Vectors Multivariable Calculus X. Du

- Have **magnitude** and **direction**.
- Contrast to scalar quantities
- **Magnitude**: $|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}} = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2 + ... + a_n{}^2}$ Pythagorean Theorem
  - **Unit vectors**: have magnitude of 1
  - **Standard basis unit vectors** in three-space: $\hat{i} = \langle 1, 0, 0\rangle$, $\hat{j} = \langle 0, 1, 0\rangle$, $\hat{k} = \langle 0, 0, 1\rangle$
- **Zero Vector**: vector of all 0's. $\vec{0} = \langle 0, 0, \ldots, 0\rangle$.
- Vector arithmetic:
  - Let $\vec{a} = \langle a_1, a_2, a_3, ..., a_n\rangle$ and $\vec{b} = \langle b_1, b_2, b_3, ..., b_n\rangle$ in *n*-space.
  - $\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3, ..., a_n + b_n\rangle$
  - $c\vec{a} = \langle ca_1, ca_2, ca_3, ..., ca_n\rangle$
- **Dot product**:
  - Let $\vec{a} = \langle a_1, a_2, a_3, ..., a_n\rangle$ and $\vec{b} = \langle b_1, b_2, b_3, ..., b_n\rangle$.
  - $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3 + ... + a_nb_n$
  - Scalar!
  - $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ Proof with Law of Cosines
- **Cross product**:
  - Only valid in three-space: Let $\vec{a} = \langle a_1, a_2, a_3\rangle$ and $\vec{b} = \langle b_1, b_2, b_3\rangle$.
  - $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1\rangle$
  - Vector! Direction - use right hand rule.
  - $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$
- $\vec{a}$ and $\vec{b}$ are perpendicular iff $\vec{a}\cdot\vec{b} = 0$
- $\vec{a}$ and $\vec{b}$ are parallel iff $\vec{a}\times\vec{b} = \vec{0}$
- **Projections**:
  - $comp_{\vec{a}}\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$ Scalar!
  - $proj_{\vec{a}}\vec{b} = \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\left(\dfrac{\vec{a}}{|\vec{a}|}\right)$ Vector! - $\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)$ gives magnitude, $\left(\dfrac{\vec{a}}{|\vec{a}|}\right)$ gives direction
  - $proj_{\vec{a}}\vec{b} + proj_{\vec{a}}^{\perp}\vec{b} = \vec{b}$

Further notes:

- **Tensors**: extension of vectors. Scalars are 0-tensors (no direction), and vectors are 1-tensors (1 direction). An *n*-tensor has *n* directions.