AUV Docking System Localization using Model-Based Estimation

Xinyu Liu

MAE 6760 Final Project

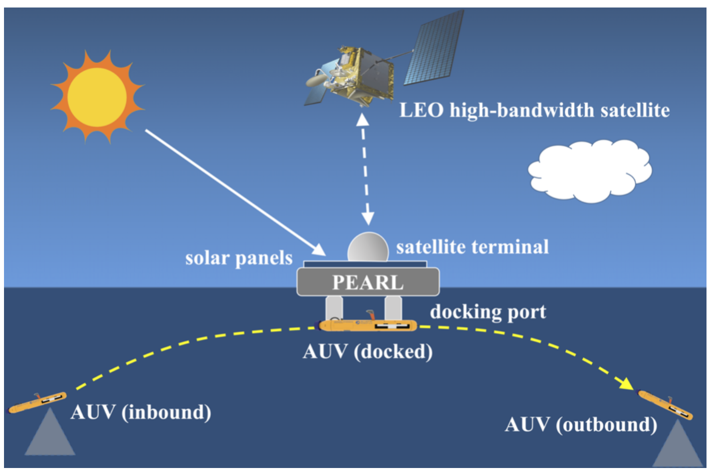


Figure 1: Overview[1]

1 Introduction

Autonomous Underwater Vehicle(AUV) missions are limited in range and duration by the vehicle's battery capacity. AUVs need to recharge and offload data frequently, and this process can cost more than $30,000 per day[1][2]. A solar-powered AUV charging platform is designed for expanding AUV's duration and range.

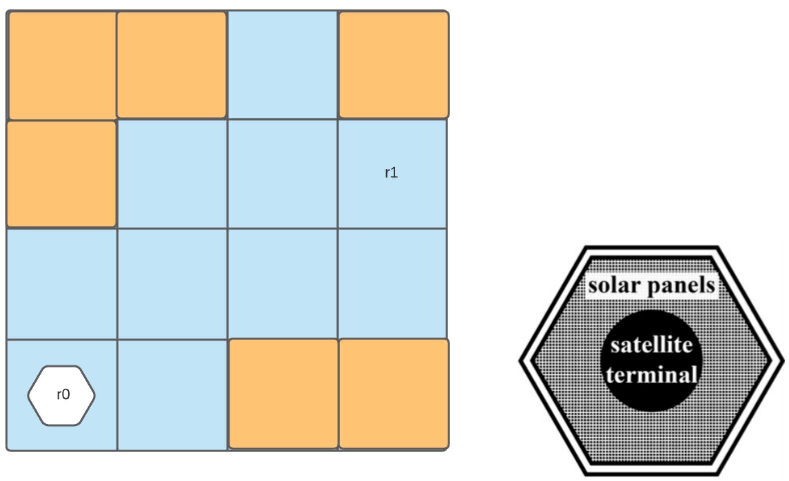
Suppose an AUV is conducting underwater surveillance under an archipelago. We have known an area of water and the distribution of obstacles. And an AUV charging platform is floating on the water surface in this area most of the time to remain stationary, and when it detects that the power of AUV is below a certain value, it will start to move in the direction of AUV and prepare for charging. I will use *robot* in this project to substitute the AUV charging platform to save words.

This project aims to design a path from the robot and the AUV charging area while avoiding obstacles. I then used two methods to localize the robot: Least squares estimation using range measurements to some way-points and particle filter.

2 Robot Work-space and Dynamics Model

Consider a work-space depicted in Figure. The blue parts are the water surface and the orange parts are obstacles. We want the robot to move from r_0 to r_1.

The top view of the Charging Dock is a hexagon, as shown in the Figure(2,b) But shape and size don't affect our analysis and execution of specification here, treat it as a point-mass model, and the four-state continuous time dynamics are given as



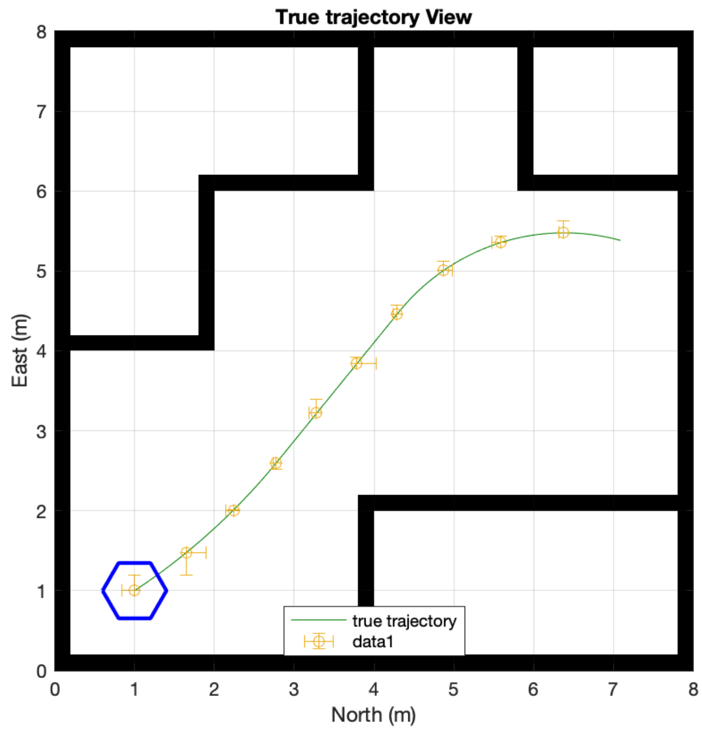
(a) (b)

Figure 2: (a) System model (b) Robot top view[1]

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{V} \\ \theta dot \end{bmatrix} = \begin{bmatrix} V cos\theta \\ V sin\theta \\ u_{acc} + \omega_{acc} \\ u_{rg} + \omega_{rg} \end{bmatrix}$$

3 Least Square Estimation

Consider localizing the robot using range measurements to three points in the obstacles. A(2,4), B(6,6), C(4,2). The true locations of the robot are sampled



every 0.1 seconds for 100 times, and three range measurements RA, RB, RC are collected at any one time. Consider Gaussian white noise in this case.

$z_k = \begin{bmatrix} R^A \\ R^B \\ R^C \end{bmatrix} + v_k$

Noise covariance $R_{vk} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The 2D location estimate can be calculated via nonlinear least squares after 10 measurements. I didn't plot the whole estimation trajectory because it looks very messy. So I chose 10 points in the true trajectory to plot the error bar between the true location and estimation location. Here are some examples.

At t=5s $(\hat{x},\hat{y})$=(1.1190,1.3564) Ture location (x,y)= (1.2635,1.1815)
At t=30s $(\hat{x},\hat{y})$=(2.6375,2.6163) Ture location (x,y)= (2.7258,2.5383)
At t=70s $(\hat{x},\hat{y})$=(4.6206,5.6778) Ture location (x,y)= (4.8076,4.9605)

4 Particle Filter Estimation

Particle filter is a probability-based estimator, a completely nonlinear state estimator.The particle filter was invented to implement the Bayesian estimator numerically, and good for non-Gaussian model. [3]

4.1 Measurements and Sensors

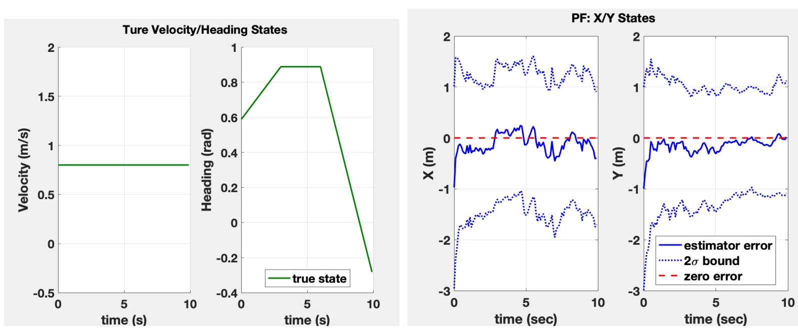
In the particle filter estimation part, creates 2D GPS measurements.

R=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$z_{gps} = \begin{bmatrix} x \\ y \end{bmatrix} + v_{gps}$

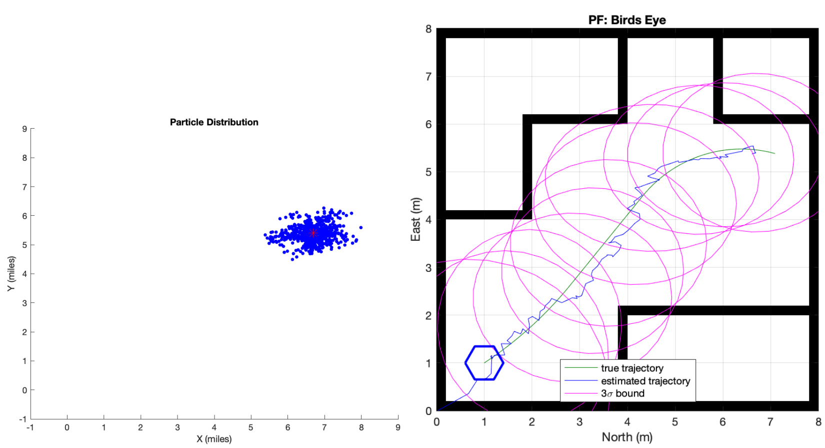
Two sensor models are used here, one is the obstacle detector, and the other is odometry. Sensors are corrupted by white noise.

4.2 Results



(a) (b)

Figure 3: (a) True velocity and heading (b) PF estimate error



(a) (b)

Figure 4: (a) PF simulation (b) Estimated trajectory

Here is the full Particle Filter simulation video link:
https://youtu.be/JrhgPvzQit0

5 Conclusion and Discussion

Comparing these two estimation methods, it is impossible to say which one is more accurate from the simulation results. The least square estimation has less calculation effort and is suitable for a simple system with no disturbance or sensor noise. The noise covariance matrix R and sample numbers n also influence the results. Estimate state using particle filter can apply to many complex system models while it has computational challenges and is not great for low probability elements.

This project can be optimized in many ways. For instance, at LS estimation, would it be better to choose three different waypoints at each sampling time rather than using the same waypoints at all times. Moreover, the particle filter results here have relatively large errors. How to design and optimize the sensor model "obstacle detector" in particle filter estimation in order to make the results more accurate?

6 Reference

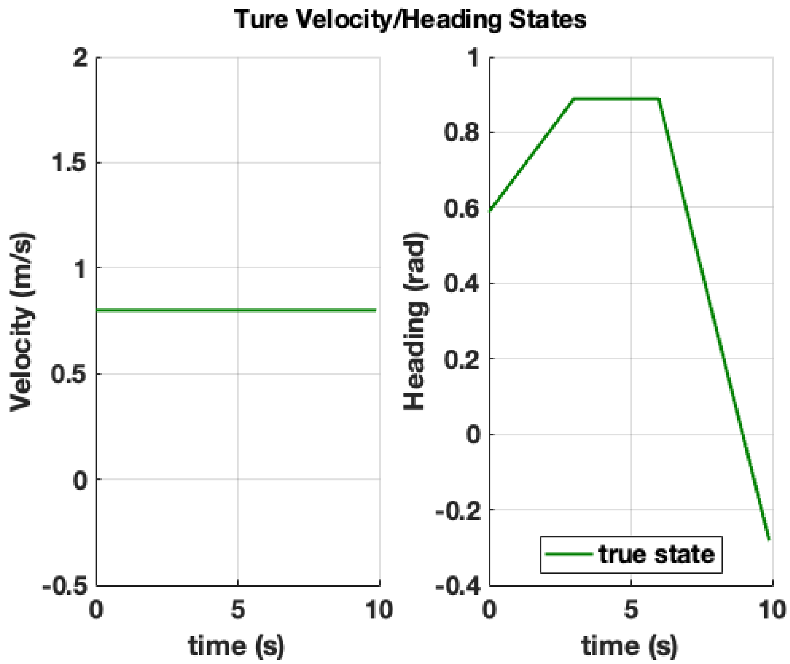
[1]Haji, M., Tran, J., Norheim, J., Weck, O. D. (2021). Design and Testing of Auv Docking Modules for a Renewably Powered Offshore Auv Servicing Platform. doi:10.1115/1.0000908v
[2]Podder, T., Sibenac, M., Bellingham, J. (2004). AUV docking system for sustainable science missions. IEEE International Conference on Robotics and Automation, 2004. Proceedings. ICRA 04. 2004. doi:10.1109/robot.2004.1302423
[3]Simon, D. (2006). Optimal state estimation: Kalman, H and nonlinear approaches. Hoboken: Wiley-Interscience. Page 461

Contents


```
%MAE 6760 Model Based Estimation %   M. Campbell
%Xinyu Liu
%Final Project
```

Problem Set--up: Define Scenario, Simulate no noise case

```
clear all;close all;
rng('default')
video_name='PF_Gaussian1.mp4';
record_video=0;
%Define trajectory
Uacc = [zeros(1,100)];
Uomega = [ones(1,30)*0.1 zeros(1,30) -ones(1,40)*0.3];
%simulate no noise system
[Xnonoise,n,t,dt,nt]=simulate_robot(Uacc,Uomega);
nk=nt;
plot_birdseyeview(Xnonoise,[],[],'True trajectory View');
% %calculate est error and plot errorbar
% i=1;xe=zeros(2,100)
% for i=(1:99)
% xe(:,i)=Elocation(Xnonoise,i)
% i=i+1
% end
nx=1:10:100;
x=Xnonoise(1,nx);
y=Xnonoise(2,nx);
% errorx=xe(1,nx)-x;
% errory=xe(2,nx)-y;
xpos=[0,1.1554,0,0,0,0.79,0,0.558,0,0];
xneg=[0.422,0,0.3021,0.0755,0.2741,0,0.0494,0,0.3838,0.1087];
ypos=[0.5887,0,0.0354,0,0.5228,0.1143,0.3003,0.1914,0.2372,0.4706];
yneg=[0,0.8959,0,0.2683,0,0,0,0,0,0];
hold on
errorbar(x,y,yneg,ypos,xneg,xpos,'o')
ii_plot=[3 4];
plot_estimator(t,Xnonoise,[],[],ii_plot,'Ture Velocity/Heading States');
%[xe]=Elocation(Xnonoise,10)
```



Create IMU measurements (accel, rate gyro) and 2D GPS

```
nw=2;
Qw=diag([0.1^2 0.04^2]);
Qsq=sqrtm(Qw);
w=sqrtm(Qw)*randn(nw,nk);
Zacc=Uacc+w(1,:);
Zrg=Uomega+w(2,:);
%Create 2D GPS like measurements
nz=2;
R=eye(nz)*1^2;
v=sqrtm(R)*randn(nz,nt);
Z=[Xnonoise(1:2,:)] + v;
Hgps=[eye(2) zeros(2,2)]; %output matrix is linear
```

SENSOR MODEL: Obstalce detector

```
mx=0;
sx=1;
pdfx = makedist('Normal',mx,sx^2); %creates a probability distribution object
Lx =makedist('Normal',0,sx^2);
```

SENSOR MODEL: Odometry

```
my=0;
sy=1;
pdfy = makedist('Normal',my,sy);%sensor likelihood model, assuming my is correct
Ly = makedist('Normal',0,sy);
```

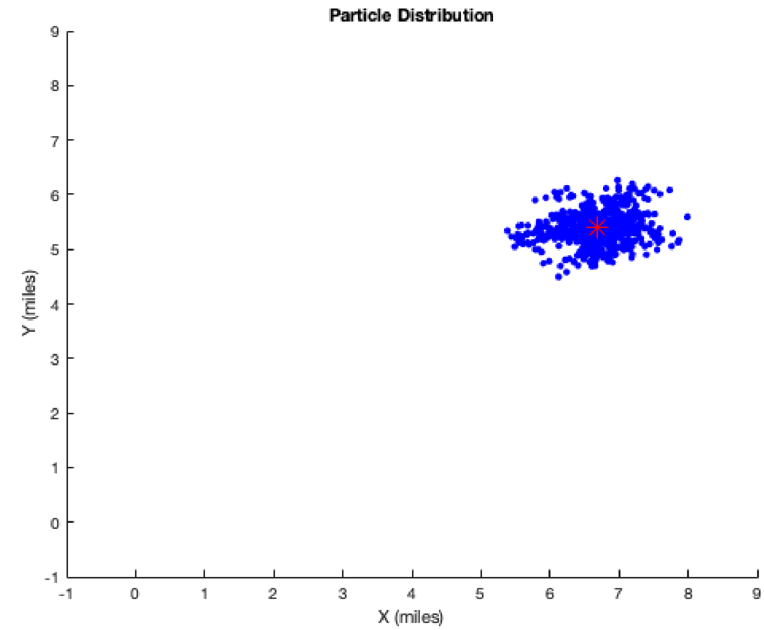
PF

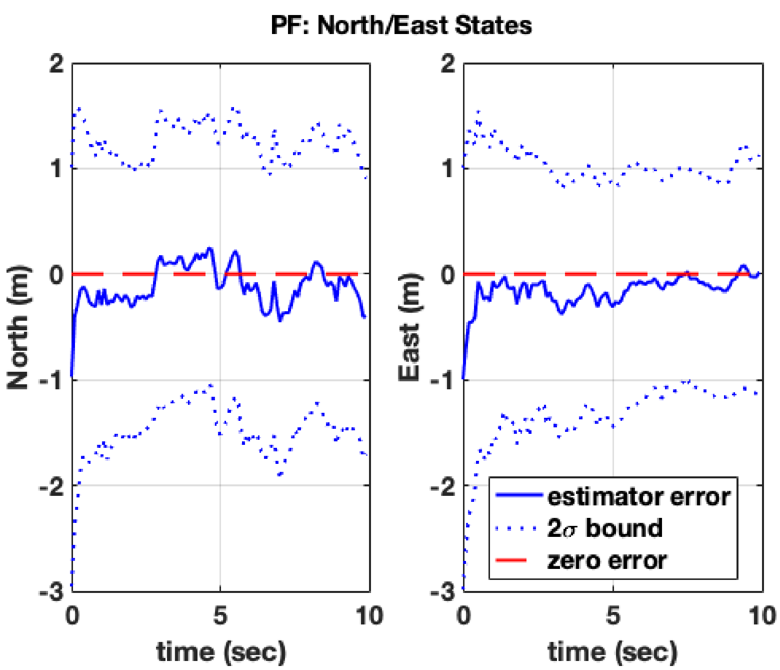
```
%Initialize state using sensor model
ns=1000;
x0=random(pdfx,ns,1);
y0=random(pdfy,ns,1);

X=zeros(ns,2,nk);
X(:,:,1)=[x0 y0];
W=zeros(ns,nk);
W(:,1) = ones(ns,1)/ns;
xEst=zeros(nk,2);xEst(1,:)=mean(X(:,:,1),1);
xSig=zeros(nk,2);xSig(1,:)=std(X(:,:,1),1);
%Plot PF
figure(10);
hold on;
```

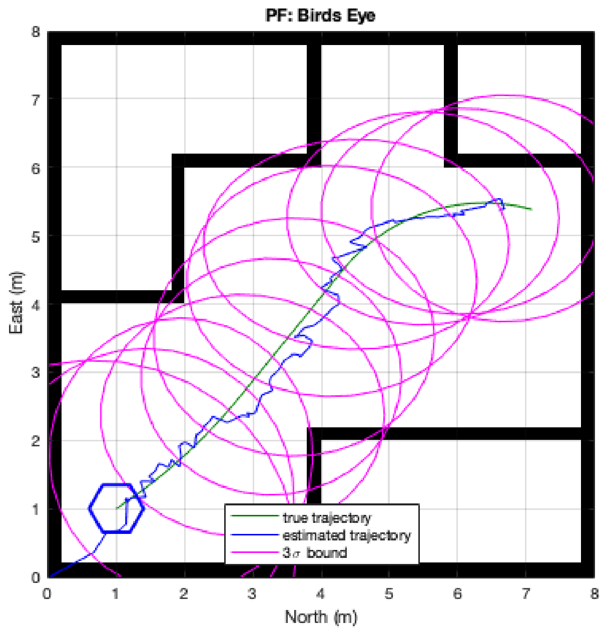
```
hPoints = plot(X(:,1,1),X(:,2,1),'b.','markersize',10);
hEst = plot(xEst(1,1),xEst(1,2),'r*','markersize',10);
hold off;
title('Particle Distribution');
xlabel('X (miles)');ylabel('Y (miles)');
axis([-1 9 -1 9]);

Neff_thresh=ns; %will always resample (enables SIR)
%Neff_thresh=ns/2; %standard practice
if record_video,
    vidfile = VideoWriter(video_name,'MPEG-4');
    vidfile.FrameRate = 10;
    open(vidfile);
    F(1) = getframe(10);
    writeVideo(vidfile,F(1));
end

for k=2:nk,

    Xprior=X(:,:,k-1);

    %PREDICTION STEP
    V=Xnonoise(3,k);
    theta=Xnonoise(4,k);
    u=ones(ns,1)*dt*[V*cos(theta) V*sin(theta)];%sample process noise
    w1=randn(ns,2)*Qsq;
    Xpred=Xprior+u+w1; %predition step for all particles: k -> k+1

    %UPDATE STEP
    Zcurrent=Z(:,k)'; %pull off current measurement
    Zhat = Xpred; %calculate estimated measurement: assumes H=eye(2) or z=x+v

    Inn = ones(ns,1)*Zcurrent - Zhat; %calculate innovations for each particle

    %calculate likelihood weighting for each particle
    for ipart=1:ns,
        L(ipart,1) = pdf(Lx,Inn(ipart,1))*pdf(Ly,Inn(ipart,2)); %likelihoods of two indepedent msts multiply
    end

    %UPDATE WEIGHTS
    Wk_unnorm=W(:,k-1).*L; %update weight with unnormalized likelihood
    Wk=Wk_unnorm/sum(Wk_unnorm); %normalize all weights to sum to 1

    %RESAMPLING
    Neff=1/[Wk'*Wk]; %calculate effective number of particles (varies between 1 and ns)
    if Neff<Neff_thresh, %resample if under threshold
        %
        CDF = cumsum(Wk)/sum(Wk); %create a running sum function of the weights
        CDF_plus=CDF+rand(ns,1)*1E-6; %for cases when a particle weight went to zero
        %randomly (uniform) choose likely (better) particles...
        iSelect  = rand(ns,1);
        %find particle corresponding to each y value
        iNextGeneration = interp1(CDF_plus,1:ns,iSelect,'nearest','extrap');
        %copy selected particles for next generation
        X(:,:,k) = Xpred(iNextGeneration,:);
        W(:,k) = ones(ns,1)/ns;
    else,
        %
        X(:,:,k) = Xpred;
        W(:,k)=Wk;
    end

    %our estimate is simply the mean of the particles
    xEst(k,:) = sum(Wk.*X(:,:,k),1);
    xSig(k,:) = sqrt([sum(Wk.*(X(:,1,k)-xEst(k,1)).^2,1) sum(Wk.*(X(:,2,k)-xEst(k,2)).^2,1)]);

figure(10);
set(hPoints,'XData',X(:,1,k));
set(hPoints,'YData',X(:,2,k));
set(hEst,'XData',xEst(k,1));
set(hEst,'YData',xEst(k,2));
drawnow;
pause(0.1);
if record_video,
    F(k)=getframe(10);
    writeVideo(vidfile,F(k));
end
```

```
end

if record_video,
    close(vidfile)
end
P = zeros(2,2,nk);
for i=1:nk
    Sx= xSig(i,1);
    Sy= xSig(i,2);
    P(:,:,i)= [Sx 0 ; 0 Sy];
end
ii_plot=[1 2];
plot_estimator_error(t,Xnonoise,xEst',P,ii_plot,'PF: North/East States');

plot_birdseyeview(Xnonoise,xEst',P,'PF: Birds Eye');
```

```
myMap =

  binaryOccupancyMap with properties:

   mapLayer Properties
              LayerName: 'binaryLayer'
               DataType: 'logical'
           DefaultValue: 0
    GridLocationInWorld: [0 0]
      GridOriginInLocal: [0 0]
     LocalOriginInWorld: [0 0]
             Resolution: 5
               GridSize: [40 40]
           XLocalLimits: [0 8]
           YLocalLimits: [0 8]
           XWorldLimits: [0 8]
           YWorldLimits: [0 8]


x =

    0.6000    0.8000    1.2000    1.4000    1.2000    0.8000    0.6000


y =

    1.0000    0.6536    0.6536    1.0000    1.3464    1.3464    1.0000
```


Particle Distribution
Y (miles)
X (miles)
9
8
7
6
5
4
3
2
1
0
-1


PF: North/East States
North (m)
East (m)
time (sec)
time (sec)
estimator error
2σ bound
zero error




```
function [Xnonoise,n,t,dt,nt]=simulate_robot(Uacc,Uomega);
dt=0.1;
nt=length(Uacc);
t=[0:dt:dt*(nt-1)];
n=4;
Xnonoise=zeros(n,nt);
Xnonoise(:,1)=[1;1;8/10;0.588]; % Given an initial position, velocity and heading
for k=1:(nt-1),
    Vk=Xnonoise(3,k);
    Tk=Xnonoise(4,k);
    Xnonoise(:,k+1) = Xnonoise(:,k) +...
        dt*[Vk*cos(Tk);Vk*sin(Tk);Uacc(k);Uomega(k)];
end
end
```

```
myMap = 

  binaryOccupancyMap with properties:

   mapLayer Properties
              LayerName: 'binaryLayer'
               DataType: 'logical'
           DefaultValue: 0
    GridLocationInWorld: [0 0]
      GridOriginInLocal: [0 0]
     LocalOriginInWorld: [0 0]
             Resolution: 5
               GridSize: [40 40]
           XLocalLimits: [0 8]
           YLocalLimits: [0 8]
           XWorldLimits: [0 8]
           YWorldLimits: [0 8]


x =

    0.6000    0.8000    1.2000    1.4000    1.2000    0.8000    0.6000


y =

    1.0000    0.6536    0.6536    1.0000    1.3464    1.3464    1.0000
```

Least Squares estimation

```
function [xe]=Elocation(Xnonoise,tn)
bA = [2;4];
bB = [6;6];
bC = [4;2];
%True location of the robot
xtrue = [Xnonoise(1,:);Xnonoise(2,:)];
%Perfect measurements
RA = sqrt((bA(1) - xtrue(1,tn)).^2 + (bA(2) - xtrue(2,tn)).^2);
RB = sqrt((bB(1) - xtrue(1,tn)).^2 + (bB(2) - xtrue(2,tn)).^2);
RC = sqrt((bC(1) - xtrue(1,tn)).^2 + (bC(2) - xtrue(2,tn)).^2);
htrue=[RA RB RC];
na=100;
Rpart = diag([10 10 10]);
va = sqrtm(Rpart)*randn(3,na);
za = [RA;RB;RC]+ va;
xhat=[0;0;0];
NLS_pass=1;J=[];Jold=1;iter=0;
while NLS_pass,
    iter=iter+1;
    xj=xhat(:,iter);
    RAhat=sqrt((bA(1) - xj(1))^2 + (bA(2) - xj(2))^2);
    RBhat=sqrt((bB(1) - xj(1))^2 + (bB(2) - xj(2))^2);
    RChat=sqrt((bC(1) - xj(1))^2 + (bC(2) - xj(2))^2);
    H=[(-bA(1)+xj(1))/RA  (-bA(2)+xj(2))/RA;
        (-bB(1)+xj(1))/RB (-bB(2)+xj(2))/RB;
        (-bC(1)+xj(1))/RC (-bC(2)+xj(2))/RC];
    M1=0;M2=0;J(iter)=0;
    for k=1:na,
        M1=M1+H'*inv(Rpart)*H;
        ek=[za(:,k)-[RAhat;RBhat;RChat]];
        M2=M2+H'*inv(Rpart)*ek;
        J(iter)=J(iter)+0.5*ek'*inv(Rpart)*ek;
    end
    Sigma_x = inv(M1);
    xhat(1:2,iter+1)=xj(1:2,:)+inv(M1)*M2;
    if abs(Jold-J(iter))<1E-3,
        NLS_pass=0;
    else,
        Jold=J(iter);
    end
end
xe=xj(1:2);
end
```

Plotting functions

```
function plot_estimator(t,x1,x2,P2,ii_plot,title_name);
% x1 is the true value or reference comparison
% x2,P2 is the estimator state and covariance
% ii_plot: 2x1 vector of which states to plot
%
axis_names={'X (miles)','Y (miles)','Velocity (m/s)','Heading (rad)','Accel Bias (m/sec^2)','RG Bias (rad/sec)'};
figure;subplot(122);
ii_x1=[];ii_x2=[];ii_P2=[]; %for legend
%
for i=1:length(ii_plot),
    ii=ii_plot(i);
    subplot(1,2,i);
    hold on;
    if ~isempty(x1),
        plot(t,x1(ii,:),'color',[0 0.5 0]);ii_x1=1;
    end
    if ~isempty(x2),
        plot(t,x2(ii,:),'b-');ii_x2=2;
    end
    if ~isempty(P2)
        plot(t,x2(ii,:)'-2*sqrt(squeeze(P2(ii,ii,:))),'b:');
        plot(t,x2(ii,:)'+2*sqrt(squeeze(P2(ii,ii,:))),'b:');ii_P2=3;
    end
    hold off
    xlabel('time (s)');ylabel(axis_names(ii));grid;
    xlim([0 10]);set(gca,'xtick',[0:5:10]);
end
legend_names={'true state','estimate','2\sigma bound'};
legend(legend_names{ii_x1},legend_names{ii_x2},legend_names{ii_P2},'Location','South');
```

```
%
sgtitle(title_name);
PrepFigPresentation(gcf);
end

function plot_estimator_error(t,x1,x2,P2,ii_plot,title_name);
% x1 is the true value or reference comparison
% x2,P2 is the estimator state and covariance
% ii_plot: 2x1 vector of which states to plot
%
axis_names={'North (m)','East (m)','Velocity (m/sec)','Heading (rad)','Accel Bias (m/sec^2)','RG Bias (rad/sec)'};
figure;subplot(122);
%
for i=1:length(ii_plot),
    ii=ii_plot(i);
    subplot(1,2,i);
    err=x2(ii,:)-x1(ii,:);
    plot(t,err,'b-');
    hold on;
    if ~isempty(P2)
        plot(t,err'-2*sqrt(squeeze(P2(ii,ii,:))),'b:');
        plot(t,zeros(length(t),1),'r--');
        plot(t,err'+2*sqrt(squeeze(P2(ii,ii,:))),'b:');
    end
    hold off
    xlabel('time (sec)');ylabel(axis_names(ii));grid;
    xlim([0 10]);set(gca,'xtick',[0:5:10]);
end
legend('estimator error','2\sigma bound','zero error','Location','South');
%
sgtitle(title_name);
PrepFigPresentation(gcf);
end

function plot_birdseyeview(x1,x2,P2,title_name);
% x1 is the true value or reference comparison
% x2,P2 is the estimator state and covariance
%
ii_x1=[];ii_x2=[];ii_P2=[]; %for legend
figure;
myMap=binaryOccupancyMap(8,8,5)
walls=zeros(40,40);
walls(1,:)=1;
walls(end,:)=1;
walls(:,1)=1;
walls(:,end)=1;
walls(10:20,10)=1;
walls(1:10,20)=1;
walls(30:40,20)=1;
walls(20,1:10)=1;
walls(10,10:20)=1;
walls(30,20:40)=1;
walls(10,30:40)=1;
walls(1:10,30)=1;
setOccupancy(myMap,[1 1],walls,"grid")
show(myMap)
hold on
if ~isempty(x1),
    plot(x1(1,:),x1(2,:),'color',[0 0.5 0]);ii_x1=1;
end
hold on;
if ~isempty(x2),
    plot(x2(1,:),x2(2,:),'b-');ii_x2=2;
end
if ~isempty(P2),
    iell=[2 10:10:100];
    for i=1:length(iell),
        ii=iell(i);
        [Xe,Ye] = calculateEllipseCov(x2([1 2],ii),P2([1 2],[1 2],ii),3);
        plot(Xe,Ye,'m-');
    end
    ii_P2=3;
end
xlabel('North (m)');ylabel('East (m)');grid;
hold on;
[robot] = hexagon(0.4,1,1);
hold off
```

```
legend_names={'true trajectory','estimated trajectory','3\sigma bound'};
legend(legend_names{ii_x1},legend_names{ii_x2},legend_names{ii_P2},'Location','South')
%
title(title_name);
%PrepFigPresentation(gcf);
end

function [Xe,Ye] = calculateEllipseCov(X, P, nsig, steps)
    %# This functions returns points to draw an ellipse
    %#
    %#  @param X     x,y coordinates
    %#  @param P     covariance matrix
    %#

    error(nargchk(2, 3, nargin));
    if nargin<3, nsig = 1; end
    if nargin<4, steps = 36; end

    [U,S,V]=svd(P);
    s1=sqrt(S(1,1));s2=sqrt(S(2,2));angle=acos(U(1,1))*180/pi;
    x=X(1);
    y=X(2);

    %scale by nsig
    s1=nsig*s1;
    s2=nsig*s2;

    beta = angle * (pi / 180);
    sinbeta = sin(beta);
    cosbeta = cos(beta);

    alpha = linspace(0, 360, steps)' .* (pi / 180);
    sinalpha = sin(alpha);
    cosalpha = cos(alpha);

    Xe = (x + (s1 * cosalpha * cosbeta - s2 * sinalpha * sinbeta));
    Ye = (y + (s1 * cosalpha * sinbeta + s2 * sinalpha * cosbeta));

end

function PrepFigPresentation(fignum);
%
% prepares a figure for presentations
%
% Fontsize: 14
% Fontweight: bold
% LineWidth: 2
%

figure(fignum);
fig_children=get(fignum,'children'); %find all sub-plots

for i=1:length(fig_children),

    set(fig_children(i),'FontSize',16);
    set(fig_children(i),'FontWeight','bold');

    fig_children_children=get(fig_children(i),'Children');
    set(fig_children_children,'LineWidth',2);
end
end
% Define robot model
function [robot]=hexagon(cote,x0,y0)
   %cote= side size;,(x0,y0) exagon center coordinates;
   x=cote*[-1 -0.5 0.5 1 0.5 -0.5 -1]+x0
   y=cote*sqrt(3)*[0 -0.5 -0.5 0 0.5 0.5 0]+y0
   robot=plot(x,y,'b','Linewidth',2);
   end
```

Published with MATLAB® R2021b