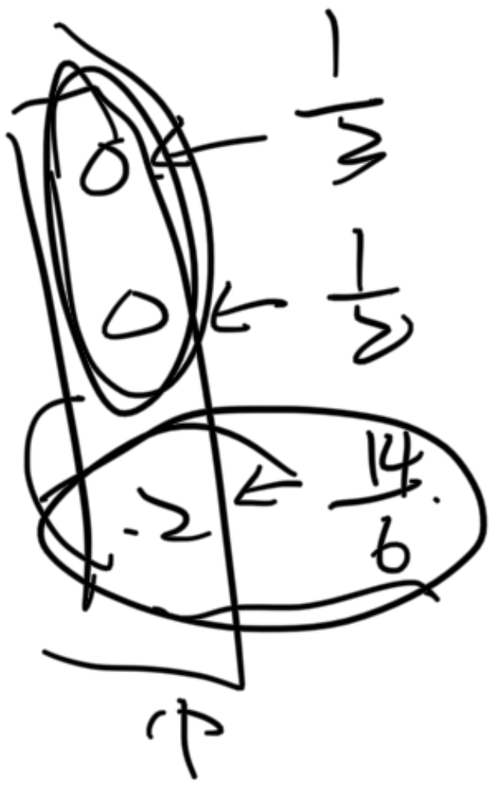
3

1 1

2 3

7 4

4 6




struct {

10

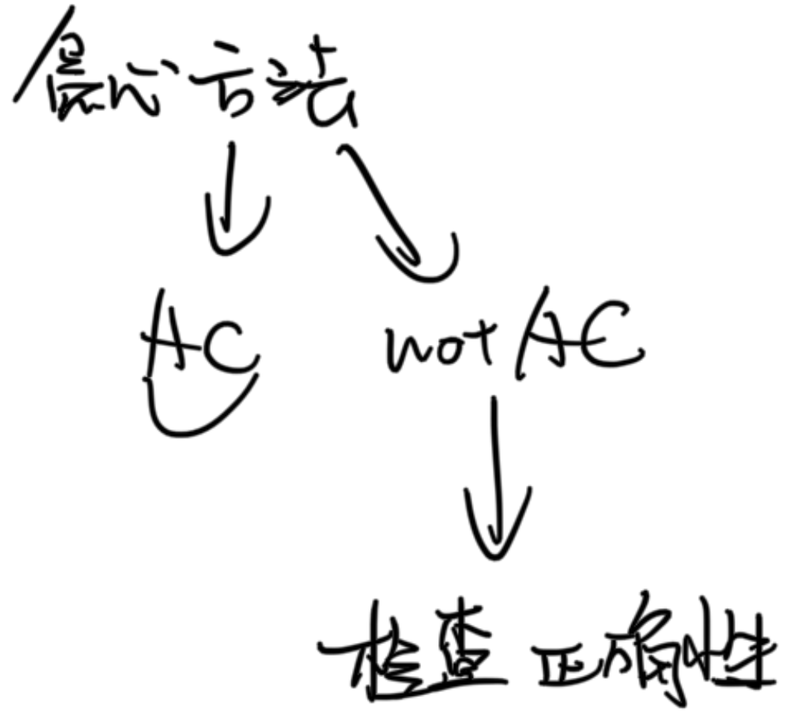
a b

}

$a_i$ $b_i$

$a_1 \cdots\cdots a_{i-1}$

$b_i$



$a_i$ $b_i$

$$\max \frac{a'_1 \times \cdots \times a'_i}{b'_i}$$

证明

例子

错误 → 反例

| | | | |
|---|---|---|---|
| 1 | 1 | | |
| 4 | 7 | $\frac{1}{7}$ | → 0 |
| 2 | 1 | 4 | → 4 |
| 3 | 1 | 8 | → 8 |

无法 → 反证

| | | | |
|---|---|---|---|
| 1 | 1 | | 8 |
| 4 | 7 | $\frac{1}{7}$ | → 0 |
| 3 | 1 | 4 | → 4 |
| 2 | 1 | 12 | → 12 |

最优性

⇒ 反推

$\{a_1,b_1\}$ $\{a_2,b_2\}$ ——— $\{a_n,b_n\}$

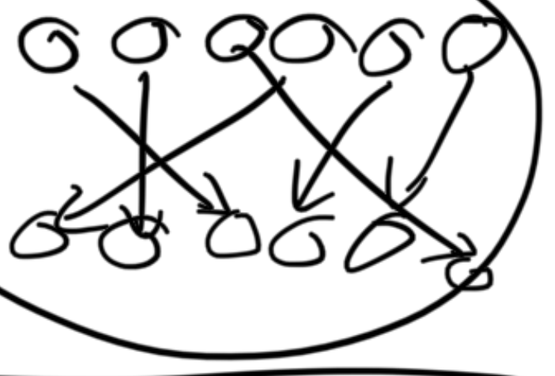
$a_0b_0$ 通过排列 $\{a_1b_1\}$ $\{a_2,b_2\}$ ——— $\{a_n,b_n\}$ 下标顺序

使得 $\left(\max_{1\le i\le n}\frac{a_0\times\cdots\times a_{i-1}}{b_i}\right)$ 最大

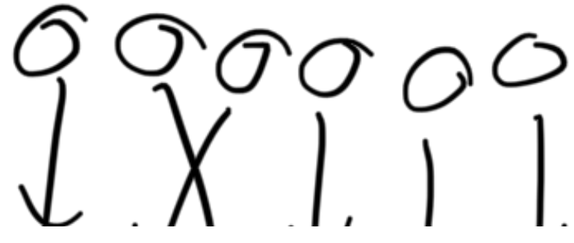
$>$ $a_1^*,b_1^*$ $a_2^*b_2^*$ ——— $a_n^*b_n^*$ 最优解

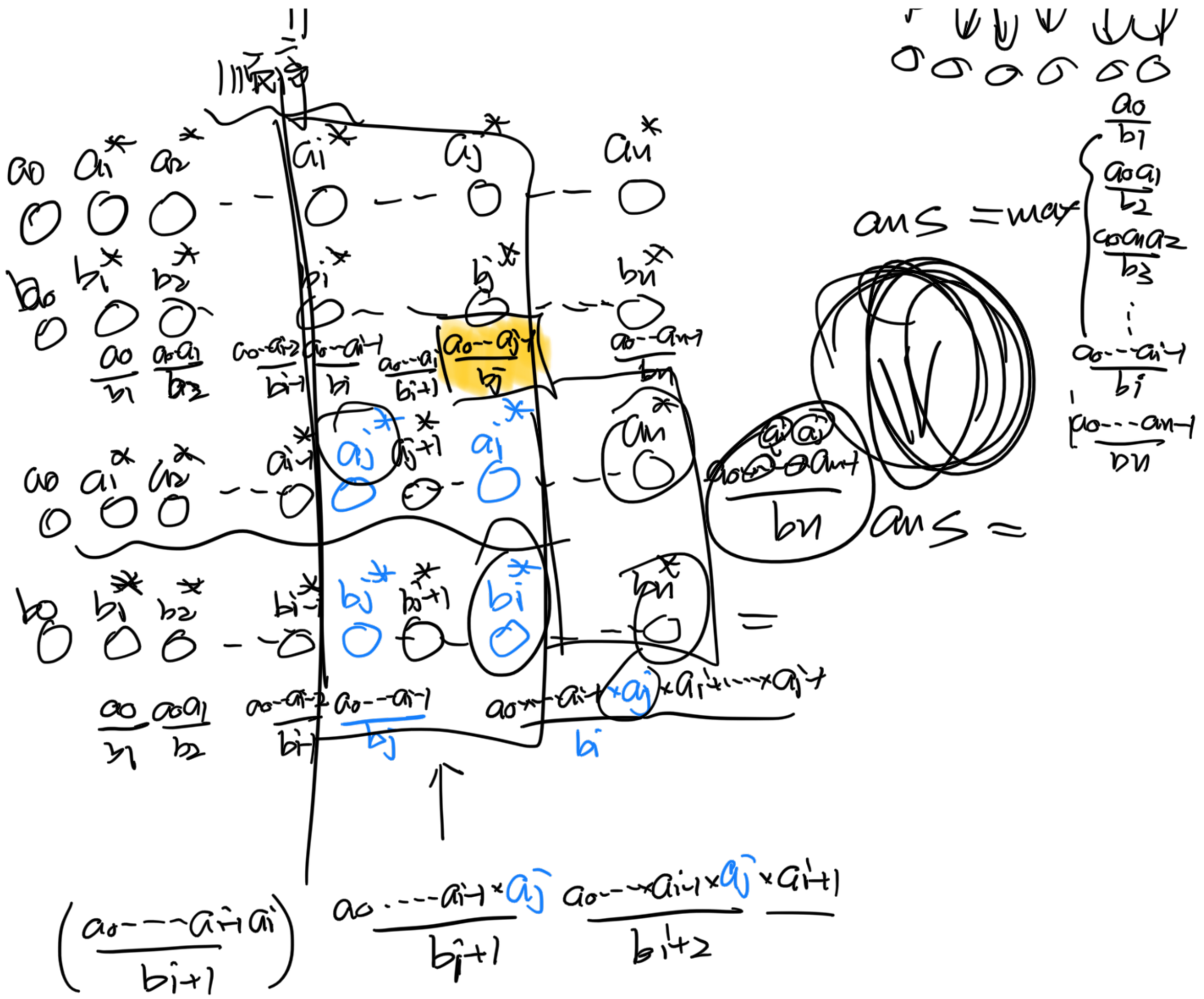
任意打乱后，答案会变小

（不会变大）



交换任意 $\{a_i^*,b_i^*\}$ $\{a_j^*,b_j^*\}$ 后答案会变小




ans = max
ans =
=

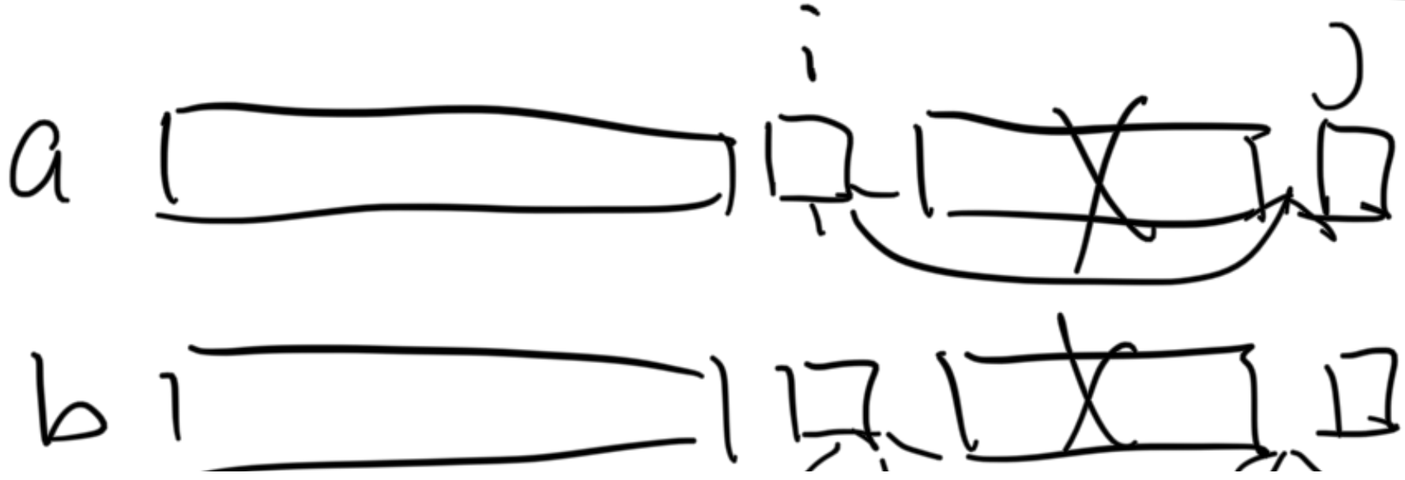
$$\max\left(\frac{a_0 \times \cdots \times a_{i-1}}{b_i} \quad \frac{a_0 \times \cdots \times a_i}{b_{i+1}} \quad \cdots \quad \frac{a_0 \times a_i \times a_{j-1}}{b_j}\right)$$

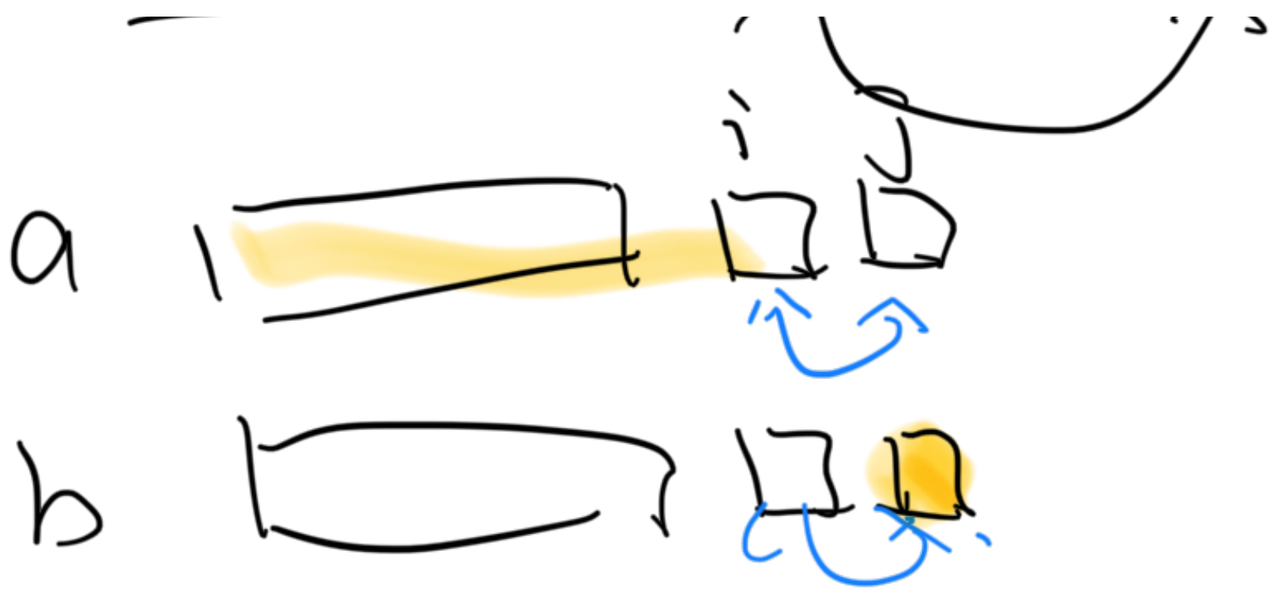
$$\geq \max\left(\frac{a_0 \times \cdots \times a_{i-1}}{b_j} \quad \frac{a_0 \times \cdots \times a_{i-1} \times a_j}{b_{i+1}} \quad \cdots \quad \frac{a_0 \times \cdots \times a_{i-1} \times a_j \times a_{i+1} \times \cdots a_{j-1}}{b_i}\right)$$

$j = i+1$

$$\max\left(\frac{a_0 \times \cdots \times a_{i-1}}{b_i}, \frac{a_0 \times \cdots \times a_{i-1} \times a_i}{b_{i+1}}\right)$$

$$\geq \max\left(\frac{a_0 \times \cdots \times a_{i-1}}{b_{i+1}}, \frac{a_0 \times \cdots \times a_{i-1} \times a_{i+1}}{b_i}\right)$$







$$\max\left(\frac{1}{b_i}, \frac{a_i}{b_{i+1}}\right) \geq \max\left(\frac{1}{b_{i+1}}, \frac{a_{i+1}}{b_i}\right)$$

正整数

$\times b_i \times b_{i+1}$

$$\max(b_{i+1}, a_i b_i) \geq \max(b_i, a_{i+1} b_{i+1})$$

正整数

$\geq$

$\leq$

$a_i \geq 1$

$a_i b_i \geq b_i$

$\max(b_{i+1}, a_i b_i)$

$\max(\ ,\ )$

$\geq \max(a_{i+1}b_{i+1}, b_i)$

$\max(a_{i+1}b_{i+1}, b_i)$

$a_ib_i \geq a_{i+1}b_{i+1}$

最优性原理

| | $b_{i+1} > a_ib_i$ | $b_{i+1} < a_ib_i$ |
|---|---|---|
| $a_{i+1}b_{i+1} > b_i$ | $b_{i+1}$ 的 $> a_{i+1}b_{i+1}$ | $a_ib_i$ 的 $\rightarrow a_{i+1}b_{i+1}$ |
| $a_{i+1}b_{i+1} < b_i$ | $b_{i+1} > b_i$ | $a_ib_i > b_i$ |

I $\begin{cases} b_{i+1} > a_{i+1}b_{i+1} \\ b_{i+1} > a_ib_i \\ a_{i+1}b_{i+1} > b_i \end{cases}$ or II $\begin{cases} a_ib_i > a_{i+1}b_{i+1} \\ b_{i+1} < a_ib_i \\ a_{i+1}b_{i+1} > b_i \end{cases}$

$\max(b_{i+1}, a_ib_i) > \max(a_{i+1}b_{i+1}, b_i)$

$b_{i+1} > a_ib_i$

$b_{i+1} < a_ib_i$

$b_{i+1} > \max(a_{i+1}b_{i+1}, b_i)$

$a_ib_i > \max(a_{i+1}b_{i+1}, b_i)$

$a_{i+1}b_{i+1} > b_i$

$a_{i+1}b_{i+1} < b_i$

$a_{i+1}b_{i+1} > b_i$

$a_{i+1}b_{i+1} < b_i$

$a_ib_i > a_{i+1}b_{i+1}$

$a_ib_i > b_i$

① $b_{i+1} > a_{i+1}b_{i+1}$

$1 > a_{i+1}$ ✗

② $b_{i+1} > b_i$

③ $a_ib_i > a_{i+1}b_{i+1}$

④ $a_ib_i > b_i$

$b_{i+1} < a_ib_i$

$\begin{cases} a_{i+1}b_{i+1} < b_i \\ a_ib_i > b_i \end{cases}$

$a_i > 1$

$a_{i+1}b_{i+1} < b_i < b_{i+1}$

$\begin{cases} b_{i+1} > a_ib_i \\ a_{i+1}b_{i+1} < b_i \\ b_{i+1} > b_i \end{cases}$

$a_{i+1} < 1$

(2) $\rightarrow b_{i+1}$

③

$a_i b_i > b_i$

$a_{i+1} b_{i+1} > b_i$

$a_i b_i > a_{i+1} b_{i+1}$

$a_{i+1} b_{i+1} < b_i < a_i b_i$

or

④

$b_{i+1} < a_i b_i$

$a_{i+1} b_{i+1} < b_i$

$a_i b_i > b_i$

$a_i b_i > b_{i+1}$

~~$a_i b_i > b_i$~~

$a_i b_i > a_{i+1} b_{i+1}$

$a_i b_i \geq a_{i+1} b_{i+1}$

Sort (

fun.

$a_i b_i > a_{i+1} b_{i+1}$