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Structural balance and dynamics over signed BA scale-free network

Yue Wu [a,*], Lanlin Gao [a], Yi Zhang [b], Xi Xiong [c]

[a] *School of Computer and Software Engineering, Xihua University, Chengdu 610039, China*
[b] *School of Economics and Management, Yibin University, Yibin 644000, China*
[c] *School of Information Security Engineering, Chengdu University of Information Technology, Chengdu 610225, China*

HIGHLIGHTS

- Structural balance of a binary group is defined.
- A co-evolution model based on structural balance, opinions dynamics and relationships dynamics is proposed.
- Intermediate state structure has the main effect on reducing the number of changed relations.
- For any acyclic network, the global balance can always be achieved.
- In a network with cycles, the global structural balance index can be zero, a non-zero constant and fluctuate as well.

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ABSTRACT

By exploring the influence between dynamics and structural balance, we divide a binary group(with two nodes and a directed edge) into three categories as balanced, intermediate and unbalanced structure. On this basis, a co-evolution model considering the dynamic mechanics of both opinions and relationships is designed. Global structural balance in four basic structures are explained. In the simulation experiments over signed BA scale-free network, the impact of negative relationships on polarizing opinion, the effect of intermediate state structure, and the influences of five strategies on relaxation time are analyzed. By introducing an unbalanced cycle, two kinds of interesting results are discussed.

1. Introduction

Various complex social relationships can be represented as signed networks, such as international relations and racial tensions. In these signed networks, nodes represent objects and edges represent the interactions among them. The positive/negative edges are used to stand for the friendly/hostile, cooperative/non-cooperative, supported/opposed, or like/dislike relationships.

For signed networks, the research mainly focuses on the balance structure of the networks [1]. Balance theory is derived from sociology by Heider in 1944 [2], which indicates that a balanced structure exists if the opinions towards the causal units which consist of persons and acts are similar. Cartwright and Harary [3] developed Heider's theory by using signed triangle structure and made it quantitative. This research is of great significance for the follow-up studies, then the subsequent researches are mainly about weak balance theory [4], structural balance of reciprocal, asymmetric

* Corresponding author.
E-mail address: wuyue@mail.xhu.edu.cn (Y. Wu).

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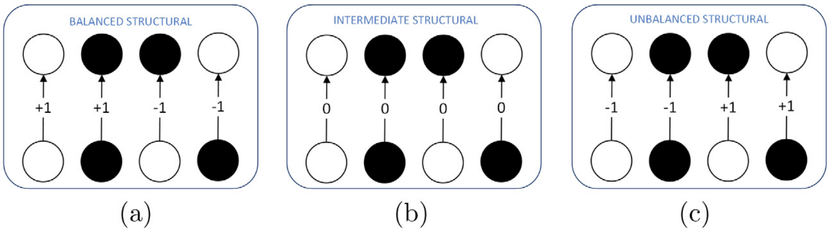


Fig. 1. (a) Balanced, (b) intermediate and (c) unbalanced structure of binary groups.

and non-joined relationships [5], global structural balance of real online social networks [6], fast algorithm to compute global structural balance [7], new interpretable models for evaluating structural balance over signed networks [8], and the data-driven verification of structural balance of multiplex networks [9].

In addition to the study about structural balance in signed networks, many researchers are interested in the dynamics over signed networks. Some researchers are concerned with the dynamics of node states, especially how the individuals' states are altered by their neighbors [10], and the interplay between consensus and coherence [11]. Meanwhile, some researchers focus on the dynamics of signed edges, such as the repelling rule [12], the opposing rule [13], and DeGroot's rule of social interactions [14]. Shi [15] reviewed some fundamental convergence results for such dynamics over deterministic and random signed networks.

At present, several scholars have combined the structural balance and dynamics over signed network. For instance, Zelazo [16] studied the effects of negative edge weights on the stability of consensus networks. The results indicate that the resulting network will not be stable, if any negative weight edge splits the networks, or its magnitude is less than the inverse of the effective resistance. Macy [17] studied the polarization of social networks with the Hopfield model, and found that networks can be organized by themselves into two rival cliques, without the purpose of the individuals.

Similar to Ref. [16,17], this paper also studies the structural balance and dynamics over signed network. The difference is that, three novel types for classifying the structures of binary groups are defined. Furthermore, the social pressure index is designed according to the neighbor states, the signed edges and the out-degrees. Global structural balance is discussed in both directed acyclic networks and cycles. Simulation experiments are carried out on BA scale-free network. And a series of macroscopic parameters such as global structural balance index, iteration traversal times, proportion of positive opinions, percentage of positive relationships, number of individuals who changed their opinions or the relationships are analyzed.

2. Structural balance

2.1. Social relationship

A signed social network usually stand for a network with positive ("+") and negative ("−") links [18]. Positive relationships are often used to represent homogenous relationships in a group, for instance, like, cooperation, support, trust, friendly, etc. While negative relationships represent heterogeneous relationships between different groups, such as dislike, non-cooperation, nonsupport, distrust, hostility, etc. In general, positive relationships are necessary for reaching a consensus, and negative relations often results in polarization. In real life, there are more than two kinds of relationships between people, organizations, countries and so on. For example, in a social network, a person sometimes posts to support an official media, yet sometimes criticizes it. If the proportions of the two cases are similar, then the relationship between them could be seemed as a neutral one on a long-term perspective. Therefore, besides positive and negative relations, we defined another concept named neutral relationship ("0"), which is an intermediate status between the positive and negative relations. When long-time friends have opposite opinions or long-time enemies make an agreement temporarily, their relationships are represented as "0".

2.2. Binary structural balance

In classical balance theory [2], people are supposed to be motivated to keep "balance" in their relationships, such that two persons who are positively linked to one another will get nervous when they hold opposing opinions on the third object. If this kind of relationship is put in a binary group, we will get a new balanced structure, as shown in Fig. 1(a). Take a binary group as an example, suppose the two nodes in the binary group are i and j. If i and j have the same opinion, at the same time, i supports j, then the binary group is balanced. However, if i always opposes j, but i and j have the same opinion about a certain topic, then the directed binary group is unbalanced. Four types of unbalanced structures of binary groups are shown in Fig. 1(c). It can be seen that if the product of the three parameters in a binary group (two individuals' opinions, and their relationship) is positive, then the structure is balanced. Whereas if the product is negative, the structure is unbalanced. Moreover, if the relationship is equal to "0", it means the two-tuple group is in a temporary intermediate state between the balanced and unbalanced one. The intermediate structure has an impact on the signed edges dynamics, which will be analyzed in Sections 3.3 and 5.1.

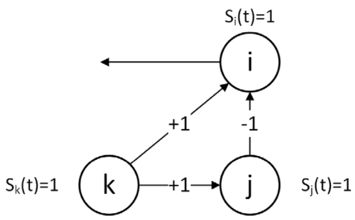


Fig. 2. The graph for explaining the evolution process of opinions.

2.3. Global structural balance index

The global structural balance index h_s is shown in Eq. (1) as paper [6] mentioned. s_i is the opinion of i, $s_i = +1$ means i holds a positive opinion, $s_i = -1$ means i holds a negative opinion. J_{ij} is a little different from which in the paper [6]. In this paper, J_{ij} is the one-sided relationship from i to j. $J_{ij} = +1$ indicates that i has a positive impact on j, $J_{ij} = -1$ means i has a negative effect on j, and $J_{ij} = 0$ implies i has no influence on j. When the index h_s becomes zero, the network is global structural balanced.

$$h_s = \sum_{(i,j)} (1 - J_{ij} s_i s_j)/2 \tag{1}$$

3. Co-evolution model of opinions and relations

In order to construct a co-evolution model of opinions and social relations, an index of social cumulative pressure is designed. Based on it, evolution rules of opinions and relationships are put forward.

3.1. Social cumulative pressure

Social cumulative pressure index is shown in Eq. (2). N_i is the in-degree of i, representing the number of individuals who exert social pressure on i. D_j is the out-degree of j, indicates the number of individuals affected by j. In a real social network, a person with large out-degree always has more influence on others, so D_j is considered as an important parameter in social cumulative pressure.

$$P_i(t) = \sum_{j=1}^{N_i} D_j J_{ji}(t) s_i(t) s_j(t) \tag{2}$$

3.2. Evolutionary rule of opinions

Evolutionary rule of opinions is shown in Eq. (3). If the social cumulative pressure of i is greater than zero, it is a positive motivation. And if it equals to zero, the external positive and negative pressures are offset. In both cases, i will keep the original opinion. However, if the social cumulative pressure is negative, i will change its opinion.

$$s_i(t+1) = \begin{cases} s_i(t), & \text{if} \quad P_i(t) \geq 0 \\ -s_i(t), & \text{if} \quad P_i(t) < 0 \end{cases} \tag{3}$$

To explain it more clearly, an example is given as seen in Fig. 2. If we want to calculate $s_i(t+1)$ in Fig. 2, the out-degrees of i, j, k should be computed firstly, they are 1, 2 and 1 respectively. Then the social cumulative pressure of i need to be calculated, that is, $P_i(t) = P_{ki}(t) + P_{ji}(t) = 2 - 1 = 1$. For $P_i(t) > 0$, according to Eq. (3), $s_i(t+1) = s_i(t)$. This process is coincide with our life experience, people would like to keep the original opinion when they gain great support.

3.3. Evolutionary rule of relations

The relationship evolutionary rule is shown in Eq. (4), which is similar to the strategy proposed in the literature [17]. According to Eq. (4), the one-direction relationship from j to i depends not only on the previous relationship, but also on the opinions of i and j. In the first case, if i and j have the same opinion, meanwhile, j does not oppose i, at the next tick of the clock, j will support i. In the second case, if the two individuals have opposite opinions, and j does not support i, then j will oppose i in the next time step. Except for the two cases above, j will hold a neutral relationship with i temporarily.

$$J_{ji}(t+1) = \begin{cases} +1, & \text{if} \quad J_{ji}(t) + s_i(t)s_j(t) > 0 \\ 0, & \text{if} \quad J_{ji}(t) + s_i(t)s_j(t) = 0 \\ -1, & \text{if} \quad J_{ji}(t) + s_i(t)s_j(t) < 0 \end{cases} \tag{4}$$

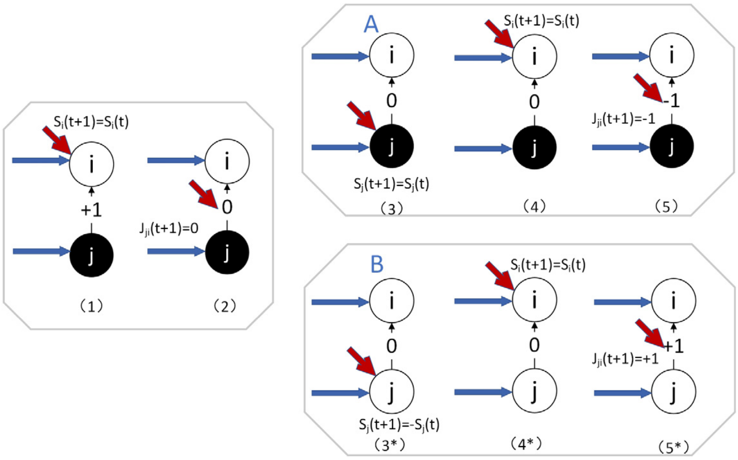


Fig. 3. An illustration for explaining the evolution of relationships.

Table 1
Initialization parameters.

Parameters	Description
N	Network size
$Round$	The number of iterations
R_{s+}	The proportion of positive opinions
R_{s-}	The proportion of negative opinions
R_{r+}	The proportion of positive relationships
R_{r-}	The proportion of negative relationships
R_{r0}	The proportion of neutral relationships

Roles of neutral relationship and intermediate structure are illustrated in Fig. 3, by five stages in the synergistic process of binary opinions and relationships. Blue arrows in Fig. 3 represent the external social pressure, red arrows are used to mark the selected objects in different time steps. Two representative cases are given respectively. In case A, neither i nor j changes opinion, so the one-direction relationship changes from "$+1$" to "0", then to "-1" finally. In case B, j shifts its view, so the relationship in stage(5*) is not changed compared with stage(1). It can be seen that, the unbalanced structure will develop to a balanced one in both instances, no matter what the altered object is. However, if the neutral relationship and intermediate structure are not taken into account, the unbalanced structure will be changed to a balanced state immediately by turning the relationship from one side to the opposite one. Therefore, intermediate structure helps individuals have more time to communicate with their neighbors, and then decide whether the shift in opinions or relations is needed.

3.4. Model algorithm

In this paper, we model the dynamics of opinions and relations based on the social cumulative pressure. In an acyclic graph, the iteration will not stop until the global structural balance is reached. Model algorithm is defined as follows, and the initialization parameters are shown in Table 1. In the algorithm, Round represents the traversal times. In each Round, all of the nodes are selected according to a designed constant sequence. If a node updates its status, all its adjacent edges will update their status, then move to another node. This algorithm is similar with the process of posting messages in a social network. After a user posts his opinion, all the followers would consider how to update their relationships, to be friends or enemies, or just wait. Users post one after another, when all of them have expressed their opinions, a *Round* is finished.

4. Global structural balance analysis

4.1. Characteristics of the model

In this novel model, the system evolves according to Eqs. (4) and (5) as time goes by. The process of evolution will not be ended until the global structural balanced is reached ($h_s(Round) = 0$), which means that even if only one binary group is unbalanced, it will disturb the global structure. A question thus arises as to in which circumstances does global structural balance occur. For answer this question, three main characteristics of the model are given.

Algorithm 1: Model implementation process

Input: N, $R_{s+}(0)$, $R_{s-}(0)$, $R_{r+}(0)$, $R_{r-}(0)$, $R_{r0}(0)$. Generate a signed BA scale-free network of N nodes, and the distributions of positive and negative opinions are initialized by R_{s+} and R_{s-}, the proportions of relations in {+1,0,-1} are decided by R_{r+}, R_{r-}, R_{r0}.
Output: $Round$, $h_s(Round)$, $R_{s+}(Round)$, $R_{r+}(Round)$.

```
Stra ← Selection{Stra1, Stra2, Stra3, Stra4, Stra5};
Generate a Sequence according to Stra,(ie,Sequence ← Stra2|{1, 2, 3, ..., N});
Round ← 1;
repeat
    h_s(Round) ← 0;
    Tcounter ← 1;
    repeat
        for each node i from the Sequence in turn do
            if P_i(t) < 0 then
                s_i(t + 1) ← −s_i(t);
            for each node j pointing to i do
                if J_ji(t) + s_i(t)s_j(t) > 0 then
                    J_ji(t + 1) ← 1;
                else if J_ji(t) + s_i(t)s_j(t) == 0 then
                    J_ji(t + 1) ← 0;
                else
                    J_ji(t + 1) ← -1;
        Tcounter ← Tcounter + 1;
    until Tcounter == N;
    for each Tuple i and j do
        h_s(Round) ← h_s(Round) + (1 − J_ji(t) ∗ s_i(t) ∗ s_j(t))/2;
    Round ← Round + 1;
until h_s(Round) == 0;
```

Character 1: In the process of evolution, the root-nodes, nodes with zero in-degree, will not change their status forever. The reversion of node status is possible only if it is a non-root-node.
Character 2: If the state of a non-root-node is reversed, it is possible that the status of other nodes will be impacted and changed. This phenomenon, nodes reverse their status one after another is called a chain reaction in this paper. The chain reaction will not be ended until it meets a particular node which is satisfied one of the following three conditions. In Fig. 4, node m is designed as the particular node.
Character 2-a: If the particular node is a leaf-node, with zero out-degree. Then the chain reaction will be ended when the leaf-node reverses its status, as indicated in Fig. 4(a).
Character 2-b: If there is big enough cumulative social pressure on the particular node. Then the chain reaction will be over when the particular node changes its relative edge status, as illustrated in Fig. 4(b).
Character 2-c: If the binary groups constructed by the particular node and its parent nodes are unbalanced. Then the chain reaction will be over when the particular node alters its status, as seen in Fig. 4(c).
Character 3: Reversion of edge status can only transform an unbalanced binary group to a balanced one, but this transformation does not cause a chain reaction. The nodes and edges that are outside of the group would not be impacted.

4.2. Global structural balance in basic structures

Based on the above three characteristics, global structural balance is analyzed in the following basic structures.
Structure 1: A original global structural balanced network.

For an initial global structural balanced network, if the status of any node is reversed, the network will be global structural balanced again after the states of all the nodes are reversed.

Explanation 1: Any global structural balanced network can be reconstructed into a node, a global structural balanced network constructed by the rest of nodes, and an edge that links the two parts. We regard the global structural balanced network as a super node SN, thus any balanced network can be represented as Unit A or Unit B, as shown in Fig. 5(a) and (b). In Fig. 5(a), if the status of node i is reversed, according to $Character1$, the status of SN will keep the same. But from Eq. (5), we know that the status of the edge J_{SNi} will be altered. Due to $s_i * s_{SN} * J_{SNi} = 1$, then$(-s_i) * s_{SN} * (-J_{SNi}) = 1$. Therefore, the system will be balanced again based on Eq. (1). In Fig. 5(b), the structure of SN can be deconstructed again, as illustrated in Fig. 5(c) and (d). If SN is divided as the form of Unit A, as Fig. 5(c) shows, from $Character2 - b$, the status of SN will not be changed, and only the edge J_{iSN} will be altered, after that, the system will be balanced. But if SN has to be divided as the form of Unit B, the evolution will be more complicated. In Fig. 5(d), if node i changes its status, then node i' has to be altered to keep the local balance, which leads to a chain reaction. If Unit A is obtained in every process of deconstructions, the chain reaction will continue until the particular leaf-node is deconstructed base on $Character2$. Because every global balanced structure can be represented as combinations of Unit A and Unit B, and every balanced

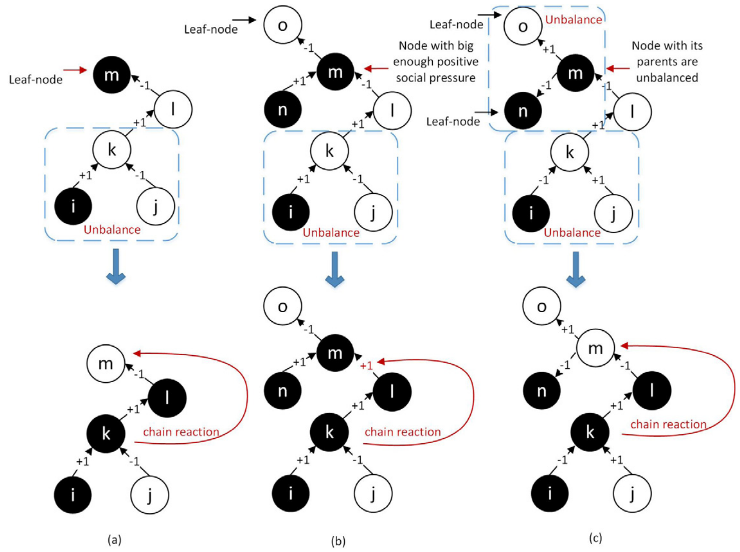


Fig. 4. Three types of particular nodes server as terminators for the chain reaction.

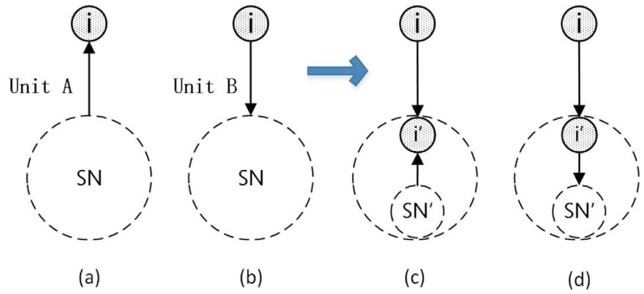


Fig. 5. Deconstruction of a global structural balanced network.

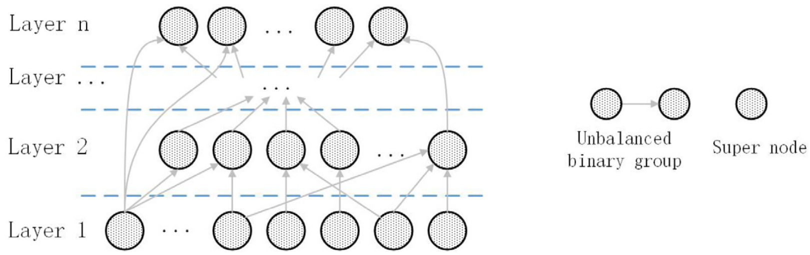


Fig. 6. A directed acyclic network is represented as a hierarchy structure.

chain has at least one leaf-node. Therefore, the chain reaction will be eventually finished, and the system will be recovered to global structural balance again.

In conclusion, the global structural balance of a structure will always be recovered if it used to be balanced. In the following study, our focus is the unbalanced structure. To simplify the research, each balanced structure is represented as a super node instead, and any network is recomposed of unbalanced binary groups.

Structure 2: A directed acyclic network.

For any directed acyclic network without circles in it, the global structural balance can always be reached eventually.

Explanation 2: Any directed acyclic network can be represented as a hierarchy structure, as shown in Fig. 6. In a hierarchy graph, the nodes on the first layer are root-nodes with zero in-degree. Then on the second layer are the nodes with zero in-degree when the nodes on the first layer are removed. By that analogy, all of the nodes can be put on their corresponding layers. The nodes on the top layer are leaf-nodes with zero out-degree. As *Character* 1 shows, the root-nodes will not change their status. Therefore, if the nodes are picked from layer2 to the top layer, all of the unbalanced binary

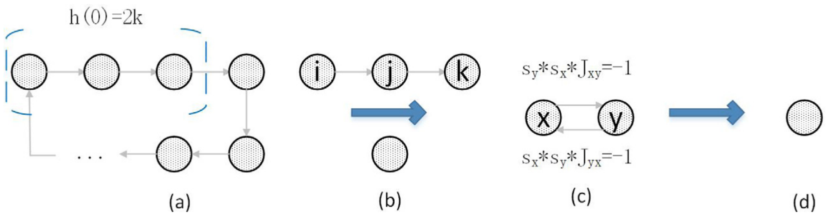


Fig. 7. The process of evolution in a directed cycle with $h_s(0) = 2k$.

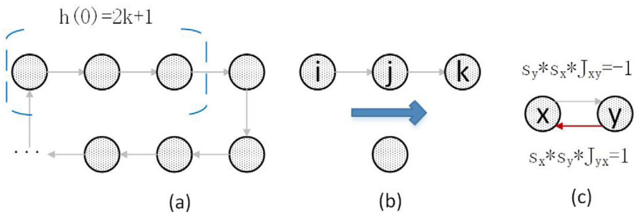


Fig. 8. The process of evolution in a directed cycle with $h_s(0) = 2k + 1$.

groups will be transformed to be balanced from down to up. By *Character*1 and *Character*$2 - c$, there is no chain reaction in this kinds of graphs, so it can be predicted that when the leaf-nodes on the top layer are iterated, the system is global structural balanced at the same time.

Structure 3: A directed cycle with $h_s(0) = 2k$, k is an integer.

For any directed cycle, if its initial global structural balance index is an even number, then the global structural balance will be eventually reached.

Explanation 3: In a directed cycle, any two linked binary groups can be represented as a chain with three nodes and two edges, as Fig. 7(b) shows. By the definition of unbalanced and balanced binary group, if $s_i*s_j*J_{ij} = -1$, $s_j*s_k*J_{jk} = -1$, then $s_i*(-s_j)*J_{ij} = 1$, $(-s_j)*s_k*J_{jk} = 1$. Therefore, the structure of two linked unbalanced binary group can be balanced as long as the middle node j changes its status. When a new balanced structure is obtained, we replace it with a super node. In this way, the number of nodes is decreased, as well as the edges. Because the number of unbalanced groups is even, in the end, the structure will be a small cycle with two nodes and two edges as Fig. 7(c) indicates. For $s_x * s_y * J_{xy} = -1$ and $s_y * s_x * J_{yx} = -1$, if one node changes its status, then the global structural balance will be obtained. In fact, if a node is randomly picked as the start-node, and the status of every other node is changed iteratively, then after k times, the whole cycle will be global structural balanced.

Structure 4: A directed cycle with $h_s(0) = 2k + 1$, k is an integer.

For any directed cycle without enough external social pressure, if its initial global structural balance index is an odd number, then the it will be unbalanced forever.

Explanation 4: With the same deconstructed method, a cycle with odd unbalanced binary groups will be simply represented as a cycle with an unbalanced binary group and a balanced binary group, as shown in Fig. 8(c). Formally, that is $s_x * s_y * J_{xy} = -1$ and $s_x * s_y * J_{yx} = 1$. According to Eq. (4), if node y changes its status, then $s_x * (-s_y) * J_{xy} = 1$ and $s_x * (-s_y) * J_{yx} = -1$. In the process of evolution, J_{xy} and J_{yx} cannot be altered, and $J_{xy} = -J_{yx}$, so $s_x * s_y * J_{xy}$ and $s_x * s_y * J_{yx}$ cannot be equal to one at the same time, thus the global structural balance will not be obtained forever.

Structure 5: A directed cycle with a bridge node

For any unbalanced cycle, if there is a bridge node pointing to one of the nodes in the cycle, the cycle will become global balance. A bridge node can be any node in a balanced global structure with a relatively big out-degree.

Explanation 5: In a complicated network with cycles, balanced and unbalanced structure are usually connected with each other. To break unbalance in Structure 4, a bridge node is designed as Fig. 9 illustrates. When a bridge node k is pointed to a random node j in the unbalanced cycle. According to Eqs. (2)–(4), as long as $D_k \geq D_i$ (i is the son-node of j, D_i is the out-degree of node i). Then the status of the node j will not change its status forever, and the chain reaction will be ended by altering the state of edge from node i to node j. Eventually, if the edge state is altered accordingly, the whole cycle will be global structural balanced.

5. Experimental results

For making an acyclic network, we conducted simulation experiments based on BA scale-free network [19]. Choosing this network lies in two aspects: On one hand, BA scale-free network has a good growth state, which allows us to restrict the directions of edges. In this paper, all of the edges are required to point to the new nodes. Thus, it is easy to analyze the global structural balance, and add designed cycles conveniently. On the other hand, the heterogeneity of BA scale-free

Table 2
Strategies for asynchronous operation of N nodes in each Round.

Strategies	Description
*Stra*1	Non-repetitive random order
*Stra*2	According to the order of layers of nodes
*Stra*3	Reverse order of *Stra*2
*Stra*4	In descending order of out-degrees of nodes
*Stra*5	Reverse order of *Stra*4

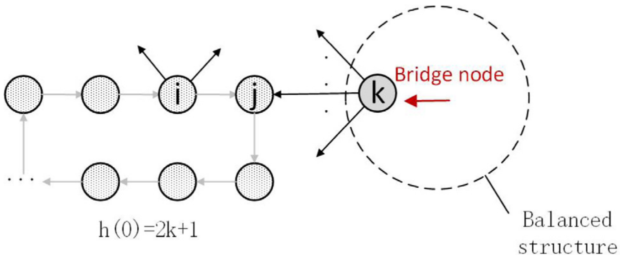
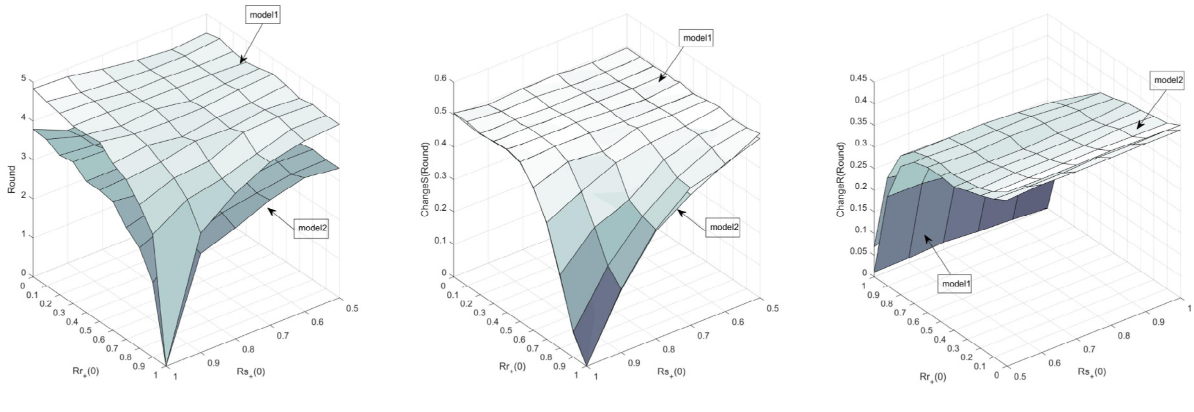


Fig. 9. A bridge node breaks the unbalanced cycle.



(a) (b) (c)

Fig. 10. (a) *Round* of model1 and model2, (b) *ChangeS*(*Round*) of model1 and model2, (c). *ChangeR*(*Round*) of model1 and model2.

network has an impact on the structural balance, which helps us to do deep analysis on the relations among network topology, dynamics, and structural balance.

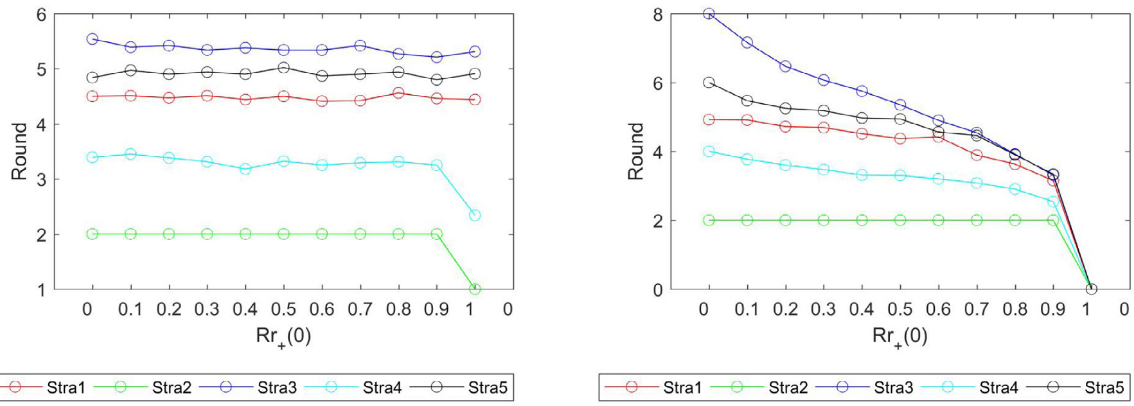
Considering both the growth and scale-free characteristics of BA scale-free network, five strategies are designed, as shown in Table 2. Experiments repeated 100 times with $N = 100$ and data averaged.

5.1. Intermediate state structure analysis of acyclic networks

Affected by neutral relationship and intermediate state structure, the rule of relational evolution is designed as Eq. (4). Assume that a relationship is changed immediately without any intermediate state when its structure is not balanced, the rule will be revised as Eq. (5). Evolutionary models of Eqs. (4) and (5) are labeled as *model*1 and *model*2 respectively. In the following experiments, similarities and differences between the two models are analyzed, such as the number of iterations(*Round*), the number of changed opinions(*ChangeS*), and the number of changed relations(*ChangeR*).

$$J_{ji}(t+1) = \begin{cases} J_{ji}(t), & if \quad J_{ji}(t) * s_i(t) * s_j(t) = 1 \\ -J_{ji}(t), & if \quad J_{ji}(t) * s_i(t) * s_j(t) = -1 \end{cases} \tag{5}$$

As shown in Fig. 10(a), *Round* of *model*1 is always higher than that of *model*2. This is natural because in *model*2, without a neutral status, the system evolve faster. It can be also noted that, similar in trend, both *Round* in *model*1 and *model*2 is equal to zero when the initial majority has dominated. The data is in line with reality. From Fig. 10(b), the final proportions of opinions of the two models are the same. In Fig. 10(c), it is obviously that the number of changed relationships of *model*2 is higher than that of *model*1. On the basis of this fact, we may draw the conclusion that, as the intermediate structure



(a) (b)

Fig. 11. (a) *Round* with increased $R_{r+}(0)$ when $R_{s+}(0) = 0.5$, $R_{r0}(0) = 0$, (b) *Round* with increased $R_{r+}(0)$ when $R_{s+}(0) = 1$, $R_{r0}(0) = 0$.

is introduced, fewer individuals will change their relationships. This is because individuals have to spend more time to communicate in the intermediate state, and consider how to properly handle the relationships with varied neighbors. In reality, reversing relationships is usually more difficult than just keeping it.

5.2. Global structural balance analysis of acyclic networks

The number of iterations (*Round*), the percentages of positive opinion ($R_{s+}(Round)$) and positive relationship ($R_{r+}(Round)$) at the steady state are calculated with five strategies above.

Compared with Fig. 11(a) and (b), it can be seen that *Round* in the five strategies is significantly different, but in the same order. According to the order of *Round* from low to high, the five strategies are *Stra*2, *Stra*4, *Stra*1, *Stra*5, and *Stra*3 respectively, which illustrates that the traversal strategies based on node properties are influential on the relaxation time. Among the strategies, *Stra*2 minimizes *Round* while *Stra*3 maximize it, *Stra*1 is in an intermediate position. Note that if the order of picked nodes is accordance with their layers, the global structural balance can be reached within two *Round*. Conversely, if the order is reversed, the relaxation time will be much longer. *Round* of *Stra*4 is between *Stra*1 and *Stra*2, which means that selecting nodes with big degrees preferentially is better than random selection, the former one can reduce the iteration time, but its effect is a little worse than *Stra*1. This results are accordance with some rumor-controlled strategies. For example, when conducting rumor regulation, it is usually best to trace the source of rumor but not to wield the individuals who have many fans.

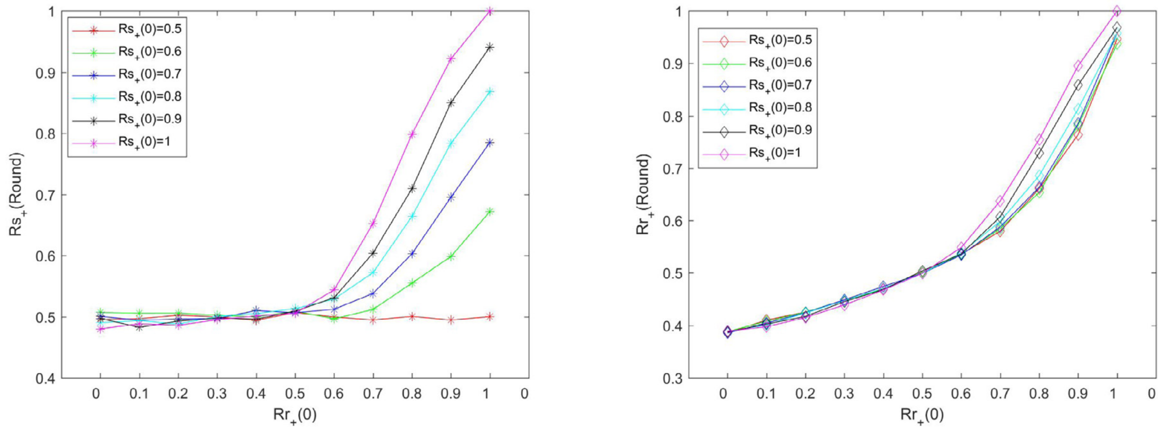
In Fig. 11(a), when the initial opinions are uniformly distributed, *Round* is substantially unchanged with varied $R_{r+}(0)$, except for *Stra*2 and *Stra*4 when $R_{r+}(0) = 1$. It can be seen from Fig. 11(b) that, when the group opinion is unified initially, *Round* of the five strategies decreases gradually as the percentage of positive relationships increases. When all of the individuals have the same opinion and positive relationships, *Round* is equal to zero.

As seen in Fig. 12(a), $R_{r+}(Round)$ fluctuates slightly around 0.5 when $R_{r+}(0) \leq 0.5$. However, it rises as $R_{r+}(0)$ goes up, when $R_{r+}(0) > 0.5$. The growth rate of $R_{s+}(Round)$ increases with the growth of $R_{s+}(0)$. Meanwhile, it can be noted that, as long as $R_{r+}(0) \neq 1$, the consensus cannot be reached, which is accordance with Ref. [16]. In Fig. 12(b), $R_{r+}(Round)$ grows as $R_{r+}(0)$ increases. When $R_{s+}(0)$ arises, the growth rate of $R_{r+}(Round)$ increases slightly. Furthermore, $R_{r+}(Round)$ approaches 40% when $R_{r+}(0) = 0$, which illustrates the proportion of positive relationships is essential to maintain a global structural balance.

5.3. Global structural balance analysis of networks with cycles

In order to explore the actual effect of an unbalanced cycle, a binary group is selected randomly in the BA Scale-free network, and a reversed-edge (edge pointing from a higher layer to a lower layer) is added. Fig. 13 illustrates an example of an initialized signed network with a hierarchy structure and a designed reversed edge. With $R_{s+}(0) = 1$ and $R_{r+}(0) = 0$, two kinds of different results arise. One is the final global structural balance, another is the unbalanced cycle evolves permanently in circulation. The orders of picked nodes are followed by the five strategies before the reversed edge is added.

From Fig. 14(a), it can be seen that after a few *Round*, all of the nodes reverse their status. This is because of that the designed reversed edge is pointed to the root-node on the first layer. Therefore, when the root-node changes its status, the chain reaction which is related to the whole system occurs. Fig. 14(b) illustrates the dynamic processes of the changed edges, all of the edges keep their status after a few *Round*. In *Stra*2, *ChangeR*(*Round*) arrives its maximum value in the first



(a) (b)

Fig. 12. (a) $R_{s+}(Round)$ with increased $R_{r+}(0)$ and varied $R_{s+}(0)$, (b) $R_{r+}(Round)$ with increased $R_{r+}(0)$ and varied $R_{s+}(0)$.

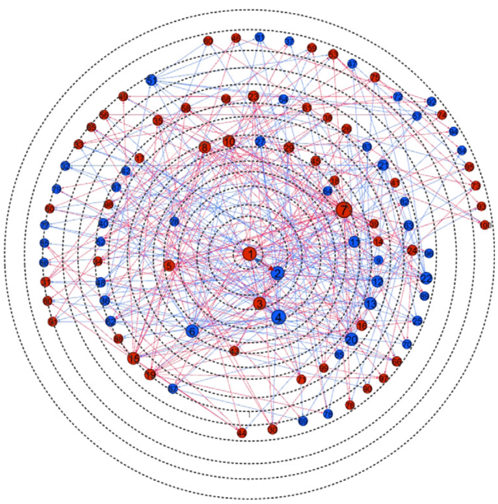


Fig. 13. Initialization of a directed signed network by introducing a reversed edge(when the reversed edge from node 2 to node 1 is added, the binary groups of node 1 and node 2 make up an unbalanced cycle).

Round, and keeps the same value in the second *Round*. The reason is that in the first *Round*, all of the status of the edges which should be revised have been altered to the neutral states, in the second *Round*, these altered edges change their status, then no edge status can be altered in the following time. When the status of edges cannot be altered, only the status of nodes can be changed to carry on the evolution. So when $ChangeR(Round)$ reaches zero for the first time, the *Round* order of the five strategies is consistent with the order at which $ChangeS(Round) = 100\%$. Fig. 14(c) shows how $h_s(Round)$ changes. The order of stable $h_s(Round)$ from low to high is as the same as which in Fig. 14(a) and (b). With *Stra*2, $h_s(Round)$ comes down to one at *Round*2, which means all of the binary groups are balanced except one in the cycle. When the order of picked node is reversed, see *Stra*3, $h_s(Round)$ remains unchanged at the value of 323, which is the number of binary groups minus one, that is, only one binary group is balanced. Similarly, when $h_s(Round)$ remains constant, the sum of $h_s(Round)$ in *Stra*4 and in *Stra*5 is equal to the number of binary groups too. Fig. 14(d) displays the proportion of the neutral relations in the process of evolution. It is shown that all of the neutral relations are temporary. In *Stra*3 and *Stra*5, the proportion of the neutral relations increases at first but then drops, while in the other three strategies, $R_{r0}(Round)$ decreases quickly.

Different cycles with different number of unbalanced binary groups are also tested, the results are accordance with which are presented in Section4. Interestingly, it is found that there is a periodic fluctuation in the evolution of $h_s(Round)$, as shown in Fig. 15(a). By analyzing the evolution system, any system in this model will repeat its state every other *Round*, when the nodes are selected with the same random order in each *Round*. Moreover, when a particular cycle occurs, the cyclic variation happens. Fig. 15(b) shows one example of a network with a cycle. In Fig. 15(b), when the state in *Round*1 happens, all of the status of the nodes are reversed in the following every other *Round*, and $h_s(Round)$ changes periodicity.

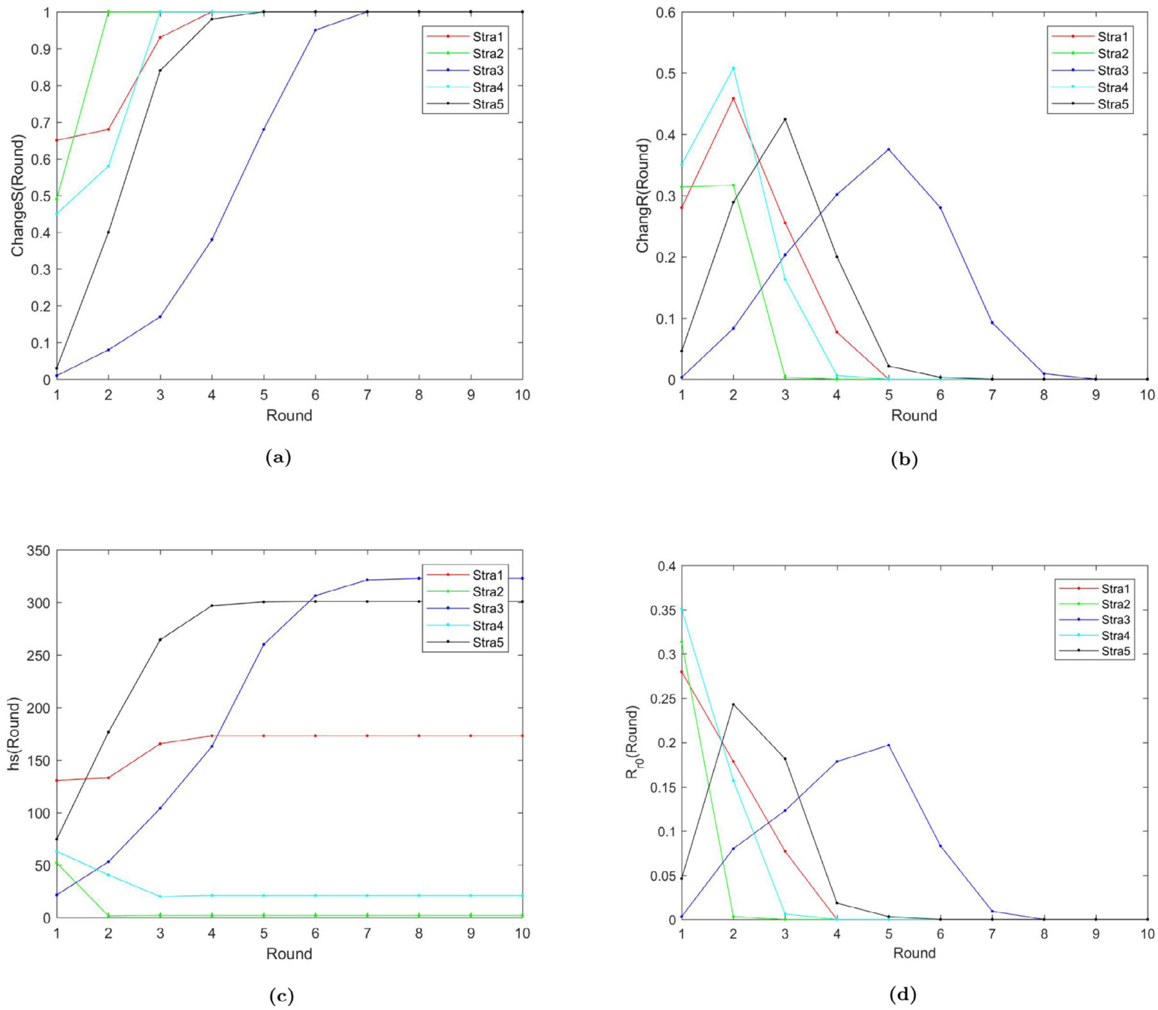


Fig. 14. The dynamic processes of four parameters. (a) *ChangeS*(*Round*) changes with time, (b) *ChangeR*(*Round*) changes with time, (c) $h_s(Round)$ changes with time, (d) $R_{r0}(Round)$ changes with time.

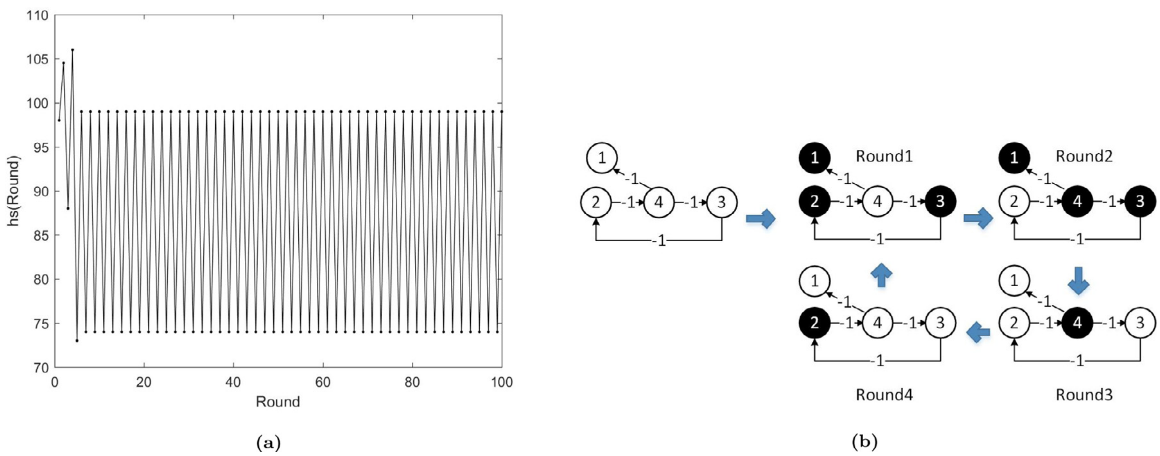


Fig. 15. (a) Periodic fluctuation of $h_s(Round)$, (b) A example for explaining the periodic fluctuation of $h_s(Round)$.

6. Conclusion

To conclude this paper, three types of the binary groups are defined, as balanced, unbalanced, and intermediate structure. Global structural balance of four kinds of basic networks are analyzed. They are acyclic networks, directed cycles with $h_s(0) = 2k$, $h_s(0) = 2k + 1$ and a bridge node. The results show that the global balance can be achieved in many networks. In simulation experiments, the evolution process of opinions and relations under the cumulative social pressure are further studied. Experimental results show that with the introduction of intermediate state, the probability of relational conversion is reduced effectively. Results also illustrate that, as long as there is an edge of negative relationship, the public opinions will be polarized eventually. With varied strategies of picking nodes, *Round* is quite different. Specifically, when the order of picking nodes is line with their layer, *Round* will reach the minimum. In addition, the index of global structural balance is analyzed. By introducing an unbalanced cycle, two different results are get. In one case, the global structural balance is reached. In another case, all of the states of nodes will be reversed in each *Round* or every other *Round*, and the global structural balance index can be consistent and fluctuate over time, but cannot be zero.

In a real network, the status of edges between people can be computed according to the historical posts over a long period of time, and the status of a node can be considered as one's opinion to a specified topic. For a signed network with some given states of the root-nodes, the topic-opinion of other people can be forecasted quickly by our model with *Stra*2. Besides that, the future status of the edges can be predicted by computing the cumulative social pressure. Our future research will focus on the application of the model, especially its accuracy and efficiency of prediction on people's relations and topic-opinions.

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