数字信号处理

周治国

2023.11

第五章 数字滤波器

FIR数字滤波器

窗函数设计法

三、窗函数方法：Windowing Method

设计原理：

$$H_d'(e^{j\omega}) = \begin{cases} 1 \ , & |\omega| \le \omega_c \\ 0 \ , \ \omega_c & |\omega| \quad \pi \end{cases}$$

$$h_d'(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega = \frac{\sin \omega_c n}{\pi n}$$

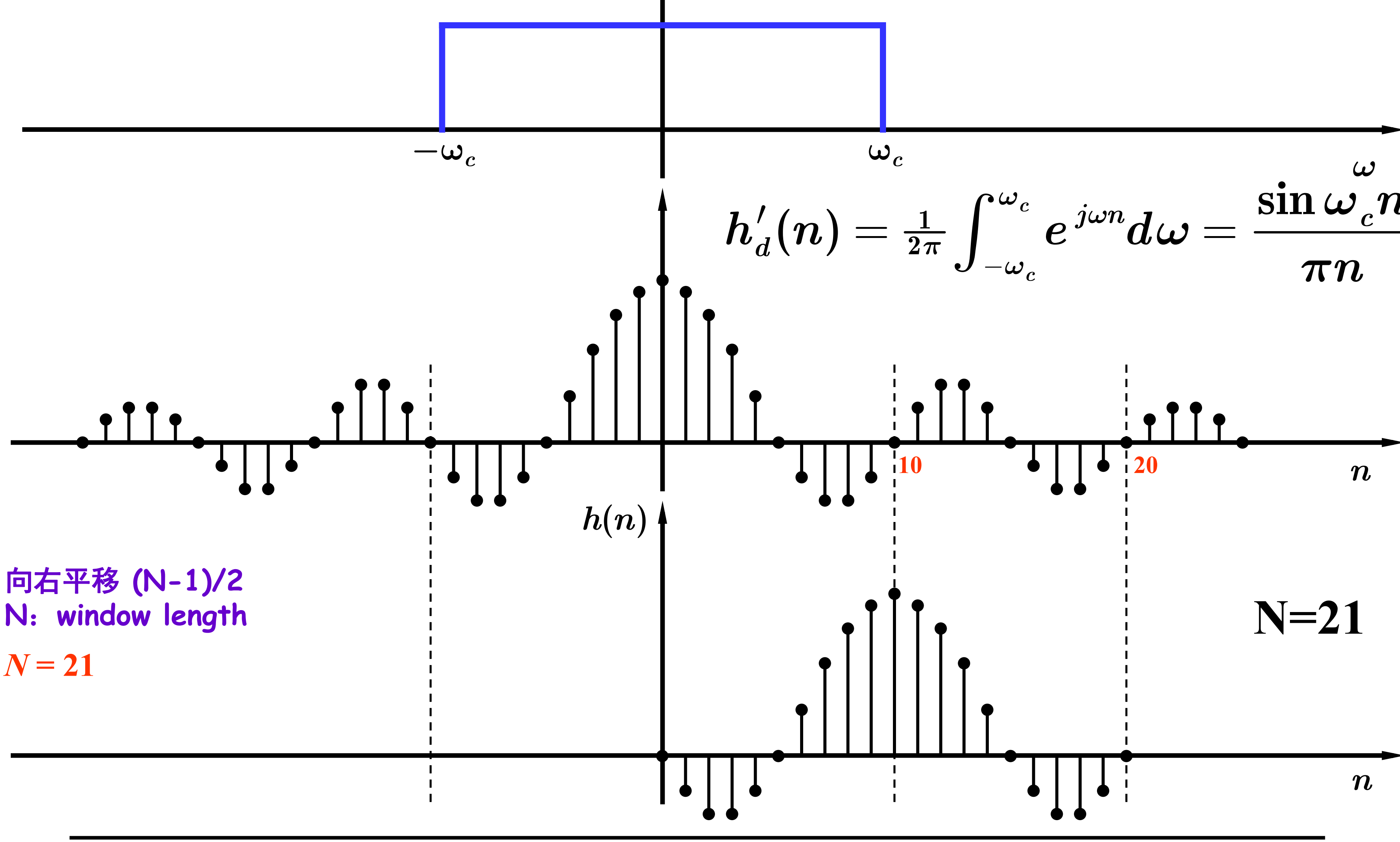
$$H_d(e^{j\omega}) = \begin{cases} e^{-j\frac{N-1}{2}\omega} \ , & |\omega| \le \omega_c \\ 0 \ , \ \omega_c & |\omega| \quad \pi \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\frac{N-1}{2}\omega} e^{j\omega n} d\omega = \frac{\sin \ \omega_c |n - \dfrac{N-1}{2}|}{\pi |n - \dfrac{N-1}{2}|}$$

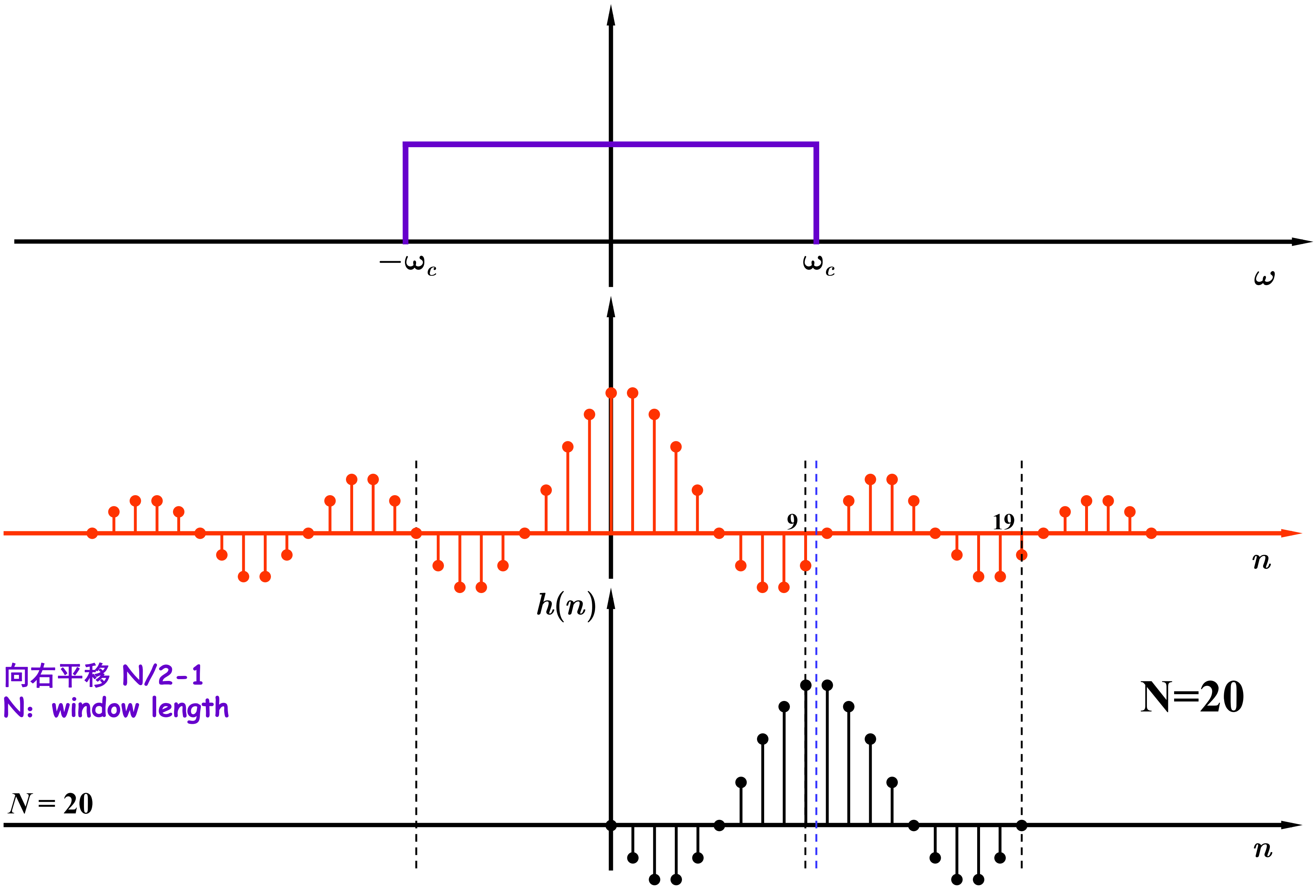
$$h(n) = h_d(n) R_N(n) = \begin{cases} h_d(n) \ , & 0 \quad n \quad N-1 \\ 0 \ , \ \text{其他} & \end{cases}$$

$$H_d'(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c \quad |\omega| \quad \pi \end{cases}$$

$$h_d'(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega = \frac{\sin \omega_c n}{\pi n}$$



EXAMPLE：截止频率为ω_c的线性相位理想低通滤波器： $h(n)$ 偶对称，奇数点



EXAMPLE：截止频率为ω_c的线性相位理想低通滤波器：$h(n)$ 偶对称，偶数点

矩形窗截断的影响：

$$h(n)=h_d(n)R_N(n)$$

$$W_R(e^{j\omega})\Leftrightarrow R_N(n)$$

$$H(e^{j\omega})=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(e^{j\omega})W_R\ \ e^{j(\omega-\theta)}\ \ d\theta$$

$$\begin{cases} W_R(e^{j\omega})=\dfrac{\sin|\dfrac{\omega N}{2}|}{\sin\dfrac{\omega}{2}}e^{-j\omega|\frac{N-1}{2}|}=W_R(\omega)e^{-j\omega\alpha} \\ \alpha=\dfrac{N-1}{2},W_R(\omega)=\dfrac{\sin|\dfrac{\omega N}{2}|}{\sin\dfrac{\omega}{2}} \end{cases}$$

矩形窗截断的影响：

$$
\begin{cases}
H_d(e^{j\omega}) = H_d(\omega)e^{-j\omega\alpha} \\
H_d(\omega) = \begin{cases} 1\ , & |\omega| \le \omega_c \\ 0\ , \quad \omega_c & |\omega| \quad \pi \end{cases},
\end{cases}
$$

$$
H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\theta)e^{-j\theta\alpha}W_R(\omega-\theta)e^{j(\omega-\theta)}d\theta
$$

$$
= e^{-j\omega\alpha}\ \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\theta)W_R(\omega-\theta)d\theta
$$

$$
= H(\omega)e^{-j\omega\alpha}
$$

$$
H(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\theta)W_R(\omega-\theta)d\theta
$$

截断效应：吉布斯现象 用加窗技术减小截断效应

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega}, & |\omega| \le \omega_c \\ 0, & \omega_c \le |\omega| \le \pi \end{cases} = H_d(\omega)e^{-j\alpha\omega}, \alpha = \frac{N-1}{2}$$

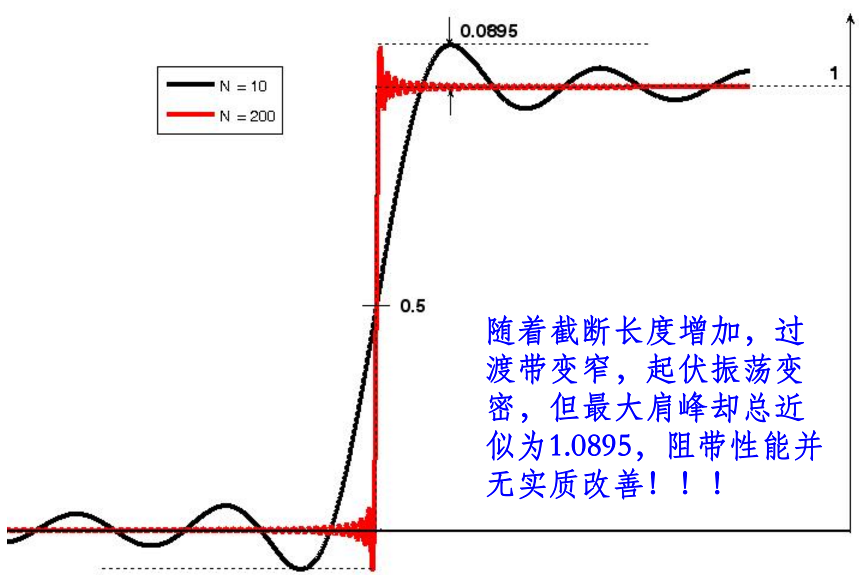
N：窗长

$$H_d(\omega) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c \le |\omega| \le \pi \end{cases}$$

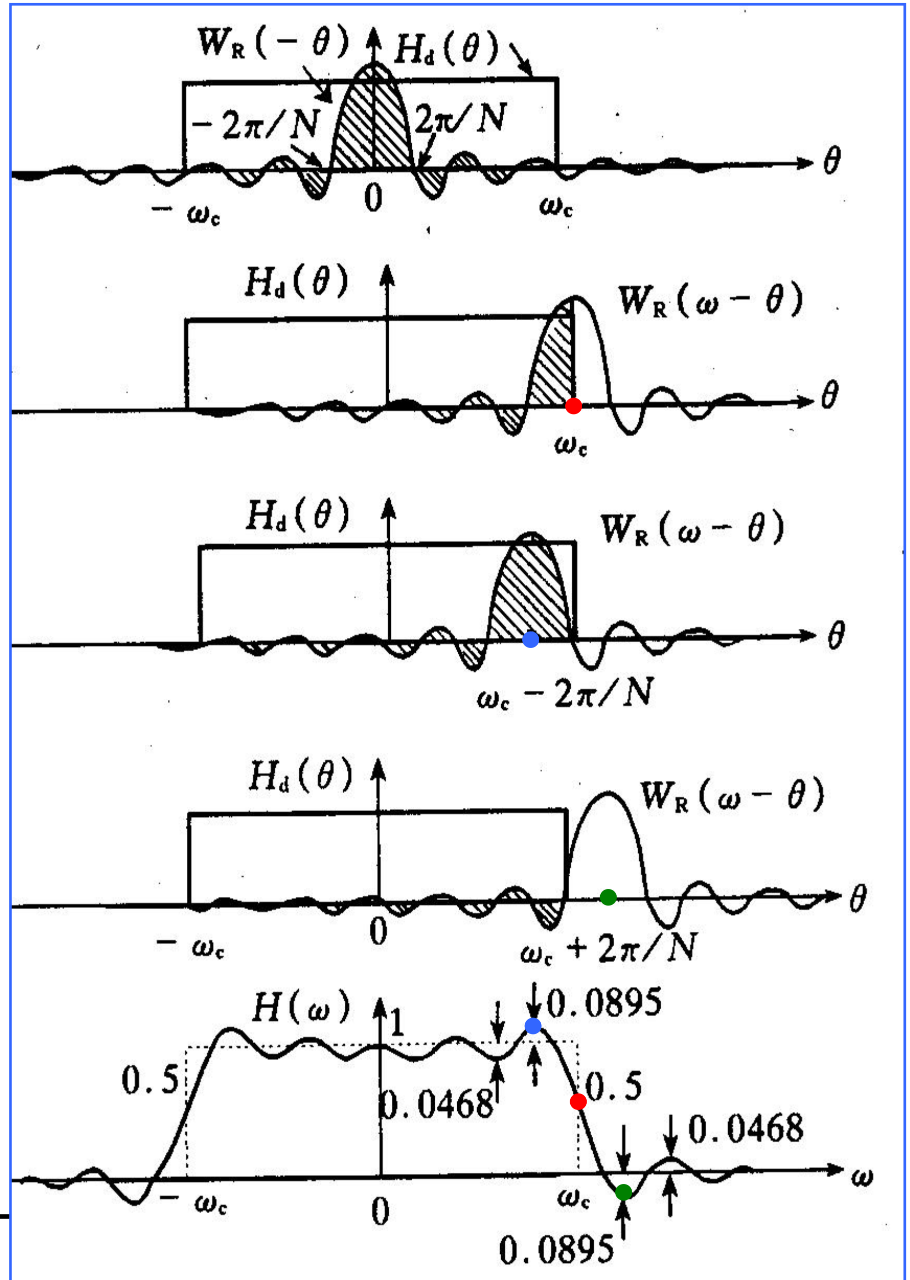
$$W_R(e^{j\omega}) = \frac{\sin(\omega N/2)}{\sin(\omega/2)} e^{-j\alpha\omega} = W_R(\omega)e^{-j\alpha\omega}$$

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta}) W_R[e^{j(\omega-\theta)}]d\theta$$

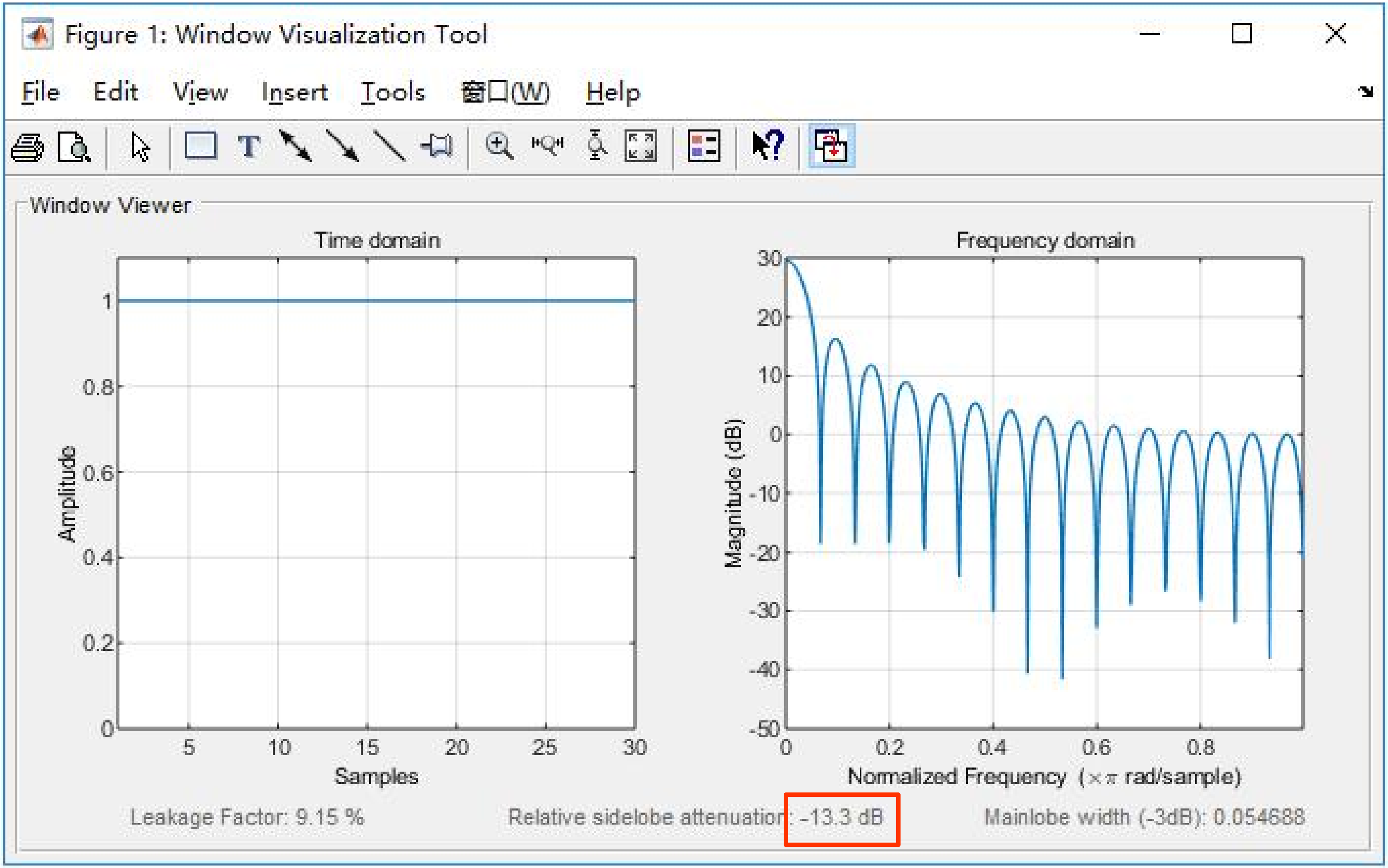
$$= \left[\frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\theta) W_R(\omega-\theta)d\theta\right] e^{-j\alpha\omega} = H(\omega)e^{-j\alpha\omega}$$



随着截断长度增加，过渡带变窄，起伏振荡变密，但最大肩峰却总近似为1.0895，阻带性能并无实质改善！！！

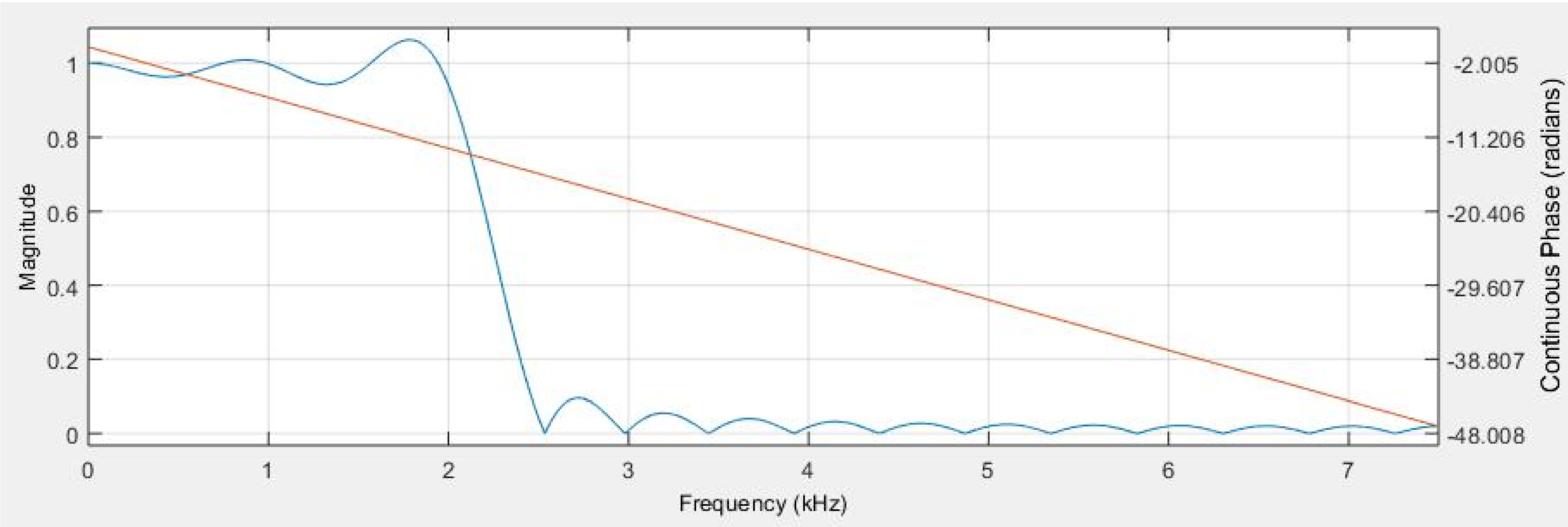


$20\log 0.0895 = -21\text{dB}$

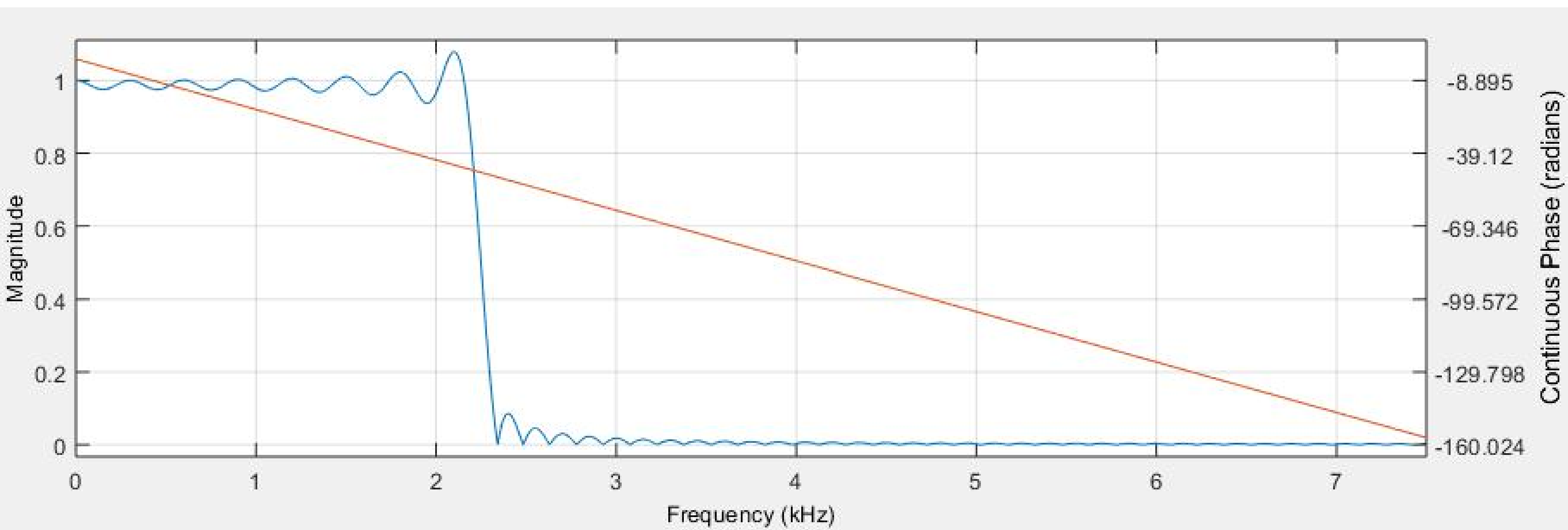


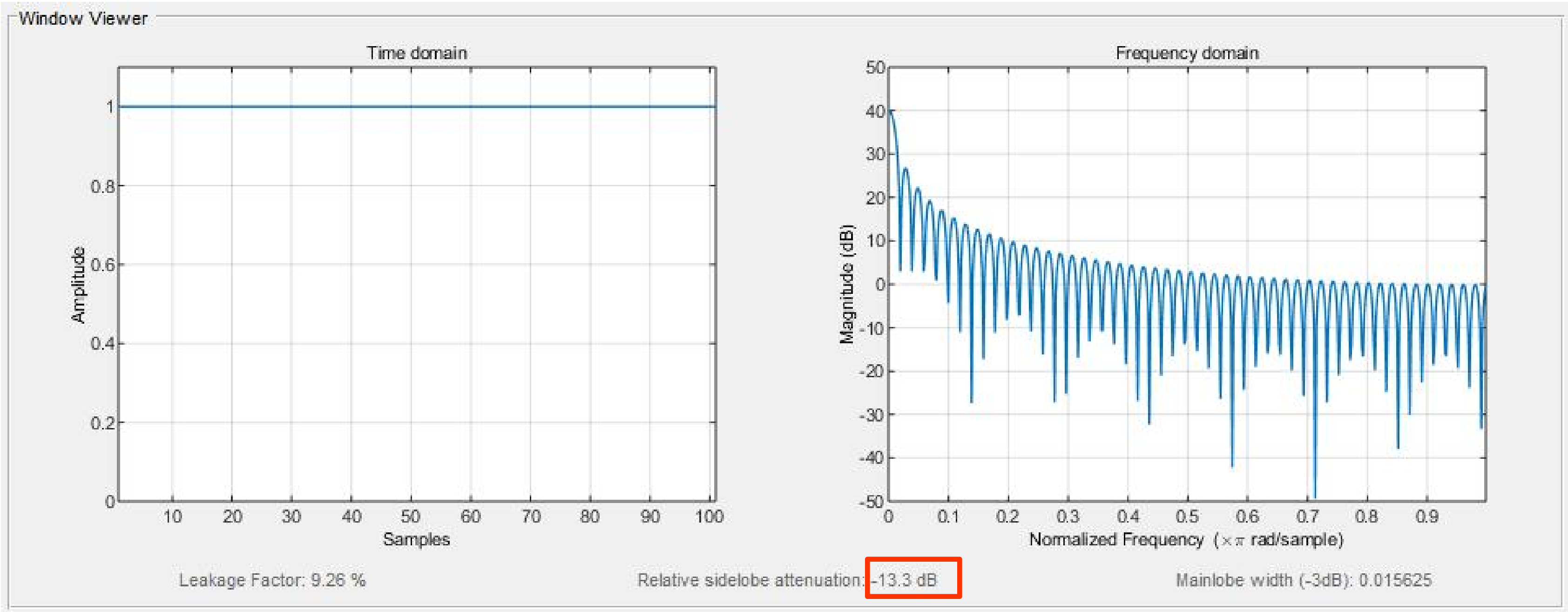
N=31

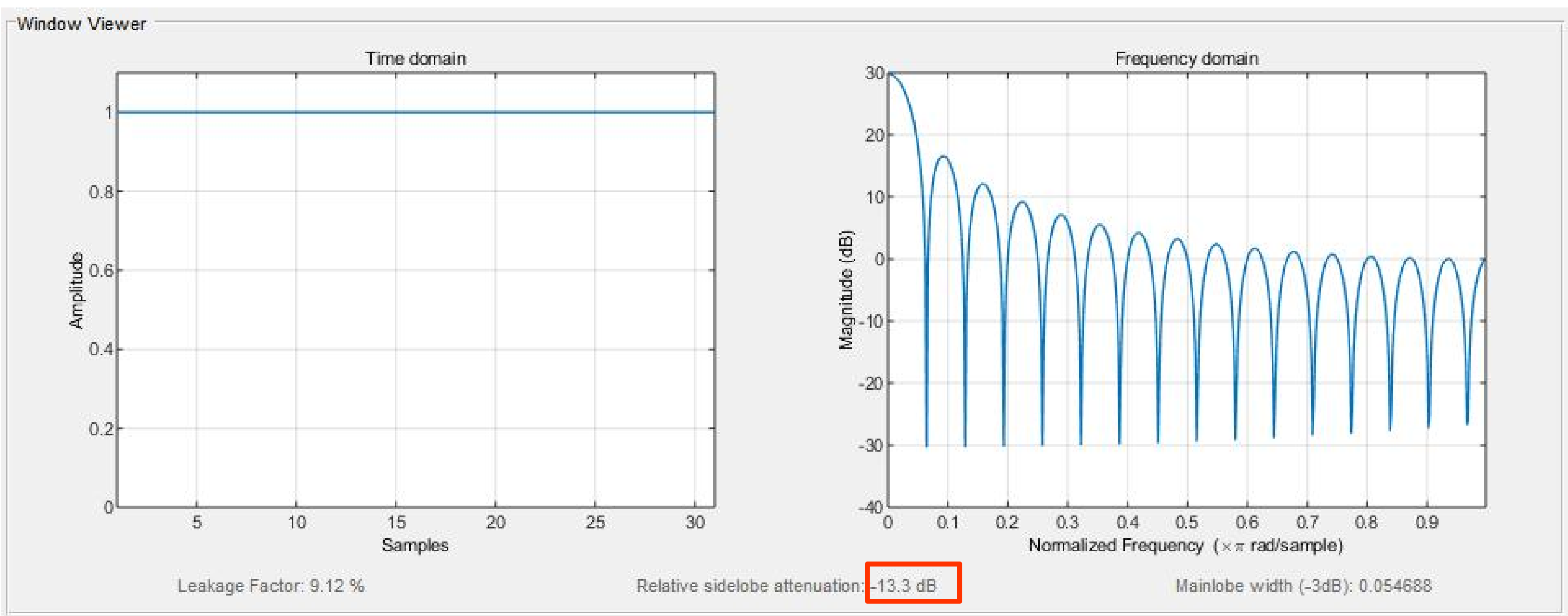
Rectangular



N=101

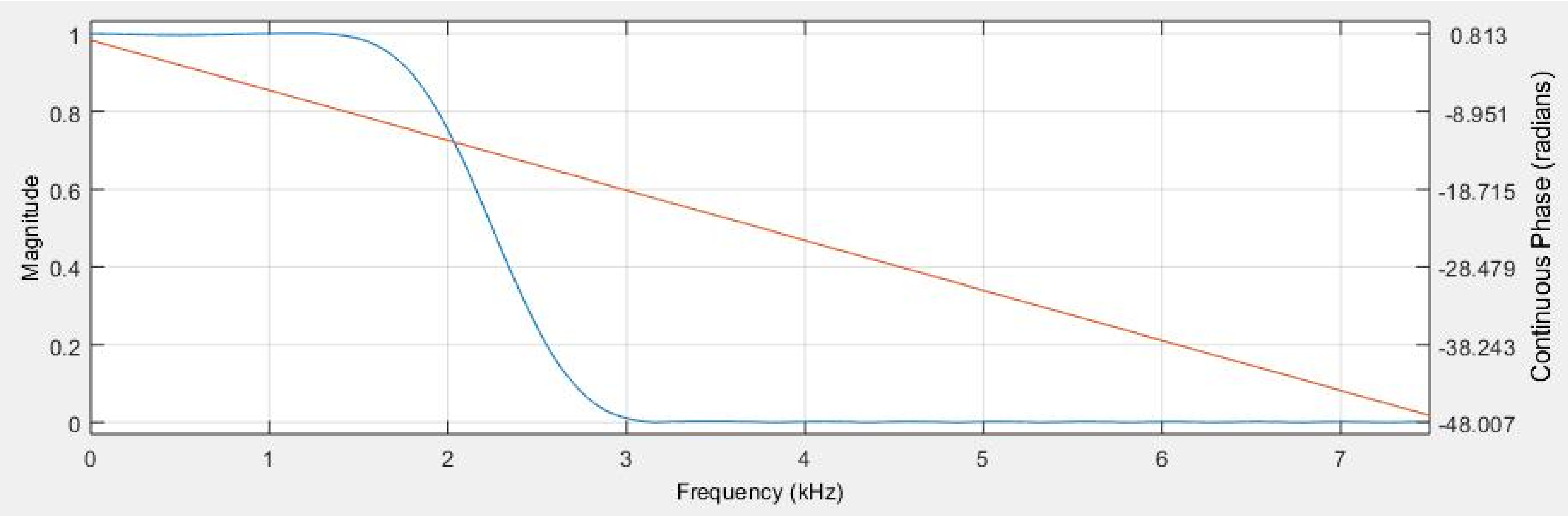



Window Viewer
Time domain
Amplitude
Samples
Frequency domain
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Leakage Factor: 9.26 %
Relative sidelobe attenuation: -13.3 dB
Mainlobe width (-3dB): 0.015625

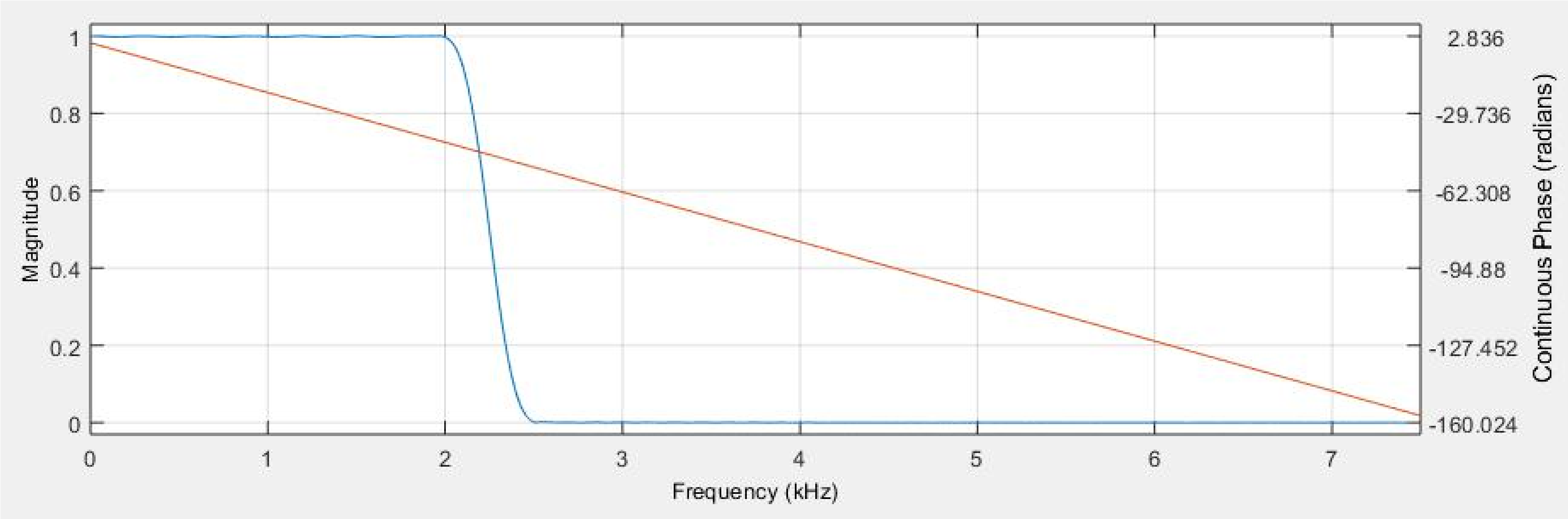

Window Viewer
Time domain
Amplitude
Samples
Frequency domain
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Leakage Factor: 9.12 %
Relative sidelobe attenuation: -13.3 dB
Mainlobe width (-3dB): 0.054688

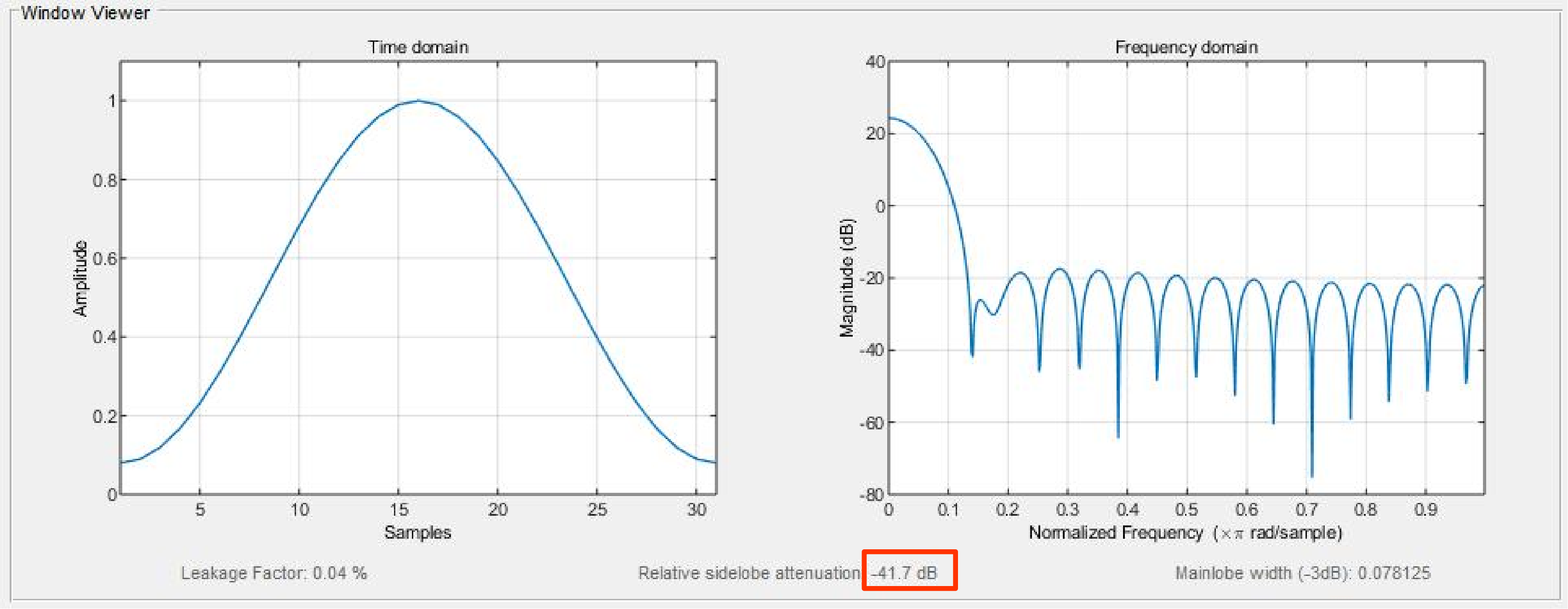
Hamming

N=31

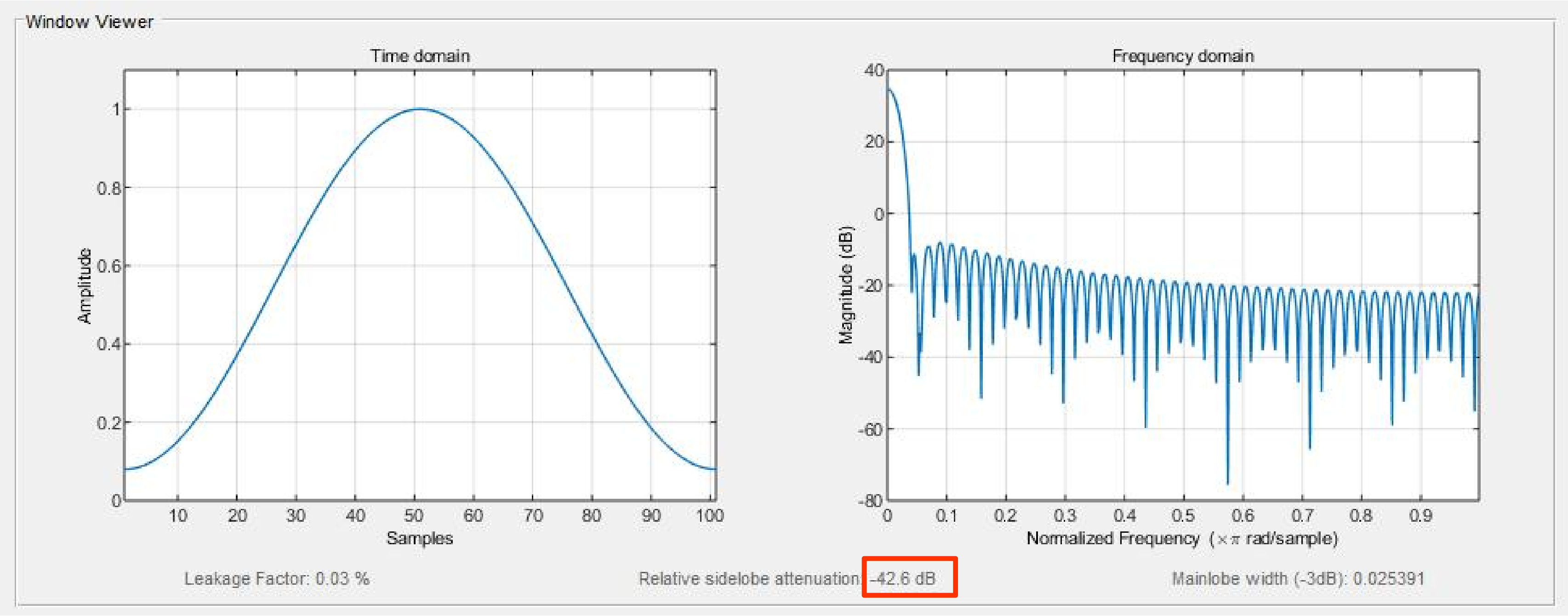


N=101





Window Viewer
Time domain
Amplitude
Samples
Frequency domain
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Leakage Factor: 0.04 %
Relative sidelobe attenuation: -41.7 dB
Mainlobe width (-3dB): 0.078125



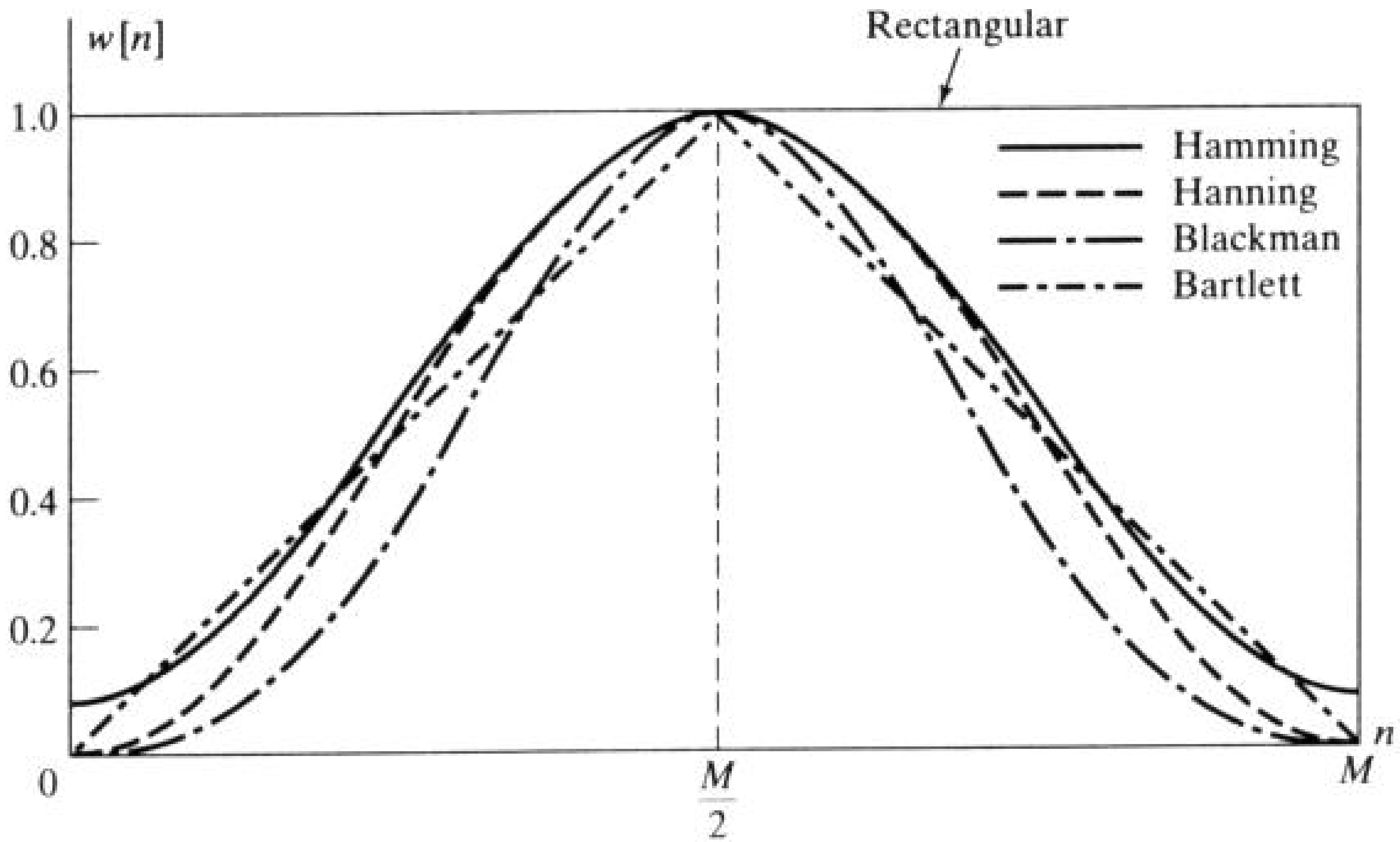
Window Viewer
Time domain
Amplitude
Samples
Frequency domain
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Leakage Factor: 0.03 %
Relative sidelobe attenuation: -42.6 dB
Mainlobe width (-3dB): 0.025391

Window Functions for FIR Filter Design

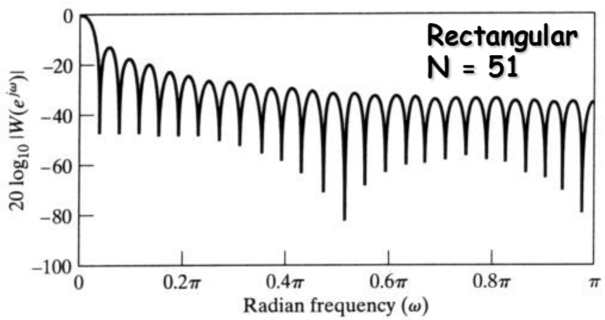
Window Type	Time-Domain Sequence
Rectangular	$w[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$
Bartlett (Triangular)	$w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2 \\ 2-2n/M, & M/2 < n \le M \\ 0, & \text{otherwise} \end{cases}$
Hanning	$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$
Hamming	$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$
Blackman	$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$
Kaiser	$w[n] = \begin{cases} I_0[\beta(1 - \{(n-\alpha)/\alpha\}^2)^{1/2}]/I_0(\beta), & 0 \le n \le M,\ \alpha = M/2 \\ 0, & \text{otherwise} \end{cases}$

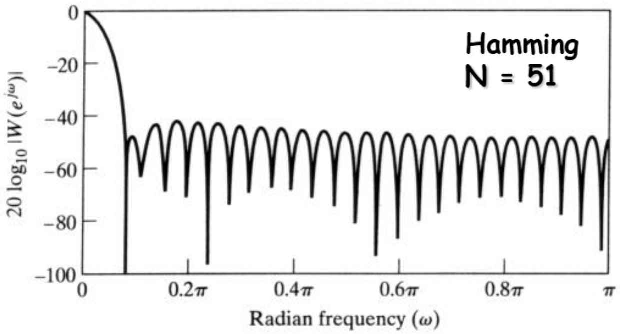
$I_0(.)$ is zero order modified Bessel function of the first kind, β is window shape parameter. $M = N-1$.

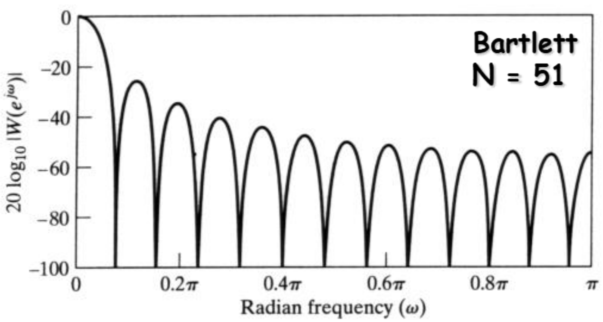
Shape of commonly used window functions

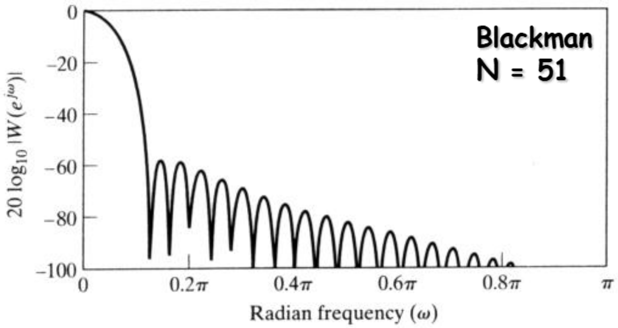


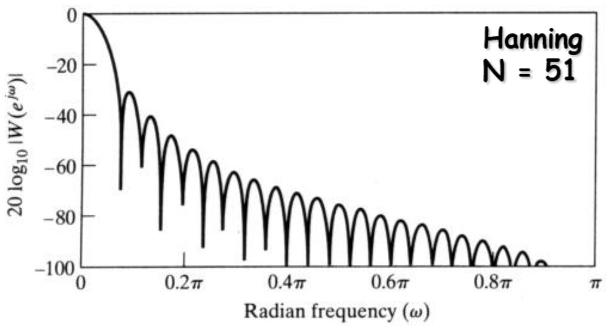
主瓣宽度v.s.副瓣高度

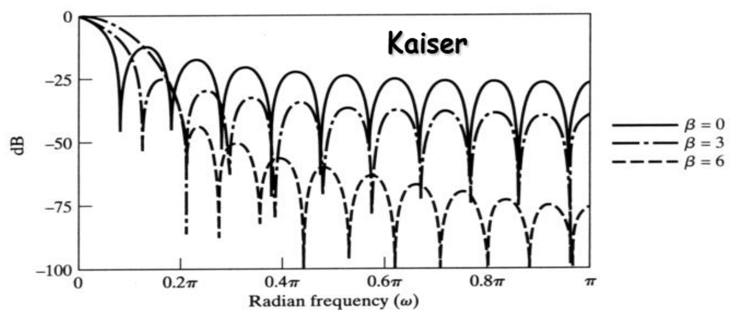

Rectangular
N = 51
20 log10 |W(e^jω)|
Radian frequency (ω)


Hamming
N = 51
20 log10 |W(e^jω)|
Radian frequency (ω)


Bartlett
N = 51
20 log10 |W(e^jω)|
Radian frequency (ω)


Blackman
N = 51
20 log10 |W(e^jω)|
Radian frequency (ω)


Hanning
N = 51
20 log10 |W(e^jω)|
Radian frequency (ω)


Kaiser
dB
Radian frequency (ω)
β = 0
β = 3
β = 6

六种窗函数的基本参数 表 5—7

窗函数	过 渡 带 $\Delta\omega$	最小阻带衰减(dB)
矩形窗	$4\pi/N$	—21
三角窗	$8\pi/N$	—25
汉宁窗	$8\pi/N$	—44
海明窗	$8\pi/N$	—53
布拉克曼窗	$12\pi/N$	—74
凯塞—贝塞尔窗	$10\pi/N$	—80

表 7-3 六种窗函数基本参数的比较

窗函数	窗谱性能指标		加窗后滤波器性能指标	
	旁瓣峰值 /dB	主瓣宽度 /$(2\pi/N)$	过滤带宽 $\Delta\omega$ /$(2\pi/N)$	阻带最小衰减 /dB
矩形窗	—13	2	0.9	—21
三角形窗	—25	4	2.1	—25
汉宁窗	—31	4	3.1	—44
海明窗	—41	4	3.3	—53
布拉克曼窗	—57	6	5.5	—74
凯泽窗 (β=7.865)	—57		5	—80

注意：窗和滤波器不一样

窗是时域h(n)的频域H(e^jw)；
滤波器是窗与理想滤波器频域卷积；

形状不一样：
1，过渡带
2，衰减

三、窗函数方法：Windowing Method

设计步骤：

(1)给定要求的频率响应函数$H_d(e^{j\omega})$

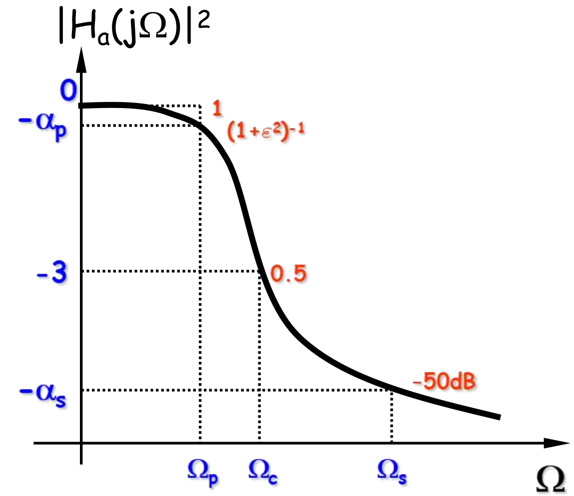
(2)根据$h_d(n)=\dfrac{1}{2\pi}\int_{-\pi}^{\pi}H_d(e^{j\omega})e^{j\omega n}d\omega$计算$h_d(n)$

(3)根据过渡带及阻带最小衰减要求，选定窗和N

(4)根据$h(n)=h_d(n)R_N(n)$求得$h(n)$

线性相位FIR低通滤波器的设计

例： 设计一个线性相位FIR低通滤波器，满足下列条件：抽样频率为15KHz；通带截止频率为1.5KHz；阻带起始频率为3KHz；阻带衰减不小于50dB，幅度特性如右图所示



解： 1）确定模拟指标对应的数字频率

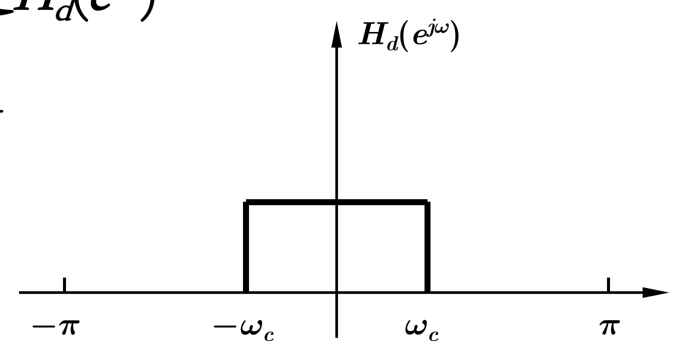
$$\omega_p = 2\pi f_p / F_s = 0.2\pi ; \omega_s = 2\pi f_s / F_s = 0.4\pi$$

$$\alpha_s = -50\text{dB}$$

2）根据过渡带设定选截止频率ω_c，由此确定$H_d(e^{j\omega})$

$$\omega_c = 2\pi \tfrac{1}{2}(f_p + f_s) / F_s = 0.3\pi; \tau = \frac{N-1}{2}$$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & -\omega_c \quad \omega \quad \omega_c \\ 0 & -\pi \quad \omega \quad -\omega_c, \omega_c \quad \omega \quad \pi \end{cases}$$

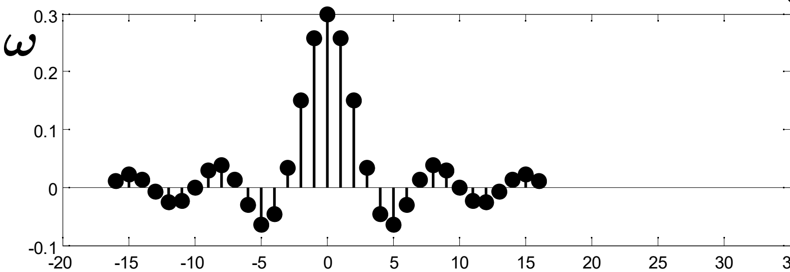


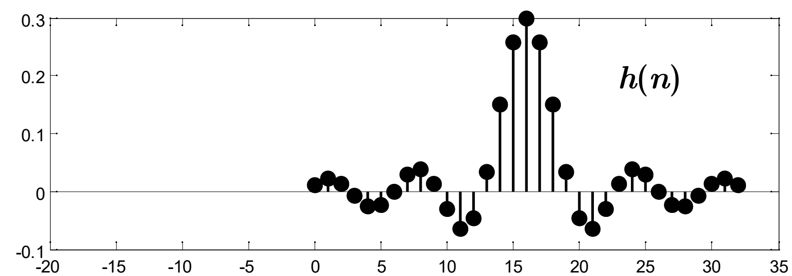
线性相位FIR低通滤波器的设计

$$h_d(n) = IDTFT\{H_d(e^{j\omega})\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega\tau} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(n-\tau)} d\omega$$

$$= \begin{cases} \dfrac{1}{\pi(n-\tau)}\sin[\omega_c(n-\tau)] & n \neq \tau \\ \dfrac{\omega_c}{\pi} & n = \tau \end{cases}$$





3） 求$h_d(n)$

4）据阻带衰减要求选择窗函数：由$\alpha_s = 50$dB，确定海明窗（-53dB）

$$w(n) = 0.54 - 0.46\cos\left|\frac{2\pi n}{N-1}\right| R_N(n)$$

5）据过渡带宽要求确定窗长N（海明窗：$\Delta\omega = 6.6\pi/N$）

$$\Delta\omega = 2\pi(f_s - f_p)/F_s = 0.2\pi;\ N = A/\Delta\omega = 6.6\pi/0.2\pi = 33$$

线性相位FIR低通滤波器的设计

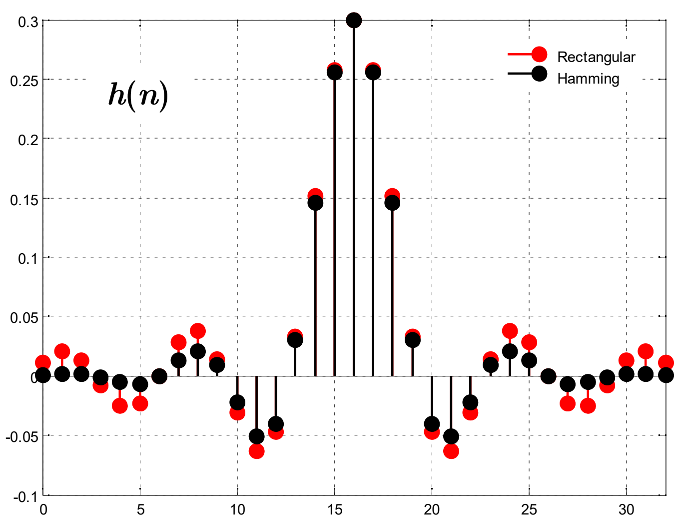
6）确定FIR滤波器的单位抽样响应$h(n)$

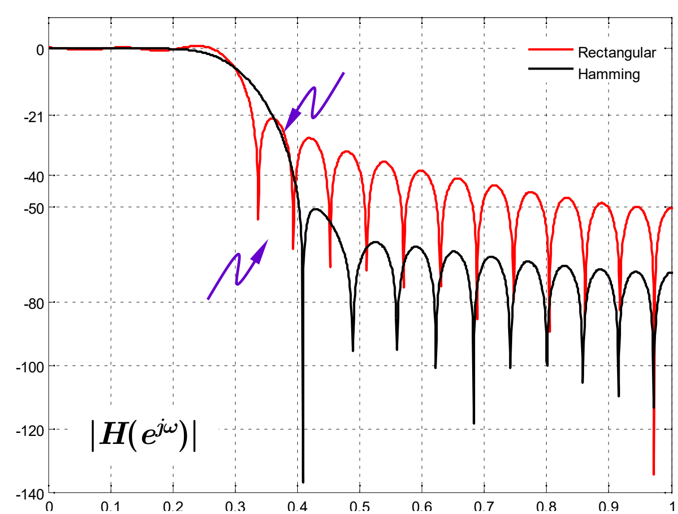
$$\tau = (N-1)/2 = 16$$

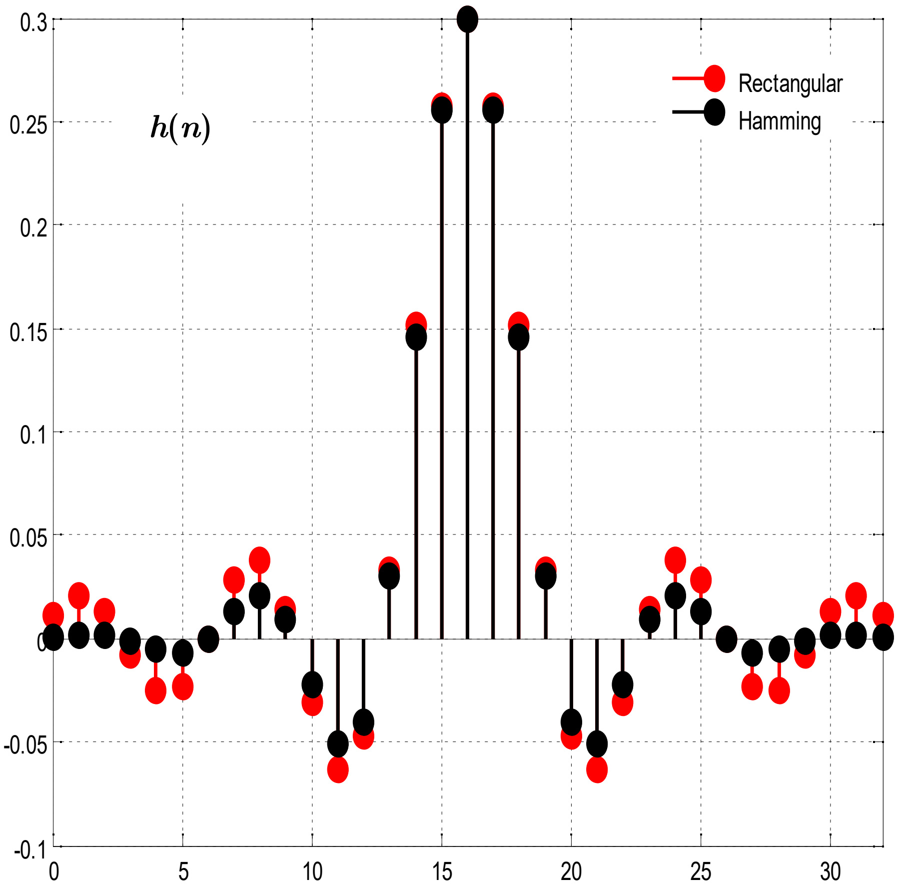
$$h(n) = h_d(n)w(n)$$

$$= \frac{\sin\left[0.3\pi(n-16)\right]}{\pi(n-16)} \cdot \left[0.54 - 0.46\cos\frac{\pi n}{16}\right] R_{33}(n)$$

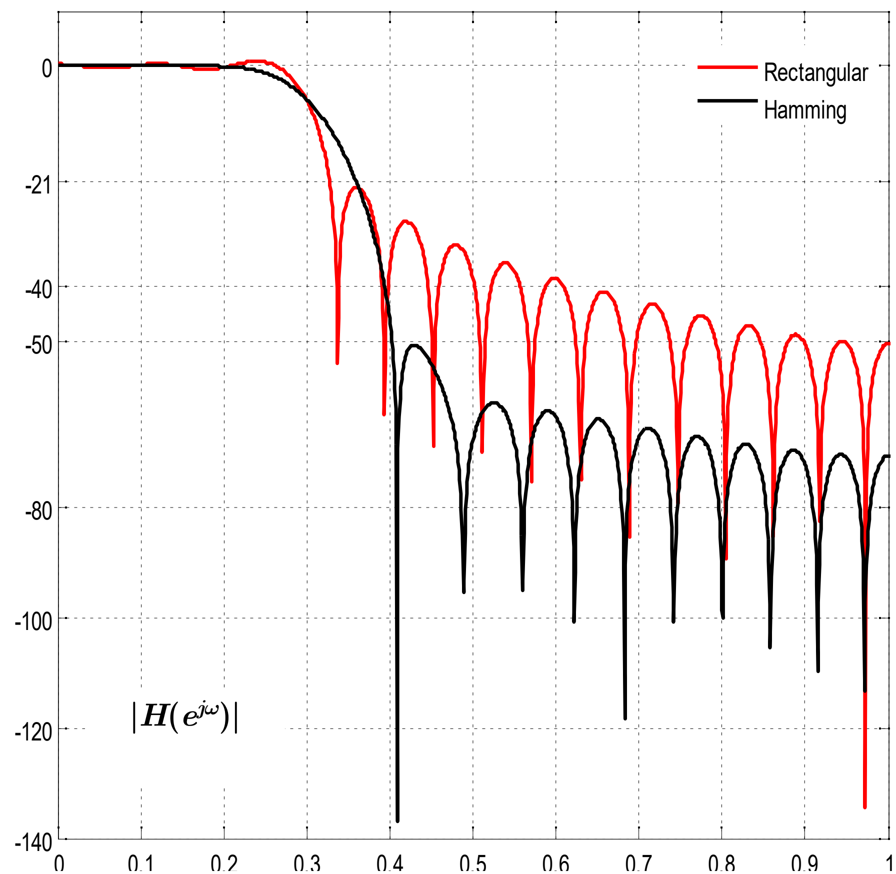
7）求$H(e^{j\omega})$，并检验性能是否满足预定指标。若不满足，则改变N或窗形状重新设计



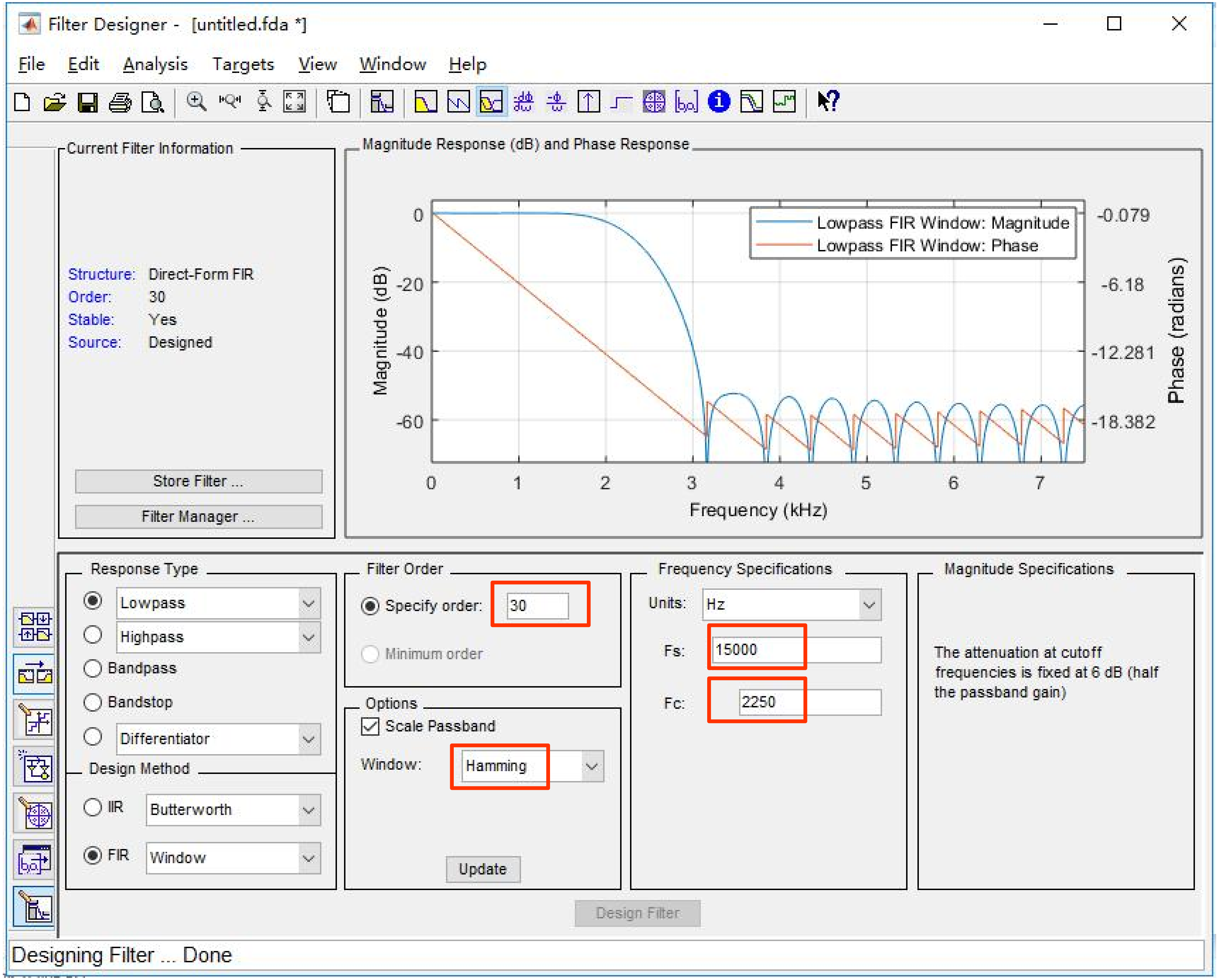


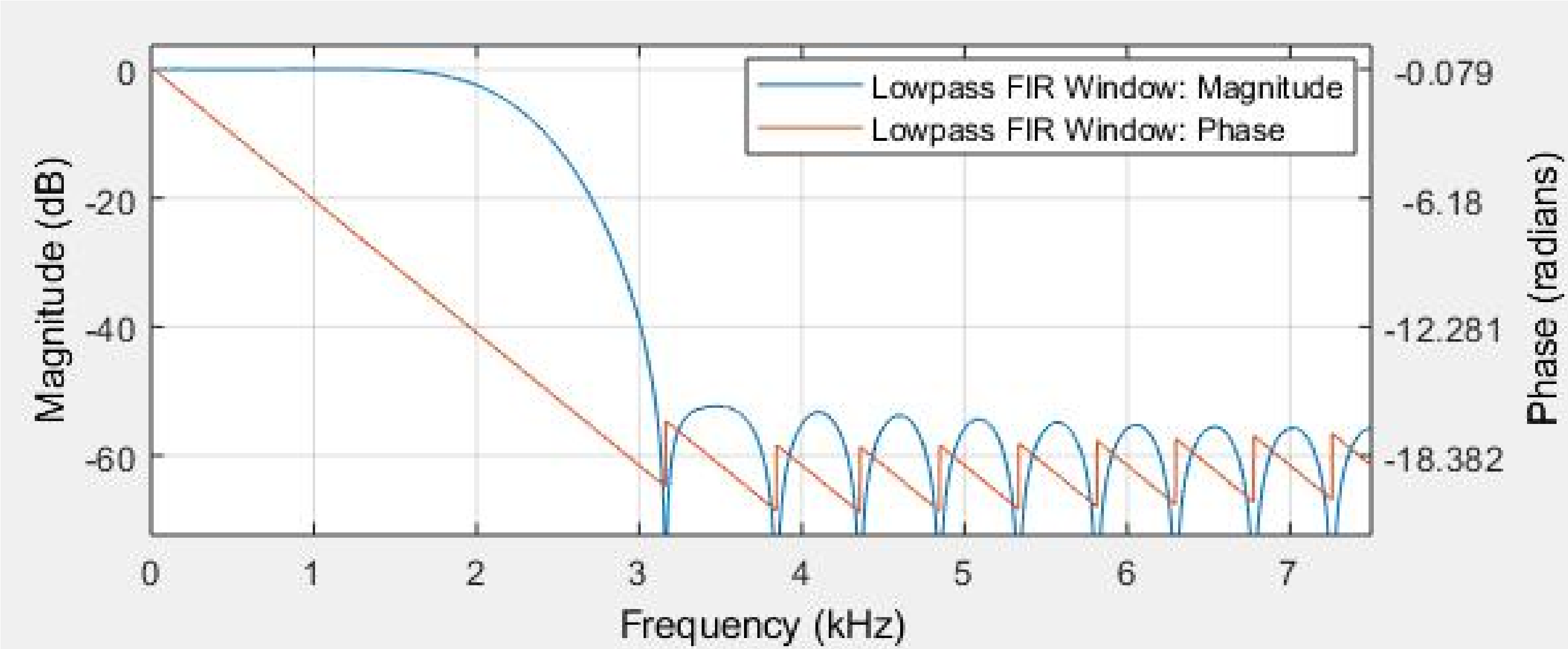


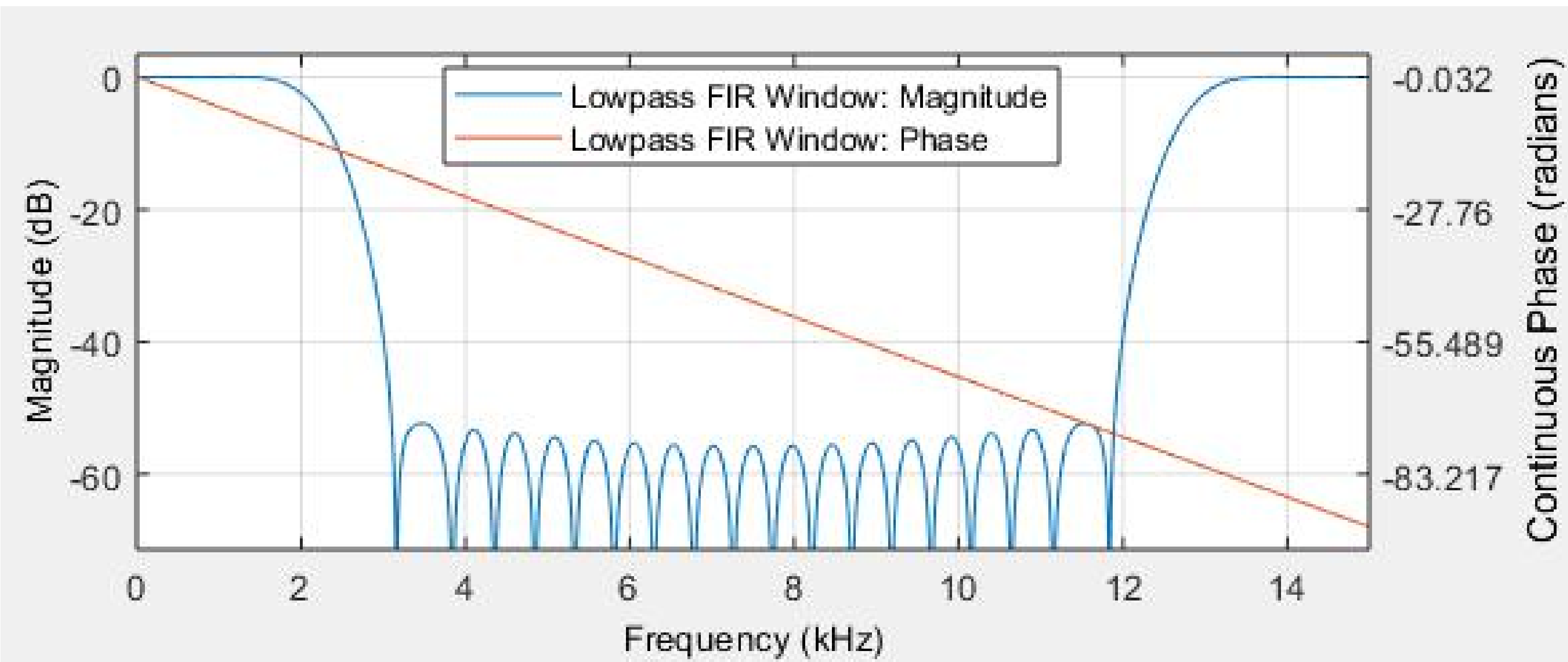
$h(n)$
Rectangular
Hamming
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
5
10
15
20
25
30

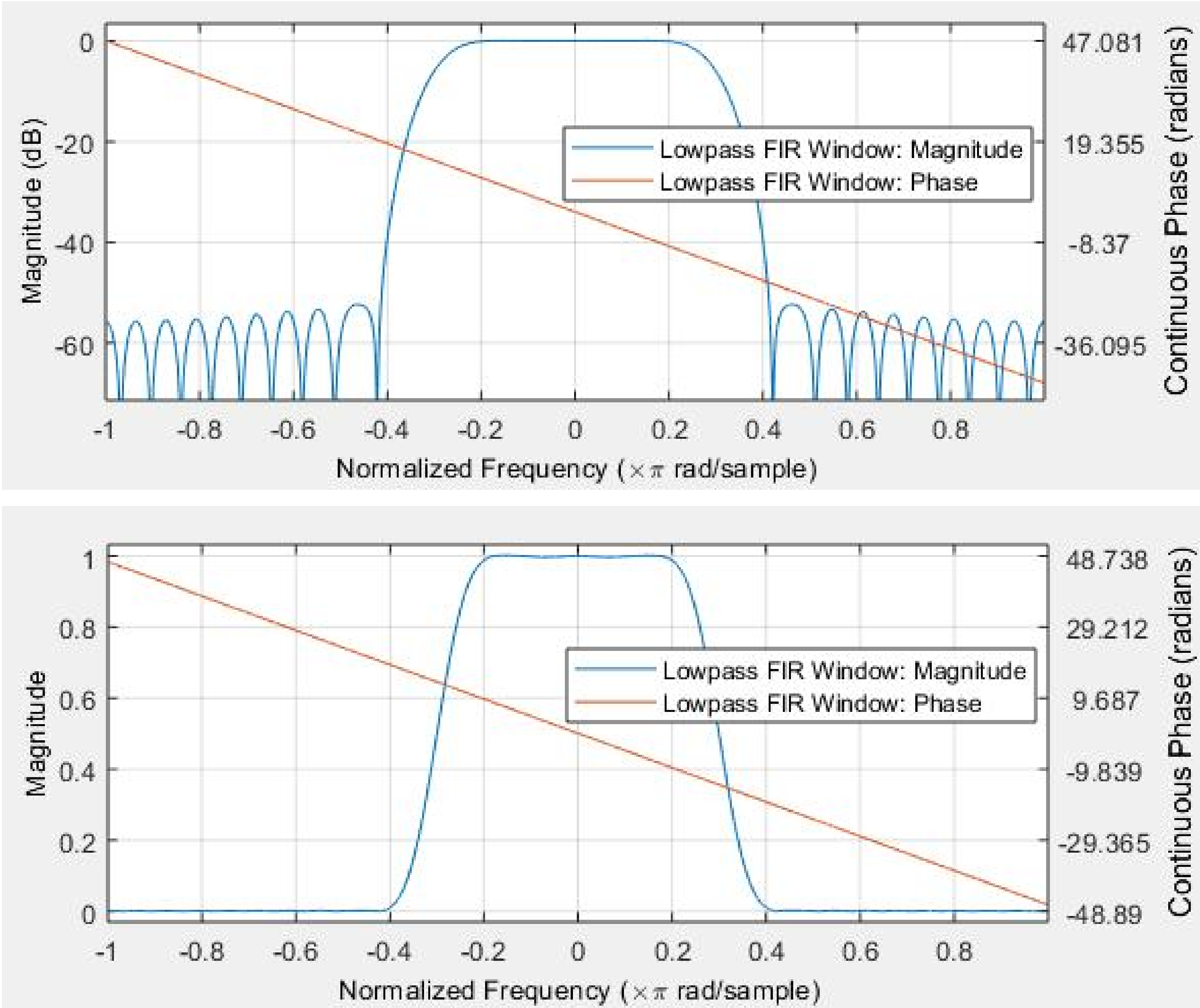


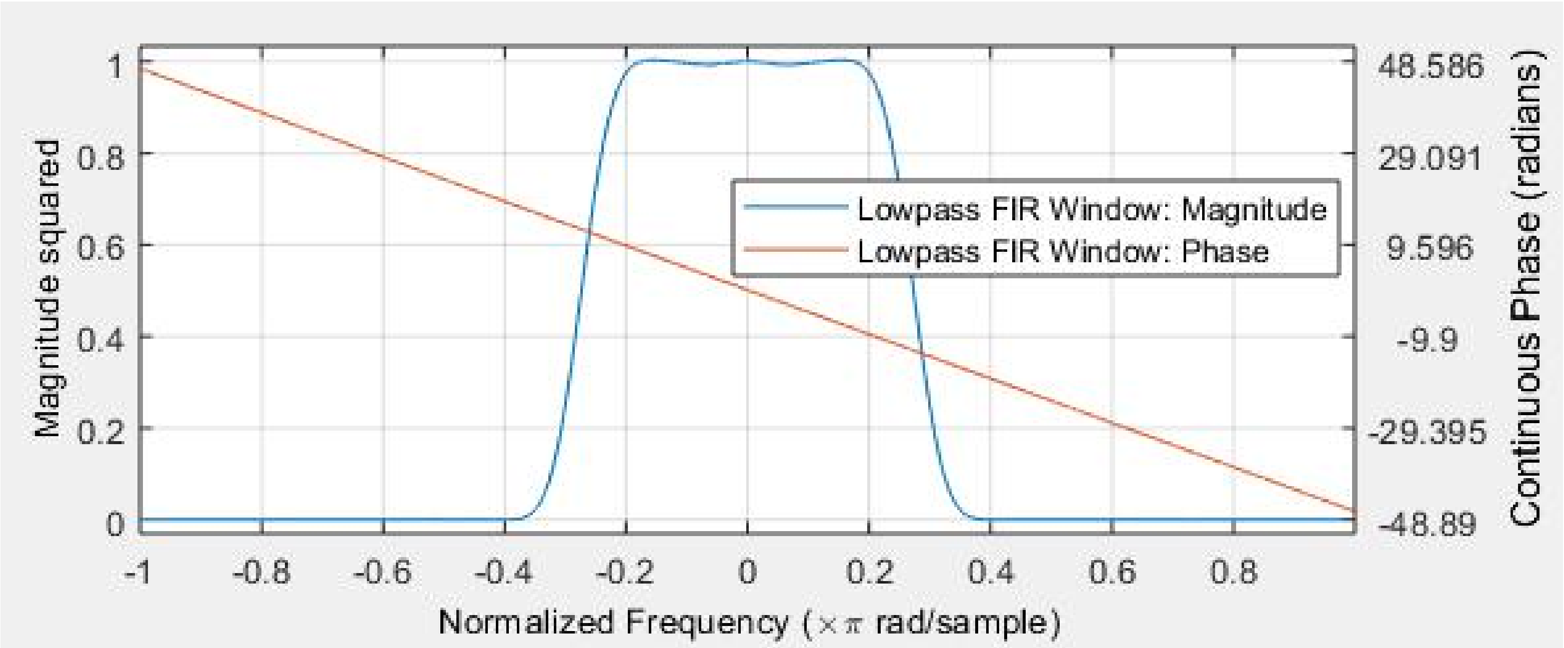
$|H(e^{j\omega})|$
Rectangular
Hamming
0
-21
-40
-50
-80
-100
-120
-140
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 30
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB) and Phase Response
Lowpass FIR Window: Magnitude
Lowpass FIR Window: Phase
Magnitude (dB)
Phase (radians)
Frequency (kHz)
0
-20
-40
-60
-0.079
-6.18
-12.281
-18.382
0 1 2 3 4 5 6 7
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Window
Filter Order
Specify order: 30
Minimum order
Options
Scale Passband
Window: Hamming
Update
Frequency Specifications
Units: Hz
Fs: 15000
Fc: 2250
Magnitude Specifications
The attenuation at cutoff frequencies is fixed at 6 dB (half the passband gain)
Design Filter
Designing Filter ... Done

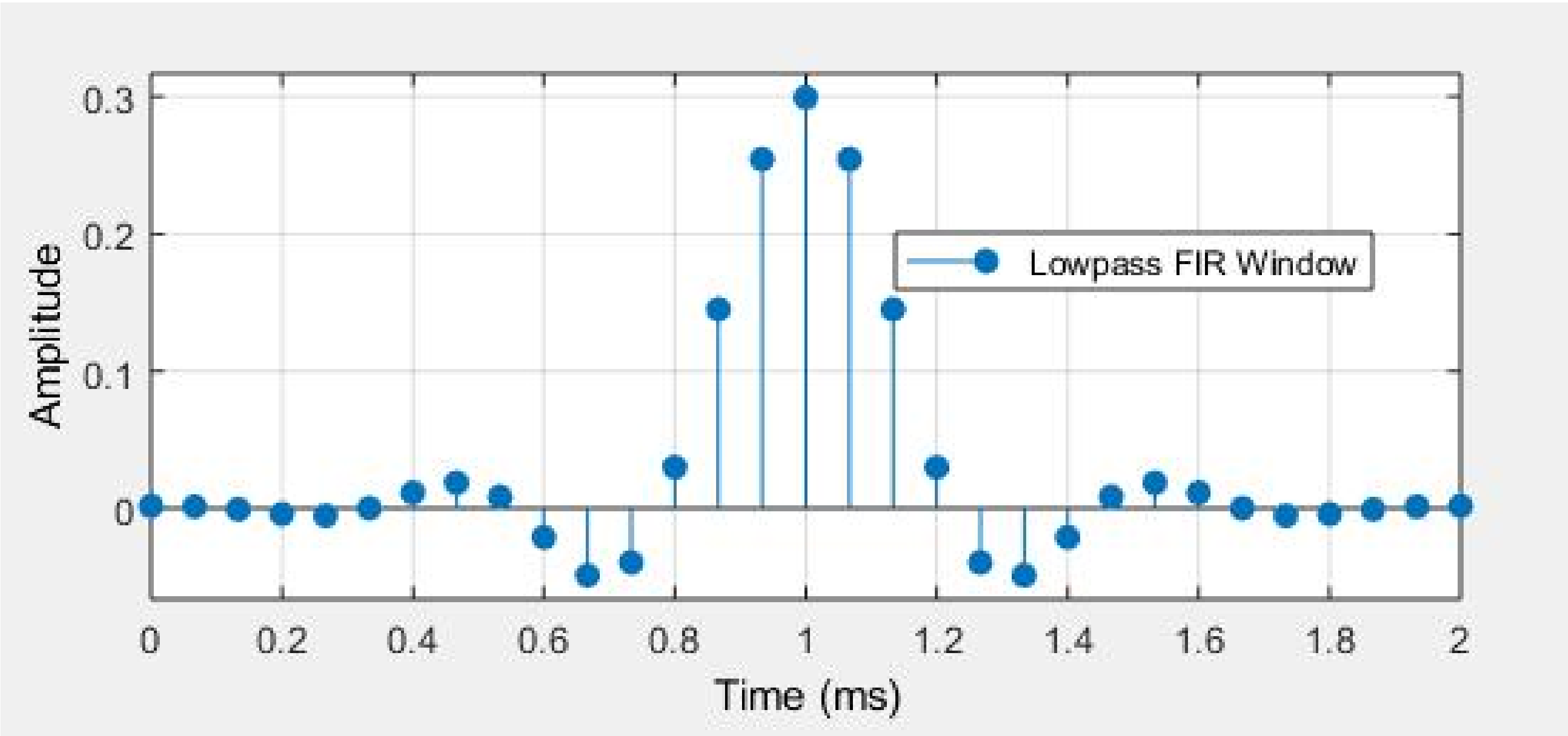

Lowpass FIR Window: Magnitude
Lowpass FIR Window: Phase
Magnitude (dB)
Phase (radians)
Frequency (kHz)
0
-20
-40
-60
-0.079
-6.18
-12.281
-18.382
0 1 2 3 4 5 6 7


Lowpass FIR Window: Magnitude
Lowpass FIR Window: Phase
Magnitude (dB)
Continuous Phase (radians)
Frequency (kHz)
0
-20
-40
-60
-0.032
-27.76
-55.489
-83.217
0 2 4 6 8 10 12 14


Magnitude (dB)
Continuous Phase (radians)
Lowpass FIR Window: Magnitude
Lowpass FIR Window: Phase
0
-20
-40
-60
47.081
19.355
-8.37
-36.095
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Normalized Frequency (×π rad/sample)
Magnitude
Continuous Phase (radians)
Lowpass FIR Window: Magnitude
Lowpass FIR Window: Phase
1
0.8
0.6
0.4
0.2
0
48.738
29.212
9.687
-9.839
-29.365
-48.89
Normalized Frequency (×π rad/sample)



Lowpass FIR Window: Magnitude
Lowpass FIR Window: Phase
Magnitude squared
Continuous Phase (radians)
Normalized Frequency (×π rad/sample)
1
0.8
0.6
0.4
0.2
0
48.586
29.091
9.596
-9.9
-29.395
-48.89
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8



Lowpass FIR Window
Amplitude
Time (ms)
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

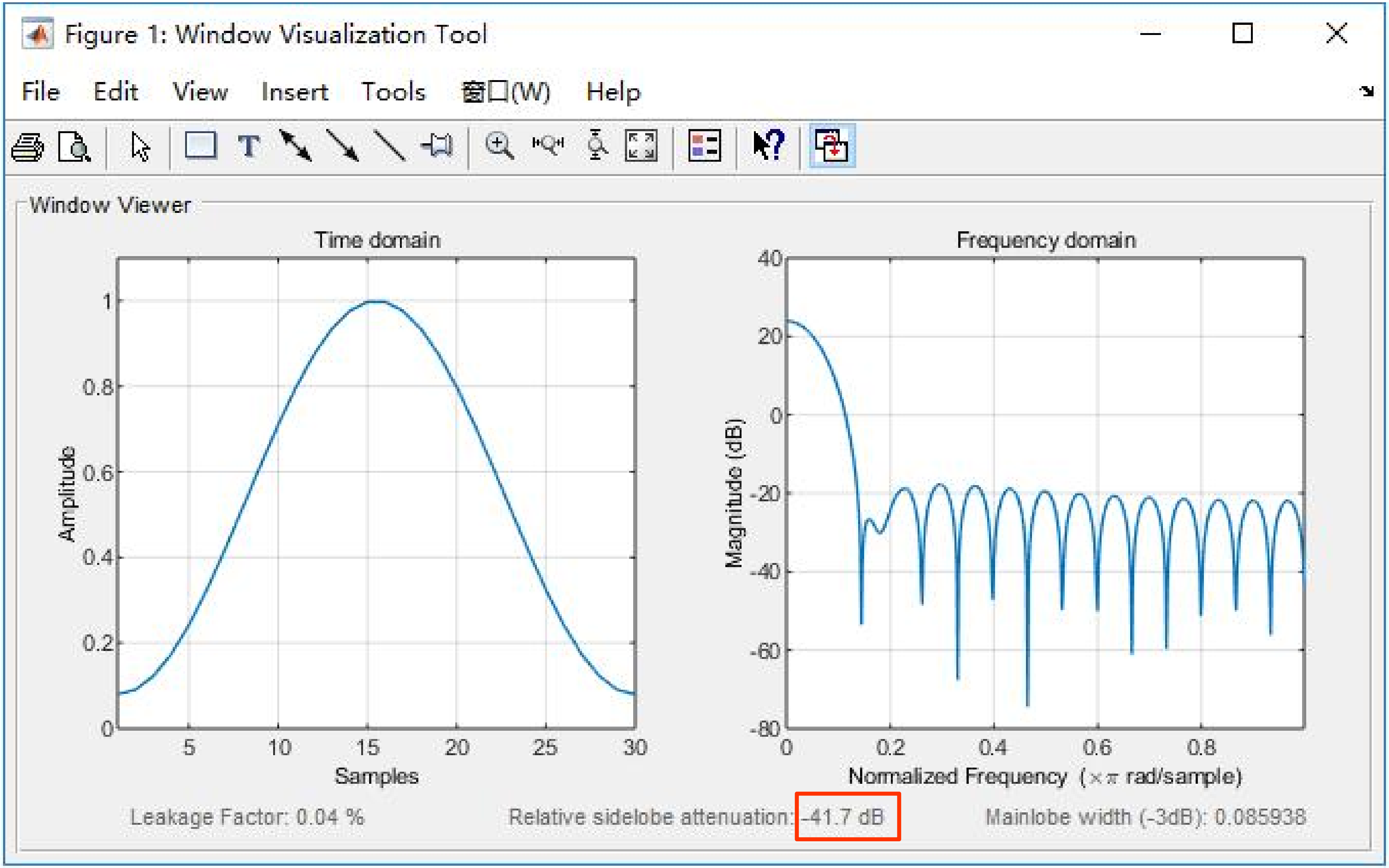
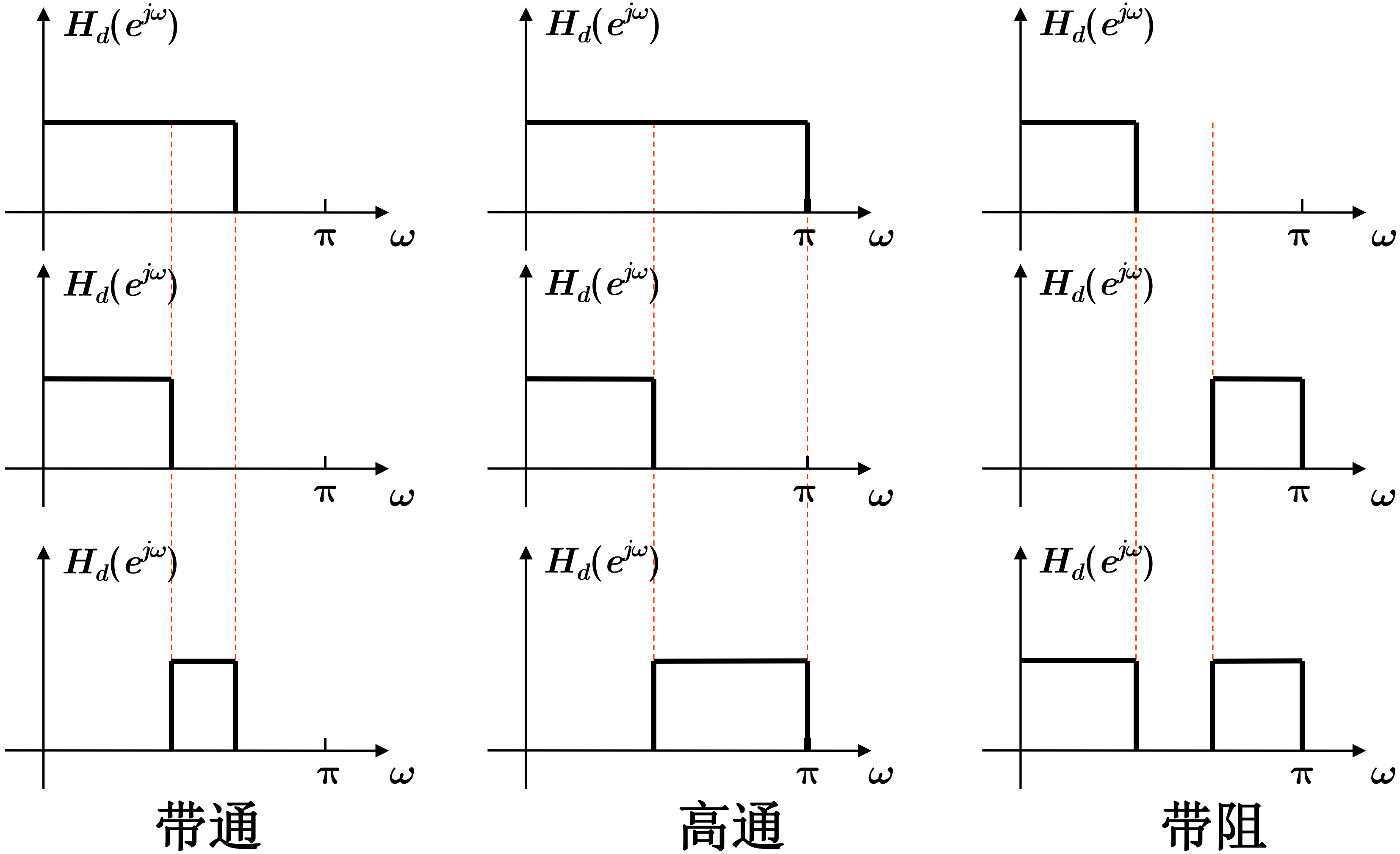


Figure 1: Window Visualization Tool
File Edit View Insert Tools 窗口(W) Help
Window Viewer
Time domain
Frequency domain
Amplitude
Samples
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Leakage Factor: 0.04 %
Relative sidelobe attenuation: -41.7 dB
Mainlobe width (-3dB): 0.085938

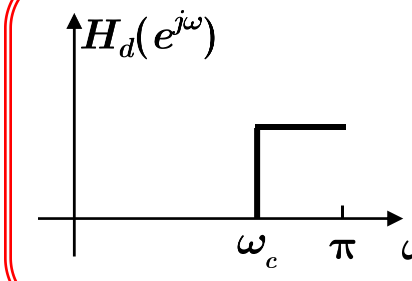
From low pass to band pass, high pass, band stop ...



线性相位FIR高通滤波器的设计公式

— 理想高通的频响：

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & \omega_c \quad |\omega| \quad \pi \\ 0 & otherwise \end{cases} \qquad \tau = \frac{N-1}{2}$$



其单位抽样响应：

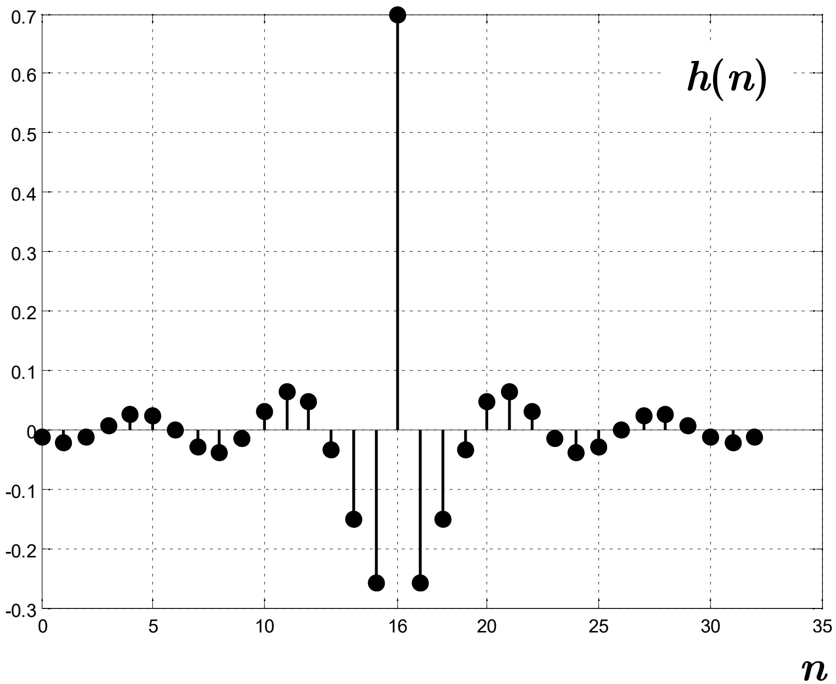
$$h_d(n) = \frac{1}{2\pi}\left[{}_{-\pi}^{-\omega_c} e^{j\omega(n-\tau)}d\omega + {}_{\omega_c}^{\pi} e^{j\omega(n-\tau)}d\omega \right]$$

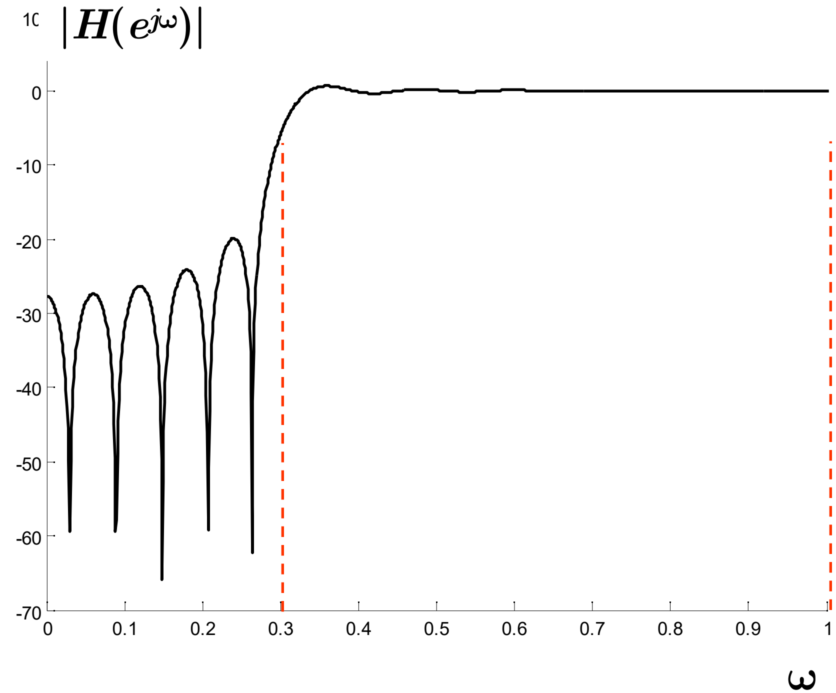
$$= \begin{cases} \dfrac{1}{\pi(n-\tau)}\left\{\sin\left[\pi(n-\tau)\right] - \sin\left[\omega_c(n-\tau)\right]\right\} & n \neq \tau \\ \dfrac{1}{\pi}(\pi - \omega_c) & n = \tau \end{cases}$$

$$\text{高通滤波器}(\omega_c) = \text{全通滤波器} - \text{低通滤波器}(\omega_c)$$

线性相位FIR高通滤波器的设计公式

$$\frac{\sin[\pi(n-16)]-\sin[0.3\pi(n-16)]}{\pi(n-16)}$$





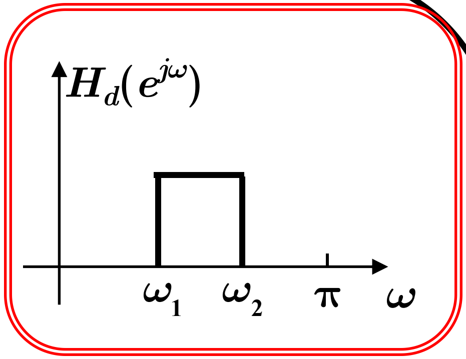
N=33

0.3π

线性相位FIR带通滤波器的设计公式

— 理想带通的频响：

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & 0 < \omega_1 \quad |\omega| \quad \omega_2 < \pi \\ 0 & otherwise \end{cases} \qquad \tau = \frac{N-1}{2}$$



其单位抽样响应：

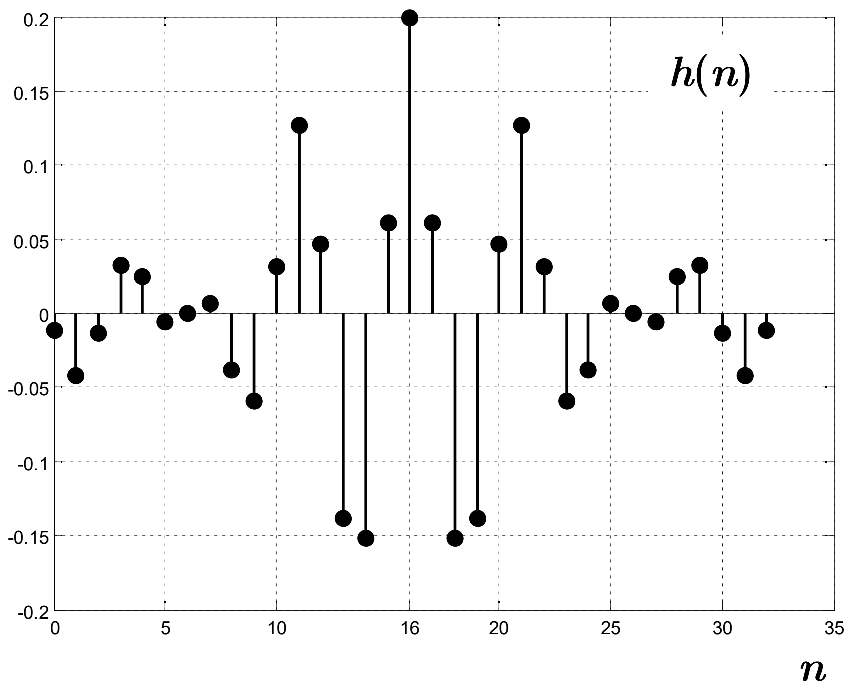
$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_2}^{-\omega_1} e^{j\omega(n-\tau)}d\omega + \int_{\omega_1}^{\omega_2} e^{j\omega(n-\tau)}d\omega$$

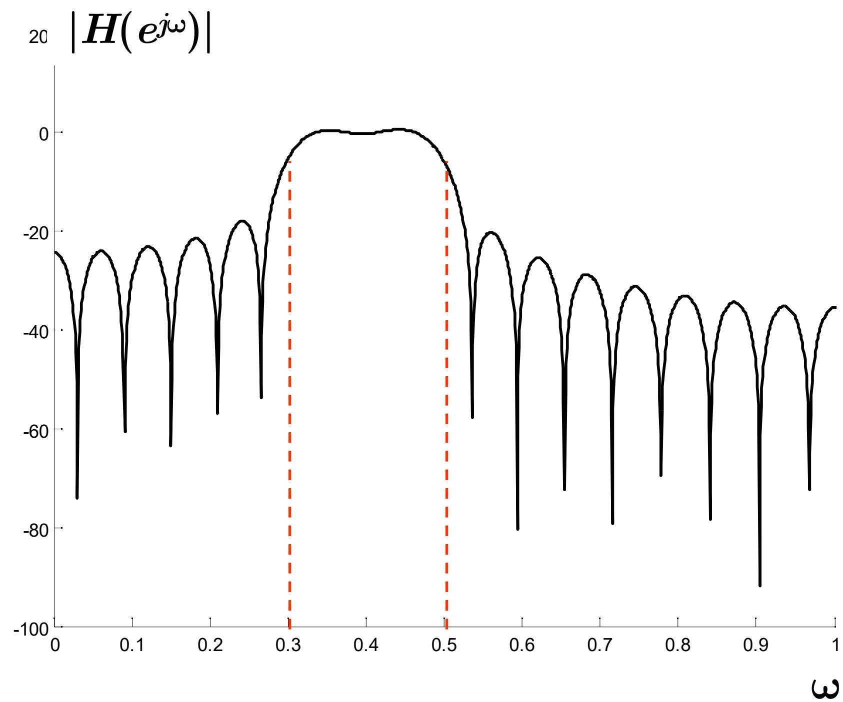
$$= \begin{cases} \dfrac{1}{\pi(n-\tau)}\left\{\sin\ \omega_2(n-\tau) - \sin\ \omega_1(n-\tau)\right\} & n \neq \tau \\ \dfrac{1}{\pi}(\omega_2 - \omega_1) & n = \tau \end{cases}$$

带通滤波器(ω_1, ω_2) = 低通滤波器(ω_2) − 低通滤波器(ω_1)

线性相位FIR带通滤波器的设计公式

$$\frac{\sin[0.5\pi(n-16)]-\sin[0.3\pi(n-16)]}{\pi(n-16)}$$





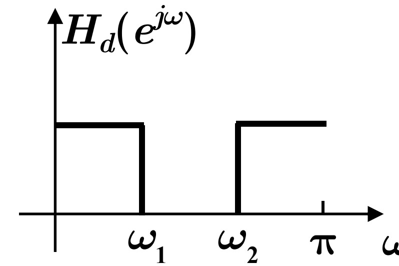
N=33

0.3π, 0.5π

线性相位FIR带阻滤波器的设计公式

— 理想带阻的频响：

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & 0 \quad |\omega| \quad \omega_1, \omega_2 \quad |\omega| \quad \pi \\ 0 & otherwise \end{cases} \qquad \tau = \frac{N-1}{2}$$



其单位抽样响应：

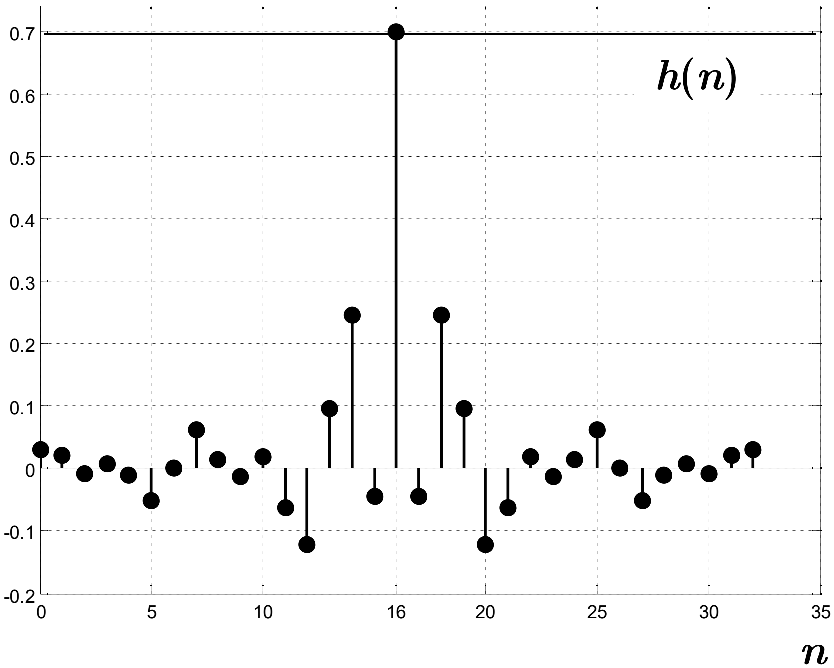
$$h_d(n) = \frac{1}{2\pi}\left[{}_{-\pi}^{-\omega_2} e^{j\omega(n-\tau)} d\omega + {}_{-\omega_1}^{\omega_1} e^{j\omega(n-\tau)} d\omega + {}_{\omega_2}^{\pi} e^{j\omega(n-\tau)} d\omega \right]$$

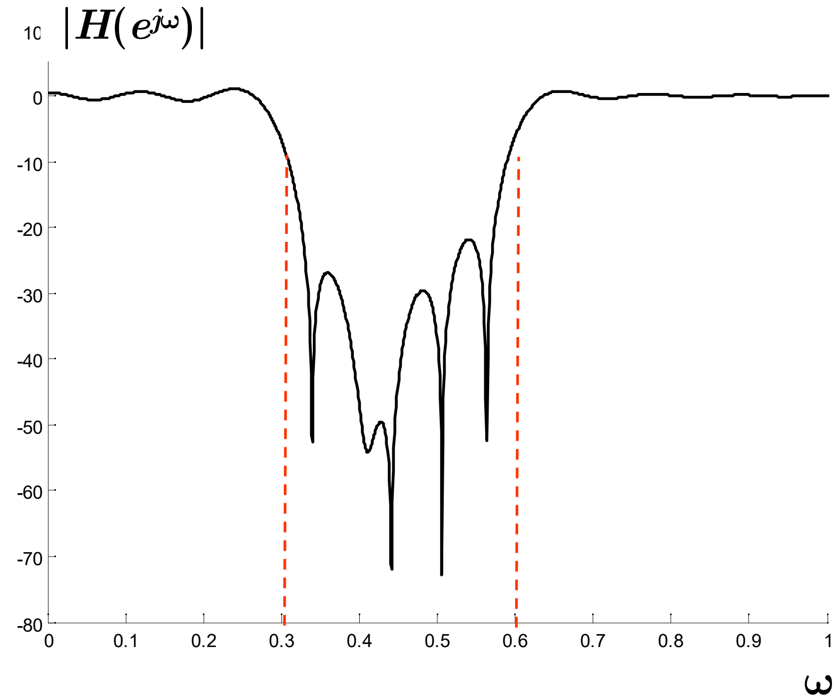
$$= \begin{cases} \dfrac{1}{\pi(n-\tau)}\left\{\sin[\pi(n-\tau)] + \sin[\omega_1(n-\tau)] - \sin[\omega_2(n-\tau)]\right\} & n \neq \tau \\ \dfrac{1}{\pi}(\pi + \omega_1 - \omega_2) & n = \tau \end{cases}$$

$$\text{带阻滤波器}(\omega_1, \omega_2) = \text{高通滤波器}(\omega_2) + \text{低通滤波器}(\omega_1)$$

线性相位FIR带阻滤波器的设计公式

$$\frac{\sin[\pi(n-16)]+\sin[0.3\pi(n-16)]-\sin[0.6\pi(n-16)]}{\pi(n-16)}$$



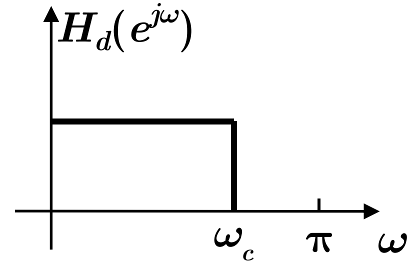


N=33

0.3π, 0.6π

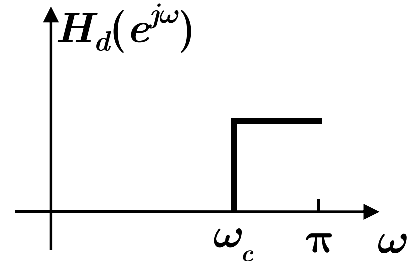
低通滤波器(ω_c)

$$h_d(n)=\begin{cases}\dfrac{1}{\pi(n-\tau)}\sin[\omega_c(n-\tau)] & n\neq\tau\\[2ex] \dfrac{\omega_c}{\pi} & n=\tau\end{cases}$$



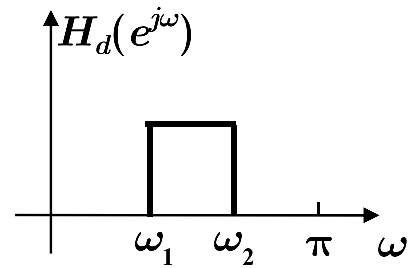
高通滤波器(ω_c) = 全通滤波器 − 低通滤波器(ω_c)

$$h_d(n)=\begin{cases}\dfrac{1}{\pi(n-\tau)}\left\{\sin\ \pi\left(n-\tau\right)-\sin\ \omega_c\left(n-\tau\right)\right\} & n\neq\tau\\[2ex] \dfrac{1}{\pi}(\pi-\omega_c) & n=\tau\end{cases}$$



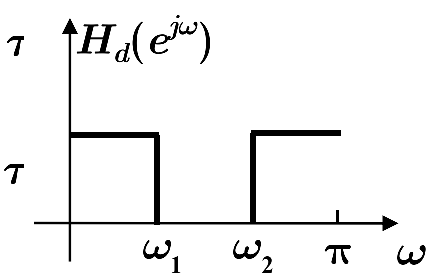
带通滤波器(ω_1,ω_2) = 低通滤波器(ω_2) − 低通滤波器(ω_1)

$$h_d(n)=\begin{cases}\dfrac{1}{\pi\left(n-\tau\right)}\left\{\sin\ \omega_2\left(n-\tau\right)-\sin\ \omega_1\left(n-\tau\right)\right\} & n\neq\tau\\[2ex] \dfrac{1}{\pi}\left(\omega_2-\omega_1\right) & n=\tau\end{cases}$$



带阻滤波器(ω_1,ω_2) = 高通滤波器(ω_2) + 低通滤波器(ω_1)

$$h_d(n)=\begin{cases}\dfrac{1}{\pi\left(n-\tau\right)}\left\{\sin\ \pi\left(n-\tau\right)+\sin\ \omega_1\left(n-\tau\right)-\sin\ \omega_2\left(n-\tau\right)\right\} & n\neq\tau\\[2ex] \dfrac{1}{\pi}\left(\pi+\omega_1-\omega_2\right) & n=\tau\end{cases}$$



FIR滤波器设计1—习题集P108

用矩形窗函数方法设计一个FIR线性相位数字低通滤波器，已知$\omega_c = 0.5\pi, N = 21$。

(1) 确定单位抽样响应序列$h(n), n = 0, 1, \cdots, N-1$

(2) 确定滤波器的系统函数$H(z)$

(3) 确定滤波器的频率响应$H(e^{j\omega})$

(4)给出滤波器的任意一种结构实现形式

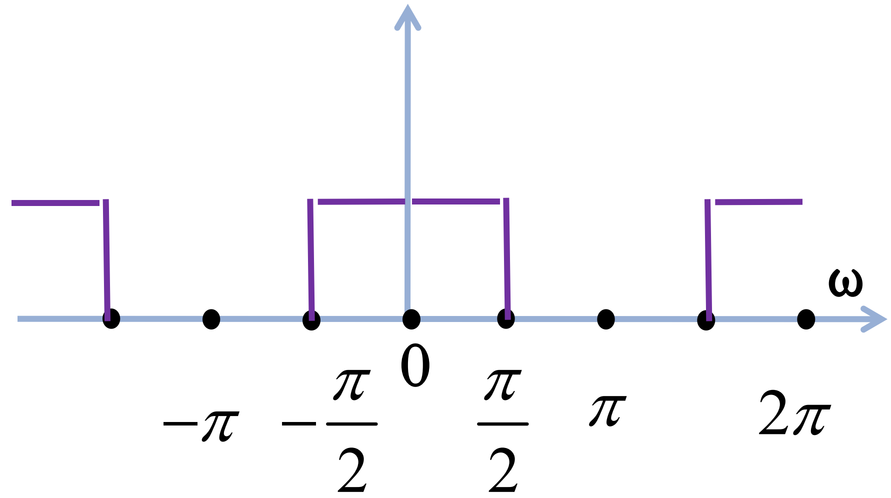
理想数字低通滤波器的幅频响为

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & -\omega_c \le |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$$

√

解：理想数字低通滤波器的幅频响应为

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & -\omega_c \le \omega \le \omega_c \\ 0 & \text{其他} \end{cases}$$



$$\Rightarrow \tau = \frac{N-1}{2} = 10,\ \ \omega_c = \frac{\pi}{2}$$

$$(1)h_d(n) = \begin{cases} \frac{1}{\pi(n-\tau)}\sin[\omega_c(n-\tau)] & n \ne \tau \\ \frac{\omega_c}{\pi} & n = \tau \end{cases} = \begin{cases} \frac{1}{\pi(n-10)}\sin[\frac{\pi}{2}(n-10)] & n \ne 10 \\ \frac{1}{2} & n = 10 \end{cases}$$

$$h(n) = h_d(n)w(n) = \begin{cases} \frac{1}{\pi(n-10)}\sin[\frac{\pi}{2}(n-10)], 0 \le n \le 20, n \ne 10 \\ \frac{1}{2}, & n = 10 \\ 0, & n\text{为其他} \end{cases}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-10)}\sin[\dfrac{\pi}{2}(n-10)], 0\le n\le 20, n\ne 10\\ \dfrac{1}{2}, & n=10\\ 0, & n\text{为其他}\end{cases}$$

$$h(0)=0; h(1)=\frac{1}{9\pi}=0.035;$$

$$h(2)=0; h(3)=\frac{-1}{7\pi}=-0.045;$$

$$h(4)=0; h(5)=\frac{1}{5\pi}=0.064$$

$$h(6)=0; h(7)=\frac{-1}{3\pi}=-0.106;$$

$$h(8)=0; h(9)=\frac{1}{\pi}=0.318;$$

$$h(11)=\frac{1}{\pi}=0.318; h(12)=0;$$

$$h(13)=\frac{-1}{3\pi}=-0.106; h(14)=0;$$

$$h(15)=\frac{1}{5\pi}=0.064; h(16)=0;$$

$$h(17)=\frac{-1}{7\pi}=-0.045; h(18)=0;$$

$$h(19)=\frac{1}{9\pi}=0.035; h(20)=0;$$

$$h(10)=\frac{1}{2}$$

(2) $H(z)=\sum_{n=0}^{N-1} h(n)z^{-n}$

(3) $H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1} h(n)e^{-j\omega n}$

(4)给出滤波器的任意一种结构实现形式

直接型

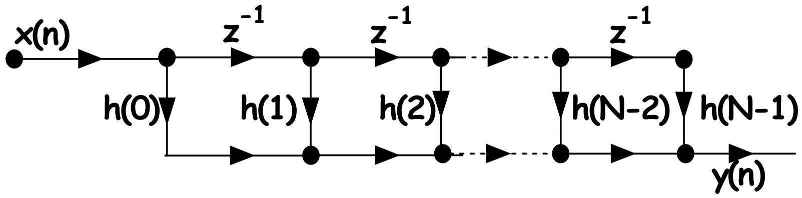
```
figure;
ord = 20;
b = fir1(ord,0.5,'low',rectwin(ord+1));
fvtool(b,1);
Hz = filt(b,1);
```

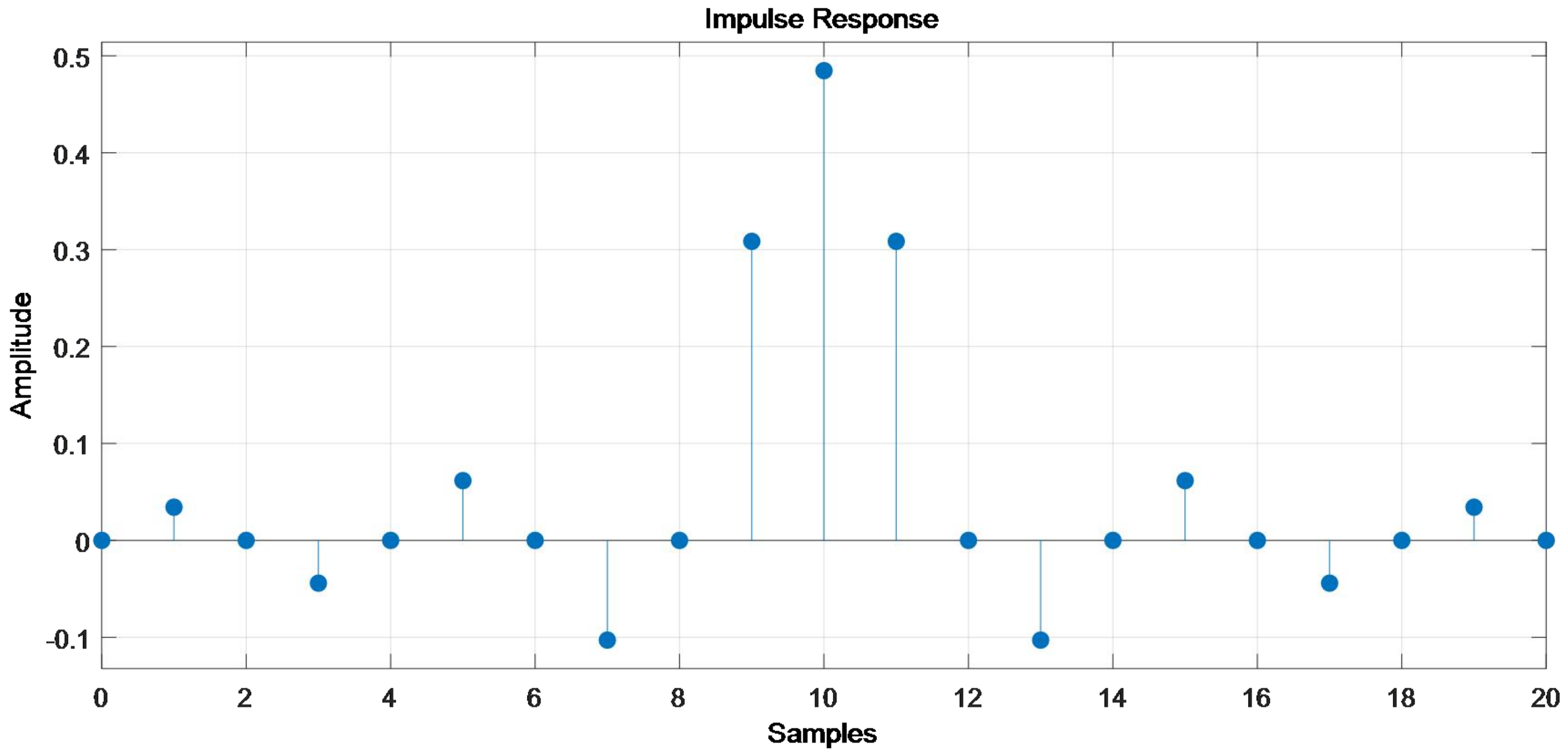
b =

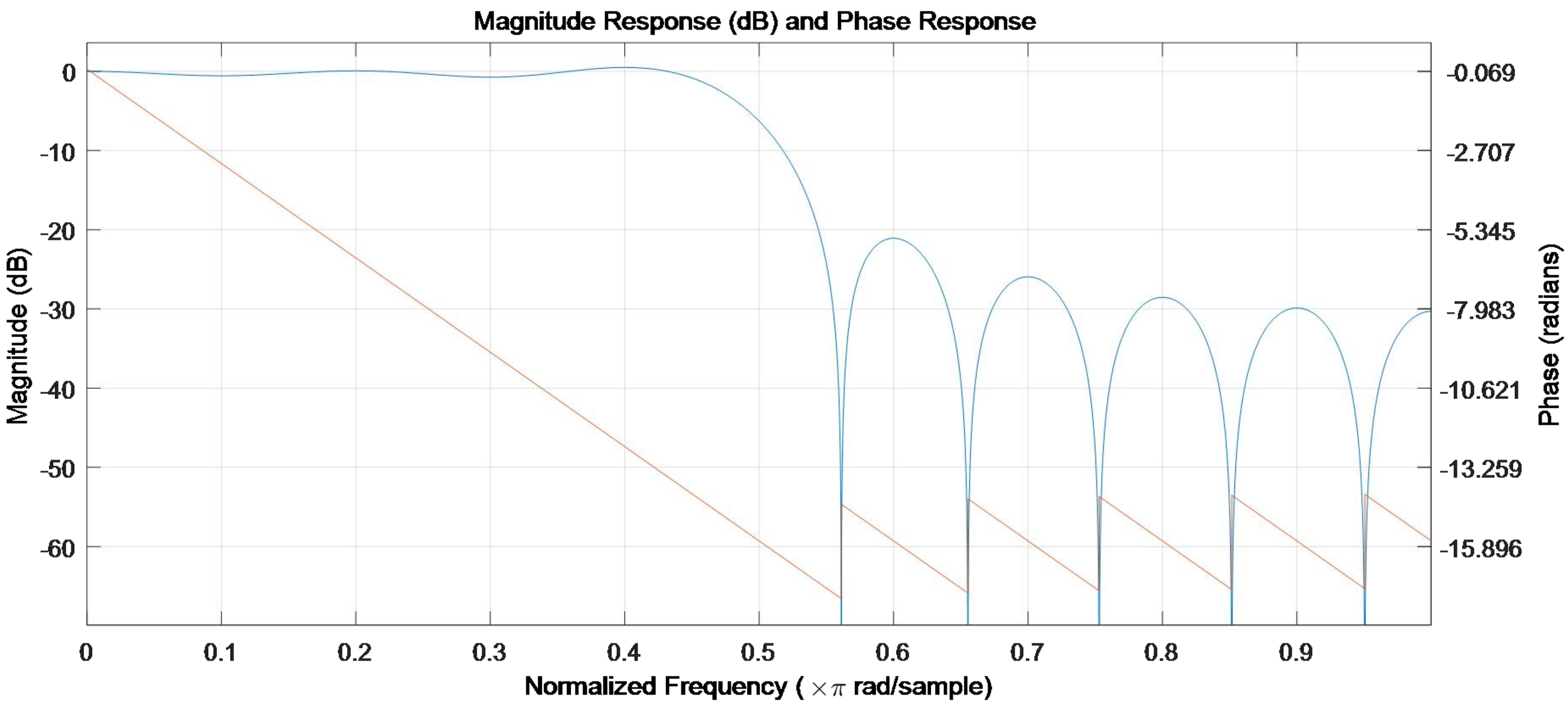
0.0000 0.0343 -0.0000 -0.0441 0.0000 0.0617 -0.0000 -0.1029 0.0000 0.3086
0.4847
0.3086 0.0000 -0.1029 -0.0000 0.0617 0.0000 -0.0441 -0.0000 0.0343 0.0000

Hz =

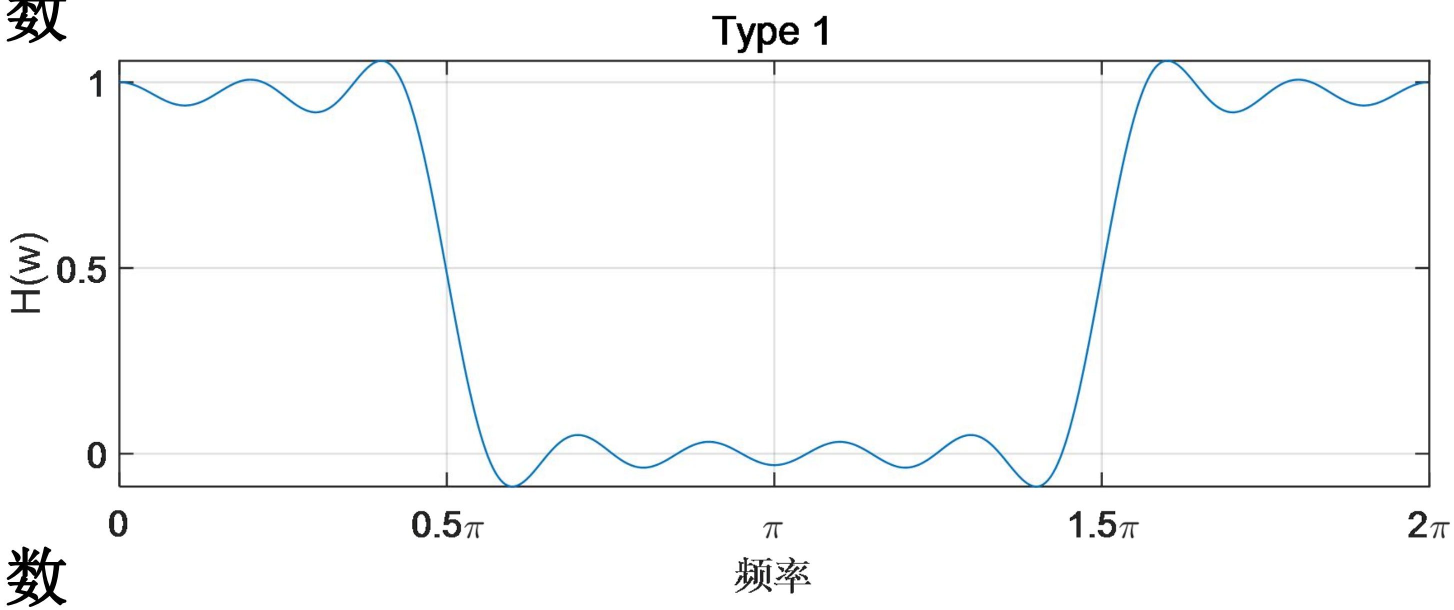
0.03429 z^-1 - 0.04408 z^-3 + 0.06172 z^-5 - 0.1029 z^-7 + 0.3086 z^-9 + 0.3086 z^-11 - 0.1029 z^-13 + 0.06172 z^-15 - 0.04408 z^-17 + 0.03429 z^-19



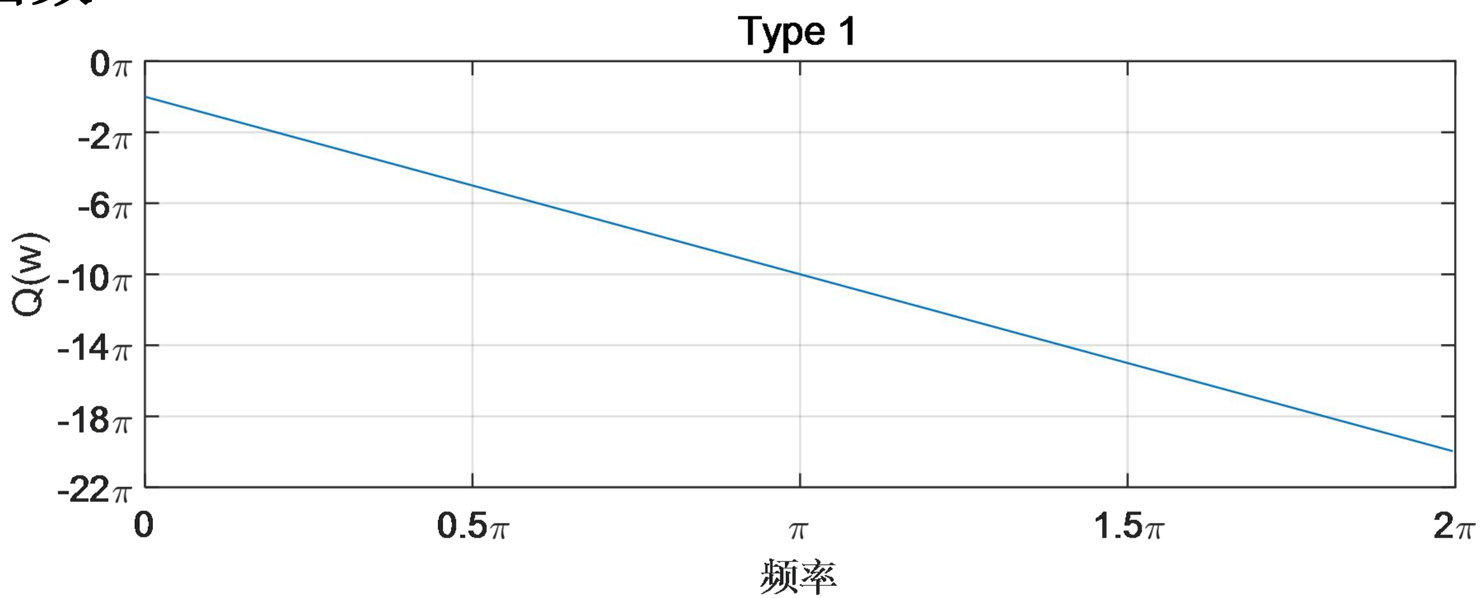

Impulse Response
Amplitude
Samples
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
2
4
6
8
10
12
14
16
18
20


Magnitude Response (dB) and Phase Response
Magnitude (dB)
Phase (radians)
Normalized Frequency (×π rad/sample)
0
-10
-20
-30
-40
-50
-60
-0.069
-2.707
-5.345
-7.983
-10.621
-13.259
-15.896
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

幅度函数


Type 1
1
0.5
0
H(w)
0
0.5π
π
1.5π
2π
频率

相位函数


Type 1
0π
-2π
-6π
-10π
-14π
-18π
-22π
Q(w)
0
0.5π
π
1.5π
2π
频率

FIR滤波器设计2--往年真题

设理想数字高通滤波器的幅频响应为

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/2 \le |\omega| \le \pi \\ 0 & |\omega| < \pi/2 \end{cases}$$

用矩形窗函数方法设计一个$N=11$时FIR线性相位数字高通滤波器，

(1) 确定单位抽样响应序列$h(n), n = 0,1,\cdots,N-1$

(2) 确定滤波器的系统函数$H(z)$

(3) 确定滤波器的频率响应$H(e^{j\omega})$

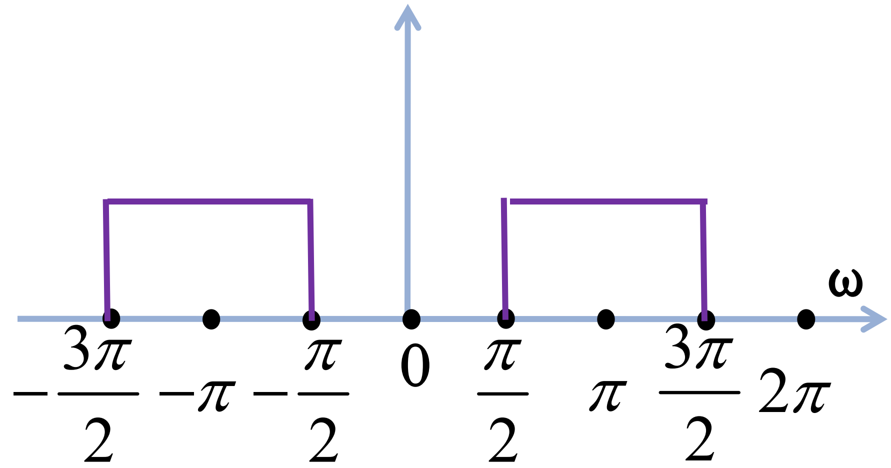
(4)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

√

03A(05-06)

解：理想数字高通滤波器的幅频响为



$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & \omega_c \le \omega \le \omega_c \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow \tau = \frac{N-1}{2} = 5,\ \ \omega_c = \frac{\pi}{2}$$

$$(1)\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/2 \le |\omega| \le \pi \\ 0 & |\omega| < \pi/2 \end{cases}$$

$$h_d(n) = \begin{cases} \dfrac{1}{\pi(n-\tau)}\left\{\sin\left[\pi\left(n-\tau\right)\right] - \sin\left[\omega_c\left(n-\tau\right)\right]\right\} & n \ne \tau \\ \dfrac{1}{\pi}(\pi - \omega_c) & n = \tau \end{cases}$$

$$= \begin{cases} \dfrac{1}{\pi(n-5)}\left\{\sin\left[\pi\left(n-5\right)\right] - \sin\left[\dfrac{\pi}{2}\left(n-5\right)\right]\right\} & n \ne 5 \\ \dfrac{1}{\pi}(\pi - \dfrac{\pi}{2}) & n = 5 \end{cases}$$

$$h_d(n)=\begin{cases}\dfrac{1}{\pi(n-5)}\left\{\sin\left[\pi\left(n-5\right)\right]-\sin\left[\dfrac{\pi}{2}\left(n-5\right)\right]\right\} & n\neq 5\\ \dfrac{1}{\pi}(\pi-\dfrac{\pi}{2}) & n=5\end{cases}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-5)}\left\{\sin\left[\pi\left(n-5\right)\right]-\sin\left[\dfrac{\pi}{2}\left(n-5\right)\right]\right\}, & 0\le n\le 10, n\neq 5\\ \dfrac{1}{\pi}(\pi-\dfrac{\pi}{2}), & n=5\\ 0, & n\text{为其他}\end{cases}$$

$$h(0)=-\frac{1}{5\pi}=-0.064;\quad h(1)=0;\quad h(2)=\frac{1}{3\pi}=0.106;\quad h(3)=0;\quad h(4)=-\frac{1}{\pi}=-0.318;$$

$$h(5)=\left.\left((-1)^{n-5}\frac{\sin[\frac{\pi}{2}(n-5)]}{\pi(n-5)}\right)'\right|_{n=5}=\frac{1}{2};$$

$$h(6)=-\frac{1}{\pi}=-0.318;\quad h(7)=0;\quad h(8)=\frac{1}{3\pi}=0.106;\quad h(9)=0;\quad h(10)=-\frac{1}{5\pi}=-0.064$$

$$(2)H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}$$

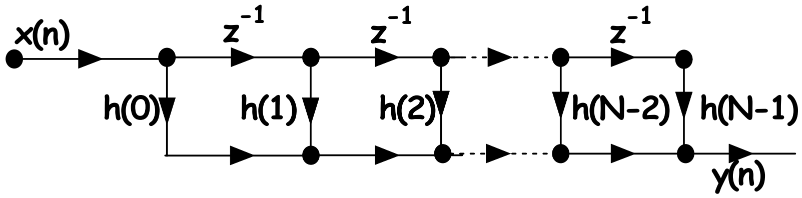
$$(3)H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1}h(n)e^{-j\omega n}$$

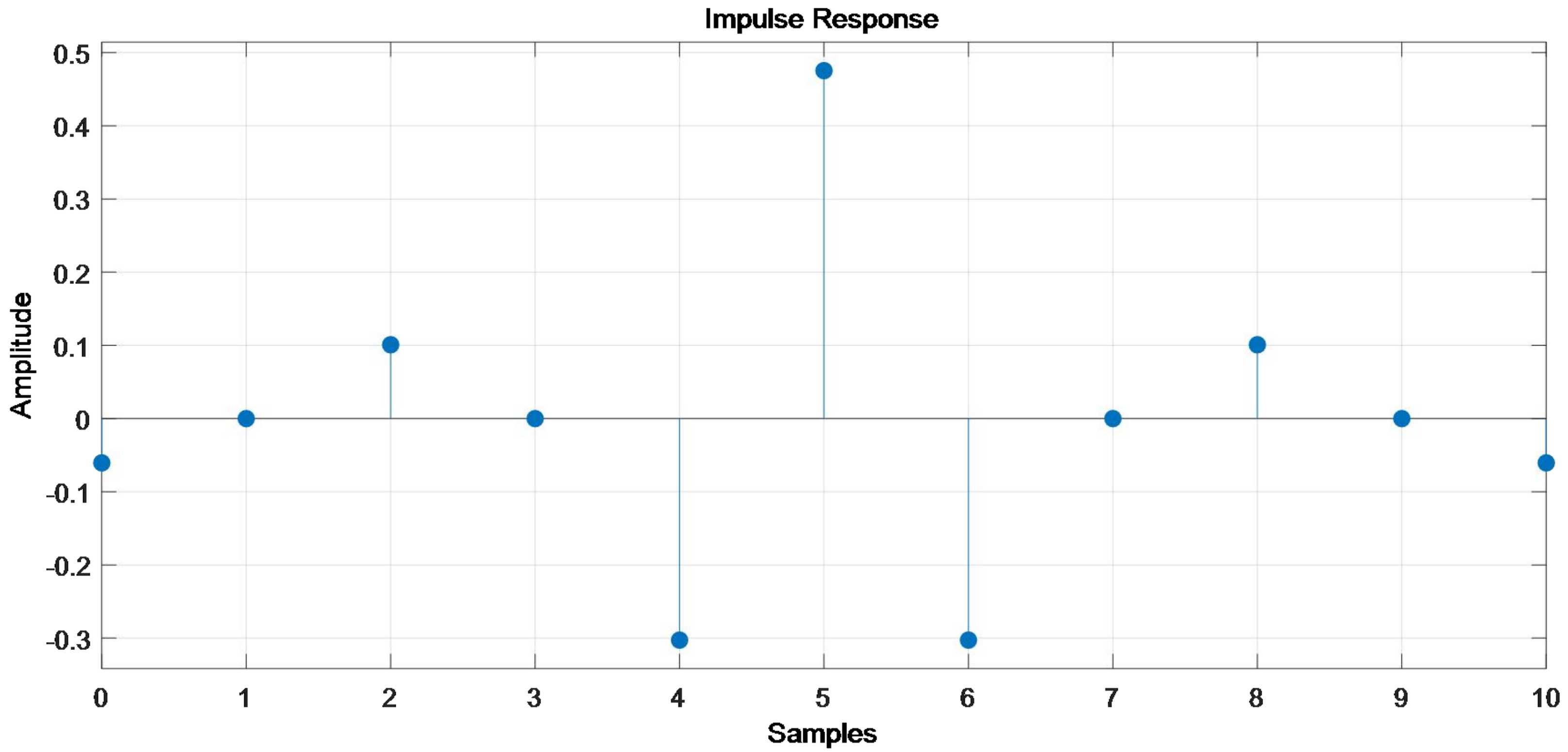
(4)给出滤波器的任意一种结构实现形式

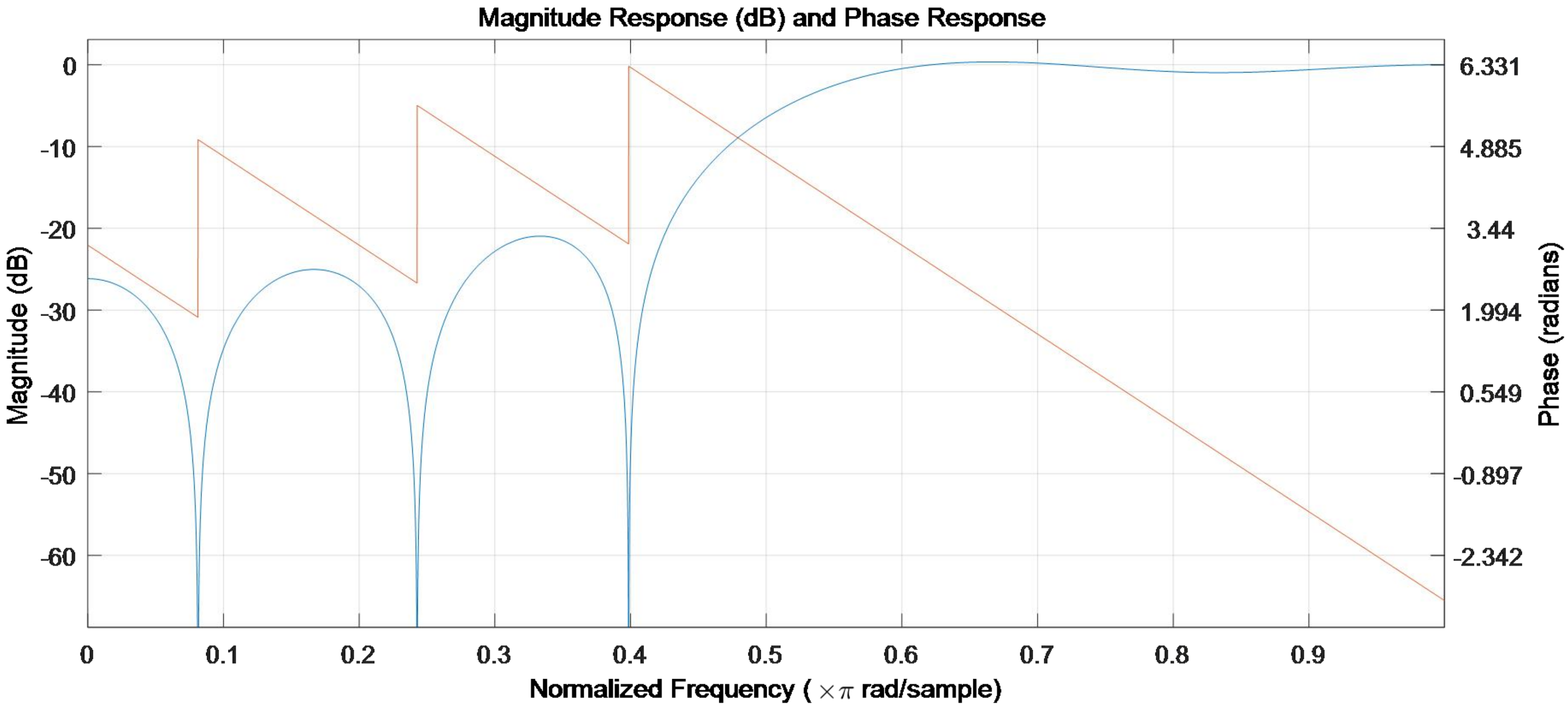
直接型

```
figure;
ord = 10;
b = fir1(ord,0.5,'high',rectwin(ord+1));
fvtool(b,1);
Hz = filt(b,1);
```

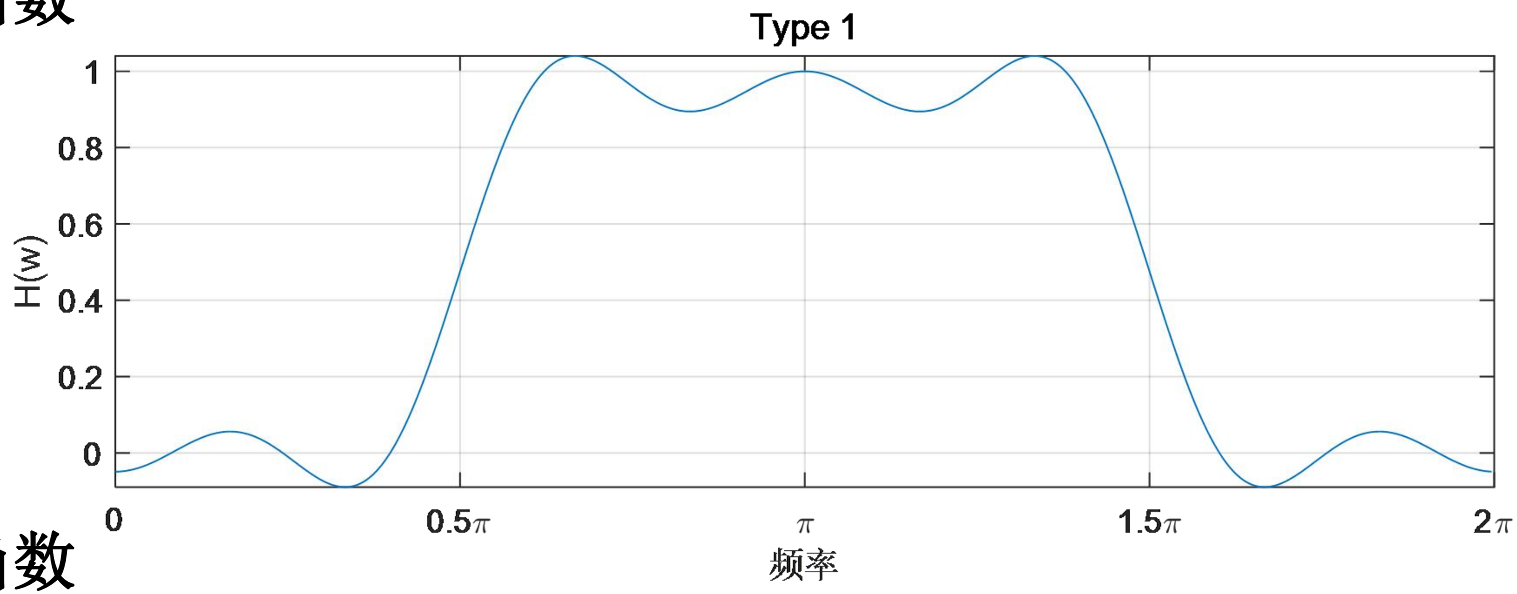
```
b =

   -0.0605   -0.0000    0.1009   -0.0000   -0.3027    0.4754   -0.3027   -0.0000    0.1009   -0.0000   -0.0605

Hz =

  -0.06053 + 0.1009 z^-2 - 0.3027 z^-4 + 0.4754 z^-5 - 0.3027 z^-6 + 0.1009 z^-8 - 0.06053 z^-10
```



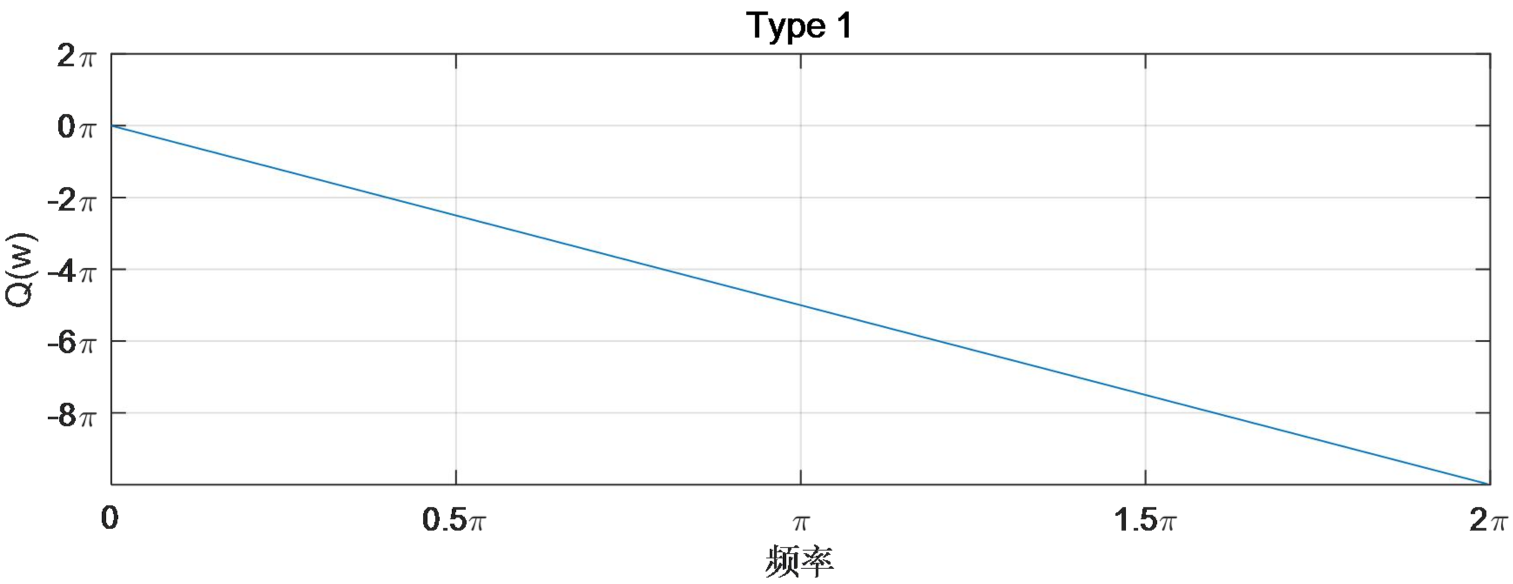

Impulse Response
Amplitude
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
1
2
3
4
5
6
7
8
9
10
Samples


Magnitude Response (dB) and Phase Response
0
-10
-20
-30
-40
-50
-60
Magnitude (dB)
6.331
4.885
3.44
1.994
0.549
-0.897
-2.342
Phase (radians)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)

幅度函数


Type 1
1
0.8
0.6
0.4
0.2
0
H(w)
0
0.5π
π
1.5π
2π
频率

相位函数


Type 1
2π
0π
-2π
-4π
-6π
-8π
Q(w)
0
0.5π
π
1.5π
2π
频率

FIR滤波器设计3-1--往年真题

设理想数字带通滤波器的幅频响应为

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/4 \le |\omega| \le \pi/2 \\ 0 & \pi/2 \le |\omega| \le \pi, |\omega| \le \pi/4 \end{cases}$$

用矩形窗函数方法设计一个$N=9$时FIR线性相位数字带通滤波器，

(1)确定滤波器单位抽样响应序列$h(n), n = 0,1,\cdots,N-1$

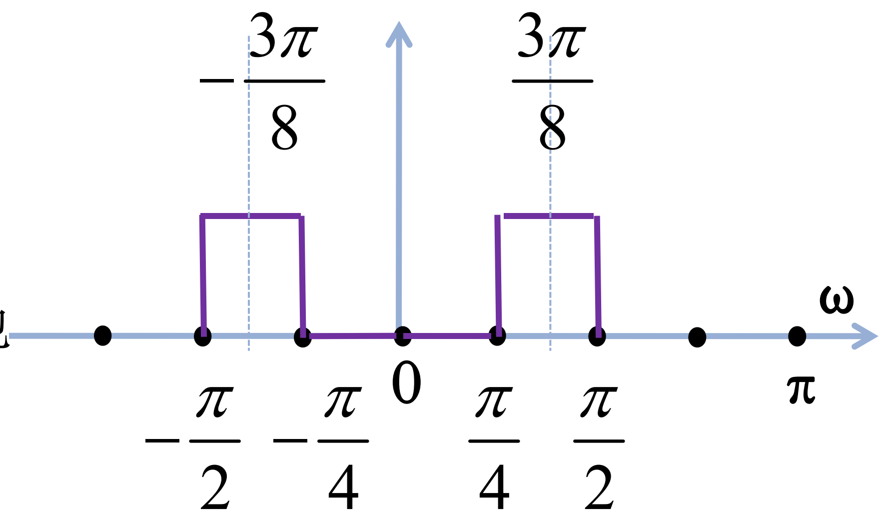
(2)确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

解：理想数字带通滤波器的幅频响为

$$H_d(e^{j\omega})=\begin{cases} e^{-j\omega\alpha} & -\dfrac{\pi}{8}\le\omega\pm\dfrac{3\pi}{8}\le\dfrac{\pi}{8} \\ 0 & \text{其他} \end{cases}$$



$$\Rightarrow \alpha=\frac{N-1}{2}=4,\ \ \omega_1=\frac{\pi}{4},\ \ \omega_2=\frac{\pi}{2}$$

$$(1)h_d(n)=\begin{cases} \dfrac{1}{\pi(n-\alpha)}\left\{\sin\left[\omega_2(n-\alpha)\right]-\sin\left[\omega_1(n-\alpha)\right]\right\} & n\ne\alpha \\ \dfrac{1}{\pi}(\omega_2-\omega_1) & n=\alpha \end{cases}$$

$$=\begin{cases} \dfrac{1}{\pi(n-4)}\left\{\sin\left[\dfrac{\pi}{2}(n-4)\right]-\sin\left[\dfrac{\pi}{4}(n-4)\right]\right\} & n\ne 4 \\ \dfrac{1}{4} & n=4 \end{cases}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-4)}\left\{\sin\left[\dfrac{\pi}{2}(n-4)\right]-\sin\left[\dfrac{\pi}{4}(n-4)\right]\right\}, 0\le n\le 8, n\ne 4\\ \dfrac{1}{4}, n=4\\ 0, n\text{为其他}\end{cases}$$

$$h(0)=0; h(1)=\frac{1}{-3\pi}(1-\frac{\sqrt{2}}{2})=-0.03;$$

$$h(2)=\frac{1}{-2\pi}(-1)=0.16; h(3)=\frac{1}{-\pi}(-1-\frac{\sqrt{2}}{2})=0.54;$$

$$h(4)=\frac{1}{4};$$

$$h(5)=\frac{1}{\pi}(1+\frac{\sqrt{2}}{2})=0.54; h(6)=\frac{1}{2\pi}(1)=0.16;$$

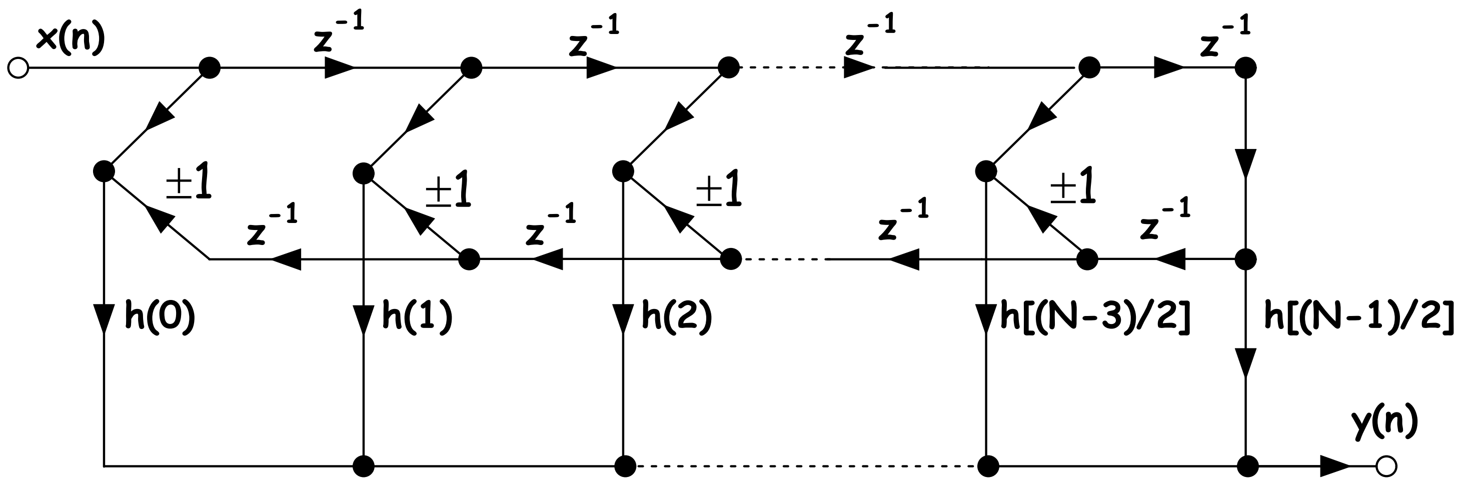
$$h(7)=\frac{1}{3\pi}(-1+\frac{\sqrt{2}}{2})=-0.03; h(8)=0;$$

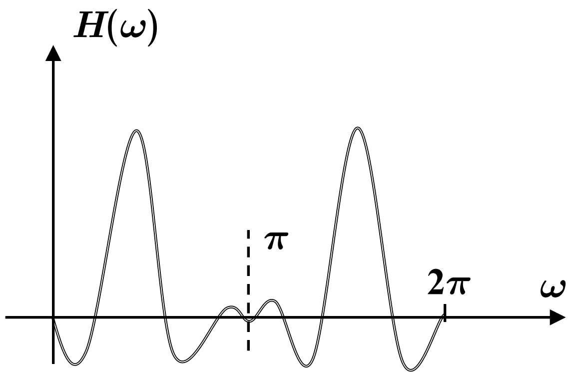
$$(2) H(z)=\sum_{n=0}^{N-1} h(n) z^{-n}$$

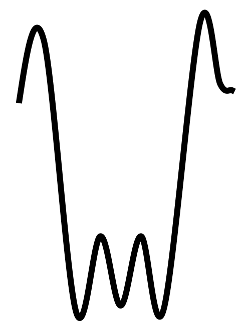
$$(3) H(e^{j\omega})=H(z)\Big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

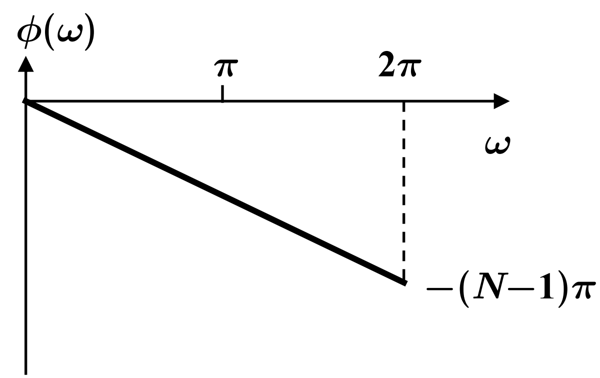
(4)给出滤波器的任意一种结构实现形式

直接型


x(n)
z-1
z-1
z-1
z-1
±1
±1
±1
±1
z-1
z-1
z-1
z-1
h(0)
h(1)
h(2)
h[(N-3)/2]
h[(N-1)/2]
y(n)


$H(\omega)$
π
2π
ω




$\phi(\omega)$
π
2π
ω
$-(N-1)\pi$

FIR滤波器设计3-2--往年真题

用矩形窗函数方法设计一个FIR线性相位数字带通滤波器，

要求上下边带截止频率分别为$\omega_1 = 0.6\pi$，$\omega_2 = 0.3\pi$，窗长$N = 11$。

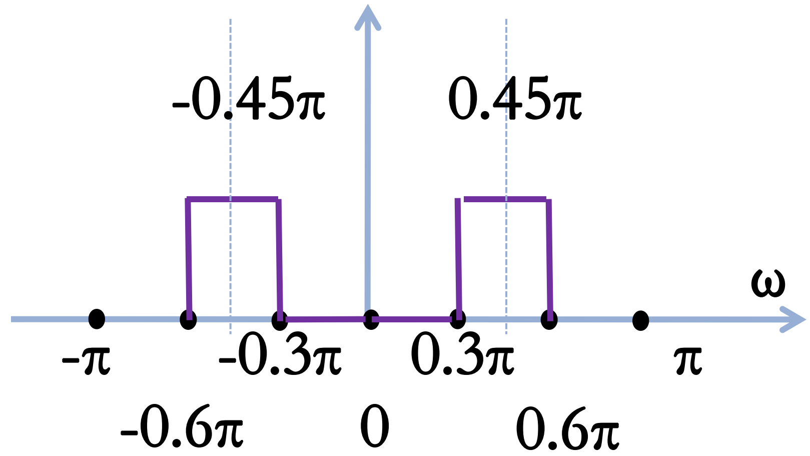
(1) 确定滤波器单位抽样响应序列$h(n)$

(2) 确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

解：理想数字带通滤波器的幅频响为



$$H_d(e^{j\omega})=\begin{cases} e^{-j\omega\alpha} & -0.15\pi \le \omega \pm 0.45\pi \le 0.15\pi \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow \alpha=\frac{N-1}{2}=\frac{11-1}{2}=5,\ \ \omega_1=0.3\pi，\ \ \omega_2=0.6\pi$$

$$(1)h_d(n)=\begin{cases} \dfrac{1}{\pi(n-\alpha)}\left\{\sin\left[\omega_2(n-\alpha)\right]-\sin\left[\omega_1(n-\alpha)\right]\right\} & n\neq\alpha \\ \dfrac{1}{\pi}(\omega_2-\omega_1) & n=\alpha \end{cases}$$

$$=\begin{cases} \dfrac{1}{\pi(n-5)}\left\{\sin\left[0.6\pi(n-5)\right]-\sin\left[0.3\pi(n-5)\right]\right\} & n\neq 5 \\ 0.3 & n=5 \end{cases}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-5)}\left\{\sin\left[0.6\pi(n-5)\right]-\sin\left[0.3\pi(n-5)\right]\right\} & 0\le n\le 10, n\ne 5\\ 0.3 & n=5\\ 0, & n\text{为其他}\end{cases}$$

$h(0)=0.064;$

$h(1)=0.123;$

$h(2)=-0.095;$

$h(3)=-0.245;$

$h(4)=0.045;$

$h(5)=0.3;$

$h(6)=0.045;$

$h(7)=-0.245;$

$h(8)=-0.095;$

$h(9)=0.123;$

$h(10)=0.064$

$$(2)H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}$$

$$(3)H(e^{j\omega})=H(z)\Big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1}h(n)e^{-j\omega n}$$

(4)给出滤波器的任意一种结构实现形式

直接型

FIR滤波器设计4

用矩形窗函数方法设计一个FIR线性相位数字带阻滤波器，要求上下边带截止频率分别为0.3π和0.6π，窗长$N=11$。

(1) 确定滤波器单位抽样响应序列$h(n)$

(2) 确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

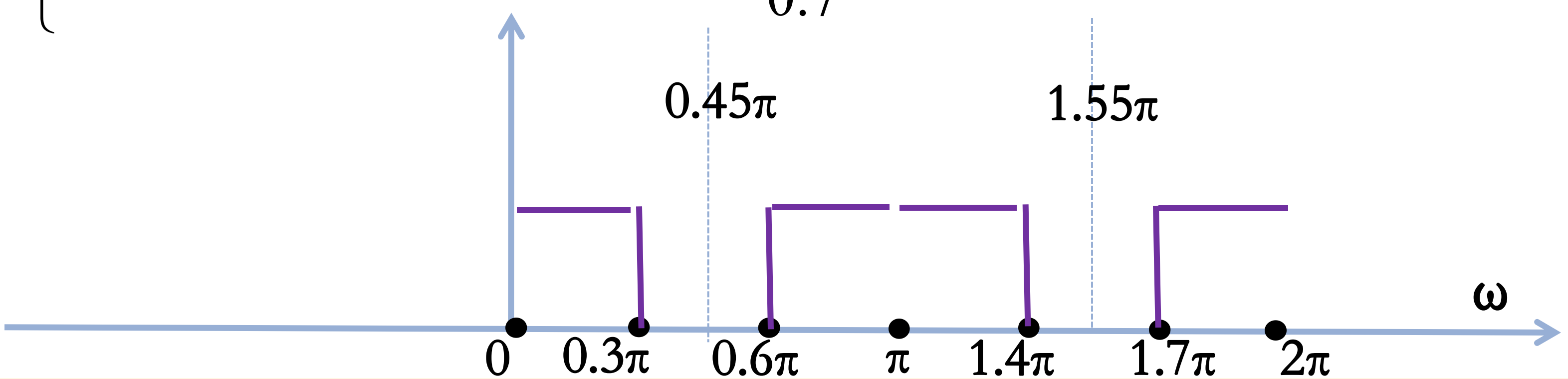
解：理想数字带通滤波器的幅频响为

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & 0 \le \omega \le 0.3\pi;\ 1.7 \le \omega \le 2\pi;\ 0.6\pi \le \omega \le 1.4\pi; \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow \alpha = \frac{N-1}{2} = \frac{11-1}{2} = 5,\ \omega_1 = 0.3\pi，\ \omega_2 = 0.6\pi$$

$$h_d(n) = \begin{cases} \dfrac{1}{\pi(n-\alpha)}\left\{\sin\left[\pi(n-\alpha)\right] + \sin\left[\omega_1(n-\alpha)\right] - \sin\left[\omega_2(n-\alpha)\right]\right\} & n \ne \alpha \\ \dfrac{1}{\pi}(\pi + \omega_1 - \omega_2) & n = \alpha \end{cases}$$

$$= \begin{cases} \dfrac{1}{\pi(n-5)}\left\{\sin\left[\pi(n-5)\right] + \sin\left[0.3\pi(n-5)\right] - \sin\left[0.6\pi(n-5)\right]\right\} & n \ne 5 \\ 0.7 & n = 5 \end{cases}$$



$$h(n)=h_d(n)w(n)$$

$$=\begin{cases}\dfrac{1}{\pi(n-5)}\left\{\sin\left[\pi(n-5)\right]+\sin\left[0.3\pi(n-5)\right]-\sin\left[0.6\pi(n-5)\right]\right\} & 0\le n\le 10, n\ne 5\\ 0.7 & n=5\\ 0, & n\text{为其他}\end{cases}$$

$h(0)=-0.064;$

$h(1)=-0.123;$

$h(2)=0.095;$

$h(3)=0.245;$

$h(4)=-0.045;$

$h(5)=0.7;$

$h(6)=-0.045;$

$h(7)=0.245;$

$h(8)=0.095;$

$h(9)=-0.123;$

$h(10)=-0.064$

$$(2)H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}$$

$$(3)H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1}h(n)e^{-j\omega n}$$

(4)给出滤波器的任意一种结构实现形式

直接型

以下是自己计算

FIR滤波器设计3-1--往年真题

设理想数字带通滤波器的幅频响应为

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/4 \le |\omega| \le \pi/2 \\ 0 & \pi/2 \le |\omega| \le \pi, |\omega| \le \pi/4 \end{cases}$$

用矩形窗函数方法设计一个$N=9$时FIR线性相位数字$\boxed{\text{带通}}$滤波器，

(1) 确定滤波器单位抽样响应序列$h(n), n = 0, 1, \cdots, N-1$

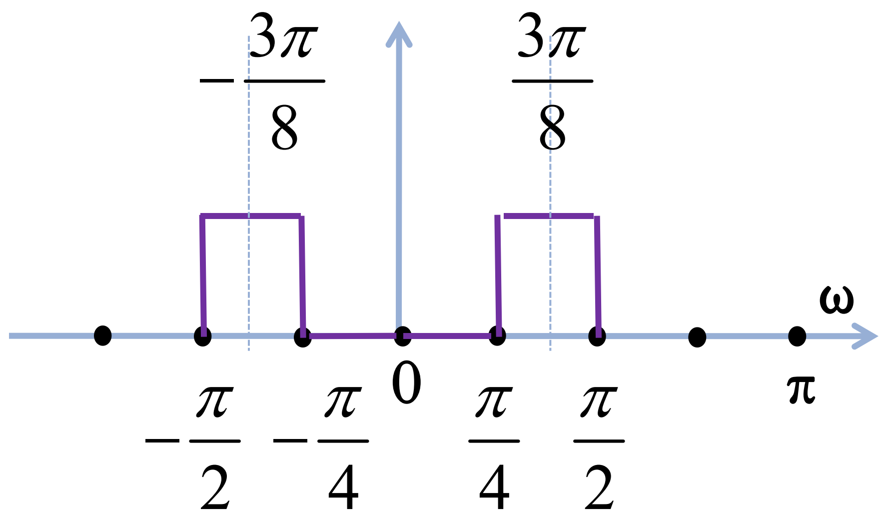
(2) 确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

04(06-07)

解：理想数字带通滤波器的幅频响为



$$H_d(e^{j\omega})=\begin{cases} e^{-j\omega\alpha} & -\omega_c \le \omega \pm \omega_0 \le \omega_c \\ 0 & \text{其他} \end{cases}$$

$$H_d(e^{j\omega})=\begin{cases} e^{-j\omega\alpha} & -\frac{\pi}{8} \le \omega \pm \frac{3\pi}{8} \le \frac{\pi}{8} \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow \alpha=\frac{N-1}{2}=4,\ \ \omega_c=\frac{\pi}{8},\ \ \omega_0=\frac{3\pi}{8}$$

$$(1)h_d(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(e^{j\omega})e^{j\omega n}d\omega=\frac{1}{2\pi}\int_{-\omega_0-\omega_c}^{-\omega_0+\omega_c}e^{-j\omega\alpha}e^{j\omega n}d\omega+\frac{1}{2\pi}\int_{\omega_0-\omega_c}^{\omega_0+\omega_c}e^{-j\omega\alpha}e^{j\omega n}d\omega$$

(注意积分区间，此时为$-\pi\sim\pi$)

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left(e^{j\omega(n-\alpha)}\Big|_{-\omega_0-\omega_c}^{-\omega_0+\omega_c}+e^{j\omega(n-\alpha)}\Big|_{\omega_0-\omega_c}^{\omega_0+\omega_c}\right)$$

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left\{\left[e^{j(-\omega_0+\omega_c)(n-\alpha)}-e^{j(-\omega_0-\omega_c)(n-\alpha)}\right]+\left[e^{j(\omega_0+\omega_c)(n-\alpha)}-e^{j(\omega_0-\omega_c)(n-\alpha)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left\{\left[e^{j(-\omega_0+\omega_c)(n-\alpha)}-e^{j(-\omega_0-\omega_c)(n-\alpha)}\right]+\left[e^{j(\omega_0+\omega_c)(n-\alpha)}-e^{j(\omega_0-\omega_c)(n-\alpha)}\right]\right\}$$

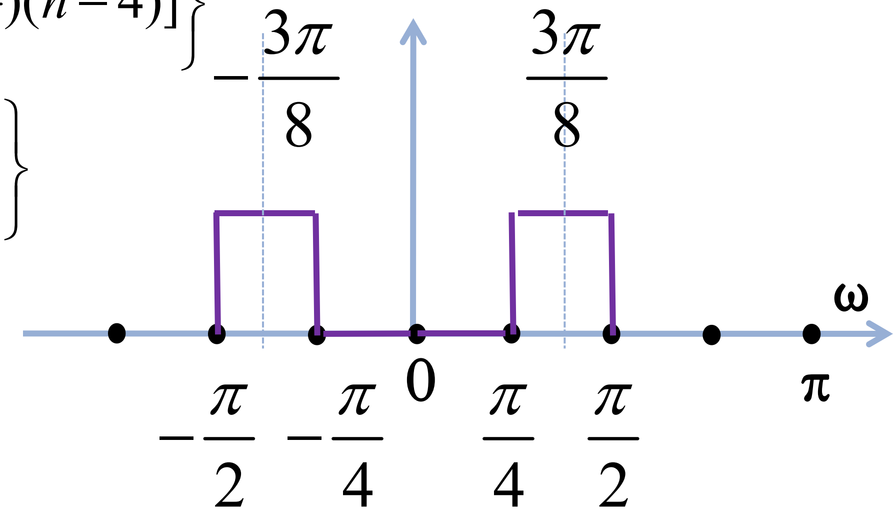
$$=\frac{1}{2\pi}\frac{1}{j(n-4)}\left\{\left[e^{j(-\frac{\pi}{4})(n-4)}-e^{j(-\frac{\pi}{2})(n-4)}\right]+\left[e^{j(\frac{\pi}{2})(n-4)}-e^{j(\frac{\pi}{4})(n-4)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-4)}\left\{\left[e^{j(-\frac{\pi}{4})(n-4)}-e^{j(\frac{\pi}{4})(n-4)}\right]+\left[e^{j(\frac{\pi}{2})(n-4)}-e^{j(-\frac{\pi}{2})(n-4)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-4)}\left\{2j\sin[(-\frac{\pi}{4})(n-4)]+2j\sin[(\frac{\pi}{2})(n-4)]\right\}$$

$$=\frac{1}{2\pi}\frac{2j}{j(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}$$

$$=\frac{1}{\pi(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}$$



$$h(n)=h_d(n)w(n)=\begin{cases}\frac{1}{\pi(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}, & 0\le n\le 8\\ 0, & n\text{为其他}\end{cases}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-4)}\left\{\sin\left[\dfrac{\pi}{2}(n-4)\right]-\sin\left[\dfrac{\pi}{4}(n-4)\right]\right\}, & 0\le n\le 8\\ 0, & n\text{为其他}\end{cases}$$

$$h(0)=0; h(1)=\frac{1}{-3\pi}(1-\frac{\sqrt{2}}{2})=-0.03;$$

$$h(2)=\frac{1}{-2\pi}(-1)=0.16; h(3)=\frac{1}{-\pi}(-1-\frac{\sqrt{2}}{2})=0.54;$$

$$h(4)=\frac{1}{4};$$

$$h(5)=\frac{1}{\pi}(1+\frac{\sqrt{2}}{2})=0.54; h(6)=\frac{1}{2\pi}(1)=0.16;$$

$$h(7)=\frac{1}{3\pi}(-1+\frac{\sqrt{2}}{2})=-0.03; h(8)=0;$$

$$(2)H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}$$

$$=\sum_{n=0}^{8}\frac{1}{\pi(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}z^{-n}$$

$$(3)H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1}h(n)e^{-j\omega n}$$

$$=\sum_{n=0}^{8}\frac{1}{\pi(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}e^{-j\omega n}$$

(4)给出滤波器的任意一种结构实现形式

直接型

FIR滤波器设计2--往年真题

设理想数字高通滤波器的幅频响应为

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/2 \le |\omega| \le \pi \\ 0 & |\omega| < \pi/2 \end{cases}$$

用矩形窗函数方法设计一个$N=11$时FIR线性相位数字高通滤波器，

(1) 确定单位抽样响应序列$h(n), n=0,1,\cdots,N-1$

(2) 确定滤波器的系统函数$H(z)$

(3) 确定滤波器的频率响应$H(e^{j\omega})$

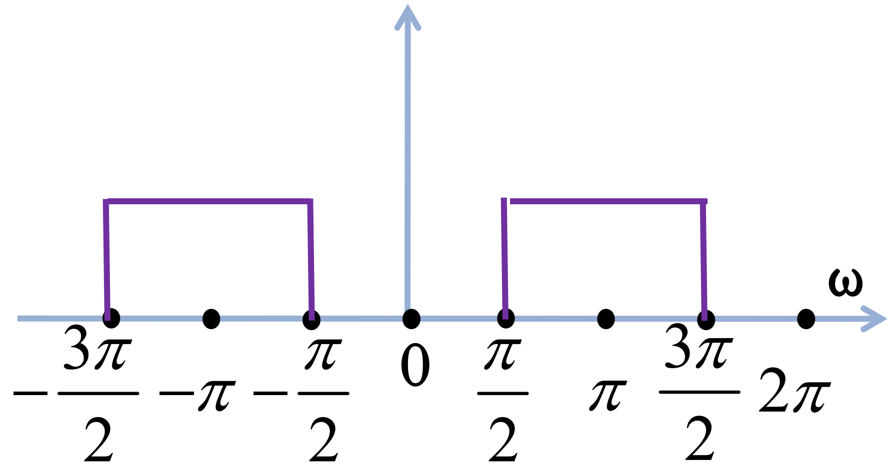
(4)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

√

解：理想数字高通滤波器的幅频响为

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & \pi - \omega_c \le \omega \le \pi + \omega_c \\ 0 & \text{其他} \end{cases}$$



$$\Rightarrow \alpha = \frac{N-1}{2} = 5,\ \ \omega_c = 0.5\pi$$

$$(1)\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/2 \le |\omega| \le \pi \\ 0 & |\omega| < \pi/2 \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_0^{2\pi} H_d(e^{j\omega})e^{j\omega}d\omega = \frac{1}{2\pi}\int_{\pi-\omega_c}^{\pi+\omega_c} e^{-j\omega\alpha}e^{j\omega n}d\omega$$

(注意积分区间，此时为$0 \sim 2\pi$)

$$= \frac{1}{2\pi}\frac{1}{j(n-\alpha)}e^{j\omega(n-\alpha)}\Big|_{\pi-\omega_c}^{\pi+\omega_c} = \frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left[e^{j(\pi+\omega_c)(n-\alpha)} - e^{j(\pi-\omega_c)(n-\alpha)}\right]$$

$$= \frac{1}{2\pi}\frac{1}{j(n-\alpha)}e^{j\pi(n-\alpha)}\left[e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}\right]$$

$$= \frac{1}{2\pi}\frac{1}{j(n-\alpha)}(-1)^{(n-\alpha)}2j\sin[\omega_c(n-\alpha)]$$

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}(-1)^{(n-\alpha)}2j\sin[\omega_c(n-\alpha)]$$

$$=(-1)^{(n-\alpha)}\frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}=(-1)^{n-5}\frac{\sin[\frac{\pi}{2}(n-5)]}{\pi(n-5)}$$

$$h(n)=h_d(n)w(n)=\begin{cases}(-1)^{n-5}\dfrac{\sin[\frac{\pi}{2}(n-5)]}{\pi(n-5)}, & 0\le n\le 10\\ 0, & n\text{为其他}\end{cases}$$

$$h(0)=-\frac{1}{5\pi}=-0.064;\quad h(1)=0;\quad h(2)=\frac{1}{3\pi}=0.106;\quad h(3)=0;\quad h(4)=-\frac{1}{\pi}=-0.318;$$

$$h(5)=\left.\left((-1)^{n-5}\frac{\sin[\frac{\pi}{2}(n-5)]}{\pi(n-5)}\right)'\right|_{n=5}=\frac{1}{2};$$

$$h(6)=-\frac{1}{\pi}=-0.318;\quad h(7)=0;\quad h(8)=\frac{1}{3\pi}=0.106;\quad h(9)=0;\quad h(10)=-\frac{1}{5\pi}=-0.064$$

$$(2)H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}=\sum_{n=0}^{10}(-1)^{n-5}\frac{\sin[\frac{\pi}{2}(n-5)]}{\pi(n-5)}z^{-n}$$

$$(3)H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1}h(n)e^{-j\omega n}=\sum_{n=0}^{10}(-1)^{n-5}\frac{\sin[\frac{\pi}{2}(n-5)]}{\pi(n-5)}e^{-j\omega n}$$

(4)给出滤波器的任意一种结构实现形式

直接型

FIR滤波器设计3-1--往年真题

设理想数字带通滤波器的幅频响应为

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1 & \pi/4 \le |\omega| \le \pi/2 \\ 0 & \pi/2 \le |\omega| \le \pi, |\omega| \le \pi/4 \end{cases}$$

用矩形窗函数方法设计一个$N=9$时FIR线性相位数字带通滤波器，

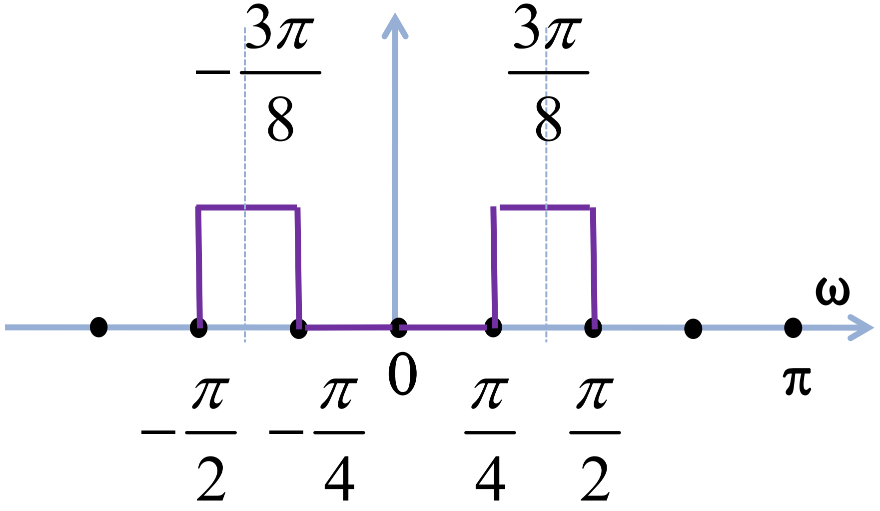
(1) 确定滤波器单位抽样响应序列$h(n), n=0,1,\cdots,N-1$

(2) 确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

解：理想数字带通滤波器的幅频响为



$$H_d(e^{j\omega})=\begin{cases} e^{-j\omega\alpha} & -\omega_c \le \omega \pm \omega_0 \le \omega_c \\ 0 & \text{其他} \end{cases}$$

$$H_d(e^{j\omega})=\begin{cases} e^{-j\omega\alpha} & -\frac{\pi}{8} \le \omega \pm \frac{3\pi}{8} \le \frac{\pi}{8} \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow \alpha=\frac{N-1}{2}=4,\ \ \omega_c=\frac{\pi}{8},\ \ \omega_0=\frac{3\pi}{8}$$

$$(1) h_d(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega=\frac{1}{2\pi}\int_{-\omega_0-\omega_c}^{-\omega_0+\omega_c} e^{-j\omega\alpha}e^{j\omega n}d\omega+\frac{1}{2\pi}\int_{\omega_0-\omega_c}^{\omega_0+\omega_c} e^{-j\omega\alpha}e^{j\omega n}d\omega$$

(注意积分区间，此时为$-\pi \sim \pi$)

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left(e^{j\omega(n-\alpha)}\Big|_{-\omega_0-\omega_c}^{-\omega_0+\omega_c}+e^{j\omega(n-\alpha)}\Big|_{\omega_0-\omega_c}^{\omega_0+\omega_c}\right)$$

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left\{\left[e^{j(-\omega_0+\omega_c)(n-\alpha)}-e^{j(-\omega_0-\omega_c)(n-\alpha)}\right]+\left[e^{j(\omega_0+\omega_c)(n-\alpha)}-e^{j(\omega_0-\omega_c)(n-\alpha)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left\{\left[e^{j(-\omega_0+\omega_c)(n-\alpha)}-e^{j(-\omega_0-\omega_c)(n-\alpha)}\right]+\left[e^{j(\omega_0+\omega_c)(n-\alpha)}-e^{j(\omega_0-\omega_c)(n-\alpha)}\right]\right\}$$

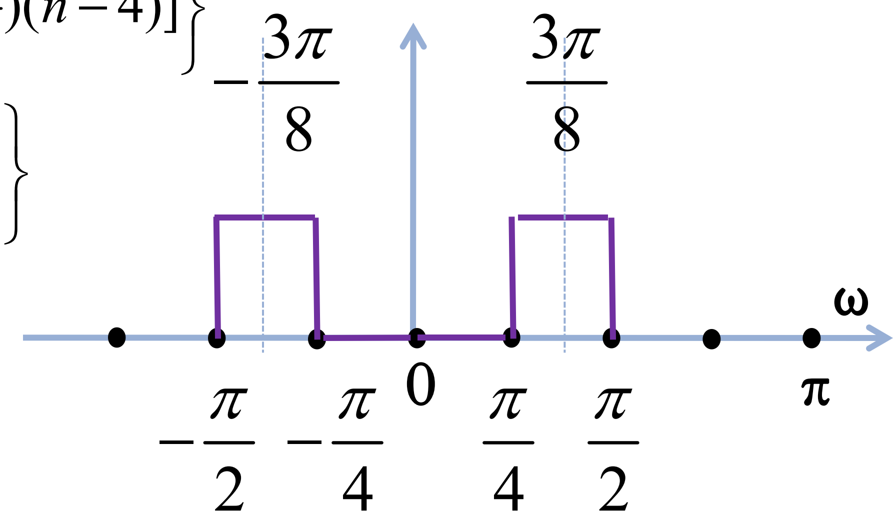
$$=\frac{1}{2\pi}\frac{1}{j(n-4)}\left\{\left[e^{j(-\frac{\pi}{4})(n-4)}-e^{j(-\frac{\pi}{2})(n-4)}\right]+\left[e^{j(\frac{\pi}{2})(n-4)}-e^{j(\frac{\pi}{4})(n-4)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-4)}\left\{\left[e^{j(-\frac{\pi}{4})(n-4)}-e^{j(\frac{\pi}{4})(n-4)}\right]+\left[e^{j(\frac{\pi}{2})(n-4)}-e^{j(-\frac{\pi}{2})(n-4)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-4)}\left\{2j\sin[(-\frac{\pi}{4})(n-4)]+2j\sin[(\frac{\pi}{2})(n-4)]\right\}$$

$$=\frac{1}{2\pi}\frac{2j}{j(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}$$

$$=\frac{1}{\pi(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}$$



$$h(n)=h_d(n)w(n)=\begin{cases}\frac{1}{\pi(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}, & 0\le n\le 8\\ 0, & n\text{为其他}\end{cases}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-4)}\left\{\sin\left[\dfrac{\pi}{2}(n-4)\right]-\sin\left[\dfrac{\pi}{4}(n-4)\right]\right\}, & 0\le n\le 8\\ 0, & n\text{为其他}\end{cases}$$

$$h(0)=0;h(1)=\frac{1}{-3\pi}(1-\frac{\sqrt{2}}{2})=-0.03;$$

$$h(2)=\frac{1}{-2\pi}(-1)=0.16;h(3)=\frac{1}{-\pi}(-1-\frac{\sqrt{2}}{2})=0.54;$$

$$h(4)=\frac{1}{4};$$

$$h(5)=\frac{1}{\pi}(1+\frac{\sqrt{2}}{2})=0.54;h(6)=\frac{1}{2\pi}(1)=0.16;$$

$$h(7)=\frac{1}{3\pi}(-1+\frac{\sqrt{2}}{2})=-0.03;h(8)=0;$$

$$(2)H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}$$

$$=\sum_{n=0}^{8}\frac{1}{\pi(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}z^{-n}$$

$$(3)H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1}h(n)e^{-j\omega n}$$

$$=\sum_{n=0}^{8}\frac{1}{\pi(n-4)}\left\{\sin\left[\frac{\pi}{2}(n-4)\right]-\sin\left[\frac{\pi}{4}(n-4)\right]\right\}e^{-j\omega n}$$

(4)给出滤波器的任意一种结构实现形式

直接型

FIR滤波器设计3-2--往年真题

用矩形窗函数方法设计一个FIR线性相位数字带通滤波器，要求上下边带截止频率分别为$\omega_1 = 0.6\pi$，$\omega_2 = 0.3\pi$，窗长$N = 11$。

(1) 确定滤波器单位抽样响应序列$h(n)$

(2) 确定滤波器的系统函数$H(z)$

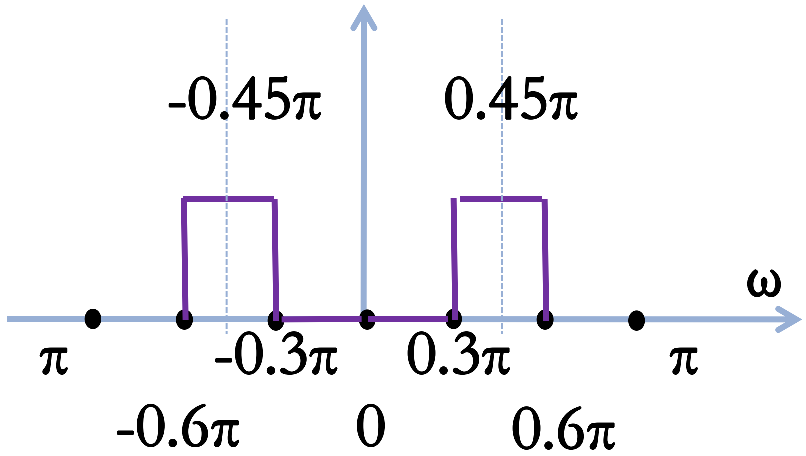
(3)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

解：理想数字带通滤波器的幅频响为

$$H_d(e^{j\omega})=\begin{cases} e^{-j\omega\alpha} & -\omega_c \le \omega \pm \omega_0 \le \omega_c \\ 0 & \text{其他} \end{cases}$$

$$H_d(e^{j\omega})=\begin{cases} e^{-j\omega\alpha} & -0.15\pi \le \omega \pm 0.45\pi \le 0.15\pi \\ 0 & \text{其他} \end{cases}$$



$$\Rightarrow \alpha=\frac{N-1}{2}=\frac{11-1}{2}=5,\ \ \omega_c=0.15\pi，\ \ \omega_0=0.45\pi$$

$$(1) h_d(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(e^{j\omega})e^{j\omega n}d\omega=\frac{1}{2\pi}\int_{-\omega_0-\omega_c}^{-\omega_0+\omega_c}e^{-j\omega\alpha}e^{j\omega n}d\omega+\frac{1}{2\pi}\int_{\omega_0-\omega_c}^{\omega_0+\omega_c}e^{-j\omega\alpha}e^{j\omega n}d\omega$$

(注意积分区间，此时为$-\pi \sim \pi$)

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left(e^{j\omega(n-\alpha)}\Big|_{-\omega_0-\omega_c}^{-\omega_0+\omega_c}+e^{j\omega(n-\alpha)}\Big|_{\omega_0-\omega_c}^{\omega_0+\omega_c}\right)$$

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left\{\left[e^{j(-\omega_0+\omega_c)(n-\alpha)}-e^{j(-\omega_0-\omega_c)(n-\alpha)}\right]+\left[e^{j(\omega_0+\omega_c)(n-\alpha)}-e^{j(\omega_0-\omega_c)(n-\alpha)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left\{\left[e^{j(-\omega_0+\omega_c)(n-\alpha)}-e^{j(-\omega_0-\omega_c)(n-\alpha)}\right]+\left[e^{j(\omega_0+\omega_c)(n-\alpha)}-e^{j(\omega_0-\omega_c)(n-\alpha)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-5)}\left\{\left[e^{j(-0.3\pi)(n-5)}-e^{j(-0.6\pi)(n-5)}\right]+\left[e^{j(0.6\pi)(n-5)}-e^{j(0.3\pi)(n-5)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-5)}\left\{\left[e^{j(-0.3\pi)(n-5)}-e^{j(0.3\pi)(n-5)}\right]+\left[e^{j(0.6\pi)(n-5)}-e^{j(-0.6\pi)(n-5)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-5)}\{2j\sin[(-0.3\pi)(n-5)]+2j\sin[(0.6\pi)(n-5)]\}$$

$$=\frac{1}{2\pi}\frac{2j}{j(n-5)}\left\{\sin\left[\frac{3\pi}{5}(n-5)\right]-\sin\left[\frac{3\pi}{10}(n-5)\right]\right\}$$

$$=\frac{1}{\pi(n-5)}\left\{\sin\left[\frac{3\pi}{5}(n-5)\right]-\sin\left[\frac{3\pi}{10}(n-5)\right]\right\}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-5)}\left\{\sin\left[\dfrac{3\pi}{5}(n-5)\right]-\sin\left[\dfrac{3\pi}{10}(n-5)\right]\right\}, & 0\le n\le 10\\ 0, & n\text{为其他}\end{cases}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-5)}\left\{\sin\left[\dfrac{3\pi}{5}(n-5)\right]-\sin\left[\dfrac{3\pi}{10}(n-5)\right]\right\}, & 0\le n\le 10\\ 0, & n\text{为其他}\end{cases}$$

$$h(0)=\frac{1}{-5\pi}(-1)=0.064;$$

$$h(1)=\frac{1}{-4\pi}\left[\sin(-\frac{12\pi}{5})-\sin(-\frac{12\pi}{10})\right]=\frac{1}{-4\pi}\left[-0.951-0.588\right]=0.123;$$

$$h(2)=\frac{1}{-3\pi}\left[\sin(-\frac{9\pi}{5})-\sin(-\frac{9\pi}{10})\right]=\frac{1}{-3\pi}\left[0.588+0.309\right]=-0.095;$$

$$h(3)=\frac{1}{-2\pi}\left[\sin(-\frac{6\pi}{5})-\sin(-\frac{6\pi}{10})\right]=\frac{1}{-2\pi}\left[0.588+0.951\right]=-0.245;$$

$$h(4)=\frac{1}{-\pi}\left[\sin(-\frac{3\pi}{5})-\sin(-\frac{3\pi}{10})\right]=\frac{1}{-\pi}\left[-0.951+0.809\right]=0.045;$$

$$h(5)=\left(\frac{1}{\pi(n-5)}\left\{2\sin\left[\frac{3\pi}{5}(n-5)\right]-\sin\left[\frac{3\pi}{10}(n-5)\right]\right\}\right)'=\frac{3}{10};$$

$$h(6)=0.045;h(7)=-0.245;h(8)=-0.095;h(9)=0.123;h(10)=0.064$$

$$(2)H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}$$

$$=\sum_{n=0}^{10}\frac{1}{\pi(n-5)}\left\{\sin\left[\frac{3\pi}{5}(n-5)\right]-\sin\left[\frac{3\pi}{10}(n-5)\right]\right\}z^{-n}$$

$$(3)H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1}h(n)e^{-j\omega n}$$

$$=\sum_{n=0}^{10}\frac{1}{\pi(n-5)}\left\{\sin\left[\frac{3\pi}{5}(n-5)\right]-\sin\left[\frac{3\pi}{10}(n-5)\right]\right\}e^{-j\omega n}$$

(4)给出滤波器的任意一种结构实现形式

直接型

FIR滤波器设计4

用矩形窗函数方法设计一个FIR线性相位数字带阻滤波器，要求上下边带截止频率分别为0.6π和0.3π，窗长$N=11$。

(1) 确定滤波器单位抽样响应序列$h(n)$

(2) 确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

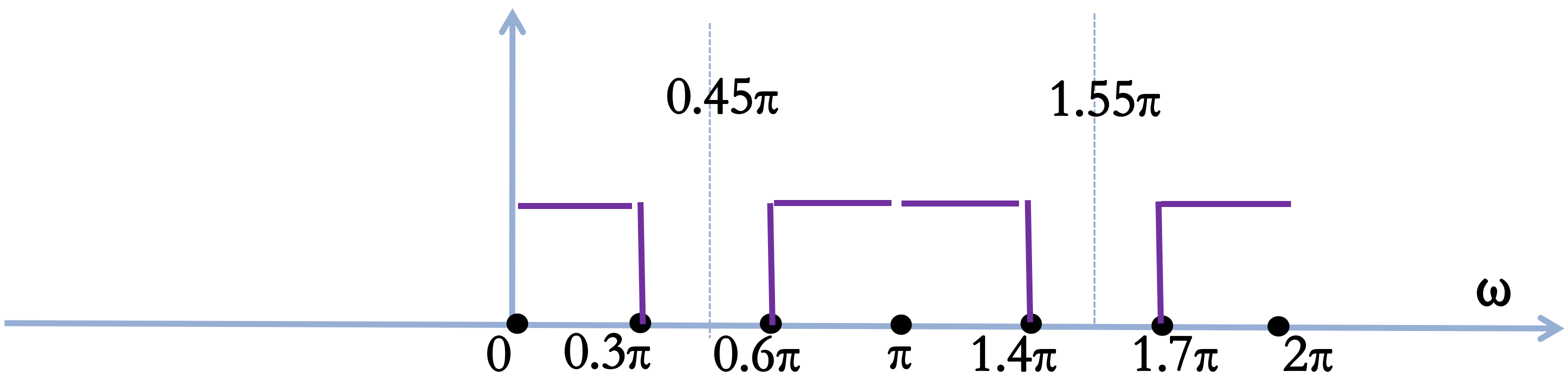
注：四舍五入到小数点后2位

解：理想数字带通滤波器的幅频响为

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & 0 \le \omega \le \omega_{c1};\ \ 2\pi - \omega_{c1} \le \omega \le 2\pi;\ \ \pi - \omega_{c2} \le \omega \le \pi + \omega_{c2}; \\ 0 & \text{其他} \end{cases}$$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & 0 \le \omega \le 0.3\pi;\ 1.7 \le \omega \le 2\pi;\ 0.6\pi \le \omega \le 1.4\pi; \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow \alpha = \frac{N-1}{2} = \frac{11-1}{2} = 5,\ \ \omega_{c1} = 0.3\pi,\ \ \omega_{c2} = 0.4\pi$$



$$H_d(e^{j\omega})=\begin{cases} e^{-j\omega\alpha} & 0\le\omega\le 0.3\pi;\ 1.7\le\omega\le 2\pi;\ 0.6\pi\le\omega\le 1.4\pi; \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow \alpha=\frac{N-1}{2}=\frac{11-1}{2}=5,\ \ \omega_{c1}=0.3\pi,\ \ \omega_{c2}=0.4\pi$$

$$(1)h_d(n)=\frac{1}{2\pi}\int_0^{2\pi}H_d(e^{j\omega})e^{j\omega n}d\omega=\frac{1}{2\pi}\int_0^{\omega_{c1}}e^{-j\omega\alpha}e^{j\omega n}d\omega$$

$$+\frac{1}{2\pi}\int_{\pi-\omega_{c2}}^{\pi+\omega_{c2}}e^{-j\omega\alpha}e^{j\omega n}d\omega+\frac{1}{2\pi}\int_{2\pi-\omega_{c1}}^{2\pi}e^{-j\omega\alpha}e^{j\omega n}d\omega$$

(注意积分区间，此时为$0\sim 2\pi$)

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left(e^{j\omega(n-\alpha)}\Big|_0^{\omega_{c1}}+e^{j\omega(n-\alpha)}\Big|_{\pi-\omega_{c2}}^{\pi+\omega_{c2}}+e^{j\omega(n-\alpha)}\Big|_{2\pi-\omega_{c1}}^{2\pi}\right)$$

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left\{\left[e^{j\omega_{c1}(n-\alpha)}-1\right]+\left[e^{j(\pi+\omega_{c2})(n-\alpha)}-e^{j(\pi-\omega_{c2})(n-\alpha)}\right]+\left[1-e^{j(2\pi-\omega_{c1})(n-\alpha)}\right]\right\}$$

$$=\frac{1}{2\pi}\frac{1}{j(n-\alpha)}\left\{\left[e^{j\omega_{c1}(n-\alpha)}-e^{-j\omega_{c1}(n-\alpha)}\right]+\left[e^{j(\pi+\omega_{c2})(n-\alpha)}-e^{j(\pi-\omega_{c2})(n-\alpha)}\right]\right\}$$

$$=\frac{\sin[\omega_{c1}(n-\alpha)]}{\pi(n-\alpha)}+\frac{(-1)^{(n-\alpha)}\sin[\omega_{c2}(n-\alpha)]}{\pi(n-\alpha)}$$

$$h_d(n)=\frac{\sin[\omega_{c1}(n-\alpha)]}{\pi(n-\alpha)}+\frac{(-1)^{(n-\alpha)}\sin[\omega_{c2}(n-\alpha)]}{\pi(n-\alpha)}$$

$$=\frac{1}{\pi(n-5)}\left\{\sin\left[0.3\pi(n-5)\right]+(-1)^{n-5}\sin\left[0.4\pi(n-5)\right]\right\}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-5)}\left\{\sin\left[0.3\pi(n-5)\right]+(-1)^{n-5}\sin\left[0.4\pi(n-5)\right]\right\}, & 0\le n\le 10\\ 0, & n\text{为其他}\end{cases}$$

$$h(n)=h_d(n)w(n)=\begin{cases}\dfrac{1}{\pi(n-5)}\left\{\sin\left[0.3\pi(n-5)\right]+(-1)^{n-5}\sin\left[0.4\pi(n-5)\right]\right\}, & 0\le n\le 10\\ 0, & n\text{为其他}\end{cases}$$

$$h(0)=\frac{1}{-5\pi}\left[1+0\right]=-0.064;$$

$$h(1)=\frac{1}{-4\pi}\left[\sin(-1.2\pi)+\sin(-1.6\pi)\right]=\frac{1}{-4\pi}\left[0.588+0.951\right]=-0.123;$$

$$h(2)=\frac{1}{-3\pi}\left[\sin(-0.9\pi)-\sin(-1.2\pi)\right]=\frac{1}{-3\pi}\left[-0.309-0.588\right]=0.095;$$

$$h(3)=\frac{1}{-2\pi}\left[\sin(-0.6\pi)+\sin(-0.8\pi)\right]=\frac{1}{-2\pi}\left[-0.951-0.588\right]=0.245;$$

$$h(4)=\frac{1}{-\pi}\left[\sin(-0.3\pi)-\sin(-0.4\pi)\right]=\frac{1}{-\pi}\left[0.809+0.951\right]=-0.045;$$

$$h(5)=\left(\frac{1}{\pi(n-5)}\left\{\sin\left[0.3\pi(n-5)\right]+(-1)^{n-5}\sin\left[0.4\pi(n-5)\right]\right\}\right)'\Bigg|=0.3+0.4=0.7;$$

$$h(6)=-0.045;h(7)=0.245;h(8)=0.095;h(9)=-0.123;h(10)=-0.064$$

$$(2)H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}$$

$$=\sum_{n=0}^{10}\left(\frac{1}{\pi(n-5)}\left\{\sin\left[0.3\pi(n-5)\right]+(-1)^{n-5}\sin\left[0.4\pi(n-5)\right]\right\}\right)z^{-n}$$

$$(3)H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}=\sum_{n=0}^{N-1}h(n)e^{-j\omega n}$$

$$=\sum_{n=0}^{10}\left(\frac{1}{\pi(n-5)}\left\{\sin\left[0.3\pi(n-5)\right]+(-1)^{n-5}\sin\left[0.4\pi(n-5)\right]\right\}\right)e^{-j\omega n}$$

(4)给出滤波器的任意一种结构实现形式

直接型