Methods and initial Results

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This is a theoretical paper of the **efficiency of estate tax**. I built two main models to calibrate the efficiency from Macroeconomics prospect and Microeconomics prospect separately. **Please note that this is not a reproduce of literature.**

Models

1. Efficiency model of estate taxes

This model is based on national accounting theory. Suppose the inheritance tax does not distort any decisions then it would have to be more efficient than other taxes. From this model, we can see the efficiency of inheritance taxes comparing to lump-sum taxes.

Suppose there is a closed economy with two generation, the parent and the child. There is neither export nor import. Thus, we have:

$$Y = C + I + G$$

where Y is Total Income, C is consumption, I is investment and G is government purchase.

According to Barro's Permanent Income model, we have:

$$B = (1 + r)\left(Y_{parent} - T_{paranet} - C_{parent}\right)$$

$$C_{child} = Y_{child} - T_{child} + B$$

where B is the legacy of the parent, $T_{paranet}$ and T_{child} is the total taxes paid by the parent and the child separately, r is the interest rate $(r > 0)$.

Specifically, we can derive:

$$T_{paranet} = tY_{parent} + T_0$$

$$T_{child} = tY_{child} + T_0$$

where T_0 is the lump-sum tax and t is the quantity tax rate $(0 < t < 1)$.

$$C_{parent} = cYD_{parent} + C_0$$

$$C_{child} = YD_{child}$$

where YD_{parent} and YD_{child} are the disposable personal income of the parent and the child separately, c is the marginal propensity to consume$(0 < c < 1)$, C_0 is the fixed consumption. And for the child, they use all their disposable income to consume,

Let S_{parent} to be the saving of the parent. The parent put all their savings into legacy to the child. But actually, S_{parent} just equal to the legacy in quantity, it is the balance of Y_{parent} and C_{parent}, T_{parent}. Taxing on legacy won't affect the consumption-saving decision of the parent.

$$Y_{parent} = C_{parent} + S_{parent} + T_{parent}$$

$$= C_{parent} + \frac{B}{(1+r)} + T_{parent}$$

$$= YD_{parent} + T_{parent}$$

Now, the government will tax on the legacy the child can get, suppose it is a lump-sum tax and the quantity is equal to $|\Delta B_0|$ $(\Delta B_0 < 0)$, suppose the government expenditure G is exogenous and fixed, and the government has a balanced budget, which means, there is no government budget surplus nor deficit.

For the government anticipates that the total taxation will increase from the estate tax on legacy, thus, the government will reduce the taxation on the parent through selling bond by $|\Delta T_0|$ and also, the government will reduce the taxation on the child directly.

$$\Delta T_0 = \frac{\Delta B_0}{(1+r)} < 0$$

Because reducing lump-sum tax can simulate consumption thus affect national income and total taxation, thus the increase part of taxation through quantity tax, say, $t\Delta Y_{parent}$, will be used to reduce the total taxation on the child.

Thus, for the parent we have:

$$\Delta T_{parent} = t\Delta Y_{parent} + \Delta T_0$$

$$= t\Delta Y_{parent} + \frac{\Delta B_0}{(1+r)}$$

And from the assumption we can derive:

$$\Delta Y_{parent} = \Delta C_{parent} = \Delta YD_{parent} = c(\Delta Y_{parent} - \Delta T_{parent})$$

$$(1-c)\Delta Y_{parent} = -c\Delta T_{parent} = -c\left[t\Delta Y_{parent} + \frac{\Delta B_0}{(1+r)}\right]$$

$$[1-c(1-t)]\Delta Y_{parent} = -\frac{c\Delta B_0}{(1+r)}$$

Recall $\Delta B_0 < 0$, thus we have:

$$\Delta Y_{parent} = -\frac{c\Delta B_0}{(1+r)[1-c(1-t)]} > 0$$

$$\Delta C_{parent} = -\frac{c\Delta B_0}{(1+r)[1-c(1-t)]} > 0$$

Thus, the national income and consumption of the parent increase.

What the child can get from the parent changes by

$$\begin{aligned}\Delta B &= (1+r)\left(\Delta Y_{parent} - \Delta T_{paranet} - \Delta C_{parent}\right) + \Delta B_0 = -(1+r)\Delta T_{paranet} + \Delta B_0 \\ &= -(1+r)\left(t\Delta Y_{parent} + \Delta T_0\right) + \Delta B_0 \\ &= -(1+r)\left(t\Delta Y_{parent} + \frac{\Delta B_0}{(1+r)}\right) + \Delta B_0 \\ &= -(1+r)t\Delta Y_{parent} - \Delta B_0 + \Delta B_0 = \frac{ct\Delta B_0}{1-c(1-t)} < 0\end{aligned}$$

For the child doesn't save, they use all their money to consume, thus, increase or decrease consumption won't affect the national income of the child. And recall that the government will reduce the taxation on the child by $t\Delta Y_{parent}$.

Thus, for the child we have:

$$\begin{aligned}\Delta C_{child} &= \Delta T_{child} + \Delta B = t\Delta Y_{parent} + \Delta B = -\frac{ct\Delta B_0}{(1+r)[1-c(1-t)]} + \frac{ct\Delta B_0}{1-c(1-t)} \\ &= \frac{rct\Delta B_0}{(1+r)[1-c(1-t)]} < 0\end{aligned}$$

We discount the change of consumption of the child to the parent, the discount factor $\delta = \frac{1}{(1+r)}$, then we can get the change of consumption of the total society increases.

$$\Delta C = \Delta C_{parent} + \delta \Delta C_{child} = -\frac{c\Delta B_0}{(1+r)[1-c(1-t)]} + \frac{1}{(1+r)} \frac{rct\Delta B_0}{(1+r)[1-c(1-t)]}$$

$$= -\frac{c[1+r(1-t)]\Delta B_0}{(1+r)^2[1-c(1-t)]} > 0$$

Thus, estate taxes on legacy can increase the total consumption thus the total utility of the whole economy holding the total taxation fixed. Thus, comparing to lump-sum taxes, estate taxes is quite efficient.

2. Utility model of estate taxes

2.1 two generations

This model is based on the utility of household and the wholes society. We can see the effect of inheritance taxes on wealth distribution both inter-temporal and intra-temporal and the function in social evolution.

Still suppose that it is a closed economy with two generation, the parent and the child. We have a stand-in household that faces a labor-leisure decision and consumption savings decision in two different periods, thus the preference function also changes overtime. Besides, we also have our government who cares about the welfare of the whole society. The government needs to balance wealth distribution and she raises taxes and sells bonds and spends the money as a lump-sum transfer. We should notice that, the decision maker of the government also changes overtime, thus, the object function of the government also changes overtime. Besides, we should specify that there is no financial sector in our model economy.

2.1.1 the parent

Let's begin with the decisions of government and representative household during the time period of the parent.

The household faces the consumption-leisure decision and consumption-leisure decision. The utility function of the representative household is

$$U(c_{parent}, x_{parent}, h_{parent}) = \log c_{parent} + \alpha \log[100(1-h_{parent})] + \beta \log x_{parent}$$

where c_{parent} denotes consumption, h_{parent} denotes the percentage of the hours supplied to the marked per person per week. Given that on a per person basis a household has about 100 hours of productive time per week, nonmarket productive time is $100(1-h_{parent})$ hours per week per working-age person in the household, the production of which cannot be taxed. x_{parent} denotes saving, which also serves as the legacy that the parent leave to the child. The parameter $\alpha > 0$ specifies the value of the nonmarket productive time for the household, which is also known as leisure, relative to household consumption. The parameter $\beta > 0$ specifies the extent the parent care about their offspring.

Since the parent cannot forecast the future, they relate the welfare of the child directly through the legacy they leave to the child, no matter what form of the legacy it is in.

During the time period of the parent, since capital accumulated through saving, there is no capital at the beginning. Thus the whole economy has a linear labor-augmenting production function. It works like a pure agricultural sector, the more labor you devote, the more income you will get. Thus, there is no firm, and the production function of the whole economy is

$$y_{parent} = Ah_{parent}$$

where y_{parent} denotes the income, A denotes the constant technology parameter.

The only reason that the government raises money is to balance wealth distribution and maximize the utility of the whole society. During the time period of the parent, since the government knows that the households care about their offspring and they will save and leave they savings as legacy to the child, the government can sell bond to the household. The parent use part of their savings to buy government bond and leave the bond as cash equivalent to the child and leave their savings except the bond as capital to the child.

The government faces the decision of the quantity of the bond issued and the tax rate on the labor income of the parent. Thus, the object function of government is

$$V(B, \tau_{parent}) = \log c_{parent} + \gamma \log(100h_{parent})$$

where $B > 0$ denotes the quantity of the bond issued, τ_{parent} denotes the tax rate of the labor income, the parameter $\gamma > 0$ specifies the value of the labor supply for the government relative to consumption of the economy.

Considering the object of the social planer and the production function of the economy during the time period of the parent, the government won't raise tax but will only issue bond because the government knows that the parent will leave money to the child and the government only cares about the instantaneous social welfare. Taxation on labor income will decrease the amount of labor supply while the government can smooth the wealth distribution through the bond they issue. Thus we have

$$\tau_{parent} = 0$$

Thus we can rewrite the objective function of the government as

$$V(B) = \log c_{parent} + \gamma \log\left(100h_{parent}\right)$$

For the decision maker of the government only cares about the social welfare during his tenure and doesn't pay any attention to the future, he wants to issue as much bond as he can, but he faces budget constraint

$$B \leq x_{parent}$$

Note that, the bond issued by the government today will be paid buy the successor of the government. That is, the government during the time period of the child has to pay for the bond. Since there is only two generations in our model economy, the government of the child can choose to default or not. But anyway, during the time period of the parent, since there is no capital rental activity and firm, the household is indifferent between buying government bond or just put their savings there as storage. Thus the riskless interest rate in this economy equals to 0. The government won't promise to pay any interest of the bond.

Also note that, the government sells bond to raise money and uses the money as lump-sum transfer to the parent. Thus, the budget constraint faced by the household is

$$c_{parent} + x_{parent} = y_{parent} + B$$

That is

$$c_{parent} + x_{parent} = Ah_{parent} + B$$

We should notice that the people are shortsighted. They have no idea of the big picture of the government. They never notice that they actually consume part of their savings in the form of transfer from the government. Thus, getting lump-sum transfer from the government won't affect their valuation of their inheritance. They think the government of their offspring will pay the child equal amount of the bond they buy today in cash. So they think bond is equivalent to their savings to their children.

Now the household wants to maximize its utility under the budget constraint. To see their decision, we can rewrite the household decision problem as

$$\max_{c_{parent}, x_{parent}, h_{parent}} \log c_{parent} + \alpha \log\left[100\left(1 - h_{parent}\right)\right] + \beta \log x_{parent}$$

$s.t.$

$$c_{parent} + x_{parent} = Ah_{parent} + B$$

And setup Lagrangian function

$$\mathcal{L} = \log c_{parent} + \alpha \log\left[100\left(1 - h_{parent}\right)\right] + \beta \log x_{parent} + \lambda\left(Ah_{parent} + B - c_{parent} - x_{parent}\right)$$

where $\lambda \geq 0$ is Lagrangian Multiplier.

Take derivative to the Lagrangian function with respect to $c_{parent}, x_{parent}, h_{parent}$ one by one, then we can get

$$\begin{cases} \dfrac{1}{c_{parent}} = \lambda \\ \dfrac{\alpha}{1 - h_{parent}} = \lambda A \\ \dfrac{\beta}{x_{parent}} = \lambda \end{cases} \Rightarrow \begin{cases} x_{parent} = \beta c_{parent} \\ h_{parent} = 1 - \dfrac{\alpha c_{parent}}{A} \end{cases}$$

where $\frac{1}{1+\beta}$ is the marginal propensity to consume.

Put these equations into the household budget constraint, then we can derive $c_{parent}, x_{parent}, h_{parent}$ as a function of B.

$$\begin{cases} c_{parent} = \dfrac{A+B}{1+\alpha+\beta} \\ x_{parent} = \dfrac{\beta(A+B)}{1+\alpha+\beta} \\ h_{parent} = \dfrac{A(1+\beta)-\alpha B}{A(1+\alpha+\beta)} \end{cases}$$

Then recall the government's problem

$$\max_B \log c_{parent} + \gamma \log(100 h_{parent})$$

$s.t.$

$$B \leq x_{parent}$$

Put these results we got from the household problem into the government's problem, then we can rewrite the government's problem as

$$\max_B \log\left(\frac{A+B}{1+\alpha+\beta}\right) + \gamma \log\left\{100\left[\frac{A(1+\beta)-\alpha B}{A(1+\alpha+\beta)}\right]\right\}$$

$$\Leftrightarrow$$

$$\max_B \log(A+B) + \gamma \log[A(1+\beta)-\alpha B]$$

$s.t.$

$$B \leq \frac{\beta(A+B)}{1+\alpha+\beta} \quad \Leftrightarrow \quad B \leq \frac{\beta A}{1+\alpha}$$

And setup Karush-Kuhn-Tucker Conditions

$$\frac{1}{A+B} - \frac{\alpha\gamma}{A(1+\beta)-\alpha B} + \mu\left(B - \frac{\beta A}{1+\alpha}\right) = 0$$

$$\mu\left(B - \frac{\beta A}{1+\alpha}\right) = 0$$

where $\mu \geq 0$ is Multiplier which satisfies dual feasibility.

Thus there are two solutions to this problem.

When $\alpha\gamma \leq 1$, the government budget constraint is binding, that is $\mu > 0, B = \frac{\beta A}{1+\alpha}$, the household during the generation of the parent rents all their savings to the government and leaves no capital to the child. Thus we have

$$B = \frac{\beta A}{1+\alpha}$$

$$\begin{cases} c_{parent} = \dfrac{A}{1+\alpha} \\ x_{parent} = \dfrac{\beta A}{1+\alpha} \\ h_{parent} = \dfrac{1}{1+\alpha} \end{cases}$$

Note that $x_{parent} = B$ is satisfied under this situation.

Under this circumstance, since the parent leaves no capital to the child and actually consumes all their labor income, actually, there is no inheritance in this economy and the child will face a similar production function as the parent. This is kind of unrealistic in our world now. But this is quite like the economy of the ancient agricultural civilization when there is neither firm nor capitalism.

When $\alpha\gamma > 1$, the government budget constraint is not binding, that is $\mu = 0, B = \frac{A(1+\beta-\alpha\gamma)}{\alpha(1+\gamma)}$, the household during the generation of the parent use part of their savings to buy government bond and leaves the rest of their savings as capital to the child. Thus we have

$$B = \frac{A(1+\beta-\alpha\gamma)}{\alpha(1+\gamma)}$$

$$\begin{cases} c_{parent} = \dfrac{A}{\alpha(1+\gamma)} \\ x_{parent} = \dfrac{\beta A}{\alpha(1+\gamma)} \\ h_{parent} = \dfrac{\gamma}{1+\gamma} \end{cases}$$

Note that the capital left by the parent to the child is $\left(x_{parent} - B\right)$. We denote this amount as k_{child}. That is

$$k_{child} = x_{parent} - B = \frac{A(\alpha\gamma - 1)}{\alpha(1+\gamma)}$$

Under this circumstance, the child can benefit from the capital accumulation process of the parent. Suppose there is no inter-temporal depreciation. From this capital, we can see there will evolve a firm which produces in Cobb-Douglas production function. This is kind of like capitalism. The child gains from renting the capital left by the parent to the firm.

Whether there occurs the process of capital accumulation depends on the relative valuation of the household of the leisure and the relative valuation of the government of the labor supply. The less the household and the government value today's consumption, the more likely the process of capital accumulation occurs.

2.1.2 the child

Now we come to the problem of the child. Notice that there are only two generations in our economy, thus the government has the motivation to default for the government can't raise bond during the time period of the child. But when we further expand our model to infinite horizon, the motivation of the government to default will be weakened for once the government default, the current generation will realize that the government is not reliable and they won't borrow their savings to the government any more. That is, the government can't issue bond. This may hurt the welfare of the society during the current tenure period. We will discuss the infinitely horizon model later. At this point, we let the government of the child default.

And also, in our two-generations economy, the child will consume all their labor income and the inheritance they get from the parent. That is, there is no savings of the child.

The utility function of the representative household now is

$$U(c_{child}, h_{child}) = \log c_{child} + \alpha' \log[100(1 - h_{child})]$$

where c_{child} denotes consumption, h_{child} denotes the percentage of the hours supplied to the marked per person per week. The parameter $\alpha' > 0$ specifies the value of the nonmarket productive time for the household, which is also known as leisure, relative to household consumption. Since the generation changes, the child may have different mind to the parent and may value consumption and leisure differently. Under most circumstances, human beings become lazier and lazier generation by generation. As is the case, we may find $\alpha' > \alpha$. Thus won't affect the outcome of our discussion.

At this point, in our two generations model economy, we can divide our discussion into two circumstances. First, government default, no capital accumulation, second, government default, capital accumulates.

2.1.2.1 government default, no capital accumulation

This is the case when $\alpha\gamma \leq 1$. That is, the whole society values consumption relatively more than the labor supply or leisure.

Under this situation, since there is no capital accumulation during the time period of the parent. The whole economy during the time period of the child now has a linear labor-augmenting production function similar to the parent. That is, there is still no firm, and the production function of the whole economy is

$$y_{child} = A' h_{child}$$

where y_{child} denotes the income, A' denotes the constant technology parameter during the period of the child. As is often the case, the child may find some now technic or new method and get a higher technology parameter. That is $A' > A$.

Recall that the only reason that the government raises money is to balance wealth distribution and maximize the utility of the whole society. Under this circumstance, whether the government taxes labor income or not won't affect the labor-leisure decision of the household. But it's better for the government not to punish those people that works hard too much or it will hurt the incentive for those hardworking people to provide labor supply. And also, during the time period of the child, the government cannot issue a bond and he decides to default.

The government faces the decision of the tax rate on the labor income of the child. Thus, the object function of government is

$$V(\tau_{child}) = \log c_{child} + \gamma' \log(100 h_{child})$$

where τ_{child} denotes the tax rate of the labor income, the parameter $\gamma' > 0$ specifies the value of the labor supply for the government relative to consumption of the economy.

Considering the object of the social planer and the production function of the economy during the time period of the child under this circumstance, the government won't raise tax for it will decrease the amount of labor supply and total product, thus the consumption of the model economy. This obeys the object of the government.

$$\tau_{child} = 0$$

The child has no actual inheritance and will only consume the labor income of the current economy. Thus, the budget constraint faced by the household is

$$c_{child} == A' h_{child} = y_{child}$$

Now the household wants to maximize its utility under the budget constraint. To see their decision, we can rewrite the household decision problem as

$$\max_{c_{child}, h_{child}} \log c_{child} + \alpha' \log[100(1 - h_{child})]$$

$s.t.$

$$c_{child} = A' h_{child}$$

And setup Lagrangian function

$$\mathcal{L} = \log c_{child} + \alpha' \log[100(1 - h_{child})] + \lambda'(A' h_{child})$$

where $\lambda' \geq 0$ is Lagrangian Multiplier.

Take derivative to the Lagrangian function with respect to c_{child}, h_{child} separately, then we can get

$$\begin{cases} \dfrac{1}{c_{child}} = \lambda' \\ \dfrac{\alpha'}{1 - h_{child}} = \lambda' A' \end{cases} \Rightarrow h_{child} = 1 - \frac{\alpha' c_{child}}{A'}$$

Put these equations into the household budget constraint, then we can derive c_{child}, h_{child}

$$\begin{cases} c_{child} = \dfrac{A'}{1 + \alpha'} \\ h_{child} = \dfrac{1}{1 + \alpha'} \end{cases}$$

However, this ancient agricultural civilization is not what we want.

2.1.2.2 government default, capital accumulates

This is the case when $\alpha\gamma > 1$. That is, the society in total cares relatively less about consumption than the labor supply or leisure.

Under this situation, since there is capital accumulation during the time period of the parent. There evolves a firm during the time period of the child and the whole economy now has a Cobb-Douglas production function. In the model economy, the household owns the capital and rents it to the firm.

Recall that the capital owned by the child equals to the inheritance they get from the parent as savings minus the bond. Recall that the capital rental and using activities only occur during the time period of the child, there is no inter-temporal depreciation.

$$k_{child} = \frac{A(\alpha\gamma - 1)}{\alpha(1+\gamma)}$$

The production function of the representative firm is

$$y_{child} = A' {k_{child}}^{\theta} {h_{child}}^{1-\theta}$$

where y_{child} denotes the income, A' denotes the total factor productivity parameter during the period of the child. The parameter $0 < \theta < 1$ specifies the capital share.

The firm makes profit maximization decision and pays the labor and capital at their marginal product respectively. That is:

$$w_{child} = \frac{(1-\theta)y_{child}}{h_{child}}$$

$$r_{child} = \frac{\theta y_{child}}{k_{child}}$$

where w_{child} denotes the wage and r_{child} denotes the rental price of the capital, which can also seen as the interest rate.

Thus we also have

$$y_{child} = w_{child}h_{child} + r_{child}k_{child}$$

Recall that the only reason that the government raises money is to balance wealth distribution and maximize the utility of the whole society. We should notice that the tax on the inheritance as capital directly occurs at the beginning of the economy. And the government will rent this tax revenue to the firm and use the capital rents of this part as lump-sum transfer at the end of the period. Thus, tax on the inheritance directly is just the same as tax on the capital income. Thus, in our model economy, to simplify our discussion, we will let the capital income tax serves as estate tax.

Recall that during the time period of the child, the government decide to default. That is, the decision maker of the government refuses to pay for the bond issued by the predecessor. This is because in our model economy, the child doesn't save, thus the government cannot issue bond and sell it to the child.

As stated above, the government faces the decision of the tax rate on labor income and the tax rate on the capital income. Thus, the object function of government is

$$V(\tau_l, \tau_k) = \log c_{chlid} + \gamma' \log(100 h_{child})$$

where τ_l denotes the tax rate of labor income, τ_k denotes the tax rate of the capital income. The parameter $\gamma' > 0$ specifies the value of the labor supply for the government relative to consumption of the economy.

Suppose the amount of the government spending during the time period of the child is exogenous and is used as lump-sum transfer. That is

$$T_{child} = \tau_l * w_{child} h_{child} + \tau_k * r_{child} k_{child} = \bar{T}$$

where T_{child} is the total amount of lump-sum transfer, $\bar{T}$ is a constant.

Thus, the budget constraint faced by the representative household is

$$\begin{aligned} c_{child} &= (1-\tau_l) * w_{child} h_{child} + (1-\tau_k) * r_{child} k_{child} + T_{child} \\ &= (1-\tau_l) * w_{child} h_{child} + (1-\tau_k) * r_{child} k_{child} + \bar{T} \end{aligned}$$

Note that the child consumes all the income they get and the capital will depreciate fully at the end of the time period. That is the depreciation rate in our model economy is 1.

Now the household during the time period of the child faces the consumption-leisure decision only. Since the child doesn't save and wants to maximize its utility under the budget constraint. To see their decision, we can rewrite the household decision problem as

$$\max_{c_{child}, h_{child}} \log c_{child} + \alpha' \log[100(1-h_{child})]$$

$s.t.$

$$c_{child} = (1-\tau_l) * w_{child} h_{child} + (1-\tau_{\boldsymbol{k}}) * \boldsymbol{r_{child} k_{child}} + \boldsymbol{\bar{T}}$$

And setup Lagrangian function

$$\mathcal{L} = \log c_{child} + \alpha' \log[100(1 - h_{child})] + \lambda'[(1-\tau_l) * w_{child} h_{child} + (1-\tau_k) * r_{child} k_{child} + \overline{T} - c_{child}]$$

where $\lambda' \geq 0$ is Lagrangian Multiplier.

Take derivative to the Lagrangian function with respect to c_{child} and h_{child} separately, then we can get

$$\begin{cases} \dfrac{1}{c_{child}} = \lambda' \\ \dfrac{\alpha'}{1 - h_{child}} = \lambda'(1-\tau_l) * w_{child} \end{cases} \Rightarrow \quad \frac{(1-\tau_l) * w_{child}}{c_{child}} = \frac{\alpha'}{1 - h_{child}}$$

Or we can use Euler Equation directly and get

$$\frac{(1-\tau_l) * w_{child}}{c_{child}} = \frac{\alpha'}{1 - h_{child}}$$

Recall that the firm makes profit-maximizing decision

$$w_{child} = \frac{(1-\theta) y_{child}}{h_{child}}$$

Substituting it into Euler Equation and we can get

$$\frac{(1-\theta)(1-\tau_l) y_{child}}{c_{child}} = \frac{\alpha' h_{child}}{1 - h_{child}}$$

Recall that

$$\begin{aligned} c_{child} &= (1-\tau_l) * w_{child} h_{child} + (1-\tau_k) * r_{child} k_{child} + T_{child} \\ &= (1-\tau_l) * w_{child} h_{child} + (1-\tau_k) * r_{child} k_{child} \\ &+ (\tau_l * w_{child} h_{child} + \tau_k * r_{child} k_{child}) = w_{child} h_{child} + r_{child} k_{child} \\ &= y_{child} \end{aligned}$$

Thus we have

$$h_{child} = \frac{(1-\theta)(1-\tau_l)}{(1-\theta)(1-\tau_l) + \alpha'} = \frac{1}{1 + \dfrac{\alpha'}{(1-\theta)(1-\tau_l)}}$$

We can clearly see that the labor supply of the child decreases with the increase of tax rate on labor income.

Considering the object function of the government and the production function of the model economy, it's trivial that if $\boldsymbol{\bar{T} \leq r_{child} k_{child}}$, the government will set $\boldsymbol{\tau_l = 0}$ and only tax on the capital income. Then we have

$$\boldsymbol{h_{child} = \frac{1-\theta}{1-\theta+\alpha'}}$$

Recall $\boldsymbol{k_{child} = \frac{A(\alpha\gamma-1)}{\alpha(1+\gamma)}}$, then we have

$$\boldsymbol{c_{child} = y_{child} = A' k_{child}{}^{\theta} h_{child}{}^{1-\theta} = A' \left[\frac{A(\alpha\gamma-1)}{\alpha(1+\gamma)}\right]^{\theta} \left(\frac{1-\theta}{1-\theta+\alpha'}\right)^{1-\theta}}$$

Thus the tax rate on capital income is

$$\boldsymbol{\tau_k = \frac{\bar{T}}{r_{child} k_{child}} = \frac{\bar{T}}{\theta y_{child}} = \frac{\bar{T}}{\theta * A' \left[\frac{A(\alpha\gamma-1)}{\alpha(1+\gamma)}\right]^{\theta} \left(\frac{1-\theta}{1-\theta+\alpha'}\right)^{1-\theta}}}$$

And if $\boldsymbol{\bar{T} > r_{child} k_{child}}$, the government will set $\boldsymbol{\tau_k = 1}$ and fully tax the capital income. Then we have

$$\boldsymbol{h_{child} = \frac{(1-\theta)(1-\tau_l)}{(1-\theta)(1-\tau_l)+\alpha'}}$$

Recall $\boldsymbol{k_{child} = \frac{A(\alpha\gamma-1)}{\alpha(1+\gamma)}}$, then we have

$$c_{child} = y_{child} = A' k_{child}{}^{\theta} h_{child}{}^{1-\theta} = A' \left[\frac{A(\alpha\gamma-1)}{\alpha(1+\gamma)}\right]^{\theta} \left[\frac{(1-\theta)(1-\tau_l)}{(1-\theta)(1-\tau_l)+\alpha'}\right]^{1-\theta}$$

Thus the tax rate on labor income is

$$\tau_l = \frac{\bar{T} - r_{child} k_{child}}{w_{child} h_{child}} = \frac{\bar{T}}{(1-\theta) y_{child}} - \frac{\theta}{1-\theta}$$

$$= \frac{\bar{T}}{(1-\theta) * A' \left[\frac{A(\alpha\gamma-1)}{\alpha(1+\gamma)}\right]^{\theta} \left[\frac{(1-\theta)(1-\tau_l)}{(1-\theta)(1-\tau_l)+\alpha'}\right]^{1-\theta}} - \frac{\theta}{1-\theta}$$

Recall the object function of the government

$$V(\tau_l, \tau_k) = \log c_{chlid} + \gamma' \log(100 h_{child})$$

It's trivial that it would be better for the government not to set $\bar{T}$ too high and not to tax the labor income. It would be convenient to set $\tau_k \in [0,1]$, $\tau_l = 0$ in our model economy.

2.2 intuition of the model

This inter-generational model calculates the optimal tax rate on labor and bequest under both circumstance of capital accumulation and non-accumulation of capital. It based on the utility of the household and government of the parent and the child. The utility affects the decisions of the four agents. Through the model we find that, when there is no capital accumulation, it would be good for the government not to tax in our model economy while under the circumstance of capital accumulation it will be better for the government not tax on the labor income and the government can tax at any rate on the bequest to make transfers since bequest tax is efficient and won't affect the labor supply decision of the household.

Data

Next, I'll do model simulation with the data of the United States. All my data are downloaded from Fred (Federal Reserve Bank: https://fred.stlouisfed.org).