1st-Order Linear Non-Homogeneous Systems of ODEs Differential Equations X. Du

- Example (first-order): $\vec{x}' = A\vec{x} + \vec{r}(t)$
- Theorem: Let $\vec{x}_c$ be the general solution to the associated homogeneous system $\vec{x}' = A\vec{x}$, and let $\vec{x}_p$ be any particular solution to $\vec{x}' = A\vec{x} + \vec{r}(t)$. Then the general solution of $\vec{x}' = A\vec{x} + \vec{r}(t)$ is $\vec{x} = \vec{x}_c + \vec{x}_p$.
  - Note the similarities to the previous theorem regarding non-homogeneous ODEs.
  - There are two main methods to find particular solutions for systems: 1) method of undetermined coefficients, 2) variation of parameters
- **Method of Undetermined Coefficients**: Guess something with similar structure to $\vec{r}(t)$, and then solve with a system of equations.
  - However, this doesn't always work. In this case, keep on adding a linear factor of *t* until something works.
- **Variation of Parameters**: Let Ψ represent the fundamental matrix of the associated homogeneous system. Then $\vec{x}_p = \Psi \int \Psi^{-1} \vec{r}\, dt$