$$I_C(t) + I_J(t) = I(z=0,t) \tag{1}$$

$$I_C = \frac{d(CV)}{dt} = C\frac{d^2\Phi}{dt^2} = C\varphi_0\frac{d^2\delta(t)}{dt^2} \tag{2}$$

$$\delta = \Phi/\varphi_0 \tag{3}$$

$$\varphi_0 = \hbar/2e \tag{4}$$

$$I_J = I_0\sin(\delta(t)) \tag{5}$$

$$I(z=0,t) = I_{in}(t) - I_{out}(t) \tag{6}$$

$$V(z=0,t) = V_{out}(t) + V_{in}(t) = (I_{in}(t) + I_{out}(t))Z_c \tag{7}$$

$$I(z=0,t) = 2I_{in}(t) - V(z=0,t)/Z_c = 2I_{in}(t) - \frac{\varphi_0}{Z_c}\frac{d\delta(t)}{dt} \tag{8}$$

$$I_{in}(t) = I_p\cos(\omega_p t) \tag{9}$$

$$C\varphi_0\frac{d^2\delta(t)}{dt^2} + \frac{\varphi_0}{Z_c}\frac{d\delta(t)}{dt} + I_0\sin(\delta(t)) = 2I_{in}(t) \tag{10}$$

$$\frac{d^2\delta(t)}{dt^2} + 2\gamma\frac{d\delta(t)}{dt} + \omega_0^2(\delta(t) - \frac{\delta^3(t)}{6}) = \frac{2I_p}{C\varphi_0}\cos(\omega_p t) \tag{11}$$

$$\gamma = \frac{1}{2Z_cC} \tag{12}$$

$$\omega_0^2 = \frac{I_0}{C\varphi_0} \tag{13}$$

$$\sin(\delta(t)) \approx \delta(t) - \frac{\delta^3(t)}{6} \tag{14}$$

$$\tag{15}$$