My R Markdown Document

Modeling and Forecasting the Morocco Stock Index 20

```
# Loading libraries
# Date manipulation
suppressPackageStartupMessages(library(lubridate))
# Descriptive statistics
suppressPackageStartupMessages(library(fBasics))
## Warning: le package 'fBasics' a été compilé avec la version R 4.4.2
# Coefficient significance tests
suppressPackageStartupMessages(library(lmtest))
## Warning: le package 'lmtest' a été compilé avec la version R 4.4.2
# Unit root test
suppressPackageStartupMessages(library(urca))
## Warning: le package 'urca' a été compilé avec la version R 4.4.2
# Visualisation
suppressPackageStartupMessages(library(ggplot2))
# Return calculations
suppressPackageStartupMessages(library(quantmod))
## Warning: le package 'quantmod' a été compilé avec la version R 4.4.2
## Warning: le package 'xts' a été compilé avec la version R 4.4.2
## Warning: le package 'TTR' a été compilé avec la version R 4.4.2
suppressPackageStartupMessages(library(PerformanceAnalytics))
## Warning: le package 'PerformanceAnalytics' a été compilé avec la version R
## 4.4.2
suppressPackageStartupMessages(library(forecast))
## Warning: le package 'forecast' a été compilé avec la version R 4.4.2
# Changements structurels
suppressPackageStartupMessages(library(strucchange))
## Warning: le package 'strucchange' a été compilé avec la version R 4.4.2
## Warning: le package 'sandwich' a été compilé avec la version R 4.4.2
# Loading the 'tseries' library for ADF testing
library(tseries)
```

Warning: le package 'tseries' a été compilé avec la version R 4.4.2

```
library(timeSeries)
## Warning: le package 'timeSeries' a été compilé avec la version R 4.4.2
## Le chargement a nécessité le package : timeDate
##
## Attachement du package : 'timeDate'
## Les objets suivants sont masqués depuis 'package:PerformanceAnalytics':
##
##
       kurtosis, skewness
##
## Attachement du package : 'timeSeries'
## L'objet suivant est masqué depuis 'package:zoo':
##
##
       time<-
## Les objets suivants sont masqués depuis 'package:graphics':
##
##
       lines, points
library(xts)
library(pastecs)
## Warning: le package 'pastecs' a été compilé avec la version R 4.4.2
## Attachement du package : 'pastecs'
## Les objets suivants sont masqués depuis 'package:xts':
##
       first, last
# For MLP model
library(nnfor)
## Warning: le package 'nnfor' a été compilé avec la version R 4.4.2
## Le chargement a nécessité le package : generics
## Attachement du package : 'generics'
## L'objet suivant est masqué depuis 'package:sandwich':
##
       estfun
## L'objet suivant est masqué depuis 'package:lubridate':
##
##
       as.difftime
## Les objets suivants sont masqués depuis 'package:base':
##
       as.difftime, as.factor, as.ordered, intersect, is.element, setdiff,
##
       setequal, union
library(readr)
dailyMSI20 <- read.csv("MSI20_update.csv", head = TRUE)</pre>
head(dailyMSI20)
```

```
Date
                  Open Close High
## 1 2021-01-04 924.78 928.96 928.97 922.30
## 2 2021-01-05 925.99 919.52 926.79 919.52
## 3 2021-01-06 919.52 915.67 921.57 915.01
## 4 2021-01-07 915.67 921.52 924.79 915.67
## 5 2021-01-08 921.52 919.60 924.31 918.90
## 6 2021-01-12 919.60 923.89 923.89 916.78
# Clean data
# Convert Date column to time format
dailyMSI20$Date <- as.Date(dailyMSI20$Date)</pre>
# Check updated format
str(dailyMSI20)
                    590 obs. of 5 variables:
## 'data.frame':
## $ Date : Date, format: "2021-01-04" "2021-01-05" ...
## $ Open : num 925 926 920 916 922 ...
## $ Close: num 929 920 916 922 920 ...
## $ High : num 929 927 922 925 924 ...
## $ Low : num
                 922 920 915 916 919 ...
head(dailyMSI20)
##
                  Open Close
           Date
                              High
                                        Low
## 1 2021-01-04 924.78 928.96 928.97 922.30
## 2 2021-01-05 925.99 919.52 926.79 919.52
## 3 2021-01-06 919.52 915.67 921.57 915.01
## 4 2021-01-07 915.67 921.52 924.79 915.67
## 5 2021-01-08 921.52 919.60 924.31 918.90
## 6 2021-01-12 919.60 923.89 923.89 916.78
tail(dailyMSI20)
                    Open Close
                                  High
             Date
## 585 2023-04-27 838.25 840.12 843.95 838.25
## 586 2023-04-28 840.12 848.13 848.13 840.12
## 587 2023-05-02 848.13 840.54 852.01 840.54
## 588 2023-05-03 840.54 838.55 841.44 834.71
## 589 2023-05-04 838.55 837.13 838.55 831.77
## 590 2023-05-05 837.13 835.44 840.93 833.74
dim(dailyMSI20)
## [1] 590
#count the total number of missing values in the dataset
sum(is.na(dailyMSI20))
## [1] O
t(stat.desc(dailyMSI20))
        nbr.val nbr.null nbr.na
                                      min
                                               max range
                                                                 sum
                                                                        median
                       0
                               0 18631.00 19482.00 851.00 11243991.0 19059.500
## Date
             590
## Open
             590
                        0
                               0
                                   775.38 1140.69 365.31
                                                            575589.3
                                                                       982,695
## Close
             590
                        0
                               0
                                   775.38 1140.69 365.31
                                                            575563.4
                                                                       982.695
## High
             590
                        0
                               0
                                   797.01 1142.56 345.55 578174.3
                                                                       985.980
```

```
590
                      0 0 775.38 1135.86 360.48 573290.4 980.045
## Low
##
               mean SE.mean CI.mean.0.95
                                                  var
                                                       std.dev
                                                                  coef.var
## Date 19057.6119 10.157295 19.948925 60870.679 246.71984 0.01294600
        975.5752 3.543246
                                 6.958935 7407.210 86.06515 0.08821991
## Open
## Close 975.5312 3.550503 6.973188 7437.583 86.24142 0.08840458
## High 979.9565 3.512745 6.899031 7280.233 85.32428 0.08706946
           971.6786 3.552519 6.977147 7446.033 86.29040 0.08880549
## Low
#Prices in time series format
Close_Price <- xts(dailyMSI20[,3],order.by=as.Date(dailyMSI20[,1]))</pre>
head(Close_Price)
##
                [,1]
## 2021-01-04 928.96
## 2021-01-05 919.52
## 2021-01-06 915.67
## 2021-01-07 921.52
## 2021-01-08 919.60
## 2021-01-12 923.89
tail(Close_Price)
##
                [,1]
## 2023-04-27 840.12
## 2023-04-28 848.13
## 2023-05-02 840.54
## 2023-05-03 838.55
## 2023-05-04 837.13
## 2023-05-05 835.44
autoplot(Close_Price) +
  labs(title = "MSI 20 daily closing price",
       x = "Date: January 4, 2021 to May 5, 2023",
       y = "Closing price")
```

MSI 20 daily closing price

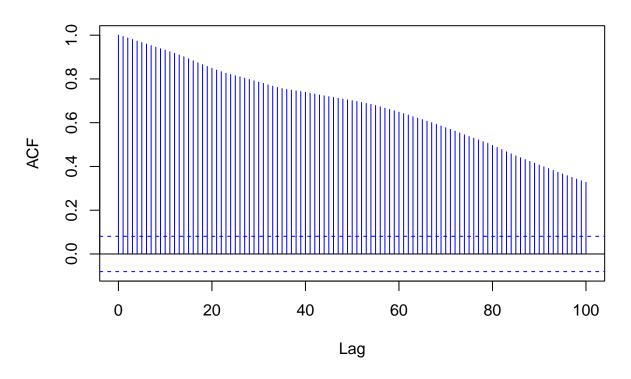


Here we see the corresponding chart, as produced by chartSeries in the quantmod package.

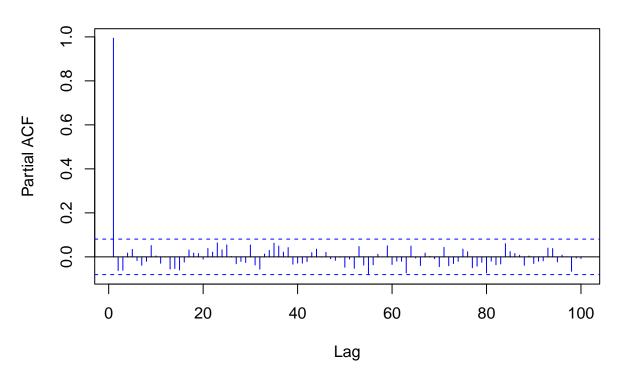
chart_Series(Close_Price, name = deparse(substitute(Daily_closing_price_of_MSI_20)),col = "blue")



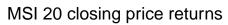
MSI 20 autocorrelation function (ACF)

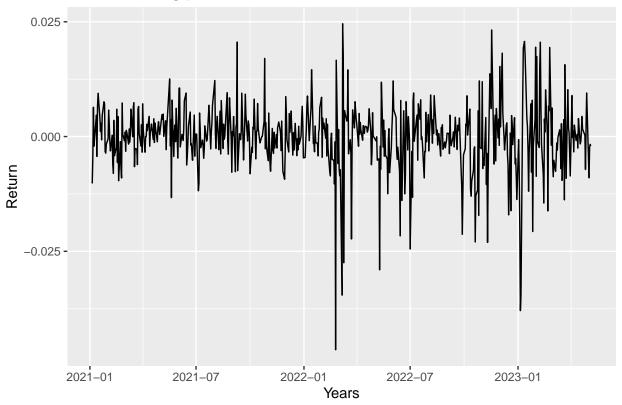


MSI 20 partial autocorrelation function (PACF)

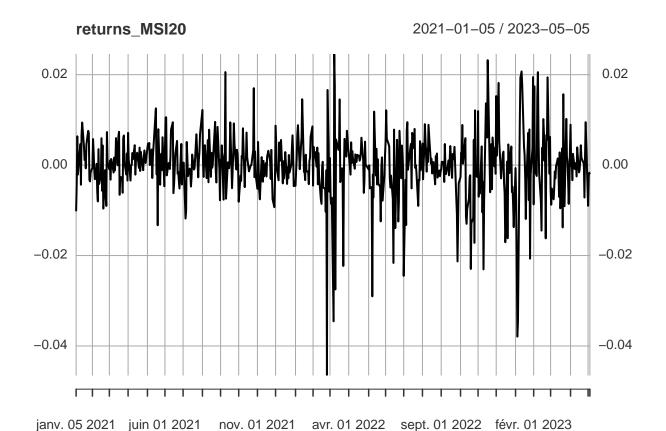


```
#Perform ADF test
adf_test <- adf.test(Close_Price)</pre>
#Print test results
print(adf_test)
##
    Augmented Dickey-Fuller Test
##
## data: Close_Price
## Dickey-Fuller = -1.701, Lag order = 8, p-value = 0.7049
## alternative hypothesis: stationary
returns_MSI20 <- diff(log(Close_Price))[-1,]</pre>
head(returns_MSI20)
                       [,1]
## 2021-01-05 -0.010213886
## 2021-01-06 -0.004195757
## 2021-01-07 0.006368443
## 2021-01-08 -0.002085688
## 2021-01-12 0.004654224
## 2021-01-13 -0.004338919
autoplot(returns_MSI20) +
  labs(x = "Years",
       y = "Return",
       title = "MSI 20 closing price returns")
```





plot(returns_MSI20)



library(gridExtra)

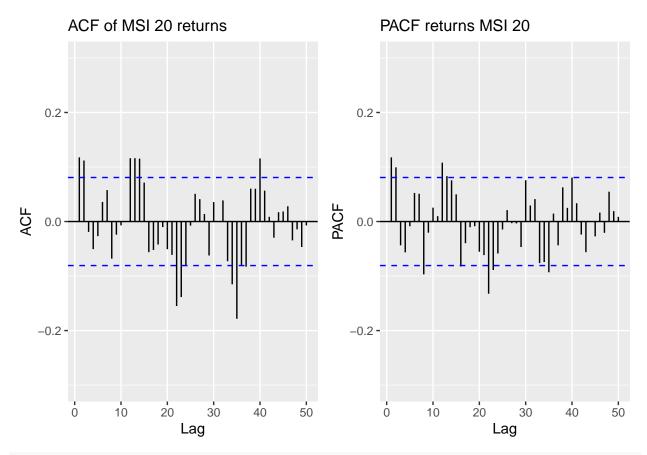
```
## Warning: le package 'gridExtra' a été compilé avec la version R 4.4.2

ACF_plot <- ggAcf(returns_MSI20, main="ACF of MSI 20 returns",lag=50, ylim=c(-0.3,0.3))+
    theme(plot.title = element_text(size = 12))

## Warning in ggplot2::geom_segment(lineend = "butt", ...): Ignoring unknown
## parameters: `main` and `ylim`

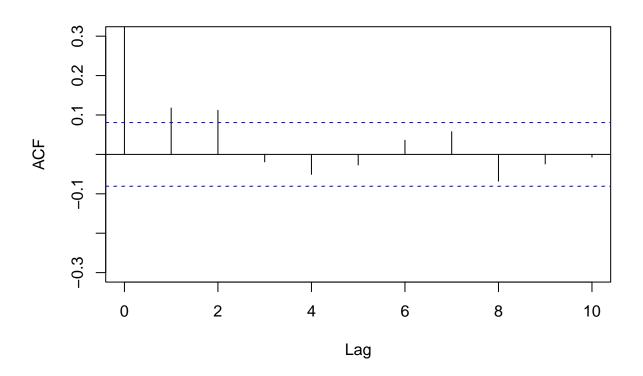
PACF_plot <- ggPacf(returns_MSI20, main= "PACF returns MSI 20", lag=50,ylim=c(-0.3,0.3))+
    theme(plot.title = element_text(size = 12))

## Warning in ggplot2::geom_segment(lineend = "butt", ...): Ignoring unknown
## parameters: `main` and `ylim`
grid.arrange(ACF_plot, PACF_plot, nrow = 1)</pre>
```



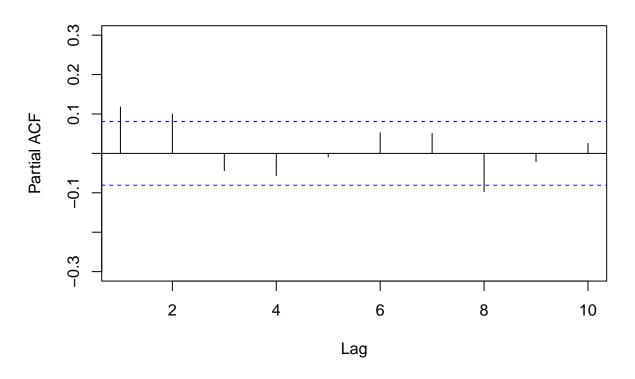
ACF_plot <- acf(returns_MSI20, main="ACF of MSI 20 returns",lag=10, ylim=c(-0.3,0.3))+ theme(plot.title = element_text(size = 12),las=1)

ACF of MSI 20 returns



```
PACF_plot <- pacf(returns_MSI20, main= "PACF returns MSI 20", lag=10,ylim=c(-0.3,0.3))+ theme(plot.title = element_text(size = 12))
```

PACF returns MSI 20



```
pacf(returns_MSI20, main= "PACF returns MSI 20", lag=10,ylim=c(-0.3,0.3))+
theme(plot.title = element_text(size = 12))
```

PACF returns MSI 20

```
Partial ACF

-0.3

-0.1

0.1

0.0 0.3

1.0 0.5

2

4

6

8

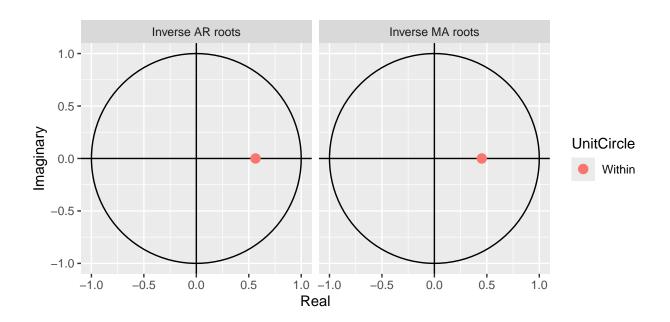
10

Lag
```

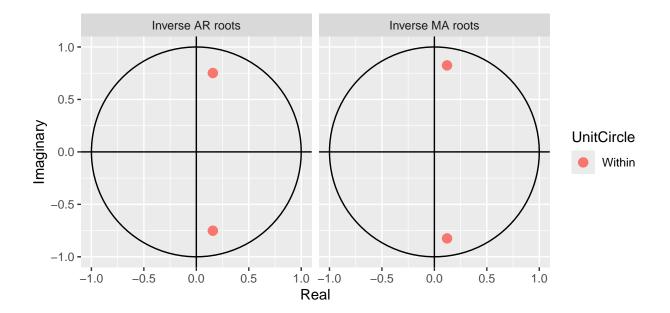
```
## NULL
# Phillips-Perron (PP) test
pp_test <- pp.test(returns_MSI20)</pre>
## Warning in pp.test(returns_MSI20): p-value smaller than printed p-value
print(pp_test)
##
  Phillips-Perron Unit Root Test
##
##
## data: returns_MSI20
## Dickey-Fuller Z(alpha) = -531.84, Truncation lag parameter = 6, p-value
## = 0.01
## alternative hypothesis: stationary
# Kwiatkowski-Phillips-Schmidt-Shin-Test (KPSS)
kpss_test <- kpss.test(returns_MSI20)</pre>
print(kpss_test)
##
   KPSS Test for Level Stationarity
##
##
## data: returns_MSI20
## KPSS Level = 0.36255, Truncation lag parameter = 6, p-value = 0.0933
# Augmented Dickey-Fuller (ADF) test
Adf_test <- adf.test(returns_MSI20)</pre>
```

```
## Warning in adf.test(returns_MSI20): p-value smaller than printed p-value
print(Adf_test)
##
   Augmented Dickey-Fuller Test
##
## data: returns_MSI20
## Dickey-Fuller = -8.4448, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
# Augmented Dickey-Fuller (ADF) tests
adf test <- ur.df(returns MSI20)
summary(adf_test)
##
## # Augmented Dickey-Fuller Test Unit Root Test #
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##
                  1Q
                        Median
        Min
                                     3Q
                                             Max
## -0.045280 -0.003468 0.000159 0.003296 0.028929
##
## Coefficients:
            Estimate Std. Error t value Pr(>|t|)
##
## z.lag.1
            -0.79472 0.05459 -14.559
                                       <2e-16 ***
## z.diff.lag -0.09993
                        0.04107 -2.433
                                        0.0153 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.007468 on 585 degrees of freedom
## Multiple R-squared: 0.4471, Adjusted R-squared: 0.4452
## F-statistic: 236.5 on 2 and 585 DF, p-value: < 2.2e-16
##
##
## Value of test-statistic is: -14.5593
## Critical values for test statistics:
        1pct 5pct 10pct
## tau1 -2.58 -1.95 -1.62
# splitting into train and test data
train_MSI20 <- Close_Price[1:531] # 2021-01-04 --> 2023-02-09 (531 Obs.)
test_MSI20 <- Close_Price[532:590] # 2023-02-10 -->2023-05-05 (59 Obs.)
model1 <- arima(train_MSI20, order=c(1,1,1),method="CSS")</pre>
library(lmtest)
coeftest(model1)
```

```
## z test of coefficients:
##
##
       Estimate Std. Error z value Pr(>|z|)
## ar1 0.49937
                 0.18379 2.7171 0.006585 **
## ma1 -0.38338
                  0.19092 -2.0081 0.044637 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
calculate bic <- function(arima model, data) {</pre>
  # Calculate the number of parameters in the model
  k <- length(coef(arima_model))</pre>
  # Calculate BIC
  bic <- -2 * logLik(arima_model) + k * log(length(data))</pre>
  return(bic)
}
model1 <- arima(train_MSI20, order=c(1,1,1),method="ML")</pre>
coeftest(model1)
##
## z test of coefficients:
##
       Estimate Std. Error z value Pr(>|z|)
                 0.20093 2.8087 0.004975 **
## ar1 0.56434
## ma1 -0.45181
                   0.21607 -2.0910 0.036524 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
summary(model1)
##
## Call:
## arima(x = train_MSI20, order = c(1, 1, 1), method = "ML")
## Coefficients:
##
            ar1
                     ma1
         0.5643 -0.4518
## s.e. 0.2009
                 0.2161
##
## sigma^2 estimated as 52.32: log likelihood = -1800.74, aic = 3607.47
## Training set error measures:
##
                                RMSE
                                          MAE
                                                      MPE
                                                                MAPE
                                                                          MASE
## Training set -0.08627774 7.226294 4.952053 -0.01168927 0.5111407 0.9948819
##
## Training set -0.01533004
bic_model1 <- calculate_bic(model1, train_MSI20)</pre>
print(bic_model1)
## 'log Lik.' 3614.02 (df=3)
library(knitr)
opts_knit$set(global.par = TRUE)
```



```
model2 <- arima(train_MSI20, order=c(2,1,2),method="ML")</pre>
coeftest(model2)
##
## z test of coefficients:
##
##
      Estimate Std. Error z value Pr(>|z|)
## ar1 0.31483 0.19427 1.6206 0.1051081
## ar2 -0.58963
               0.20794 -2.8356 0.0045747 **
0.20822 3.3369 0.0008471 ***
## ma2 0.69482
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
summary(model2)
##
## Call:
## arima(x = train_MSI20, order = c(2, 1, 2), method = "ML")
## Coefficients:
##
           ar1
                   ar2
                          ma1
                                  ma2
##
        0.3148 -0.5896 -0.2415 0.6948
## s.e. 0.1943 0.2079 0.1605 0.2082
```



```
model3 <- arima(train_MSI20, order=c(2,1,0),method="ML")
coeftest(model3)

##

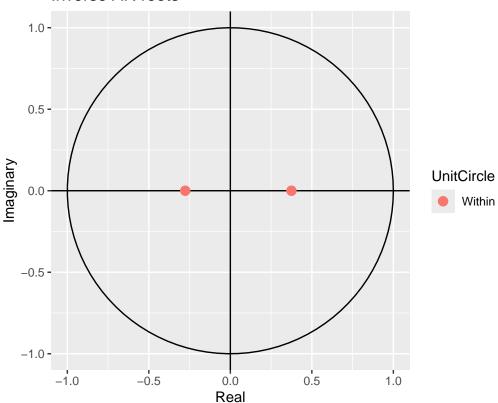
## z test of coefficients:
##

## Estimate Std. Error z value Pr(>|z|)
## ar1 0.098045  0.043223  2.2683  0.02331 *
## ar2 0.103647  0.043439  2.3861  0.01703 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

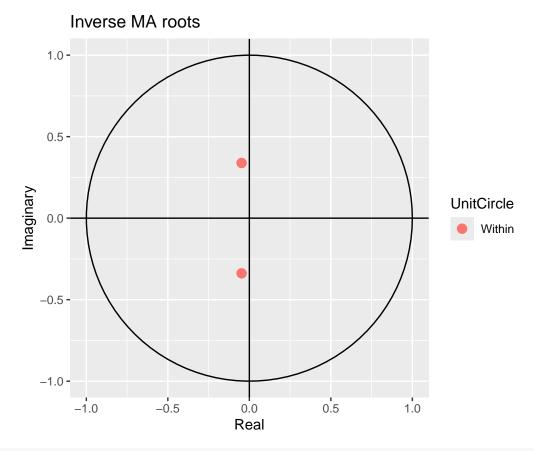
```
summary(model3)
##
## Call:
## arima(x = train_MSI20, order = c(2, 1, 0), method = "ML")
##
## Coefficients:
##
            ar1
                    ar2
         0.0980 0.1036
##
## s.e. 0.0432 0.0434
##
## sigma^2 estimated as 52.09: log likelihood = -1799.58, aic = 3605.15
##
## Training set error measures:
##
                                RMSE
                                                       MPE
                                                                MAPE
                                                                           MASE
                         ME
                                           MAE
## Training set -0.08740403 7.210452 4.929905 -0.01181901 0.5087557 0.9904324
##
                        ACF1
## Training set 0.0003644483
bic_model3 <- calculate_bic(model3, train_MSI20)</pre>
print(bic_model3)
## 'log Lik.' 3611.704 (df=3)
par(mar=c(5,5,0,0))
```

Inverse AR roots

autoplot(model3)

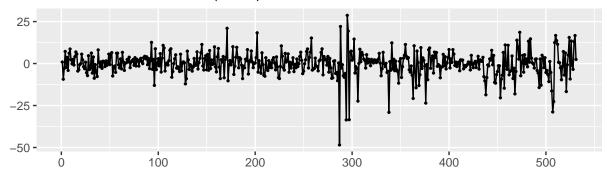


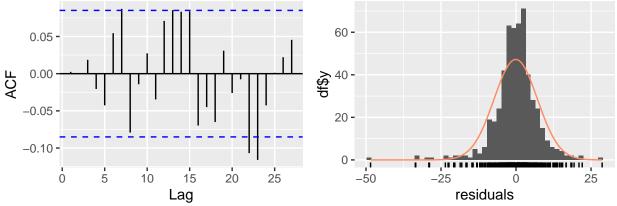
```
model4 <- arima(train_MSI20, order=c(0,1,2),method="ML")</pre>
coeftest(model4)
## z test of coefficients:
##
##
      Estimate Std. Error z value Pr(>|z|)
## ma1 0.095003 0.043070 2.2058 0.02740 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
summary(model4)
##
## Call:
## arima(x = train_MSI20, order = c(0, 1, 2), method = "ML")
##
## Coefficients:
##
           ma1
                  ma2
        0.0950 0.1168
##
## s.e. 0.0431 0.0441
## sigma^2 estimated as 52.07: log likelihood = -1799.5, aic = 3604.99
## Training set error measures:
                              RMSE
                                        MAE
                                                   MPE
                                                            MAPE
                                                                     MASE
                       ME
## Training set -0.09243934 7.209352 4.926385 -0.01250443 0.5083449 0.9897251
##
## Training set 0.002415873
bic_model4 <- calculate_bic(model4, train_MSI20)</pre>
print(bic_model4)
## 'log Lik.' 3611.543 (df=3)
par(mar=c(5,5,0,0))
autoplot(model4)
```



checkresiduals(model4)

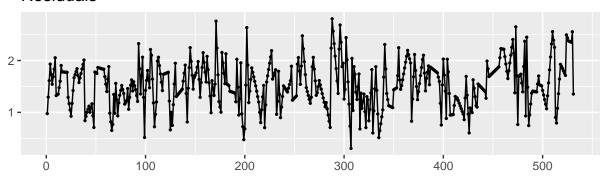
Residuals from ARIMA(0,1,2)

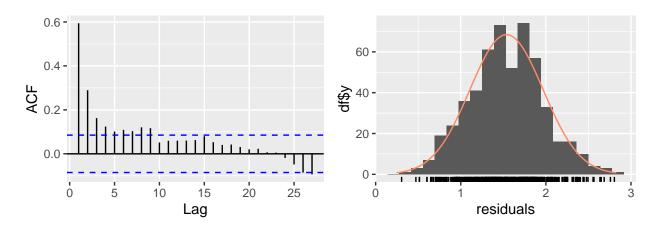




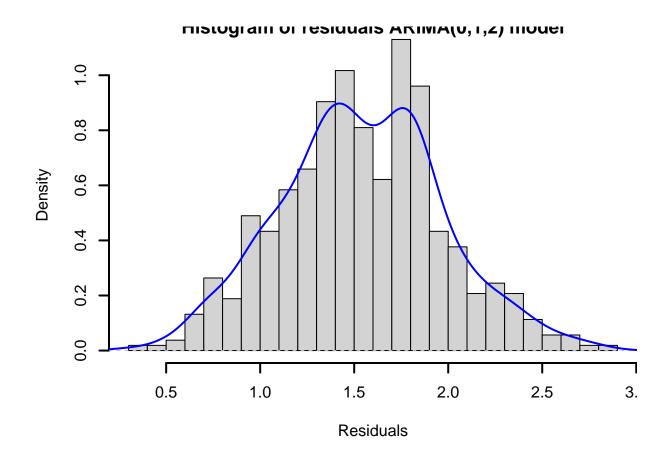
```
##
##
   Ljung-Box test
##
## data: Residuals from ARIMA(0,1,2)
## Q* = 11.005, df = 8, p-value = 0.2014
##
                  Total lags used: 10
## Model df: 2.
resi.ima <- residuals(model4)</pre>
tsoutliers(resi.ima)
## $index
## [1] 287 294 295 297 338 376 507
##
## $replacements
                    2.544107 10.948620 13.014059 -1.741477 -1.093739 -19.484213
## [1] 11.210394
resid_ts_clean_ima = tsclean(resi.ima)
#resid_ts_clean_ima_tra = sqrt(resid_ts_clean_ima)
resid_ts_clean_ima_tra = (resid_ts_clean_ima)^(1/3)
# Assuming resid_ts_clean_ima_tra is a time series object
# Impute missing values using linear interpolation
resid_imputed <- na.approx(resid_ts_clean_ima_tra)</pre>
```

Residuals





```
##
## Ljung-Box test
##
## data: Residuals
## Q* = 289.95, df = 10, p-value < 2.2e-16
## Model df: 0. Total lags used: 10
hist(resid_imputed, prob=TRUE, 24, main = paste("Histogram of residuals ARIMA(0,1,2) model"),
    xlab ="Residuals", lwd=2) # histogram
# Grid below plot
# Vertical grid
abline(v = seq(-0.02, 0.02, 0.0025),
      lty = 2, col = "gray")
# Horizontal grid
abline(h = seq(0, 80, 10),
      lty = 2, col = "gray")
lines(density(resid_imputed), type="l", col="blue", lwd=2) # smooth it - ?density for details
```



```
library(ggpubr)

## Warning: le package 'ggpubr' a été compilé avec la version R 4.4.2

##

## Attachement du package : 'ggpubr'

## L'objet suivant est masqué depuis 'package:forecast':

##

## gghistogram

ggqqplot(resid_imputed,title = "Q-Q plot of residuals from ARIMA(0,1,2) model", merge = FALSE)

## Don't know how to automatically pick scale for object of type <ts>. Defaulting

## to continuous.

## Don't know how to automatically pick scale for object of type <ts>. Defaulting

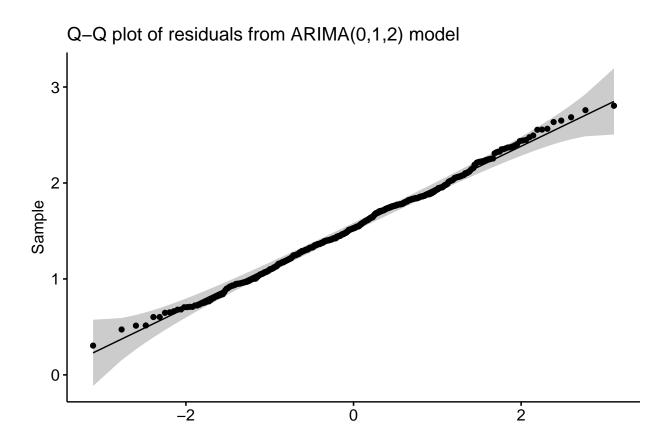
## to continuous.

## Don't know how to automatically pick scale for object of type <ts>. Defaulting

## to continuous.

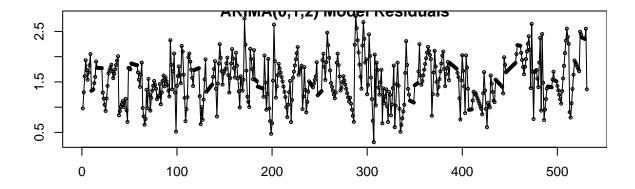
## Don't know how to automatically pick scale for object of type <ts>. Defaulting

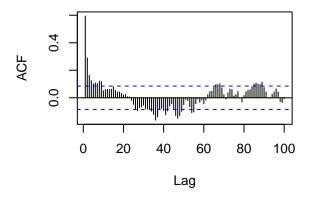
## to continuous.
```

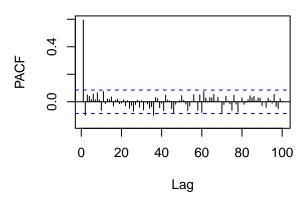


Theoretical

tsdisplay(resid_imputed, lag.max=100, main="ARIMA(0,1,2) Model Residuals")







```
#Tests auto-correlation of order greater than 1
Box.test(resid_imputed, lag = 1, type = c("Box-Pierce", "Ljung-Box"), fitdf = 0)
```

```
##
    Box-Pierce test
##
## data: resid_imputed
## X-squared = 187.32, df = 1, p-value < 2.2e-16
# Test of normality
# Perform the Jarque-Bera test on the imputed time series
result_test_jb <- jarque.bera.test(resid_imputed)</pre>
print(result_test_jb)
##
##
    Jarque Bera Test
##
## data: resid_imputed
## X-squared = 0.75552, df = 2, p-value = 0.6854
# Perform the Shapiro-Wilk test
resultat_test_sh <- shapiro.test(resid_imputed)</pre>
# Show test results
print(resultat_test_sh)
```

##
Shapiro-Wilk normality test

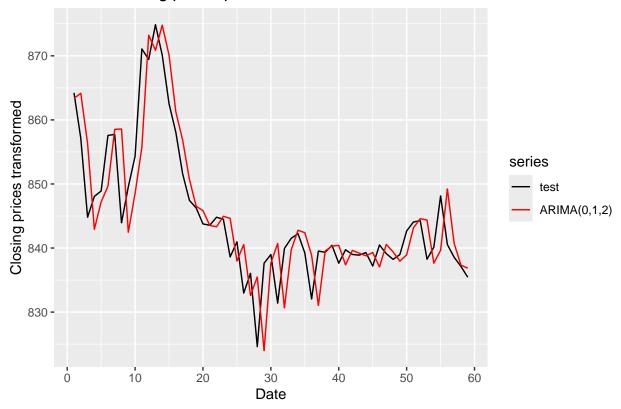
##

```
##
## data: resid_imputed
## W = 0.99693, p-value = 0.4171

# Load package
library(nortest)
forecast_test <- Arima(test_MSI20,model=model4)$fitted
autoplot(ts(test_MSI20), series = 'test',xlab ="Date",ylab="Closing prices transformed") +
autolayer(forecast_test, series = 'ARIMA(0,1,2)',PI =FALSE) +
ggtitle("MSI 20 closing prices prediction")+
scale_colour_manual(values = c('test'='black', 'ARIMA(0,1,2)'='red'),
breaks = c('test', 'ARIMA(0,1,2)'))</pre>
```

Warning in ggplot2::geom_line(ggplot2::aes(x = .data[["timeVal"]], y =
.data[["seriesVal"]], : Ignoring unknown parameters: `PI`

MSI 20 closing prices prediction



```
## Create a time series object
## create the zoo object as before
forecast_ts <- zoo(forecast_test , dailyMSI20[532:590,1]) ### 2023-02-10
#to 2023-05-05 (59 Obs.)
head(forecast_ts)</pre>
```

2023-02-10 2023-02-13 2023-02-14 2023-02-15 2023-02-16 2023-02-17 ## 863.3658 864.1510 856.3266 842.9247 847.2119 849.6599

```
tail(forecast_ts)
## 2023-04-27 2023-04-28 2023-05-02 2023-05-03 2023-05-04 2023-05-05
                                                 837.3308
              839.6428
                          849.2266
                                     840.7059
## 837.6344
forecast_ts <- ts(forecast_test,</pre>
           start = c(2023, as.numeric(format(dailyMSI20[532:590,1], "%j"))))
head(forecast_ts)
## Time Series:
## Start = 2063
## End = 2068
## Frequency = 1
## [1] 863.3658 864.1510 856.3266 842.9247 847.2119 849.6599
# merge into multivariate time series
ts.merge <- merge(test_MSI20, as.zoo(forecast_ts))</pre>
colnames(ts.merge) <- c("ts_test", "ts_forecast")</pre>
#as.ts(ts.merge)
head(ts.merge)
              ts_test ts_forecast
## 2023-02-10 864.23
                        863.3658
## 2023-02-13 857.13
                        864.1510
## 2023-02-14 844.81 856.3266
## 2023-02-15 848.06 842.9247
## 2023-02-16 848.90
                        847.2119
## 2023-02-17 857.58
                        849.6599
tail(ts.merge)
              ts_test ts_forecast
## 2023-04-27 840.12
                        837.6344
## 2023-04-28 848.13
                        839.6428
## 2023-05-02 840.54 849.2266
## 2023-05-03 838.55
                        840.7059
## 2023-05-04 837.13
                        837.3308
## 2023-05-05 835.44
                        836.8592
forecast_model4 <- forecast(model4, h = 59)</pre>
accuracy(forecast_model4,test_MSI20)
                         ME
                                  RMSE
                                            MAE
                                                         MPE
                                                                  MAPE
                                                                            MASE
## Training set -0.09243934 7.209352 4.926385 -0.01250443 0.5083449 0.9897251
## Test set
              -23.28370316 25.474242 23.718416 -2.77102972 2.8208400 4.7650993
                       ACF1
## Training set 0.002415873
## Test set
```