Recitation 1
Background

CMU 10-601: Machine Learning (Spring 2019)

Friday, Jan 18th, 2019

1 Probability and Statistics

1. Two events, A and B, are considered disjoint (mutually exclusive). $P(A) = 0.5$, $P(B) = 0.5$.

   ☐ What is the $P(A \cup B)$?

   ☐ What is the $P(A \cap B)$?

   ☐ What is the $P(A|B)$?

2. Now, instead, the two events A and B are not disjoint, but they are independent.

   ☐ What is the $P(A \cup B)$?

   ☐ What is the $P(A \cap B)$?

   ☐ What is the $P(A|B)$?

3. A student is looking at her activity tracker (Fitbit/Apple Watch) data and she notices that she seems to sleep better on days that she exercises. They observe the following:

Exercise	Good Sleep	Probability
Yes	Yes	0.3
Yes	No	0.2
No	No	0.4
No	Yes	0.1

   ☐ What is the $P(GoodSleep = Yes|Exercise = Yes)$?

   ☐ Why doesn't $P(GoodSleep = Yes \cap Exercise = Yes) = P(GoodSleep = Yes) \cdot P(Exercise = Yes)$?

   ☐ The student merges her activity tracker data with her food logs and finds that the $P(Eatwell = Yes|Exercise = Yes \cap GoodSleep = Yes)$ is 0.25. What is the probability of all three happening on the same day?

4. What is the E[X] where X is a single roll of a fair 6-sided dice (S = {1,2,3,4,5,6})? What is the Var[X]?

5. Imagine that we had a new dice where the sides were S = {3,4,5,6,7,8}. How does the E[X] and Var[X] compare to our original dice?

2 Calculus

1. If $f(x) = x^3 \sin x$, find $f'(x)$.
2. If $f(x, y) = (2 - x)^2 + 4x^3y - 2$, evaluate $\frac{\partial f(x,y)}{\partial x}$ at the point $(1, 2)$.
3. Find $\frac{\partial}{\partial w_j}\mathbf{x}^T\mathbf{w}$.

3 Vectors, Matrices, and Geometry

1. **Inner Product:** $\mathbf{u} = \begin{bmatrix}6 & 1 & 2\end{bmatrix}, \mathbf{v} = \begin{bmatrix}3 & -10 & -2\end{bmatrix}$, what is the inner product of $\mathbf{u}$ and $\mathbf{v}$? What is the geometric interpretation?
2. **Cauchy-Schwarz inequality** (Optional): Given $\mathbf{u} = \begin{bmatrix}3 & 1 & 2\end{bmatrix}, \mathbf{v} = \begin{bmatrix}3 & -1 & 4\end{bmatrix}$, what is $||\mathbf{u}||_2$ and $||\mathbf{v}||_2$? What is $\mathbf{u} \cdot \mathbf{v}$? How do $\mathbf{u} \cdot \mathbf{v}$ and $||\mathbf{u}||_2||\mathbf{v}||_2$ compare? Is this always true?
3. **Matrix algebra.** Most generally, $(AB)_{ij} = \sum_k A_{ik}B_{kj}$, if $\mathbf{A} \in \mathbb{R}^{m\times n}$ and $\mathbf{B} \in \mathbb{R}^{n\times p}$, then $\mathbf{AB} \in \mathbb{R}^{m\times p}$.

   Given $\mathbf{A} = \begin{bmatrix}1 & 2 & 5\\ 0 & 2 & 2\\ 0 & 0 & 4\end{bmatrix}, \mathbf{B} = \begin{bmatrix}4 & -3 & 2\\ 1 & 1 & -1\\ 3 & -2 & 2\end{bmatrix}, \mathbf{u} = \begin{bmatrix}1\\ 2\\ 5\end{bmatrix}, \mathbf{v} = \begin{bmatrix}3\\ 2\\ 1\end{bmatrix}$

   - What is $\mathbf{AB}$? What about $\mathbf{Bu}$?
   - What is $tr(\mathbf{A})$, $det(\mathbf{A})$, and rank of $\mathbf{A}$? What is $tr(\mathbf{AB})$ and $tr(\mathbf{BA})$?
   - What is $\mathbf{A}^T$?
   - Calculate $\mathbf{uv}^T$.
   - What are the eigenvalues of $\mathbf{A}$? (Optional) How do you calculate eigenvalues in general for square matrices?
4. **Positive Definiteness:** (Optional) Given $\mathbf{A} = \begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}, \mathbf{v} = \begin{bmatrix}3\\ 0\\ -2\end{bmatrix}$, what is $\mathbf{v}^T\mathbf{Av}$? Is the result positve/zero/negative? Is this true for all vectors in $\mathbb{R}^3$? Why? (Hint: anything special about the eigenvalues of $\mathbf{A}$?)
5. **Geometry:** Given a linear function $2x + y = 2$,
   - If a given point (x_1, y_1) satisfies $2x_1 + y_1 > 2$, where does it lie relative to the line?

- What is the relationship of vector $\mathbf{v} = (2, 1)$ to this line?
- What is the distance of point $(1, 2)$ to this line?

4 CS Fundamentals

1. For each (f, g) functions below, is $f(n) = \mathcal{O}(g(n))$ or $g(n) = \mathcal{O}(f(n))$ or both?
   - ☐ $f(n) = ln(n)$, $g(n) = log_2(n)$
   - ☐ $f(n) = \frac{n}{50}$, $g(n) = log_{10}(n)$
   - ☐ $f(n) = n^{50}$, $g(n) = 50^n$
2. Find the DFS traversal and BFS traversal of the following binary tree.