

長庚大學期中、期末考試答案用紙

科目

學年度 第 學期 考

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[1] (a)  $f_z(z) = \frac{1}{\sqrt{\pi}} e^{-\frac{1}{\pi}(z-1)^2} = \frac{1}{\sqrt{\pi}} e^{-\frac{1}{\pi}z^2}$

(b)  $P(-1 \leq z \leq 1) = \text{st.norm.cdf}(-1, 0, 1) - 0.6827$

(c)  $x = \text{st.norm.ppf}(0.975, 0, 1) = 1.96$

(d)  $f_Q(q) = \frac{1}{\sqrt{2\pi}q} e^{-\frac{q^2}{2}} \cdot e^{-\frac{q^2}{2}} = \frac{1}{\sqrt{2\pi}q} \cdot e^{-\frac{q^2}{2}}$

$f_Q(q) = \begin{cases} \frac{1}{\sqrt{2\pi}q} e^{-\frac{q^2}{2}}, & q > 0 \\ 0, & q \leq 0 \end{cases}$

(e)  $E[Q] = 1$

(f)  $\text{std}[Q] = \sqrt{2-1} = 1$

(g)  $P(Q \leq 1) = \text{st.gamma.cdf}(x=1, a=0.5, \text{scale}=2) = 0.6827$

[2] (a)  $f_T(t) = \begin{cases} e^{-t} & t > 0 \\ 0 & t \leq 0 \end{cases} \quad (\alpha=1, \beta=1)$

(b)  $E[T] = \beta = 1$

(c)  $\text{std}[T] = \sqrt{\beta^2} = 1$

(d)  $P(T > 1) = \text{st.gamma.sf}(x=1, a=1, \text{scale}=1) = 0.3679$

(e)  $\alpha=3, \beta=1$

$f_{T_2}(t) = \frac{1}{\Gamma(3)} t^{3-1} e^{-t} = \frac{1}{2} t^2 e^{-t}$

$f_{T_2}(t) = \begin{cases} \frac{1}{2} t^2 e^{-t}, & t > 0 \\ 0, & t \leq 0 \end{cases}$

(f)  $E[T_2] = \alpha\beta = 3 \cdot 1 = 3$

(g)  $\text{std}[T_2] = \sqrt{\alpha\beta^2} = \sqrt{3 \cdot 1^2} = \sqrt{3}$

(h)  $P(T_2 > 3) = \text{st.gamma.sf}(x=3, a=3, \text{scale}=1) = 0.4232$

(i)  $P(T_2 > 7) = \text{st.gamma.sf}(x=7, a=3, \text{scale}=1) = 0.0296$  假設成立