Multivariable Functions Multivariable Calculus X. Du

- Example: $f(x,y,z)$, $f(x_1,x_2,...x_n)$
- **Level Set**: $f(x_1,x_2,...x_n)=k$
  - **Level Curve**: $f(x,y)=k$
    - **Contour map**: Shows $f(x,y)=k$ for successive values of *k*.
  - **Level Surface**: $f(x,y,z)=k$
  - The level set of a function of *n* free variables $f(x_1,x_2,...x_n)$ has $n-1$ dimensions
- Formal definition of a **limit** of a function of two variables: Let $f(x,y)$ be a function defined throughout an open disk about (a,b) except possibly at (a,b) itself.
  Then, $\lim_{(x,y)\to(a,b)} f(x,y)=L$ iff for every $\varepsilon>0$, there is some number $\delta>0$ such that if $0<\sqrt{(x-a)^2+(y-b)^2}<\delta$, then $\left|f(x,y)-L\right|<\varepsilon$.
- Formal definition of a **limit** of a multivariable function: Let $f(x_1,x_2,...x_n)$ be a function defined throughout an open *n*-disk about $(a_1,a_2,...a_n)$ in *n*-space except possibly at $(a_1,a_2,...a_n)$ itself. Then, $\lim_{(x_1,x_2,...x_n)\to(a_1,a_2,...a_n)} f(x_1,x_2,...x_n)=L$ iff for every $\varepsilon>0$, there is some number $\delta>0$ such that if $0<\sqrt{(x_1-a_1)^2+(x_2-a_2)^2+...(x_n-a_n)^2}<\delta$, then $\left|f(x_1,x_2,...x_n)-L\right|<\varepsilon$.
- **Squeeze Theorem**: Let $\left|f(x_1,x_2,...x_n)-L\right|\le g(x_1,x_2,...x_n)$ for all $(x_1,x_2,...x_n)$ in a region about $(a_1,a_2,...a_n)$ except possibly $(a_1,a_2,...a_n)$ itself. If $\lim_{(x_1,x_2,...x_n)\to(a_1,a_2,...a_n)} g(x_1,x_2,...x_n)=0$, then $\lim_{(x_1,x_2,...x_n)\to(a_1,a_2,...a_n)} f(x_1,x_2,...x_n)=L$.
- **Continuity**: Suppose $f(x_1,x_2,...x_n)$ is defined throughout an open *n*-disk about $(a_1,a_2,...a_n)$. Then $f(x_1,x_2,...x_n)$ is continuous at $(a_1,a_2,...a_n)$ iff
  $\lim_{(x_1,x_2,...x_n)\to(a_1,a_2,...a_n)} f(x_1,x_2,...x_n)=f(a_1,a_2,...a_n)$
- **Composition Limit Rule**: If $f(x_1,x_2,...x_n)$ is continuous at $(a_1,a_2,...a_n)$ and $g(t)$ is continuous at $f(a_1,a_2,...a_n)$, then $g(f(x_1,x_2,...x_n))$ is continuous at $(a_1,a_2,...a_n)$.