Homework 1

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Abstract

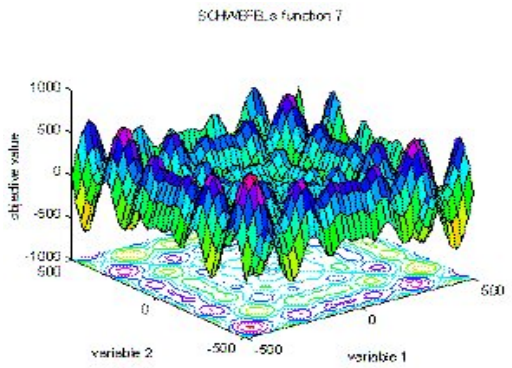
The report comprises the analysis of heuristic algorithms purposely designed to determine extreme points of functions in the case of deterministic methods/algorithms being not suitable. The inquiry cover observations of their results given different methods, strategies and initial parameters.

1 Introduction

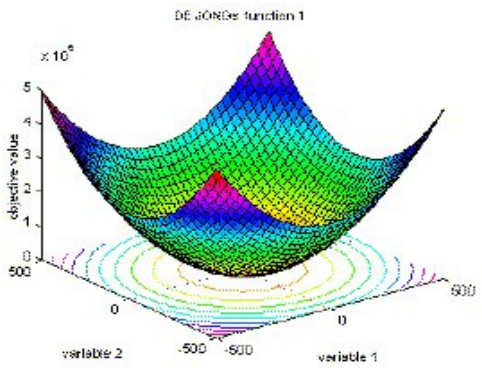
The sections of the report include implementation methods, experiment results of the algorithms, comparisons and conclusions.

The problem is concerned with optimizing multidimensional functions. Our problem at hand is finding the global minimum of such functions. The problem can become intricate especially if the functions that are desired to be optimized are deliberately designed or simply have many local minimum points.

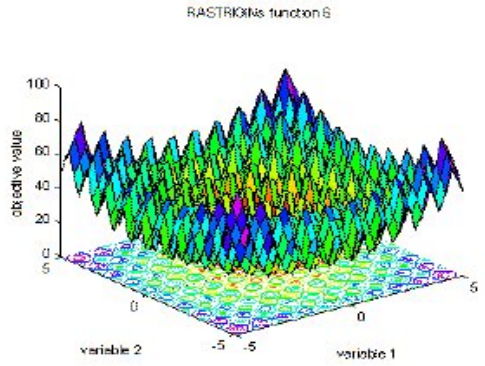
The algorithms are tested on four functions: Schwefel, DeJong, Rastrigin and Michalewicz. The functions are a fitting choice for testing these algorithms, since they have various number of local minimum.



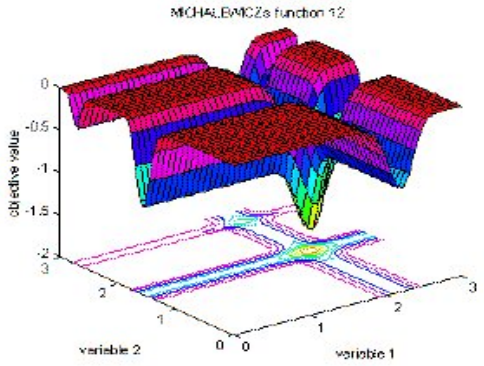
(a) Schwefel's function



(b) DeJong's function



(a) Rastrigin's function



(b) Michalewicz's function

2 Methods

The algorithms used are: Hill Climbing - Best and First improvement and Simulated Annealing. One solution is represented by bit-strings which converted to natural numbers than afterwards mapped to real values serve as input for the functions; The number of bit-strings being equal to the number of dimensions.

In the implementation a bit-string is shaped as a 32 bit unsigned integer. With respects to the function tested and each of their corresponding intervals a 32 bit unsigned integer is enough to store the bits that constitute after processing, real numbers from a given interval with a precision of $10^{-}5$.

If we imagine the bit-strings placed one after another as a single bit-string; we can say that the neighbourhood of this consists in the set of all bit-strings at hamming distance one.

Regarding the initialization, generating random bit-strings is the method used for all algorithms. An instance of the Hill Climbing stops when the neighbouring solutions do not perform better that the current solution, hence corresponding to a local extreme point. This process is executed a certain number of iterations. As to SA, the halting condition is fulfilled once the temperature decreases beyond a threshold of $10^{-}8$.

3 Experiment Results

Each of the three algorithms was ran 30 times on every function and for a certain amount of iterations. Below are three tables for 5, 10 and 30 dimensions. One cells from the tables represents: minimum, maximum, mean, standard deviation and time. The iterations for SA are the inner iterations; the steps performed before updating the temperature.

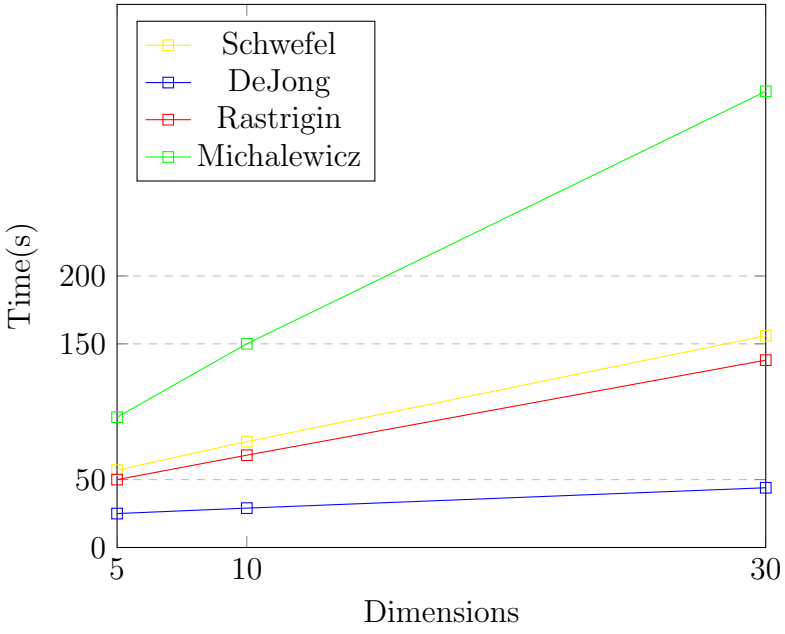
Dimensions: 5			
Iterations	10000	10000	500
Function	HCBI	HCFI	SA
Schwefel	0.00000	0.41454	0.00000
	0.10400	27.2979	34.3396
	0.00812	20.0634	1.27330
	0.02563	11.8175	6.14089
	8.2m	8.5m	57s
De Jong	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000
	30s	16s	25s
Rastrigin	0.00000	0.00000	0.00000
	0.00000	1.98992	4.22071
	0.00000	0.84519	1.39972
	0.00000	0.46089	1.05096
	2m	4.5m	50s
Michalewicz	-4.68766	-4.68623	-4.68766
	-4.68765	-4.68584	-4.47870
	-4.68766	-4.68617	-4.63777
	0.00000	0.00012	0.06318
	4m	5m	1.6m

Dimensions: 10			
Iterations	10000	10000	500
function	HCBI	HCFI	SA
Schwefel	0.10888	96.08010	0.10400
	103.124	203.0940	68.6787
	27.0088	153.6540	6.02869
	29.4537	25.21300	15.5257
	15.3m	18.3m	1.3m
De Jong	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000
	2m	1m	29s
Rastrigin	1.00005	3.46663	0.99496
	3.23083	10.1905	5.96976
	2.40042	7.72923	2.65548
	0.56704	1.71172	1.34253
	7.8m	10.1m	68s
Michalewicz	−9.60915	−9.58894	−9.61473
	−9.42190	−8.94970	−8.91998
	−9.53463	−9.19747	−9.37160
	0.04305	0.12866	0.16396
	15m	14m	2.5m

Dimensions: 30			
Iterations	1000	1000	500
function	HCBI	HCFI	SA
Schwefel	928.91400	1199.94000	1.04199
	1441.92	1677.05	526.048
	1232.48	1455.04	210.689
	115.581	112.289	130.672
	11.8m	29.7m	2.6m
De Jong	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000
	1.7m	54s	44s
Rastrigin	23.78490	39.05330	15.66730
	32.21150	57.67380	29.26290
	28.06330	52.55050	20.78260
	2.161440	5.500070	3.07867
	6.6m	21.4m	2.3m
Michalewicz	−27.63280	−25.42070	−28.69110
	−26.49230	−23.74020	−27.54690
	−27.00870	−24.4564	−28.17060
	0.23669	0	0.25159
	12m	24.8m	5.6m

It can be observed the fact that the increase in dimensions leads to bigger errors and inaccurate results. We can derive from these results that a larger number of iterations would take more time, however give better results.

Time-Dimensions plot, Simulated Annealing



4 Comparisons

Hill Climbing always chooses a better solution being the first or the best one, in order to advance. This has a drawback, since in that way HC often converges to a local point, usually not the global one. SA uses a different strategy and can deliberately pick worse solutions with a certain probability in order to avoid getting stuck in a local minimum. Yet this probability of picking worse solutions decreases along with temperature and thus becoming more selective over time. Reffering to the comparison between HCBI and HCFI, it can be concluded from the experiments that BI is a better solution both in terms of the quality of the results and time efficency.

5 Conclusions

In terms of time, SA runs much faster than HC giving good results, while HC gives more precise results in a larger amount of time. The matter of optimizing functions at first sight seems a somewhat difficult task to tackle and after some reflection and testing it certainly turns out to be a delicate one.

References

[1] https://en.wikipedia.org/wiki/Hill_climbing

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[3] https://www.baeldung.com/cs/simulated-annealing

[4] http://www.geatbx.com/docu/fcnindex-01.html

[5] https://stackoverflow.com/questions/686353/random-float-number-generation