

長庚大學期中、期末考試答案用紙

學年度 第 學期 考 卷 1 系 姓名 張 浩 宇 學號 B0729022

3. (a) Convolution Theorem 中文名為 迴積分定理

是傅立葉變換滿足的一個重要性質

$$\begin{aligned} (b) \mathcal{F}[f(t) * g(t)] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) g(t-\tau) d\tau e^{-i\omega t} dt \\ &= \int_{-\infty}^{\infty} f(\tau) \int_{-\infty}^{\infty} g(t-\tau) e^{-i\omega t} dt d\tau \\ &= \int_{-\infty}^{\infty} f(\tau) \int_{-\infty}^{\infty} g(x) e^{-i\omega(x+\tau)} dx d\tau \\ &= \int_{-\infty}^{\infty} g(x) e^{-i\omega x} dx \int_{-\infty}^{\infty} f(\tau) e^{-i\omega \tau} d\tau \\ &= G(\omega) \cdot F(\omega) \end{aligned}$$

```
1. import math
def iexp(n):
    return complex(math.cos(n), math.sin(n))
def is_pow2(n):
    return False if n==0 else (n==1 or is_pow2(n>>1))
def dft(xs):
    "naive dft"
    n = len(xs)
    return [sum(xs[k] * iexp(-2 * math.pi * i * k / n) for k in range(n))
            for i in range(n)]
def dft_inv(xs):
    "naive dft"
    n = len(xs)
    return [sum((xs[k] * iexp(2 * math.pi * i * k / n) for k in range(n))) / n
            for i in range(n)]
if __name__ == "__main__":
    wave1 = [1, 0, 0, 0, 0, 0, 0, 0]
    wave2 = [1, 1, 1, 1, 1, 1, 1, 1]
    wave3 = [1, -1, 1, -1, 1, -1, 1, -1]
    wave4 = [3, 0, 2, 0, 2, 0, 2, 0]
    dfreq1 = dft(wave1)
    dfreq2 = dft(wave2)
    dfreq3 = dft(wave3)
    dfreq4 = dft(wave4)
    dfreq5 = [1, 1, 0, 0, 0, 0, 1, 1]
    dfreq6 = [1, 1, 0, 0, 0, 0, 1, 1]
    wave5 = dft_inv(dfreq5)
    wave6 = dft_inv(dfreq6)
    print(dfreq1)
    print(dfreq2)
    print(dfreq3)
    print(dfreq4)
    print(wave5)
    print(wave6)
```

→ 背面

(請翻面繼續作答)

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科目

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(a) $[(1+0j), (1+0j), (1+0j), (1+0j), (1+0j), (1+0j), (1+0j), (1+0j)]$

(b) $[(8+0j), (-5.5511...e-16+2.2204...e-16j), (-4.28623...e-16-4.440892...e-16j), (-2.2204...e-16+8.881784...e-16j), (-4.89858719...e-16j), (-2.109423...e-15-1.2212453...e-15j), (2.293296835...e-15-6.66133814...e-16j), (-5.2180482153...e-15-2.664535...e-15j)]$

(c) $[0j, (1.11022...e-16-1.11022...e-16j), (9.555947...e-17-1.110222...e-16j), (8.8817...e-16-1.5554...e-16j), (8+7.429011037...e-15j), (-2.664535...e-15+1.110223...e-16j), (2.932968...e-15-6.661338...e-16j), (-5.218048...e-15-2.6645352...e-15j)]$

(d) $[(9+0j), (0.9999...7-4.440892...e-16j), (1-4.898587...e-16j), (1.00...7-6.661338...e-16j), (9+2.93291523...e-15j), (0.999...5-1.3322676...e-15j), (1-1.469576...e-15j), (0.999...81-1.37644...e-15j)]$

(e)

(f) $x_6[n]$ 為逆變換序列

(g) $x_7[k] = [2, -0j, 1, -1j, 0, -0j, 1, +1j]$

$x_8[k] = [2, -0j, 1, 707-0.707j, 1, -1j, 0, 293-0.707j, 0, -0j, 0, 293+0.707j, 1, +1j, 1, 707+0.707j]$

4. $x = \text{np.array}([1, 1, 0, 0, 0, 0, 0, 1])$

$x = \text{fp.fft}(x)$

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$N = x.size$

$n = \text{np.arange}(N).reshape(-1, 1)$

$k = \text{np.arange}(N).reshape(1, -1)$

$\text{dft Mat} = \text{np.exp}(-1j * 2 * \pi / N * n @ k)$

dft Mat