Sorbonne Université
Master ANDROIDE

Optimisation distribuée pour la recharge de véhicules électriques

Rapport d'avancement

Réalisé par :

Xinyu HUANG

Encadré par :

Cheng Wan, EDF
Nadia Oudjane, EDF
Guilhem Dupuis, EDF
Laurent Pfeiffer, INRIA et L2S, CentraleSupélec

Référent :
Bruno Escoffier, LIP6, Sorbonne Université

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Chapter 1

General context

This internship concerning a research project financed by the Programme Gaspard Monge pour l'Optimisation (PGMO), is co-supervised on one side by Laurent Pfeiffer, researcher at L2S, INRIA-Sacaly, on the other side by Nadia Oudjane, Cheng Wan, and Guilhem Dupuis, researchers at EDF Lab Saclay.

1.1 General Model

We want to investigate a large scale aggregative optimization problem [1], which can be expressed by the minimisation problem below :

$$\inf_{x\in\mathcal{X}} J(x) = f\Big(\frac{1}{N}\sum_{i=1}^{N} g_i(x_i)\Big) + \frac{1}{N}\sum_{i=1}^{N} h_i(x_i) \tag{$\mathcal{P}$}$$

where $x = (x_1, \dots, x_N) \in \mathcal{X} = \prod_{i=1}^{N} \mathcal{X}_i$.

We are given the following data:

- $g_i : \mathcal{X}_i \to \mathbb{R}^n, i = 1, \dots, N$
- $h_i : \mathcal{X}_i \to \mathbb{R}, i = 1, \dots, N$
- $f : \mathbb{R}^n \to \mathbb{R}$

The problem can be interpreted as a multi-agent model:

- N, the number of agents
- $\mathcal{X}_i$: the decision set of agent i
- $h_i(x_i)$: individual cost function of agent i
- $g_i(x_i)$: contribution of agent i to a common good
- $\frac{1}{N}\sum_{i=1}^{N} g_i(x_i)$: a common good, referred to as aggregate
- f: a social cost associated with the aggregate

We make the following assumptions:

- f is convex

- f is continuously differentiable and there exists $D > 0$ such that ∇f is D-Lipschitz continuous
- $\forall i = 1, \ldots, N, diam(g_i(\mathcal{X}_i)) \leq D$.

1.2 Application

We want to apply this model to an electrical vehicle charging problem. We give a short description of this problem here, the complete modele can be found in the appendix in Section B.
In the electrical vehicle charging problem, we have $x = (x_1, \ldots, x_N)$ where x represents the planning of charging of the whole recharge system and x_i represent the planning of the i-th electric vehicle.
So we want to implement these terms as variables for each vehicle and time step:

- state of charge
- charge power
- discharge power

In the realistic situation, the charging site not only provides electricity for vehicle charging(Baseline), but also for other potential uses. So we call the charging behavior when we provide electricity for other uses as Service.

In Problem $\mathcal{P}$, $h_i(x_i)$ corresponds to the charging cost of the vehicle number i. The term $g_i(x_i)$ is a vector, representing the difference in charging between the "baseline" and the "service" mode at every time step. So the term $\frac{1}{N}\sum_{i=1}^{N} g_i(x_i)$ describes the average difference of charging for the whole fleet, between the two modes. Ideally, the difference of charging should be close to reference values $(y_t^{up})_{t=1,\ldots,T}$. The function f penalizes differences of charging which are far from y_t^{up}.

1.3 Algorithm SFW

We want to solve this problem with Stochastic Frank-Wolfe algorithm, which was introduced in [1]. It is described in detail in Section A. This algorithm was specifically designed to solve problems of the form ($\mathcal{P}$). It is based on the classical Frank-Wolfe algorithm [2]. It is an iterative algorithm, which solves at each iteration a sub-problem concerning only one single agent. the context of the vehicle charging problem, each sub-problem is a problem for one specific vehicle, which is much easier to solve than the general problem.

1.4 State of art

The subject of this internship is based on the article [1]. Although this kind of problems can be tackled by existing solvers, such as CPLEX, our algorithm has the advantage of decomposing the problem into sub-problems, which enables to solve it with large values of N, possibly in a faster fashion. There are various algorithms in the literature for the decomposition of convex optimization problems, for example Uzawa's algorithm. The specificity of the Algorithm proposed in [1] is that it can be deal with the non-convexity of the cost function J (only the function f needs to be convex).

Chapter 2

Objectives and calendar

2.1 Initial objectives

- Understand the reference article [1] and the SFW (stochastic Frank&Wolfe) algorithm (see Section A). (✓)
- Test the SFW algorithm for a simple example called MIQP (Mixed-integer quadratic problem). (✓)
- Model the vehicles charging problem (see Section B), in collaboration with the partners from EDF. (✓)
- Implement the SFW for the vehicles charging problem. (In progress)
- Evaluate the efficiency of the SFW algorithm, in comparison with direct resolutions with CPLEX, for various instances of the data problem (in particular, for more or less large values of N). (In progress)
- Find limitations of the algorithm. (May)

2.2 Objectives for the second part of the internship

- Propose, implement and evaluate variants and improved versions of the SFW algorithm. (June-August)
- Formulate the final report and prepare for the oral defense. (August-September)

Appendix A

Stochastic Frank-Wolfe algorithm

We give the full description of the SFW algorithm in Algorithm 1. The first step of the inner loop of the algorithm requires to solve a sub-problem, done with Algorithm 2.

Algorithm 1: Stochastic Frank-Wolfe Algorithm

Input:
$\bar{x}^{(0)}$: vector containing $(\bar{x}_1^{(0)}, \ldots, \bar{x}_i^{(0)}, \ldots, \bar{x}_N^{(0)})$, each representing the state of an agent
$(n_k)_{k \in N}$: a list of integers, where n_k represents the number of drawings for the k-th iteration.

for $k \leftarrow 1$ ***to*** K **do**
  Find a solution to the sub-problem x^k with Algorithm 2 using $\bar{x}^k$ as input ;
  Set $\delta_k = 2/(k+2)$;
  for $j \leftarrow 1$ ***to*** n_k **do**
    for $i \leftarrow 1$ ***to*** N **do**
      Draw the state of each agent $\tilde{x}_i^j$ according to

$$\tilde{x}_i^j = \begin{cases} \bar{x}_i^k & \text{if } 0 < Z_{i,j,k} \leq 1 - \delta_k \\ x_i^k & \text{if } 1 - \delta_k < Z_{i,j,k} \leq 1, \end{cases}$$

      Here $Z_{i,j,k}$ is a random variable with uniform distribution in $[0,1]$.
      Set $\tilde{x}^j = (\tilde{x}_1^j, \ldots, \tilde{x}_i^j, \ldots, \tilde{x}_N^j)$;
    Find $\bar{x}^{(k+1)} \in \arg\min_{\forall j \in \{1,\cdots,nk\}, x \in \{\tilde{x}_1^j,\ldots,\tilde{x}_N^j\}} J(x)$;

Output: $\bar{x}^{K+1}$, the approximate solution

Algorithm 2: Resolution of the linear programming problem

Input:
$\bar{x}$: vector containing $(\bar{x}_1, \ldots, \bar{x}_i, \ldots, \bar{x}_N)$, each representing the state of an agent

Set $\lambda = \nabla f(\frac{1}{N} \sum_{i=1}^N g_i(\bar{x}_i))$;
for $i \leftarrow 1$ ***to*** N **do**
  Find a solution x_i to

$$\inf_{x_i \in X_i} \langle \lambda, g_i(x_i) \rangle + h_i(x_i)$$

Output: $(x_1, \ldots, x_i, \ldots, x_N)$

Appendix B

Model

For the subject of the internship, we focus on applying our algorithm to the problem of smart charging of electric vehicles, which is $x = (x_1, \dots, x_N)$ where x represents the planning of charging of the whole recharge system and x_i represent the planning of the i-th electric vehicle:

$$x_i = (s_i^{t'}, c_i^t, d_i^t, \bar{s}_i^{t'}, \bar{c}_i^t, \bar{d}_i^t, u_i^t, \bar{u}_i^t, v_i^t, \bar{v}_i^t, n_i^t, p_i^t, \bar{n}_i^t, \bar{p}_i^t), \quad t \in \{1, \cdots, T\},\ t' \in \{0, \cdots, T\}.$$

Variables:

Variables (baseline)	Variables (service)	Description	Range	Time
s_i^t	$\hat{s}_i^t$	The state of charge of the electric vehicle.	$[0, s_i^{max}]$	$\{0, \cdots, T\}$
c_i^t	$\hat{c}_i^t$	The charging power of the electric vehicle.	$\{0\} \cup [c_i^{min}, c_i^{max}]$	$\{1, \cdots, T\}$
d_i^t	$\hat{d}_i^t$	The discharging power of the electric vehicle.	$\{0\} \cup [d_i^{min}, d_i^{max}]$	$\{1, \cdots, T\}$
u_i^t	$\hat{u}_i^t$	The decision variable characterizes whether the vehicle is being charged or not.	{0,1}	$\{1, \cdots, T\}$
v_i^t	$\hat{v}_i^t$	The decision variable characterizes whether the vehicle is being discharged or not.	{0,1}	$\{1, \cdots, T\}$
n_i^t	$\hat{n}_i^t$	The negative part of the difference between the state of charge(Baseline) and $s_i^{t,min}$.	$\mathbb{R}^+$	$\{1, \cdots, T\}$
p_i^t	$\hat{p}_i^t$	The positive part of the difference between the state of charge(Baseline) and s_i^{max}.	$\mathbb{R}^+$	$\{1, \cdots, T\}$

Table B.1: Variables introduced in the origin problem.

Parameters: We classify parameters into three categories : parameters introduced in x_i, introduced in the function h_i and parameters introduced in the function f:

- Parameters introduced in x_i:
  - c_i^{max} : The maximal charging power of the electric vehicle.
  - c_i^{min} : The minimal charging power of the electric vehicle.
  - d_i^{max} : The maximal discharging power of the electric vehicle.
  - d_i^{min} : The minimal discharging power of the electric vehicle.
  - $s_{i,init}$: The initial state of charge of the electric vehicle.
- Parameters introduced in the function h_i:
  - $s_i^{t,min} = \max(s_i^{min}, s_i^{final} - (T-t)c_i^{max})$: The minimum state of charge for the i-th electric vehicle at the time step t capable of achieving the expected final charging level.
  - $\bar{s}_i^{max}$: The maximum state of charge for the electric vehicle.
  - s_i^{max} : The technical maximum state of charge for the electric vehicle, which is smaller than $\bar{s}_i^{max}$
  - s_i^{final} : The expected state of charge for the electric vehicle to achieve at the last time step T.
  - δ_t : The duration between two successive time steps.
  - π_t^{elec} : The price of electric production at the time step t.
  - γ : A coefficient on the penalty of the final state of charge.
- Parameters introduced in the function f_i:
  - y_t^{up} : The maximal reserve that may not be used by electric vehicle at the time step t.

Objective function:

$$\inf_{x=(x_1,\ldots,x_N), x_i\in\mathcal{X}_i} f\left(\frac{1}{N}\sum_{i=1}^{N} g_i(x_i)\right) + \frac{1}{N}\sum_{i=1}^{N} h_i(x_i)$$

with

$$g_i(x_i) = \begin{bmatrix} (c_i^1 - d_i^1) - (\hat{c}_i^1 - \hat{d}_i^1) \\ (c_i^2 - d_i^2) - (\hat{c}_i^2 - \hat{d}_i^2) \\ \vdots \\ (c_i^T - d_i^T) - (\hat{c}_i^T - \hat{d}_i^T) \end{bmatrix}$$

$$f(y) = \sum_{t=1}^{T} \left(y_t - \frac{y_t^{up}}{N}\right)^2$$

and

$$
\begin{aligned}
h_i(x_i) = \beta_{min} \sum_{t=1}^{T}((s_i^t - s_i^{t,min})^-)^2 + \beta_{max} \sum_{t=1}^{T}((s_i^t - s_i^{max})^+)^2 \\
+\beta_{min} \sum_{t=1}^{T}((\hat{s}_i^t - s_i^{t,min})^-)^2 + \beta_{max} \sum_{t=1}^{T}((\hat{s}_i^t - s_i^{max})^+)^2 \\
+\sum_{t=1}^{T}(c_i^t - d_i^t)\delta_t \pi_t^{elec} - s_i^T \gamma \pi_T^{elec}
\end{aligned}
$$

Constraints:

Baseline	Service	Description
$s_i^0 = s_{i,init}$ (1)	$\hat{s}_i^0 = s_{i,init}$ (2)	Initial SoC
$0 \leq s_i^t \leq \bar{s}_i^{max}$ (3)	$0 \leq \hat{s}_i^t \leq \bar{s}_i^{max}$ (4)	Boundary SoC
$u_i^t + v_i^t \leq 1$ (5)	$\hat{u}_i^t + \hat{v}_i^t \leq 1$ (6)	Charge or discharge
$c_i^t \leq c_i^{max} u_i^t$ (7)	$\hat{c}_i^t \leq c_i^{max} \hat{u}_i^t$ (8)	Upper bound(c)
$d_i^t \leq d_i^{max} v_i^t$ (9)	$\hat{d}_i^t \leq d_i^{max} \hat{v}_i^t$ (10)	Upper bound(d)
$u_i^t c_i^{min} \leq c_i^t$ (11)	$\hat{u}_i^t c_i^{min} \leq \hat{c}_i^t$ (12)	Lower bound(c)
$v_i^t d_i^{min} \leq d_i^t$ (13)	$\hat{v}_i^t d_i^{min} \leq \hat{d}_i^t$ (14)	Lower bound(d)
$s_i^{t+1} = s_i^t + (c_i^{t+1} - d_i^{t+1})$(15)	$\hat{s}_i^{t+1} = s_i^t + (\hat{c}_i^{t+1} - \hat{d}_i^{t+1})\delta_t$(16)	Production balance
$p_i^t \geq (s_i^t - s_i^{max})$ (17)	$\hat{p}_i^t \geq (\hat{s}_i^t - s_i^{max})$ (18)	Positive part
$n_i^t \geq -(s_i^t - s_i^{t,min})$ (19)	$\hat{n}_i^t \geq -(\hat{s}_i^t - s_i^{t,min})$ (20)	Negative part
$y_t = \sum_{i=1}^{N} c_i^t - d_i^t - \hat{c}_i^t + \hat{d}_i^t$ (21)		Facilitate calculations

Table B.2: Constraints introduced in the origin problem.

Constraints (1) and (2) specify the initialization of the state of charge for each electric vehicle. Constraints (3) and (4) limit the state of charge between the vehicle's maximal state of charge and 0. Constraints (5) and (6) state that an electric vehicle can not be charged or discharged simultaneously. Constraints (7)-(14) specify that the charge/discharge power can be any value between the minimum and the maximum charge_power, or 0. Constraints (15) and (16) specify that the difference in charge level between two successive time steps should be equal to the difference between the production and consumption. Constraints (17)-(20) determine the positive/negative part of the difference between the actual state of charge and the maximal/minimal state of charge of a given time step. Constraint (21) facilitate the calculation of function f.
With constraints (17)-(20), we can reformulate $h_i(x_i)$ as:

$$
\begin{aligned}
h_i(x_i) = \beta_{min} \sum_{t=1}^{T}(n_i^t)^2 + \beta_{max} \sum_{t=1}^{T}(p_i^t)^2 \\
+\beta_{min} \sum_{t=1}^{T}(\hat{n}_i^t)^2 + \beta_{max} \sum_{t=1}^{T}(\hat{p}_i^t)^2 \\
+\sum_{t=1}^{T}(c_i^t - d_i^t)\delta_t \pi_t^{elec} - s_i^T \gamma \pi_T^{elec}.
\end{aligned}
$$

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[2] Martin Jaggi. "Revisiting Frank-Wolfe: Projection-free sparse convex optimization". In: *International conference on machine learning*. PMLR. 2013, pp. 427–435 (page 2).