

**make
history.**



Other recursive algorithms

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Binary search

Benefits:

- halve the search space every time
- don't have to analyse any element
- complexity $O(\log n)$
- sorted data can be searched faster
- sort once, search a lot



Recursive Fibonacci

```
int fib(int n) {  
    if(n<=1)  
        return 1;  
    else  
        return fib(n-1)+fib(n-2);  
}
```

$$\frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1 - \sqrt{5}}{2} \right)^n$$

Recursive Fibonacci is $O(2^n), \Omega(2^{n/2})$



The greatest common divisor (Euclid's algorithm)

```
int recursiveGCD (int a, int b) {  
    if(b==0) return a;  
    return recursiveGCD(b, a%b);  
}
```

```
int gcd(int a, int b) {  
    while(b!=0) {  
        int remainder = a%b;  
        a = b;  
        b = remainder;  
    }  
    return a; }
```



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```

Values a and b are monotonically decreasing;

After the first two iterations $a \leq \min \{a, b\}$;

After any two iterations the value of a is at most half of what it has been before;

Hence, the time complexity is $O(\min\{a,b\})$.



Maximum subsequence sub-problem

Find subsequence with maximum sum:

-1, 2, 3, 6, -12, **13** → 13

- 1, 2 ,3, 6, -8, **13** → 13

There are different algorithms to solve this problem and their performance varies significantly.



Algorithm 1

```
for(size_t i = 0; i < s.size(); i++) {  
    for (size_t j = i; j < s.size(); j++) {  
        int sum = 0;  
        for (size_t k = i; k <= j; k++) {  
            sum += s.at(k);  
        }  
        if (sum > max) { max = sum;}  
    }  
}  
  
return max;
```



Algorithm 2

```
for(size_t i = 0; i < s.size(); i++) {  
    int sum = 0;  
    for (size_t j = i; j < s.size(); j++) {  
        sum += s.at(j);  
        if (sum > max) { max = sum;}  
    }  
}  
  
return max;
```



Algorithm 3: Divide and conquer

```
int maxSubArray(vector<int> s, int start, int end) {  
    if (start == end) {return s.at(0);}   
    int mid = start + (end-start)/2;  
    int leftMaxSum = maxSubArray(s, start, mid); int rightMaxSum = maxSubArray(s, mid + 1, end);  
    int sum = 0; int leftMidMax = 0;  
    for (int i = mid; i >=start; i--) {  
        sum+=s.at(i);  
        if (sum > leftMidMax) {  
            leftMidMax = sum;  
        }  
    }  
    sum = 0; int rightMidMax = 0;  
    for (int i = mid + 1; i <=end; i++) {  
        ...  
    }  
    int centerSum = leftMidMax + rightMidMax;  
    return max(centerSum, max(leftMaxSum, rightMaxSum));  
}
```



Algorithm 4: Kadan's algorithm

```
int maxSubArray(vector<int> s) {  
    int sum, maxSum = 0;  
    for (int i = 0; i < s.size(); i++) {  
        sum+=s.at(i);  
        if (sum > maxSum) {  
            maxSum = sum;  
        } else if (sum < 0) {  
            sum =0;  
        } return maxSum;  
    }  
}
```