– **1. Predictive accuracy and cross-validation:** compute AIC, DIC, WAIC, and cross-validation for the logistic regression fit to the bioassay example of section 3.7. The code is under the directory code/ch7/

1. AIC
   The maximum likelihood estimate for $(\hat{\alpha}, \hat{\beta}) = (0.8431, 7.7258)$. Since we have 2 parameters, hence, the value of $\hat{\text{elpd}}_{\text{AIC}}$ is
   $$-5.89 - 2 = -7.89,$$
   and AIC $= -2\hat{\text{elpd}}_{\text{AIC}} = 15.78$.
2. DIC
   Following the example given in the book, we need to first calculate
   $$p_{\text{DIC}} = 2(\log p(y|E_{\text{post}}(\alpha, \beta)) - E_{\text{post}}(\log p(y|\alpha, \beta))).$$
   The second of these terms can be calculated as
   $$E_{\text{post}}(y|\alpha, \beta) = \frac{1}{S}\sum_{s=1}^{S}\sum_{i=1}^{4} \log p(y_i|\alpha^s, \beta^s, x_i, n_i) = -7.018$$
   based on a large number S of simulation draws.
   The first term
   $$\log p\left(y|E_{\text{post}}(\theta)\right) = \sum_{i=1}^{4} \log p(y_i|E_{\text{post}}(\alpha|y), E_{\text{post}}(\beta|y), x_i, n_i) = -6.119,$$
   which gives $p_{\text{DIC}} = 2(-6.119 - (-7.108)) = 1.7214$, $\hat{\text{elpd}}_{\text{DIC}} = \log p\left(y|E_{\text{post}}(\theta)\right) - p_{\text{DIC}} = -7.817$, and DIC $= -2\hat{\text{elpd}}_{\text{DIC}} = 15.635$.
3. WAIC
   The log pointwise predictive probability of the observed data under the fitted model is
   $$\text{llpd} = \sum_{i=1}^{4} \log\left(\frac{1}{S}\sum_{i=1}^{S}\left([\text{logit}^{-1}(\alpha^s + \beta^s x_i)]^{y_i}[1 - \text{logit}^{-1}(\alpha^s + \beta^s x_i)]^{n_i - y_i}\right)\right) = -6.651.$$
   The effective number of the parameters can be caculated as
   $$p_{\text{WAIC1}} = 2(\text{llpd} - E_{\text{post}}(y|\alpha, \beta)) = 0.670$$
   or
   $$p_{\text{WAIC2}} = \sum_{i=1}^{4} \text{var}_{\text{post}}(\log p(y_i|\alpha, \beta)),$$
   which can be computed as (Eq. 7.12, *want to take a note here too, so I copy the formula over*)
   $$\text{computed } p_{\text{WAIC2}} = \sum_{i=1}^{4} V_{s=1}^{S}\left(\log p(y_i|\alpha^s, \beta^s)\right) = 1.073,$$
   where $V_{s=1}^{S} a_s = \frac{1}{S-1}(a_s - \bar{a})^2$, the sample variance.
   Then $\hat{\text{elpd}}_{\text{WAIC1}} = \text{llpd} - p_{\text{waic1}} = -7.321$, $\hat{\text{elpd}}_{\text{WAIC2}} = \text{llpd} - p_{\text{waic2}} = -7.724$, so WAIC is 14.642 or 15.448.
4. cross-validation