Tentative Classification of Methods and Bibliography on Solving Systems of Linear Equations

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在数值分析领域, 乔治 · 埃尔默 · 福赛斯（George Elmer Forsythe, 1917-1972）是和威尔金森 (James Hardy Wilkinson, 1919-1986) 齐名的奠基人之一. 他是斯坦福大学计算机科学系的创始人兼负责人, 是美国早期倡导 “计算机科学” 的领导人之一. 是他将高德纳 (Donald Knuth) 引进了斯坦福大学计算机科学系. 他培养了许多如今居学术领导地位的博士生, 他的关门弟子 Cleve Moler (在他去世前还没有毕业) 是 MATLAB 语言的创立者.

Tentative Classification of Methods and Bibliography on Solving Systems of Linear Equations 是 1953 年美国国家标准局举办的数值分析专题研讨会上的主题论文. 作者试图对 $Ax = b$ 的各种迭代解法做一个分类, 并对当时已经非常庞大的文献做一个梳理. 大半个世纪过去了, 数值分析领域已经发生了突飞猛进的发展, 但这篇论文如今仍然具有一定的参考意义. 这里把它的前三节重新排版出来 (略去文献部分的 IV, V 两节), 以供学习参考. 虽然编者经过仔细的校对, 但排版的过程中难免存在错漏, 所以把原文影印件也附在后面, 供读者对比查阅.

关于迭代法的最新进展和历史回顾, 可以参考 2019 年 Yousef Saad 的短文*Iterative methods for linear systems of equations: A brief historical journey*.

— 编者 (5070319@qq.com)

2020.01.06 — 2020.01.07

1 Introduction

The scope of this article is indicated in the introductions to §§2, 4, and especially §5. In the same places will be found acknowledgments to some of the persons who have helped build the bibliography.

This article is a collection of working notes, and not a polished bibliography. Because there is apparently nothing comparable in the literature, it seems better to publish it as it is than to wait for a state of completion which might never arrive. Although many of the references in §5 have been checked in the course of the preparation of the manuscript, time has not permitted a systematic editing and search for errors[1]. If readers will send in lists of errors and omissions, it may later be possible to issue a corrected edition of this article. Please address the compiler at the National Bureau of Standards, 405 Hilgard Avenue, Los Angeles 24, Calif.

2 Basis of the Tentative Classification; Symbols Used in the Outline

We are here dealing with methods for getting the solution $A^{-1}b$ of a system of linear equations $Ax = b$, where A is a nonsingular square matrix of order n and b is a column vector. We are equally concerned with methods for obtaining the inverse matrix A^{-1}; where such methods simply parallel those for getting $A^{-1}b$, they will not ordinarily be mentioned. Since the only known precedent for a classification of methods, Jensen 1944[2], covers but a fraction of the methods considered here, it was necessary to devise some basis for the outline of §3. The basis was developed after conversations with Dr. Theodore S. Motzkin, and is regarded as only tentative.

Consider, for the moment, an iterative process for getting $A^{-1}b$. Starting from b, A, and some initial vector $\xi^{(0)}$, a set of arithmetic operations is prescribed, which, if carried out without round-off or other error, will yield a new vector $\xi^{(1)} : \xi^{(1)} = F_1(b, A; \xi^{(0)})$. Similarly, $\xi^{(2)}$ is obtained by another set of operations, carried out without error. In general, $\xi^{(k)}$ is obtained from $b, A, \xi^{(k-1)}, \xi^{(k-2)}, \cdots, \xi^{(0)}$ by applying a set of exact arithmetic operations which we assume to be uniquely described by a formula of type

$$\xi^{(k)} = F_k(b, A; \xi^{(k-1)}, \xi^{(k-2)}, \cdots, \xi^{(0)}).$$

In this manner there is defined a theoretically determined sequence of vectors

[1]In reading the galley proofs, the non-bracketed parts of the entries in §5 have been corrected against the original papers or their reviews In MR, Zb, Fs, or SA (see §5), except for the perhaps 15 to 20 titles neither contained nor reviewed In the NBS Institute for Numerical Analysis.

[2]Names in capitals refer to the bibliography in §5.

$$\xi^{(0)}, \xi^{(1)}, \xi^{(2)}, \cdots, \xi^{(k)}, \cdots.$$

However, round-off errors occur in almost all numerical processes (there are exceptions). As a cumulative result of these errors, one actually obtains a vecctor $x^{(1)}$ instead of $\xi^{(1)}$. In the next step, one ordinarily uses b, A, and $x^{(1)}$ in the algorithm designed to compute $\xi^{(2)}$. Suppose that $\eta^{(2)}$ would be the result of errorless application of the algorithm:

$$\eta^{(2)} = F_2(b, A; x^{(1)}, x^{(0)}),$$

where $x^{(0)} = \xi^{(0)}$. Because of round-off errors, one really obtains not $\eta^{(2)}$, but another uniquely defined vector $x^{(2)}$:

$$x^{(2)} = \hat{F}_2(b, A; x^{(1)}, x^{(0)}).$$

Here $\hat{F}_2$ is the function F_2, altered in accord with each of the round-off errors entering the calculation.

In this manner will be determined a theorectical sequence of vectors

$$\eta^{(0)}(= \xi^{(0)}), \eta^{(1)}, \eta^{(2)}, \cdots, \eta^{(k)}, \cdots,$$

and a practical sequence

$$x^{(0)}(= \xi^{(0)}), x^{(1)}, x^{(2)}, \cdots, x^{(k)}, \cdots.$$

Each vector $x^{(k)}$ is determined from its precedessors by the "rounded-off function" $\hat{F}_k$:

$$x^{(k)} = \hat{F}_k(b, A; x^{(k-1)}, x^{(k-2)}, \cdots, x^{(0)}).$$

Each vector $\eta^{(k)}$ is the result of errorless application of the theorectical function F_k to the "contaminated" sequence $x^{(0)}, \cdots, x^{(k-1)}$:

$$\eta^{(k)} = F_k(b, A; x^{(k-1)}, x^{(k-2)}, \cdots, x^{(0)}).$$

We thus have three sequences in mind: (1) the sequence $\{\xi^{(k)}\}$ of vectors free from round-off error in all generations (it is these theoretical vectors with which convergence proofs generally deal), (2) the sequence $\{x^{(k)}\}$ of vectors which the computer actually obtains while making the prescribed round-off errors, and (3) the sequence

$\{\eta^{(k)}\}$ of vectors which have one "generation" of errorless calculation, based on the computed vectors $x^{(0)}, \cdots, x^{(k-1)}$.

We have supposed heretofore that we deal with an iterative process, but the same analysis applies also to a direct process — for example, elimination. The result of an entire elimination solution is theoretically the vector $\xi^{(1)} = A^{-1}b$. In practice one makes round-off errors and obtains $x^{(1)}$ instead. Since $b - Ax^{(1)} = r^{(1)} \neq 0$, it is customary to solve the equation $A(\eta^{(2)} - x^{(1)}) = r^{(1)}$ for the correction to be added to $x^{(1)}$ to obtain $\eta^{(2)}$. Here $\eta^{(2)} = A^{-1}b$, but round-off errors cause the approximate solution $x^{(2)}$ to be obtained instead, etc. Thus the earlier model describes also the "direct" solution of the system $Ax = b$ whenever round-off errors are made in the actual solution.

The present classification of methods is based on two trichotomies.

First trichotomy: The sequence $\{\eta^{(k)}\}$ is a priori bound to be of one of the following three types:

I). All $\eta^{(k)}$ are necessarily equal to $A^{-1}b$, i.e., all are exact.

II). Some $\eta^{(k)}$, but not all, are necessarily equal to $A^{-1}b$.

III). No $\eta^{(k)}$ is equal to $A^{-1}b$, except for special choices of $\eta^{(0)}, b, A$.

Typical processes of these three types are I, the elimination process of GAUSS 1826; II, the finite iteration of LANCZOS 1951; and III, the infinite iteration of SEIDEL 1874.

Second trichotomy: The sequence $\{F_k\}$ of functions is bound a priori to fall into one of the following three classes:

α). All functions F_k have in fact the same number of arguments and are identical, i.e., there exists an integer, r, such that

$$(*) \qquad F_k(b, A; x^{(k-1)}, \cdots, x^{(0)}) \equiv F(b, A; x^{(k-1), \cdots, x^{(k-r)}}) \text{ (all } k).$$

Such processes are called *stationary*.

β). There are finite number (greater than one) of distinct functions F_k, each of type $(*)$. Such processes are called *partly stationary*.

γ). There are an infinite number of distinct functions F_k. Such processes are called *nonstationary*.

Typical processes of these types are α, (iterated) elimination; β, iterated elimination, occasionally interrupted by an acceleration procedure; γ, weighted averages of the successive iterates of Seidel's process, with weights equal to the coefficients of the Chebyshev polynomials of order $1, 2, 3, \cdots$ (Divisions between processes of types β and γ could be made in many different ways.)

The double trichotomy gives us a priori a nine-way classification of methods for getting $A^{-1}b$ (or indeed of estimating any quantity by a numerical process). Three of the possible types Iβ, Iγ, and IIβ, do not seem to occur in the literature on getting $A^{-1}b$, but the other six form a convenient structure on which to hang an outline of methods of solving linear equations:

	α Stationary	β Partly stationary	γ Nonstationary
I. All η^k exact	Iα		
II. Some η^k exact	IIα	IIβ	
III. No η^k exact	IIIα	IIIβ	IIIγ

It is not presumed that this particular nine-way classification is necessarily a "good" one; it merely provides one of many possible frameworks on which to spread out the known solution processes.

The outline in §3 serves two purposes: (1) It contains a tentative classification of methods for obtaining A^{-1} or $A^{-1}b$; (2) it provides subject headings for the bibliography of §5. Purpose (1) is achieved by the portion of the outline in which the six major headings Iα, IIα, IIβ, IIIα, IIIβ, IIIγ are subdivided. In these the methods are classified according to the nature of the iteration functions found in the literature. No strenuous effort has been made to bring all closely related methods into a single entry, even where this could be done within the conventional outline structure, for it is felt that purpose (2) is better served without too great a condensation. There are some methods that belong at two distinct points of the outline (e. g., Seidel's at 10c and at 13a (1 & 5)).

Purpose (2) requires not only the above subdivisions, but also the heading 0 (surveys), and those under IV (miscellaneous relevant topics) and V (contents unknown).

A good many words could be said in explanation of the outline and its notation, but it is hoped that the reader will be able to make it out well enough. Let it suffice to define the symbols used:

- $A = (a_{ij})$: nonsigular square matrix of order n;
- b: column vector;
- $f(A)$: characteristic polynomial of A;
- I: unit matrix;
- A^{-1}: inverse of A;
- A^T: transpose of A;
- $A > 0$: means A is positive definite;
- δ^2 process: see Aitken 1925, 1937b, 1950;
- $x^{(k)}$: k-th approximating vector to $A^{-1}b$;
- B: matrix approximating to A which is effectively inverted in processes of type 10;
- $|x|_R = \sqrt{x^T R x}$: length of x in R metric;

- e_i: unit column vector with 1 for i-th component and 0 for all other components;
- $X^{(k)}$: k-th approximating matrix to A^{-1};
- d_i: some column vector;
- $\Delta x^{(k)} = x^{(k+1)} - x^{(k)}$;
- P, Q: nonsingular square matrices of order n.

3 Tentative Outline of Methods

0. Survey of methods
   a. Mainly iterative methods.
   b. Mainly direct methods.
   c. Contains references not included in §5.

Iα : *All answers theorectically exact. Stationary process.*

1. Solve explicitly.
   a. By determinants.
      1. Efficient evaluation of determinants (often same as 2a).
   b. Otherwise.
2. Triangularize A and solve.
   a. By elimination (almost same as 3a1).
      1. Inexact. A. Efficient arrangements.
      2. Exact. A. Controlled magnitude of numbers.
   b. By extension $\equiv$ escalator processes (related to 4a).
      1. By A-orthogonalization of unit vectors.
   c. By generalized Cholesky method (A symmetrical or not).
      1. Square-root method (symmetric A).
   d. Any of above, with search for pivots.
   e. Any of above, with "below-the-line" arrangement.
   f. Any of above, using Cracovians.
3. Orthogonalize A and solve.
   a. By triangularization.
      1. Elimination (almost same as 2a).
      2. Complete elimination.
      3. Above, with search for pivots.
   b. By orthogonalizing rows or columns of A.
4. Partition A into blocks, and carry out processes 2 or 3 on the blocks.
   a. Smaller matrices inverted by a direet process.
   b. Smaller matrices inverted by iterative process.
5. Iterate any of above.

IIα : *Some answers theorectically exact. Stationary process.*

6. Methods related to characteristic polynomial f of some matrix H.

a. Calculate $f(A)$ from determinant $d(A - \lambda I)$.
    1. Use Newton's identities.
b. Finite iteration to get $f(A)$.
    1. Matrix iteration.
    2. Vector iteration.
c. Process 6b2 applied to iterates of an infinite iteration, to get $f(H)$.
d. Methods related to successively orthogonalizing the vectors $Ax, A^2x, A^3x, \cdots$.
    1. In I metric.
    2. In A^{-1} metric (when $A = A^T > 0$).

7. Not related to characteristic polynomials.
    a. Use of theory of congruences in n dimensions.
    b. Replacement methods (closely related to 2).
        1. Simplex method.
        2. Replace one column of A at a time.

IIβ : *Some answers theorectically exact. Partly stationary process.*

8. Occasionally interrupt the iteration of process of type Iα by accelerating with
    a. δ^2 process (21b1).
    b. linear combinations of prior answers (i.e., apply 21a1).
    c. other process.
9. Occasionally interrupt the iterative cycles of process of type IIα by accelerating
    a. with δ^2 process (21b1)
    b. with linear combinations of prior answers (i.e., apply 21a1).
    c. by starting over from current residual (for process 6d).
    d. with other process.

IIIα : *No answers theorectically exact. Stationary process.*

10. Use a linear iteration function: $x^{(k+1)} = x^{(k)} + B^{-1}(b - Ax^{(k)})$.
    a. General process of first degree.
    b. $B =$diagonal of A (Jacobi).
    c. $B =$lower triangle of A (Seidel[3]).
    d. $B =$scalar matrix (e.g., Neumann series).
    e. B built of submatrix blocks.
        1. Blocks inverted by type I process.
        2. Blocks inverted by type II process.
    f. Reflection in hyperplanes.
    g. Linear processes of second or higher degree (actually 21a1).
11. Use a nonlinear iteration function.
    a. Use a least-squares process with hermitian matrix R (here $\pi = \pi(x)$ determines the subspace in which one minimizes $|Ax - b|_R$).
        1. $\pi = Ax - b$ (one dimension).

[3] Added in proof: This is commonly called Seidel's process. Professor Ostrowski has pointed out, however, that **SEIDEL** 1871 advocated only the process 11b1 (C & F & I), while the process 10c was apparently first advocated by **NEKRASOV** 1881 (see Second Supplement), who attributed it to Seidel.

2. $\pi = A^T(Ax - b)$ (one dimension).
3. π has 2 or more dimensions.
4. Systematically undershoot or overshoot point of minimum $|Ax-b|_R$.
5. $R = I$.
6. $R = A^{-1}$.
7. $R = (AA^T)^{-1}$.
8. Other R.

b. Use nonhermitian metric or nonmetric process.

1. Use piecewise linear iteration (relaxation methods); For some i maximizing

(choice of one) $\begin{cases} \text{A. } (Ax-b)_i, \text{ change} \\ \text{B. } |(Ax-b)_i|/|a_{ii}|, \text{ change} \\ \text{C. } |(Ax-b)_i|/|a_{ii}|^{\frac{1}{2}}, \text{ change} \\ \text{D. } |[A^T(Ax-b)]_i|/(A^TA)_{ii}^{\frac{1}{2}}, \text{ change} \\ \text{E. something else, change} \end{cases}$

(choice of one) $\begin{cases} \text{F. } e_i \text{ (unit vector) so as to bring} \\ \text{G. } A^Te_i \text{ so as to bring} \\ \text{H. something else so as to bring} \end{cases}$

(choice of one) $\begin{cases} \text{I. } (Ax-b)_i \text{ approximately to 0.} \\ \text{J. } [A^T(Ax-b)]_i \text{ approximately to 0.} \\ \text{K. something else to 0.} \end{cases}$

L. With any scheme above, systematically overshoot or undershoot point where residual becomes 0 (over and underrelaxing).

2. Use piecewise linear iteration (block relaxation methods): For some two or more i, change two or more

(choice of one) $\begin{cases} \text{A. } e_i, \text{ so as to minimize something.} \\ \text{B. } A^Te_i, \text{ so as to minimize something.} \end{cases}$

c. Get A^{-1} with polynomial iteration function:

1. Use $X^{(k+1)} = X^{(k)}(2I - AX^{(k)})$.
2. Modify 1.
3. Use related polynomials of third or higher order.

IIIβ : *No answers theorectically exact. Partly stationary process.*

12. Occasionally interrupt the iteration of process of type IIIα by accelearting with

a. δ^2 process (21b1).
b. step of type 11a4 (in types 11a).
c. change of π.
d. step of type 11b2 (in types 11b1).
e. change of B (in type 10).
f. linear combination of prior answers (i.e., apply 21a1).

13. Use m iteration functions in cyclic order (when consolidated in cycles, process becomes type IIIα.)
    a. Use least-squares process with hermitian metric R, minimizing along directions $d_1, d_2, \cdots, d_n$, and repeat.
        1. $d_i = e_i$ (unit vectors).
        2. $d_i = A^T e_i$.
        3. Systematically undershoot or overshoot point of minimum $|Ax - b|_R$.
        4. $R = I$.
        5. $R = A^{-1}$ (when $A = A^T > 0$, 13a (1 & 5) = Seidel process 10c).
        6. $R = (AA^T)^{-1}$.
        7. Other R.
    b. Use least-squares process with hermitian metric R, minimizing along directions $d_1, d_2, \cdots, d_n, d_{n-1}, \cdots, d_2$, and repeat.
        1. $d_i = e_i$ (unit vectors), $R = A^{-1}$ (back-and-forth Seidel process).
        2. $d_i = e_i$, general R.
    c. Use m linear iteration functions of type 10a and repeat.
        1. Use them in a "back-and-forth" order.
    d. Use m nonlinear iteration fucntions.
14. Use one of n separate processes of type 10a in arbitrary order.
    a. Any process of type 11b1 can be regarded as a repeated choice among n processes of type 10a.
    b. Use some permutation of order $1, 2, \cdots, n$ at each cycle.

IIIγ : *No answers theorectically exact. Nonstationary process.*

15. Use infinitely many distinct iteration fucntions.
    a. Use polynomials in A of orders $1, 2, 3, \cdots$.
        1. Generalized Chebyshev polynomials.
        2. Other polynomials.
    b. Polynomials in other matrices connected with iterative processes.
    c. Monte Carlo method (estimates certain population means related to A^{-1}, by samples of size $1, 2, 3, \cdots$).
        1. Based on Neumann series 10d.

IV: *Miscellaneous relevant topics.*

16. Analyses of errors.
    a. Not related to specific method.
        1. "Condition".
        2. Stability of solution, i.e., variation of A^{-1} (or $A^{-1}b$), as A varies (or A and b vary).
    b. Related to specific method.
        1. Truncation errors.
        2. Round-off errors. A. Worst possible. B. Probabilistic.
        3. Control of blunders.

17. “preconditioning” (i.e., changing the system $Ax = b$).
    - a. By enlarging A.
        1. By Gauss’ transformation.
        2. By homogenizing the system (special case of 1).
    - b. By premultiplication of A.
        1. By A^T (normalization).
        2. By some polynomial in A.
        3. By some approximation to A^{-1}.
    - c. By approximate use of some method of solution.
        1. Orthogonalization.
        2. Elimination.
    - d. By taking high power of A (after 17a2).
    - e. By successively rotating off-diagonal elements of A into diagonal (when $A = A^T$).

18. Methods for punched-card equipment.

19. Methods for automatic digital computing machinery.

19A. Methods for analogue computing machinery.

20. Special computing tricks.
    - a. Cumulating solution as $\sum \Delta x^{(k)}$.
        1. Using few significant digits at once.
    - b. Getting A^{-1} by solving n systems $Ax = e_i$ simultaneously (operating on I).
    - c. Getting $A^{-1}B$ by applying to B the row operations that reduce A to I.
    - d. If $PAQ = I$, then $A^{-1} = QP$.
    - e. Splitting real parts of A from imaginary parts of A.

21. Methods for accelerating convergence, applicable to many problems of numerical analysis.
    - a. Linear processes.
        1. Use of linear combinations of sequences of $x^{(k)}$.
    - b. Nonlinear processes.
        1. Use of δ^2 process.
        2. Use of generalized δ^2 process.

22. Counts of operations, time, labor, etc.

23. Methods applicable to special matrices.
    - a. Almost trangular matrices.
    - b. Matrices of large order.

V: *Contents or part of contents unknown*

4 Cross References From the Outline to the Bibliography (略)

5 Bibliography, With Cross-References to the Outline (略)

I. Tentative Classification of Methods and Bibliography on Solving Systems of Linear Equations*

By George E. Forsythe†

I. Introduction

The scope of this article is indicated in the introductions to sections II, IV, and especially V. In the same places will be found acknowledgments to some of the persons who have helped build the bibliography.

This article is a collection of working notes, and not a polished bibliography. Because there is apparently nothing comparable in the literature, it seems better to publish it as it is than to wait for a state of completion which might never arrive. Although many of the references in section V have been checked in the course of the preparation of the manuscript, time has not permitted a systematic editing and search for errors.** If readers will send in lists of errors and omissions, it may later be possible to issue a corrected edition of this article. Please address the compiler at the National Bureau of Standards, 405 Hilgard Avenue, Los Angeles 24, Calif.

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Consider, for the moment, an iterative process for getting $A^{-1}b$. Starting from b, A, and some initial vector $\xi^{(0)}$, a set of arithmetic operations is prescribed, which, if carried out without round-off or other error, will yield a new vector $\xi^{(1)}$: $\xi^{(1)}=F_1(b,A;\xi^{(0)})$. Similarly, $\xi^{(2)}$ is obtained by another set of operations, carried out without error. In general, $\xi^{(k)}$ is obtained from b, A, $\xi^{(k-1)}$, $\xi^{(k-2)}$, . . ., $\xi^{(0)}$ by applying a set of exact arithmetic operations which we assume to be uniquely described by a formula of type

$$\xi^{(k)}=F_k(b,A;\xi^{(k-1)},\xi^{(k-2)}, \ldots,\xi^{(1)},\xi^{(0)}).$$

In this manner there is defined a theoretically determined sequence of vectors

$$\xi^{(0)},\xi^{(1)},\xi^{(2)}, \ldots,\xi^{(k)}, \ldots.$$

However, round-off errors occur in almost all numerical processes (there are exceptions). As a cumulative result of these errors, one actually obtains a vector $x^{(1)}$ instead of $\xi^{(1)}$. In the next step, one ordinarily uses b, A, and $x^{(1)}$ in the algorithm designed to compute $\xi^{(2)}$. Suppose that $\eta^{(2)}$ would be the result of errorless application of the algorithm:

$$\eta^{(2)}=F_2(b,A;x^{(1)},x^{(0)}),$$

*The preparation of this paper was sponsored (in part) by the Office of the Air Comptroller, USAF, and the Office of Naval Research.

†National Bureau of Standards, Los Angeles, Calif.

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where $x^{(0)}=\xi^{(0)}$. Because of round-off errors, one really obtains not $\eta^{(2)}$, but another uniquely defined vector $x^{(2)}$:

$$x^{(2)}=\hat{F}_2(b,A;x^{(1)},x^{(0)}).$$

Here $\hat{F}_2$ is the function F_2, altered in accord with each of the round-off errors entering the calculation.

In this manner will be determined a theoretical sequence of vectors

$$\eta^{(0)}(=\xi^{(0)}),\eta^{(1)},\eta^{(2)},\ .\ .\ .,\ \eta^{(k)},\ .\ .\ .,$$

and a practical sequence

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We thus have three sequences in mind: (1) the sequence $\{\xi^{(k)}\}$ of vectors free from round-off error in all generations (it is these theoretical vectors with which convergence proofs generally deal), (2) the sequence $\{x^{(k)}\}$ of vectors which the computer actually obtains while making the prescribed round-off errors, and (3) the sequence $\{\eta^{(k)}\}$ of vectors which have one "generation" of errorless calculation, based on the computed vectors $x^{(0)},\ .\ .\ .,\ x^{(k-1)}$.

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Typical processes of these three types are I, the elimination process of Gauss 1826; II, the finite iteration of Lanczos 1951; and III, the infinite iteration of Seidel 1874.

Second trichotomy: The sequence $\{F_k\}$ of functions is bound a priori to fall into one of the following three classes:

α. All functions F_k have in fact the same number of arguments and are identical, i. e., there exists an integer, r, such that

$$F_k(b,A;x^{(k-1)},.\ .\ .,x^{(0)})\equiv F(b,A;x^{(k-1)},.\ .\ .,x^{(k-r)})\ \text{(all } k). \tag{*}$$

Such processes are called *stationary*.

β. There are a finite number (greater than one) of distinct functions F_k, each of type (*). Such processes are called *partly stationary*.

γ. There are an infinite number of distinct functions F_k. Such processes are called *nonstationary*. Typical processes of these types are α, (iterated) elimination; β, iterated elimination, occasionally interrupted by an acceleration procedure; γ, weighted averages of the successive iterates of Seidel's process, with weights equal to the coefficients of the Chebyshev polynomials of order 1, 2, 3, ... (Divisions between processes of types β and γ could be made in many different ways.)

The double trichotomy gives us a priori a nine-way classification of methods for getting $A^{-1}b$ (or indeed of estimating any quantity by a numerical process). Three of the possible types, Iβ, Iγ, and

IIβ, do not seem to occur in the literature on getting $A^{-1}b$, but the other six form a convenient structure on which to hang an outline of methods of solving linear equations:

	α Stationary process	β Partly stationary process	γ Nonstationary process
I. All $\eta^{(k)}$ exact	Iα	----------	----------
II. Some $\eta^{(k)}$ exact	IIα	IIβ	----------
III. No $\eta^{(k)}$ exact	IIIα	IIIβ	IIIγ

It is not presumed that this particular nine-way classification is necessarily a "good" one; it merely provides one of many possible frameworks on which to spread out the known solution processes.

The outline in section III serves two purposes: (1) It contains a tentative classification of methods for obtaining A^{-1} or $A^{-1}b$; (2) it provides subject headings for the bibliography of section V. Purpose (1) is achieved by the portion of the outline in which the six major headings Iα, IIα, IIβ, IIIα, IIIβ, IIIγ are subdivided. In these the methods are classified according to the nature of the iteration functions found in the literature. No strenuous effort has been made to bring all closely related methods into a single entry, even where this could be done within the conventional outline structure, for it is felt that purpose (2) is better served without too great a condensation. There are some methods that belong at two distinct points of the outline (e. g., Seidel's at 10c and at 13a (*1* & *5*)).

Purpose (2) requires not only the above subdivisions, but also the heading 0 (surveys), and those under IV (miscellaneous relevant topics) and V (contents unknown).

A good many words could be said in explanation of the outline and its notation, but it is hoped that the reader will be able to make it out well enough. Let it suffice to define the symbols used:

$A=(a_{ij})$=nonsingular square matrix of order n;
b=column vector;
$f(A)$=characteristic polynomial of A;
I=unit matrix;
A^{-1}=inverse of A;
A^T=transpose of A;
"$A>0$" means A is positive definite;
δ^2 process: see AITKEN 1925, 1937b, 1950;
$x^{(k)}$=kth approximating vector to $A^{-1}b$;
B=matrix approximating to A which is effectively inverted in processes of type 10;
$|x|_R=(x^TRx)^{\frac{1}{2}}$=length of x in R metric;
e_i=unit column vector with 1 for ith component and 0 for all other components;
$X^{(k)}$ is kth approximating matrix to A^{-1};
d_i=some column vector;
$\Delta x^{(k)}=x^{(k+1)}-x^{(k)}$;
P, Q=nonsingular square matrices of order n.

III. Tentative Outline of Methods

0. Survey of methods.
    a. Mainly iterative methods.
    b. Mainly direct methods.
    c. Contains references not included in section V.

Iα. *All answers theoretically exact. Stationary process.*

1. Solve explicitly.
    a. By determinants.
        1. Efficient evaluation of determinants (often same as 2a).
    b. Otherwise.

2. Triangularize A and solve.
   a. By elimination (almost same as 3a*1*).
      1. Inexact.
         A. Efficient arrangements.
      2. Exact.
         A. Controlled magnitude of numbers.
   b. By extension≡escalator processes (related to 4a).
      1. By A-orthogonalization of unit vectors.
   c. By generalized Cholesky method (A symmetrical or not).
      1. Square-root method (symmetric A).
   d. Any of above, with search for pivots.
   e. Any of above, with "below-the-line" arrangement.
   f. Any of above, using Cracovians.
3. Orthogonalize A and solve.
   a. By triangularization.
      1. Elimination (almost same as 2a).
      2. Complete elimination.
      3. Above, with search for pivots.
   b. By orthogonalizing rows or columns of A.
4. Partition A into blocks, and carry out processes 2 or 3 on the blocks.
   a. Smaller matrices inverted by a direct process.
   b. Smaller matrices inverted by iterative process.
5. Iterate any of above.

IIα. *Some answers theoretically exact. Stationary process.*

6. Methods related to characteristic polynomial f of some matrix H.
   a. Calculate $f(A)$ from determinant $d(A-\lambda I)$.
      1. Use Newton's identities.
   b. Finite iteration to get $f(A)$.
      1. Matrix iteration.
      2. Vector iteration.
   c. Process 6b*2* applied to iterates of an infinite iteration, to get $f(H)$.
   d. Methods related to successively orthogonalizing the vectors Ax, A^2x, A^3x, A^4x, . . .
      1. In I metric.
      2. In A^{-1} metric (when $A=A^T>0$).
7. Not related to characteristic polynomials.
   a. Use of theory of congruences in n dimensions.
   b. Replacement methods (closely related to 2).
      1. Simplex method.
      2. Replace one column of A at a time.

IIβ. *Some answers theoretically exact. Partly stationary process.*

8. Occasionally interrupt the iteration of process of type Iα by accelerating with . . .
   a. . . . δ^2 process (21b*1*).
   b. . . . linear combinations of prior answers (i. e., apply 21a*1*).
   c. . . . other process.
9. Occasionally interrupt the iterative cycles of process of type IIα by accelerating . . .
   a. . . . with δ^2 process (21b*1*).
   b. . . . with linear combinations of prior answers (i. e., apply 21a*1*).
   c. . . . by starting over from current residual (for process 6d).
   d. . . . with other process.

IIIα. *No answer theoretically exact. Stationary process.*

10. Use a linear iteration function: $x^{(k+1)}=x^{(k)}+B^{-1}(b-Ax^{(k)})$.
   a. General process of first degree.
   b. B=diagonal of A (Jacobi).

c. B=lower triangle of A (Seidel).*
d. B=scalar matrix (e. g., Neumann series).
e. B built of submatrix blocks.
1. Blocks inverted by type I process.
2. Blocks inverted by type II process.
f. Reflection in hyperplanes.
g. Linear processes of second or higher degree (actually 21a*1*).

11. Use a nonlinear iteration function.
a. Use a least-squares process with hermitian matric R (here $\pi=\pi(x)$ determines the subspace in which one minimizes $|Ax-b|_R$.).
1. $\pi=Ax-b$ (one dimension).
2. $\pi=A^T(Ax-b)$ (one dimension).
3. π has 2 or more dimensions.
4. Systematically undershoot or overshoot point of minimum $|Ax-b|_R$.
5. $R=I$.
6. $R=A^{-1}$ (when $A=A^T>0$).
7. $R=(AA^T)^{-1}$.
8. Other R.
b. Use nonhermitian metric or nonmetric process.
1. Use piecewise linear iteration (relaxation methods): For some i maximizing . . .

(choice of one)
A. . . . $(Ax-b)_i$, change . . .
B. . . . $|(Ax-b)_i|/|a_{ii}|$, change . . .
C. . . . $|(Ax-b)_i|/|a_{ii}|^{\frac{1}{2}}$, change . . .
D. . . . $|[A^T(Ax-b)]_i|/(A^TA)_{ii}^{\frac{1}{2}}$, change . . .
E. . . . something else, change . . .

(choice of one)
F. . . . e_i (unit vector) so as to bring . . .
G. . . . A^Te_i so as to bring . . .
H. . . . something else so as to bring . . .

(choice of one)
I. . . . $(Ax-b)_i$ approximately to 0.
J. . . . $[A^T(Ax-b)]_i$ approximately to 0.
K. . . . something else to 0.

L. With any scheme above, systematically overshoot or undershoot point where residual becomes 0 (over and underrelaxing).
2. Use piecewise linear iteration (block relaxation methods): For some two or more i, change two or more . . .

(choice of one)
A. . . . e_i so as to minimize something.
B. . . . A^Te_i so as to minimize something.

c. Get A^{-1} with polynomial iteration function:
1. Use $X^{(k+1)}=X^{(k)}\ (2I-AX^{(k)})$.
2. Modify *1.*
3. Use related polynomials of third or higher order.

IIIβ. *No answer theoretically exact. Partly stationary process.*

12. Occasionally interrupt the iteration of process of type IIIα by accelerating with . . .
a. . . . δ^2 process (21b*1*).
b. . . . step of type 11a*4* (in types 11a).
c. . . . change of π.
d. . . . step of type 11b*2* (in types 11b*1*).
e. . . . change of B (in type 10).
f. . . . linear combination of prior answers (i. e., apply 21a*1*).

* Added in proof: This is commonly called Seidel's process. Professor Ostrowski has pointed out, however, that SEIDEL 1874 advocated only the process 11b*1* (*C* & *F* & *I*), while the process 10c was apparently first advocated by NEKRASOV 1884 (see Second Supplement), who attribute it to Seidel.

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13. Use m iteration functions in cyclic order (when consolidated in cycles, process becomes typ IIIα.)

a. Use least-squares process with hermitian metric R, minimizing along directions d $d_2, \ldots, d_n$, and repeat.

1. $d_i = e_i$ (unit vectors).

2. $d_i = A^T e_i$.

3. Systematically undershoot or overshoot point of minimum $|Ax-b|_R$.

4. $R=I$.

5. $R=A^{-1}$ (when $A=A^T>0$, 13a (*1* & *5*)=Seidel process 10c).

6. $R=(AA^T)^{-1}$.

7. Other R.

b. Use least-squares process with hermitian metric R, minimizing along directions d $d_2, \ldots, d_n, d_{n-1}, \ldots, d_2$, and repeat.

1. $d_i = e_i$ (unit vectors), $R=A^{-1}$ (back-and-forth Seidel process).

2. $d_i = e_i$, general R.

c. Use m linear iteration functions of type 10a and repeat.

1. Use them in a "back-and-forth" order.

d. Use m nonlinear iteration functions.

14. Use one of n separate processes of type 10a in arbitrary order.

a. Any process of type 11b*1* can be regarded as a repeated choice among n processes o type 10a.

b. Use some permutation of order $1, 2, \ldots, n$ at each cycle.

IIIγ. *No answer theoretically exact. Nonstationary process.*

15. Use infinitely many distinct iteration functions.

a. Use polynomials in A of orders $1, 2, 3, \ldots$.

1. Generalized Chebyshev polynomials.

2. Other polynomials.

b. Polynomials in other matrices connected with iterative processes.

c. Monte Carlo method (estimates certain population means related to A^{-1}, by sam ples of size $1, 2, 3, \ldots$).

1. Based on Neumann series 10d.

IV. *Miscellaneous relevant topics.*

16. Analyses of errors.

a. Not related to specific method.

1. "Condition".

2. Stability of solution, i. e., variation of A^{-1} (or $A^{-1}b$), as A varies (or A and b vary)

b. Related to specific method.

1. Truncation errors.

2. Round-off errors.

A. Worst possible.

B. Probabilistic.

3. Control of blunders.

17. "Preconditioning" (i.e., changing the system $Ax=b$).

a. By enlarging A.

1. By Gauss' transformation.

2. By homogenizing the system (special case of *1*).

b. By premultiplication of A.

1. By A^T (normalization).

2. By some polynomial in A.

3. By some approximation to A^{-1}.

c. By approximate use of some method of solution.

1. Orthogonalization.

2. Elimination.

d. By taking high power of A (after 17a*2*).
e. By successively rotating off-diagonal elements of A into diagonal (when $A=A^T$).
18. Methods for punched-card equipment.
19. Methods for automatic digital computing machinery.
19A. Methods for analogue computing machinery.
20. Special computing tricks.
a. Cumulating solution as $\Sigma\Delta x^{(k)}$.
1. Using few significant digits at once.
b. Getting A^{-1} by solving n systems $Ax=e_i$ simultaneously (operating on I).
c. Getting $A^{-1}B$ by applying to B the row operations that reduce A to I.
d. If $PAQ=I$, then $A^{-1}=QP$.
e. Splitting real parts of A from imaginary parts of A.
21. Methods for accelerating convergence, applicable to many problems of numerical analysis.
a. Linear processes.
1. Use of linear combinations of sequence of $x^{(k)}$.
b. Nonlinear processes.
1. Use of δ^2 process.
2. Use of generalized δ^2 process.
22. Counts of operations, time, labor, etc.
23. Methods applicable to special matrices.
a. Almost triangular matrices.
b. Matrices of large order.
V. *Contents or part of contents unknown.*

IV. Cross References From the Outline to the Bibliography

In this section the subject heading (numbers and letters like 11c*1*) correspond to the outline in section III. Opposite each subject heading are given the references from the bibliography (section V) that deal with the subject. There is an inconsistency in the presentation about which the user should be warned. For example, Kunz 1947 treats several of the methods 2a, 2b, 2c, . . ., but is listed only under the general heading 2, whereas Bodewig 1947b will be found under 2a, 2b, and 2c. (The unavailability of Kunz 1947 at the time of compiling section IV caused the difference in treatment.)

A question mark after an entry indicates doubt as to whether the title deals with the subject, and does not query the date as such.

0. Bargmann-Montgomery-von Neumann 1946, Bodewig 1947b, Engineering Research Associates 1950, Forsythe 1951, Fox 1950a, Frazer-Duncan-Collar 1938, Grossman 1950?, Hartree 1949b, Householder 1949, 1950, Kunz 1947, Pipes 1948, Salvadori 1948.

a. Geiringer 1949, Hotelling 1943a, 1949, Panov 1934, Reich 1948, Zurmühl 1950.

b. Dwyer 1951, Jensen 1944, Kunz 1947, Turing 1948, Zurmühl 1950.

c. Dunlap 1939, Dwyer 1941a, 1942, 1951, Frame 1945, Griffin 1931, Higgins 1949, Jenne 1949, Kantorovich-Krylov 1948, International Business Machines Corp. 1950, Southwell 1940, 1946, Taussky 1951, Tolley-Ezekiel 1927, zur Capellen 1948.

1α. Adams 1915?

1.

a. Bargmann-Montgomery-von Neumann 1946, Bodewig 1947a, Cassina 1948, Dickson 1922, Ingraham 1937, Leppert 1947, Milne 1949, Ostrowski 1938, Salvadori 1948, Schmeidler 1949, H. Schulz 1938.

1. Aitken 1937a, Chiò 1853, Collatz 1948a, Dwyer 1951, Goodwin 1950, Jensen 1944, Milne-Thomson 1941, Pipes 1948, Rice 1920, Smith 1927, Walther 1944, Whittaker-Robinson 1924.

b. Bisshopp 1945, Collar 1939, Jenne 1949, Nörlund 1940.

2. Kunz 1947.

a. Aitken 1932, 1935, Andersen 1947, Andree 1951, Baetslé 1951, Bargmann-Montgomery-von Neumann 1946, Bauschinger 1901, Bodewig 1947b, Cassina 1948, Cassinis 1946, Collatz 1948a, Couffignal 1944, Deming 1928, Dwyer 1941a, 1951, Encke 1835, Forsythe 1951, Fox 1950a, Fox-Huskey-Wilkinson 1948a, Frazer-Duncan-Collar 1938, Fröberg 1953, Gauss 1826, Harvard Computation Lab. 1946, Herzberger 1949, Herzberger-Morris 1947, Hoel 1941, Householder 1949, 1950, Jensen 1944, 1948, Jürgens 1886, Kolmogoroff 1946?, Margenau-Murphy 1943, Mehmke 1930, Milne 1949, Mitchell 1948, Parkes 1950, Polachek 1948, Renner 1946, Reynolds 1934, Ricci 1949, D. F. Richardson 1946, Salvadori 1948, Scarborough 1950, H. Schulz 1938, Seidel 1874, Spoerl 1943, 1944, Verzuh 1949, Walther 1944, Whittaker-Robinson 1924, Willers 1928, 1947, Worch 1932, Wren 1937, Zurmühl 1950.

1. Fox 1950a, Goldstine-von Neumann 1951, Pipes 1948, Turing 1948.

A. Black 1949, Bruner 1947, Doolittle 1878, Dwyer 1941d, 1951, Fox 1950a, Kurtz 1936, Leavens 1947, Nielsen-Goldstein 1947, Satterthwaite 1944, Tollay-Ezekiel 1927, Waugh-Dwyer 1945, Rosser 1952.

2. Dodgson 1866, Dwyer 1951, Waugh-Dwyer 1945, Rosser 1952.

b. Aitken 1932, Bodewig 1947b, Boschan 1946, Cochran 1938?, Collatz 1948a, Dwyer 1951, Frazer 1947, Householder 1949, 1950, Jossa 1938?, 1940?, Morris 1946, 1947, Neville 1948, Turing 1948.

1. Fox 1950a, Fox-Huskey-Wilkinson 1948a, Turing 1948.

c. Andersen 1947, Baetslé 1951, Banachiewicz 1951, Bodewig 1947b, 1950, Cassina 1948, Cassinis 1946, Crout 1941, Dwyer 1951, Forsythe 1951, Fox 1950a, 1950b, Fox-Huskey-Wilkinson 1948a, Herzberger 1949, Householder 1949, 1950, Istituto per le Applicazioni del Calcolo #4 (date unknown), Jensen 1944, Milne 1949, Picone (date unknown), Ricci 1949, Turing 1948, Waugh-Dwyer 1945, Zurmühl 1949, 1950.

1. Asplund 1945, Banachiewicz 1938a?, 1938b, Benoit 1924, D. B. Duncan-Kenney 1946, Dwyer 1942, 1944, 1945, Istituto per le Applicazioni del Calcolo #3 (date unknown), Jensen 1944, Kunz 1947, Laderman 1948, Rubin 1926.

d. Dwyer 1951, Fox 1950a, Fox-Huskey-Wilkinson 1948a, Frazer-Duncan-Collar 1938, Herzberger 1949, Herzberger-Morris 1947, Waugh-Dwyer 1945, Zurmühl 1950.

e. Andree 1951, Baetslé 1951, Dwyer 1951, Forsythe 1951, Fox 1950a, Hoel 1941, Verzuh 1949.

f. Backman 1946, Banachiewicz 1937b, 1937c, 1942, 1951, Kamela 1943.

3.

a. Basile 1949, Kunz 1947?

1. Bodewig 1947b, Fox-Huskey-Wilkinson 1948a, Frazer-Duncan-Collar 1938, Jürgens 1886

2. Clasen 1888, Dreyer 1943, Forsythe 1951, Fox 1950a, Householder 1949, Jensen 1944, Jordan 1920, Petrie 1953, Samuelson 1950, Turing 1948, Volta 1950, Whittaker-Robinson 1924, Worch 1932.

3. Forsythe 1951.

b. Bargmann-Montgomery-von Neumann 1946, Cassinis 1946, Forsythe 1951, Fox 1950a, Householder 1950, Kunz 1947?, Picone (date unknown), Ricci 1949, Roma 1946, 1947, E. Schmidt 1908, Shreĭder 1951, Turing 1948, Unger 1951.

4. BANACHIEWICZ 1937a, 1937b, 1937c?, BARGMANN-MONTGOMERY-VON NEUMANN 1946, BODEWIG 1947a, 1947b, BOLTZ 1923, COLLATZ 1948a, W. J. DUNCAN 1944, FORSYTHE 1951, FOX 1950a, FRAZER-DUNCAN-COLLAR 1938, (GRAM 1883), GUTTMAN 1946, HOTELLING 1943a, 1943b, 1949, INGRAHAM 1937, JOSSA 1938, 1940, JENSEN 1944, KRÜGER 1905, KUNZ 1947?, MORRIS 1946, 1947, OSTROWSKI 1938, PIPES 1941, SAIBEL 1944, SCHUR 1917, TURETSKY 1951, WAUGH 1945, WOODBURY 1950.

a. (Many of references under 4.)

b. NEVILLE 1948.

5. BODEWIG 1947b, FORSYTHE 1951, POLACHEK 1948, RUNGE-KÖNIG 1924, SALVADORI 1948, THOMSON 1878, TURING 1948, ZURMÜHL 1944, 1950.

IIα.

6.

a. DWYER 1951.

1. HOTELLING 1943a.

b.

1. BINGHAM 1941, DWYER 1951, FRAME 1949, PIPES 1948.

2. FORSYTHE 1951, FREEMAN 1943, MILNE 1951.

c. FREEMAN 1943, R. J. SCHMIDT 1941.

d. HESTENES 1951, LANCZOS 1951, STIEFEL 1951, 1952, ROSSER 1953.

7.

a. LEWY (unpublished), ROBINSON 1951.

b.

1. DANTZIG 1951.

2. SHERMAN 1951, SHERMAN-MORRISON 1949.

IIβ.

8.

9.

c. HESTENES (unpublished).

IIIα.

10. ARGELANDER 1844, BAUSCHINGER 1901, CHEREPKOV 1936, GROSSMAN 1948, KANTOROVICH 1939, KUNZ 1947, WADDELL 1916, ZYLEV 1939.

a. AITKEN 1950, CESARI 1937a, 1937b, COLLATZ 1950b, FRAZER-DUNCAN-COLLAR 1938, FORSYTHE 1951, GEIRINGER 1949, HOTELLING 1943, HOUSEHOLDER 1949, MILNE 1951, MORRIS 1935?, PIPES 1948, REICH 1948, WITTMEYER 1936b.

b. ANÉR 1926, BLACK (date unknown), BRAND 1935?, CASSINIS 1946, CHEN 1944?, COLLATZ 1942, 1948a, 1949, 1950a, 1950b, CROSS 1932, FORSYTHE 1951, FOX 1950a, FRAZER-DUNCAN-COLLAR 1938, GEIRINGER 1949, HOUSEHOLDER 1949, 1950, IVANOV 1939, JACOBI 1845, KORMES 1943?, 1947?, MORRIS 1947, RUNGE-KÖNIG 1924, SCHOTT 1855, STEIN-ROSENBERG 1948, VON MISES POLLACZEK-GEIRINGER 1929, YOUNG 1950.

c. AITKEN 1950, BOWIE 1950, CASSINIS 1946, COLLATZ 1942, 1948a, 1949, 1950a, 1950b, FORSYTHE 1951, FOX 1950a, FRANKEL 1950, FRAZER-DUNCAN-COLLAR 1938, GATTO 1949, GEIRINGER 1949, HOTELLING 1933, 1943a, HOUSEHOLDER 1949, 1950, IVANOV 1939, KANTOROVICH-KRYLOV 1948, LIEBMANN 1918, MEHMKE 1892, MEHMKE-NEKRASSOF 1892?, MILLER 1947, MORRIS 1935, 1947, OLDENBURGER 1940b, PIPES 1948, POLLACZEK-GEIRINGER 1928, REICH 1948, 1949, RUNGE 1899, SALVADORI 1948, SASSENFELD 1951, SCHMEIDLER 1949, SEIDEL 1862, 1874,* SHORTLEY-WELLER 1938, SHORTLEY-WELLER-FRIED 1940, SNYDER-LIVINGSTON 1949, P. STEIN 1951, STEIN-ROSENBERG 1948, VON MISES-POLLACZEK-GEIRINGER 1929, WHITTAKER-ROBINSON 1924, WILLERS 1928, 1947, YOUNG 1950, ZURMÜHL 1950.

d. BIEZENO 1924, BODEWIG 1947b, FORSYTHE 1951, FRANKEL 1950, GEIRINGER 1949, HELLINGER-TOEPLITZ 1924, HOLLEY 1951, HOTELLING 1943a, HOUSEHOLDER 1949, MILNE 1951, NEWING 1941, PIPES 1948, PLUNKETT 1950, QUADE 1947, RICHARDSON 1910, SCHMEIDLER 1949, VON MISES-POLLACZEK-GEIRINGER 1929, WALSH 1920, WAUGH 1950.

e.

1. GAUSS 1826, GEIRINGER 1942, 1949, HERTWIG 1912, MEHMKE 1892.

f. BODEWIG 1947b, CIMMINO 1938, FORSYTHE 1951, HOUSEHOLDER 1949.

g. FRANKEL 1950.

11.

a (comprehensive).

FORSYTHE 1951, HOUSEHOLDER 1949, ROSSER 1949, 1950, M. STEIN 1952.

(1 & 6). BIRMAN 1950?, HOUSEHOLDER 1950, KANTOROVICH 1945, 1947, MYSOVSKIKH 1950?, PIPES 1948, TEMPLE 1939.

(2 & 5). BOOTH 1949, CAUCHY 1847, CURRY 1944, FORSYTHE-MOTZKIN 1950, 1951a, 1951b, HARTREE 1948, 1949b, KANTOROVICH 1948, KANTOROVICH-KRYLOV 1948, REICH 1948.

(3 & 5). FORSYTHE-MOTZKIN 1950, 1951b.

(3 & 6). KANTOROVICH 1947.

(2 & 4 & 5). CAUCHY 1847, M. STEIN 1952.

4. HARTREE 1948?, M. STEIN 1952.

b.

1. BLACK-SOUTHWELL 1938, BOWIE 1947?, FOX 1947, HENDRIKZ (date unknown)?, MENZIES (date unknown), NIKOLAEVA 1949, L. WRIGHT 1943.

(A & F & I). BLACK 1938, BLACK (date unknown), FORSYTHE 1951, FOX 1948, 1950b, GASKELL 1943, HOUSEHOLDER 1950, MILNE 1951, REICH 1948, SOUTHWELL 1940, 1946.

(D & G & J). HOUSEHOLDER 1949.

(B & F & I). DEDEKIND 1901, FOX 1948, GAUSS 1823, GERLING 1843, JÜRGENS 1886, RAINSFORD (date unknown), SCHAEFER 1927, SCHOTT 1855, SOUTHWELL 1940, 1946, TEMPLE 1939, THOMPSON (date unknown), WHITTAKER-ROBINSON 1924, ZURMÜHL 1950.

L. FOX 1948, GERLING 1843, SOUTHWELL 1940, 1946.

(C & F & I). FOX 1950a, HOUSEHOLDER 1949, SEIDEL 1874, SYNGE 1944.

2. BLACK (date unknown), FOX 1948, 1950b, SOUTHWELL 1940, 1946.

*See footnote, p. 5.

c.

1. Bargmann-Montgomery-von Neumann 1946, Bodewig 1947b, Fox 1950a, Frazer-Duncan-Collar 1938, Hotelling 1943a, 1949, Householder 1949, Kunz 1947?, Lonseth 1949, Kantorovich 1949, Neville 1948, Pipes 1948, G. Schulz 1933, Turing 1948, Ullman 1944.

2. Bargmann-Montgomery-von Neumann 1946.

3. Bodewig 1947b, Schröder 1870, Ullman 1944.

IIIβ.

12. Ljusternik 1947, Shortley-Weller 1938, Shortley-Weller-Fried 1940.

a. Forsythe 1951.

b. Hestenes (unpublished).

c. Forsythe-Motzkin 1951b.

d. (References under 11b*2*.)

e. Forsythe 1951, Richardson 1910.

f. Fox 1950b, Milne 1951.

13.

a (comprehensive). Householder 1949.

(*1* & *4*). de la Garza 1951, Householder 1949, Rosser 1949.

(*2* & *6*). Bodewig 1947b, Bottema 1950, Kaczmarz 1937, Rosser 1949, Tompkins 1949.

(*1* & *5*). Frankel 1950, Householder 1950.

(*1* & *3* & *5*). Frankel 1950.

(*2* & *4*). Householder 1949.

b.

1. Aitken 1950.

c. Bückner 1950, Geiringer 1949, Richardson 1910.

1. Aitken 1950, Rosser 1949, 1950.

d. Kelley-Salisbury 1926.

14.

a. Hestenes (unpublished).

b. Geiringer 1949, Reich 1948.

IIIγ

15.

a. Flanders-Shortley 1950.

b.

c.

1. FORSYTHE-LEIBLER 1950, OPLER 1951, SWIFT-TIKSON 1951, TODD 1951a, 1951b, WASOW 1952.

IV.

16. DWYER 1953, OPITZ-WILLERS (date unknown), PIRLET 1909, H. SCHULZ 1938.

a. HERTWIG 1905?, PRICE 1951, SHERMAN-MORRISON 1950.

1. HARTREE 1948, JÜRGENS 1888, TAUSSKY 1949, 1950, TODD 1949a, 1949b, TURING 1948.

2. BARGMANN-MONTGOMERY-VON NEUMANN 1946, BERKSON 1936, BARTLETT 1951, BLUMENTHAL 1914, COLLATZ 1949, W. E. DEMING 1937, DWYER 1951, ETHERINGTON 1932, FORSYTHE 1951, HOTELLING 1943a, JANET 1920a, 1920b, LONSETH 1942, 1944, 1947, MILNE 1949, MORGENSTERN-WOODBURY 1950, MOULTON 1913, 1936, OSTROWSKI 1937a, 1937b, 1950, REDHEFFER 1948, ROESSLER 1936, SCARBOROUGH 1950, TUCKERMAN 1941, TURING 1948, WALSH 1920, WILLERS 1928, 1947, WITTMEYER 1934, 1936a, WOODBURY 1949, 1950, ZURMÜHL 1950.

b. HOTELLING 1949, HOUSEHOLDER 1949, LEAVENS 1947.

1. COLLATZ 1950b, WITTMEYER 1936b.

2. BARGMANN-MONTGOMERY-VON NEUMANN 1946, DWYER 1951, GOLDSTINE-VON NEUMANN 1951, NEVILLE 1948, PARKES 1950, POLACHEK 1948, REICH (date unknown), REICH 1948, SALVADORI 1948, SATTERTHWAITE 1944, SCARBOROUGH 1950, TUCKERMAN 1941, TURING 1948, WAUGH 1950.

A. HOEL 1940, HOTELLING 1943a, 1943b.

B. HOEL 1940, HOTELLING 1943a, ULLMAN 1944.

3. DWYER 1951, GAUSS 1823.

17.

a.

1. FORSYTHE-MOTZKIN 1952, GAUSS 1823, GERLING 1843, ZURMÜHL 1950.

b.

1. BARGMANN-MONTGOMERY-VON NEUMANN 1946, BLACK-SOUTHWELL 1938, CASSINIS 1946, COLLATZ 1942, 1948a, HOTELLING 1943a, HOUSEHOLDER 1950, KANTOROVICH 1948, MILNE 1951, SEIDEL 1874, TAUSSKY 1949, 1950, TEMPLE 1939, VON MISES-POLLACZEK-GEIRINGER 1929, ZURMÜHL 1950.

2. BODEWIG 1947b, CESARI 1937b.

3. SATTERTHWAITE 1944.

c.

1.

2. BODEWIG 1947b, BOWIE 1950, JÜRGENS 1886.

d. GEIRINGER 1949, HOTELLING 1936a, 1943a.

e. JACOBI 1845, SEIDEL 1874.

18. AKUSHSKY 1946a, 1946b, 1946c, ALT 1946, BASILE 1949, ECKERT 1940, ENGINEERING RESEARCH ASSOCIATES 1950, FLANAGAN 1940, FORSYTHE 1951, FOX-HUSKEY-WILKINSON 1948b, GROSSMAN 1948, HARTLEY 1946, IBM 1950 (complete), (IBM educational forums), KORMES 1943, LEPPERT 1947, OPLER 1951, PETRIE 1953, RENNER 1946, SHERMAN 1951, SNEDECOR 1928, TUCKER 1940, VERZUH 1949.

19. ENGINEERING RESEARCH ASSOCIATES 1950, FRÖBERG 1953, GOLDSTINE-VON NEUMANN 1951, HARTREE 1949b, HARVARD COMPUTATION LAB. 1946, MCPHERSON 1948, MITCHELL 1948, ROSSER 1950, SNYDER-LIVINGSTON 1949, STIEFEL 1951, 1952.

19A. ADCOCK 1948, ANONYMOUS 1934, BERRY-WILCOX-ROCK-WASHBURN 1946, BOWIE (date unknown), BROWN 1947, DENNIS 1948, ENGINEERING RESEARCH ASSOCIATES 1950, FAIRTHORNE 1944, FRAME 1945, FUCHS 1914, GORUSHKIN 1948, GRADSTEÏN 1947, HARDY-DENCH 1948, HAUPT 1950, HAZEN-SCHURIG-GARDNER 1930, KORN 1949, KRON 1945, MALLOCK 1933, MANY 1950, MCCANN 1949, MEEROVICH 1947, MUIRHEAD 1912, MURRAY 1947, 1949, NÄBAUER 1910, PARKER 1941, 1945, PICCHI 1948, PIPES 1948, PROSHKO 1947, SAMSSONOW 1943, SCHUMANN 1940, SPANGENBERG-WALTERS-SCHOTT 1949, WILBUR 1936, ZUR CAPELLEN 1948.

20.

a. BLACK (date unknown), DEDEKIND 1901, CROSS 1932, GAUSS 1823, JACOBI 1845, SALVADORI 1948.

1. FORSYTHE 1951, FOX 1948, 1950a?, 1950b, SOUTHWELL 1940, 1946, ZURMÜHL 1950.

b. AITKEN 1932, ALBERT 1941, DWYER 1951, FISHER 1938, FORSYTHE 1951, FRAZER-DUNCAN-COLLAR 1938, HOEL 1941, HOTELLING 1943a, HOUSEHOLDER 1950, MILNE 1949, G. SCHULZ 1933.

c. BURGESS 1916, DWYER 1941c, 1951, FORSYTHE 1951.

d. FORSYTHE 1951.

e. FOX 1950a, GOODWIN 1950, PIPES 1941, ZURMÜHL 1950.

21.

a.

1. FORSYTHE 1951, FOX 1948, SOUTHWELL 1940?, 1946?

b.

1. AITKEN 1926, 1937b, 1950, FORSYTHE 1951, FOX 1950a, SAMUELSON 1945, SHANKS 1949.

2. FORSYTHE 1951, SAMUELSON 1945, SHANKS 1949.

22. BARGMANN-MONTGOMERY-VON NEUMANN 1946, BODEWIG 1947b, CASSINA 1948, COLLATZ 1948a, FRANKEL 1950, HECK 1946, KAMELA 1943, PARKES 1950, RICCI 1949.

23.

a. BISSHOPP 1945, GOOD 1950.

b. GOLDSTINE-VON NEUMANN 1951, JENNE 1949, KERKHOFS 1947.

V. BALLANTINE 1931, W. E. DEMING 1935, D. B. DUNCAN-KENNEY 1946, DUNLAP 1939, DWYER 1941b, 1946, ENCKE 1835, FLANAGAN 1940, FOX 1947, 1950a, FOX-HUSKEY-WILKINSON 1948b, GRIFFIN 1931, GROSSMAN 1948, 1950, HALBERSTADT 1914, HARTLEY 1946, HECK 1946, HENDRIKZ (date unknown), HORST 1941a, HRUŠKA 1943, JENNE 1949, JORDAN 1920, JOSSA 1938, 1940, KANTOROVICH-KRYLOV 1941, KERKHOFS 1947, KOLMOGOROFF 1946, KORMES 1943, 1945, LÜSTERNIK 1947, MAGNIER 1948, OPPOKOV 1939, PANOV 1938, PICONE 1940, REICH (date unknown), SCHAEFER 1927, H. SCHULZ 1938, SHAW 1946, TURTON 1945, WEGNER (date unknown).

V. Bibliography, With Cross-References to the Outline

In this section are given approximately 450 titles on the solution of linear equations, taken from the compiler's card file on numerical matrix methods. The card file has been collected during the past two years as part of an investigation at the National Bureau of Standards, Los Angeles, of old and new matrix methods suited to automatic digital computers.

The bibliography is most nearly complete in recent titles dealing with mathematical methods for obtaining A^{-1} or $A^{-1}b$, where A is a nonsingular square matrix. The bibliographies of BODEWIG 1947b, DWYER 1951, HARVARD COMPUTATION LABORATORY 1946, HIGGINS 1949, HOTELLING 1943, HOUSEHOLDER 1949, KANTOROVICH 1948, KANTOROVICH-KRYLOV 1948, and OSTROWSKI-TODD-TODD 1949 proved invaluable in starting and augmenting the list. Original papers were consulted when reasonably obtainable, and their references were added to the card file. Certain papers which appear to be covered in subsequent books have been omitted; some titles containing lists of such papers are given under heading 0c of section IV.

Because much of the research on solving linear equations has been ancillary to the solution of numerical problems in pure mathematics, statistics, physics, astronomy, geodesy, psychology, economics, engineering, etc., many good references are from these diverse fields. The present bibliography is less complete in these related fields, some of which have their own bibliographies (e. g., HIGGINS 1949 on numerical solution of partial differential equations).

There are several mathematical fields which are related to our subject, but which have not been included here. Three of these are known to be subjects for bibliographies now in preparation: MOTZKIN 1951 (on linear inequalities and applications); TAUSSKY 1951 (on bounds for eigenvalues); SCHWERDTFEGER 1951 (on iteration in general; listed at end of bibliography).

At the last minute an effort was made to include some representative titles on analogue machinery for solving linear equations, starting with references in FRAME 1945 and TRYON 1951. However, in regard to analogue machinery the bibliographies of ENGINEERING RESEARCH ASSOCIATES 1950 and ZUR CAPELLEN 1948 are far more extensive than the few titles included below. Also, the bibliography of INTERNATIONAL BUSINESS MACHINES 1950 (mostly reproduced in ENGINEERING RESEARCH ASSOCIATES 1950) is more complete on uses of IBM equipment for matrix problems than the present bibliography.

The numbers in brackets after each title in the bibliography refer to the outline in section III, and summarize the contents of the reference. The notation $11bI(B\&F\&I)$ means that the paper treats the subcase of subject $11bI$, in which B, F, and I are simultaneously fulfilled (logical product).

A circle (°) is placed before each title which the compiler has not examined personally, insofar as he can remember.

The abbreviations MR, Fs, Zb, SA after titles refer to abstract journals:

MR = *Mathematical Reviews;*
Fs = *Jahrbuch über die Fortschritte der Mathematik;*
Zb = *Zentralblatt der Mathematik und ihre Grenzgebiete;*
SA = *Science Abstracts, Section A.*

References to abstract journals are listed when they happen to be known; there is no uniform policy.

The following journal abbreviation has been used in the references: *MTAC, Mathematical Tables and Other Aids to Computation.*

Attention is invited to the supplementary bibliographies at the end, containing papers noted too late for inclusion in the main list.

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