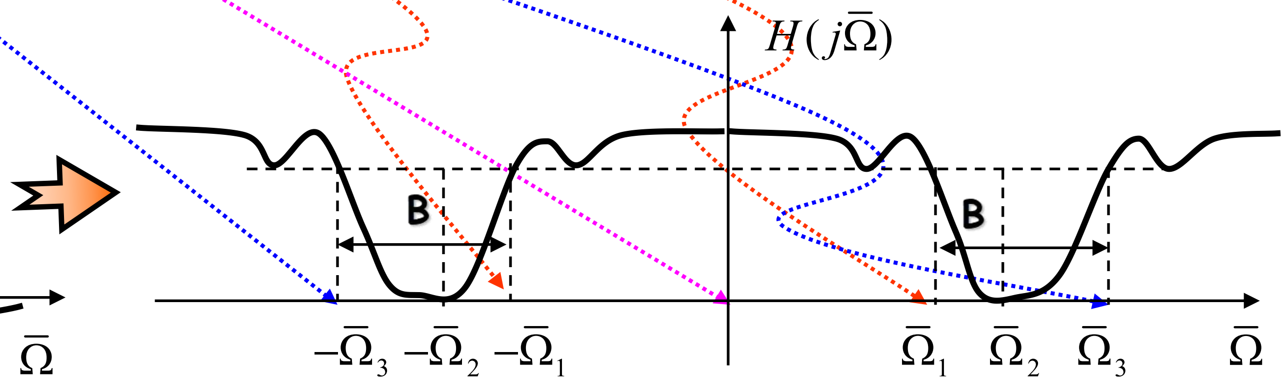
$$\begin{cases}\Omega_{pc}=\dfrac{\bar{\Omega}_2^2\bar{\Omega}_1}{\bar{\Omega}_2^2-\bar{\Omega}_1^2}\\ -\Omega_{pc}=\dfrac{\bar{\Omega}_2^2\bar{\Omega}_3}{\bar{\Omega}_2^2-\bar{\Omega}_3^2}\end{cases}\Rightarrow\begin{cases}\bar{\Omega}_2=\sqrt{\bar{\Omega}_1\bar{\Omega}_3}\\ B=\bar{\Omega}_3-\bar{\Omega}_1=\Omega_{pc}\end{cases}$$



由模拟低通滤波器到模拟带阻滤波器变换：

$$H_{br}(\bar{s})=H_{lp}(p)\Big|_{p=\frac{\bar{\Omega}_2^2\bar{s}}{\bar{s}^2+\bar{\Omega}_2^2}}$$





数字带阻滤波器设计

利用双线性变换将模拟带阻滤波器转换为数字带阻滤波器

$$H(z)=H_{br}(\overline{s})\Big|_{\overline{s}=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

推导：利用双线性变换将模拟低通滤波器转换为数字带阻滤波器

$$\overline{s}=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\Rightarrow p=\frac{\overline{\Omega}_2^2\overline{s}}{\overline{s}^2+\overline{\Omega}_2^2}=\frac{\overline{\Omega}_2^2\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)}{\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\overline{\Omega}_2^2}$$

$$p=\frac{\overline{\Omega}_2^2\overline{s}}{\overline{s}^2+\overline{\Omega}_2^2},\begin{cases}p=\sigma_p+j\Omega_p\\ \overline{s}=\overline{\sigma}+j\overline{\Omega}\end{cases}$$

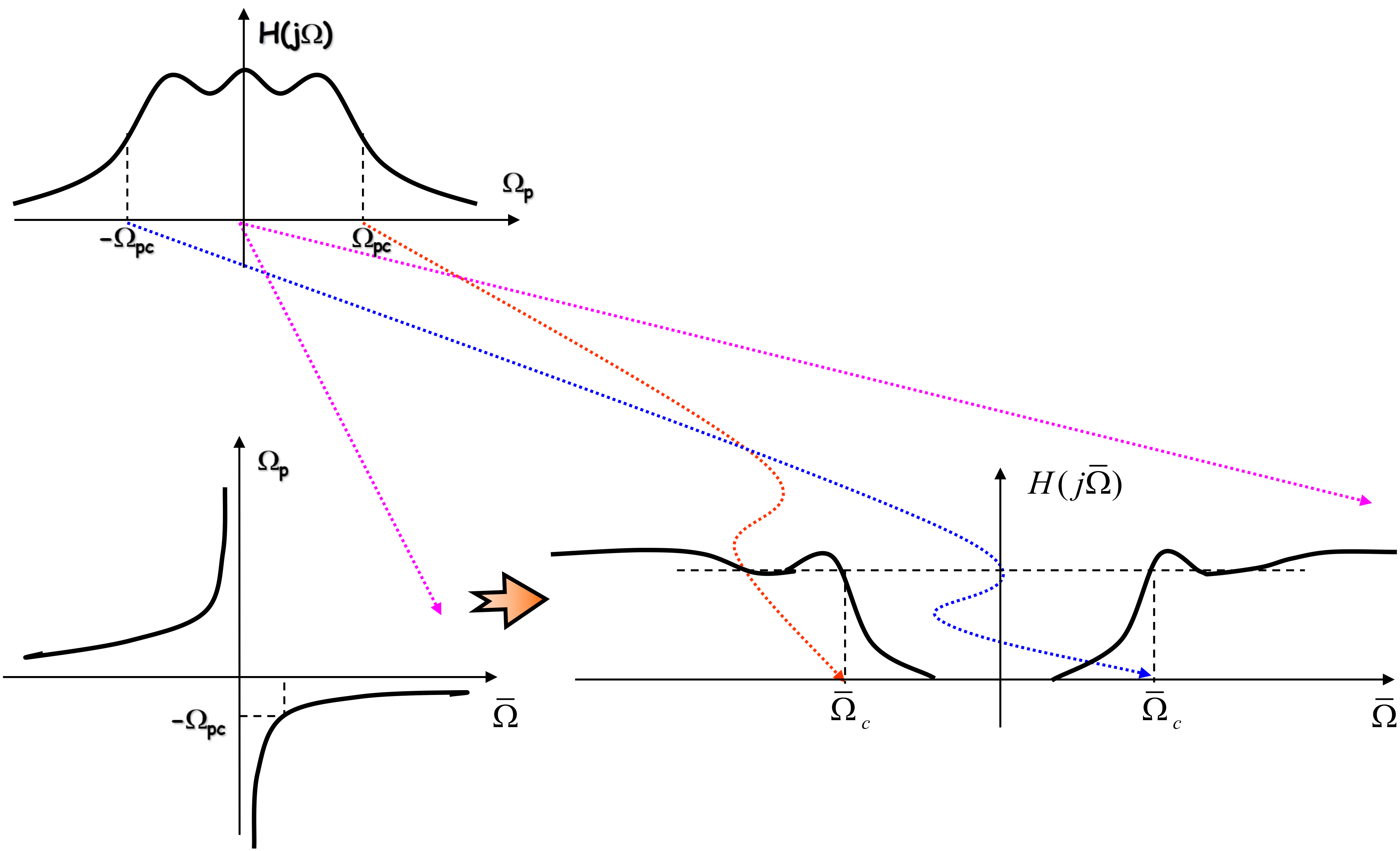
$$\Rightarrow\Omega_p=D_1\frac{\sin\Omega T}{\cos\Omega T-E_1/2}$$

(用来确定低通原型滤波器截止频率Ω_{pc})

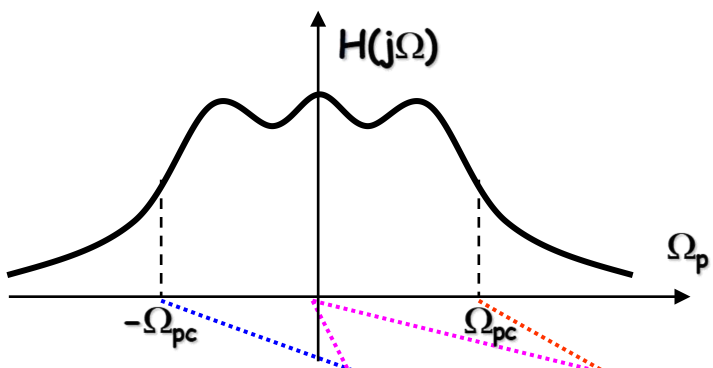
$$由\begin{cases}D_1=\Omega_{pc}tg\left(\dfrac{\Omega_3-\Omega_1}{2}\right)T\\ E_1=2\dfrac{\left(\frac{2}{T}\right)^2-\overline{\Omega}_2^2}{\left(\frac{2}{T}\right)^2+\overline{\Omega}_2^2}=2\cos(\Omega_2T)\end{cases}\Rightarrow p=D_1\left[\frac{(1-z^{-2})}{1-E_1z^{-1}+z^{-2}}\right]$$

$$\Rightarrow H(z)=H_{lp}(p)\Big|_{p=D_1\left[\frac{(1-z^{-2})}{1-E_1z^{-1}+z^{-2}}\right]}$$

3 模拟原型方法：模拟低通-〉模拟高通



3 模拟原型方法：模拟低通-〉模拟高通



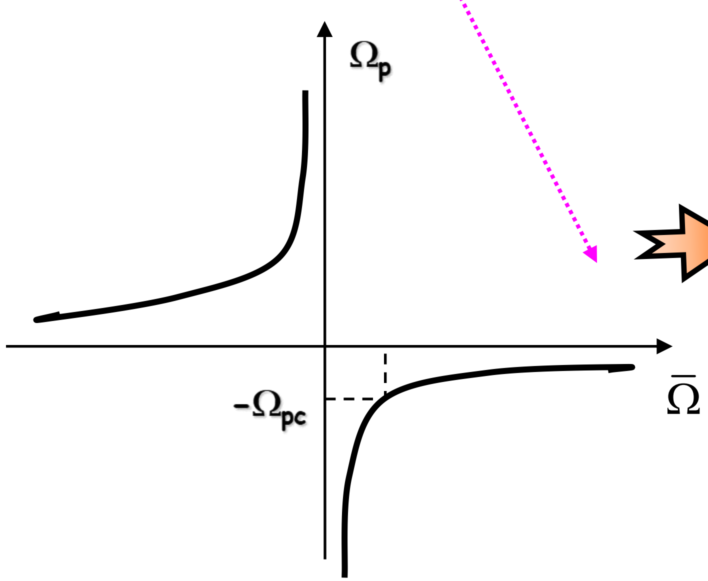
低通(p平面)到高通($\bar{s}$平面)的变换是

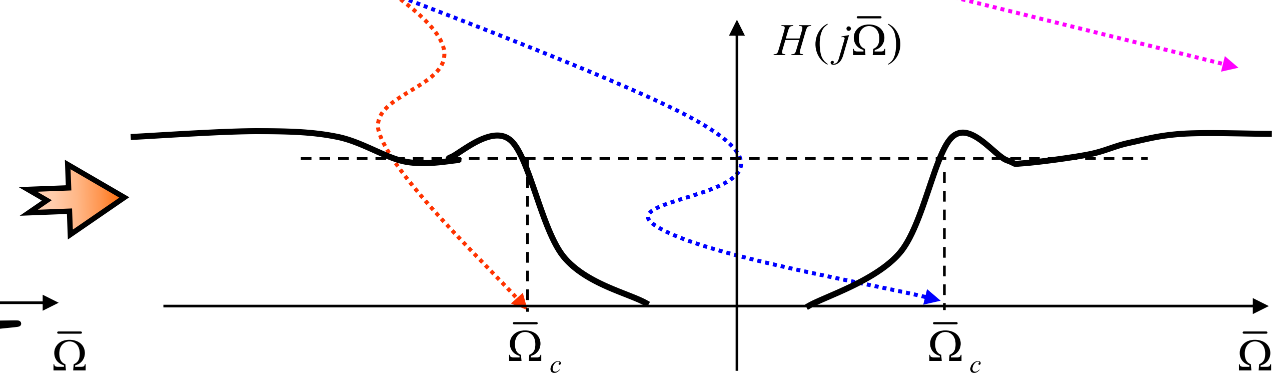
$$p=\frac{\Omega_{pc}\bar{\Omega}_c}{\bar{s}},\begin{cases}p=\sigma_p+j\Omega_p\\ \bar{s}=\bar{\sigma}+j\bar{\Omega}\end{cases}$$

$$\Rightarrow \Omega_p=-\frac{\bar{\Omega}_c}{\bar{\Omega}}\Omega_{pc}$$

由模拟低通滤波器到模拟高通滤波器变换：

$$H_{hp}(\bar{s})=H_{lp}(p)\Big|_{p=\frac{\Omega_{pc}\bar{\Omega}_c^2}{\bar{s}}}$$





数字高通滤波器设计

利用双线性变换将模拟高通滤波器转换为数字高通滤波器

$$H(z)=H_{hp}(\bar{s})\Big|_{\bar{s}=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

推导：利用双线性变换将模拟低通滤波器转换为数字高通滤波器

$$\bar{s}=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\Rightarrow p=\frac{\Omega_{pc}\bar{\Omega}_c}{\bar{s}}=\frac{\Omega_{pc}\bar{\Omega}_c}{\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)}=c_1\frac{1+z^{-1}}{1-z^{-1}}$$

$$\Rightarrow H(z)=H_{lp}(p)\Big|_{p=c_1\frac{1+z^{-1}}{1-z^{-1}}}$$

$$p=\frac{\Omega_{pc}\bar{\Omega}_c}{\bar{s}},\begin{cases}p=\sigma_p+j\Omega_p\\ \bar{s}=\bar{\sigma}+j\bar{\Omega}\end{cases}$$

$$\Rightarrow \Omega_p=-\frac{\bar{\Omega}_c}{\bar{\Omega}}\Omega_{pc}$$

由 $\begin{cases}\bar{\Omega}=\frac{2}{T}tg\frac{\Omega T}{2}\\ c_1=\Omega_{pc}tg\frac{\Omega_c T}{2}\end{cases}$

$$\Rightarrow \Omega_p=-c_1 ctg\frac{\Omega T}{2}$$

(用来确定低通原型滤波器截止频率Ω_{pc})

频率变换1—数字带通滤波器设计

设一取样频率为2kHz的数字带通滤波器，满足如下要求：
通带范围为300Hz到400Hz，在300Hz和400Hz处衰减不大于3dB，在200Hz和500Hz频率处衰减不小于18dB；用双线性变换法设计一个满足下述指标要求的数字巴特沃斯带通滤波器，

(1) 确定滤波器的阶数N

(2) 确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

解：（1）

$$D=\Omega_{pc}ctg\left(\frac{\Omega_3-\Omega_1}{2}T\right)=\Omega_{pc}ctg\left(\frac{2\pi(400-300)}{2}\frac{1}{2000}\right)=6.31\Omega_{pc}$$

$$E=\frac{2\cos\left(\dfrac{\Omega_3+\Omega_1}{2}T\right)}{\cos\left(\dfrac{\Omega_3-\Omega_1}{2}T\right)}=\frac{2\cos(0.35\pi)}{\cos(0.05\pi)}=\frac{2\times0.45}{0.99}=0.92$$

$$\Omega_p=D\frac{E/2-\cos\Omega T}{\sin\Omega T}$$

Ω_{ps}为满足所设计的数字带通滤波器要求的模拟原型的阻带起始频率

$$\begin{cases}-\Omega_{ps}=D\dfrac{E/2-\cos(2\pi\times200\times\dfrac{1}{2000})}{\sin(2\pi\times200\times\dfrac{1}{2000})}=6.31\Omega_{pc}\dfrac{0.46-0.81}{0.59}=-3.74\Omega_{pc}\\ \\ \Omega_{ps}=D\dfrac{E/2-\cos(2\pi\times500\times\dfrac{1}{2000})}{\sin(2\pi\times500\times\dfrac{1}{2000})}=6.31\Omega_{pc}\dfrac{0.46-0}{1}=-2.90\Omega_{pc}\end{cases}$$

取$\Omega_{ps}=2.90\Omega_{pc}$

已经预畸

(2)由已知条件列出对模拟低通滤波器的衰减要求

$$A^2(\Omega)=\left|H_a(j\Omega)\right|^2=\frac{1}{1+\left(\dfrac{\Omega}{\Omega_{pc}}\right)^{2N}}$$

$$\Rightarrow 20\lg\left|H_a(j\Omega)\right|=-10\lg\left[1+\left(\frac{\Omega}{\Omega_{pc}}\right)^{2N}\right]$$

$$\Rightarrow\begin{cases}20\lg\left|H_a(j\Omega_{pc})\right|\geq -3dB\\ 20\lg\left|H_a(j\Omega_{ps})\right|\leq -18dB\end{cases}$$

$$\begin{cases}-10\lg\left[1+\left(\dfrac{\Omega_{pc}}{\Omega_{pc}}\right)^{2N}\right]\geq -3dB\\ -10\lg\left[1+\left(\dfrac{\Omega_{ps}}{\Omega_{pc}}\right)^{2N}\right]\leq -18dB\end{cases}$$

$$-10\lg\left[1+\left(\frac{\Omega_{ps}}{\Omega_{pc}}\right)^{2N}\right]=-18dB$$

$$\Rightarrow 1+(2.9)^{2N}=10^{1.8}$$

$$\Rightarrow N=1.94,\text{取}N=2$$

(3)直接由表5-1

$$H_{LP}(p)=\frac{{\Omega_{pc}}^2}{p^2+\sqrt{2}\Omega_{pc}p+{\Omega_{pc}}^2}$$

$$(4)H_{LP}(p)=\frac{\Omega_{pc}{}^{2}}{p^{2}+\sqrt{2}\Omega_{pc}p+\Omega_{pc}{}^{2}}$$

$$p=D\left[\frac{1-Ez^{-1}+z^{-2}}{1-z^{-2}}\right]=6.31\Omega_{pc}\left[\frac{1-0.92z^{-1}+z^{-2}}{1-z^{-2}}\right]$$

$$H(z)=H_{LP}(p)\Big|_{p=D\left[\frac{1-Ez^{-1}+z^{-2}}{1-z^{-2}}\right]}$$

$$=\frac{\Omega_{pc}{}^{2}}{p^{2}+\sqrt{2}\Omega_{pc}p+\Omega_{pc}{}^{2}}=\frac{1}{6.31^{2}\left[\frac{1-0.92z^{-1}+z^{-2}}{1-z^{-2}}\right]^{2}+\sqrt{2}\times 6.31\left[\frac{1-0.92z^{-1}+z^{-2}}{1-z^{-2}}\right]+1}$$

$$=\frac{(1-z^{-2})^{2}}{39.82(1-0.92z^{-1}+z^{-2})^{2}+8.92(1-0.92z^{-1}+z^{-2})(1-z^{-2})+(1-z^{-2})^{2}}$$

$$=\frac{0.02(1-z^{-2})^{2}}{1-1.64z^{-1}+2.34z^{-2}-1.31z^{-3}+0.64z^{-4}}$$

$$=\frac{0.02-0.04z^{-2}+0.02z^{-4}}{1-1.64z^{-1}+2.34z^{-2}-1.31z^{-3}+0.64z^{-4}}$$

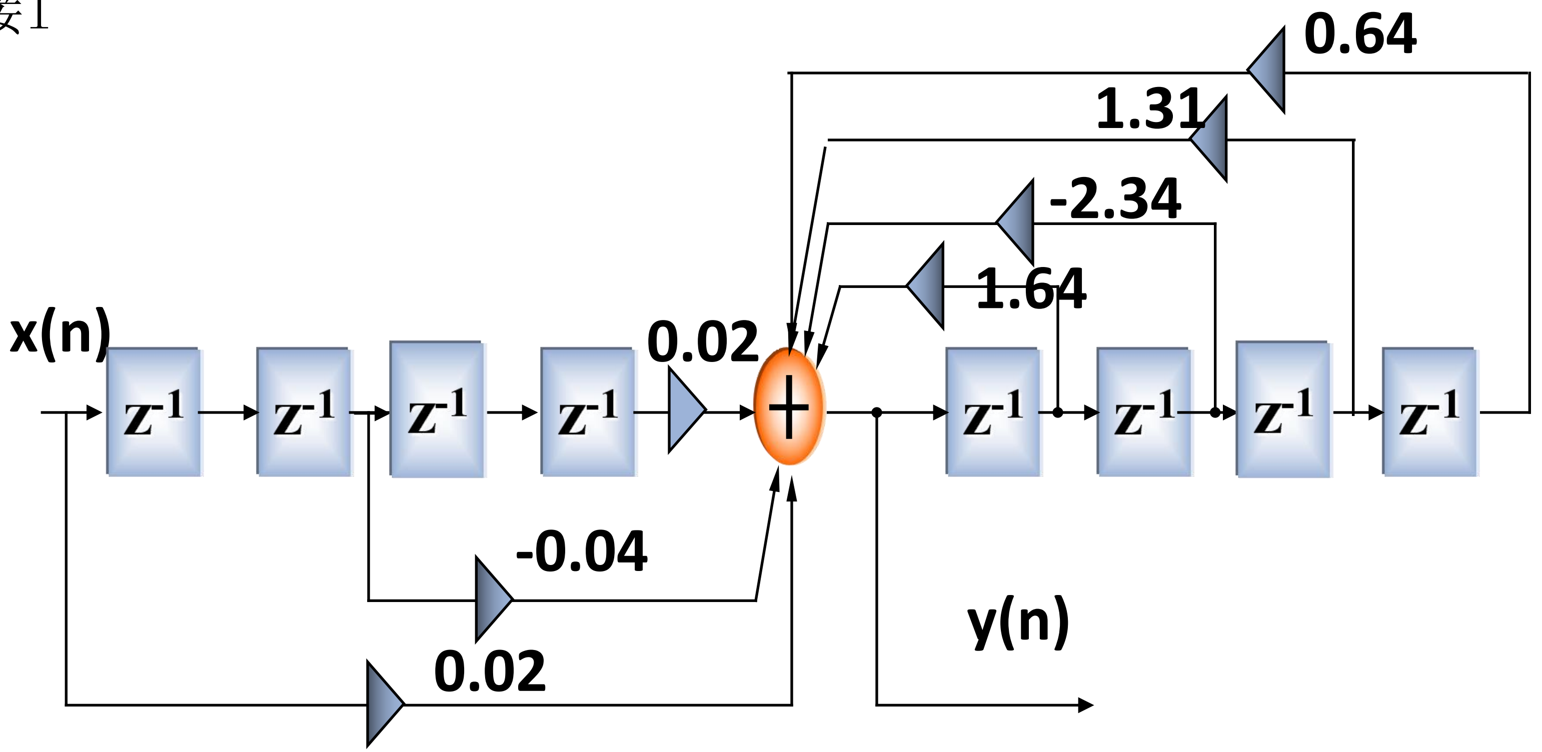
$$H(z)=\frac{0.02-0.04z^{-2}+0.02z^{-4}}{1-1.64z^{-1}+2.34z^{-2}-1.31z^{-3}+0.64z^{-4}}$$

(4)频率响应

$$H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}$$

(5)滤波器结构

直接I



频率变换2—数字带阻滤波器设计

设计一取样频率为100kHz的二阶巴特沃斯数字带阻滤波器，其3dB带边频率分别为12.5kHz,22.5kHz

解：由于设计二阶带阻数字滤波器，所以模拟原型系统函数为

$$H_{LP}(p)=\frac{\Omega_{pc}}{\Omega_{pc}+p}$$

$$D_1=\Omega_{pc}tg\left(\frac{\Omega_3-\Omega_1}{2}T\right)=\Omega_{pc}tg\left(\frac{2\pi(22.5-12.5)}{2}\frac{1}{100}\right)=0.32\Omega_{pc}$$

$$E_1=\frac{2\cos\left(\frac{\Omega_3+\Omega_1}{2}T\right)}{\cos\left(\frac{\Omega_3-\Omega_1}{2}T\right)}=\frac{2\cos\left(\frac{2\pi(22.5+12.5)}{2}\frac{1}{100}\right)}{\cos\left(\frac{2\pi(22.5-12.5)}{2}\frac{1}{100}\right)}=0.95$$

$$p=D_1\left[\frac{(1-z^{-2})}{1-E_1z^{-1}+z^{-2}}\right]=0.32\Omega_{pc}\frac{(1-z^{-2})}{1-0.95z^{-1}+z^{-2}}$$

$$H(z)=H_{LP}(p)\Big|_{p=D_1\left[\frac{(1-z^{-2})}{1-E_1z^{-1}+z^{-2}}\right]}$$

$$=\frac{\Omega_{pc}}{\Omega_{pc}+p}\Bigg|_{p=0.32\Omega_{pc}\frac{(1-z^{-2})}{1-0.95z^{-1}+z^{-2}}}=\frac{\Omega_{pc}}{\Omega_{pc}+0.32\Omega_{pc}\frac{(1-z^{-2})}{1-0.95z^{-1}+z^{-2}}}=\frac{1}{1+0.32\frac{(1-z^{-2})}{1-0.95z^{-1}+z^{-2}}}$$

$$=\frac{1-0.95z^{-1}+z^{-2}}{1-0.95z^{-1}+z^{-2}+0.32(1-z^{-2})}=\frac{1-0.95z^{-1}+z^{-2}}{1.32-0.95z^{-1}+0.68z^{-2}}=\frac{0.76-0.72z^{-1}+0.76z^{-2}}{1-0.72z^{-1}+0.52z^{-2}}$$

频率变换3—数字带阻滤波器设计

设一取样频率为2kHz的数字带阻滤波器，满足如下要求：

阻带范围为300Hz到400Hz，在300Hz和400Hz处衰减不小于16dB，

在200Hz和500Hz频率处衰减不大于3dB；用双线性变换法设计

一个满足下述指标要求的数字巴特沃斯带阻滤波器，

(1) 确定滤波器的阶数N

(2) 确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

解：（1）

$$D_1=\Omega_{pc}tg\left(\frac{\Omega_3-\Omega_1}{2}T\right)=\Omega_{pc}tg\left(\frac{2\pi(500-200)}{2}\frac{1}{2000}\right)=0.51\Omega_{pc}$$

$$E_1=\frac{2\cos\left(\frac{\Omega_3+\Omega_1}{2}T\right)}{\cos\left(\frac{\Omega_3-\Omega_1}{2}T\right)}=\frac{2\cos(\frac{2\pi(500+200)}{2}\frac{1}{2000})}{\cos(\frac{2\pi(500-200)}{2}\frac{1}{2000})}=\frac{2\times0.45}{0.89}=1.01$$

$$\Omega_p=D_1\frac{\sin\Omega T}{\cos\Omega T-E_1/2}$$

Ω_{ps}为满足所设计的数字带阻滤波器要求的模拟原型的阻带起始频率

$$\begin{cases}\Omega_{ps}=D_1\dfrac{\sin(2\pi\times300\times\frac{1}{2000})}{\cos(2\pi\times300\times\frac{1}{2000})-E_1/2}=0.51\Omega_{pc}\dfrac{0.81}{0.59-0.50}=4.59\Omega_{pc}\\ -\Omega_{ps}=D_1\dfrac{\sin(2\pi\times400\times\frac{1}{2000})}{\cos(2\pi\times400\times\frac{1}{2000})-E_1/2}=0.51\Omega_{pc}\dfrac{0.95}{0.31-0.50}=-2.55\Omega_{pc}\end{cases}$$

取$\Omega_{ps}=2.55\Omega_{pc}$

已经预畸

(2)由已知条件列出对模拟低通滤波器的衰减要求

$$A^2(\Omega)=\left|H_a(j\Omega)\right|^2=\frac{1}{1+\left(\dfrac{\Omega}{\Omega_{pc}}\right)^{2N}}$$

$$\Rightarrow 20\lg\left|H_a(j\Omega)\right|=-10\lg\left[1+\left(\frac{\Omega}{\Omega_{pc}}\right)^{2N}\right]$$

$$\Rightarrow\begin{cases}20\lg\left|H_a(j\Omega_{pc})\right|\geq -3dB\\ 20\lg\left|H_a(j\Omega_{ps})\right|\leq -16dB\end{cases}$$

$$\begin{cases}-10\lg\left[1+\left(\dfrac{\Omega_{pc}}{\Omega_{pc}}\right)^{2N}\right]\geq -3dB\\ -10\lg\left[1+\left(\dfrac{\Omega_{ps}}{\Omega_{pc}}\right)^{2N}\right]\leq -16dB\end{cases}$$

$$-10\lg\left[1+\left(\frac{\Omega_{ps}}{\Omega_{pc}}\right)^{2N}\right]=-16dB$$

$$\Rightarrow 1+(2.55)^{2N}=10^{1.6}$$

$$\Rightarrow N=1.95,\text{取}N=2$$

(3)直接由表5−1

$$H_{LP}(p)=\frac{{\Omega_{pc}}^2}{p^2+\sqrt{2}\Omega_{pc}p+{\Omega_{pc}}^2}$$

$$(4)H_{LP}(p)=\frac{\Omega_{pc}{}^{2}}{p^{2}+\sqrt{2}\Omega_{pc}p+\Omega_{pc}{}^{2}}$$

$$p=D_{1}\left[\frac{(1-z^{-2})}{1-E_{1}z^{-1}+z^{-2}}\right]=0.51\Omega_{pc}\frac{(1-z^{-2})}{1-1.01z^{-1}+z^{-2}}$$

$$H(z)=H_{LP}(p)\Big|_{p=D_{1}\left[\frac{(1-z^{-2})}{1-E_{1}z^{-1}+z^{-2}}\right]}=\frac{\Omega_{pc}{}^{2}}{p^{2}+\sqrt{2}\Omega_{pc}p+\Omega_{pc}{}^{2}}\Bigg|_{p=0.51\Omega_{pc}\frac{(1-z^{-2})}{1-1.01z^{-1}+z^{-2}}}$$

$$=\frac{1}{0.51^{2}\left[\frac{(1-z^{-2})}{1-1.01z^{-1}+z^{-2}}\right]^{2}+\sqrt{2}\times0.51\left[\frac{(1-z^{-2})}{1-1.01z^{-1}+z^{-2}}\right]+1}$$

$$=\frac{(1-1.01z^{-1}+z^{-2})^{2}}{0.51^{2}(1-z^{-2})^{2}+\sqrt{2}\times0.51(1-z^{-2})(1-1.01z^{-1}+z^{-2})+(1-1.01z^{-1}+z^{-2})^{2}}$$

$$=\frac{1-2.02z^{-1}+3.02z^{-2}-2.02z^{-3}+z^{-4}}{1.98-2.57z^{-1}+2.5z^{-2}-1.29z^{-3}+0.54z^{-4}}$$

$$=\frac{0.51-1.02z^{-1}+1.53z^{-2}-1.02z^{-3}+0.51z^{-4}}{1-1.3z^{-1}+1.26z^{-2}-0.65z^{-3}+0.27z^{-4}}$$

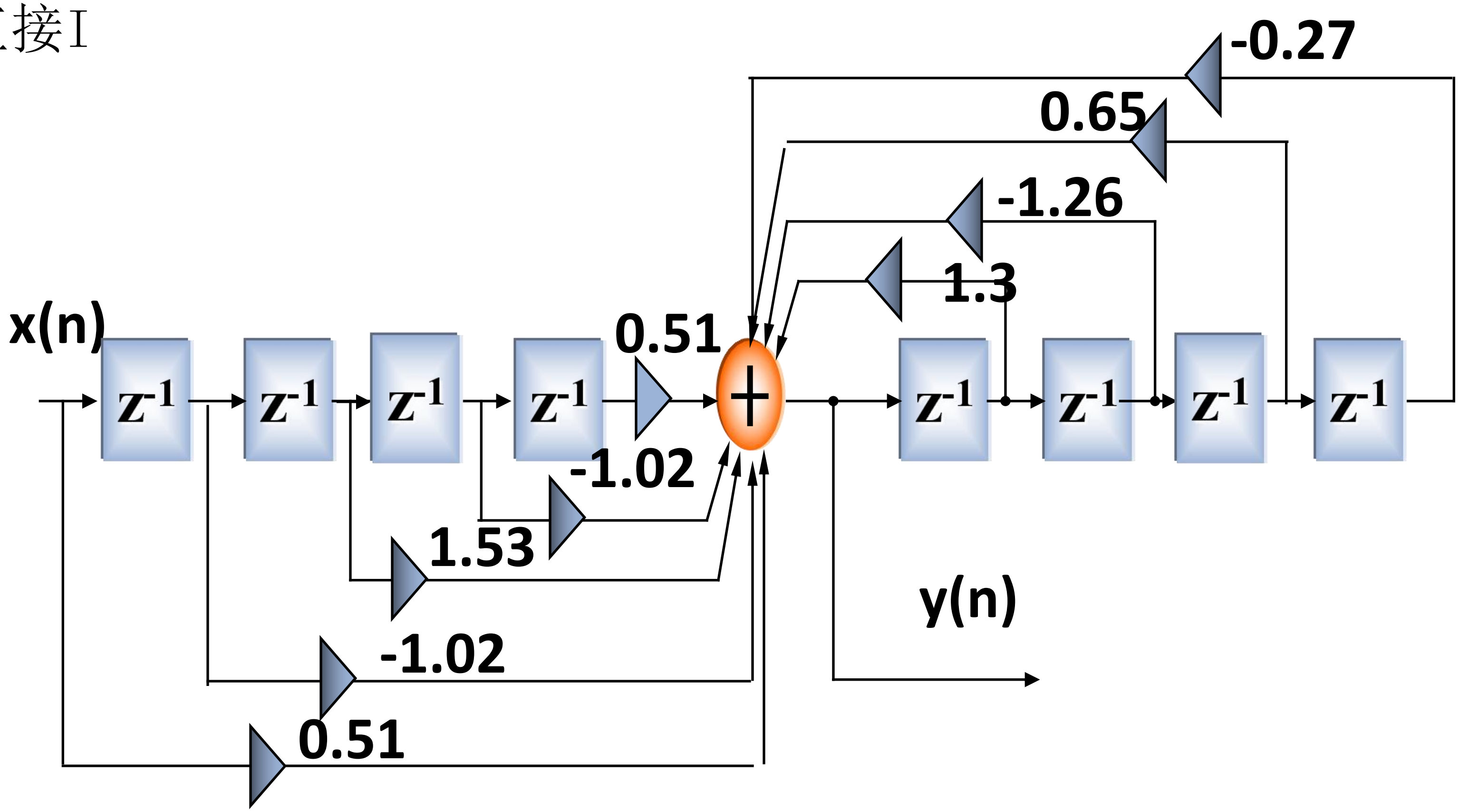
$$H(z)=\frac{0.51-1.02z^{-1}+1.53z^{-2}-1.02z^{-3}+0.51z^{-4}}{1-1.3z^{-1}+1.26z^{-2}-0.65z^{-3}+0.27z^{-4}}$$

(4)频率响应

$$H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}$$

(5)滤波器结构

直接I



频率变换4—数字高通滤波器设计

设计一取样频率为10kHz的二阶巴特沃斯数字高通滤波器，其3dB截止频率分别为2kHz。

解：由于设计二阶高通数字滤波器，所以模拟原型系统函数为

$$H_{LP}(p)=\frac{\Omega_{pc}{}^{2}}{p^{2}+\sqrt{2}\Omega_{pc}p+\Omega_{pc}{}^{2}}$$

$$c_1=\Omega_{pc}tg\left(\frac{\Omega_c}{2}T\right)=\Omega_{pc}tg\left(\frac{2\pi\times 2000}{2}\times\frac{1}{10000}\right)=0.73\Omega_{pc}$$

$$p=c_1\frac{1+z^{-1}}{1-z^{-1}}=0.73\Omega_{pc}\frac{1+z^{-1}}{1-z^{-1}}$$

$$H(z)=H_{LP}(p)\Big|_{p=c_1\frac{1+z^{-1}}{1-z^{-1}}}=\frac{\Omega_{pc}{}^{2}}{p^{2}+\sqrt{2}\Omega_{pc}p+\Omega_{pc}{}^{2}}\Bigg|_{p=0.73\Omega_{pc}\frac{1+z^{-1}}{1-z^{-1}}}$$

$$=\frac{\Omega_{pc}{}^{2}}{\left(0.73\Omega_{pc}\frac{1+z^{-1}}{1-z^{-1}}\right)^{2}+\sqrt{2}\Omega_{pc}\left(0.73\Omega_{pc}\frac{1+z^{-1}}{1-z^{-1}}\right)+\Omega_{pc}{}^{2}}$$

$$=\frac{1}{\left(0.73\frac{1+z^{-1}}{1-z^{-1}}\right)^{2}+\sqrt{2}\left(0.73\frac{1+z^{-1}}{1-z^{-1}}\right)+1}=\frac{(1-z^{-1})^{2}}{\left(0.73(1+z^{-1})\right)^{2}+\sqrt{2}\left(0.73(1+z^{-1})(1-z^{-1})\right)+(1-z^{-1})^{2}}$$

$$=\frac{0.39-0.78z^{-1}+0.39z^{-2}}{1-0.37z^{-1}+0.2z^{-2}}$$

频率变换5—数字高通滤波器设计

如果所要设计的数字高通滤波器满足下列条件：

(a) 在$\omega \le \pi/8$的通带范围内幅度衰减不小于$20dB$，

(b) 在$\pi/2 \le \omega \le \pi$的阻带范围内幅度变化不大于$3dB$，

试用双线性变换法，设计相应的数字巴特沃斯高通滤波器，

(1) 确定滤波器的阶数N

(2) 确定滤波器的系统函数$H(z)$

(3)确定滤波器的频率响应$H(e^{j\omega})$

(4)给出滤波器的任意一种结构实现形式

设：$T=1$

√

03B(05-06)改

解:(1)$c_1=\Omega_{pc}tg\left(\frac{\Omega_c}{2}T\right)=\Omega_{pc}tg\left(\frac{\pi/2}{2}\right)=\Omega_{pc}$

(2)列出对模拟原型滤波器的衰减要求

由$\Omega_p=-c_1ctg\frac{\Omega T}{2}$

$$\Rightarrow \Omega_{ps}=-c_1ctg\frac{-\Omega_sT}{2}=-\Omega_{pc}ctg\frac{-\pi/8}{2}$$

$$=5.03\Omega_{pc}$$

$$A^2(\Omega)=\left|H_a(j\Omega)\right|^2=\frac{1}{1+\left(\frac{\Omega}{\Omega_{pc}}\right)^{2N}}$$

$$\Rightarrow 20\lg\left|H_a(j\Omega)\right|=-10\lg\left[1+\left(\frac{\Omega}{\Omega_{pc}}\right)^{2N}\right]$$

$$\Rightarrow\begin{cases}20\lg\left|H_a(j\Omega_{pc})\right|\geq-3dB\\20\lg\left|H_a(j\Omega_{ps})\right|\leq-20dB\end{cases}$$

$$-10\lg\left[1+\left(\frac{\Omega_{ps}}{\Omega_{pc}}\right)^{2N}\right]\leq-20dB$$

$$\Rightarrow 1+(5.03)^{2N}=10^2$$

解出：$N=1.42$，取$N=2$

直接由表5-1

$$H_{LP}(p)=\frac{\Omega_{pc}{}^2}{p^2+\sqrt{2}\Omega_{pc}p+\Omega_{pc}{}^2}$$

$$(3)\, p = c_1 \frac{1+z^{-1}}{1-z^{-1}} = \Omega_{pc} \frac{1+z^{-1}}{1-z^{-1}}$$

$$H(z) = H_{LP}(p)\Big|_{p=c_1 \frac{1+z^{-1}}{1-z^{-1}}} = \frac{\Omega_{pc}{}^2}{p^2 + \sqrt{2}\Omega_{pc} p + \Omega_{pc}{}^2}\Bigg|_{p=\Omega_{pc} \frac{1+z^{-1}}{1-z^{-1}}}$$

$$= \frac{\Omega_{pc}{}^2}{\left(\Omega_{pc} \frac{1+z^{-1}}{1-z^{-1}}\right)^2 + \sqrt{2}\Omega_{pc}\left(\Omega_{pc} \frac{1+z^{-1}}{1-z^{-1}}\right) + \Omega_{pc}{}^2}$$

$$= \frac{1}{\left(\frac{1+z^{-1}}{1-z^{-1}}\right)^2 + \sqrt{2}\left(\frac{1+z^{-1}}{1-z^{-1}}\right) + 1} = \frac{(1-z^{-1})^{-2}}{\left(1+z^{-1}\right)^2 + \sqrt{2}(1+z^{-1})(1-z^{-1}) + (1-z^{-1})^{-2}}$$

$$= \frac{1-2z^{-1}+z^{-2}}{3.41+0.39z^{-2}} = \frac{0.29-0.59z^{-1}+0.29z^{-2}}{1+0.11z^{-2}}$$

$$H(z)=\frac{0.29-0.59z^{-1}+0.29z^{-2}}{1+0.11z^{-2}}$$

(4)频率响应

$$H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}$$

(5)滤波器结构

直接I