Topos

March 20, 2022

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Contents

1 Topos

For other uses, see Topos (disambiguation)[1].

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In mathematics[12], a **topos** (UK[13]: /ˈtɒpɒs/[14], US[15]: /ˈtoʊpoʊs, ˈtoʊpɒs/[16]; plural **topoi** /ˈtoʊpɔɪ/[17] or /ˈtɒpɔɪ/[18], or **toposes**) is a category[19] that behaves like the category of sheaves[20] of sets[21] on a topological space[22] (or more generally: on a site[23]). Topoi behave much like the category of sets[24] and possess a notion of localization; they are a direct generalization of point-set topology[25].[1] The **Grothendieck topoi** find applications in algebraic geometry[26]; the more general **elementary topoi** are used in logic[27].

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1   https://en.wikipedia.org/wiki/Topos_(disambiguation)
2   https://en.wikipedia.org/wiki/Wikipedia:Verifiability
3   https://en.wikipedia.org/w/index.php?title=Topos&action=edit
4   https://en.wikipedia.org/wiki/Help:Referencing_for_beginners
5   http://www.google.com/search?as_eq=wikipedia&q=%22Topos%22
6   http://www.google.com/search?tbm=nws&q=%22Topos%22+-wikipedia&tbs=ar:1
7   http://www.google.com/search?&q=%22Topos%22&tbs=bkt:s&tbm=bks
8   http://www.google.com/search?tbs=bks:1&q=%22Topos%22+-wikipedia
9   http://scholar.google.com/scholar?q=%22Topos%22
10 https://www.jstor.org/action/doBasicSearch?Query=%22Topos%22&acc=on&wc=on
11 https://en.wikipedia.org/wiki/Help:Maintenance_template_removal
12 https://en.wikipedia.org/wiki/Mathematics
13 https://en.wikipedia.org/wiki/British_English
14 https://en.wikipedia.org/wiki/Help:IPA/English
15 https://en.wikipedia.org/wiki/American_English
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19 https://en.wikipedia.org/wiki/Category_(mathematics)
20 https://en.wikipedia.org/wiki/Sheaf_(mathematics)
21 https://en.wikipedia.org/wiki/Set_(mathematics)
22 https://en.wikipedia.org/wiki/Topological_space
23 https://en.wikipedia.org/wiki/Site_(mathematics)
24 https://en.wikipedia.org/wiki/Category_of_sets
25 https://en.wikipedia.org/wiki/Point-set_topology
26 https://en.wikipedia.org/wiki/Algebraic_geometry
27 https://en.wikipedia.org/wiki/Mathematical_logic

1.1 Grothendieck topos (topos in geometry)

Since the introduction of sheaves into mathematics in the 1940s, a major theme has been to study a space by studying sheaves on a space. This idea was expounded by Alexander Grothendieck[28] by introducing the notion of a "topos". The main utility of this notion is in the abundance of situations in mathematics where topological heuristics are very effective, but an honest topological space is lacking; it is sometimes possible to find a topos formalizing the heuristic. An important example of this programmatic idea is the étale topos[29] of a scheme[30]. Another illustration of the capability of Grothendieck toposes to incarnate the "essence" of different mathematical situations is given by their use as bridges for connecting theories which, albeit written in possibly very different languages, share a common mathematical content.[2][3]

1.1.1 Equivalent definitions

A Grothendieck topos is a category[31] C which satisfies any one of the following three properties. (A theorem[32] of Jean Giraud[33] states that the properties below are all equivalent.)

- There is a small category[34] D and an inclusion $C \hookrightarrow \mathrm{Presh}(D)$ that admits a finite-limit-preserving[35] left adjoint[36].
- C is the category of sheaves on a Grothendieck site[37].
- C satisfies Giraud's axioms, below.

Here $\mathrm{Presh}(D)$ denotes the category of contravariant functors[38] from D to the category of sets; such a contravariant functor is frequently called a presheaf[39].

Giraud's axioms

Giraud's axioms for a category[40] C are:

- C has a small set of generators[41], and admits all small colimits[42]. Furthermore, fiber products[43] distribute over coproducts. That is, given a set I, an I-indexed coproduct mapping to A, and a morphism $A' \to A$, the pullback is an I-indexed coproduct of the pullbacks:

28 https://en.wikipedia.org/wiki/Alexander_Grothendieck
29 https://en.wikipedia.org/wiki/%C3%89tale_topos
30 https://en.wikipedia.org/wiki/Scheme_(mathematics)
31 https://en.wikipedia.org/wiki/Category_(mathematics)
32 https://en.wikipedia.org/w/index.php?title=Giraud%27s_theorem&action=edit&redlink=1
33 https://en.wikipedia.org/wiki/Jean_Giraud_(mathematician)
34 https://en.wikipedia.org/wiki/Category_(mathematics)#Small_and_large_categories
35 https://en.wikipedia.org/wiki/Adjoint_functor#Limit_preservation
36 https://en.wikipedia.org/wiki/Left_adjoint
37 https://en.wikipedia.org/wiki/Grothendieck_site
38 https://en.wikipedia.org/wiki/Contravariant_functor#Covariance_and_contravariance
39 https://en.wikipedia.org/wiki/Presheaf_(category_theory)
40 https://en.wikipedia.org/wiki/Category_(mathematics)
41 https://en.wikipedia.org/wiki/Generator_(category_theory)
42 https://en.wikipedia.org/wiki/Colimit
43 https://en.wikipedia.org/wiki/Fiber_product

$$(_{i \in I} B_i) \times_A A' \cong_{i \in I} (B_i \times_A A').$$

- Sums in C are disjoint. In other words, the fiber product of X and Y over their sum is the initial object[44] in C.
- All equivalence relations[45] in C are effective[46].

The last axiom needs the most explanation. If X is an object of C, an "equivalence relation" R on X is a map $R \to X \times X$ in C such that for any object Y in C, the induced map Hom(Y, R) $\to$ Hom(Y, X) $\times$ Hom(Y, X) gives an ordinary equivalence relation on the set Hom(Y, X). Since C has colimits we may form the coequalizer[47] of the two maps $R \to X$; call this X/R. The equivalence relation is "effective" if the canonical map

$$R \to X \times_{X/R} X$$

is an isomorphism.

1.1.2 Examples

Giraud's theorem already gives "sheaves on sites" as a complete list of examples. Note, however, that nonequivalent sites often give rise to equivalent topoi. As indicated in the introduction, sheaves on ordinary topological spaces motivate many of the basic definitions and results of topos theory.

Category of sets and G-sets

The category[48] of sets is an important special case: it plays the role of a point in topos theory. Indeed, a set may be thought of as a sheaf on a point since functors on the singleton category with a single object and only the identity morphism are just specific sets in the category of sets.

Similarly, there is a topos BG for any group[49] G which is equivalent to the category of G-sets. We construct this as the category of presheaves on the category with one object, but now the set of morphisms is given by the group[50] G. Since any functor must give a G-action on the target, this gives the category of G-sets. Similarly, for a groupoid[51] $\mathcal{G}$ the category of presheaves on $\mathcal{G}$ gives a collection of sets indexed by the set of objects in $\mathcal{G}$, and the automorphisms of an object in $\mathcal{G}$ has an action on the target of the functor.

Topoi from ringed spaces

More exotic examples, and the *raison d'être* of topos theory, come from algebraic geometry. The basic example of a topos comes from the Zariksi topos of a scheme[52]. For each scheme X

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44 https://en.wikipedia.org/wiki/Initial_and_terminal_objects
45 https://en.wikipedia.org/wiki/Equivalence_relation
46 https://en.wikipedia.org/wiki/Regular_category#Exact_(effective)_categories
47 https://en.wikipedia.org/wiki/Coequalizer
48 https://en.wikipedia.org/wiki/Category_(mathematics)
49 https://en.wikipedia.org/wiki/Group_(mathematics)
50 https://en.wikipedia.org/wiki/Group_(mathematics)
51 https://en.wikipedia.org/wiki/Groupoid
52 https://en.wikipedia.org/wiki/Scheme_(mathematics)

there is a site $\text{Open}(X)$ (of objects given by open subsets and morphisms given by inclusions) whose category of presheaves forms the Zarisksi topos $(X)_{Zar}$. But once distinguished classes of morphisms are considered, there are multiple generalizations of this which leads to non-trivial mathematics. Moreover, topoi give the foundations for studying schemes purely as functors on the category of algebras.

To a scheme and even a stack[53] one may associate an étale[54] topos, an fppf[55] topos, or a Nisnevich[56] topos. Another important example of a topos is from the crystalline site[57]. In the case of the étale topos, these form the foundational objects of study in anabelian geometry[58], which studies objects in algebraic geometry that are determined entirely by the structure of their étale fundamental group[59].

Pathologies

Topos theory is, in some sense, a generalization of classical point-set topology. One should therefore expect to see old and new instances of pathological[60] behavior. For instance, there is an example due to Pierre Deligne[61] of a nontrivial topos that has no points (see below for the definition of points of a topos).

1.1.3 Geometric morphisms

If X and Y are topoi, a *geometric morphism $u : X \to Y$* is a pair of adjoint functors[62] (u^*, u_*) (where $u^* : Y \to X$ is left adjoint to $u_* : X \to Y$) such that u^* preserves finite limits. Note that u^* automatically preserves colimits by virtue of having a right adjoint.

By Freyd's adjoint functor theorem[63], to give a geometric morphism $X \to Y$ is to give a functor $u^* : Y \to X$ that preserves finite limits and all small colimits. Thus geometric morphisms between topoi may be seen as analogues of maps of locales[64].

If X and Y are topological spaces and u is a continuous map between them, then the pullback and pushforward operations on sheaves yield a geometric morphism between the associated topoi for the sites $\text{Open}(X), \text{Open}(Y)$.

Points of topoi

A point of a topos X is defined as a geometric morphism from the topos of sets to X.

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53 https://en.wikipedia.org/wiki/Stack_(mathematics)
54 https://en.wikipedia.org/wiki/%C3%89tale_topology
55 https://en.wikipedia.org/wiki/Flat_topology
56 https://en.wikipedia.org/wiki/Nisnevich_topology
57 https://en.wikipedia.org/wiki/Crystal_(mathematics)
58 https://en.wikipedia.org/wiki/Anabelian_geometry
59 https://en.wikipedia.org/wiki/%C3%89tale_fundamental_group
60 https://en.wikipedia.org/wiki/Pathological_(mathematics)
61 https://en.wikipedia.org/wiki/Pierre_Deligne
62 https://en.wikipedia.org/wiki/Adjoint_functor
63 https://en.wikipedia.org/wiki/Adjoint_functors#Existence
64 https://en.wikipedia.org/wiki/Frames_and_locales

If X is an ordinary space and x is a point of X, then the functor that takes a sheaf F to its stalk F_x has a right adjoint (the "skyscraper sheaf" functor), so an ordinary point of X also determines a topos-theoretic point. These may be constructed as the pullback-pushforward along the continuous map $x: 1 \to X$.

For the etale topos $(X)_{et}$ of a space X, a point is a bit more refined of an object. Given a point $x : \mathrm{Spec}(\kappa(x)) \to X$ of the underlying scheme X a point x' of the topos $(X)_{et}$ then given by a separable field extension k of $\kappa(x)$ such that the associated map $x' : \mathrm{Spec}(k) \to X$ factors through the original point x. Then, the factorization map

$$\mathrm{Spec}(k) \to \mathrm{Spec}(\kappa(x))$$

is an etale morphism[65] of schemes.

More precisely, those are the *global* points. They are not adequate in themselves for displaying the space-like aspect of a topos, because a non-trivial topos may fail to have any. *Generalized* points are geometric morphisms from a topos Y (the *stage of definition*) to X. There are enough of these to display the space-like aspect. For example, if X is the classifying topos[66] $S[T]$ for a geometric theory T, then the universal property says that its points are the models of T (in any stage of definition Y).

Essential geometric morphisms

A geometric morphism (u^*, u_*) is *essential* if u^* has a further left adjoint $u_!$, or equivalently (by the adjoint functor theorem) if u^* preserves not only finite but all small limits.

1.1.4 Ringed topoi

Main article: Ringed topos[67] A **ringed topos** is a pair *(X,R)*, where X is a topos and R is a commutative ring object[68] in X. Most of the constructions of ringed spaces[69] go through for ringed topoi. The category of R-module[70] objects in X is an abelian category[71] with enough injectives. A more useful abelian category is the subcategory of quasi-coherent[72] R-modules: these are R-modules that admit a presentation.

Another important class of ringed topoi, besides ringed spaces, are the étale topoi of Deligne–Mumford stacks[73].

65   https://en.wikipedia.org/wiki/%C3%89tale_morphism
66   https://en.wikipedia.org/wiki/Classifying_topos
67   https://en.wikipedia.org/wiki/Ringed_topos
68   https://en.wikipedia.org/wiki/Ring_object
69   https://en.wikipedia.org/wiki/Ringed_space
70   https://en.wikipedia.org/wiki/Module_(mathematics)
71   https://en.wikipedia.org/wiki/Abelian_category
72   https://en.wikipedia.org/wiki/Coherent_sheaf
73   https://en.wikipedia.org/wiki/Algebraic_stack

1.1.5 Homotopy theory of topoi

Michael Artin[74] and Barry Mazur[75] associated to the site underlying a topos a pro-simplicial set[76] (up to homotopy[77]).[4] (It's better to consider it in Ho(pro-SS); see Edwards) Using this inverse system[78] of simplicial sets one may *sometimes* associate to a homotopy invariant[79] in classical topology an inverse system of invariants in topos theory. The study of the pro-simplicial set associated to the étale topos of a scheme is called étale homotopy theory[80].[5] In good cases (if the scheme is Noetherian[81] and geometrically unibranch[82]), this pro-simplicial set is pro-finite[83].

1.2 Elementary topoi (topoi in logic)

1.2.1 Introduction

Since the early 20th century, the predominant axiomatic foundation of mathematics has been set theory[84], in which all mathematical objects are ultimately represented by sets (including functions[85], which map between sets). More recent work in category theory allows this foundation to be generalized using topoi; each topos completely defines its own mathematical framework. The category of sets forms a familiar topos, and working within this topos is equivalent to using traditional set-theoretic mathematics. But one could instead choose to work with many alternative topoi. A standard formulation of the axiom of choice[86] makes sense in any topos, and there are topoi in which it is invalid. Constructivists[87] will be interested to work in a topos without the law of excluded middle[88]. If symmetry under a particular group[89] G is of importance, one can use the topos consisting of all G-sets[90].

It is also possible to encode an algebraic theory[91], such as the theory of groups, as a topos, in the form of a classifying topos[92]. The individual models of the theory, i.e. the groups in our example, then correspond to functors[93] from the encoding topos to the category of sets that respect the topos structure.

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74   https://en.wikipedia.org/wiki/Michael_Artin
75   https://en.wikipedia.org/wiki/Barry_Mazur
76   https://en.wikipedia.org/wiki/Pro-simplicial_set
77   https://en.wikipedia.org/wiki/Homotopy_category
78   https://en.wikipedia.org/wiki/Inverse_system
79   https://en.wikipedia.org/wiki/Homotopy#Homotopy_invariance
80   https://en.wikipedia.org/wiki/%C3%89tale_homotopy_theory
81   https://en.wikipedia.org/wiki/Noetherian_scheme
82   https://en.wikipedia.org/wiki/Geometrically_unibranch
83   https://en.wikipedia.org/wiki/Pro-finite
84   https://en.wikipedia.org/wiki/Set_theory
85   https://en.wikipedia.org/wiki/Function_(mathematics)
86   https://en.wikipedia.org/wiki/Axiom_of_choice
87   https://en.wikipedia.org/wiki/Mathematical_constructivism
88   https://en.wikipedia.org/wiki/Law_of_excluded_middle
89   https://en.wikipedia.org/wiki/Group_(mathematics)
90   https://en.wikipedia.org/wiki/Group_action_(mathematics)
91   https://en.wikipedia.org/wiki/Universal_algebra
92   https://en.wikipedia.org/wiki/Classifying_topos
93   https://en.wikipedia.org/wiki/Functor

1.2.2 Formal definition

When used for foundational work a topos will be defined axiomatically; set theory is then treated as a special case of topos theory. Building from category theory, there are multiple equivalent definitions of a topos. The following has the virtue of being concise:

A topos is a category that has the following two properties:

- All limits[94] taken over finite index categories exist.
- Every object has a power object. This plays the role of the powerset[95] in set theory.

Formally, a **power object** of an object X is a pair $(PX, \ni_X)$ with $\ni_X \subseteq PX \times X$, which classifies relations, in the following sense. First note that for every object I, a morphism $r \colon I \to PX$ ("a family of subsets") induces a subobject $\{(i, x) \mid x \in r(i)\} \subseteq I \times X$. Formally, this is defined by pulling back $\ni_X$ along $r \times X \colon I \times X \to PX \times X$. The universal property of a power object is that every relation arises in this way, giving a bijective correspondence between relations $R \subseteq I \times X$ and morphisms $r \colon I \to PX$.

From finite limits and power objects one can derive that

- All colimits[96] taken over finite index categories exist.
- The category has a subobject classifier[97].
- The category is Cartesian closed[98].

In some applications, the role of the subobject classifier is pivotal, whereas power objects are not. Thus some definitions reverse the roles of what is defined and what is derived.

1.2.3 Logical functors

A *logical functor* is a functor between toposes that preserves finite limits and power objects. Logical functors preserve the structures that toposes have. In particular, they preserve finite colimits, subobject classifiers[99], and exponential objects[100].[6]

1.2.4 Explanation

A topos as defined above can be understood as a Cartesian closed category for which the notion of subobject of an object has an elementary[101] or first-order definition. This notion, as a natural categorical abstraction of the notions of subset[102] of a set, subgroup[103] of a group, and more generally subalgebra[104] of any algebraic structure[105], predates the notion

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94 https://en.wikipedia.org/wiki/Limit_(category_theory)
95 https://en.wikipedia.org/wiki/Powerset
96 https://en.wikipedia.org/wiki/Limit_(category_theory)
97 https://en.wikipedia.org/wiki/Subobject_classifier
98 https://en.wikipedia.org/wiki/Cartesian_closed_category
99 https://en.wikipedia.org/wiki/Subobject_classifier
100 https://en.wikipedia.org/wiki/Exponential_object
101 https://en.wikipedia.org/wiki/First-order_logic
102 https://en.wikipedia.org/wiki/Subset
103 https://en.wikipedia.org/wiki/Subgroup
104 https://en.wikipedia.org/wiki/Subalgebra
105 https://en.wikipedia.org/wiki/Algebraic_structure

of topos. It is definable in any category, not just topoi, in second-order[106] language, i.e. in terms of classes of morphisms instead of individual morphisms, as follows. Given two monics m, n from respectively Y and Z to X, we say that $m \leq n$ when there exists a morphism p : $Y \to Z$ for which $np = m$, inducing a preorder[107] on monics to X. When $m \leq n$ and $n \leq m$ we say that m and n are equivalent. The subobjects of X are the resulting equivalence classes of the monics to it.

In a topos "subobject" becomes, at least implicitly, a first-order notion, as follows.

As noted above, a topos is a category C having all finite limits and hence in particular the empty limit or final object 1. It is then natural to treat morphisms of the form $x\colon 1 \to X$ as *elements* $x \in X$. Morphisms $f\colon X \to Y$ thus correspond to functions mapping each element $x \in X$ to the element $fx \in Y$, with application realized by composition.

One might then think to define a subobject of X as an equivalence class of monics $m\colon X' \to X$ having the same image[108] $\{ mx \mid x \in X' \}$. The catch is that two or more morphisms may correspond to the same function, that is, we cannot assume that C is concrete in the sense that the functor $C(1,\text{-})\colon C \to \textbf{Set}$ is faithful. For example the category **Grph** of graphs[109] and their associated homomorphisms[110] is a topos whose final object 1 is the graph with one vertex and one edge (a self-loop), but is not concrete because the elements $1 \to G$ of a graph G correspond only to the self-loops and not the other edges, nor the vertices without self-loops. Whereas the second-order definition makes G and the subgraph of all self-loops of G (with their vertices) distinct subobjects of G (unless every edge is, and every vertex has, a self-loop), this image-based one does not. This can be addressed for the graph example and related examples via the Yoneda Lemma[111] as described in the Further examples[112] section below, but this then ceases to be first-order. Topoi provide a more abstract, general, and first-order solution.

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106 https://en.wikipedia.org/wiki/Second-order_logic
107 https://en.wikipedia.org/wiki/Preorder
108 https://en.wikipedia.org/wiki/Image_(mathematics)
109 https://en.wikipedia.org/wiki/Multidigraph
110 https://en.wikipedia.org/wiki/Homomorphism
111 https://en.wikipedia.org/wiki/Yoneda_Lemma
112 https://en.wikipedia.org/wiki/Topos#Further_examples

$$X' \xrightarrow{\quad m \quad} X$$
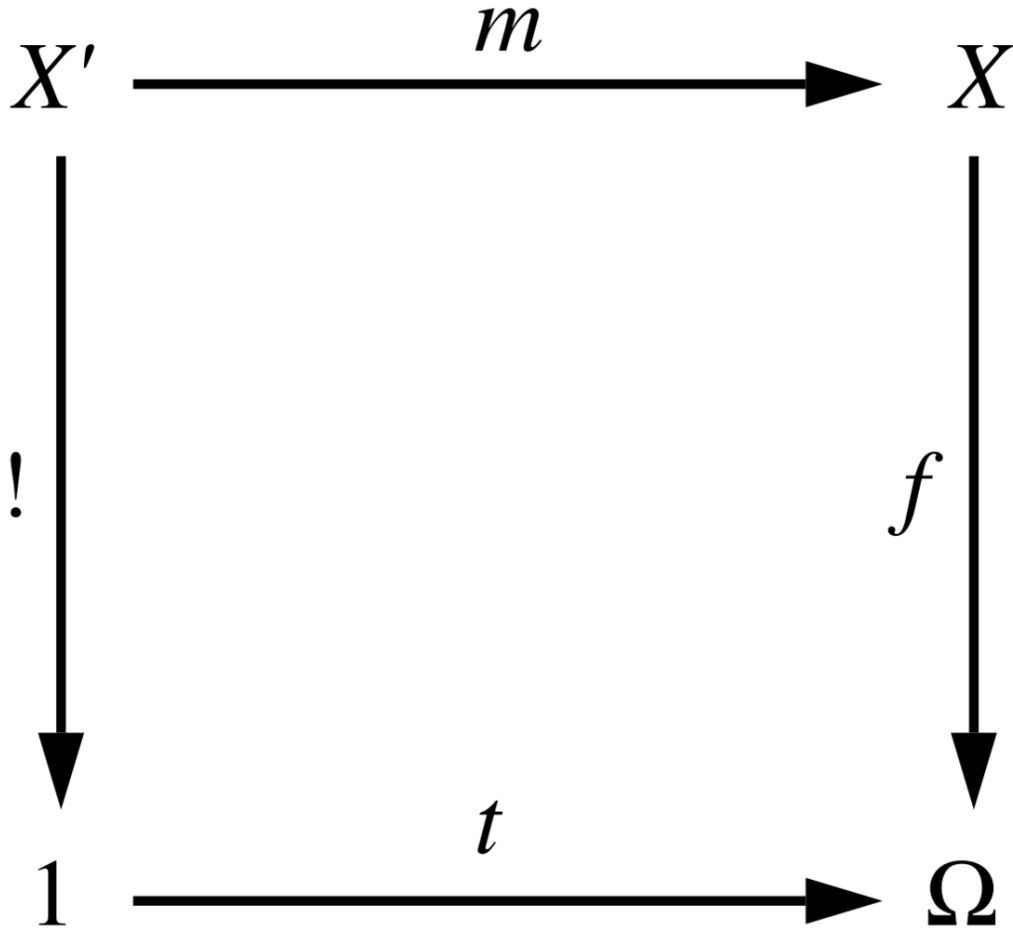
Figure 2   Figure 1. m as a pullback of the generic subobject t along f.

As noted above, a topos C has a subobject classifier Ω, namely an object of C with an element $t \in \Omega$, the *generic subobject* of C, having the property that every monic[113] $m\colon X' \to X$ arises as a pullback of the generic subobject along a unique morphism $f\colon X \to \Omega$, as per Figure 1. Now the pullback of a monic is a monic, and all elements including t are monics since there is only one morphism to 1 from any given object, whence the pullback of t along $f\colon X \to \Omega$ is a monic. The monics to X are therefore in bijection with the pullbacks of t along morphisms from X to Ω. The latter morphisms partition the monics into equivalence classes each determined by a morphism $f\colon X \to \Omega$, the characteristic morphism of that class, which we take to be the subobject of X characterized or named by f.

All this applies to any topos, whether or not concrete. In the concrete case, namely $C(1,-)$ faithful, for example the category of sets, the situation reduces to the familiar behavior of functions. Here the monics $m\colon X' \to X$ are exactly the injections (one-one functions) from X' to X, and those with a given image $\{\, mx \mid x \in X' \,\}$ constitute the subobject of X

113 https://en.wikipedia.org/wiki/Monomorphism

corresponding to the morphism $f\colon X \to \Omega$ for which $f^{-1}(t)$ is that image. The monics of a subobject will in general have many domains, all of which however will be in bijection with each other.

To summarize, this first-order notion of subobject classifier implicitly defines for a topos the same equivalence relation on monics to X as had previously been defined explicitly by the second-order notion of subobject for any category. The notion of equivalence relation on a class of morphisms is itself intrinsically second-order, which the definition of topos neatly sidesteps by explicitly defining only the notion of subobject *classifier* Ω, leaving the notion of subobject of X as an implicit consequence characterized (and hence namable) by its associated morphism $f\colon X \to \Omega$.

1.2.5 Further examples and non-examples

Every Grothendieck topos is an elementary topos, but the converse is not true (since every Grothendieck topos is cocomplete, which is not required from an elementary topos).

The categories of finite sets, of finite G-sets (actions of a group[114] G on a finite set), and of finite graphs are elementary topoi that are not Grothendieck topoi.

If C is a small category, then the functor category[115] $\mathbf{Set}^C$ (consisting of all covariant functors from C to sets, with natural transformations[116] as morphisms) is a topos. For instance, the category $\mathbf{Grph}$ of graphs of the kind permitting multiple directed edges between two vertices is a topos. Such a graph consists of two sets, an edge set and a vertex set, and two functions s,t between those sets, assigning to every edge e its source $s(e)$ and target $t(e)$. $\mathbf{Grph}$ is thus equivalent[117] to the functor category $\mathbf{Set}^C$, where C is the category with two objects E and V and two morphisms $s,t\colon E \to V$ giving respectively the source and target of each edge.

The Yoneda lemma[118] asserts that C^{op} embeds in $\mathbf{Set}^C$ as a full subcategory. In the graph example the embedding represents C^{op} as the subcategory of $\mathbf{Set}^C$ whose two objects are V' as the one-vertex no-edge graph and E' as the two-vertex one-edge graph (both as functors), and whose two nonidentity morphisms are the two graph homomorphisms from V' to E' (both as natural transformations). The natural transformations from V' to an arbitrary graph (functor) G constitute the vertices of G while those from E' to G constitute its edges. Although $\mathbf{Set}^C$, which we can identify with $\mathbf{Grph}$, is not made concrete by either V' or E' alone, the functor $U\colon \mathbf{Grph} \to \mathbf{Set}^2$ sending object G to the pair of sets $(\mathbf{Grph}(V',G), \mathbf{Grph}(E',G))$ and morphism $h\colon G \to H$ to the pair of functions $(\mathbf{Grph}(V',h), \mathbf{Grph}(E',h))$ is faithful. That is, a morphism of graphs can be understood as a *pair* of functions, one mapping the vertices and the other the edges, with application still realized as composition but now with multiple sorts of *generalized* elements. This shows that the traditional concept of a concrete category as one whose objects have an underlying set can be generalized to cater for a wider range of topoi by allowing an object to have multiple underlying sets, that is, to be multisorted.

114 https://en.wikipedia.org/wiki/Group_action
115 https://en.wikipedia.org/wiki/Functor_category
116 https://en.wikipedia.org/wiki/Natural_transformation
117 https://en.wikipedia.org/wiki/Equivalent_categories
118 https://en.wikipedia.org/wiki/Yoneda_lemma

The category of pointed sets[119] with point-preserving functions is *not* a topos, since it doesn't have power objects: if PX were the power object of the pointed set X, and 1 denotes the pointed singleton, then there is only one point-preserving function $r \colon 1 \to PX$, but the relations in $1 \times X$ are as numerous as the pointed subsets of X. The category of abelian groups[120] is also not a topos, for a similar reason: every group homomorphism must map 0 to 0.

1.3 See also

- Mathematics portal[121]

- History of topos theory[122]
- Homotopy hypothesis[123]
- Intuitionistic type theory[124]
- ∞-topos[125]
- Quasitopos[126]
- Geometric logic[127]

1.4 Notes

1. Illusie 2004[128]
2. CARAMELLO, OLIVIA[129] (2016). *Grothendieck toposes as unifying 'bridges' in Mathematics*[130] (PDF) (HDR). PARIS DIDEROT UNIVERSITY (PARIS 7).
3. CARAMELLO, OLIVIA (2017). *Theories, Sites, Toposes: Relating and studying mathematical theories through topos-theoretic 'bridges'*[131]. OXFORD UNIVERSITY PRESS. DOI[132]:10.1093/oso/9780198758914.001.0001[133]. ISBN[134] 9780198758914[135].

119 https://en.wikipedia.org/wiki/Pointed_set
120 https://en.wikipedia.org/wiki/Category_of_abelian_groups
121 https://en.wikipedia.org/wiki/Portal:Mathematics
122 https://en.wikipedia.org/wiki/History_of_topos_theory
123 https://en.wikipedia.org/wiki/Homotopy_hypothesis
124 https://en.wikipedia.org/wiki/Intuitionistic_type_theory
125 https://en.wikipedia.org/wiki/%E2%88%9E-topos
126 https://en.wikipedia.org/wiki/Quasitopos
127 https://en.wikipedia.org/wiki/Geometric_logic
128 #CITEREFIllusie2004
129 https://en.wikipedia.org/wiki/Olivia_Caramello
130 https://www.oliviacaramello.com/Unification/HDROliviaCaramello.pdf
131 https://oxford.universitypressscholarship.com/view/10.1093/oso/9780198758914.001.0001/oso-9780198758914
132 https://en.wikipedia.org/wiki/Doi_(identifier)
133 https://doi.org/10.1093%2Foso%2F9780198758914.001.0001
134 https://en.wikipedia.org/wiki/ISBN_(identifier)
135 https://en.wikipedia.org/wiki/Special:BookSources/9780198758914

4. ARTIN, MICHAEL[136]; MAZUR, BARRY[137] (1969). *Etale homotopy.* Lecture Notes in Mathematics. Vol. 100. Springer-Verlag[138]. doi[139]:10.1007/BFb0080957[140]. ISBN[141] 978-3-540-36142-8[142].

5. FRIEDLANDER, ERIC M.[143] (1982), *Étale homotopy of simplicial schemes*, Annals of Mathematics Studies, vol. 104, Princeton University Press[144], ISBN[145] 978-0-691-08317-9[146]

6. McLarty 1992[147], p. 159[148]

1.5 References

Some gentle papers

- EDWARDS, D.A.; HASTINGS, H.M. (SUMMER 1980). "ČECH THEORY: ITS PAST, PRESENT, AND FUTURE"[149]. *Rocky Mountain Journal of Mathematics.* **10** (3): 429–468. doi[150]:10.1216/RMJ-1980-10-3-429[151]. JSTOR[152] 44236540[153].

- BAEZ, JOHN[154]. "TOPOS THEORY IN A NUTSHELL"[155]. A gentle introduction.

- Steven Vickers[156]: "Toposes pour les nuls[157]" and "Toposes pour les vraiment nuls.[158]" Elementary and even more elementary introductions to toposes as generalized spaces.

- ILLUSIE, LUC[159] (2004). "WHAT IS...A TOPOS?"[160] (PDF). *Notices of the AMS.* **51** (9): 160–1.

The following texts are easy-paced introductions to toposes and the basics of category theory. They should be suitable for those knowing little mathematical logic and set theory, even non-mathematicians.

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136 https://en.wikipedia.org/wiki/Michael_Artin
137 https://en.wikipedia.org/wiki/Barry_Mazur
138 https://en.wikipedia.org/wiki/Springer-Verlag
139 https://en.wikipedia.org/wiki/Doi_(identifier)
140 https://doi.org/10.1007%2FBFb0080957
141 https://en.wikipedia.org/wiki/ISBN_(identifier)
142 https://en.wikipedia.org/wiki/Special:BookSources/978-3-540-36142-8
143 https://en.wikipedia.org/wiki/Eric_Friedlander
144 https://en.wikipedia.org/wiki/Princeton_University_Press
145 https://en.wikipedia.org/wiki/ISBN_(identifier)
146 https://en.wikipedia.org/wiki/Special:BookSources/978-0-691-08317-9
147 #CITEREFMcLarty1992
148 https://books.google.com/books?id=V8cON1x39bIC&dq=%22Logical+functor%22
149 https://doi.org/10.1216%2FRMJ-1980-10-3-429
150 https://en.wikipedia.org/wiki/Doi_(identifier)
151 https://doi.org/10.1216%2FRMJ-1980-10-3-429
152 https://en.wikipedia.org/wiki/JSTOR_(identifier)
153 http://www.jstor.org/stable/44236540
154 https://en.wikipedia.org/wiki/John_Baez
155 http://math.ucr.edu/home/baez/topos.html
156 https://en.wikipedia.org/wiki/Steve_Vickers_(computer_scientist)
157 http://www.cs.bham.ac.uk/~sjv/papers.php
158 http://www.cs.bham.ac.uk/~sjv/TopPLVN.pdf
159 https://en.wikipedia.org/wiki/Luc_Illusie
160 http://www.ams.org/notices/200409/what-is-illusie.pdf

- LAWVERE, F. WILLIAM[161]; SCHANUEL, STEPHEN H. (1997). *Conceptual Mathematics: A First Introduction to Categories*[162]. CAMBRIDGE UNIVERSITY PRESS. ISBN[163] 978-0-521-47817-5[164]. An "introduction to categories for computer scientists, logicians, physicists, linguists, etc." (cited from cover text).
- LAWVERE, F. WILLIAM; ROSEBRUGH, ROBERT (2003). *Sets for Mathematics*[165]. CAMBRIDGE UNIVERSITY PRESS. ISBN[166] 978-0-521-01060-3[167]. Introduces the foundations of mathematics from a categorical perspective.

Grothendieck foundational work on toposes:

- GROTHENDIECK, A.[168]; VERDIER, J.L.[169] (1972). *Théorie des Topos et Cohomologie Etale des Schémas*. Lecture notes in mathematics. Vol. 269. Springer. doi[170]:10.1007/BFb0081551[171]. ISBN[172] 978-3-540-37549-4[173]. *Tome 2* **270** doi[174]:10.1007/BFb0061319[175] ISBN[176] 978-3-540-37987-4[177]

The following monographs include an introduction to some or all of topos theory, but do not cater primarily to beginning students. Listed in (perceived) order of increasing difficulty.

- MCLARTY, COLIN[178] (1992). *Elementary Categories, Elementary Toposes*[179]. CLARENDON PRESS. ISBN[180] 978-0-19-158949-2[181]. A nice introduction to the basics of category theory, topos theory, and topos logic. Assumes very few prerequisites.
- GOLDBLATT, ROBERT[182] (2013) [1984]. *Topoi: The Categorial Analysis of Logic*[183]. COURIER CORPORATION. ISBN[184] 978-0-486-31796-0[185]. A good start. Available online[186] at Robert Goldblatt's homepage.[187]

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161 https://en.wikipedia.org/wiki/F._William_Lawvere
162 https://books.google.com/books?id=o1tHw4W5MZQC
163 https://en.wikipedia.org/wiki/ISBN_(identifier)
164 https://en.wikipedia.org/wiki/Special:BookSources/978-0-521-47817-5
165 https://archive.org/details/setsformathemati0000lawv
166 https://en.wikipedia.org/wiki/ISBN_(identifier)
167 https://en.wikipedia.org/wiki/Special:BookSources/978-0-521-01060-3
168 https://en.wikipedia.org/wiki/Grothendieck
169 https://en.wikipedia.org/wiki/Jean-Louis_Verdier
170 https://en.wikipedia.org/wiki/Doi_(identifier)
171 https://doi.org/10.1007%2FBFb0081551
172 https://en.wikipedia.org/wiki/ISBN_(identifier)
173 https://en.wikipedia.org/wiki/Special:BookSources/978-3-540-37549-4
174 https://en.wikipedia.org/wiki/Doi_(identifier)
175 https://doi.org/10.1007%2FBFb0061319
176 https://en.wikipedia.org/wiki/ISBN_(identifier)
177 https://en.wikipedia.org/wiki/Special:BookSources/978-3-540-37987-4
178 https://en.wikipedia.org/wiki/Colin_McLarty
179 https://books.google.com/books?id=V8cON1x39bIC
180 https://en.wikipedia.org/wiki/ISBN_(identifier)
181 https://en.wikipedia.org/wiki/Special:BookSources/978-0-19-158949-2
182 https://en.wikipedia.org/wiki/Robert_Goldblatt
183 https://books.google.com/books?id=hLPDAgAAQBAJ
184 https://en.wikipedia.org/wiki/ISBN_(identifier)
185 https://en.wikipedia.org/wiki/Special:BookSources/978-0-486-31796-0
186 http://historical.library.cornell.edu/cgi-bin/cul.math/docviewer?did=Gold010&id=3
187 http://www.mcs.vuw.ac.nz/~rob/

- BELL, JOHN L.[188] (2001). "THE DEVELOPMENT OF CATEGORICAL LOGIC"[189]. In GABBAY, D.M.; GUENTHNER, FRANZ (EDS.). *Handbook of Philosophical Logic*. Vol. 12 (2nd ed.). Springer. pp. 279–. ISBN[190] 978-1-4020-3091-8[191]. Version available online[192] at John Bell's homepage.[193]
- MACLANE, SAUNDERS[194]; MOERDIJK, IEKE[195] (2012) [1994]. *Sheaves in Geometry and Logic: A First Introduction to Topos Theory*[196]. SPRINGER. ISBN[197] 978-1-4612-0927-0[198]. More complete, and more difficult to read.
- BARR, MICHAEL[199]; WELLS, CHARLES[200] (2013) [1985]. *Toposes, Triples and Theories*[201]. SPRINGER. ISBN[202] 978-1-4899-0023-4[203]. (Online version). More concise than *Sheaves in Geometry and Logic*, but hard on beginners.

Reference works for experts, less suitable for first introduction

- EDWARDS, D.A.; HASTINGS, H.M. (1976). *Čech and Steenrod homotopy theories with applications to geometric topology*. Lecture Notes in Maths. Vol. 542. Springer-Verlag. doi[204]:10.1007/BFb0081083[205]. ISBN[206] 978-3-540-38103-7[207].
- BORCEUX, FRANCIS (1994). *Handbook of Categorical Algebra: Volume 3, Sheaf Theory*[208]. ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS. VOL. 52. CAMBRIDGE UNIVERSITY PRESS. ISBN[209] 978-0-521-44180-3[210]. The third part of "Borceux' remarkable magnum opus", as Johnstone has labelled it. Still suitable as an introduction, though beginners may find it hard to recognize the most relevant results among the huge amount of material given.

188 https://en.wikipedia.org/wiki/John_Lane_Bell
189 https://books.google.com/books?id=yObMqG9EcCEC&pg=PA279
190 https://en.wikipedia.org/wiki/ISBN_(identifier)
191 https://en.wikipedia.org/wiki/Special:BookSources/978-1-4020-3091-8
192 http://publish.uwo.ca/~jbell/catlogprime.pdf
193 http://publish.uwo.ca/~jbell/
194 https://en.wikipedia.org/wiki/Saunders_Mac_Lane
195 https://en.wikipedia.org/wiki/Ieke_Moerdijk
196 https://books.google.com/books?id=LZWLBAAAQBAJ
197 https://en.wikipedia.org/wiki/ISBN_(identifier)
198 https://en.wikipedia.org/wiki/Special:BookSources/978-1-4612-0927-0
199 https://en.wikipedia.org/wiki/Michael_Barr_(mathematician)
200 https://en.wikipedia.org/wiki/Charles_Wells_(mathematician)
201 http://www.tac.mta.ca/tac/reprints/articles/12/tr12abs.html
202 https://en.wikipedia.org/wiki/ISBN_(identifier)
203 https://en.wikipedia.org/wiki/Special:BookSources/978-1-4899-0023-4
204 https://en.wikipedia.org/wiki/Doi_(identifier)
205 https://doi.org/10.1007%2FBFb0081083
206 https://en.wikipedia.org/wiki/ISBN_(identifier)
207 https://en.wikipedia.org/wiki/Special:BookSources/978-3-540-38103-7
208 https://books.google.com/books?id=7jgots78faUC
209 https://en.wikipedia.org/wiki/ISBN_(identifier)
210 https://en.wikipedia.org/wiki/Special:BookSources/978-0-521-44180-3

- JOHNSTONE, PETER T.[211] (2014) [1977]. *Topos Theory*[212]. COURIER. ISBN[213] 978-0-486-49336-7[214]. For a long time the standard compendium on topos theory. However, even Johnstone describes this work as "far too hard to read, and not for the faint-hearted."

- JOHNSTONE, PETER T. (2002). *Sketches of an Elephant: A Topos Theory Compendium*[215]. VOL. 2. CLARENDON PRESS. ISBN[216] 978-0-19-851598-2[217]. As of early 2010, two of the scheduled three volumes of this overwhelming compendium were available.

- CARAMELLO, OLIVIA (2017). *Theories, Sites, Toposes: Relating and studying mathematical theories through topos-theoretic 'bridges*[218]. OXFORD UNIVERSITY PRESS. DOI[219]:10.1093/oso/9780198758914.001.0001[220]. ISBN[221] 9780198758914[222].

Books that target special applications of topos theory

- PEDICCHIO, MARIA CRISTINA; THOLEN, WALTER; ROTA, G.C., EDS. (2004). *Categorical Foundations: Special Topics in Order, Topology, Algebra, and Sheaf Theory*[223]. ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS. VOL. 97. CAMBRIDGE UNIVERSITY PRESS. ISBN[224] 978-0-521-83414-8[225]. Includes many interesting special applications.

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211 https://en.wikipedia.org/wiki/Peter_Johnstone_(mathematician)
212 https://books.google.com/books?id=zO8WAgAAQBAJ
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223 https://books.google.com/books?id=WHudOFwuMn8C
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36 https://en.wikipedia.org/wiki/User:DumZiBoT
37 https://en.wikipedia.org/wiki/User:Ed_g2s
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41 https://en.wikipedia.org/wiki/User:FlaBot
42 https://en.wikipedia.org/wiki/User:Galoubet
43 https://en.wikipedia.org/wiki/User:Gauge
44 https://en.wikipedia.org/wiki/User:GeoffreyT2000
45 https://en.wikipedia.org/wiki/User:Ghewgill

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51 https://en.wikipedia.org/wiki/User:Gwern
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53 https://en.wikipedia.org/wiki/User:Hamaryns
54 https://en.wikipedia.org/wiki/User:Helpful_Pixie_Bot
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62 https://en.wikipedia.org/wiki/User:Jonesey95
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64 https://en.wikipedia.org/wiki/User:KSmrq
65 https://en.wikipedia.org/wiki/User:Kaptain-k-theory
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67 https://en.wikipedia.org/wiki/User:Lavidav
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69 https://en.wikipedia.org/wiki/User:Loom91
70 https://en.wikipedia.org/wiki/User:LordAnubisBOT

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71 https://en.wikipedia.org/w/index.php%3ftitle=User:ManSpan&action=edit&redlink=1
72 https://en.wikipedia.org/wiki/User:Mark_viking
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79 https://en.wikipedia.org/wiki/User:Michael_Hardy
80 https://en.wikipedia.org/wiki/User:Michael_Slone
81 https://en.wikipedia.org/wiki/User:Mika58
82 https://en.wikipedia.org/wiki/User:Monkbot
83 https://en.wikipedia.org/w/index.php%3ftitle=User:Multiverse_Guy&action=edit&redlink=1
84 https://en.wikipedia.org/wiki/User:NathanoNL
85 https://en.wikipedia.org/wiki/User:NeilOnWiki
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92 https://en.wikipedia.org/wiki/User:Pedant17
93 https://en.wikipedia.org/wiki/User:Penarc
94 https://en.wikipedia.org/wiki/User:Phys
95 https://en.wikipedia.org/w/index.php%3ftitle=User:Pit-trout&action=edit&redlink=1

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