Series: Convergence and Divergence Tests Calculus of a Single Variable X. Du

- ***n*^th^ term** - $\lim_{n\to\infty} a_n \neq 0 \Rightarrow$ diverges
- **Integral** - Given a series $\sum_{n=1}^{\infty} a_n$ and $a_n = f(x)$
  - $\int_1^{\infty} f(x)dx$ diverges $\Rightarrow \sum_{n=1}^{\infty} a_n$ diverges
  - $\int_1^{\infty} f(x)dx$ converges $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges
- **Root** - $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1 \Rightarrow$ convergence $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1 \Rightarrow$ divergence
  - $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1 \Rightarrow$ inconclusive
- **Ratio** - $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1 \Rightarrow$ convergence $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1 \Rightarrow$ divergence
  - $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1 \Rightarrow$ inconclusive
- **Direct Comparison**
  - $0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges.
  - $0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges $\Rightarrow \sum_{n=1}^{\infty} a_n$ diverges.
- **Limit Comparison**
  - $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ and $\sum_{n=1}^{\infty} b_n$ converges $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges.
  - $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ and $\sum_{n=1}^{\infty} b_n$ diverges $\Rightarrow \sum_{n=1}^{\infty} a_n$ diverges.