Mathematics 322 — Final — 180 minutes

June 25th 2021

- The test consists of 9 pages and 7 questions worth a total of 65 marks.
- This is a closed-book examination. None of the following are allowed: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- No work on this page will be marked.
- Fill in the information below before turning to the questions.
- You will have 15 minutes to upload your work on Assign2 through eClass after 180 minutes exam time.
- Any delay must be immediately notified to the instructor at shukla2@ualberta.ca
- No need to upload the first page since your identity is authenticated through SEM.

I affirm that I will not give or receive any unauthorized help during this quiz, that all work will be my own, and that I will abide by any special rules for conduct set out by the examiner.

Student number								
Section	2	2	3	0	3			
Name								
Signature								

1. $\boxed{\text{7 marks}}$ Indicate True or False. No justification is required.

If $L(G)$ is Eulerian then G is Eulerian too.

$e = uv$ $\quad$ u e v

$d(e) = \text{even}$ $\quad$ $d(u) - d(v) - 2 =$

F

If G contains a cut-vertex then it also contains a bridge.

F

If G has a strong orientation then reversing the orientation of all the edges also gives a strong orientation.

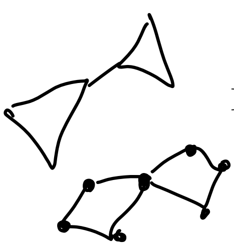
T

If a connected weighted graph (G, w) has minimum spanning tree T. Consider a new weight function $w'(e) = w(e) + 1$. Then T is also a minimum spanning tree for (G, w').

F

If $\delta(G) \geq \dfrac{n-1}{2}$ then G is connected.

T



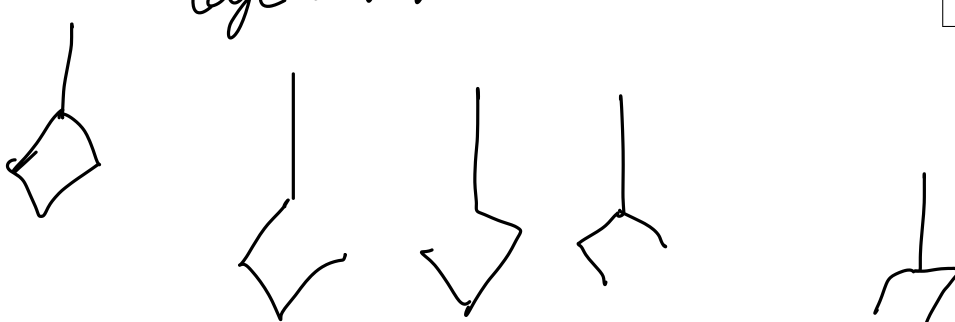
If $L(G)$ has a bridge then G has a bridge too.

F

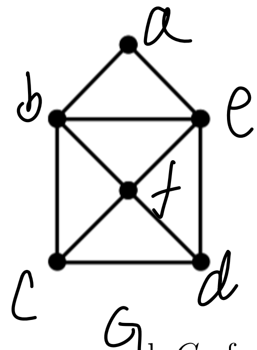
Let G be a connected unicyclic graph of order $n \geq 3$ which contains C_k as a subgraph. Then $\tau(G) = k$.

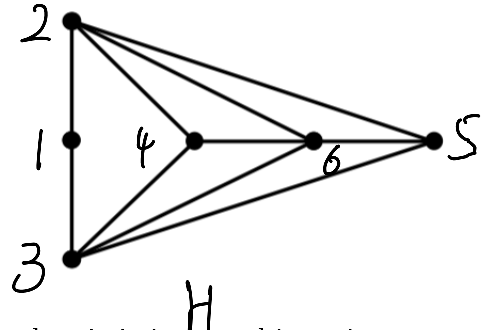
edge = vertex

T



10 marks (a) Determine if the following two graphs are isomorphic.





(b) Draw a graph G of order $n \geq 2$ such that it is isomorphic to its complement $\overline{G}$.

(a)

$$G = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & 0 & 1 & 0 & 0 & 1 & 0 \\ b & 1 & 0 & 1 & 0 & 1 & 1 \\ c & 0 & 1 & 0 & 1 & 0 & 1 \\ d & 0 & 0 & 1 & 0 & 1 & 1 \\ e & 1 & 1 & 0 & 1 & 0 & 1 \\ f & 0 & 1 & 1 & 1 & 1 & 0 \end{array} \qquad H = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 0 & 1 & 1 & 1 \\ 4 & 0 & 1 & 1 & 0 & 0 & 1 \\ 5 & 0 & 1 & 1 & 0 & 0 & 1 \\ 6 & 0 & 1 & 1 & 1 & 1 & 0 \end{array}$$

$$H' = \begin{array}{c|cccccc} & a & e & c & b & d & f \\ \hline a & 0 & 1 & 0 & 1 & 0 & 0 \\ b & 1 & 1 & 1 & 0 & 0 & 1 \\ e & 1 & 0 & 0 & 1 & 1 & 1 \\ d & 0 & 1 & 1 & 0 & 0 & 1 \\ c & 0 & 0 & 0 & 1 & 1 & 1 \\ f & 0 & 1 & 1 & 1 & 1 & 0 \end{array}$$

compare G and H'
some are correspond.
but others are not

it is not isomorphic.

if we stasify triangle Δabe, there must be $d(a)=2$ $d(b)=4$ $d(e)=4$ and b & e connect

we can not find such triangle in right graph.

also we cannot find such matrix H' are equal G

(b)

	G	$\overline{G}$

$n=2$

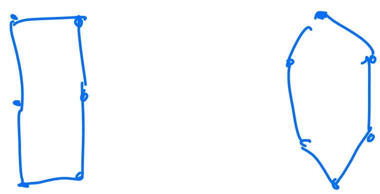
$n=3$

$n=4$

$n=5$

they are isomorphic

$n=6$



3

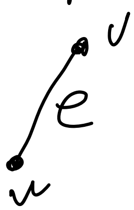
$\boxed{\text{10 marks}}$ (a) For $k \geq 1$, let G be a connected k-regular graph. Show that the line graph $L(G)$ is Eulerian.

(b) Prove or disprove: Suppose H_1 and H_2 are Hamiltonian graphs. The graph $H_1 \cdot H_2$ is Hamiltonian.

[Recall that $H_1 \cdot H_2$ is obtained from H_1 and H_2 by identifying two vertices w_1 and w_2 in H_1 and H_2, respectively]

(a)

G L(G)



• e(uv)

$\because$ G is connected k-regular graph.

$\therefore$ degree is k on each vertex

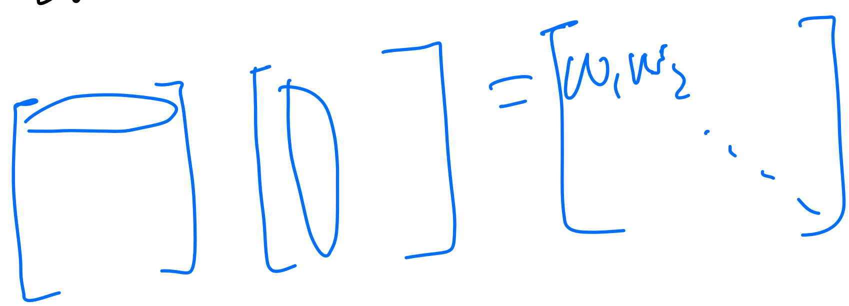
$\because d(e) = d(uv) = d(u) + d(v) - 2 = 2(k-1)$

in line graph. each edge adjiont $2(k-1)$

$\therefore 2(k-1)$ is even

$\therefore$ L(G) is Eulerian.

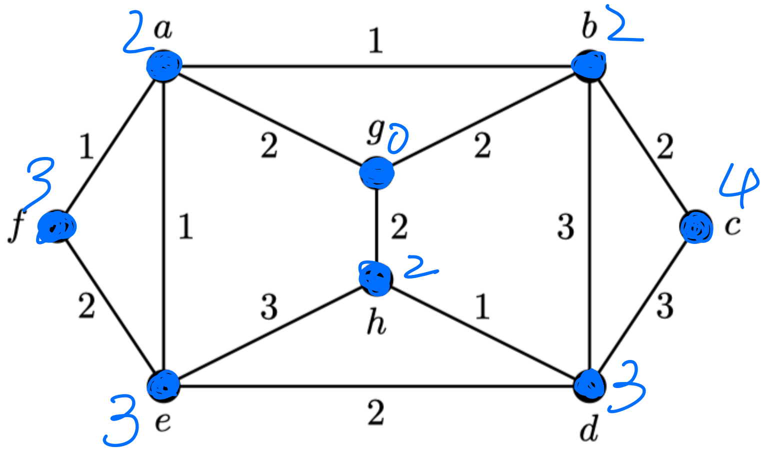
(b) $H_1 \cdot H_2$ =



The graph $H_1 \cdot H_2$ is not Hamiltonian

4.

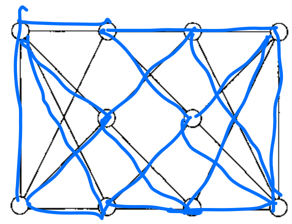
$\boxed{\text{8 marks}}$ Find the shortest distance from **vertex g** to all other vertices using Dijkstra's algorithm.



vertex	dist	pre	fixed
a	~~∞~~ 2	~~—~~ g	~~F~~ T
b	~~∞~~ 2	~~—~~ g	~~F~~ T
c	~~∞~~ 4	~~—~~ b	~~F~~ T
d	~~∞~~ ~~5~~ 3	~~—~~ ~~b~~ h	~~F~~ T
e	~~∞~~ 3	~~—~~ a	~~F~~ T
f	~~∞~~ 3	~~—~~ a	~~F~~ T
g	0	—	~~F~~ T
h	~~∞~~ 2	~~—~~ g	~~F~~ T

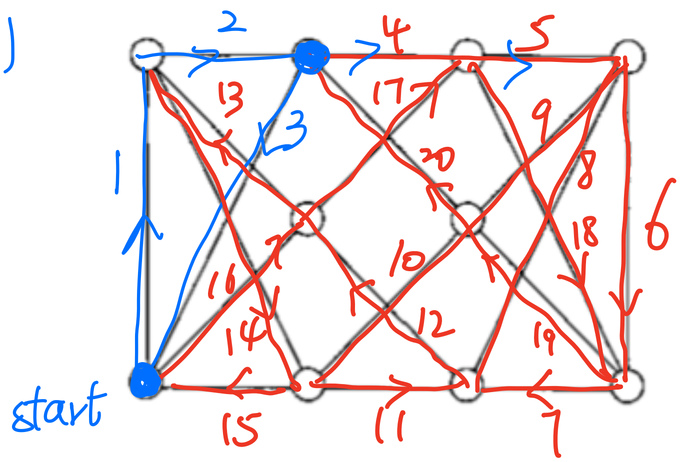
5.

$\boxed{\text{10 marks}}$ (a) Construct an Euler circuit for this graph using any algorithm. Label the edges numerically in the order they appear in the Euler circuit.



(b) Suppose G is a connected Eulerian graph. Is it possible to assign directions to the edges in such a way that at each vertex, the number of incoming edges is equal to the number of outgoing edges? If so, explain how this can be done. If not, explain why not.

(a)



1 2 3

1 2 (4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20) 3

(b) ∵ G is connected Eulerian graph

It is traverse all edge to produce closed cycle (no repeated edge)

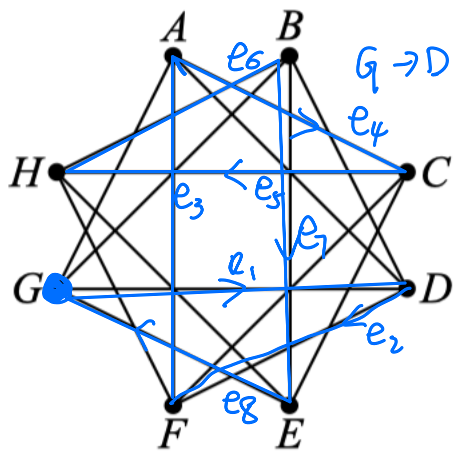
All vertex degree are even.

∴ It always incoming edges equal outgoing edges

Thus. it is possible to assign direction to the edges

6

10 marks 1. Is the following graph Hamiltonian? If yes then either prove it is Hamiltonian or demonstrate a Hamiltonian cycle. Otherwise justify why it is not Hamiltonian.



start G and G

$G \to D \to F \to A \to C \to H \to B \to E \to G$

It is Hamiltonian cycle

Yes, it is Hamiltonian

2. For $p \geq q \geq p-1 \geq 1$, show that the graph $K_{p,q}$ contain a Hamiltonian path.

To show it contain Hamiltonian path

need satisfy $d(v) \geq \frac{n-1}{2}$

$\because p \geq q \geq p-1 \geq 1$ in $K_{p,q}$

$\therefore p-1 \leq q$, q is $d(v)$

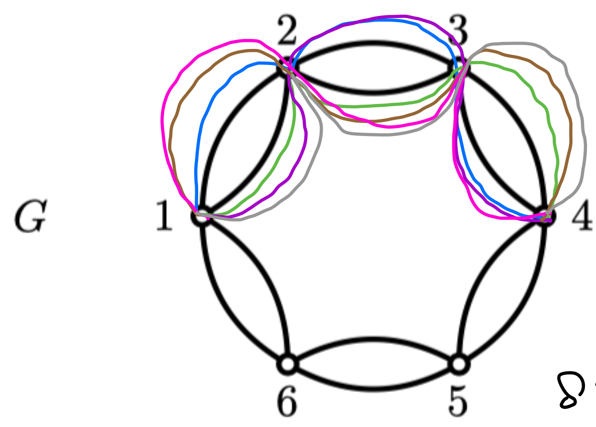
$\therefore \frac{n-1}{2} = \frac{p+q-1}{2} \leq \frac{q+q}{2} = q = d(v)$

$\therefore d(v) \geq \frac{n-1}{2}$

Thus $K(p,q)$ contain Hamiltonian path

7.

10 marks Consider the graph below and let A be its adjacency matrix with respect to the given numbering of its vertices.



$2^3 = 8$

$8+8=16$

1. Write down the $(1,4)^{th}$ entry of A^3.
2. Write down the $(1,2)^{th}$ entry of A^{10}.

$$A = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 2 & 0 & 0 & 0 & 2 \\ 2 & 2 & 0 & 2 & 0 & 0 & 0 \\ 3 & 0 & 2 & 0 & 2 & 0 & 0 \\ 4 & 0 & 0 & 2 & 0 & 2 & 0 \\ 5 & 0 & 0 & 0 & 2 & 0 & 2 \\ 6 & 2 & 0 & 0 & 0 & 2 & 0 \end{array}$$

1. $(1,4)^{th}$ entry of A^3 is # of walks between 1 and 4 of length 3.
$\therefore\ 2^3 + 2^3 = 8 + 8 = \boxed{16}$

2. $(1,2)^{th}$ entry of $A^{10} = \boxed{0}$

Since $(1,2)^{th}$ entry of A^{10} means # of walks between 1 and 2 of lenth 10.
There is no correspond walks
so $(1,2)^{th}$ entry of A^{10} is 0.

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