Gambling for Redemption or Ripoff, and the Impact of Superpriority*

Philip H. Dybvig[†]   Xinyu Hou[‡]

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ABSTRACT

Myers (1977) described how firms can gamble using asset substitution, which is switching to a less efficient and more volatile project. Gambling using derivatives is a sharper instrument, allowing the owners to gamble just to what is needed, and with negligible efficiency loss. In our model, "gambling for redemption" operates at small scale and is socially beneficial, while "gambling for ripoff" operates at large scale and is socially inefficient but benefits firm owners at the expense of bondholders. Superpriority laws grant exemptions of derivatives for bankruptcy law, which makes more funds available for gambling. This reduces firm value due to difficulty borrowing in the face of more gambling for ripoff.

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[†]Dybvig is the Boatmen's Bancshares Professor of Banking and Finance at the Olin Business School of Washington University in St. Louis. E-mail: dybvig@wustl.edu

[‡]Hou is a Research Associate at Cambridge Endowment for Research in Finance (CERF), Cambridge Judge Business School. E-mail: x.hou@jbs.cam.ac.uk

1 Introduction

In the early days of Federal Express, the company's cash once dwindled to $5,000, too little to cover the $24,000 jet fuel bill due the following Monday. With the firm hanging on the edge, the founder Frederick Smith flew to Las Vegas over the weekend and played blackjack to convert the $5,000 into $32,000, enough to keep the company afloat for another week.[1] This gambling was obviously beneficial to the firm's owners since it provided a positive probability to avoid bankruptcy, and it was probably also beneficial to the other claimants including the fuel company, who were unlikely to receive much in bankruptcy. Gambling by a firm can also benefit owners at the expense of creditors as in the asset substitution studied in Myers (1977) because owners receive most of the upside of large gambles but most of the downside is borne by the creditors. In this paper, we study pure gambling by the firm using derivatives, which allows more control over the payoff distribution and negligible inefficiency of investment. We can understand the impact of this gambling through two polar cases. Gambling for redemption, which means gambling just enough to stay in business as in the Federal Express example, is good for the owners, the creditors, and for overall efficiency. Gambling for ripoff, which is at a larger scale, benefits the owners at the expense of the creditors and overall economic efficiency.

Gambling for ripoff is of special current interest because of controversial legislation before the financial crisis that exempts repos and other derivative securities from important provisions of bankruptcy, including the automatic stay and clawbacks, causing some people to call them superpriority claims.[2] In the United States, it has traditionally been difficult to redeploy assets for gambling. While common law allows for asset seizure in satisfaction of debts, seizure or sales can be clawed back in bankruptcy,[3] and bond covenants can be used to trigger bankruptcy in response

[1] Frock (2006)

[2] Roe (2010) argues that these laws accelerated the financial crisis by undermining creditors' incentives to monitor the firm and creating the too-big-to-fail problem.

[3] According to the original bankruptcy law, an asset sale in satisfaction of a particular claim within 90 days (or in some cases longer. The clawback extends back one year for a preferential transfer to an insider, or up to two years for a fraudulent conveyance.) before bankruptcy is considered preferential if the firm cannot satisfy all claimants and can be clawed back (reversed by the court), which tends to make asset seizure or sale pointless.

to seizure or sales.[4] Consequently, any promise by the firm to transfer assets to pay off on a failed gamble would not be credible unless the gambling counterparties are sure that the firm will not be pushed into bankruptcy. However, the exemption from bankruptcy law for "superpriority" claims sidesteps these laws.[5] The owners can now pledge their assets, because the gambling counterparties know that the assets can be collected without being stayed in the firm's estate in bankruptcy.[6] With the access to more funds to gamble, we show in our result that gambling for ripoff becomes more appealing to the owners. Besides gambling for redemption or ripoff in the single-period model, we use a multi-period model to show that superpriority laws can reduce firm value by encouraging gambling. In our single-period model, superpriority laws seem good for firm owners, but perhaps only because the amount of debt and the continuation value are both exogenous. In a multi-period model that endogenizes these variables and some others, superpriority typically reduces firm value because bond investors realize that superpriority increases the likelihood of gambling for ripoff, and this is reflected in bond pricing.

Consider an example of gambling for redemption by a firm with continuation value 120 (ex-

[4]Bonds often contain covenants to restrict asset sales, typically placing the firm in default on the loan if the covenant is violated. Together with cross-default clauses in bonds saying that a default on one bond places the company in default on all its bonds, this would normally result in the firm entering bankruptcy.

[5]Superpriority protects the contractual right of derivatives counterparties to "terminate, liquidate, or accelerate" a derivatives contract before the commencement of the case. Though bankruptcy procedures for different firms are governed by different laws, the laws generally grant or expand superpriority rights to the derivatives contracts. For example, see 11 U.S. Code §362(b)(6), §546(e). Besides Chapter 7 and Chapter 11 Bankruptcy Code, which govern all persons with some exceptions, Chapter 15 applies to foreign debtors and expressly provides safe harbor to QFCs. "Financial institutions" defined by FDICIA are governed by rules under FDIA (Federal Deposit Insurance Act, applies to financial institutions insured by FDIC), FCUA (Federal Credit Union Act, applies to federally insured credit unions), and HERA (Housing and Economic Recovery Act of 2008, applies to mortgage related firms). These three have nearly identical superpriority rules. Stockholders who are members of Securities Investor Protection Corporation (SIPC) are liquidated under SIPA with similar rules, instead of the Code, see 15 U.S. Code §78eee(b)(2)(C). For systemically important financial companies, OLA (Orderly Liquidation Authority) is in charge of the bankruptcy procedure. Unlike under the Code, resolutions under OLA still grant a 24-hour stay of assets, and superpriority rights are enforced unless the receiver transfers all QFCs to another financial institutions and provides a notice to the counterparties within this 24 hours. The bankruptcy resolution of insurance companies are governed by states laws and there are discrepancies whether superpriority laws apply to insurance firms in different states. For more details, see *Qualified Financial Contracts and Netting under U.S. Insolvency Laws* by Cleary Gottlieb Steen & Hamilton LLP downloaded from `https://www.clearygottlieb.com/-/media/organize-archive/cgsh/files/2017/publications/qualified-financial-contracts-and-netting-under-us-insolvency-laws.pdf` on August 15, 2022. For details of OLA, see Treasury (2018). In 2018, Treasury proposed in Treasury (2018) a new Chapter of Bankruptcy Code, Chapter 14, to replace the OLA to handle bankruptcy procedures of large, interconnected firms.

[6]This can also be possible due to poor specification or enforcement of property rights and bankruptcy law in under-developed countries. An exception is perfected collateral, which are still honored under UCC Article 9, a commonly adopted state law.

cluding cash flow), cash flow 40, and required payment 100. Without gambling, the firm defaults and the owners obtain 0 in bankruptcy. We can think of a fair gamble which allows the owners to win 60 with probability 0.4 and to lose cash flow 40 with probability 0.6. We assume either risk neutrality or that the expectations are interpreted as the risk-neutral (martingale) probabilities of Cox and Ross (1976). When the owners win 60, the total cash flow 100 can cover the required payment and the continuation value 120 is maintained. The owners are strictly better off with expected value $0.4 \times 120 = 48 > 0$. Interestingly, this gamble does not hurt bondholders since receiving 100 with probability 0.4 and 0 with probability 0.6 (with gambling) has the same expected value as receiving 40 with probability 1 (without gambling). Considering bankruptcy costs paid by bondholders, gambling for redemption makes bondholders strictly better off. This is consistent with Federal Express's gambling. In this example, larger gambles that rip off bondholders are not attractive to the owners. For instance, by taking a large gamble that wins 360 with probability 0.1 and loses 40 with probability 0.9, the owners have expected value $0.1 \times (120 + 360 + 40 - 100) = 42$, smaller than 48, the value that they would get if they gambled for redemption. Thus, the owners prefer gambling for redemption (expected value 48) over gambling for ripoff (expected value 42).

Now consider an example of gambling for ripoff in a firm whose continuation value is 80 instead of 120. With the same cash flow 40 and required payment 100, a fair gamble that wins 360 with probability 0.1 and loses 40 with probability 0.9 allows the owners to obtain expected value $0.1 \times (80 + 360 + 40 - 100) = 38$, higher than the value $0.4 \times 80 = 32$ from gambling for redemption. Bondholders get fully repaid only 10% of the time, with expected value $0.1 \times 100 = 10$, and are therefore ripped off compared to gambling for redemption or no gambling. In the previous example, continuation is good for the owners because the continuation value 120 exceeds the required debt payment 100, while in this example, continuation is bad for the owners because the required debt payment 100 exceeds the continuation value 80.

It may seem strange that bondholders are indifferent about gambling for redemption if there are no costs, given that we have learned from Myers (1977) that in a single-period model, their payoff is concave in firm value, so Jensen's inequality implies that gambling makes them worse off. However, the bond payoff is linear below the face value of debt, and linear above, so the strict concavity is only at the point of meeting the required payment exactly. For gambles that stay (weakly) on one side or the other of the point of strict concavity, bondholders are risk neutral. When gambling for redemption, the owners find it optimal to gamble just enough to achieve the required debt payment, which leaves bondholders indifferent absent costs, and better off considering costs. By contrast, gambling for ripoff crosses the point of concavity and the bondholders are worse off absent costs, and worse off even considering costs if the scale of gambling is large enough.

In Myers (1977), asset substitution is inefficient because the firms undertake inefficient activities in order to add noise to the payoff distribution. Asset substitution is also imprecise, and bondholders are worse off because the added noise can move the cash across the threshold. In most of the previous literature, the noise is usually added to the whole distribution so that selections of risk are constrained by the shapes. The choice of risk is oftentimes the choice of the variance of a normal distribution, such as in Ericsson (1997), Ross et al. (1998), Leland (1998), Gong (2004) and Della Seta et al. (2020).[7] Under normality, increasing risk increases the probability of both tails. However, gambling with derivatives is a sharper tool for gambling just what is needed, allowing gambling only in the left tail. For example, a firm can buy a digital option paying off exactly the payment due this period. In our framework, gambling is fairly priced and is more flexible and precise. the owners can concentrate the probability in a targeted payoff, e.g., the amount needed to repay the debt. If the current required payment is less than the equity's continuation value, the owners want the firm to survive and will gamble up to the face value of debt (if possible) but not further, to maximize the probability of survival.

We start our analysis in a single-period model. The required payment on debt coming due may or may not be covered by the incoming cash flow. Managers' incentives are assumed to be aligned with the owners' to maximize equity value.[8] Bankruptcy has two costs: the loss of the

[7]Ericsson (1997) studies firm's one-time choice of risk which is either at a high level or at a low level. Gong (2004), Ross et al. (1998), Leland (1998) and Della Seta et al. (2020) extend the choice of variance to an interval.

[8]Our firms are more like proprietorships than corporations in order to focus on the role of gambling. "The owners" in the model act as a single entity that makes decisions as a whole. Similarly, "bondholders" are also considered as a whole.

continuation value and an administrative cost paid out of the surviving assets. In the model, the loss of continuation value is borne primarily by the owners and the administrative cost is borne primarily by the bondholders. Before paying off the maturing debt or going into bankruptcy, the owners can choose to undertake a fair gamble subject to having enough cash flow to handle the downside. If the owners gambles for redemption, just to the level needed to repay the debt, the gambling benefits both the owners and the bondholders by minimizing both costs. Absent the administrative cost paid out of the surviving assets, the bondholders can be just indifferent about gambling for redemption since we remain on the linear part of their payoff (as shown in the previous example, a simplified version of our single-period model, receiving 40 cents on the dollar 100% of the time and receiving 100 cents on the dollar 40% of the time have the same expected value). With the administrative cost, gambling for redemption is actually better for the bondholders because it allows them to avoid the cost. If part of debt has been rolled over, gambling for redemption also makes bondholders better off since they are paid in full with higher probability when the owners gamble to cover a lower required due payment. However, whenever the face value of debt is greater than the continuation value lost in bankruptcy, the firm owners prefer for the firm not to continue since the lost continuation value is trumped by not having to pay off the debt. The firm owners may simply "take the money and run," and gambling provides a legal way of extracting the value. With gambling for ripoff, the bondholders are worse off compared to gambling for redemption (only receiving full repayment of a hundred 10% of the time and zero 90% of the time as in the previous example, compared to receiving 40 for sure without gambling).[9]

If the firm also has liquidation value, and the value can be gambled away (for example, due to superpriority), gambling for ripoff can be more attractive because it also transfers part or all of the asset value (which would be subject to a stay or a clawback absent superpriority) to the owners.

Consider again the example of gambling for redemption. The firm has continuation value 120 (excluding cash), cash flow 40, and required payment 100. To illustrate the impact of superpriority,

[9]Gambling for ripoff makes bondholders worse off compared to no gambling when the administrative costs in bankruptcy is not too big, otherwise the bondholders would likely get nothing if the firm does not gamble. However, gambling for redemption is always better for the bondholders than gambling for ripoff since redemption saves bankruptcy costs.

we also assume that the liquidation value of the firm (excluding cash) is 50. Without superpriority, we know that the owners cannot gamble the liquidation value, then gambling for redemption would still be preferred to gambling for ripoff. Now, if the firm can pledge the liquidation value 50 to gamble, we consider the following four fair gambles: a) gambling for redemption without gambling away liquidation value, that is, winning 60 with probability 0.4 and losing cash flow 40 with probability 0.6, b) gambling for ripoff without gambling away liquidation value, that is, winning 360 with probability 0.1 and losing 40 with probability 0.9, c) gambling for redemption with gambling away liquidation value, that is, winning 60 with probability 0.6 and losing 90 (cash flow plus liquidation value) with probability 0.4, and d) gambling for ripoff with gambling away liquidation value, that is, winning 360 with probability 0.2 and losing 90 (cash flow plus liquidation value) with probability 0.8. Without superpriority, we know that the owners prefer gamble a) to gamble b), and the calculation is the same as in the previous example. If the firm can gamble away liquidation value of the assets 50, gambles c) and d) are available. In gamble a), the owners obtain expected value $0.4 \times 120 = 48$. Bondholders obtain expected value $0.4 \times 100 + 0.6 \times 50 = 70$. Gamble b) makes the owners better off, with the owners obtaining expected value $0.6 \times 120 = 72 > 48$ and the bondholders obtaining expected value $0.6 \times 100 = 60$. However, gamble c) is even better for the owners who would get expected value $0.2 \times (120 + 360 + 40 - 100) = 84 > 72 > 48$, and the bondholders have expected value $0.2 \times 100 = 20 < 70$. This example shows that being able to gamble away liquidation value of assets (possibly due to superpriority law) can push the owners towards gambling for ripoff.

In the single-period model, the face value of maturing debt is exogenous. This might be a good assumption at the time of the superpriority legislation, if the legislation is a surprise to the bondholders with debt in place. It is perhaps more interesting to think about the impact of the law once it is understood by bondholders and is priced out in the lending decision. For this, we have a multi-period model that endogenizes the level of borrowing, equity's continuation value, and some other variables that are fixed in the single period model. In each period of the multi-period model, the owners choose gambling and new financing after the realization of a shock.

After the shock, if the continuation value is greater than debt coming due but current cash flow plus potential borrowing and liquidation value do not cover the debt, the owners will gamble for redemption. However, if the shock leaves the continuation value small enough, the owners prefer to lose the continuation value rather than pay the debt. Gambling for ripoff avoids paying off the debt most of the time while still capturing the cash flow on average by concentrating the gamble's payoff in a small set of states. In general, whether gambling is beneficial depends on how often there is gambling for redemption and ripoff. Our main result of the multi-period model shows that if there is significant liquidation value (for example due to superpriority), being able to gamble against assets reduces the maximum amount the owners can borrow, and also reduces the market value of equity. This suggests that superpriority can benefit firm owners if it is a surprise at the time of passage, but not once the lenders understand that the law can make large gambles optimal for the firm owners.

The "superpriority" claims we are talking about obtained their exemption from bankruptcy in a series of laws passed between 1978 and 2006. See Schwarcz and Sharon (2014) for a detailed history of the law. The game changer seems to be the 2005 amendment to bankruptcy code (BAPCA), which extends the exemption, which started with some commodity futures and previously extended to repos and swaps, to all derivative securities. Taken together, these laws exempt qualified contracts (including securities contracts, commondity contracts, forward contracts, repos, swaps, and contracts, etc.) from the automatic stay and clawbacks of bankruptcy.[10] BAPCA and the subsequent 2006 Act also added and extended protections for "master netting agreements," an arrangement between counterparties to net or set off any qualified contracts described above. If a counterparty and the firm owed each other one dollar, without netting, when the firm is in bankruptcy the counterparty has to repay the one dollar and may receive only 50 cents out of the dollar from the firm. With a netting agreement, the counterparty can set off beforehand and be paid 100 cents out of a dollar. This treatment makes gambling even easier.

[10] Supepriority also favors derivatives by exempting clawback of constructive (but not actual) fraudulent transfers. See Vasser (2005). However, the exemption from avoidance of fraudulent transfer may not apply in the context of gambling, since the transfer is in satisfaction of an existing claim and reflects a fair value.

The superpriority treatment has drawn a lot of attention since the 2008 financial crisis. Roe (2010) observes a soaring volume of interest rate derivatives from \$13 trillion in 1994 to \$430 trillion in 2009, an increase by a factor of nearly 40. During this period, the private business debt only tripled from \$11 trillion to \$34 trillion.[11] Baily et al. (2008) also shows an exponential increase in value of CDS outstanding since 2001. Roe suggests that this is because superpriority provides a cheaper way of financing, facilitating more liquidity that otherwise would not occur. This shifts the firms away from using traditional financing and lower the incentives of derivatives counterparties to monitor the firm. As a consequence of the expansion of market, the "too big to fail" problem is worsened if the superpriority claims are heavily used by the systemically important firms. Besides the costs, Duffie and Skeel (2012) claims benefits to the safe harbor exemption on QFCs, such as increasing reliance of firms using critical hedges and reducing self-fulfilling security runs, although it is also possible that superpriority causes runs to grab assets that were previously deterred by the automatic stay and clawbacks. Previous economics literature also focus on the repo market fire sales, which dilute the collateral value for the secured creditors.[12] Our paper excludes the above factors in superpriority, focusing instead on pure gambling by firms. We think that uperpriority makes it possible for a financially shaky firm on a path to bankruptcy to gamble their assets. Therefore, gambling using superpriority can operate at a much larger scale and in the presence of accounting controls, and derivative securities makes it easier to shape the exact distribution of the gamble a firm chooses.

In normal times (when it is beneficial for the firm owners to continue the firm), gambling would not be a problem because the firm with sufficient cash to pay debt would not gamble. Even at times when transient negative shocks decline the firm's cash flow, the owners prefer gambling for redemption, increasing values for the owners and bondholders. The firm always achieves the highest probability of survival, minimizing bankruptcy costs. Gambling becomes a problem when the firm's continuation value is small compared to debt that cannot be rolled over. The owners would prefer gambling for ripoff, which maximizes owners' benefits by looting the value that should have

[11] Roe (2010) Figure 1.

[12] See Infante (2013), Oehmke (2014), Antinolfi et al. (2015) and Auh et al. (2018).

been collected by the bondholders. Interestingly, if the owners benefit from bankruptcy, they favor such extreme gambling regardless having enough cash to cover debt or not. When cash is insufficient, gambling for ripoff transfers value from the bondholders to the owners; but with enough cash, gambling for ripoff also dissipates equity's continuation value. In the gambling for ripoff scenario, providing liquidity in order to save the firm may help to keep the firm temporarily, but it may not be useful enough to change the risk taking by the owners. Rather, policies that increase firms' continuation value or block large-scale gambling may be more socially efficient.

If the firm owners are potentially worse off because of the laws, as claimed in our multi-period model, the owners would have incentives to use more defensive measures (operating leverage, secured debt, short-term debt, and even repos) to protect against the laws. Also, the bondholders cannot rely on protections in bankruptcy through negative pledge covenants which preclude asset sales, but may rely more on perfected security interests (collateral) which are still honored under UCC Article 9. This is supported by some empirical evidence. For example, Benmelech et al. (2020) documents an increase of secured debt over total debt since 1995 and an upward jump in 2005.[13] Baily et al. (2008) shows that the issuance of total value of short term (with 1-4 days maturity) asset-backed commercial paper has increased significantly from 2005 to mid 2007, whereas the commercial paper with longer terms (with 21-40 days and >40 days maturities) stayed steady during the period.[14]. There was also a surge in the growth in the market for repurchase agreements, a much higher growth rate compared to the total debt in the financial sector, particularly after 1999 (Roe (2010)). In other empirical studies of repo, Ganduri (2016) finds a surge of the number of repurchase agreements after BAPCPA went into effect in 2005, whereas the number of loans plummeted during the same period; Lewis (2020) provides causal evidence of expansion of repo collateral rehypothecation as a result of the law and estimates a money multiplier of private-label mortgage collateral to be 4.5 times that of Treasuries. However, using perfected collateral to prevent large gambles also carries its own costs, including damaging the flexibility of assets redeployment and constraining future borrowing and investment claimed by Donaldson et al. (2019,

[13]See Benmelech et al. (2020) Figure 8a.

[14]Baily et al. (2008) Figure 6.

2020).[15]

Like superpriority for derivatives, superpriority for medical claims may also increase incentives for gambling. Gambling for ripoff was more attractive after Congress enacted Chapter 11 section 1114 to give retiree medical benefits special priority in 1988, because an increase of debt obligation will reduce the net gain that the owners receive upon continuation. Consequently, gambling for ripoff wipes out most of the assets and the assets dilution effect may be larger than purely having additional debt. This highly risky behavior may make it harder for the firm to raise funds, and bondholders may only be willing to lend if the firm promises to file for Chapter 7 liquidation to evade the legislation with underfunded retiree insurance benefits, exacerbating the issues pointed at by Keating (1990, 1991).

The paper is organized as follows. Section 2 focuses on the two polar cases of *"gambling for redemption"* and *"gambling for ripoff"* by examining a stripped-down single-period model. Section 3 presents a multi-period model using the building block in Section 2 and with endogenous decision making to study the ex ante effect of gambling with and without superpriority. Section 4 characterizes equilibrium properties of the model and provides numerical examples to illustrate the results, and Section 5 concludes.

2 Optimal gambling: the single-period model

We start with a stripped-down model in which we show there can be two different forms of gambling: "gambling for redemption" and "gambling for ripoff." Gambling for redemption occurs when the firm cannot pay the debt immediately, and the owners would suffer a net loss from bankruptcy. In this case, bankruptcy is bad for firm owners, so they will minimize the probability of bankruptcy by gambling just to what is needed to pay the debt, and the firm owners and the bondholders are all better off. By contrast, *gambling for ripoff* occurs when the owners would gain in net from bankruptcy. In this case, bankruptcy is good for the owners, so they will maximize the

[15] In turn, these costs can be mitigated somewhat by issuing collateralized debt with a call provision or a short maturity.

probability of bankruptcy to evade debt obligations but still collect the firm's value by gambling to a large payoff, benefiting the owners at the expense of the bondholders. Superpriority claims reduce the net loss to owners because they allow them to collect part of the firm's asset value directly without paying the bondholders most of the time, which allows bigger gambles.

To focus on gambling, the single period model in this section assumes that everything except for the firm's gambling decision is exogenous. The multi-period model in Section 3 endogenizes the continuation value, the amount of debt, and investment choice (which affects liquidation value and cash flow of the firm), but they are all exogenous in this section. The firm undergoes a liquidation (as in Chapter 7) if the owners do not come up with enough to make the current required payment. The firm has continuation value $C > 0$ in excess of current cash flow π. Total debt $P+F$ matures at the end of the period. An amount $P \geq 0$ must be paid off now or the firm will be liquidated, and $F \geq 0$ is the value of debt. We assume that P and F have the same priority in bankruptcy.[16] If the firm is liquidated, L is the liquidation value of the firm's assets excluding cash flow π. We assume that $L \leq F$, which is the interesting case because otherwise the owners can probably borrow for at least a little while to defer part of the current payment P. C, F, P, L and π are all exogenous in the single-period model we are studying now, but are endogenous in the multi-period model in Section 3. Absent gambling, if the current cash flow π exceeds the current required payment P, the payment is made and the value to owners is $\pi - P + C$. However, if $\pi < P$ and there is no gambling, then the firm fails, the owners lose the continuation value, and bondholders collect a fraction $1-c$ of the remaining value $(\pi + L)$. In bankruptcy, $\pi < P$ and $L \leq F$, implying that $\pi + L < P + F$, i.e., what is available to pay the bondholders is less than what they are owed. The constant $c \in [0,1]$ is the proportional bankruptcy costs taken out of the bankrupt firm before paying financial claimants. The total cost of bankruptcy is therefore $C + (\pi + L)c$. These assumptions imply it is efficient (from the perspective of total value of financial claimants) to continue the firm.[17]

With gambling, the owner can gamble π plus the fraction of the liquidation value that is avail-

[16] If different bondholders have different priorities in bankruptcy, the total payoff to bondholders is as in our analysis, but different bondholders will be affected differently by what the owners do.

[17] To be efficient to continue the firm, the total value of continuing the firm has to be greater than the total value not continuing. The total value if the firm is continued is $F + C + \pi + \mathbf{g}$. A feasible gamble $\mathbf{g}$ is described later. The total

able for gambling. Denote this fraction by $\gamma \in [0,1]$. Superpriority allows the owners to make larger gambles and hence increases γ. In this section, for simplicity we assume that superpriority increases the amount available for gambling from 0 to the entire liquidation value L. Gambling is required to be fair and feasible, meaning that the owners can purchase mean-zero marketed claims include all random variables on some nonatomic continuum of states, subject to the limitation of what is available to pay. We assume an upper bound $\bar{g}$ that avoids a closure problem if there is no upper limit of gambling, but we can compute limits of payoffs as $\bar{g} \uparrow \infty$. We denote by $\mathbf{g}(\tilde{x})$ the stochastic gambling payoff, implying a cash flow after gambling of $\pi + \mathbf{g}(\tilde{x})$, where $\tilde{x} \sim_d U(0,1)$ is the underlying gambling randomness. For example, it would be sufficient if it is possible to buy all functions of a random variable $\tilde{x} \sim_d U(0,1)$ with finite mean. We think of $\bar{g} >> 0$, but we require at a minimum that $\bar{g} \geq P - \pi$, or $\bar{g} + \pi \geq P$, meaning that gambling is at least possible to cover the required payment. The two rules for gambling are

$$\text{gambling is fair:} \qquad \mathrm{E}[\mathbf{g}(\tilde{x})] = 0, \quad \text{and} \tag{1}$$

$$\text{gambling is feasible:} \; -\gamma L - \pi \leq \mathbf{g}(\tilde{x}) \leq \bar{g} \tag{2}$$

where

$$\gamma \equiv \begin{cases} 1, & \text{with superpriority} \\ 0, & \text{absent superpriority.} \end{cases}$$

"Fair gambling" assumes no transaction costs, which is reasonable because the cost of gambling trading derivatives is tiny provided a derivative in a liquid market is chosen. Either agents are risk neutral and $\mathrm{E}[\cdot]$ in (1) indicates the common beliefs, or $\mathrm{E}[\cdot]$ indicates risk-neutral expectations given agents' shared valuations. Note that not gambling can be regarded as the particular choice $\mathbf{g}(\tilde{x}) \equiv 0$.

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value if the firm is not continued is $(1-c)(L+\pi+\mathbf{g})$. Since $F \geq L$, $C > 0$ and $c \in [0,1]$, we have

$$F + C + \pi + \mathbf{g} > L + \pi + \mathbf{g} \geq (1-c)(L+\pi+\mathbf{g}),$$

so it is always efficient to continue.

Rule (2) suggests that market players cannot make "empty promises" of payment beyond their ability to pay. Absent superpriority, $\gamma = 0$ and $\mathbf{g}(\tilde{x}) \geq -\pi$, the owners can gamble away at most the cash flow π, because gambling counterparties know that the owners cannot reliably promise more than π. However, with superpriority, $\gamma = 1$ and $\mathbf{g}(\tilde{x}) \geq -L - \pi$, saying that the liquidation value L of other assets in the firm is available for gambling. Of course, γ is also impacted by the amount of perfected collateral which is protected even with superpriority law. In this paper, we assume no perfected collateral, which is the most interesting case.[18] This rule and the assumption of no perfected collateral indicate that superpriority laws make larger scale gambling feasible.

We show an example to illustrate the rules. Absent superpriority, if the firm has \$100 in cash and uses it to gamble, the gambling counterparty would not gamble with the owners if the firm promised to pay \$200 in some contingency, or to put it another way, the counterparty would not believe the promise of \$200. Even if the owners pledged its other assets with market value of \$100, the promise to repay \$200 is still not credible because of the clawback rules in bankruptcy.

We have in mind that the gambling is very short-term, which is essential if the proceeds are to be used to pay current liabilities. Very short-term gambles are definitely possible when gambling using derivatives. Very short-term gambling means it is reasonable that the liquidation value after the gamble is assumed to be known before the gamble. It makes sense for the firm's gambling counterparties to understand the firm's condition in the short run and assumption that the liquidation value L is an exogenous constant.

The firm owners maximize the expected payoff to equity, and the owners' problem is formally stated as follows.

2.1 Firm's problem

Given the cash flow π, debt P due now, the continuation value C, the liquidation value L, and the face value of ongoing debt F, the owners' problem is to choose a fair gamble $\mathbf{g}(\tilde{x})$, where $\tilde{x}$ is the

[18] If the owners pledge all the assets as collateral, then superpriority makes no difference to the gambling choices. However, as said before, the use of collateral also comes with other costs as suggested in Donaldson et al. (2019, 2020). The optimal choice of collateral would also be interesting to study in this scenario.

underlying gambling randomness: $\tilde{x} \sim_d U(0,1)$, maximizing[19]

$$\mathrm{E}\big[\big(\pi + \mathbf{g}(\tilde{x}) \geq P\big)\big(\pi + \mathbf{g}(\tilde{x}) - P + C\big)\big],^{20} \tag{3}$$

subject to the gamble being fair,

$$\mathrm{E}[\mathbf{g}(\tilde{x})] = 0, \tag{4}$$

and the constraint of a feasible gambling outcome

$$-\gamma L - \pi \leq \mathbf{g}(\tilde{x}) \leq \bar{g}. \tag{5}$$

Given any choice of gambling $\mathbf{g}$, we have

$$\textit{bond value} = \mathrm{E}\Big[(\pi + \mathbf{g}(\tilde{x}) < P)(1-c)(\pi + \mathbf{g}(\tilde{x}) + L) + (\pi + \mathbf{g}(\tilde{x}) \geq P)(P + F)\Big].$$

Available liquidation value L may be small because of the nature of the firm and its capital, but we are mostly interested in its availability for gambling. In our simple model, we think of L as the liquidation value of the firm and also the available liquidation value for gambling under superpriority. Below we show that the gambling behavior of the owners is quite different depending on whether P is bigger than $C - \gamma L$. In particular, if $P < C - \gamma L$, the owners will gamble at a small scale which benefits bondholders; whereas if $P > C - \gamma L$, the owners will gamble at a large scale

[19]The notations we use are defined as follows:

$$(aRb) \equiv \begin{cases} 1, & \text{for } aRb \\ 0, & \text{otherwise} \end{cases}$$
$$a \wedge b \equiv \min\{a, b\},$$
$$(A)^+ \equiv \begin{cases} A, & \text{for } A \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

[20]In bankruptcy, since the cash flow and liquidation value cannot cover the total face value of debt, all the value subtracting the bankruptcy costs go to the bondholders and hence the owners get zero. In detail, the owners' value (or equity value) is

$$\mathrm{E}\big[\big(\pi + \mathbf{g}(\tilde{x}) < P\big)(1-c)\big(\pi + \mathbf{g}(\tilde{x}) + L - P - F\big)^+ + \big(\pi + \mathbf{g}(\tilde{x}) \geq P\big)\big(\pi + \mathbf{g}(\tilde{x}) - P + C\big)\big].$$

When $\pi + \mathbf{g}(\tilde{x}) < P$ and since $L < F$, we have $\pi + \mathbf{g}(\tilde{x}) + L - P - F < 0$. The first term is zero.

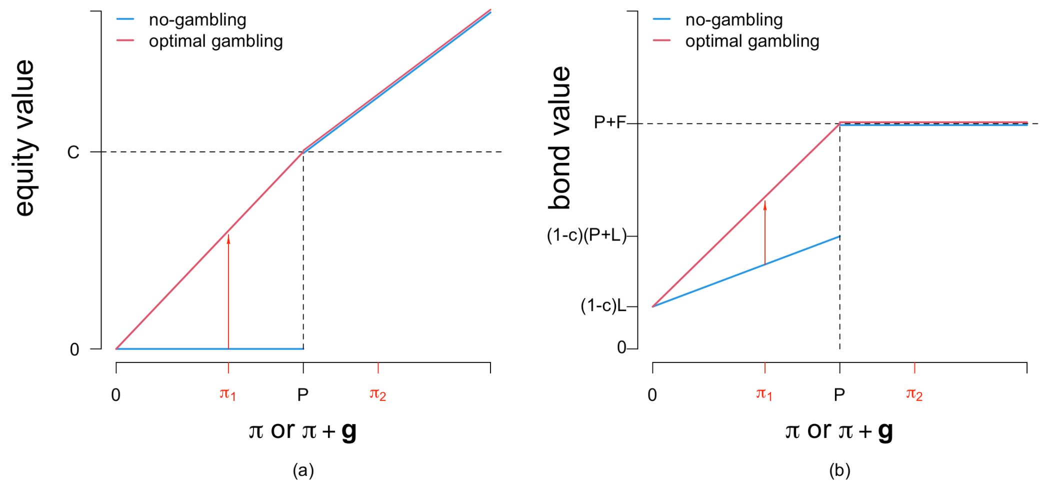

Figure 1: ***When $P < C$, the owners gamble for redemption. Graphs (a) and (b) show dependence of equity value and bond value on cash flow π or $\pi + \mathbf{g}(\tilde{x})$, respectively.*** *Gambling using derivatives is precise. The optimal gambling depicted by the red lines "concavifies" the equity value function. The red arrow shows the change of the expected value through optimal gambling given cash flow π. When bankruptcy costs for bondholders are zero ($c = 0$) and liquidation value can cover the debt rolled-over ($L = F$), gambling for redemption stays on a linear segment of the debt payoff (between zero and the required payment) where the bondholders are indifferent about gambling.*

which rips off the bondholders. With superpriority, γ is larger, implying that the owners gambles for ripoff more often.

Example 1: gambling for redemption (absent superpriority, $\gamma = 0$)

Without superpriority, $\gamma = 0$. Figure 1 demonstrates the equity value and bond value as functions of cash flow when $P < C$, i.e., debt maturing now is less than the equity's continuation value. The blue lines represent the values without gambling: if cash flow π is below P, the owner loses all the continuation value in bankruptcy and bondholders lose a fraction $1 - c$ of the remaining assets $\pi + L$; if cash flow π is above P, the continuation value C is maintained and the total bond value is $P + F$.

To reach the maximal expected equity value, the value of an optimal gambling strategy, shown by the red lines, should "concavify" the blue curves. As shown in Figure 1, if the firm starts with cash flow $\pi_2 \geq P$, the firm is sound and the owners will only gamble along the 45 degree segment and will never gamble down below P, and none of those gambles change the expected payoff for the owners or bondholders. However, if the firm starts with cash flow $\pi_1 < P$, equity is worthless unless there is gambling. In this case, optimal gambling retains the continuation value as often as possible, and the after-gamble cash flow $\pi_1 + \mathbf{g}(\tilde{x})$ achieves P, with probability $\frac{\pi_1}{P}$, and 0, with probability $1 - \frac{\pi_1}{P}$. The owners get C with probability $\frac{\pi_1}{P}$, expected value $\frac{\pi_1}{P}C > 0$. The bondholders get $P+F$ with probability $\frac{\pi_1}{P}$ and $(1-c)L$ with probability $\frac{\pi_1}{P}$, expected value $L + \pi_1 + \frac{\pi_1}{P}(F-L) \geq L + \pi_1$. The owners and bondholders will achieve values along the "concavified" value functions and are better off than not gambling.

To conclude, gambling for redemption adds value to the owners and the bondholders due to less frequent value loss in bankruptcy. When there is no fractional bankruptcy cost ($c = 0$) and the face value of ongoing debt is equal to the liquidation value ($F = L$), the bondholders are indifferent to whether the owners gamble or not.

Example 2: gambling for ripoff (absent superpriority, $\gamma = 0$)

However, when the face value of the maturing debt P is greater than the value lost in bankruptcy C, *"gambling for redemption"* is no longer optimal. The dashed red lines in Figure 2 give the payoffs of fair Bernoulli gambles

$$\mathbf{g}(\tilde{x}) = \begin{array}{l} \xrightarrow{\pi/(\bar{g}+\pi)} \bar{g} \\ \xrightarrow[\bar{g}/(\bar{g}+\pi)]{} -\pi \end{array}$$

As the payoff $\bar{g}$ increases, the probability of winning declines but the owners have a larger value because not paying P is more important to them than not receiving C. The above Bernoulli gamble concavifies the owners' original value function and is the optimal gamble. In this gamble, the owners obtain $\bar{g}$ with probability $\frac{\pi}{\bar{g}+\pi}$ and 0 with probability $1 - \frac{\pi}{\bar{g}+\pi}$. Therefore, the maximum that the owners can achieve in gambling in the limit is $\lim_{\bar{g}\uparrow\infty} \frac{\pi}{\bar{g}+\pi}(\bar{g} - P + C) = \pi$ and the bond value is $\lim_{\bar{g}\uparrow\infty} \left(\frac{\pi}{\bar{g}+\pi}(P+F) + \frac{\bar{g}}{\bar{g}+\pi}(1-c)L \right) = (1-c)L$. The bondholders almost always only

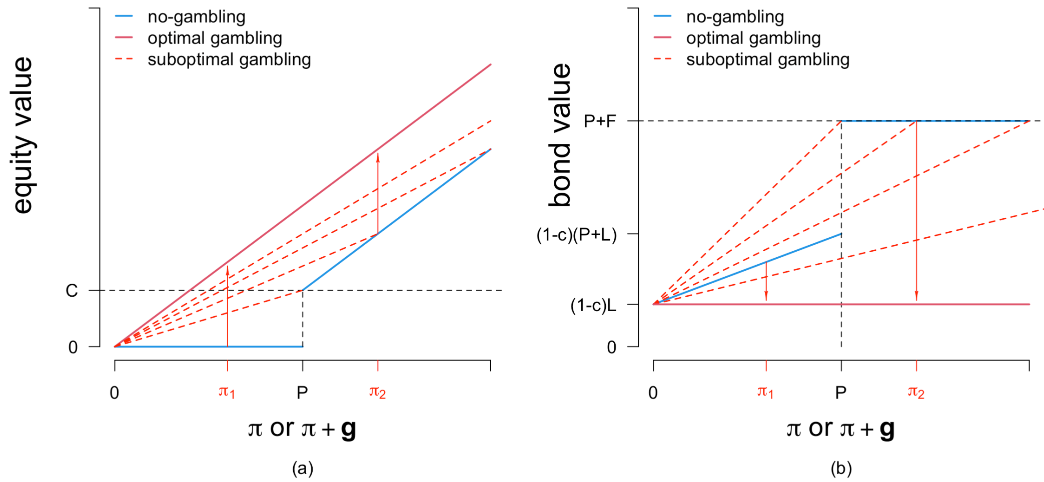

Figure 2: ***When $P > C$, the owners gamble for ripoff.*** *Similarly, the equity value after optimal gambling (red lines) is a concavification of the original value function (blue lines). The bond value decreases to $(1-c)L$, the amount that cannot be gambled away by the owners. Interestingly, whether the firm is out of money ($\pi = \pi_1$) or in the money ($\pi = \pi_2$), gambling for ripoff is optimal for the owners.*

receive part of the liquidation value. In this case, *gambling for redemption* would increase the total value of bond and equity because the continuation value would be preserved as often as possible, but the owners would rather choose a larger gamble which gives them a higher value at the expense of bondholders. We have shown that $\pi = \pi_1$, if the firm has cash flow $< P$, gambling for ripoff transfers value from the bondholders to the owners. Interestingly, gambling for ripoff is also optimal for equity in this example even if $\pi = \pi_2 > P$. so the firm has enough to payoff the debt P without gambling.

Example 3: with superpriority $C - L < P < C$

Positive available liquidation value to gamble will change the shape of gambling if $C > P > C - L$, as illustrated in Figure 3. Superpriority makes the liquidation value available for gambling, allowing firm owners to gamble down to $-L$ instead of 0. In Figure 3, the continuation value C is greater than the payment due P, so that absent superpriority the owners will gamble its cash flow for redemption of the value of the payment due and obtain $\frac{\pi}{P}C$, as depicted by Figure 3(a). Figure 3(b)

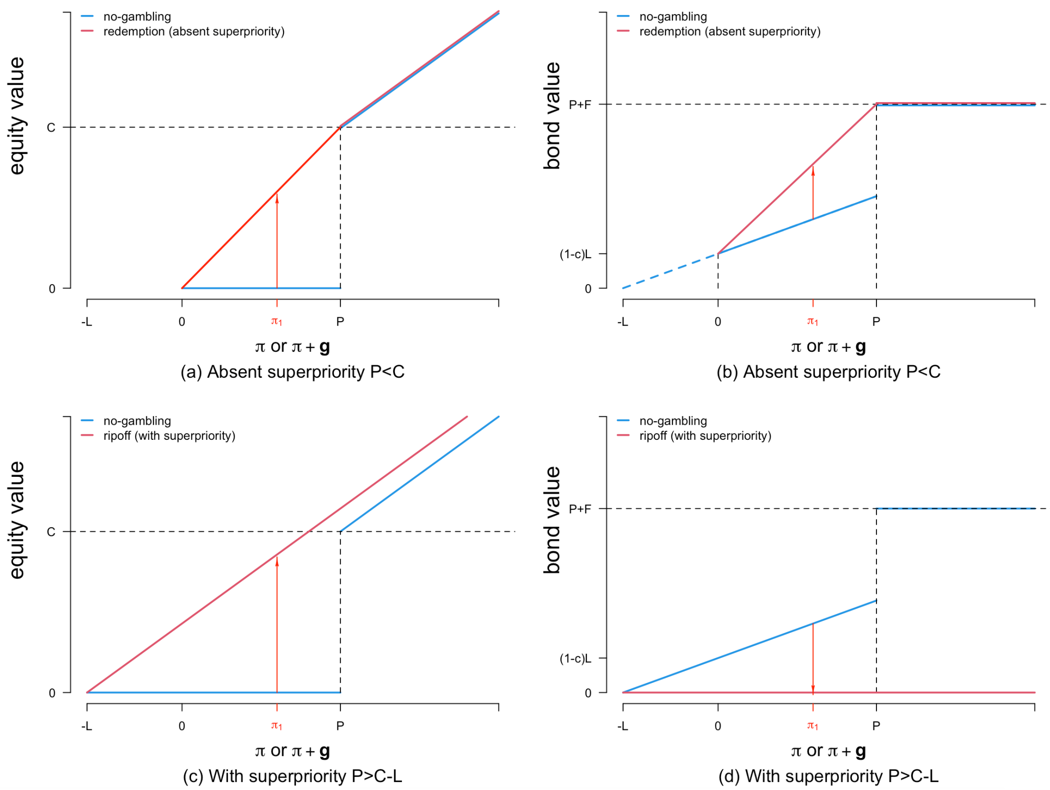

Figure 3: ***Superpriority makes gambling for ripoff more often.*** *When $C - L < P < C$, the owners gamble for redemption absent superpriority, shown by (a)(b); the owners gamble for ripoff with superpriority, shown by (c)(d).*

shows the relevant bond value and bondholders are also better off. By allowing gambling with superpriority claims, firms can gamble down to $-L$, and superpriority gambling yields greater benefits when the owners *"gambles for ripoff,"* as shown in Figure 3(c). However, larger gambles make bondholders worse as in Figure 3(d). It is straightforward in the graph that the determinant of gambling for redemption or ripoff falls to the comparison of $L+P$ and C, or P and $C-L$, where $C-L$ is the value lost in bankruptcy.

These graphic observations are formally stated by the following propositions:

PROPOSITION 2.1 *when $P < C - \gamma L$ (the payment due now is less than the value lost in bankruptcy), it is optimal to **gamble for redemption**. Under this parameter restriction, gambling strictly increases the value of both bond and equity when $\pi < P$, and leaves both unchanged when $\pi \geq P$. Specifically,* *(1) If the cash flow before gambling is insufficient to make the current debt payment ($\pi < P$), all optimal gambles have the same distribution. In particular, an optimal gamble is*

$$\mathbf{g}^*(\tilde{x}) = \begin{cases} P - \pi, & \textit{for } 0 < x \leq \frac{\pi+\gamma L}{P+\gamma L} \\ -\gamma L - \pi, & \textit{for } \frac{\pi+\gamma L}{P+\gamma L} < x < 1. \end{cases} \tag{6}$$

(2) If the cash flow before gambling is sufficient to make the current debt payment ($\pi \geq P$), the optimal gambles are the feasible gambles that never reduce cash below the current debt payment P. The set of solutions is

$$\left\{ \mathbf{g}^*(\tilde{x}) \middle| \, \pi + \mathbf{g}^*(\tilde{x}) \geq P \textit{ and } \mathrm{E}[\mathbf{g}^*(\tilde{x})] = 0 \right\}.$$

In particular, not gambling ($\mathbf{g}^*(\tilde{x}) \equiv 0$) is always optimal, and it is the only solution if $\pi = P$.

(3) The payoffs are

$$\textit{equity value} = \begin{cases} \pi - P + C, & \textit{for } \pi \geq P \\ \frac{\pi+\gamma L}{F+\gamma L} C, & \textit{for } \pi < P \end{cases}$$

$$\textit{bond value} = \begin{cases} P + F, & \textit{for } \pi \geq P \\ \frac{\pi+\gamma L}{P+\gamma L}(P+F) + (1 - \frac{\pi+\gamma L}{P+\gamma L})(1-c)(1-\gamma)L, & \textit{for } \pi < P \end{cases}$$

$$\textit{bond+equity} = \begin{cases} \pi + C + F, & \textit{for } \pi \geq P \\ \frac{\pi+\gamma L}{P+\gamma L}(C+P+F) + (1 - \frac{\pi+\gamma L}{P+\gamma L})(1-c)(1-\gamma)L, & \textit{for } \pi < P \end{cases}$$

Proof. (Sketch) Following Aumann and Perles (1965), we first concavify the objective function and use Kuhn-Tucker conditions to solve for the concavified problem. Since the constructed solution(s) for the concavified problem is(are) also feasible for the original problem, and the concavified objective function is greater than the original function, we can conclude that the solution(s) for the concavified problem also solve the original problem. Details see Appendix A. ■

In Proposition 2.1(2), if we believe that gambling is costly, or if we are not using risk neutral probabilities (the owners are risk averse), then $\mathbf{g}^*(\tilde{x}) \equiv \pi$ (no gambling) should be the unique solution. However, in Proposition 2.1(1), gambling is still optimal in the face of a sufficiently small cost.

PROPOSITION 2.2 *when $P > C - \gamma L$, it is optimal for the owners to* ***gamble for ripoff.*** *Gambling for ripoff transfers value from bondholders to the owners when $\pi < P$, and also destroys continuation value when $\pi \geq P$. Specifically,*

(1) The optimal gambling is

$$\mathbf{g}^*(\tilde{x}) = \begin{cases} \bar{g}, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{\bar{g}+\pi+\gamma L} \\ -\gamma L - \pi, & \text{for } \frac{\pi+\gamma L}{\bar{g}+\pi+\gamma L} < x < 1 \end{cases} \tag{7}$$

(2) Owners' payoff is $\frac{\pi+\gamma L}{\bar{g}+\pi+\gamma L}(\bar{g} - P + C)$, which increases to $\pi + \gamma L$ as $\bar{g} \to \infty$. The value of the bonds is $\frac{\pi+\gamma L}{\bar{g}+\pi+\gamma L}(P+F) + \frac{\bar{g}}{\bar{g}+\pi+\gamma L}(1-c)(1-\gamma)L$, and declines to $(1-c)(1-\gamma)L$ as $\bar{g} \to \infty$. For any $\pi > 0$, the total value of bond and equity is always $\pi + L - (1-\gamma)cL$ when $\bar{g} \to \infty$.

Proof. See Appendix B. ■

Since it is efficient to continue the firm, gambling for redemption maximizing the probability of continuation is socially beneficial while gambling for ripoff minimizes this probability is socially damaging. In the trade-offs between *gambling for redemption* and *gambling for ripoff*, superpriority plays an important role. It transfers owners the liquidation value which should go to bondholders, making *gambling for ripoff* more appealing to the owners. With more "ripoff" cases, continuation value is lost more often.

There is also a knife-edge case when $P = C - \gamma L$. In this case, any fair gamble with outcomes distributed long the 45-degree linear segment would yield the same expected value. That is to say, *gambling for redemption* and *gambling for ripoff* give the same outcome for the owners, and anything in between the two polar cases is also optimal. Though these optimal gambles generate different values for the bondholders (for example, we still have *gambling for redemption* makes the bondholders better-off and *gambling for redemption* worse-off), we don't want to go into the details of what the equilibrium(equilibria) is(are) because it is reasonable to believe that $P = C - \gamma L$ almost never happen.

In this paper we assume the absence of workouts before or during bankruptcy. We can think of workouts as infeasible if there is a large number of diverse claimants. Even if workouts are possible, gambling for ripoff tends to be robust because the owners would prefer gambling to workouts and bankruptcy. Under gambling for ripoff, the bondholders receive little assets especially when superpriority is available. In a hypothetical workout, this threat of gambling for ripoff allows the owners to appropriate most of the bondholders' value, making the payoffs similar to gambling for ripoff even if the gambling does not actually happen in equilibrium. Gambling for redemption is less robust to the availability of workouts, but the advantage of workouts can be undermined by the high costs.

3 The Dynamic Model with Endogenous Debt and Continuation Value

Warning: the multiperiod model is under revision. In particular, the plots doesn't match the text.

Thus far, we have examined the conditions for gambling for redemption and ripoff, depending on the net loss for the owners in bankruptcy. We can think the single-period model may represent owners' risk taking when the superpriority law is a surprise. If the superpriority legislation induces gambling for ripoff in the single-period model, it probably also induces gambling for ripoff in the multi-period model of which it is a snapshot, given that the continuation value is likely to be reduced by the announcement.

Taking the continuation value and the amount of borrowing as exogenous is a shortcoming of the single-period model, while we expect that both will be affected by the presence of superpriority law. In this section, we consider a multi-period model in which both are endogenous. Lenders anticipate the impact of superpriority on the owners' behavior, and we find that a significant amount of superpriority tends to reduce firm value because the increased incentive to gamble is anticipated and reflected in the terms on which the owners can borrow.

In this section, the firm is liquidated and ceases to exit either because of bankruptcy – it fails to meet the debt payment – or because of exogenous disappearance of the firm's market. All the Debt in this model has duration of one period, and the owners have to repay the face value of debt in full to avoid bankruptcy. Exogenous disappearance happens with conditional probability ρ after gambling and before borrowing (for the purpose of simplification), with randomness drawn independently of the other shocks in the model. We will write the objective function as the expectation taken over gambling and the distribution of exogenous ending dates (due to disappearance of industry) and endogenous ending dates (due to bankruptcy). There are two objective functions for the owners: before gambling and after gambling. Choice variables, state variables, and realization of shocks will be conditional on the firm still existing.

Below shows the timeline of the model at time t:

The firm enters time t with capital K_t^{new} and face value of debt F_t
Nature chooses whether to terminate with exogenous shock with probability ρ
Nature chooses multiplicative capital shock δ_t, capital after the shock $K_t \equiv \delta_t K_t^{new}$
Cash flow vK_t
Owners choose gambling payoff $\mathbf{G_t}$, value function $C^{b4}(K_t, F_t)$
Nature chooses realization of the gamble G_t, "surplus" $S_t \equiv vK_t - F_t + G_t$
Owners choose debt proposal $(B_t^{offer}, F_{t+1}^{offer})$, value function $C^{after}(K_t, S_t)$
Bondholders accept or not, indicator ι_t.
Actual debt $B_t \equiv \iota_t B_t^{offer}$ and actual face value $F_{t+1} \equiv \iota_t F_{t+1}^{offer}$.
If $S_t + B_t < 0$, not enough cash to cover F_t, terminate
Remaining cash augments capital by no more than the growth rate $g > 0$
Carry augmented capital K_{t+1}^{new} and debt F_{t+1} into the next period

In each period $t \in [0,T]$, the firm enters with capital K_t^{new} and a maturing debt F_t. Nature chooses whether to terminate the firm with exogenous probability ρ and in this case, the owners receive liquidation value $\theta K_t^{new} - F_t$. With probability $1-\rho$ the firm continues. Capital is subject to an i.i.d. shock $\tilde{\delta}_t > 0$ with $\mathrm{E}[\tilde{\delta}_t] = 1$, so capital is $K_t = \tilde{\delta}_t K_t^{new}$ after the shock.[21] This assumption reflects the fluctuation of firm's asset value under economic uncertainty: in good times, the asset value increases and so does the firm's continuation value; when the economy goes south, the negative shock can drive down the firm's continuation value. The shock can be industry or firm-specific. After the shock, capital pays a cash flow $vK_t > 0$.

Depending on which case we are considering, after the capital shock is realized, the owners can gamble in a frictionless competitive gambling market. The value function denoted $C^{b4}(K_t, F_t)$ is the owners' continuation value "before" gambling. A frictionless competitive gambling market means that gambling choice $\mathbf{G_t}$ has mean 0, a fair gamble as in the single-period model, with $\mathbf{G_t} \equiv 0$ if there is no gambling. Absent superpriority, the owners gamble with only the cash flow $vK_t + (F_t)^-$,[22] which includes savings (negative borrowings) from the previous period. With superpriority, the owners can deploy $vK_t + (F_t)^- + \gamma\theta K_t$, the cash flow plus a fraction $\gamma \in [0,1]$ of the liquidation value θK_t. The constant parameter γ is the proportion of assets that are not protected, for example, when they are not pledged as perfected collateral. For our analysis γ is exogenous, but in a richer model superpriority laws could induce firms to increase γ since the owners and bondholders would have incentives to seek protection of the firm's assets. Liquidation can be costly, and the liquidation value is $\theta \in [0,1]$ per unit of capital. So, $\gamma\theta K_t$ is part of capital that the owners can gamble away. We assume that new borrowing representing forecast cash flow cannot be used

[21] Any depreciation has been included in $\tilde{\delta}$. We assume i.i.d. distribution for simplicity. It may be interesting to model correlated shocks or temporary shocks to better mirror the reality, but a case is i.i.d. In our model, cash flow in this period is affected by the capital shock, which simplifies the expression.

[22] We follow the convention that for any $a \in \mathbb{R}$, $a^+ \equiv \max\{a, 0\}$ and $a^- \equiv \max\{-a, 0\}$.

to gamble, since the firm cannot credibly commit to borrow.[23] Formally, $\mathbf{G_t}$ satisfies

$$\begin{aligned}
&\mathrm{E}\left[\mathbf{G_t}\right] = 0, \text{ and} \\
&\bar{g}K_t \geq \mathbf{G_t} \geq -\nu K_t - (F_t)^- - \gamma\theta K_t \ (\gamma = 0 \text{ absent superpriority}) \qquad (*)
\end{aligned}$$

It is worth emphasizing that gambling in this paper has a short duration. We think that it is optimal for the owners to use short-maturity derivatives since they have to acquire current information about the various positions at the time the derivative matures.[24]

After gambling is realized, the owners have net cash $S_t \equiv \nu K_t - F_t + G_t$. Then they propose a take-it-or-leave-it bond offer with amount B_t^{offer} and face value F_{t+1}^{offer}, where $B_t^{offer} < 0$ is interpreted as risk-free investment. The value function denoted $C^{after}(K_t, S_t)$ is the owners' continuation value "after" gambling. The bond market is also frictionless and competitive. Bondholders choose whether to accept the bond offer with indicator $\iota_t \in \{0,1\}$.[25] We denote the actual debt $B_t \equiv \iota_t B_t^{offer}$, and the actual face value is $F_{t+1} \equiv \iota_t F_{t+1}^{offer}$. If there is not enough cash after borrowing to cover all the debt due, i.e. $S_t + B_t < 0$, bankruptcy occurs and the owners receive $(1-c)(\theta K_t + S_t)^+$, value of assets after selling at a discount price and a deduction of fractional bankruptcy cost. Bondholders obtains $(1-c)[F_t^{offer} \wedge (\theta K_t + \nu K_t)]$.[26] If $S_t + B_t \geq 0$ there is enough cash after clearing all the debt, the firm continues and may also increase the capital at a growth rate capped by g per period[27], and we require $(1+g)(1-\rho) < 1$ to ensure that firm value is finite. The new capital K_{t+1}^{new} entering the next period is the sum of remaining capital and new

[23]This simplification may be without loss of generality in our setting, because there seems to be no point of gambling if the firm can survive a loss.

[24]Firms' motivations for holding derivatives (either to gamble or to hedge) might be distinguished by duration of derivatives. Long-dated derivatives are probably for hedging, while short-dated derivatives are more likely for gambling.

[25]A mixed strategy of the bondholders would not survive in equilibrium since the owners can always provide infinitesimal amount more to the bondholders to break the indifference. Since the bond market is competitive, in an equilibrium the owners can always fairly price the bonds such that the bondholders choose $\iota_t \equiv 1$ for every t. This is useful in finding an equilibrium.

[26]This assumes that bondholders are also subject to a fractional bankruptcy cost even after they received full repayment.

[27]The cap g is a reasonable assumption because acquisitions are usually time consuming, and firms usually have limited capacity to expand within a period of time. We also need this assumption to rule out infinite borrowing.

investment which comes from net cash and new borrowing, and it is constrained by a maximum growth rate g of capital

$$K_{t+1}^{new} = K_t + S_t + B_t \leq K_t(1+g).$$

We will look for an equilibrium that is Markov in a short list of state variables. Specifically, the owners' value functions before gambling depends on capital after shock K_t and outstanding debt F_t. The value function after gambling depends on capital after the shock K_t and net cash after gambling S_t. There are different time points in a period at which we can look at the equity's continuation values, but we pick the two moments right before agent's choices of gambling. We state the owners' problems sequentially in a form of Bellman equation.

3.1 Bellman equations

(Gambling node) At time t after capital shock, given capital K_t and debt outstanding F_t, the owners choose adapted gambling $\mathbf{G_t}(\tilde{x}) \in \mathscr{G}$ to maximize expected value. The Bellman equation is

$$C^{b4}(K_t, F_t) = \max_{\mathbf{G_t}(\tilde{x}) \in \mathscr{G}_t} \mathrm{E}\left[C^{after}(K_t, vK_t - F_t + \mathbf{G_t}(\tilde{x}))\right], \tag{8}$$

the set $\mathscr{G}_t$ of feasible gambles is given by

$$\mathscr{G}_t \equiv \left\{\mathbf{G_t} : [0,1] \to \mathscr{O}_t \,\middle|\, \mathrm{E}[\mathbf{G_t}(x)] = 0\right\},$$

given the feasible gambling outcomes

$$\mathscr{O}_t = \begin{cases} \{0\}, & \text{no gambling} \\ [-vK_t - (F_t)^- - \gamma\theta K_t, \bar{g}K_t], & \text{otherwise} \end{cases}$$

Note that $\gamma = 0$ if there is no superpriority.

(Borrowing proposal node) At time t after gambling is realized, given capital after gambling K_t and net cash after gambling $S_t \equiv vK_t - F_t + G_t$, the owners choose adapted new borrowing and face value $(B_t^{offer}, F_{t+1}^{offer})$ to maximize expected value. The Bellman equation is

$$\begin{aligned}
C^{after}(K_t, S_t) = \max_{(B_t^{offer}, F_{t+1}^{offer})} & \left(S_t + \iota_t(B_t^{offer}, F_{t+1}^{offer}, K_t, S_t)B_t^{offer} < 0\right)(1-c)(\theta K_t + S_t)^+ \\
& + \left(S_t + \iota_t(B_t^{offer}, F_{t+1}^{offer}, K_t, S_t)B_t^{offer} \geq 0\right)\Big\{\rho(\theta K_{t+1}^{new} - \iota_t(B_t^{offer}, F_{t+1}^{offer}, K_t, S_t)F_{t+1}^{offer}) \\
& + (1-\rho)\mathrm{E}\left[C^{b4}(\delta_{t+1}K_{t+1}^{new}, \iota_t(B_t^{offer}, F_{t+1}^{offer}, K_t, S_t)F_{t+1}^{offer})\right]\Big\},
\end{aligned} \tag{9}$$

subject to the expressions for capital at the start of next period

$$K_{t+1}^{new} = K_t + S_t + \iota_t(B_t^{offer}, F_{t+1}^{offer}, K_t, S_t)B_t^{offer},$$

the capital growth constraint

$$K_{t+1}^{new} \leq K_t(1+g),$$

and the function $\iota_t(B_t^{offer}, F_{t+1}^{offer}, K_t, S_t) = 1$ if the market accepts the offer or 0 otherwise. Specifically,

$$\iota_t(B_t^{offer}, F_{t+1}^{offer}, K_t, S_t) = \begin{cases} 0, & \text{if } S_t + B_t^{offer} < 0 \\ \arg\max_{\iota_t \in \{0,1\}} \iota_t \mathrm{E}\Big[\rho F_{t+1}^{offer} + (1-\rho)\Big\{\left(\tilde{S}_{t+1} + B_{t+1}^{offer} < 0\right)(1-c) & \\ \quad [F_{t+1}^{offer} \wedge (\theta K_{t+1} + vK_{t+1})] + \left(\tilde{S}_{t+1} + B_{t+1}^{offer} \geq 0\right)F_{t+1}^{offer}\Big\} - B_t^{offer}\Big], & \text{otherwise} \end{cases}$$

We assume that the bond market does not deal with a firm that is going bankrupt the current period, that is, when the proposed new borrowing does not cover the shortfall $\iota = 0$. We also assume that the firm does not have any other sources of financing. We next normalize the problems by dividing K_t, assuming homogeneity of the problem, so everything can be expressed as value per unit of capital.

3.2 Normalization

In this section, we look for a homogenous solution in our short list of state variables in each node. We normalize the model and express the variables in terms of per unit of capital. We analyze a subgame perfect Nash equilibrium in which all the optimal debt offers are accepted, i.e., $\iota_t(\cdot) \equiv 1$ (therefore, we have $(B_t^{offer}, F_{t+1}^{offer}) = (B_t, F_{t+1})$). We define

$$
\begin{aligned}
&s \equiv \frac{S_t}{K_t},\ \phi \equiv \frac{F_t}{K_t},\ b \equiv \frac{B_t}{K_t},\ f' \equiv \frac{F_{t+1}}{K_{t+1}^{new}};\\
&\tilde{s}' \equiv \frac{\tilde{S}_{t+1}}{K_{t+1}},\ \phi' \equiv \frac{F_{t+1}}{K_{t+1}},\ b' \equiv \frac{B_{t+1}}{K_{t+1}};\\
&\delta \equiv \delta_t,\ \tilde{\delta}' \equiv \tilde{\delta}_{t+1};\ \mathbf{g}(\tilde{x}) \equiv \frac{\mathbf{G_t}(\tilde{x})}{K_t},\ \mathbf{g}'(\tilde{x}) \equiv \frac{\mathbf{G_{t+1}}(\tilde{x})}{K_{t+1}}.
\end{aligned}
$$

The "prime" on any variable refers to the variable at $t+1$. For example, s is the net cash per unit of capital after gambling in this period, while s' is net cash per capital after gambling in the next period. The choice variables (b, f'), representing borrowing per capital and face value per new capital. We can compute other variables accordingly.[28]

Now we restate the firm's problem:

(Gambling node) Given debt per unit of capital ϕ, the owners choose adapted gambling $\mathbf{g}(\tilde{x}) \in \mathscr{G}$ to maximize expected value. The Bellman equation is

$$
C^{b4}(\phi) = \max_{\mathbf{g}(\tilde{x}) \in \mathscr{G}} \mathrm{E}\left[C^{after}(v - \phi + \mathbf{g}(\tilde{x}))\right], \tag{10}
$$

[28]We have

$$
\begin{aligned}
&\frac{K_{t+1}^{new}}{K_t} = 1+s+b,\ \frac{K_{t+1}}{K_t} = \tilde{\delta}'(1+s+b),\\
&\beta = \begin{cases} 1, \text{ if } 1+s=0, \\ \frac{b}{(1+s+b)}, \text{ otherwise.} \end{cases} \qquad b = \frac{(1+s)\beta}{1-\beta}, (\beta \neq 1).
\end{aligned}
$$

the set $\mathscr{G}$ of feasible gambles is given by

$$\mathscr{G} \equiv \left\{ \mathbf{g} : [0,1] \to \mathscr{O} \middle| \mathrm{E}[\mathbf{g}(x)] = 0 \right\},$$

given the feasible gambling outcomes

$$\mathscr{O} = \begin{cases} \{0\}, & \text{no gambling} \\ [-v - \phi^{-} - \gamma\theta, \bar{g}], & \text{otherwise} \end{cases}$$

Note that $\gamma = 0$ if there is no superpriority.

(Borrowing proposal node) Given net cash per capital after gambling s, the owners choose adapted new borrowing and face value (b, f') to maximize expected value. We conjecture $\mathscr{S}$ the set of s within which the firm owners optimally choose to continue the firm. If the firm's value is increasing in s (can prove?), and we assume that the owners and bondholders follow the equilibrium policy functions in each period, then $\mathscr{S}$ is of the form $\mathscr{S} = \{s | s \geq \underline{s}\}$, $\underline{s}$ is a constant threshold of default-level of cash flow. The Bellman equation is[29]

$$C^{after}(s) = \max_{(b,f')} \left(s < \underline{s}\right)(1-c)(\theta + s)^{+} + \left(s \geq \underline{s}\right)(1 + s + b)\left\{ \rho(\theta - f') + (1-\rho)\mathrm{E}\left[\tilde{\delta}' C^{b4}(f'/\tilde{\delta}')\right] \right\},$$

subject to the capital growth constraint

$$s + b \leq g,$$

29

$$C^{after}(s) = \max_{(b,f')} \left(s + b < 0\right)(1-c)(\theta + s)^{+} + \left(s + b \geq 0\right)(1 + s + b)\left\{ \rho(\theta - f') + (1-\rho)\mathrm{E}\left[\tilde{\delta}' C^{b4}(f'/\tilde{\delta}')\right] \right\},$$

and the bond is fairly priced[30]

$$f' - \mathrm{E}\left[\left(\tilde{s}' < \underline{s}\right)\left((1-c)\left(\tilde{\delta}'\theta + \tilde{\delta}'v - f'\right)^{-} + cf'\right)\right] = \frac{b}{1+s+b}$$

where

$$\tilde{s}' = v - f'/\tilde{\delta}' + \mathbf{g}'(f'/\tilde{\delta}', \tilde{x}).$$

and $\mathbf{g}'(f'/\tilde{\delta}', \tilde{x})$ solves the firm's gambling problem in the next period as in (10).

4 Equilibrium and Graphic Illustration

The owners' problem does not have a closed form solution because of the inter-reliance of gambling and (continuation) value function, but the equilibrium properties and the numerical results provide useful implications for understanding gambling by firm in a dynamic setting. We begin with propositions which are immediate derivations from the model setting.

PROPOSITION 4.1 *Given the cap of growth rate g, cash flow v per unit of capital, and ending of the firm at a rate ρ, the firm's value per unit of capital $C^{after}(s)$ and $C^{b4}(s)$ are bounded by $1+s+\frac{(1-\rho)(1+g)}{\rho-g+\rho g}v$, and $\lim_{s\to\infty} C(s) = 1+s+\frac{(1-\rho)(1+g)}{\rho-g+\rho g}v$.*

Proof. Since firm's growth is bounded by g in each period, the value is capped by growing at maximum in each period perpetually, equivalently $\frac{(1-\rho)(1+g)}{\rho-g+\rho g}v$. When the firm's net cash s is approaching infinity, the firm achieves (or tends to achieve) the maximum perpetual growth, and an increment of s raises firm's value at a one-for-one rate. ■

[30]From bondholders' problem (**??**), assume in equilibrium the solution is homogenous, debt is fairly priced and is always accepted. Then we divide everything by K_{t+1}^{new} and we have a fairly priced debt

$$\begin{aligned}
\frac{b}{1+s+b} &= \rho f' + (1-\rho)\mathrm{E}\left[\left(\tilde{s}' + b' < 0\right)(1-c)\left[f' \wedge \left(\tilde{\delta}'\theta + \tilde{\delta}'v\right)\right] + \left(\tilde{s}' + b' \geq 0\right)f'\right] \\
&= \rho f' + (1-\rho)\mathrm{E}\left[-\left(\tilde{s}' + b' < 0\right)(1-c)\left(\tilde{\delta}'\theta + \tilde{\delta}'v - f'\right)^{-} + \left(\tilde{s}' + b' < 0\right)(1-c)f' + \left(\tilde{s}' + b' \geq 0\right)f'\right] \\
&= f' - (1-\rho)\mathrm{E}\left[\left(\tilde{s}' + b' < 0\right)\left((1-c)\left(\tilde{\delta}'\theta + \tilde{\delta}'v - f'\right)^{-} + cf'\right)\right]
\end{aligned}$$

4.1 Bond pricing

For further analysis we assume $\tilde{\delta}'$ follows a uniform distribution in $(\underline{\delta}, \bar{\delta})$, without loss of generality.

The maximization of the owners' wealth should always offer a fair bond price in equilibrium that equals to the face value subtracts the loss from bankruptcy. Without gambling, it is explicitly a function of face value ϕ':

$$\beta(\phi') = \phi' + \mathrm{E}\big[\big(\frac{v-\phi'}{\tilde{\delta}'} < \underline{s}\big)\big((1-c)(v+\theta\tilde{\delta}') - \phi'\big)^{-}\big] \tag{11}$$

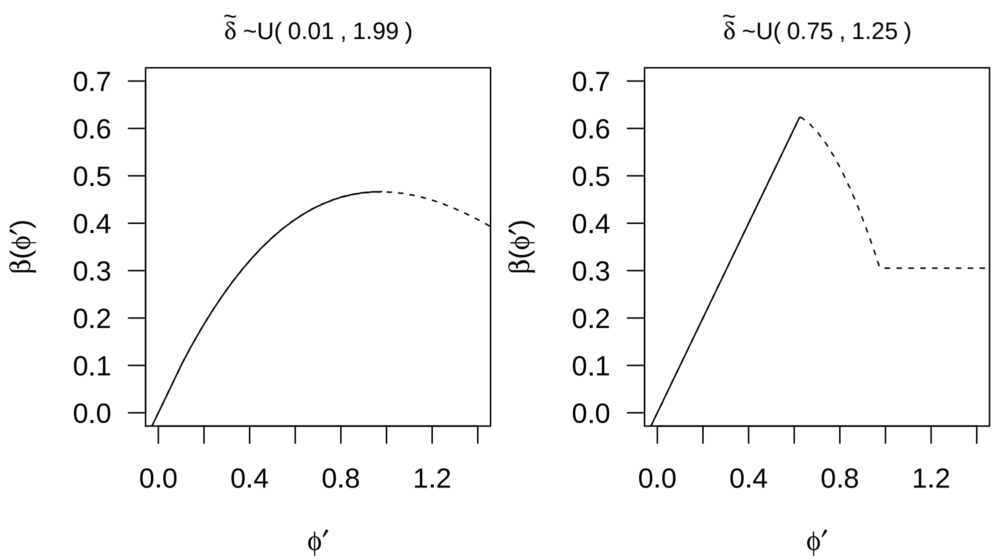

Figure 4: ***Bond pricing without gambling*** ($\theta = 0.5$, $c = 0.5$, $v = 0.1$, $\theta = 0.5$, $g = 0.05$, $\rho = 0.25$, $\underline{s} = -0.7$) bond value as a function of face value. Feasible bond pricing (solid curves) represents the rational choice set of the bond contracts that satisfy the pricing function (11).

Figure 4 shows two possibilities: risky borrowing and risk free borrowing (only). The curves represent the balance between borrowing and risk of bankruptcy. When the face value (compared to capital level) is small, borrowing can be less risky and the bondholders obtain full repayment more often and is demonstrated by the linear (or close to linear) part of the curve. As the face value becomes greater, the risk of default becomes bigger and bond value has smaller increment and can be decreasing when the face value is high enough. The black solid upward curves are efficient borrowing frontiers which represent the set of the bond contracts that the owners would choose in equilibrium. the owners can never write a contract beyond the maximum borrowing and the upper bound is an endogenous borrowing constraint. Comparing the two graphs, when capital shocks are relatively volatile (Figure 4 left graph), firms may choose risky debt; when capital is "stable," borrowing is always safe.[31]

Bond pricing with gambling requires explicit functional expression of gambling and the continuation value. Figure **??** at the beginning of Section 3 provides an example. We can observe some sensible features: when face value is small, the upward linear segment of each curve implies a risk-free bond absent bankruptcy risk; as face value is getting bigger, the growing possibility of bankruptcy generates higher costs that offset the promise of higher repayment. As can be seen, there is a maximum amount that the owners can borrow, and with greater promise of repayment, bankruptcy costs are also greater. The discrepancy between gambling with and without superpriority becomes larger when face value is getting bigger. This is likely due to the higher occurrence of "gambling for ripoff" with superpriority that depletes value.

4.2 Equity value

Table 1 shows the parameter values we use for the numerical exercise in this section:[32] We fix g, ρ and $\tilde{\delta}$ and vary v and θ. The result is shown in Figure 5.

We can observe the effects of pure gambling as well as superpriority in the figures with different profitability (v) and liquidation value of assets (θ). The comparison of the black curves and the blue curves shows the pure gambling effect without superpriority: gambling tends to be more damaging with greater liquidation value per unit of capital. The first row depicts cases in which

[31]Safe borrowing is probably not an interesting case, therefore we mainly focus on the situations in which capital volatility is large.

[32]The hazard ratio ρ is set large because we want the curves to converge quicker. The capital shock is also set with a large range because we are more interested in the case with risky borrowing.

g	0.05
ρ	0.25
c	0.5
$\tilde{\delta}$	$U(0.05, 1.95)$
v	$\{0.05, 0.1, 0.2\}$
θ	$\{0, 0.5, 0.7\}$

Table 1: Parameter values for the numerical exercise.

the firm's assets have no liquidation value and hence superpriority does not impact gambling. In this case, gambling is actually value enhancing because gambling for redemption dominates. The bondholders are also more willing to lend and the borrowing constraints are relaxed. All the value of gambling is reflected in the equity value (since bond is fairly priced). The benefit of gambling is enlarged as the cash flow is greater - the idea is that higher cash flow increases the chance of winning a fair gamble, further alleviating borrowing and improves equity value. For greater liquidation values, the black and blue curves are getting closer, indicating that the advantage of gambling tends to be smaller. Gambling plays a smaller roll in affecting equity value probably because borrowing is more determined by asset value instead of gaining from gambling. However, firm value is largely impaired when gambling assets is plausible with superpirority. Compared to gambling without superpriority, two effects may contribute to the dissipation: first, the assets are diluted because the ability of asset redeployment by the firm owners; second, "gambling for ripoff" would be more likely to prevail.

The effect of superpriority can be shown by comparing the blue solid curves and red dashed curves. Both curves coincide in the first row, because the owners' problems look exactly the same with and without superpriority when $\theta = 0$. When $\theta > 0$, equity value with superpriority tends to be smaller than that without superpriority. Of course, when the net cash s is very negative (for example in some graphs $s < -0.5$), a firm has no value with and without superpriority. With larger net cash (for example in some graphs $s > 0$), the blue curves and red curves tend to converge since the risky borrowing and gambling would have a smaller proportion. This is also shown by the convergence to the black (no gambling) curves as well as the dashed black lines which represents the maximum firm values.

4.3 Optimal gambling

The optimal gambling can be found following the same procedure as in the single-period model, that is, by "concavifying" the value function $C(s)$ for each contingency. Figure 6 shows an example. Notice that s' is the pre-gambling net cash in the next period, and it can be used to target where can the owners gamble to. For example, given any s' and $\frac{\phi'}{\delta'}$, the firm can use s' to gamble down to $-\frac{\phi'}{\delta'}$ without gambling assets (or to $-\frac{\phi'}{\delta'}-\theta$ with gambling assets) and up to a tangent point of $C(s)$ and a linear line going through the point $(-\frac{\phi'}{\delta'},0)$ (or $(-\frac{\phi'}{\delta'}-\theta,0)$ with gambling assets). By using the optimal gambling and the value function, we can compute the pre-gambling value functions of equity value and bond value, denoted as $\widehat{C}(s',\frac{\phi'}{\delta'})$ and $\widehat{\phi}(s',\frac{\phi'}{\delta'})$, respectively.

We first define

$$s_0 \equiv -1-\frac{(1-\rho)(1+g)}{\rho-g+\rho g}v, \quad s_1 \equiv \underline{s}-\frac{C(\underline{s})}{C'(\underline{s})},$$

where s_0 is the interception of the upper bound of value function (see Prop. 4.1) and the x axis, and s_1 is the smallest s on the x axis through which the tangent point on the value function $C(s)$ is the "kink." (see Figure 6.) Also define

$$i(-\frac{\phi'}{\delta'}) \equiv -\frac{\phi'}{\delta'}-\gamma\theta$$

and

$$I(-\frac{\phi'}{\delta'}) \equiv \begin{cases} +\infty, & \text{if } i(-\frac{\phi'}{\delta'}) \leq s_0 \\ \arg\max_s \frac{C(s)}{s-i(-\phi'/\delta')}, & \text{if } s_0 < i(-\frac{\phi'}{\delta'}) \end{cases}$$

where $i(-\frac{\phi'}{\delta'})$ is the minimum value that the owners can gamble. Absent superpriority, the owners can only gamble down to $-\frac{\phi'}{\delta'}$, while the owners can gamble down to $-\frac{\phi'}{\delta'}-\theta$ if there is superpriority. $I(-\frac{\phi'}{\delta'})$ is the tangent point of $C(s)$ and the straight line going through $i(-\frac{\phi'}{\delta'})$, if not infinite.

If $i(-\frac{\phi'}{\delta'})$ falls on the left of s_0, we cannot find a tangent point of $i(-\frac{\phi'}{\delta'})$ along the the value function curve $C(s)$, and hence the owners will gamble for ripoff. If $i(-\frac{\phi'}{\delta'})$ falls between s_1 and $\underline{s}$, then the tangent point is exactly the "kink" and the owners will gamble for redemption. Any point in between s_0 and s_1 always has tangent point(s) on $C(s)$, and the tangent point(s) should be the point(s) that the owners gamble towards. Proposition 4.2 formally states the gambling feature:

PROPOSITION 4.2 ***(Optimal gambling)*** *Given ϕ', δ',*

1. *the optimal gambling for the owners is*

$$\mathbf{s}^*(\tilde{x}, \phi', \delta') = \begin{cases} I(-\frac{\phi'}{\delta'}) \vee s', & \text{if } 0 < x < w(s', \frac{\phi'}{\delta'}) \\ i(-\frac{\phi'}{\delta'}), & \text{if } w(s', \frac{\phi'}{\delta'}) \le x < 1 \end{cases}$$

where $w(s', \frac{\phi'}{\delta'}) \equiv \frac{s' - i(-\phi'/\delta')}{I(-\phi'/\delta') \vee s' - i(-\phi'/\delta')}$ is the probability or weight that the equity value goes up. This result indicates that if $I(-\frac{\phi'}{\delta'}) \le s'$, then $\mathbf{s}^*(\tilde{x}, \phi', \delta') \equiv s'$ (i.e., the owners do not choose to gamble); otherwise, if $I(-\frac{\phi'}{\delta'}) > s'$, the owners gambles s' up to $I(-\frac{\phi'}{\delta'})$, and down to $i(-\frac{\phi'}{\delta'})$.

2. *for $I(-\frac{\phi'}{\delta'}) > s'$, define*

$$\psi(-\frac{\phi'}{\delta'}) \equiv \sup_{s \ge \underline{s}} \frac{C(s)}{s - i(-\phi'/\delta')}, \quad \xi(-\frac{\phi'}{\delta'}) \equiv \begin{cases} 0, & \text{if } i(-\frac{\phi'}{\delta'}) \le s_0, \\ \frac{\phi'}{I(-\phi'/\delta') - i(-\phi'/\delta')}, & \text{if } s_0 < i(-\frac{\phi'}{\delta'}). \end{cases}$$

Then the value functions of the firm and the bond before gambling are

$$\textit{\textbf{firm:}}\ \widehat{C}(s',\frac{\phi'}{\delta'}) = \begin{cases} \psi(-\frac{\phi'}{\delta'})(s'-i(-\frac{\phi'}{\delta'})), & I(-\frac{\phi'}{\delta'}) > s', \\ C(s'), & I(-\frac{\phi'}{\delta'}) \le s'. \end{cases} \tag{12}$$

$$\textit{\textbf{bond:}}\ \widehat{\phi}(s',\frac{\phi'}{\delta'}) = \begin{cases} \xi(-\frac{\phi'}{\delta'})(s'-i(-\frac{\phi'}{\delta'})), & I(-\frac{\phi'}{\delta'}) > s', \\ \phi', & I(-\frac{\phi'}{\delta'}) \le s'. \end{cases} \tag{13}$$

or, equivalently,

$$\textit{\textbf{firm:}}\ \widehat{C}(s',\frac{\phi'}{\delta'}) = w(s',\frac{\phi'}{\delta'}) \cdot C\left(I(-\frac{\phi'}{\delta'}) \vee s'\right), \tag{14}$$

$$\textit{\textbf{bond:}}\ \widehat{\phi}(s',\frac{\phi'}{\delta'}) = w(s',\frac{\phi'}{\delta'}) \cdot \phi'. \tag{15}$$

The optimal gambling has a feature different from many existing literature: the firm does not always choose extreme risks. Rather, gambling has a mixed feature of gambling for redemption and ripoff and is continuous rather than jumping between extremes.

Gambling can easily adjust for different assumptions. For example, if the firm value has a decreased margin when cash flow increases, gambling for extreme ripoff may not happen since we can find a finite tangent point on the flatter segment of the function. Yet the conclusion that gambling is bigger and more damaging with higher level of liability and liquidation value still holds.

5 Conclusions

We provided a simple framework to analyze gambling by firms. “Gambling for redemption” is a Pareto improvement and occurs when the firm owners are eager to maintain the firm, whereas “gambling for ripoff” can be socially costly and occurs when continuing a firm is beneficial socially but not to the owners. By making gambling some of the assets possible, superpriority law lowers the value lost to owners in bankruptcy and increases the incentives for the firm owners to gamble for ripoff. In the more realistic intertemporal model with endogenous borrowing and endogenous continuation value, the owners choices of gambling are intermediate between gambling for redemption and ripoff. We find that superpriority increases the scale of gambling taken by the owners and makes funding more difficult. Our results suggest an interesting empirical question: how do we distinguish "gambling for redemption" and "gambling for ripoff" ex post since they both wipe out the firm's assets in the case of failure? To know the exact gambling, we can instead look at their bets in place, compare the risk of their assets and the amount of matured debt in place.

One possible implication of superpriority law will be the adoption of financing that reduces the scale of superpriority gambling. One possibility is the adoption by bond issuers of more defensive measures that protect against superpriority claims. For example, it may be more common to protect bonds to specific perfected collateral instead of passive covenants claiming the preclusion of asset sales and security transfers. It may also incentivize firms to issue short-term bonds which have less exposure to a stay in bankruptcy, or even use repos which are also protected against bankruptcy. The substitution away from traditional financing to repo financing can cause an asset grab race which undermines the purposes of bankruptcy law to facilitate an orderly liquidation (or reorganization) and to give breathing space for the firm owners to resolve financial difficulties.

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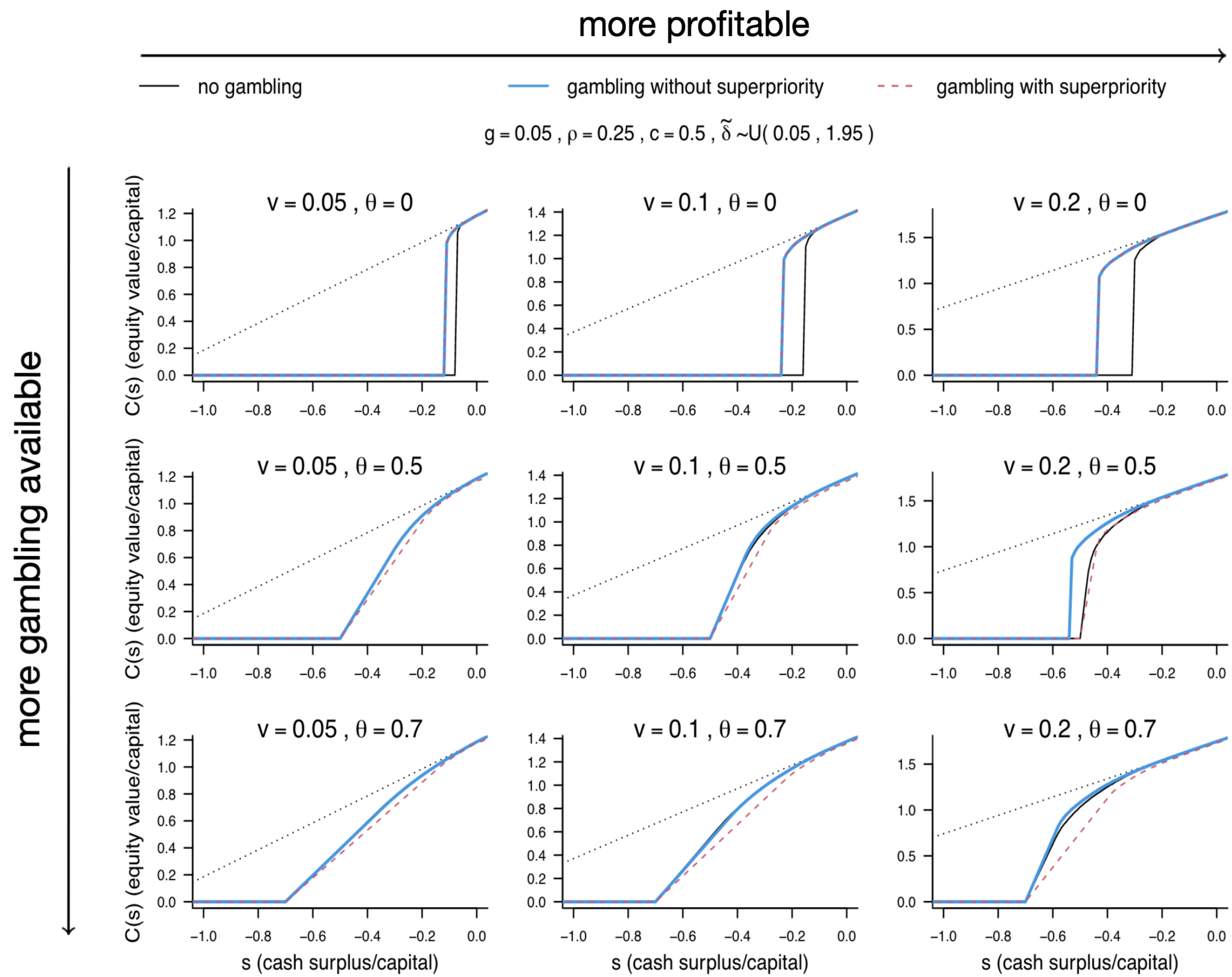

Figure 5: ***(Equity value/capital as a function of net cash/capital)***
v : cash flow/capital; θ : liquidation value available/capital. If $\theta = 0$, superpriority is irrelevant, and increasing θ implies increasing damage from superpriority, especially when v is large.

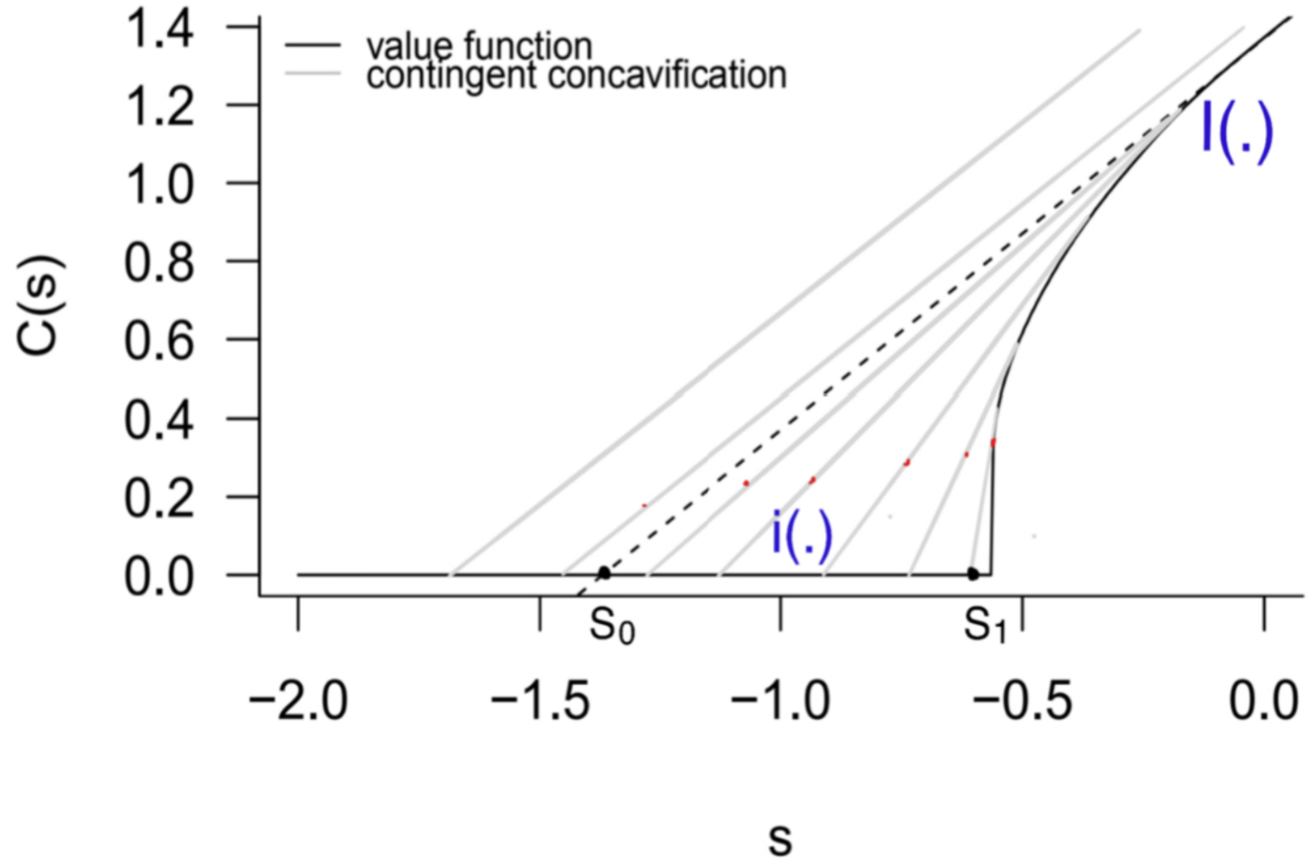

Figure 6: ***An example of gambling*** gambling has a mixed feature of “gambling for redemption” and “gambling for ripoff” and is continuous rather than jumping between extremes.

A Proof of Optimal Gambling: redemption

Given constants $P, F, C, \pi \in \mathbb{R}_{++},\ L \in \mathbb{R}_{+},\ \bar{g} \geq P - \pi$ and $\gamma \equiv \begin{cases} 1, \text{ with superpriority} \\ 0, \text{ absent superpriority,} \end{cases}$ the question becomes

$$\max_{\mathbf{g}(x)} \mathrm{E}\Big[(\pi + \mathbf{g}(x) \geq P)(\pi + \mathbf{g}(x) - P + C)\Big]$$
$$\text{s.t. } \mathrm{E}[\mathbf{g}(x)] = 0, \text{ and } -\gamma L - \pi \leq \mathbf{g}(x) \leq \bar{g}$$

Assume that $\tilde{x}$ is the underlying randomness: $\tilde{x} \sim_d U(0,1)$. W.l.o.g., assume that $\mathbf{g}(x)$ is decreasing in x. To get the necessary conditions for the solution, we first concavify the function

$$(\pi + \mathbf{g}(x) \geq P)(\pi + \mathbf{g}(x) - P + C) \tag{*}$$

to make it continuous.

Gambling for redemption: When $P < C - \gamma L$, define $H(\mathbf{g}(x))$ as the concavified function of (*)

$$H(\mathbf{g}(x)) \equiv \begin{cases} \frac{C}{P+\gamma L}(\pi + \mathbf{g}(x) + \gamma L), & \text{for } \mathbf{g}(x) < P - \pi \\ \pi + \mathbf{g}(x) - P + C, & \text{for } \mathbf{g}(x) \geq P - \pi \end{cases}$$

The subgradient of $H(\mathbf{g})$ is

$$\nabla H(\mathbf{g}) = \begin{cases} (-\infty, 1], & \text{for } \mathbf{g} = \bar{g} \\ \{1\}, & \text{for } P - \pi < \mathbf{g} < \bar{g} \\ [1, \frac{C}{P+\gamma L}], & \text{for } \mathbf{g} = P - \pi \\ \{\frac{C}{P+\gamma L}\}, & \text{for } -\pi - \gamma L < \mathbf{g} < P - \pi \\ [\frac{C}{P+\gamma L}, +\infty], & \text{for } \mathbf{g} = -\pi - \gamma L \end{cases}$$

Assume $\lambda, w_1, w_2 \geq 0$, and the first order condition of the problem is

$$\lambda - w_1 + w_2 \in \nabla H(\mathbf{g})$$

with

$$w_1 \geq 0, \ (\mathbf{g} + \pi + \gamma L) w_1 = 0$$
$$w_2 \geq 0, \ (\mathbf{g} - \bar{g}) w_2 = 0$$

We ignore the case when $\mathbf{g} \in (-\pi - \gamma L, P - \pi)$ since it has measure zero and is not on the original function. We then have

$$\mathbf{g} = \begin{cases} \bar{g}, & \text{for } -\infty < \lambda - w_1 + w_2 \leq 1 \\ [P - \pi, \bar{g}], & \text{for } \lambda - w_1 + w_2 = 1 \\ P - \pi, & \text{for } 1 \leq \lambda - w_1 + w_2 \leq \frac{C}{P+\gamma L} \\ -\pi - \gamma L, & \text{for } \frac{C}{P+\gamma L} \leq \lambda - w_1 + w_2 \leq +\infty \end{cases}$$

(1) If $\pi < P$, $\mathbf{g}(x) = -\pi - \gamma L$ is the only $\mathbf{g}(x)$ that is smaller than 0. For $\mathrm{E}[\mathbf{g}(x)] = 0$, there must be some x such that $\mathbf{g}(x) = -\pi - \gamma$. Therefore, $\lambda - w_1 + w_2 \geq \frac{C}{P+\gamma L} > 1$ since $C - \gamma L > P$. This

implies that $\lambda - w_1 + w_2 = \frac{C}{P+\gamma L}$ and $\mathbf{g}(x) = P - \pi$ or $\mathbf{g}(x) = -\pi - \gamma L$. Thus, by solving

$$0 = \int_{x=0}^{t} (P-\pi)dx + \int_{x=t}^{1} (-\pi - \gamma L)dx,$$

we have $t = \frac{\pi+\gamma L}{P+\gamma L}$. The optimal gambling is

$$\mathbf{g}^*(x) = \begin{cases} P - \pi, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{P+\gamma L} \\ -\gamma L - \pi, & \text{for } \frac{\pi+\gamma L}{P+\gamma L} < x < 1. \end{cases}$$

Since $\mathbf{g}^*(x)$ solves the concavified problem and is also feasible for the original problem, and the concavified objective function is greater than the original function, we can conclude that $\mathbf{g}^*(x)$ also solves the original problem. If we relax the condition that $\mathbf{g}$ is decreasing, then any gamble with the same distribution would also be optimal.

(2) If $\pi > P$, then we must have that for some x, $\mathbf{g}(x) \geq P$ and $\lambda - w_1 + w_2 \geq 1$. Then any $\mathbf{g}(x) \in [P - \pi, \bar{g}]$ that satisfies

$$\int_{x=0}^{1} \mathbf{g}(x)dx = 0$$

would be a possible solution.

Now we prove that these candidate solutions are the actual solutions. For any candidate solutions $\{\mathbf{g}^*(x) | \pi + \mathbf{g}^*(x) \geq P \text{ and } \mathrm{E}[\mathbf{g}^*(x)] = 0\}$,

$$\mathrm{E}[H(\mathbf{g}^*(x))] = \pi - P + C.$$

Since for any feasible solutions $\mathrm{E}[H(\mathbf{g}(x))] \leq \pi - P + C = \mathrm{E}[H(\mathbf{g}^*(x))]$, the candidate solutions are the actual solutions. For the same argument as above, the solutions for the concavified problem are also the solutions for the original problem.

B Proof of Optimal Gambling: ripoff

Gambling for ripoff: When $P > C - \gamma L$, similarly define $H(\mathbf{g}(x))$ as the concavified function of (*)

$$H(\mathbf{g}(x)) \equiv \frac{\pi + \bar{g} - P + C}{\pi + \bar{g} + \gamma L}(\pi + \mathbf{g} + \gamma L)$$

The subgradient of $H(\mathbf{g})$ is

$$\nabla H(\mathbf{g}) = \begin{cases} (-\infty, \frac{\pi+\bar{g}-P+C}{\pi+\bar{g}+\gamma L}], & \text{for } \mathbf{g} = \bar{g} \\ \frac{\pi+\bar{g}-P+C}{\pi+\bar{g}+\gamma L}, & \text{for } -\pi - \gamma L < \mathbf{g} < \bar{g} \\ [\frac{\pi+\bar{g}-P+C}{\pi+\bar{g}+\gamma L}, +\infty), & \text{for } \mathbf{g} = -\pi - \gamma L \end{cases}$$

The first order condition is the same as before. Ignoring the case in which $-\pi - \gamma L < \mathbf{g} < \bar{g}$ since the measure is zero, we have

$$\mathbf{g} = \begin{cases} \bar{g}, & \text{for } \lambda - w_1 + w_2 \leq \frac{\pi+\bar{g}-P+C}{\pi+\bar{g}+\gamma L} \\ -\pi - \gamma L, & \text{for } \lambda - w_1 + w_2 \geq \frac{\pi+\bar{g}-P+C}{\pi+\bar{g}+\gamma L} \end{cases}$$

Therefore, the Lagrange multipliers satisfy $\lambda - w_1 + w_2 = \frac{\pi+\bar{g}-P+C}{\pi+\bar{g}+\gamma L}$. By solving

$$0 = \int_{x=0}^{t} \bar{g} dx + \int_{x=t}^{1} (-\pi - \gamma L) dx,$$

we have $t = \frac{\pi+\gamma L}{\bar{g}+\pi+\gamma L}$. The optimal gambling is

$$\mathbf{g}^*(x) = \begin{cases} \bar{g}, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{\bar{g}+\pi+\gamma L} \\ -\gamma L - \pi, & \text{for } \frac{\pi+\gamma L}{\bar{g}+\pi+\gamma L} < x < 1 \end{cases}$$

For the same argument as above, the solutions for the concavified problem are also the solutions for the original problem.