ME604

~~ME5xx~~ HW#3 Gu, Xinyuan

Problem 1

Spy plots attached in the Appendix.

$\phi(x,y) = \sin(2\pi n x)\sin(2\pi n y)$

$$f(x,y) = \frac{\partial\phi}{\partial x} - \frac{1}{Re}\nabla^2\phi$$

$$= 2\pi n\cos(2\pi n x)\sin(2\pi n y) + \frac{8n^2\pi^2}{Re}\sin(2\pi n x)\sin(2\pi n y)$$

$$= \sin(2\pi n y)\left[2\pi n \times \cos(2\pi n x) + \frac{8n^2\pi^2}{Re}\times\sin(2\pi n x)\right].$$

Consider three values of n & $Re = 1$ $(n = 1, 5, 10)$.

for $n=1$, $Re=1$,

$\underline{\underline{A}}\,\underline{x} = \underline{b}$

$$\begin{cases} \underline{\underline{A}} = \frac{\partial\phi}{\partial x} - \text{DivGrad}/Re \\ \underline{x} = \underline{\phi} \\ \underline{b} = f(x,y) \end{cases} \Longrightarrow \text{SOR method, Set } \{1,\ \min(N_x, N_y) = 20\}.$$

$n = 1,\ 5,\ 10.$

$$f(x,y) = \sin(2\pi y)\left[2\pi\cos(2\pi x) + 8\pi^2\sin(2\pi x)\right].$$

$\phi(x,y) = \sin(2\pi x)\sin(2\pi y)$

	w_{opt} $n=1$	$n=5$	$n=10$
1x1	N/A		
5x5	1.15	1.3	1.2
10x10	1.35	1	1.45
15x15	1.55	1.2	1.3
20x20	1.65	1.3	0.8

following the same procedure for $n = 5, 10$.

$\Rightarrow$ plots of w vs. $\frac{|r_k|}{|r_0|}$ for $n=1,5,10$ attached.

plots of k vs. $|r_k|$ for $n=1,5,10$ attached.

For $Nx = Ny = 256$ & high n case.

$\omega_{opt} = 1.37$

plots of k vs. $|r_k|$ for $Re = 0.01, 1, 100, 1e4, 1e6, 1e8$ attached

From the plot, it can be concluded that:
the iterative solve blows up after $Re = 10000$
plots of k vs. $|r_k|$ for lower Re attached.

Consider nonlinear eqn.

$$\phi \frac{\partial \phi}{\partial x} - \frac{1}{Re} \nabla^2 \phi = f(x, y).$$

$Nx = Ny = 130$, $n = 10$

Re		k-iteration	time (s)
0.01	case a	3	265.35
0.01	case b	25	4192.5
1	case a	5	568.44
1	case b	37 (overflow & invalid)	9115.68
100	case a	8 (overflow & invalid)	979.68
100	case b	N/A	N/A.

For a LU pre-factorization operator stored in preprocessing, it could reduce the time for calculation, however the iteration number is pretty much the same.

No, because of inverting a singular matrix to be very messy.

Problem 2

$u(x,y,t) = -e^{-2t}\cos x \sin y$ $\nu = 1$

$v(x,y,t) = e^{-2t}\sin x \cos y$

$p(x,y,t) = -\frac{e^{-4t}}{4}(\cos 2x + \cos 2y)$

& periodic b.c. $\Rightarrow$ all inside nodes.

from $t=0$ to $t=1$, apply fully-explicit

Q: Briefly explain (with eqns) the suggested time-advancement strategies.

For fully-explicit,

$$\frac{\underline{u}^{n+1} - \underline{u}^n}{\Delta t} = -\frac{1}{\rho}\nabla p^{n+1} - \underline{u}^n \cdot \nabla \underline{u}^n + \nu \nabla^2 \underline{u}^n$$

properlly choose Δt by CFL condition.

First, predictor step, to calculate $\underline{u}^*$ by omitting pressure,

$$\frac{\underline{u}^* - \underline{u}^n}{\Delta t} = -\underline{u}^n \cdot \nabla \underline{u}^n$$

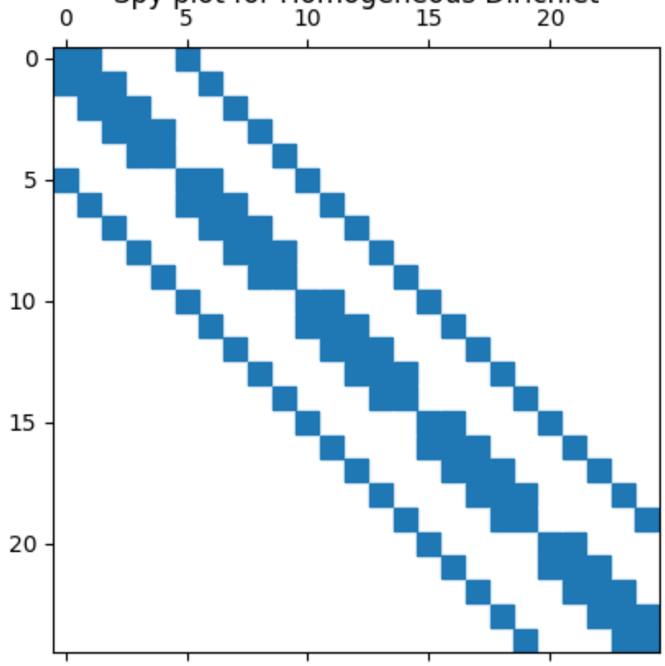
Second, corrector step, to solve for $\underline{u}^{n+1}$ with pressure,

$$\frac{\underline{u}^{n+1} - \underline{u}^*}{\Delta t} = -\frac{1}{\rho}\nabla p^{n+1}$$

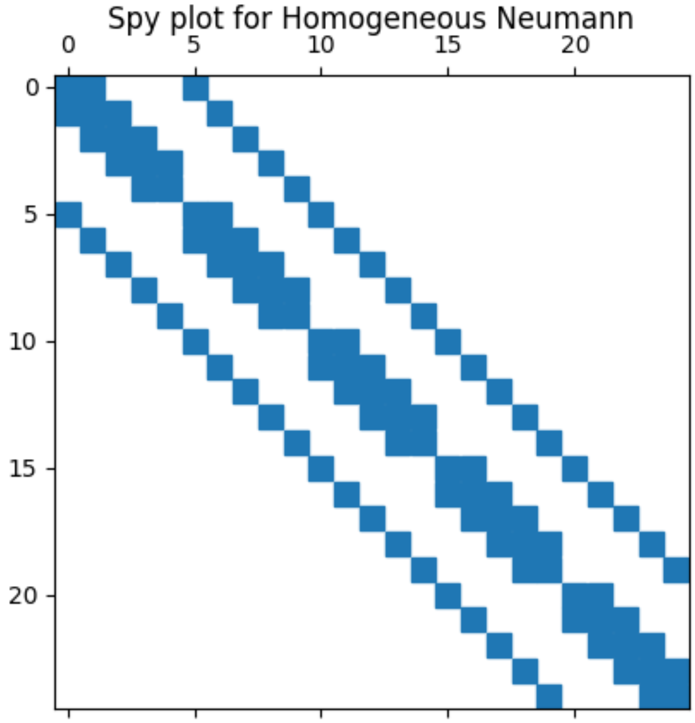
then the pressure is found,

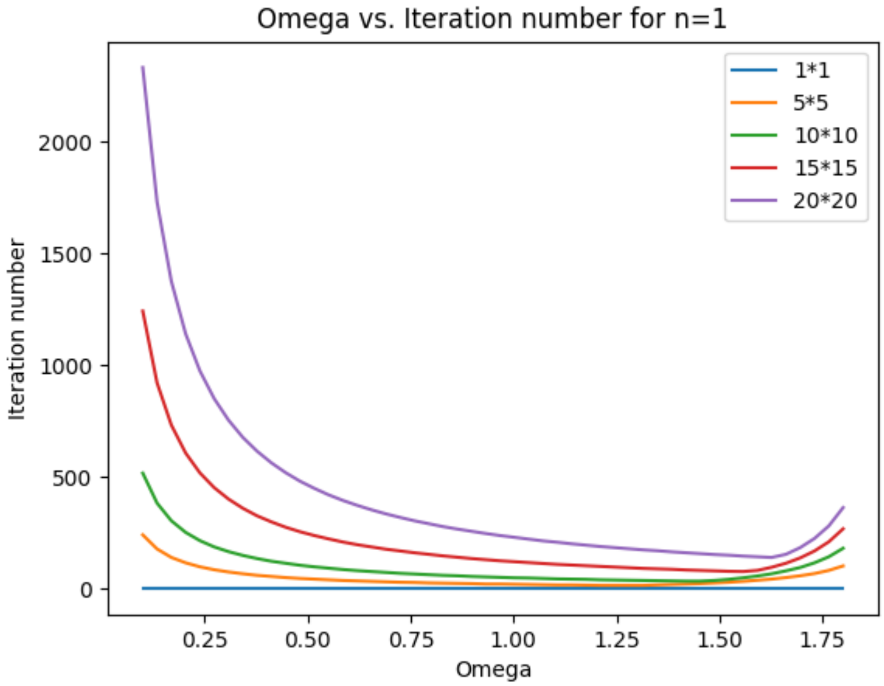
$$\begin{cases} \nabla^2 p^{n+1} = -\frac{\rho}{\Delta t}\nabla \cdot \underline{u}^* \\ \nabla \cdot \underline{u}^{n+1} = 0 \end{cases}$$

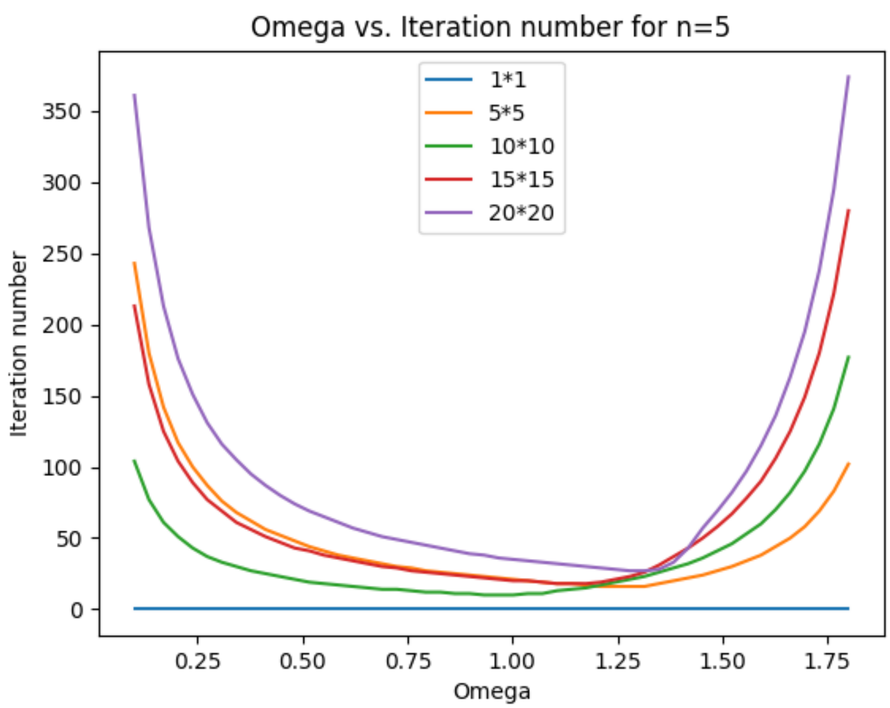
Spy plot for Homogeneous Dirichlet

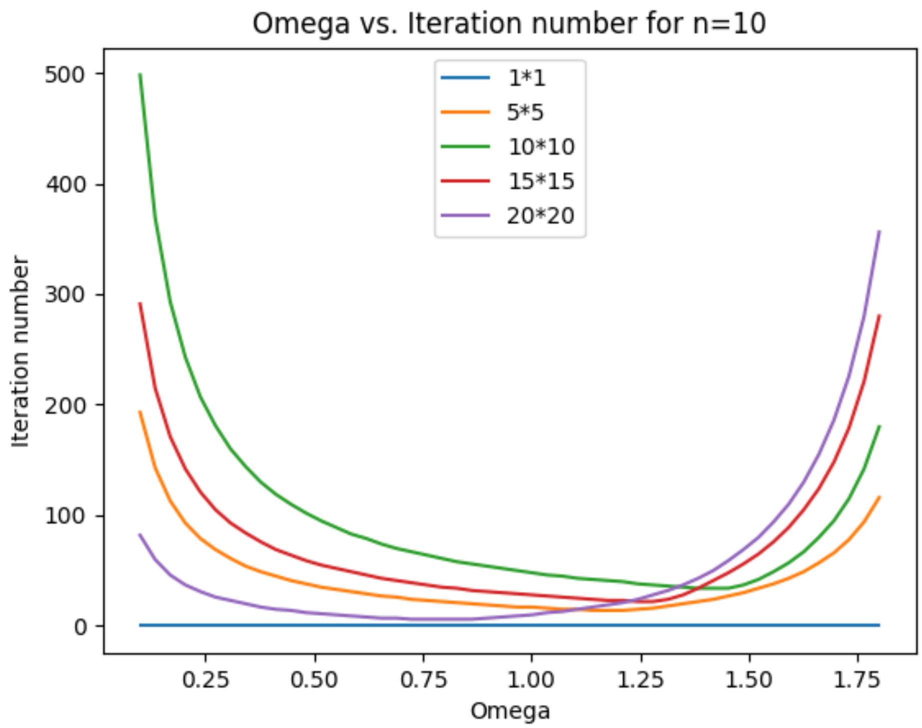


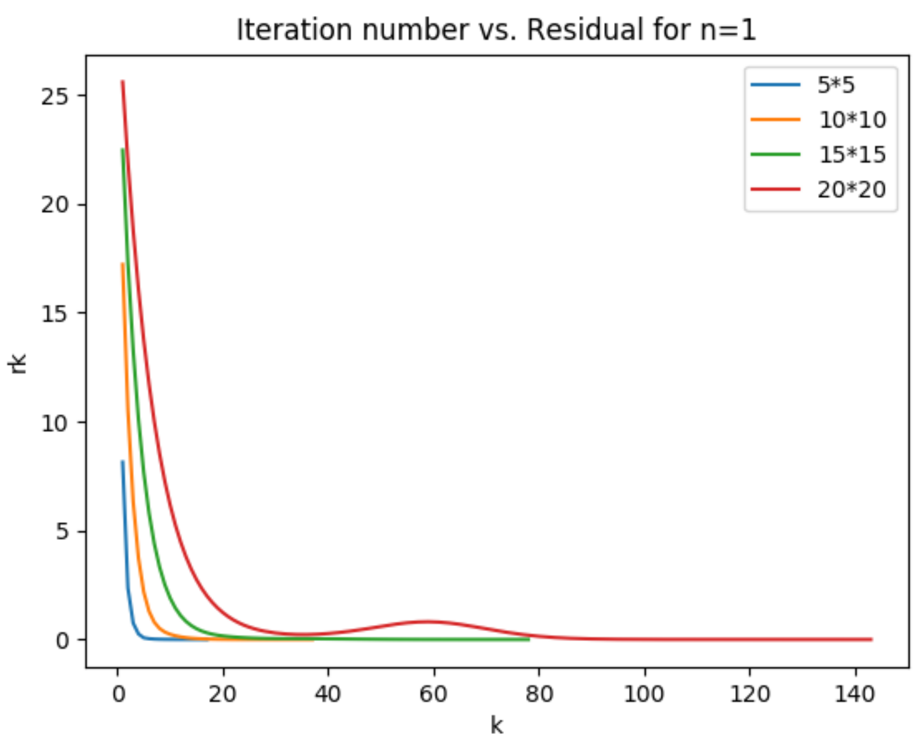
Spy plot for Homogeneous Neumann

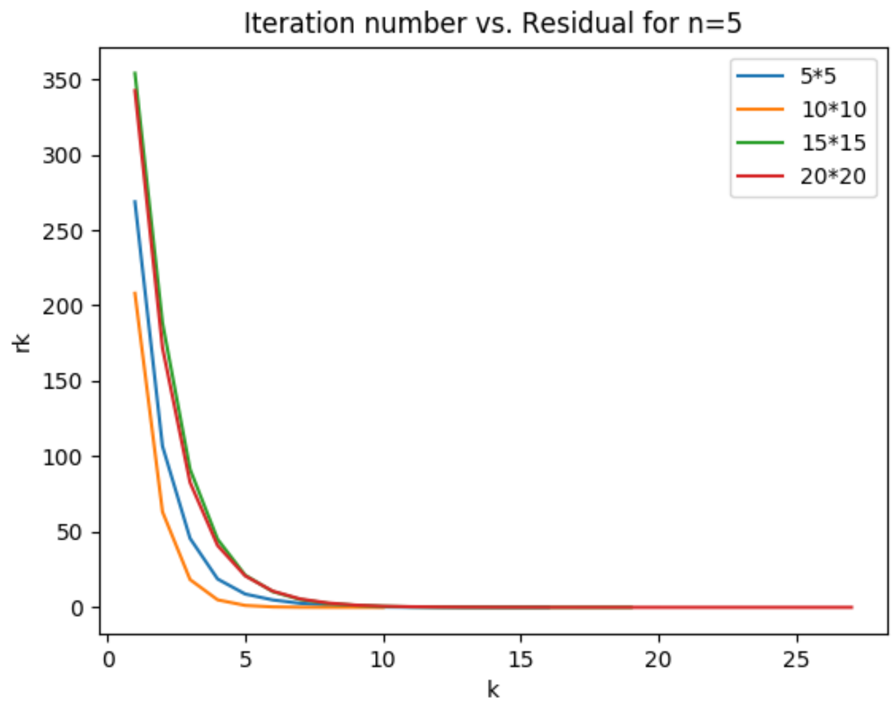


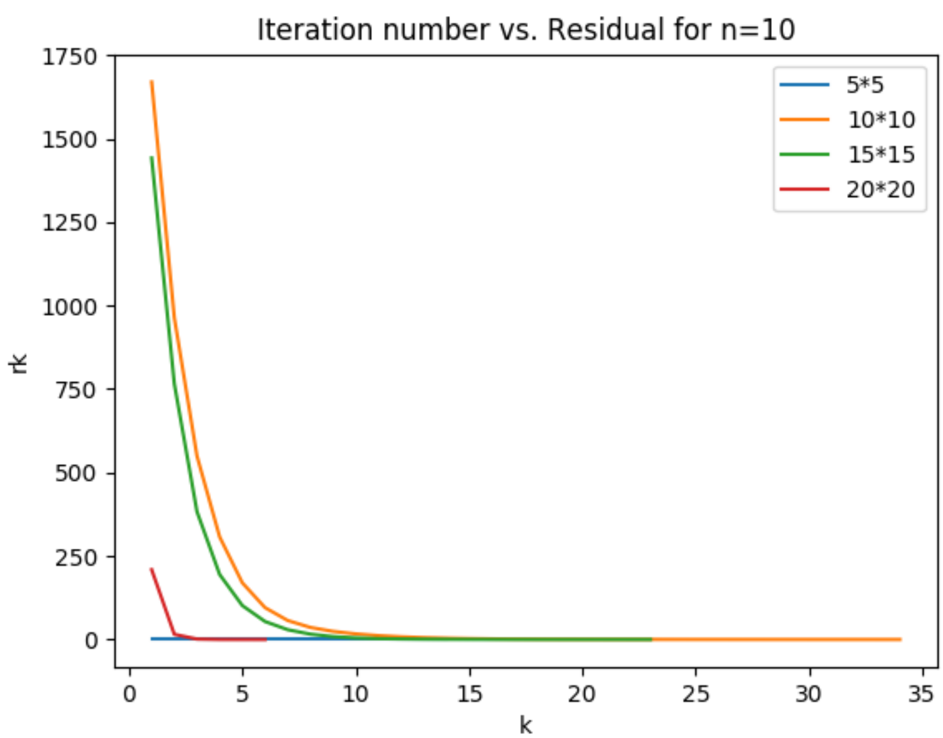

Omega vs. Iteration number for n=1
1*1
5*5
10*10
15*15
20*20
Iteration number
Omega
0
500
1000
1500
2000
0.25
0.50
0.75
1.00
1.25
1.50
1.75

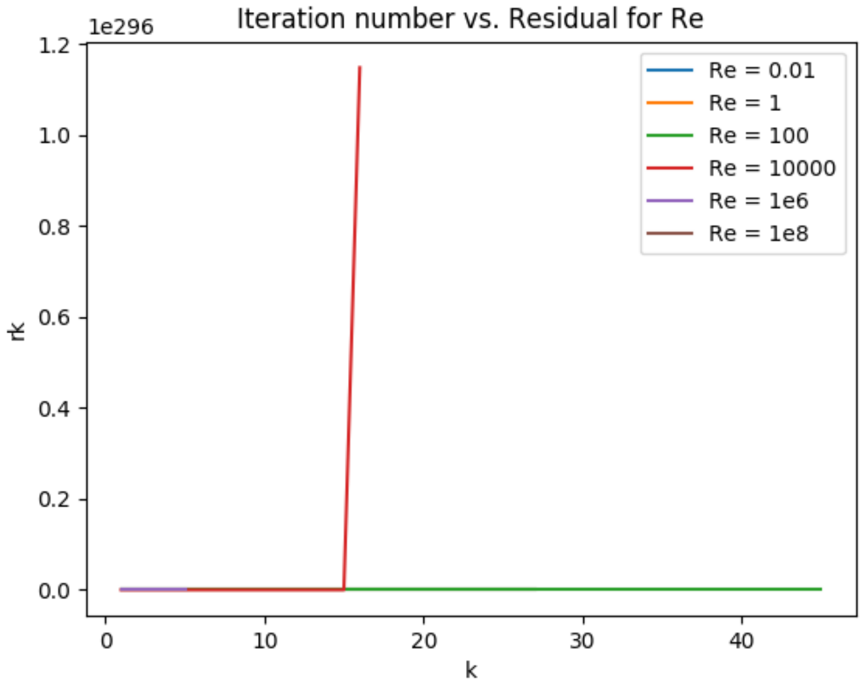

Omega vs. Iteration number for n=5
1*1
5*5
10*10
15*15
20*20
Iteration number
Omega
0
50
100
150
200
250
300
350
0.25
0.50
0.75
1.00
1.25
1.50
1.75

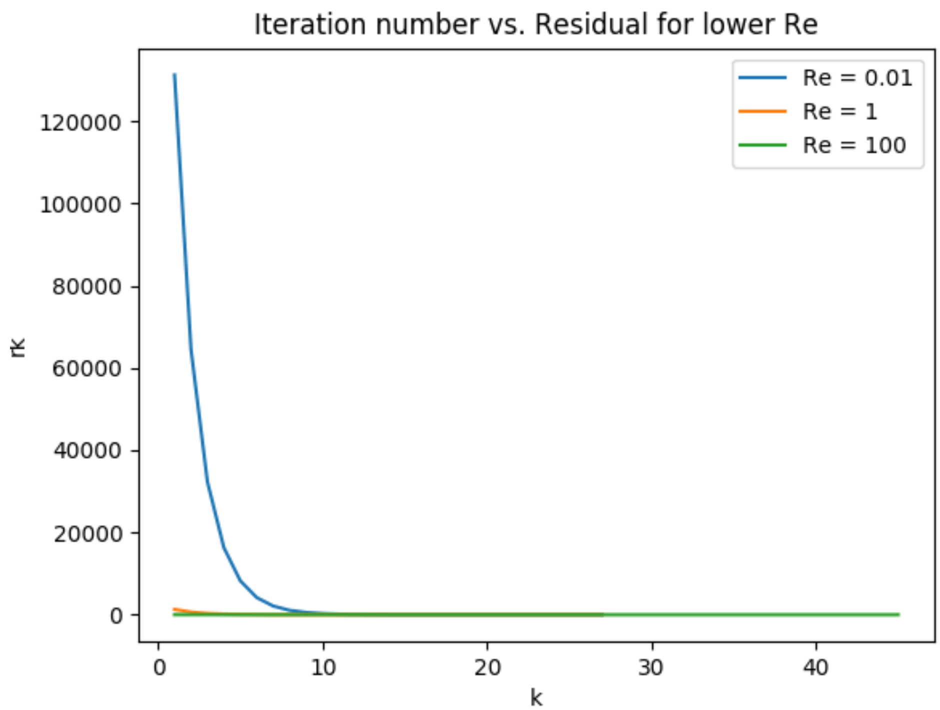

Omega vs. Iteration number for n=10
Iteration number
Omega
1*1
5*5
10*10
15*15
20*20
0
100
200
300
400
500
0.25
0.50
0.75
1.00
1.25
1.50
1.75


Iteration number vs. Residual for n=1
rk
k
5*5
10*10
15*15
20*20
0
5
10
15
20
25
0
20
40
60
80
100
120
140


Iteration number vs. Residual for n=5
5*5
10*10
15*15
20*20
rk
k
0
50
100
150
200
250
300
350
0
5
10
15
20
25


Iteration number vs. Residual for n=10
5*5
10*10
15*15
20*20
rk
k
0
250
500
750
1000
1250
1500
1750
0
5
10
15
20
25
30
35


Iteration number vs. Residual for Re
1e296
1.2
1.0
0.8
0.6
0.4
0.2
0.0
rk
0
10
20
30
40
k
Re = 0.01
Re = 1
Re = 100
Re = 10000
Re = 1e6
Re = 1e8


Iteration number vs. Residual for lower Re
120000
100000
80000
60000
40000
20000
0
rk
0
10
20
30
40
k
Re = 0.01
Re = 1
Re = 100