REPORT
ON
ASSIGNMENT 1

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Contents

1 Abstract

This report answer questions of the research and study on the assignment 1 which is the numerical analysis of the $x^{\frac{1}{3}}$. Each Question answer section consist of 2 part consist of 2 part. First part is the use of bisection method and the second part is the use of Newton's method. And the conclusion is the comparison of the two method.

2 Question 1 (my strategy)

To solve the problem. I need to define a recursive function to get the better and better precision. And a end condition predicate to stop the recursive call. The end condition is the same, so I need to choose 2 different kind of recursive method to approximate the real value.

2.1 Bisection Method

This method come into my mind cause we learn how to solve the square root in the tutorial. So I came about a similar idea to carry out this method.

This is the mathematics is topology like a binary search tree. Trying to narrow the interval of the answer area by repeatedly bisecting and comparing.

2.2 Newton's Method

And this one is what we learn in the year 2 math class. Chapter numerical method. We use Tyler series to drive the convergence rate of this method.

In numerical analysis, Newton's method (also known as the Newton–Raphson method), named after Isaac Newton and Joseph Raphson, is a method for finding successively better approximations to the roots (or zeroes) of a real-valued function. It is one example of a root-finding algorithm.

3 Question 2

let $x^{\frac{1}{3}}$ = t, so the questions is equivalent to $y = x^3 - t$

3.1 Bisection Method

In bisecting method the upper bound and the lower bound with define in Haskell type class Range, and we have 5 conditions and we have to define the upper and lower bounder separately.

The range can be classified into 5 sub- range:($-\infty$,-1),[-1,0),[0],(0,1],(1,∞^+)

Since it is symmetric, let us talk about the positive range of it.

If x $>$ 1, the lower border is obvious to be 1, and the upper range is $\frac{(x+1)}{2}$.

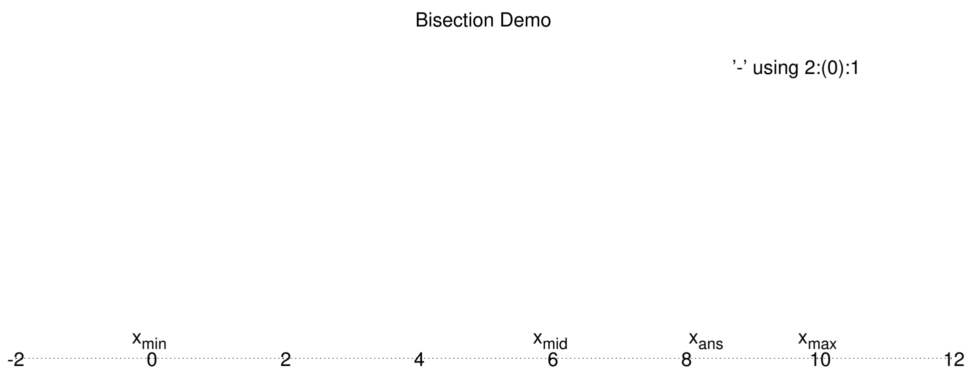
$$(\frac{(x+1)}{2})^3/x = \frac{x^3+3(x^2)+3x+1}{8}/x = \frac{x^3+3(x^2)+3x+1}{8x}$$

since $x^3 >= 1, x^2 >= 1, x >= 1$, so $\frac{x^3+3(x^2)+3x+1}{8x} >= 0$, so max border is upper than x_{ans}.

If $0 < x <= 1$, the lower bounder is obvious to be 0, and the upper range is x since $x < 1$,so $x^3 < x$.

And x $=$ 0 is obvious to be zero.

While in all range, we narrow our range to be half, so the answer series is converge.



3.2 Newton's Method

The Newton–Raphson method in one variable is implemented as follows:

The method starts with a function f defined over the real numbers x, the function's derivative f, and an initial guess x_0 for a root of the function f.

If the function satisfies the assumptions made in the derivation of the formula and the initial guess is close, then a better approximation x_1 is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Geometrically, (x_1, 0) is the intersection of the x-axis and the tangent of the graph of f at (x_0, $f(x_0)$). The process is repeated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For the correctness of Newton's Method all we need to ensure is 2 part:

$$f'x = 3x^2$$

So when x = $0, f'x = 0$, I use pattern matching to just return the 0 value to avoid the NaN happens.

$$f''x = 6x$$

So it is a monotone increasing function. The tangent line and x-xies is converge. That prove the newton's method is always correct in my code.

4 Question 3

4.1 Bisection Method

We can analysis this method by complexity and convergence rate.

4.1.1 Complexity

When the precision is fixed. And the value of n become infinity. The complexity of the bisection method is $log_2(x)$

4.1.2 Convergence Rate

When the number of precision digit is fixed we can see that:

$$\frac{precision - x_{n+1}}{precision - x_n} = \frac{1}{2}$$

So the convergence rate of bisection method is linear to be n.

4.2 Newton's Method

4.2.1 Complexity

When the precision is fixed. And the value of n become infinity.It is hard to define the complexity of the newton's Method. Because newton's method is greatly affected by the initial guess value. So we can not find the worst case. Cause if you define a worst case initial value $\mathrm{x_{worst}}$ there must exist a even worst value:

$$x_{worst} = x_{evenworst} - \frac{f(x_{evenworst})}{f'(x_{evenworst})}$$

So it is a np problem. The complexity of the Newton's is $\log_2(\mathrm{x})$.

4.2.2 Convergence Rate

By use of Tyler series we can easily prove.

$$f() = f(x_n) + f'(x_n)(\alpha - x_n) + R_1$$

$$R_1 - \frac{1}{2!}f''(\beta_n)(\alpha - x_n)$$

since α is the root, So we have:

$$0 = f(\alpha) = f(x_n) + f'(x_n)(\alpha - x_n) + \frac{1}{2}f''(\beta_n)(\alpha - x_n)^2$$

$$\underbrace{\alpha - x_{n+1}}_{\beta_{n+1}} = \frac{-f''(\beta)}{2f'(x_n)}(\alpha - x_n)^2$$

Therefor $error_{n+1} = error_n^2$ So the convergence rate of newton's method is quadratic.

5 Conclusion

Even the complexity of the 2 method is nearly the same. While the newton's method have the higher convergence rate. So I suggest to use the newton's method.