**Series and Sequences** Calculus of a Single Variable X. Du

- Sequences
  - Convergence: $\lim_{n\to\infty} a_n$ exists
  - Divergence: $\lim_{n\to\infty} a_n$ does not exist or is infinite
  - Use Squeeze Theorem if necessary
  - Monotonic sequences - always either increasing or decreasing
- Series
  - Convergence $\Rightarrow \sum_{n=1}^{\infty} a_n$ is finite.
  - Divergence $\Rightarrow \sum_{n=1}^{\infty} a_n$ does not exist or is infinite.
- Specific Types of Series and Convergence and Divergence
  - Geometric - $\sum_{n=0}^{\infty} ar^n$
    - $|r| < 1 \Rightarrow$ converges
    - $|r| > 1 \Rightarrow$ diverges
  - Telescoping - $\sum_{n=1}^{\infty} (b_n - b_{n+1})$
    - $\lim_{n\to\infty} b_n = L \Rightarrow$ converges
  - *p*-series - $\sum_{n=1}^{\infty} \frac{1}{n^p}$
    - $p > 1 \Rightarrow$ converges $\quad 0 < p \le 1 \Rightarrow$ diverges
  - Alternating - $\sum_{n=1}^{\infty} (-1)^n a_n$
    - $\lim_{n\to\infty} a_n = 0$ and $0 < |a_{n+1}| \le |a_n| \Rightarrow$ converges