1 Written Questions [25pts]

Answer the following questions in the HW2 solutions template provided. DO NOT show your work. Then upload your solutions to Gradescope.

1.1 Warm-up

First, let's think a little bit about decision trees. The following dataset consists of 7 examples, each with 3 attributes, (A, B, C), and a label, Y.

A	B	C	Y
1	1	0	0
1	1	2	1
1	0	0	1
1	1	2	1
0	0	2	0
0	1	1	0
0	0	0	0

Use the data above to answer the following questions.

A few important notes:

- All calculations should be done without rounding! After you have finished all of your calculations, write your rounded solutions in the boxes below.
- Note that, throughout this homework, we will use the convention that the leaves of the trees do not count as nodes, and as such are not included in calculations of depth and number of splits. (For example, a tree which classifies the data based on the value of a single attribute will have depth 1, and contain 1 split.)

l.	[1pt] What is the entropy of Y in bits, $H(Y)$? In this and subsequent questions, when we request the
	units in bits, this simply means that you need to use log base 2 in your calculations. (Please include
	one number rounded to the fourth decimal place, e.g. 0.1234)
2.	[1pt] What is the mutual information of Y and A in bits, $I(Y;A)$? (Please include one number rounded to the fourth decimal place, e.g. 0.1234)

¹If instead you used \log base e, the units would be *nats*; \log base 10 gives *bats*.

²In the context of decision trees, and therefore in this assignment, the terms "information gain" and "mutual information" are synonymous (e.g. they have the same meaning as one another). However, in information theory and machine learning in general, "information gain" is synonymous with "Kullback-Leibler (K-L) divergence", often referred to as relative entropy, which is not the same as mutual information. For more information, go here.

rounded to the fourth decimal place, e.g. 0.1234)
Tounded to the Tourin decimal place, e.g. 0.1254)
[1pt] What is the mutual information of Y and C in bits, $I(Y;C)$? (Please include one number rounded to the fourth decimal place, e.g. 0.1234)
[1pt] Consider the dataset given above. Which attribute $(A, B, \text{ or } C)$ would a decision tree algorithm pick first to branch on, if its splitting criterion is mutual information?
Select one:
$\bigcirc A$
$\bigcirc B$
$\bigcirc C$
[1pt] Consider the dataset given above. Which is the second attribute you would pick to branch on if its splitting criterion is mutual information? (<i>Hint:</i> Notice that this question correctly presuppose that there is <i>exactly one</i> second attribute.)
Select one:
$\bigcirc A$
$\bigcirc B$
\bigcirc C
[1pt] If the same algorithm continues until the tree perfectly classifies the data, what would the depth of the tree be?

Solution			

1.2 Empirical Questions

The following questions should be completed as you work through the programming portion of this assignment (Section ??).

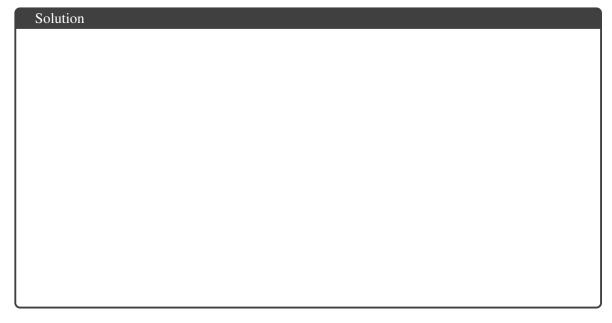
9. [2pt] Train and test your decision tree on the politician dataset and the education dataset with four different values of max-depth, {0,1,2,4}. Report your findings in the HW2 solutions template provided. A Decision Tree with max-depth 0 is simply a *majority vote classifier*; a Decision Tree with max-depth 1 is called a *decision stump*. If desired, you could even check that your answers for these two simple cases are correct using your favorite spreadsheet application (e.g. Excel, Google Sheets). (Please round each number to the fourth decimal place, e.g. 0.1234)

Dataset	Max-Depth	Train Error	Test Error
politician	0		
politician	1		
politician	2		
politician	4		
education	0		
education	1		
education	2		
education	4		

10. [3pt] For the politicians dataset, create a computer-generated plot showing error on the y-axis against depth of the tree on the x-axis. Plot *both* training error and testing error, clearly labeling which is which. That is, for each possible value of max-depth $(0, 1, 2, \ldots)$, up to the number of attributes in the dataset), you should train a decision tree and report train/test error of the model's predictions.

Solution		
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11.	[2pt] Suppose your research advisor asks you to run some model selection experiments and then
	report your results. You select the Decision Tree model's max-depth to be the one with lowest test
	error in metrics.txt and then report that model's test error as the performance of our classifier on held
	out test data. Is this a good experimental setup? If so, why? If not, why not?



12. **[2pt]** In this assignment, we used max-depth as our stopping criterion, and as a mechanism to prevent overfitting. Alternatively, we could stop splitting a node whenever the mutual information for the best attribute is lower than a threshold value. This threshold would be another hyperparameter. Theoretically, how would increasing this threshold value affect the number of nodes and depth of the learned trees?



Solution		

13. [2pt] From question 12, how would you set-up model training to choose the threshold value?

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15.	After you have completed all other components of this assignment, report your answers to the collab-
	oration policy questions detailed in the Academic Integrity Policies found here. (a) Did you receive
	any help whatsoever from anyone in solving this assignment? Is so, include full details. (b) Did you
	give any help whatsoever to anyone in solving this assignment? Is so, include full details. (c) Did you
	find or come across code that implements any part of this assignment? If so, include full details.

Solution	