Recursive Regularization for Large-scale Classification with Hierarchical and Graphical Dependencies

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Outline of the Talk

- Motivation
- Related work
- Proposed model and Optimization
- Experiments

Motivation

- Big data era - easy access to lots of structured data.
- Hierarchies and graphs provide a natural way to organize data.
- For example
  1. **Open Directory Project** - A collection of Billions of webpages into a hierarchy with $\sim$ 300,000 classes.
  2. **International Patent Taxonomy** - Millions of patents across the world follow this hierarchy.
  3. **Wikipedia pages** - Millions of wikipedia pages have associated categories which are linked to each other.

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How to scale to large number of classes ?

Scalability

Some existing datasets

Dataset	#Instances	#Labels	#Features	#Parameters
ODP subset	394,756	27,875	594,158	16,562,154,250
Wikipedia subset	2,365,436	325,056	1,617,899	525,907,777,344

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Focus

1. How to use interclass dependencies ?
2. How to scale ?

Related Work

- **Earlier works** Top-down *pachinko machine* style approaches [Dumais and Chen, 2000], [Yang et al., 2003] [Liu et al., 2005], [Koller and Sahami, 1997]
- **Large-margin methods**
  1. Maximize the margin between correct and incorrect labels based on a hierarchical loss.
  2. Discriminant functions takes contribution from all nodes along the path to root-node.

  [Tsochantaridis et al., 2006], [Cai and Hofmann, 2004], [Rousu et al., 2006], [Dekel et al., 2004], [Cesa-Bianchi et al., 2006]
- **Bayesian methods** Hierarchical Naive Bayes [McCallum et al., 1998] , Correlated Multinomial Logit [Shahbaba and Neal, 2007] , Hierarchical Bayesian logistic regression [Gopal et al., 2012]

Notations

Given training examples and hierarchy

1. Hierarchy of nodes $\mathcal{N}$ defined by parent function $\pi(n)$.

2. N training examples,
   - x_i denote i^{th} instance
   - y_{in} denotes whether x_i is labeled to node n.

3. $\mathcal{T}$ denotes set of leaf nodes.

4. C_n denotes the set of child-nodes of node n.

Proposed model

Learn a prediction function with parameters **W**. Estimate **W** as

$$\arg\min_{\mathbf{W}} \lambda(\mathbf{W}) + C \times R_{emp}$$

Each node n is associated with parameter vector w_n.

Proposed model

Define R_{emp} as the empirical loss using loss function L at the leaf-nodes.

$$R_{emp} = \sum_{i=1}^{N} \sum_{n \in \mathcal{T}} L(w_n^\top x_i, y_{in})$$

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Incorporate the hierarchy into regularization term $\lambda(\mathbf{W})$

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Incorporate the hierarchy into regularization term $\lambda(\mathbf{W})$

$$\lambda(\mathbf{W}) = \sum_{n \in \mathcal{N}} \|w_n - w_{\pi(n)}\|^2$$

With a graph with edges $E \subset \{(i,j) : i,j \in \mathcal{N}\}$,

$$\lambda(\mathbf{W}) = \sum_{(i,j) \in E} \|w_i - w_j\|^2$$

Advantages

Advantages over other works

1. Structure not used in the Empirical Risk term.
2. Multiple independent problems that can be parallelized.
3. Flexibility in choosing a loss function.

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$$\textbf{[HR-SVM]} \quad \min_{\mathbf{W}} \sum_{n \in \mathcal{N}} \frac{1}{2} ||w_n - w_{\pi(n)}||^2 + C \sum_{n \in \mathcal{T}} \sum_{i=1}^{N} (1 - y_{in} w_n^\top x_i)_+$$

$$\textbf{[HR-LR]} \quad \min_{\mathbf{W}} \sum_{n \in \mathcal{N}} \frac{1}{2} ||w_n - w_{\pi(n)}||^2 + C \sum_{n \in \mathcal{T}} \sum_{i=1}^{N} \log(1 + \exp(-y_{in} w_n^\top x_i))$$

Optimizing with Hinge-loss

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- Large-number of parameters (2 Terabytes)
- Non-differentiability of Hinge-loss

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Solution

- Block-coordinate descent to handle large number of parameters (update one w_n at a time).
- Solve dual problem within block for non-differentiability.

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$$\textbf{Dual} \quad \min_{\boldsymbol{\alpha}} \quad \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_{in} y_{jn} x_i^\top x_j - \sum_{i=1}^{N}\alpha_i(1 - y_{in} w_{\pi(n)}^\top x_i)$$

$$s.t. \quad 0 \leq \boldsymbol{\alpha} \leq C$$

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[Use co-ordinate descent again ! Update one α_i at a time]

Optimizing HR-SVM

It turns out the each α_i has closed form update.

$$G = (\sum_{j=1}^{N} \alpha_j y_{jn} x_j)^\top x_i - 1 + y_{in} w_{\pi(n)}^\top x_i$$

$$\alpha_i^{new} = \min\left(\max\left(\alpha_i^{old} - \frac{G}{x_i^\top x_i}, 0\right), C\right)$$

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New time complexity : $O(nnz(x_i))$

Recover original primal solution, $w_n = w_{\pi(n)} + \sum_{i=1}^{N} \alpha_i y_{in} x_i$.

Optimizing HR-LR

$$\textbf{[HR-LR]} \quad \min_{\mathbf{W}} \sum_{n \in \mathcal{N}} \frac{1}{2} ||w_n - w_{\pi(n)}||^2 + C \sum_{n \in \mathcal{T}} \sum_{i=1}^{N} \log(1 + \exp(-y_{in} w_n^\top x_i))$$

1. Convex and Differentiable.
2. Block co-ordinate descent to handle parameter size.
3. LBFGS for optimization.

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RECAP

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3. Block co-ordinate descent to avoid memory issues.
4. Handle non differentiability using dual space.

Parallelization

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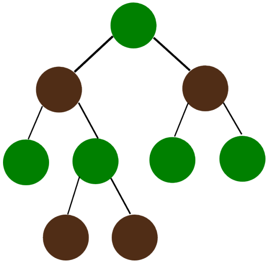
Can we update multiple blocks in parallel ?

Key point for parallelization: Parameters are only locally dependent.

1. In a hierarchy, the parameters of a node depend only parent and children.
2. In a graph, the parameters of a node depend on its neighbours.

Parallelization (cont)

- Hierarchies:
  1. Fix parameters at odd-levels, optimize even levels in parallel.
  2. Fix parameters at even-level, optimize odd levels in parallel.
  3. Repeat until convergence.



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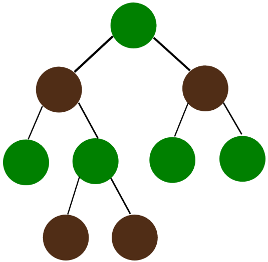
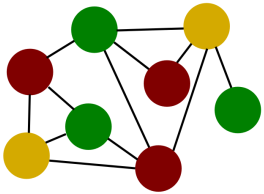


- Graphs: First find the minimum graph coloring [Np-hard]
  1. Pick a color.
  2. In parallel, optimize all nodes with that color.
  3. Repeat with a different color.



Experiments

DATASETS

Name	#Training	#Classes	#dims	Avg #labels per instance	Parameter size
CLEF	10,000	87	89	1	30 KB
RCV1	23,149	137	48,734	3.18	26 MB
IPC	46,324	552	541,869	1	1.1 GB
LSHTC-small	4,463	1,563	51,033	1	320 MB
DMOZ-2010	128,710	15,358	381,580	1	23 GB
DMOZ-2012	383,408	13,347	348,548	1	18 GB
DMOZ-2011	394,756	27,875	594,158	1.03	66 GB
SWIKI-2011	456,886	50,312	346,299	1.85	70 GB
LWIKI	2,365,436	614,428	1,617,899	3.26	2 TB

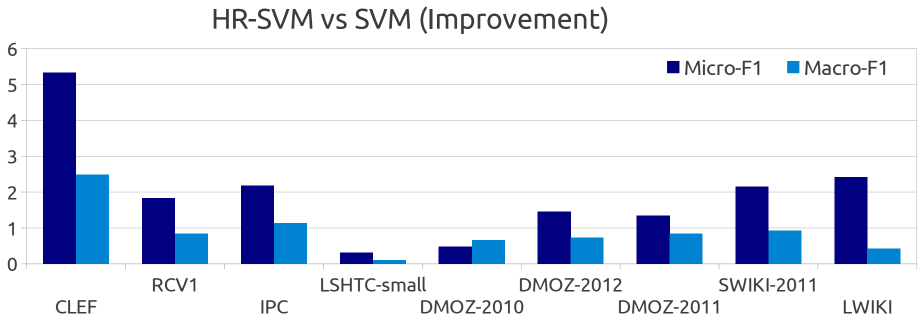
Comparison with published results

	LSHTC Published Results	HR-SVM	HR-LR
DMOZ-2010			
Macro-F_1	**34.12**	33.12	32.42
Micro-F_1	**46.76**	46.02	45.84
DMOZ-2012			
Macro-F_1	31.36	**33.05**	20.04
Micro-F_1	51.98	**57.17**	53.18
DMOZ-2011			
Macro-F_1	**26.48**	25.69	23.90
Micro-F_1	38.85	**43.73**	42.27
SWIKI-2011			
Macro-F_1	23.16	**28.72**	24.26
Micro-F_1	37.39	**41.79**	40.99
LWIKI			
Macro-F_1	18.68	**22.31**	20.22
Micro-F_1	34.67	**38.08**	37.67

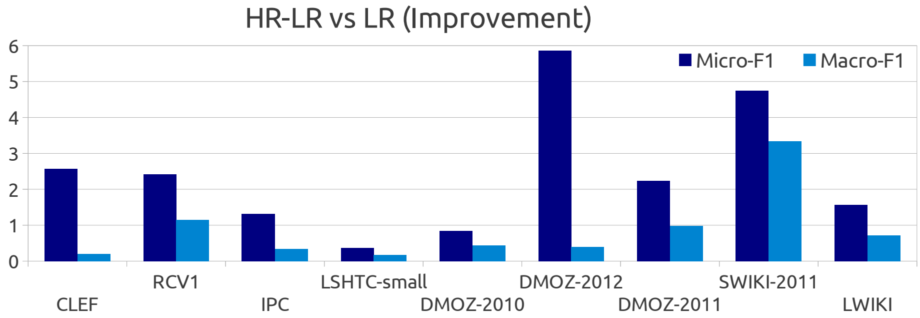
Methods for comparison

- **Flat baselines**:
  1. One-versus-rest binary Support Vector Machines (SVM)
  2. One-versus-rest regularized logistic regression (LR).
- **Hierarchical baselines**:
  1. **Hierarchical SVM** (HSVM) [Tsochantaridis et al., 2006] a large-margin discriminative method with path dependent discriminant function.
  2. **Hierarchical Orthogonal Transfer** (OT) [Zhou et al., 2011], a large-margin method enforcing orthogonality between the parent and the children.
  3. **Top-down SVM (TD)** a Pachinko-machine style SVM.
  4. **Hierarchical Bayesian Logistic Regression** (HBLR), [Gopal et al., 2012], our previous work using a fully Bayesian hierarchical model.
     1. Computationally more costly than HR-LR
     2. Not applicable for graph-based dependencies

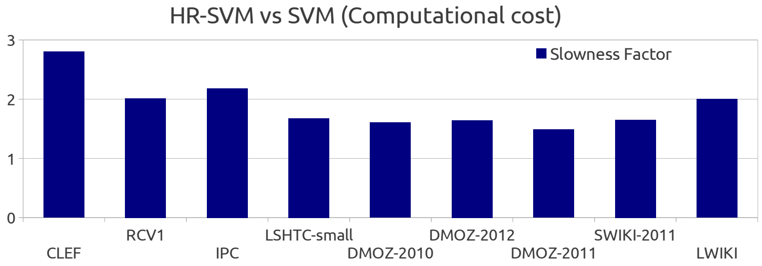
Against flat baselines

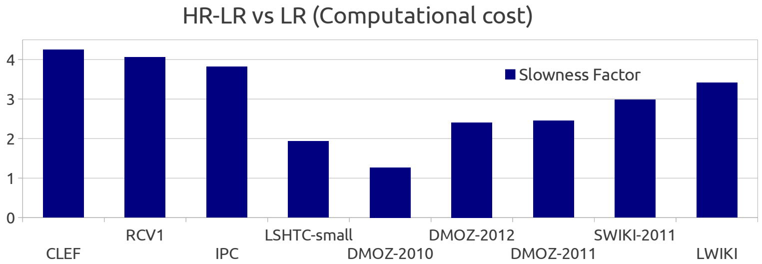


Against flat baselines



Time complexity





Conclusion

A Model that can

1. Use both hierarchial and graphical dependencies between classes to improve classification.
2. And can be scaled to real-world data.

Thanks !

Against Hierarchical Baselines

Micro-F1 comparison

Datasets	**HR-SVM**	**TD**	**HSVM**	**OT**	**HBLR**
CLEF	80.02	70.11	79.72	73.84	**81.41**
RCV1	**81.66**	71.34	NA	NS	NA
IPC	54.26	50.34	NS	NS	**56.02**
LSHTC-small	45.31	38.48	39.66	37.12	**46.03**
DMOZ-2010	**46.02**	38.64	NS	NS	NA
DMOZ-2012	**57.17**	55.14	NS	NS	NA
DMOZ-2011	**43.73**	35.91	NA	NS	NA
SWIKI-2011	**41.79**	36.65	NA	NA	NA
LWIKI	**38.08**	NA	NA	NA	NA

[NA - Not applicable, NS - Not scalable]

Time complexity

Time (in mins)

Datasets	HR-SVM	TD	HSVM	OT	HBLR
CLEF	.42	.13	3.19	1.31	3.05
RCV1	.55	.213	NA	NS	NA
IPC	6.81	2.21	NS	NS	31.2
LSHTC-small	.52	.11	289.60	132.34	5.22
DMOZ-2010	8.23	3.97	NS	NS	NA
DMOZ-2012	36.66	12.49	NS	NS	NA
DMOZ-2011	58.31	16.39	NA	NS	NA
SWIKI-2011	89.23	21.34	NA	NA	NA
LWIKI	2230.54	NA	NA	NA	NA

Conclusion

1. A **scalable** framework that can leverage class-label dependencies.
2. and that works in practice !

Cai, L. and Hofmann, T. (2004).
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