

- A set of vectors $S = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is **linearly independent** iff

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n = \vec{0} \Rightarrow c_1 = c_2 = \dots = c_n = 0$$
 - Otherwise, S is **linearly dependent**
 - Subsets of linearly independent sets are linearly independent
 - Supersets of linearly dependent sets are linearly dependent
- A vector \vec{v} is a **linear combination** of $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ iff $\exists c_1, c_2, \dots, c_n : \vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n$
 - In a linearly dependent set of vectors, there exists a vector that can be expressed as a linear combination of the other vectors
- If $\vec{0} \in S$, then S is linearly dependent
- For $S \subseteq \mathbb{R}^n$ such that $|S| = n$, S is linearly independent iff $\det(A) \neq 0$, where A is the matrix constructed of the vectors of S as either rows or columns.
- If you put a set of vectors as columns in a matrix and then perform Gaussian elimination, then the columns with pivots correspond to a linearly independent subset
- Linear independence of functions
 - **Wronskian Determinant.** Suppose there is a set of n functions $\{y_1, y_2, \dots, y_n\}$, each of which are $n-1$ times differentiable on some interval $I \subseteq \mathbb{R}$. The **Wronskian Determinant** of the functions in this set is:

$$W(y_1, y_2, \dots, y_n) = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \vdots & \vdots & \ddots & \vdots \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}$$

- If $W(x_0) \neq 0$ for some $x_0 \in I$, then $\{y_1, y_2, \dots, y_n\}$ is linearly independent on I .
- If $\{y_1, y_2, \dots, y_n\}$ is linearly dependent on I , then $W(x) = 0$ for all $x \in I$.