# Power dispatch optimization for an EV fleet

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## 1 Model

# 1.1 Variables

Index sets:

 $\mathcal{I}$ , the set of plugged EV

$$T = \{0, 1, \dots, T\}$$

 $\Sigma = \{chg, dischg\} \text{ (i.e., } \{charging, discharging\})$ 

$$\Omega = \{bsl, up, dn\} \text{ (i.e., } \{baseline, FCR_{up}, FCR_{down}\})$$

Let  $p^{max} = \max\{\overline{p^{charging}}, \overline{p^{discharging}}\}$ , then  $\forall i \in \mathcal{I}$ , we define:

$$x_{i} = \begin{pmatrix} \left(u_{i,t}^{\sigma,\omega}\right)_{\sigma,\omega,t} & \in \{0,1\}^{\Sigma \times \Omega \times (T+1)} \\ \left(p_{i,t}^{\sigma,\omega}\right)_{\sigma,\omega,t} & \in [0,p^{max}]^{\Sigma \times \Omega \times (T+1)} \\ \left(SOC_{i,t}^{\omega}\right)_{\omega,t} & \in [0,SOC^{max}]^{\Omega \times (T+1)} \end{pmatrix}$$

where

- $u_{i,t}^{\sigma,\omega} \in \{1,0\}$  signifies, at instant t, whether the EV i is charging ( $\sigma = chg$ ) (resp. discharging ( $\sigma = dischg$ )) according to its baseline power consumption ( $\omega = bsl$ ) (resp. power consumption while doing FCR up service ( $\omega = up$ ), power consumption while doing FCR down service ( $\omega = dn$ ));
- $p_{i,t}^{\sigma,\omega}$  is the baseline/FCR up service/FCR down service power consumption of EV i at instant t;
- $SOC_{i,t}^{\omega}$  is the state of charge of EV i at instant t when it has not done any FCR service till t ( $\omega = bsl$ ) (resp. it has not done till any service till t-1 but has done a FCR up service at t ( $\omega = up$ , it has not done till any service till t-1 but has done a FCR down service at t ( $\omega = dn$ ).

#### 1.2 Constraints

The domain of  $x_i$  is  $X_i$ , defined by the following constraints

$$u_{i,t}^{\sigma,\omega} \in \{0,1\}, \qquad \forall t \in T, \forall \sigma \in \Sigma, \forall \omega \in \Omega \quad (1.1a)$$

$$u_{i,t}^{chg,\omega} + u_{i,t}^{dischg,\omega} \leq 1, \qquad \forall t \in T, \forall \omega \in \Omega \quad (1.1b)$$

$$0 \leq p_{i,t}^{\sigma,\omega} \leq \overline{p_i^{\sigma}}, \qquad \forall t \in T, \forall \sigma \in \Sigma, \forall \omega \in \Omega \quad (1.1c)$$

$$p_{i,t}^{\sigma,bsl} \leq \overline{p_i^{\sigma}} \cdot u_{i,t}^{\sigma,\omega}, \qquad \forall t \in T, \forall \sigma \in \Sigma, \forall \omega \in \Omega \quad (1.1d)$$

$$p_{i,t}^{\sigma,bsl} \geq p\_{charge_{i,t}^{min}} \cdot u^{\sigma,bsl}, \qquad \forall t \in T, \forall \sigma \in \Sigma, \forall \omega \in \Omega \quad (1.1e)$$

$$\sum_{t \in T} p_{i,t}^{dischg,bsl} \leq E\_{discharge_{i}^{max}}, \qquad (1.1f)$$

$$p_{i,t}^{chg,bsl} - p_{i,t}^{dischg,bsl} \geq p_{i,t}^{chg,up} - p_{i,t}^{dischg,up}, \qquad \forall t \in T \quad (1.1g)$$

$$p_{i,t}^{chg,bsl} - p_{i,t}^{dischg,bsl} \leq p_{i,t}^{chg,dn} - p_{i,t}^{dischg,dn}, \qquad \forall t \in T \quad (1.1h)$$

$$SOC_{i,0}^{\omega} = SOC_{i,0}^{initial}, \qquad \forall \omega \in \Omega \quad (1.1i)$$

$$0 \leq SOC_{i,t}^{\omega} \leq capacity_i, \qquad \forall t \in T, \forall \omega \in \Omega \quad (1.1j)$$

$$SOC_{i,t}^{\omega} = SOC_{i,t-1}^{bsl} + \left[\rho_{ad} p_{i,t}^{chg,\omega} - \frac{1}{\rho_{da}} p_{i,t}^{dischg,\omega}\right] \delta_t, \quad \forall t \in T \setminus \{0\}, \forall \omega \in \Omega \quad (1.1k)$$

### 1.3 Objective function

$$\begin{split} FCR\_up_t &= \sum_{i \in I} \left[ \left( p_{i,t}^{chg,bsl} - p_{i,t}^{dischg,bsl} \right) - \left( p_{i,t}^{chg,up} - p_{i,t}^{dischg,up} \right) \right] \\ FCR\_down_t &= \sum_{i \in I} \left[ \left( p_{i,t}^{chg,dn} - p_{i,t}^{dischg,dn} \right) - \left( p_{i,t}^{chg,bsl} - p_{i,t}^{dischg,bsl} \right) \right] \\ penalty_{i,t}^{SOC,\omega} &= \left( \left[ SOC_{i,t}^{\omega} - SOC_{i,t}^{min} \right]^- + \left[ SOC_{i,t}^{\omega} - SOC_{i}^{max} \right]^+ \right)^2, \quad \forall \omega \in \Omega \\ customer\_energy\_cost &= \sum_{t \in T} \sum_{i} p_{i,t}^{bsl} \, \delta_t \, \pi_t^{\text{elec}} - \sum_{i} SOC_{i,T} \, \pi_T^{\text{elec}} \end{split}$$

$$g_i(x_i) = \begin{pmatrix} g_i^{up}(x_i) \\ g_i^{dn}(x_i) \end{pmatrix} = \begin{pmatrix} \left( \left( p_{i,t}^{chg,bsl} - p_{i,t}^{dischg,bsl} \right) - \left( p_{i,t}^{chg,up} - p_{i,t}^{dischg,up} \right) \right)_t \\ \left( \left( p_{i,t}^{chg,dn} - p_{i,t}^{dischg,dn} \right) - \left( p_{i,t}^{chg,bsl} - p_{i,t}^{dischg,bsl} \right) \right)_t \end{pmatrix} \in \mathbb{R}^{2(T+1)}$$

$$y = ((y_t^{up})_t, (y_t^{dn})_t) \in \mathbb{R}^{2(T+1)}$$

$$f_0(y) = \sum_t \alpha^{up} (y_t^{up} - demand_t^{up})^2 + \sum_t \alpha^{dn} (y_t^{dn} - demand_t^{dn})^2$$

$$f_0(\sum_i g_i(x_i)) = \sum_t \alpha^{up} \left( \sum_i g_{i,t}^{up}(x_{i,t}) - demand_t^{up} \right)^2 + \sum_t \alpha^{dn} \left( \sum_i g_{i,t}^{dn}(x_{i,t}) - demand_t^{dn} \right)^2$$

$$f_{i}(x_{i}) = \sum_{t \in T} \sum_{\omega \in \Omega} \beta^{min} \left( \left[ SOC_{i,t}^{\omega} - SOC_{i,t}^{min} \right]^{-} \right)^{2} + \sum_{t \in T} \sum_{\omega \in \Omega} \beta^{max} \left( \left[ SOC_{i,t}^{\omega} - SOC_{i}^{max} \right]^{+} \right)^{2} + \sum_{t \in T} p_{i,t}^{bsl} \, \delta_{t} \, \pi_{t}^{\text{elec}} - SOC_{i,T} \, \gamma \, \pi_{T}^{\text{elec}} \right)$$

Objective function is

$$F(x) = f_0(\sum_{i} g_i(x_i)) + \sum_{i} f_i(x_i)$$