

# Homotopy type theory

[en.wikipedia.org](https://en.wikipedia.org)

March 19, 2022

On the 28th of April 2012 the contents of the English as well as German Wikibooks and Wikipedia projects were licensed under Creative Commons Attribution-ShareAlike 3.0 Unported license. A URI to this license is given in the list of figures on page 25. If this document is a derived work from the contents of one of these projects and the content was still licensed by the project under this license at the time of derivation this document has to be licensed under the same, a similar or a compatible license, as stated in section 4b of the license. The list of contributors is included in chapter Contributors on page 21. The licenses GPL, LGPL and GFDL are included in chapter Licenses on page 29, since this book and/or parts of it may or may not be licensed under one or more of these licenses, and thus require inclusion of these licenses. The licenses of the figures are given in the list of figures on page 25. This PDF was generated by the L<sup>A</sup>T<sub>E</sub>X typesetting software. The L<sup>A</sup>T<sub>E</sub>X source code is included as an attachment (`source.7z.txt`) in this PDF file. To extract the source from the PDF file, you can use the `pdfdetach` tool including in the `poppler` suite, or the <http://www.pdfabs.com/tools/pdftk-the-pdf-toolkit/> utility. Some PDF viewers may also let you save the attachment to a file. After extracting it from the PDF file you have to rename it to `source.7z`. To uncompress the resulting archive we recommend the use of <http://www.7-zip.org/>. The L<sup>A</sup>T<sub>E</sub>X source itself was generated by a program written by Dirk Hünninger, which is freely available under an open source license from [http://de.wikibooks.org/wiki/Benutzer:Dirk\\_Huenniger/wb2pdf](http://de.wikibooks.org/wiki/Benutzer:Dirk_Huenniger/wb2pdf).

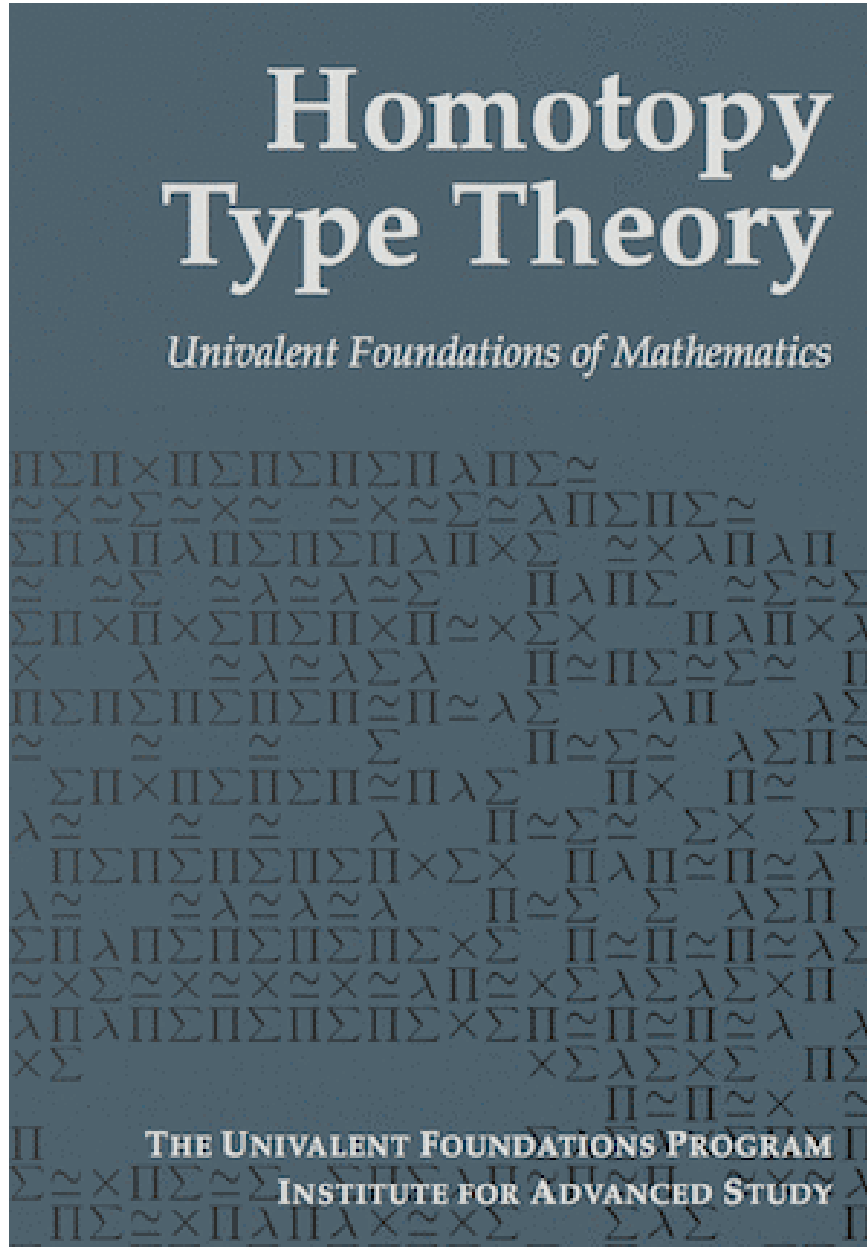
# Contents

<b>1</b>	<b>Homotopy type theory</b>	<b>3</b>
1.1	History . . . . .	4
1.2	Univalent foundations . . . . .	7
1.3	Special Year on Univalent Foundations of Mathematics . . . . .	8
1.4	Key concepts . . . . .	9
1.5	Applications . . . . .	12
1.6	See also . . . . .	13
1.7	Notes . . . . .	13
1.8	References . . . . .	13
1.9	Bibliography . . . . .	17
1.10	Further reading . . . . .	18
1.11	External links . . . . .	19
<b>2</b>	<b>Contributors</b>	<b>21</b>
	<b>List of Figures</b>	<b>25</b>
<b>3</b>	<b>Licenses</b>	<b>29</b>
3.1	GNU GENERAL PUBLIC LICENSE . . . . .	29
3.2	GNU Free Documentation License . . . . .	30
3.3	GNU Lesser General Public License . . . . .	31



# 1 Homotopy type theory

Type theory in logic and mathematics



**Figure 1** Cover of *Homotopy Type Theory: Univalent Foundations of Mathematics*.

In mathematical logic<sup>1</sup> and computer science<sup>2</sup>, **homotopy type theory (HoTT)** /hɒt/<sup>3</sup> refers to various lines of development of intuitionistic type theory<sup>4</sup>, based on the interpretation of types as objects to which the intuition of (abstract) homotopy theory<sup>5</sup> applies.

This includes, among other lines of work, the construction of homotopical and higher-categorical<sup>6</sup> models<sup>7</sup> for such type theories; the use of type theory as a logic (or internal language<sup>8</sup>) for abstract homotopy theory and higher category theory<sup>9</sup>; the development of mathematics within a type-theoretic foundation<sup>10</sup> (including both previously existing mathematics and new mathematics that homotopical types make possible); and the formalization<sup>11</sup> of each of these in computer proof assistants<sup>12</sup>.

There is a large overlap between the work referred to as homotopy type theory, and as the univalent foundations<sup>13</sup> project. Although neither is precisely delineated, and the terms are sometimes used interchangeably, the choice of usage also sometimes corresponds to differences in viewpoint and emphasis.<sup>[1]</sup> As such, this article may not represent the views of all researchers in the fields equally. This kind of variability is unavoidable when a field is in rapid flux.

## 1.1 History

### 1.1.1 Prehistory: the groupoid model

At one time the idea that types in intensional type theory<sup>14</sup> with their identity types could be regarded as groupoids<sup>15</sup> was mathematical folklore<sup>16</sup>. It was first made precise semantically in the 1998 paper of Martin Hofmann and Thomas Streicher<sup>17</sup> called "The groupoid interpretation of type theory", in which they showed that intensional type theory had a model in the category of groupoids<sup>18</sup>.<sup>[2]</sup> This was the first truly "homotopical"<sup>19</sup> model of type theory, albeit only "1-dimensional"<sup>20</sup> (the traditional models in the category of sets<sup>21</sup> being homotopically 0-dimensional).

- 
- 1 [https://en.wikipedia.org/wiki/Mathematical\\_logic](https://en.wikipedia.org/wiki/Mathematical_logic)
  - 2 [https://en.wikipedia.org/wiki/Computer\\_science](https://en.wikipedia.org/wiki/Computer_science)
  - 3 <https://en.wikipedia.org/wiki/Help:IPA/English>
  - 4 [https://en.wikipedia.org/wiki/Intuitionistic\\_type\\_theory](https://en.wikipedia.org/wiki/Intuitionistic_type_theory)
  - 5 [https://en.wikipedia.org/wiki/Homotopy\\_theory](https://en.wikipedia.org/wiki/Homotopy_theory)
  - 6 [https://en.wikipedia.org/wiki/Higher\\_category\\_theory](https://en.wikipedia.org/wiki/Higher_category_theory)
  - 7 [https://en.wikipedia.org/wiki/Model\\_\(mathematical\\_logic\)](https://en.wikipedia.org/wiki/Model_(mathematical_logic))
  - 8 [https://en.wikipedia.org/wiki/Internal\\_language](https://en.wikipedia.org/wiki/Internal_language)
  - 9 [https://en.wikipedia.org/wiki/Higher\\_category\\_theory](https://en.wikipedia.org/wiki/Higher_category_theory)
  - 10 [https://en.wikipedia.org/wiki/Foundation\\_of\\_mathematics](https://en.wikipedia.org/wiki/Foundation_of_mathematics)
  - 11 [https://en.wikipedia.org/wiki/Formal\\_proof](https://en.wikipedia.org/wiki/Formal_proof)
  - 12 [https://en.wikipedia.org/wiki/Proof\\_assistant](https://en.wikipedia.org/wiki/Proof_assistant)
  - 13 [https://en.wikipedia.org/wiki/Univalent\\_foundations](https://en.wikipedia.org/wiki/Univalent_foundations)
  - 14 [https://en.wikipedia.org/wiki/Intuitionistic\\_type\\_theory#Extensional\\_versus\\_intensional](https://en.wikipedia.org/wiki/Intuitionistic_type_theory#Extensional_versus_intensional)
  - 15 <https://en.wikipedia.org/wiki/Groupoid>
  - 16 [https://en.wikipedia.org/wiki/Mathematical\\_folklore](https://en.wikipedia.org/wiki/Mathematical_folklore)
  - 17 [https://en.wikipedia.org/wiki/Thomas\\_Streicher](https://en.wikipedia.org/wiki/Thomas_Streicher)
  - 18 <https://en.wikipedia.org/wiki/Groupoid>
  - 19 [https://en.wikipedia.org/wiki/Homotopical\\_algebra](https://en.wikipedia.org/wiki/Homotopical_algebra)
  - 20 <https://en.wikipedia.org/wiki/Dimension>
  - 21 [https://en.wikipedia.org/wiki/Category\\_of\\_sets](https://en.wikipedia.org/wiki/Category_of_sets)

Their paper also foreshadowed several later developments in homotopy type theory. For instance, they noted that the groupoid model satisfies a rule they called "universe extensionality", which is none other than the restriction to 1-types of the *univalence axiom* that Vladimir Voevodsky<sup>22</sup> proposed ten years later. (The axiom for 1-types is notably simpler to formulate, however, since a coherent<sup>23</sup> notion of "equivalence" is not required.) They also defined "categories with isomorphism as equality" and conjectured that in a model using higher-dimensional groupoids, for such categories one would have "equivalence is equality"; this was later proven by Benedikt Ahrens, Krzysztof Kapulkin, and Michael Shulman<sup>24</sup>.<sup>[3]</sup>

### 1.1.2 Early history: model categories and higher groupoids

The first higher-dimensional models of intensional type theory were constructed by Steve Awodey<sup>25</sup> and his student Michael Warren in 2005 using Quillen model categories<sup>26</sup>. These results were first presented in public at the conference FMCS 2006<sup>[4]</sup> at which Warren gave a talk titled "Homotopy models of intensional type theory", which also served as his thesis prospectus (the dissertation committee present were Awodey, Nicola Gambino and Alex Simpson). A summary is contained in Warren's thesis prospectus abstract.<sup>[5]</sup>

At a subsequent workshop about identity types at Uppsala University<sup>27</sup> in 2006<sup>[6]</sup> there were two talks about the relation between intensional type theory and factorization systems: one by Richard Garner, "Factorisation systems for type theory",<sup>[7]</sup> and one by Michael Warren, "Model categories and intensional identity types". Related ideas were discussed in the talks by Steve Awodey, "Type theory of higher-dimensional categories", and Thomas Streicher<sup>28</sup>, "Identity types vs. weak omega-groupoids: some ideas, some problems". At the same conference Benno van den Berg gave a talk titled "Types as weak omega-categories" where he outlined the ideas that later became the subject of a joint paper with Richard Garner.

All early constructions of higher dimensional models had to deal with the problem of coherence typical of models of dependent type theory, and various solutions were developed. One such was given in 2009 by Voevodsky, another in 2010 by van den Berg and Garner.<sup>[8]</sup> A general solution, building on Voevodsky's construction, was eventually given by Lumsdaine and Warren in 2014.<sup>[9]</sup>

At the PSSSL86 in 2007<sup>[10]</sup> Awodey gave a talk titled "Homotopy type theory" (this was the first public usage of that term, which was coined by Awodey<sup>[11]</sup>). Awodey and Warren summarized their results in the paper "Homotopy theoretic models of identity types", which was posted on the ArXiv<sup>29</sup> preprint server in 2007<sup>[12]</sup> and published in 2009; a more detailed version appeared in Warren's thesis "Homotopy theoretic aspects of constructive type theory" in 2008.

At about the same time, Vladimir Voevodsky was independently investigating type theory in the context of the search of a language for practical formalization of mathematics. In

22 [https://en.wikipedia.org/wiki/Vladimir\\_Voevodsky](https://en.wikipedia.org/wiki/Vladimir_Voevodsky)

23 [https://en.wikipedia.org/wiki/Coherence\\_condition](https://en.wikipedia.org/wiki/Coherence_condition)

24 [https://en.wikipedia.org/wiki/Michael\\_Shulman\\_\(mathematician\)](https://en.wikipedia.org/wiki/Michael_Shulman_(mathematician))

25 [https://en.wikipedia.org/wiki/Steve\\_Awodey](https://en.wikipedia.org/wiki/Steve_Awodey)

26 [https://en.wikipedia.org/wiki/Model\\_category](https://en.wikipedia.org/wiki/Model_category)

27 [https://en.wikipedia.org/wiki/Uppsala\\_University](https://en.wikipedia.org/wiki/Uppsala_University)

28 [https://en.wikipedia.org/wiki/Thomas\\_Streicher](https://en.wikipedia.org/wiki/Thomas_Streicher)

29 <https://en.wikipedia.org/wiki/ArXiv>

September 2006 he posted to the Types mailing list "A very short note on homotopy lambda calculus<sup>30</sup>,<sup>[13]</sup> which sketched the outlines of a type theory with dependent products, sums and universes and of a model of this type theory in Kan simplicial sets<sup>31</sup>. It began by saying "The homotopy  $\lambda$ -calculus is a hypothetical (at the moment) type system" and ended with "At the moment much of what I said above is at the level of conjectures. Even the definition of the model of TS in the homotopy category is non-trivial" referring to the complex coherence issues that were not resolved until 2009. This note included a syntactic definition of "equality types" that were claimed to be interpreted in the model by path-spaces, but did not consider Per Martin-Löf<sup>32</sup>'s rules for identity types. It also stratified the universes by homotopy dimension in addition to size, an idea that later was mostly discarded.

On the syntactic side, Benno van den Berg conjectured in 2006 that the tower of identity types of a type in intensional type theory should have the structure of an  $\omega$ -category, and indeed a  $\omega$ -groupoid, in the "globular, algebraic" sense of Michael Batanin. This was later proven independently by van den Berg and Garner in the paper "Types are weak omega-groupoids" (published 2008),<sup>[14]</sup> and by Peter Lumsdaine in the paper "Weak  $\omega$ -Categories from Intensional Type Theory" (published 2009) and as part of his 2010 Ph.D. thesis "Higher Categories from Type Theories".<sup>[15]</sup>

### 1.1.3 The univalence axiom, synthetic homotopy theory, and higher inductive types

The concept of a univalent fibration was introduced by Voevodsky in early 2006.<sup>[16]</sup> However, because of the insistence of all presentations of the Martin-Löf type theory on the property that the identity types, in the empty context, may contain only reflexivity, Voevodsky did not recognize until 2009 that these identity types can be used in combination with the univalent universes. In particular, the idea that univalence can be introduced simply by adding an axiom to the existing Martin-Löf type theory appeared only in 2009.<sup>[a][b]</sup>

Also in 2009, Voevodsky worked out more of the details of a model of type theory in Kan complexes<sup>33</sup>, and observed that the existence of a universal Kan fibration<sup>34</sup> could be used to resolve the coherence problems for categorical models of type theory. He also proved, using an idea of A. K. Bousfield, that this universal fibration was univalent: the associated fibration of pairwise homotopy equivalences between the fibers is equivalent to the path-space fibration of the base.

To formulate univalence as an axiom Voevodsky found a way to define "equivalences" syntactically that had the important property that the type representing the statement "f is an equivalence" was (under the assumption of function extensionality)  $(-1)$ -truncated (i.e. contractible if inhabited). This enabled him to give a *syntactic* statement of univalence, generalizing Hofmann and Streicher's "universe extensionality" to higher dimensions. He was also able to use these definitions of equivalences and contractibility to start developing

---

30 [https://en.wikipedia.org/wiki/Lambda\\_calculus](https://en.wikipedia.org/wiki/Lambda_calculus)

31 [https://en.wikipedia.org/wiki/Simplicial\\_set](https://en.wikipedia.org/wiki/Simplicial_set)

32 [https://en.wikipedia.org/wiki/Per\\_Martin-Löf](https://en.wikipedia.org/wiki/Per_Martin-Löf)

33 [https://en.wikipedia.org/wiki/Kan\\_complex](https://en.wikipedia.org/wiki/Kan_complex)

34 [https://en.wikipedia.org/wiki/Kan\\_fibration](https://en.wikipedia.org/wiki/Kan_fibration)



significant amounts of "synthetic homotopy theory" in the proof assistant Coq<sup>35</sup>; this formed the basis of the library later called "Foundations" and eventually "UniMath".<sup>[18]</sup>

Unification of the various threads began in February 2010 with an informal meeting at Carnegie Mellon University<sup>36</sup>, where Voevodsky presented his model in Kan complexes and his Coq code to a group including Awodey, Warren, Lumsdaine, Robert Harper<sup>37</sup>, Dan Licata, Michael Shulman<sup>38</sup>, and others. This meeting produced the outlines of a proof (by Warren, Lumsdaine, Licata, and Shulman) that every homotopy equivalence is an equivalence (in Voevodsky's good coherent sense), based on the idea from category theory of improving equivalences to adjoint equivalences. Soon afterwards, Voevodsky proved that the univalence axiom implies function extensionality.

The next pivotal event was a mini-workshop at the Mathematical Research Institute of Oberwolfach<sup>39</sup> in March 2011 organized by Steve Awodey, Richard Garner, Per Martin-Löf, and Vladimir Voevodsky, titled "The homotopy interpretation of constructive type theory".<sup>[19]</sup> As part of a Coq tutorial for this workshop, Andrej Bauer wrote a small Coq library<sup>[20]</sup> based on Voevodsky's ideas (but not actually using any of his code); this eventually became the kernel of the first version of the "HoTT" Coq library<sup>[21]</sup> (the first commit of the latter<sup>[22]</sup> by Michael Shulman notes "Development based on Andrej Bauer's files, with many ideas taken from Vladimir Voevodsky's files"). One of the most important things to come out of the Oberwolfach meeting was the basic idea of higher inductive types, due to Lumsdaine, Shulman, Bauer, and Warren. The participants also formulated a list of important open questions, such as whether the univalence axiom satisfies canonicity (still open, although some special cases have been resolved positively<sup>[23][24]</sup>), whether the univalence axiom has nonstandard models (since answered positively by Shulman), and how to define (semi)simplicial types (still open in MLTT, although it can be done in Voevodsky's Homotopy Type System (HTS), a type theory with two equality types).

Soon after the Oberwolfach workshop, the *Homotopy Type Theory website and blog*<sup>[25]</sup> was established, and the subject began to be popularized under that name. An idea of some of the important progress during this period can be obtained from the blog history.<sup>[26]</sup>

## 1.2 Univalent foundations

The phrase "univalent foundations" is agreed by all to be closely related to homotopy type theory, but not everyone uses it in the same way. It was originally used by Vladimir Voevodsky to refer to his vision of a foundational system for mathematics in which the basic objects are homotopy types, based on a type theory satisfying § the univalence axiom<sup>40</sup>, and formalized in a computer proof assistant.<sup>[27]</sup>

As Voevodsky's work became integrated with the community of other researchers working on homotopy type theory, "univalent foundations" was sometimes used interchangeably with "homotopy type theory",<sup>[28]</sup> and other times to refer only to its use as a foundational

35 <https://en.wikipedia.org/wiki/Coq>

36 [https://en.wikipedia.org/wiki/Carnegie\\_Mellon\\_University](https://en.wikipedia.org/wiki/Carnegie_Mellon_University)

37 [https://en.wikipedia.org/wiki/Robert\\_Harper\\_\(computer\\_scientist\)](https://en.wikipedia.org/wiki/Robert_Harper_(computer_scientist))

38 [https://en.wikipedia.org/wiki/Michael\\_Shulman\\_\(mathematician\)](https://en.wikipedia.org/wiki/Michael_Shulman_(mathematician))

39 [https://en.wikipedia.org/wiki/Mathematical\\_Research\\_Institute\\_of\\_Oberwolfach](https://en.wikipedia.org/wiki/Mathematical_Research_Institute_of_Oberwolfach)

40 [#The\\_univalence\\_axiom](#)

system (excluding, for example, the study of model-categorical semantics or computational metatheory).<sup>[29]</sup> For instance, the subject of the IAS special year was officially given as "univalent foundations", although a lot of the work done there focused on semantics and metatheory in addition to foundations. The book produced by participants in the IAS program was titled "Homotopy type theory: Univalent foundations of mathematics"; although this could refer to either usage, since the book only discusses HoTT as a mathematical foundation.<sup>[28]</sup>

### 1.3 Special Year on Univalent Foundations of Mathematics

Play media<sup>41</sup> <sup>42</sup>An animation showing development of the HoTT Book on the GitHub repository by the participants in the Univalent Foundations Special Year project. In 2012–13 researchers at the Institute for Advanced Study<sup>43</sup> held "A Special Year on Univalent Foundations of Mathematics".<sup>[30]</sup> The special year brought together researchers in topology<sup>44</sup>, computer science<sup>45</sup>, category theory<sup>46</sup>, and mathematical logic<sup>47</sup>. The program was organized by Steve Awodey<sup>48</sup>, Thierry Coquand<sup>49</sup> and Vladimir Voevodsky<sup>50</sup>.

During the program Peter Aczel<sup>51</sup>, who was one of the participants, initiated a working group which investigated how to do type theory informally but rigorously, in a style that is analogous to ordinary mathematicians doing set theory. After initial experiments it became clear that this was not only possible but highly beneficial, and that a book (the so-called **HoTT Book**)<sup>[28][31]</sup> could and should be written. Many other participants of the project then joined the effort with technical support, writing, proof reading, and offering ideas. Unusually for a mathematics text, it was developed collaboratively and in the open on GitHub<sup>52</sup>, is released under a Creative Commons license<sup>53</sup> that allows people to fork<sup>54</sup> their own version of the book, and is both purchasable in print and downloadable free of charge.<sup>[32][33][34]</sup>

More generally, the special year was a catalyst for the development of the entire subject; the HoTT Book was only one, albeit the most visible, result.

#### Official participants in the special year

- Peter Aczel<sup>55</sup>
- Benedikt Ahrens

---

41 [http://upload.wikimedia.org/wikipedia/commons/9/95/The\\_making\\_of\\_HoTT\\_book.webm](http://upload.wikimedia.org/wikipedia/commons/9/95/The_making_of_HoTT_book.webm)

42 [https://en.wikipedia.org/wiki/File:The\\_making\\_of\\_HoTT\\_book.webm](https://en.wikipedia.org/wiki/File:The_making_of_HoTT_book.webm)

43 [https://en.wikipedia.org/wiki/Institute\\_for\\_Advanced\\_Study](https://en.wikipedia.org/wiki/Institute_for_Advanced_Study)

44 <https://en.wikipedia.org/wiki/Topology>

45 [https://en.wikipedia.org/wiki/Computer\\_science](https://en.wikipedia.org/wiki/Computer_science)

46 [https://en.wikipedia.org/wiki/Category\\_theory](https://en.wikipedia.org/wiki/Category_theory)

47 [https://en.wikipedia.org/wiki/Mathematical\\_logic](https://en.wikipedia.org/wiki/Mathematical_logic)

48 [https://en.wikipedia.org/wiki/Steve\\_Awodey](https://en.wikipedia.org/wiki/Steve_Awodey)

49 [https://en.wikipedia.org/wiki/Thierry\\_Coquand](https://en.wikipedia.org/wiki/Thierry_Coquand)

50 [https://en.wikipedia.org/wiki/Vladimir\\_Voevodsky](https://en.wikipedia.org/wiki/Vladimir_Voevodsky)

51 [https://en.wikipedia.org/wiki/Peter\\_Aczel](https://en.wikipedia.org/wiki/Peter_Aczel)

52 <https://en.wikipedia.org/wiki/GitHub>

53 [https://en.wikipedia.org/wiki/Creative\\_Commons\\_license](https://en.wikipedia.org/wiki/Creative_Commons_license)

54 [https://en.wikipedia.org/wiki/Fork\\_\(software\\_development\)](https://en.wikipedia.org/wiki/Fork_(software_development))

55 [https://en.wikipedia.org/wiki/Peter\\_Aczel](https://en.wikipedia.org/wiki/Peter_Aczel)

- Thorsten Altenkirch<sup>56</sup>
- Steve Awodey<sup>57</sup>
- Bruno Barras
- Andrej Bauer
- Yves Bertot
- Marc Bezem
- Thierry Coquand<sup>58</sup>
- Eric Finster
- Daniel Grayson
- Hugo Herbelin
- André Joyal<sup>59</sup>
- Dan Licata
- Peter Lumsdaine
- Assia Mahboubi
- Per Martin-Löf<sup>60</sup>
- Sergey Melikhov
- Alvaro Pelayo
- Andrew Polonsky
- Michael Shulman<sup>61</sup>
- Matthieu Sozeau
- Bas Spitters
- Benno van den Berg
- Vladimir Voevodsky<sup>62</sup>
- Michael Warren
- Noam Zeilberger

*ACM Computing Reviews*<sup>63</sup> listed the book as a notable 2013 publication in the category "mathematics of computing".<sup>[35]</sup>

## 1.4 Key concepts

Intensional type theory	Homotopy theory
types $A$	spaces $A$
terms $a$	points $a$
$a : A$	$a \in A$
dependent type <sup>64</sup> $x : A \vdash B(x)$	fibration <sup>65</sup> $B \rightarrow A$
identity type <sup>66</sup> $\text{Id}_A(a, b)$	path space <sup>67</sup>

- 56 [https://en.wikipedia.org/wiki/Thorsten\\_Altenkirch](https://en.wikipedia.org/wiki/Thorsten_Altenkirch)
- 57 [https://en.wikipedia.org/wiki/Steve\\_Awodey](https://en.wikipedia.org/wiki/Steve_Awodey)
- 58 [https://en.wikipedia.org/wiki/Thierry\\_Coquand](https://en.wikipedia.org/wiki/Thierry_Coquand)
- 59 [https://en.wikipedia.org/wiki/Andr%C3%A9\\_Joyal](https://en.wikipedia.org/wiki/Andr%C3%A9_Joyal)
- 60 [https://en.wikipedia.org/wiki/Per\\_Martin-L%C3%B6f](https://en.wikipedia.org/wiki/Per_Martin-L%C3%B6f)
- 61 [https://en.wikipedia.org/wiki/Michael\\_Shulman\\_\(mathematician\)](https://en.wikipedia.org/wiki/Michael_Shulman_(mathematician))
- 62 [https://en.wikipedia.org/wiki/Vladimir\\_Voevodsky](https://en.wikipedia.org/wiki/Vladimir_Voevodsky)
- 63 [https://en.wikipedia.org/wiki/ACM\\_Computing\\_Reviews](https://en.wikipedia.org/wiki/ACM_Computing_Reviews)
- 64 [https://en.wikipedia.org/wiki/Dependent\\_type](https://en.wikipedia.org/wiki/Dependent_type)
- 65 <https://en.wikipedia.org/wiki/Fibration>
- 66 [https://en.wikipedia.org/wiki/Intuitionistic\\_type\\_theory#Equality\\_type](https://en.wikipedia.org/wiki/Intuitionistic_type_theory#Equality_type)
- 67 [https://en.wikipedia.org/w/index.php?title=Path\\_space\\_\(algebraic\\_topology\)&action=edit&redlink=1](https://en.wikipedia.org/w/index.php?title=Path_space_(algebraic_topology)&action=edit&redlink=1)

Intensional type theory	Homotopy theory
$p : \text{Id}_A(a, b)$	$\text{path}^{68} p : a \rightarrow b$
$\alpha : \text{Id}_{\text{Id}_A(a, b)}(p, q)$	$\text{homotopy}^{69} \alpha : p \Rightarrow q$

### 1.4.1 "Propositions as types"

HoTT uses a modified version of the "propositions as types"<sup>70</sup> interpretation of type theory, according to which types can also represent propositions and terms can then represent proofs. In HoTT, however, unlike in standard "propositions as types", a special role is played by 'mere propositions' which, roughly speaking, are those types having at most one term, up to propositional equality<sup>71</sup>. These are more like conventional logical propositions than are general types, in that they are proof-irrelevant.

### 1.4.2 Equality

The fundamental concept of homotopy type theory is the path<sup>72</sup>. In HoTT, the type  $a = b$  is the type of all paths from the point  $a$  to the point  $b$ . (Therefore, a proof that a point  $a$  equals a point  $b$  is the same thing as a path from the point  $a$  to the point  $b$ .) For any point  $a$ , there exists a path of type  $a = a$ , corresponding to the reflexive property of equality. A path of type  $a = b$  can be inverted, forming a path of type  $b = a$ , corresponding to the symmetric property of equality. Two paths of type  $a = b$  resp.  $b = c$  can be concatenated, forming a path of type  $a = c$ ; this corresponds to the transitive property of equality.

Most importantly, given a path  $p : a = b$ , and a proof of some property  $P(a)$ , the proof can be "transported" along the path  $p$  to yield a proof of the property  $P(b)$ . (Equivalently stated, an object of type  $P(a)$  can be turned into an object of type  $P(b)$ .) This corresponds to the substitution property of equality<sup>73</sup>. Here, an important difference between HoTT and classical mathematics comes in. In classical mathematics, once the equality of two values  $a$  and  $b$  has been established,  $a$  and  $b$  may be used interchangeably thereafter, with no regard to any distinction between them. In homotopy type theory, however, there may be multiple different paths  $a = b$ , and transporting an object along two different paths will yield two different results. Therefore, in homotopy type theory, when applying the substitution property, it is necessary to state which path is being used.

In general, a "proposition" can have multiple different proofs. (For example, the type of all natural numbers, when considered as a proposition, has every natural number as a proof.) Even if a proposition has only one proof  $a$ , the space of paths  $a = a$  may be non-trivial in some way. A "mere proposition" is any type which either is empty, or contains only one point with a trivial path space<sup>74</sup>.

Note that people write  $a = b$  for  $\text{Id}_A(a, b)$ , thereby leaving the type  $A$  of  $a, b$  implicit. Do not confuse it with  $\text{id}_A : A \rightarrow A$ , denoting the identity function on  $A$ .<sup>[c]</sup>

<sup>68</sup> [https://en.wikipedia.org/wiki/Path\\_\(topology\)](https://en.wikipedia.org/wiki/Path_(topology))

<sup>69</sup> <https://en.wikipedia.org/wiki/Homotopy>

<sup>70</sup> [https://en.wikipedia.org/wiki/Propositions\\_as\\_types\\_principle](https://en.wikipedia.org/wiki/Propositions_as_types_principle)

<sup>71</sup> [https://en.wikipedia.org/wiki/Propositional\\_equality](https://en.wikipedia.org/wiki/Propositional_equality)

<sup>72</sup> [https://en.wikipedia.org/wiki/Path\\_\(topology\)](https://en.wikipedia.org/wiki/Path_(topology))

<sup>73</sup> [https://en.wikipedia.org/wiki/First-order\\_logic#Equality\\_and\\_its\\_axioms](https://en.wikipedia.org/wiki/First-order_logic#Equality_and_its_axioms)

<sup>74</sup> [https://en.wikipedia.org/w/index.php?title=Path\\_space\\_\(algebraic\\_topology\)&action=edit&redlink=1](https://en.wikipedia.org/w/index.php?title=Path_space_(algebraic_topology)&action=edit&redlink=1)

### 1.4.3 Type equivalence

Two types  $A$  and  $B$  belonging to some universe  $U$  are defined as being *equivalent* if there exists an *equivalence* between them. An equivalence is a function

$$f : A \rightarrow B$$

which has both a left inverse and a right inverse, in the sense that for suitably chosen  $g$  and  $h$ , the following types are both inhabited:

$$Id_{B \rightarrow B}(f \circ g, id_B),$$

$$Id_{A \rightarrow A}(h \circ f, id_A).$$

i.e.

$$f \circ g =_{B \rightarrow B} id_B,$$

$$h \circ f =_{A \rightarrow A} id_A.$$

This expresses a general notion of “ $f$  has both a left inverse and right inverse”, using equality types. Note that the invertibility conditions above are equality types in the function types  $A \rightarrow A$  and  $B \rightarrow B$ . One generally assumes the function extensionality axiom, which ensures that these are equivalent to the following types that express invertibility using the equality on the domain and codomain  $A$  and  $B$ :

$$\Pi_{y:B}. Id_B((f \circ g)(y), id_B(y)),$$

$$\Pi_{x:A}. Id_A((h \circ f)(x), id_A(x)).$$

i.e. for all  $x : A$  and  $y : B$ ,

$$f(g(y)) =_B y,$$

$$h(f(x)) =_A x.$$

The functions of type

$$A \rightarrow B$$

together with a proof that they are equivalences are denoted by

$$A \simeq B.$$

### 1.4.4 The univalence axiom

Having defined functions that are equivalences as above, one can show that there is a canonical way to turn paths to equivalences. In other words, there is a function of the type

$$(A = B) \rightarrow (A \simeq B),$$

which expresses that types  $A, B$  that are equal are, in particular, also equivalent.

The **univalence axiom** states that this function is itself an equivalence.<sup>[28]:115[17]:4–6</sup> Therefore, we have

$$(A = B) \simeq (A \simeq B)$$

"In other words, identity is equivalent to equivalence. In particular, one may say that 'equivalent types are identical'."<sup>[28]:4</sup>

## 1.5 Applications

### 1.5.1 Theorem proving

HoTT allows mathematical proofs to be translated into a computer programming language<sup>75</sup> for computer proof assistants<sup>76</sup> much more easily than before. This approach offers the potential for computers to check difficult proofs.<sup>[36]</sup>

One goal of mathematics is to formulate axioms from which virtually all mathematical theorems can be derived and proven unambiguously. Correct proofs in mathematics must follow the rules of logic. They must be derivable without error from axioms<sup>77</sup> and already-proven statements.<sup>[36]</sup>

HoTT adds the univalence axiom, which relates the equality of logical-mathematical propositions to homotopy theory. An equation such as " $a=b$ " is a mathematical proposition in which two different symbols have the same value. In homotopy type theory, this is taken to mean that the two shapes which represent the values of the symbols are topologically equivalent.<sup>[36]</sup>

These topological equivalence relationships, ETH Zürich<sup>78</sup> Institute for Theoretical Studies director Giovanni Felder<sup>79</sup> argues, can be better formulated in homotopy theory because it is more comprehensive: Homotopy theory explains not only why " $a$  equals  $b$ " but also how to derive this. In set theory, this information would have to be defined additionally, which makes the translation of mathematical propositions into programming languages more difficult.<sup>[36]</sup>

### 1.5.2 Computer programming

This section **needs expansion**. You can help by adding to it<sup>80</sup>. (*December 2014*)

As of 2015, intense research work was underway to model and formally analyse the computational behavior of the univalence axiom in homotopy type theory.<sup>[37]</sup>

*Cubical type theory* is one attempt to give computational content to homotopy type theory.<sup>[38]</sup>

However, it is believed that certain objects, such as semi-simplicial types, cannot be constructed without reference to some notion of exact equality. Therefore, various *two-level type theories* have been developed which partition their types into fibrant types, which respect paths, and non-fibrant types, which do not. Cartesian cubical computational type

---

<sup>75</sup> [https://en.wikipedia.org/wiki/Computer\\_programming\\_language](https://en.wikipedia.org/wiki/Computer_programming_language)

<sup>76</sup> [https://en.wikipedia.org/wiki/Proof\\_assistant](https://en.wikipedia.org/wiki/Proof_assistant)

<sup>77</sup> <https://en.wikipedia.org/wiki/Axiom>

<sup>78</sup> [https://en.wikipedia.org/wiki/ETH\\_Z%C3%BCrich](https://en.wikipedia.org/wiki/ETH_Z%C3%BCrich)

<sup>79</sup> [https://en.wikipedia.org/wiki/Giovanni\\_Felder](https://en.wikipedia.org/wiki/Giovanni_Felder)

<sup>80</sup> [https://en.wikipedia.org/w/index.php?title=Homotopy\\_type\\_theory&action=edit&section=](https://en.wikipedia.org/w/index.php?title=Homotopy_type_theory&action=edit&section=)

theory is the first two-level type theory which gives a full computational interpretation to homotopy type theory.<sup>[39]</sup>

## 1.6 See also

- Calculus of constructions<sup>81</sup>
- Curry–Howard correspondence<sup>82</sup>
- Intuitionistic type theory<sup>83</sup>
- Homotopy hypothesis<sup>84</sup>
- Univalent foundations<sup>85</sup>

## 1.7 Notes

1. Univalence is a type, a property of the identity type<sup>86</sup>  $\text{Id}_U$  of a universe  $U$  —Martín Hötzel Escardó (2018)<sup>[17]:p.1</sup>
2. "Univalence is a type, and the univalence axiom says that this type has some inhabitant."<sup>[17]:p.1</sup>
3. Here the type theory convention is used, that type names begin with a capitalized letter, but that function names begin with a lower-case letter.

## 1.8 References

1. SHULMAN, MICHAEL<sup>87</sup> (27 JANUARY 2016). "HOMOTOPY TYPE THEORY: A SYNTHETIC APPROACH TO HIGHER EQUALITIES". ARXIV<sup>88</sup>:1601.05035v3<sup>89</sup> [MATH.LO<sup>90</sup>], footnote 1
2. HOFMANN, MARTIN; STREICHER, THOMAS<sup>91</sup> (1998). "THE GROUPOID INTERPRETATION OF TYPE THEORY"<sup>92</sup>. IN SAMBIN, GIOVANNI; SMITH, JAN M. (EDS.). *Twenty Five Years of Constructive Type Theory*. Oxford Logic Guides. Vol. 36. Clarendon Press. pp. 83–111. ISBN<sup>93</sup> 978-0-19-158903-4<sup>94</sup>. MR<sup>95</sup> 1686862<sup>96</sup>.
3. AHRENS, BENEDIKT; KAPULKIN, KRZYSZTOF; SHULMAN, MICHAEL<sup>97</sup> (2015). "UNIVALENT CATEGORIES AND THE REZK COMPLETION". *Mathematical Struc-*

---

81 [https://en.wikipedia.org/wiki/Calculus\\_of\\_constructions](https://en.wikipedia.org/wiki/Calculus_of_constructions)  
82 [https://en.wikipedia.org/wiki/Curry%E2%80%93Howard\\_correspondence](https://en.wikipedia.org/wiki/Curry%E2%80%93Howard_correspondence)  
83 [https://en.wikipedia.org/wiki/Intuitionistic\\_type\\_theory](https://en.wikipedia.org/wiki/Intuitionistic_type_theory)  
84 [https://en.wikipedia.org/wiki/Homotopy\\_hypothesis](https://en.wikipedia.org/wiki/Homotopy_hypothesis)  
85 [https://en.wikipedia.org/wiki/Univalent\\_foundations](https://en.wikipedia.org/wiki/Univalent_foundations)  
86 [https://en.wikipedia.org/wiki/Identity\\_type](https://en.wikipedia.org/wiki/Identity_type)  
87 [https://en.wikipedia.org/wiki/Michael\\_Shulman\\_\(mathematician\)](https://en.wikipedia.org/wiki/Michael_Shulman_(mathematician))  
88 [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))  
89 <http://arxiv.org/abs/1601.05035v3>  
90 <http://arxiv.org/archive/math.LO>  
91 [https://en.wikipedia.org/wiki/Thomas\\_Streicher](https://en.wikipedia.org/wiki/Thomas_Streicher)  
92 [https://books.google.com/books?id=pLnKggT\\_In4C&pg=PA83](https://books.google.com/books?id=pLnKggT_In4C&pg=PA83)  
93 [https://en.wikipedia.org/wiki/ISBN\\_\(identifier\)](https://en.wikipedia.org/wiki/ISBN_(identifier))  
94 <https://en.wikipedia.org/wiki/Special:BookSources/978-0-19-158903-4>  
95 [https://en.wikipedia.org/wiki/MR\\_\(identifier\)](https://en.wikipedia.org/wiki/MR_(identifier))  
96 <http://www.ams.org/mathscinet-getitem?mr=1686862>  
97 [https://en.wikipedia.org/wiki/Michael\\_Shulman\\_\(mathematician\)](https://en.wikipedia.org/wiki/Michael_Shulman_(mathematician))

- tures in *Computer Science*. **25** (5): 1010–1039. arXiv<sup>98</sup>:1303.0584<sup>99</sup>. doi<sup>100</sup>:10.1017/S0960129514000486<sup>101</sup>. MR<sup>102</sup> 3340533<sup>103</sup>. S2CID<sup>104</sup> 1135785<sup>105</sup>.
4. "FOUNDATIONAL METHODS IN COMPUTER SCIENCE 2006, UNIVERSITY OF CALGARY, JUNE 7TH - 9TH, 2006"<sup>106</sup>. UNIVERSITY OF CALGARY. RETRIEVED 6 JUNE 2021.
  5. WARREN, MICHAEL A. (2006). *Homotopy Models of Intensional Type Theory*<sup>107</sup> (PDF) (THESIS).
  6. "IDENTITY TYPES - TOPOLOGICAL AND CATEGORICAL STRUCTURE, WORKSHOP, UPPSALA, NOVEMBER 13-14, 2006"<sup>108</sup>. UPPSALA UNIVERSITY - DEPARTMENT OF MATHEMATICS. RETRIEVED 6 JUNE 2021.
  7. Richard Garner, Factorisation axioms for type theory<sup>109</sup>
  8. BERG, BENNO VAN DEN; GARNER, RICHARD (27 JULY 2010). "TOPOLOGICAL AND SIMPLICIAL MODELS OF IDENTITY TYPES". arXiv<sup>110</sup>:1007.4638<sup>111</sup> [MATH.LO<sup>112</sup>].
  9. LUMSDAINE, PETER LEFANU; WARREN, MICHAEL A. (6 NOVEMBER 2014). "THE LOCAL UNIVERSES MODEL: AN OVERLOOKED COHERENCE CONSTRUCTION FOR DEPENDENT TYPE THEORIES". *ACM Transactions on Computational Logic*. **16** (3): 1–31. arXiv<sup>113</sup>:1411.1736<sup>114</sup>. doi<sup>115</sup>:10.1145/2754931<sup>116</sup>. S2CID<sup>117</sup> 14068103<sup>118</sup>.
  10. "86TH EDITION OF THE PERIPATETIC SEMINAR ON SHEAVES AND LOGIC, HENRI POINCARÉ UNIVERSITY, SEPTEMBER 8-9, 2007"<sup>119</sup>. LORIA.FR. ARCHIVED FROM THE ORIGINAL<sup>120</sup> ON 17 DECEMBER 2014. RETRIEVED 20 DECEMBER 2014.
  11. Preliminary list of PSSSL86 participants<sup>121</sup>
  12. AWODEY, STEVE; WARREN, MICHAEL A. (3 SEPTEMBER 2007). "HOMOTOPY THEORETIC MODELS OF IDENTITY TYPES". *Mathematical Proceedings of the Cambridge Philosophical Society*. **146**: 45. arXiv<sup>122</sup>:0709.0248<sup>123</sup>.

98 [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))

99 <http://arxiv.org/abs/1303.0584>

100 [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))

101 <https://doi.org/10.1017%2FS0960129514000486>

102 [https://en.wikipedia.org/wiki/MR\\_\(identifier\)](https://en.wikipedia.org/wiki/MR_(identifier))

103 <http://www.ams.org/mathscinet-getitem?mr=3340533>

104 [https://en.wikipedia.org/wiki/S2CID\\_\(identifier\)](https://en.wikipedia.org/wiki/S2CID_(identifier))

105 <https://api.semanticscholar.org/CorpusID:1135785>

106 [http://pages.cpsc.ucalgary.ca/~robin/FMCS/FMCS\\_06/FMCS06.html](http://pages.cpsc.ucalgary.ca/~robin/FMCS/FMCS_06/FMCS06.html)

107 <http://mawarren.net/papers/prospectus.pdf>

108 <http://www2.math.uu.se/~palmgren/itt/>

109 <http://comp.mq.edu.au/~rgarner/Papers/Uppsala.pdf>

110 [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))

111 <http://arxiv.org/abs/1007.4638>

112 <http://arxiv.org/archive/math.LO>

113 [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))

114 <http://arxiv.org/abs/1411.1736>

115 [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))

116 <https://doi.org/10.1145%2F2754931>

117 [https://en.wikipedia.org/wiki/S2CID\\_\(identifier\)](https://en.wikipedia.org/wiki/S2CID_(identifier))

118 <https://api.semanticscholar.org/CorpusID:14068103>

119 <https://archive.today/20141217151510/http://www.loria.fr/~lamarche/psslHomeEN.html>

120 <http://www.loria.fr/~lamarche/psslHomeEN.html>

121 <http://www.loria.fr/~lamarche/listPart.html>

122 [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))

123 <http://arxiv.org/abs/0709.0248>



- Bibcode<sup>124</sup>:2008MPCPS.146...45A<sup>125</sup>. doi<sup>126</sup>:10.1017/S0305004108001783<sup>127</sup>. S2CID<sup>128</sup> 7915709<sup>129</sup>.
13. VOEVODSKY, VLADIMIR (27 SEPTEMBER 2006). "A VERY SHORT NOTE ON HOMOTOPY  $\lambda$ -CALCULUS"<sup>130</sup>. UCR.EDU. RETRIEVED 6 JUNE 2021.
  14. VAN DEN BERG, BENNO; GARNER, RICHARD (1 DECEMBER 2007). "TYPES ARE WEAK OMEGA-GROUPOIDS". *Proceedings of the London Mathematical Society*. **102** (2): 370–394. arXiv<sup>131</sup>:0812.0298<sup>132</sup>. doi<sup>133</sup>:10.1112/plms/pdq026<sup>134</sup>. S2CID<sup>135</sup> 5575780<sup>136</sup>.
  15. LUMSDAINE, PETER (2010). "HIGHER CATEGORIES FROM TYPE THEORIES"<sup>137</sup> (PDF) (PH.D.). CARNEGIE MELLON UNIVERSITY.
  16. Notes on homotopy lambda calculus, March 2006<sup>138</sup>
  17. Martín Hötzel Escardó (October 18, 2018) A self-contained, brief and complete formulation of Voevodsky's Univalence Axiom<sup>139</sup>
  18. GitHub repository, Univalent Mathematics<sup>140</sup>
  19. AWODEY, STEVE; GARNER, RICHARD; MARTIN-LÖF, PER; VOEVODSKY, VLADIMIR (27 FEBRUARY – 5 MARCH 2011). "MINI-WORKSHOP: THE HOMOTOPY INTERPRETATION OF CONSTRUCTIVE TYPE THEORY"<sup>141</sup> (PDF). *Oberwolfach Reports*. Mathematical Research Institute of Oberwolfach: 609–638. doi<sup>142</sup>:10.4171/OWR/2011/11<sup>143</sup>. Retrieved 6 June 2021.
  20. GitHub repository, Andrej Bauer, Homotopy theory in Coq<sup>144</sup>
  21. BAUER, ANDREJ; VOEVODSKY, VLADIMIR (29 APRIL 2011). "BASIC HOMOTOPY TYPE THEORY"<sup>145</sup>. GITHUB. RETRIEVED 6 JUNE 2021.
  22. GitHub repository, Homotopy type theory<sup>146</sup>
  23. SHULMAN, MICHAEL (2015). "UNIVALENCE FOR INVERSE DIAGRAMS AND HOMOTOPY CANONICITY". *Mathematical Structures in Computer Science*. **25**

<sup>124</sup> [https://en.wikipedia.org/wiki/Bibcode\\_\(identifier\)](https://en.wikipedia.org/wiki/Bibcode_(identifier))

<sup>125</sup> <https://ui.adsabs.harvard.edu/abs/2008MPCPS.146...45A>

<sup>126</sup> [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))

<sup>127</sup> <https://doi.org/10.1017%2FS0305004108001783>

<sup>128</sup> [https://en.wikipedia.org/wiki/S2CID\\_\(identifier\)](https://en.wikipedia.org/wiki/S2CID_(identifier))

<sup>129</sup> <https://api.semanticscholar.org/CorpusID:7915709>

<sup>130</sup> [http://math.ucr.edu/home/baez/Voevodsky\\_note.ps](http://math.ucr.edu/home/baez/Voevodsky_note.ps)

<sup>131</sup> [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))

<sup>132</sup> <http://arxiv.org/abs/0812.0298>

<sup>133</sup> [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))

<sup>134</sup> <https://doi.org/10.1112%2Fplms%2Fpdq026>

<sup>135</sup> [https://en.wikipedia.org/wiki/S2CID\\_\(identifier\)](https://en.wikipedia.org/wiki/S2CID_(identifier))

<sup>136</sup> <https://api.semanticscholar.org/CorpusID:5575780>

<sup>137</sup> <http://lib.rett.org.uk/~peterlefanulumsdaine/research/Lumsdaine-2010-Thesis.pdf>

<sup>138</sup> [https://github.com/vladimirias/2006\\_Mar\\_Homotopy\\_lambda\\_calculus](https://github.com/vladimirias/2006_Mar_Homotopy_lambda_calculus)

<sup>139</sup> <https://arxiv.org/pdf/1803.02294.pdf>

<sup>140</sup> <https://github.com/UniMath/UniMath>

<sup>141</sup> [https://hottheory.files.wordpress.com/2011/06/report-11\\_2011.pdf](https://hottheory.files.wordpress.com/2011/06/report-11_2011.pdf)

<sup>142</sup> [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))

<sup>143</sup> <https://doi.org/10.4171%2FOWR%2F2011%2F11>

<sup>144</sup> <https://github.com/andrejbauer/Homotopy>

<sup>145</sup> <https://github.com/HoTT/HoTT/commit/1fb4ed9e5cdc5494f26ae6d13c4ebc851b81e1ba>

<sup>146</sup> <https://github.com/HoTT/HoTT>

- (5): 1203–1277. arXiv<sup>147</sup>:1203.3253<sup>148</sup>. doi<sup>149</sup>:10.1017/S0960129514000565<sup>150</sup>. S2CID<sup>151</sup> 13595170<sup>152</sup>.
24. LICATA, DANIEL R.; HARPER, ROBERT (21 JULY 2011). "CANONICITY FOR 2-DIMENSIONAL TYPE THEORY"<sup>153</sup> (PDF). CARNEGIE MELLON UNIVERSITY. RETRIEVED 6 JUNE 2021.
25. Homotopy Type Theory and Univalent Foundations Blog<sup>154</sup>
26. Homotopy Type Theory blog<sup>155</sup>
27. Type Theory and Univalent Foundations<sup>156</sup>
28. UNIVALENT FOUNDATIONS PROGRAM (2013). *Homotopy Type Theory: Univalent Foundations of Mathematics*<sup>157</sup>. INSTITUTE FOR ADVANCED STUDY.
29. Homotopy Type Theory: References<sup>158</sup>
30. IAS school of mathematics: Special Year on The Univalent Foundations of Mathematics<sup>159</sup>
31. Official announcement of The HoTT Book, by Steve Awodey, 20 June 2013<sup>160</sup>
32. MONROE, D (2014). "A NEW TYPE OF MATHEMATICS?"<sup>161</sup>. *Comm ACM*. **57** (2): 13–15. doi<sup>162</sup>:10.1145/2557446<sup>163</sup>. S2CID<sup>164</sup> 6120947<sup>165</sup>.
33. SHULMAN, MIKE (20 JUNE 2013). "THE HOTT BOOK"<sup>166</sup>. THE N-CATEGORY CAFÉ. RETRIEVED 6 JUNE 2021 – VIA UNIVERSITY OF TEXAS.
34. BAUER, ANDREJ (20 JUNE 2013). "THE HOTT BOOK"<sup>167</sup>. MATHEMATICS AND COMPUTATION. RETRIEVED 6 JUNE 2021.
35. *ACM Computing Reviews*. "Best of 2013"<sup>168</sup>.
36. MEYER, FLORIAN (3 SEPTEMBER 2014). "A NEW FOUNDATION FOR MATHEMATICS"<sup>169</sup>. R&D MAGAZINE. RETRIEVED 29 JULY 2021.

---

147 [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))

148 <http://arxiv.org/abs/1203.3253>

149 [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))

150 <https://doi.org/10.1017%2FS0960129514000565>

151 [https://en.wikipedia.org/wiki/S2CID\\_\(identifier\)](https://en.wikipedia.org/wiki/S2CID_(identifier))

152 <https://api.semanticscholar.org/CorpusID:13595170>

153 <https://www.cs.cmu.edu/~drl/pubs/lh112tt/lh112tt.pdf>

154 <http://homotopytypetheory.org>

155 <http://homotopytypetheory.org/blog>

156 <http://homotopytypetheory.org/>

157 <http://homotopytypetheory.org/book/>

158 <http://ncatlab.org/homotopytypetheory/show/References>

159 <https://www.math.ias.edu/sp/univalent>

160 <http://homotopytypetheory.org/2013/06/20/the-hott-book/>

161 <http://cacm.acm.org/magazines/2014/2/171675-a-new-type-of-mathematics/abstract>

162 [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))

163 <https://doi.org/10.1145%2F2557446>

164 [https://en.wikipedia.org/wiki/S2CID\\_\(identifier\)](https://en.wikipedia.org/wiki/S2CID_(identifier))

165 <https://api.semanticscholar.org/CorpusID:6120947>

166 [https://golem.ph.utexas.edu/category/2013/06/the\\_hott\\_book.html](https://golem.ph.utexas.edu/category/2013/06/the_hott_book.html)

167 <http://math.andrej.com/2013/06/20/the-hott-book/>

168 [http://computingreviews.com/recommend/bestof/notableitems\\_2013.cfm](http://computingreviews.com/recommend/bestof/notableitems_2013.cfm)

169 <https://www.rdworldonline.com/a-new-foundation-for-mathematics>

37. SOJAKOVA, KRISTINA (2015). *Higher Inductive Types as Homotopy-Initial Algebras*<sup>170</sup>. POPL 2015. ARXIV<sup>171</sup>:1402.0761<sup>172</sup>. DOI<sup>173</sup>:10.1145/2676726.2676983<sup>174</sup>.
38. COHEN, CYRIL; COQUAND, THIERRY; HUBER, SIMON; MÖRTBERG, ANDERS (2015). *Cubical Type Theory: a constructive interpretation of the univalence axiom*<sup>175</sup>. TYPES 2015.
39. ANGUILI, CARLO; FAVONIA; HARPER, ROBERT (2018). *Cartesian Cubical Computational Type Theory: Constructive Reasoning with Paths and Equalities*<sup>176</sup> (PDF). COMPUTER SCIENCE LOGIC 2018. RETRIEVED 26 AUGUST 2018. (to appear)

## 1.9 Bibliography

- THE UNIVALENT FOUNDATIONS PROGRAM (2013). *Homotopy Type Theory: Univalent Foundations of Mathematics*<sup>177</sup>. PRINCETON, NJ: INSTITUTE FOR ADVANCED STUDY<sup>178</sup>. MR<sup>179</sup> 3204653<sup>180</sup>. (GitHub version<sup>181</sup> cited in this article.)
- AWODEY, S.<sup>182</sup>; WARREN, M. A. (JANUARY 2009). "HOMOTOPY THEORETIC MODELS OF IDENTITY TYPES". *Mathematical Proceedings of the Cambridge Philosophical Society*. **146** (1): 45–55. arXiv<sup>183</sup>:0709.0248<sup>184</sup>. Bibcode<sup>185</sup>:2008MPCPS.146...45A<sup>186</sup>. doi<sup>187</sup>:10.1017/S0305004108001783<sup>188</sup>. S2CID<sup>189</sup> 7915709<sup>190</sup>. As PDF<sup>191</sup>.
- AWODEY, STEVE<sup>192</sup> (2012). "TYPE THEORY AND HOMOTOPY"<sup>193</sup> (PDF). IN DYBJER, P.; LINDSTRÖM, STEN; PALMGREN, ERIK; ET AL. (EDS.). *Epistemology versus Ontology. Logic, Epistemology, and the Unity of Science*. Springer. pp. 183–201.

170 <http://dl.acm.org/citation.cfm?id=2676983>

171 [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))

172 <http://arxiv.org/abs/1402.0761>

173 [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))

174 <https://doi.org/10.1145%2F2676726.2676983>

175 <https://hal-iogs.archives-ouvertes.fr/INRIA/hal-01378906v2>

176 <https://www.cs.cmu.edu/~rwh/papers/cartesian/paper.pdf>

177 <https://books.google.com/books?id=LkDUKMv3yp0C>

178 [https://en.wikipedia.org/wiki/Institute\\_for\\_Advanced\\_Study](https://en.wikipedia.org/wiki/Institute_for_Advanced_Study)

179 [https://en.wikipedia.org/wiki/MR\\_\(identifier\)](https://en.wikipedia.org/wiki/MR_(identifier))

180 <http://www.ams.org/mathscinet-getitem?mr=3204653>

181 <https://web.archive.org/web/20170707022332/https://hott.github.io/book/nightly/hott-a4-1075-g3c53219.pdf>

182 [https://en.wikipedia.org/wiki/Steve\\_Awodey](https://en.wikipedia.org/wiki/Steve_Awodey)

183 [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))

184 <http://arxiv.org/abs/0709.0248>

185 [https://en.wikipedia.org/wiki/Bibcode\\_\(identifier\)](https://en.wikipedia.org/wiki/Bibcode_(identifier))

186 <https://ui.adsabs.harvard.edu/abs/2008MPCPS.146...45A>

187 [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))

188 <https://doi.org/10.1017%2FS0305004108001783>

189 [https://en.wikipedia.org/wiki/S2CID\\_\(identifier\)](https://en.wikipedia.org/wiki/S2CID_(identifier))

190 <https://api.semanticscholar.org/CorpusID:7915709>

191 <http://www.andrew.cmu.edu/user/awodey/preprints/homotopy.pdf>

192 [https://en.wikipedia.org/wiki/Steve\\_Awodey](https://en.wikipedia.org/wiki/Steve_Awodey)

193 <http://www.andrew.cmu.edu/user/awodey/preprints/TTH.pdf>

- CiteSeerX<sup>194</sup> 10.1.1.750.3626<sup>195</sup>. doi<sup>196</sup>:10.1007/978-94-007-4435-6\_9<sup>197</sup>. ISBN<sup>198</sup> 978-94-007-4434-9<sup>199</sup>. S2CID<sup>200</sup> 4499538<sup>201</sup>.
- AWODEY, STEVE<sup>202</sup> (2014). "STRUCTURALISM, INVARIANCE, AND UNIVALENCE". *Philosophia Mathematica*. **22** (1): 1–11. CiteSeerX<sup>203</sup> 10.1.1.691.8113<sup>204</sup>. doi<sup>205</sup>:10.1093/phimat/nkt030<sup>206</sup>.
  - HOFMANN, MARTIN; STREICHER, THOMAS<sup>207</sup> (1998). "THE GROUPOID INTERPRETATION OF TYPE THEORY"<sup>208</sup>. IN SAMBIN, G.; SMITH, J.M. (EDS.). *Twenty Five Years of Constructive Type Theory*. Clarendon Press. pp. 83–112. ISBN<sup>209</sup> 978-0-19-158903-4<sup>210</sup>. As postscript<sup>211</sup>.
  - RIJKE, EGBERT (2012). *Homotopy Type Theory*<sup>212</sup> (PDF) (MASTER'S). UTRECHT UNIVERSITY.
  - VOEVODSKY, VLADIMIR<sup>213</sup> (2006), *A Very Short Note on Homotopy Lambda Calculus*<sup>214</sup> (PDF)
  - VOEVODSKY, VLADIMIR<sup>215</sup> (2010), *The Equivalence Axiom and Univalent Models of Type Theory*, arXiv<sup>216</sup>:1402.5556<sup>217</sup>, Bibcode<sup>218</sup>:2014arXiv1402.5556V<sup>219</sup>
  - WARREN, MICHAEL A. (2008). *Homotopy Theoretic Aspects of Constructive Type Theory*<sup>220</sup> (PDF) (PH.D.). CARNEGIE MELLON UNIVERSITY.

## 1.10 Further reading

- David Corfield<sup>221</sup> (2020), *Modal Homotopy Type Theory: The Prospect of a New Logic for Philosophy*, Oxford University Press.

---

194 [https://en.wikipedia.org/wiki/CiteSeerX\\_\(identifier\)](https://en.wikipedia.org/wiki/CiteSeerX_(identifier))  
195 <http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.750.3626>  
196 [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))  
197 [https://doi.org/10.1007%2F978-94-007-4435-6\\_9](https://doi.org/10.1007%2F978-94-007-4435-6_9)  
198 [https://en.wikipedia.org/wiki/ISBN\\_\(identifier\)](https://en.wikipedia.org/wiki/ISBN_(identifier))  
199 <https://en.wikipedia.org/wiki/Special:BookSources/978-94-007-4434-9>  
200 [https://en.wikipedia.org/wiki/S2CID\\_\(identifier\)](https://en.wikipedia.org/wiki/S2CID_(identifier))  
201 <https://api.semanticscholar.org/CorpusID:4499538>  
202 [https://en.wikipedia.org/wiki/Steve\\_Awodey](https://en.wikipedia.org/wiki/Steve_Awodey)  
203 [https://en.wikipedia.org/wiki/CiteSeerX\\_\(identifier\)](https://en.wikipedia.org/wiki/CiteSeerX_(identifier))  
204 <http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.691.8113>  
205 [https://en.wikipedia.org/wiki/Doi\\_\(identifier\)](https://en.wikipedia.org/wiki/Doi_(identifier))  
206 <https://doi.org/10.1093%2Fphimat%2Fnkt030>  
207 [https://en.wikipedia.org/wiki/Thomas\\_Streicher](https://en.wikipedia.org/wiki/Thomas_Streicher)  
208 [https://books.google.com/books?id=pLnKggT\\_In4C&pg=PA83](https://books.google.com/books?id=pLnKggT_In4C&pg=PA83)  
209 [https://en.wikipedia.org/wiki/ISBN\\_\(identifier\)](https://en.wikipedia.org/wiki/ISBN_(identifier))  
210 <https://en.wikipedia.org/wiki/Special:BookSources/978-0-19-158903-4>  
211 <http://www.mathematik.tu-darmstadt.de/~streicher/venedig.ps.gz>  
212 <http://hottheory.files.wordpress.com/2012/08/hott2.pdf>  
213 [https://en.wikipedia.org/wiki/Vladimir\\_Voevodsky](https://en.wikipedia.org/wiki/Vladimir_Voevodsky)  
214 [http://www.math.ias.edu/~vladimir/Site3/Univalent\\_Foundations\\_files/Hlambda\\_short\\_current.pdf](http://www.math.ias.edu/~vladimir/Site3/Univalent_Foundations_files/Hlambda_short_current.pdf)  
215 [https://en.wikipedia.org/wiki/Vladimir\\_Voevodsky](https://en.wikipedia.org/wiki/Vladimir_Voevodsky)  
216 [https://en.wikipedia.org/wiki/ArXiv\\_\(identifier\)](https://en.wikipedia.org/wiki/ArXiv_(identifier))  
217 <http://arxiv.org/abs/1402.5556>  
218 [https://en.wikipedia.org/wiki/Bibcode\\_\(identifier\)](https://en.wikipedia.org/wiki/Bibcode_(identifier))  
219 <https://ui.adsabs.harvard.edu/abs/2014arXiv1402.5556V>  
220 <http://www.andrew.cmu.edu/user/awodey/students/warren.pdf>  
221 [https://en.wikipedia.org/wiki/David\\_Corfield](https://en.wikipedia.org/wiki/David_Corfield)

## 1.11 External links

Wikimedia Commons has media related to Homotopy type theory<sup>222</sup>.

- Homotopy Type Theory<sup>223</sup>
- Homotopy type theory<sup>224</sup> in *nLab*<sup>225</sup>
- Homotopy type theory wiki<sup>226</sup>
- Vladimir Voevodsky's webpage on the Univalent Foundations<sup>227</sup>
- Homotopy Type Theory and the Univalent Foundations of Mathematics<sup>228</sup> by Steve Awodey
- "Constructive Type Theory and Homotopy"<sup>229</sup> – Video lecture by Steve Awodey at the Institute for Advanced Study<sup>230</sup>

### 1.11.1 Libraries of formalized mathematics

- *Foundations library (2010-current)*<sup>231</sup>
- *HoTT library (2011-current)*<sup>232</sup>, 30 JANUARY 2022
- *P-adics library (2011-2012)*<sup>233</sup>
- *RezkCompletion library*<sup>234</sup>, JANUARY 2022 (now integrated into UniMath, where further development takes place)
- *Ktheory library*<sup>235</sup>
- *UniMath library (2014-current)*<sup>236</sup>, 25 JANUARY 2022

### Major topics in Foundations of Mathematics

- This page was last edited on 5 March 2022, at 10:57 (UTC).

<sup>222</sup> [https://commons.wikimedia.org/wiki/Category:Homotopy\\_type\\_theory](https://commons.wikimedia.org/wiki/Category:Homotopy_type_theory)

<sup>223</sup> <http://homotopytypetheory.org/>

<sup>224</sup> <https://ncatlab.org/nlab/show/homotopy+type+theory>

<sup>225</sup> <https://en.wikipedia.org/wiki/NLab>

<sup>226</sup> <http://ncatlab.org/homotopytypetheory/show/HomePage>

<sup>227</sup> [http://www.math.ias.edu/~vladimir/Site3/Univalent\\_Foundations.html](http://www.math.ias.edu/~vladimir/Site3/Univalent_Foundations.html)

<sup>228</sup> <http://www.andrew.cmu.edu/user/awodey/htt.html>

<sup>229</sup> <http://video.ias.edu/univalent/awodey>

<sup>230</sup> [https://en.wikipedia.org/wiki/Institute\\_for\\_Advanced\\_Study](https://en.wikipedia.org/wiki/Institute_for_Advanced_Study)

<sup>231</sup> <https://github.com/UniMath/UniMath/tree/master/UniMath/Foundations>

<sup>232</sup> <https://github.com/HoTT/HoTT>

<sup>233</sup> <https://github.com/UniMath/UniMath/tree/master/UniMath/PAdics>

<sup>234</sup> [https://github.com/benediktahrens/rezk\\_completion](https://github.com/benediktahrens/rezk_completion)

<sup>235</sup> <https://github.com/UniMath/UniMath/tree/master/UniMath/Ktheory>

<sup>236</sup> <https://github.com/UniMath/UniMath>

- Text is available under the Creative Commons Attribution-ShareAlike License 3.0<sup>237</sup><sup>238</sup>; additional terms may apply. By using this site, you agree to the Terms of Use<sup>239</sup> and Privacy Policy<sup>240</sup>. Wikipedia® is a registered trademark of the Wikimedia Foundation, Inc.<sup>241</sup>, a non-profit organization.

---

<sup>237</sup> [http://en.wikipedia.org/wiki/Wikipedia:Text\\_of\\_Creative\\_Commons\\_Attribution-ShareAlike\\_3.0\\_Unported\\_License](http://en.wikipedia.org/wiki/Wikipedia:Text_of_Creative_Commons_Attribution-ShareAlike_3.0_Unported_License)  
<sup>238</sup> <http://creativecommons.org/licenses/by-sa/3.0/>  
<sup>239</sup> [http://foundation.wikimedia.org/wiki/Terms\\_of\\_Use](http://foundation.wikimedia.org/wiki/Terms_of_Use)  
<sup>240</sup> [http://foundation.wikimedia.org/wiki/Privacy\\_policy](http://foundation.wikimedia.org/wiki/Privacy_policy)  
<sup>241</sup> <http://www.wikimediafoundation.org/>

## 2 Contributors

Edits	User
1	Alderzdev <sup>1</sup>
2	Aloha27 <sup>2</sup>
9	Ancheta Wis <sup>3</sup>
1	AndrejBauer <sup>4</sup>
7	AnomieBOT <sup>5</sup>
1	Arthur Rubin <sup>6</sup>
4	BG19bot <sup>7</sup>
1	BattyBot <sup>8</sup>
6	Ben Standeven <sup>9</sup>
2	Bender the Bot <sup>10</sup>
1	Bender235 <sup>11</sup>
1	Bibcode Bot <sup>12</sup>
1	Blablubbs <sup>13</sup>
1	Boreas93 <sup>14</sup>
2	Bytbox <sup>15</sup>
1	Chris the speller <sup>16</sup>
1	ChrisGualtieri <sup>17</sup>
5	Citation bot <sup>18</sup>
1	CitationCleanerBot <sup>19</sup>
1	Clarities <sup>20</sup>
3	Clements <sup>21</sup>

---

1	<a href="https://en.wikipedia.org/wiki/User:Alderzdev">https://en.wikipedia.org/wiki/User:Alderzdev</a>
2	<a href="https://en.wikipedia.org/wiki/User:Aloha27">https://en.wikipedia.org/wiki/User:Aloha27</a>
3	<a href="https://en.wikipedia.org/wiki/User:Ancheta_Wis">https://en.wikipedia.org/wiki/User:Ancheta_Wis</a>
4	<a href="https://en.wikipedia.org/wiki/User:AndrejBauer">https://en.wikipedia.org/wiki/User:AndrejBauer</a>
5	<a href="https://en.wikipedia.org/wiki/User:AnomieBOT">https://en.wikipedia.org/wiki/User:AnomieBOT</a>
6	<a href="https://en.wikipedia.org/wiki/User:Arthur_Rubin">https://en.wikipedia.org/wiki/User:Arthur_Rubin</a>
7	<a href="https://en.wikipedia.org/wiki/User:BG19bot">https://en.wikipedia.org/wiki/User:BG19bot</a>
8	<a href="https://en.wikipedia.org/wiki/User:BattyBot">https://en.wikipedia.org/wiki/User:BattyBot</a>
9	<a href="https://en.wikipedia.org/wiki/User:Ben_Standeven">https://en.wikipedia.org/wiki/User:Ben_Standeven</a>
10	<a href="https://en.wikipedia.org/wiki/User:Bender_the_Bot">https://en.wikipedia.org/wiki/User:Bender_the_Bot</a>
11	<a href="https://en.wikipedia.org/wiki/User:Bender235">https://en.wikipedia.org/wiki/User:Bender235</a>
12	<a href="https://en.wikipedia.org/wiki/User:Bibcode_Bot">https://en.wikipedia.org/wiki/User:Bibcode_Bot</a>
13	<a href="https://en.wikipedia.org/wiki/User:Blablubbs">https://en.wikipedia.org/wiki/User:Blablubbs</a>
14	<a href="https://en.wikipedia.org/w/index.php%3ftitle=User:Boreas93&amp;action=edit&amp;redlink=1">https://en.wikipedia.org/w/index.php%3ftitle=User:Boreas93&amp;action=edit&amp;redlink=1</a>
15	<a href="https://en.wikipedia.org/wiki/User:Bytbox">https://en.wikipedia.org/wiki/User:Bytbox</a>
16	<a href="https://en.wikipedia.org/wiki/User:Chris_the_speller">https://en.wikipedia.org/wiki/User:Chris_the_speller</a>
17	<a href="https://en.wikipedia.org/wiki/User:ChrisGualtieri">https://en.wikipedia.org/wiki/User:ChrisGualtieri</a>
18	<a href="https://en.wikipedia.org/wiki/User:Citation_bot">https://en.wikipedia.org/wiki/User:Citation_bot</a>
19	<a href="https://en.wikipedia.org/wiki/User:CitationCleanerBot">https://en.wikipedia.org/wiki/User:CitationCleanerBot</a>
20	<a href="https://en.wikipedia.org/wiki/User:Clarities">https://en.wikipedia.org/wiki/User:Clarities</a>
21	<a href="https://en.wikipedia.org/wiki/User:Clements">https://en.wikipedia.org/wiki/User:Clements</a>

2 Cyberbot II<sup>22</sup>  
1 Daniel5Ko<sup>23</sup>  
1 DeprecatedFixerBot<sup>24</sup>  
1 DerSpezialist<sup>25</sup>  
1 DesolateReality<sup>26</sup>  
1 D12000<sup>27</sup>  
1 Dubidugebra<sup>28</sup>  
1 Favonia<sup>29</sup>  
33 Foobarnix<sup>30</sup>  
1 Glubs9<sup>31</sup>  
1 GreenC bot<sup>32</sup>  
18 Greenrd<sup>33</sup>  
1 Gutworth<sup>34</sup>  
1 Hairy Dude<sup>35</sup>  
1 Hibou57<sup>36</sup>  
1 Hyperbolick<sup>37</sup>  
1 Ianushii<sup>38</sup>  
1 Imz<sup>39</sup>  
3 InternetArchiveBot<sup>40</sup>  
1 Izno<sup>41</sup>  
1 Jochen Burghardt<sup>42</sup>  
1 Jonesey95<sup>43</sup>  
1 Leschnei<sup>44</sup>  
2 Lfstevens<sup>45</sup>  
1 Magioladitis<sup>46</sup>

---

22 [https://en.wikipedia.org/wiki/User:Cyberbot\\_II](https://en.wikipedia.org/wiki/User:Cyberbot_II)  
23 <https://en.wikipedia.org/w/index.php%3ftitle=User:Daniel5Ko&action=edit&redlink=1>  
24 <https://en.wikipedia.org/wiki/User:DeprecatedFixerBot>  
25 <https://en.wikipedia.org/w/index.php%3ftitle=User:DerSpezialist&action=edit&redlink=1>  
26 <https://en.wikipedia.org/w/index.php%3ftitle=User:DesolateReality&action=edit&redlink=1>  
27 <https://en.wikipedia.org/wiki/User:D12000>  
28 <https://en.wikipedia.org/w/index.php%3ftitle=User:Dubidugebra&action=edit&redlink=1>  
29 <https://en.wikipedia.org/w/index.php%3ftitle=User:Favonia&action=edit&redlink=1>  
30 <https://en.wikipedia.org/wiki/User:Foobarnix>  
31 <https://en.wikipedia.org/w/index.php%3ftitle=User:Glubs9&action=edit&redlink=1>  
32 [https://en.wikipedia.org/wiki/User:GreenC\\_bot](https://en.wikipedia.org/wiki/User:GreenC_bot)  
33 <https://en.wikipedia.org/wiki/User:Greenrd>  
34 <https://en.wikipedia.org/wiki/User:Gutworth>  
35 [https://en.wikipedia.org/wiki/User:Hairy\\_Dude](https://en.wikipedia.org/wiki/User:Hairy_Dude)  
36 <https://en.wikipedia.org/wiki/User:Hibou57>  
37 <https://en.wikipedia.org/wiki/User:Hyperbolick>  
38 <https://en.wikipedia.org/w/index.php%3ftitle=User:Ianushii&action=edit&redlink=1>  
39 <https://en.wikipedia.org/wiki/User:Imz>  
40 <https://en.wikipedia.org/wiki/User:InternetArchiveBot>  
41 <https://en.wikipedia.org/wiki/User:Izno>  
42 [https://en.wikipedia.org/wiki/User:Jochen\\_Burghardt](https://en.wikipedia.org/wiki/User:Jochen_Burghardt)  
43 <https://en.wikipedia.org/wiki/User:Jonesey95>  
44 <https://en.wikipedia.org/wiki/User:Leschnei>  
45 <https://en.wikipedia.org/wiki/User:Lfstevens>  
46 <https://en.wikipedia.org/wiki/User:Magioladitis>



- 7 Mark viking<sup>47</sup>
- 1 Matěj Grabovský<sup>48</sup>
- 8 Mdnahas<sup>49</sup>
- 1 Michael Hardy<sup>50</sup>
- 11 Michael Shulman<sup>51</sup>
- 2 Monkbob<sup>52</sup>
- 1 Ndcroos<sup>53</sup>
- 1 OAbot<sup>54</sup>
- 14 Omnipaedista<sup>55</sup>
- 2 PeterLeFanuLumsdaine<sup>56</sup>
- 1 PhS<sup>57</sup>
- 3 Pierre Serge<sup>58</sup>
- 1 Pit-trout<sup>59</sup>
- 1 Pmetzger<sup>60</sup>
- 1 Pwagle<sup>61</sup>
- 1 Qetuth<sup>62</sup>
- 1 RDBrown<sup>63</sup>
- 1 Rayman60<sup>64</sup>
- 1 Rjwilmsi<sup>65</sup>
- 1 Rlink2<sup>66</sup>
- 21 Ruud Koot<sup>67</sup>
- 2 Siddharthist<sup>68</sup>
- 5 Steveawodey<sup>69</sup>
- 1 Strabcat<sup>70</sup>
- 4 Tanner Swett<sup>71</sup>

- 47 [https://en.wikipedia.org/wiki/User:Mark\\_viking](https://en.wikipedia.org/wiki/User:Mark_viking)
- 48 [https://en.wikipedia.org/wiki/User:Mat%25C4%259Bj\\_Grabovsk%25C3%25BD](https://en.wikipedia.org/wiki/User:Mat%25C4%259Bj_Grabovsk%25C3%25BD)
- 49 <https://en.wikipedia.org/wiki/User:Mdnahas>
- 50 [https://en.wikipedia.org/wiki/User:Michael\\_Hardy](https://en.wikipedia.org/wiki/User:Michael_Hardy)
- 51 [https://en.wikipedia.org/wiki/User:Michael\\_Shulman](https://en.wikipedia.org/wiki/User:Michael_Shulman)
- 52 <https://en.wikipedia.org/wiki/User:Monkbob>
- 53 <https://en.wikipedia.org/wiki/User:Ndcroos>
- 54 <https://en.wikipedia.org/wiki/User:OAbot>
- 55 <https://en.wikipedia.org/wiki/User:Omnipaedista>
- 56 <https://en.wikipedia.org/w/index.php%3ftitle=User:PeterLeFanuLumsdaine&action=edit&redlink=1>
- 57 <https://en.wikipedia.org/wiki/User:PhS>
- 58 [https://en.wikipedia.org/w/index.php%3ftitle=User:Pierre\\_Serge&action=edit&redlink=1](https://en.wikipedia.org/w/index.php%3ftitle=User:Pierre_Serge&action=edit&redlink=1)
- 59 <https://en.wikipedia.org/w/index.php%3ftitle=User:Pit-trout&action=edit&redlink=1>
- 60 <https://en.wikipedia.org/wiki/User:Pmetzger>
- 61 <https://en.wikipedia.org/wiki/User:Pwagle>
- 62 <https://en.wikipedia.org/wiki/User:Qetuth>
- 63 <https://en.wikipedia.org/wiki/User:RDBrown>
- 64 <https://en.wikipedia.org/wiki/User:Rayman60>
- 65 <https://en.wikipedia.org/wiki/User:Rjwilmsi>
- 66 <https://en.wikipedia.org/wiki/User:Rlink2>
- 67 [https://en.wikipedia.org/wiki/User:Ruud\\_Koot](https://en.wikipedia.org/wiki/User:Ruud_Koot)
- 68 <https://en.wikipedia.org/wiki/User:Siddharthist>
- 69 <https://en.wikipedia.org/w/index.php%3ftitle=User:Steveawodey&action=edit&redlink=1>
- 70 <https://en.wikipedia.org/w/index.php%3ftitle=User:Strabcat&action=edit&redlink=1>
- 71 [https://en.wikipedia.org/wiki/User:Tanner\\_Swett](https://en.wikipedia.org/wiki/User:Tanner_Swett)

1	TomT0m <sup>72</sup>
1	Tony Tan <sup>73</sup>
2	Toploftical <sup>74</sup>
1	Turgidson <sup>75</sup>
1	Typometer <sup>76</sup>
38	Vladimirias <sup>77</sup>
1	Wingsjo <sup>78</sup>
1	Xqbot <sup>79</sup>
7	Yobot <sup>80</sup>

---

<sup>72</sup> <https://en.wikipedia.org/wiki/User:TomT0m>

<sup>73</sup> [https://en.wikipedia.org/wiki/User:Tony\\_Tan](https://en.wikipedia.org/wiki/User:Tony_Tan)

<sup>74</sup> <https://en.wikipedia.org/wiki/User:Toploftical>

<sup>75</sup> <https://en.wikipedia.org/wiki/User:Turgidson>

<sup>76</sup> <https://en.wikipedia.org/wiki/User:Typometer>

<sup>77</sup> <https://en.wikipedia.org/wiki/User:Vladimirias>

<sup>78</sup> <https://en.wikipedia.org/wiki/User:Wingsjo>

<sup>79</sup> <https://en.wikipedia.org/wiki/User:Xqbot>

<sup>80</sup> <https://en.wikipedia.org/wiki/User:Yobot>

# List of Figures

- GFDL: Gnu Free Documentation License. <http://www.gnu.org/licenses/fdl.html>
- cc-by-sa-3.0: Creative Commons Attribution ShareAlike 3.0 License. <http://creativecommons.org/licenses/by-sa/3.0/>
- cc-by-sa-2.5: Creative Commons Attribution ShareAlike 2.5 License. <http://creativecommons.org/licenses/by-sa/2.5/>
- cc-by-sa-2.0: Creative Commons Attribution ShareAlike 2.0 License. <http://creativecommons.org/licenses/by-sa/2.0/>
- cc-by-sa-1.0: Creative Commons Attribution ShareAlike 1.0 License. <http://creativecommons.org/licenses/by-sa/1.0/>
- cc-by-2.0: Creative Commons Attribution 2.0 License. <http://creativecommons.org/licenses/by/2.0/>
- cc-by-2.0: Creative Commons Attribution 2.0 License. <http://creativecommons.org/licenses/by/2.0/deed.en>
- cc-by-2.5: Creative Commons Attribution 2.5 License. <http://creativecommons.org/licenses/by/2.5/deed.en>
- cc-by-3.0: Creative Commons Attribution 3.0 License. <http://creativecommons.org/licenses/by/3.0/deed.en>
- GPL: GNU General Public License. <http://www.gnu.org/licenses/gpl-2.0.txt>
- LGPL: GNU Lesser General Public License. <http://www.gnu.org/licenses/lgpl.html>
- PD: This image is in the public domain.
- ATTR: The copyright holder of this file allows anyone to use it for any purpose, provided that the copyright holder is properly attributed. Redistribution, derivative work, commercial use, and all other use is permitted.
- EURO: This is the common (reverse) face of a euro coin. The copyright on the design of the common face of the euro coins belongs to the European Commission. Authorised is reproduction in a format without relief (drawings, paintings, films) provided they are not detrimental to the image of the euro.
- LFK: Lizenz Freie Kunst. <http://artlibre.org/licence/lal/de>
- CFR: Copyright free use.

- EPL: Eclipse Public License. <http://www.eclipse.org/org/documents/epl-v10.php>

Copies of the GPL, the LGPL as well as a GFDL are included in chapter Licenses<sup>81</sup>. Please note that images in the public domain do not require attribution. You may click on the image numbers in the following table to open the webpage of the images in your webbrowser.

---

<sup>81</sup> Chapter 3 on page 29

1	Presented at <a href="https://github.com/HoTT/book">https://github.com/HoTT/book</a>	
2	derivative work by Thumperward <sup>82</sup> / * File:Wiki letter w.svg <sup>83</sup> : Jarkko Piiroinen <sup>84</sup> , derivative work by Thumperward <sup>85</sup> / * File:Wiki letter w.svg <sup>86</sup> : Jarkko Piiroinen <sup>87</sup>	

---

<sup>82</sup> <http://commons.wikimedia.org/wiki/User:Thumperward>

<sup>83</sup> [http://commons.wikimedia.org/wiki/File:Wiki\\_letter\\_w.svg](http://commons.wikimedia.org/wiki/File:Wiki_letter_w.svg)

<sup>84</sup> [http://commons.wikimedia.org/wiki/User:Jarkko\\_Piiroinen](http://commons.wikimedia.org/wiki/User:Jarkko_Piiroinen)

<sup>85</sup> <https://commons.wikimedia.org/wiki/User:Thumperward>

<sup>86</sup> [https://commons.wikimedia.org/wiki/File:Wiki\\_letter\\_w.svg](https://commons.wikimedia.org/wiki/File:Wiki_letter_w.svg)

<sup>87</sup> [https://commons.wikimedia.org/wiki/User:Jarkko\\_Piiroinen](https://commons.wikimedia.org/wiki/User:Jarkko_Piiroinen)



# 3 Licenses

## 3.1 GNU GENERAL PUBLIC LICENSE

Version 3, 29 June 2007

Copyright © 2007 Free Software Foundation, Inc. <<http://fsf.org/>>

Everyone is permitted to copy and distribute verbatim copies of this license document, but changing it is not allowed. Preamble

The GNU General Public License is a free, copyleft license for software and other kinds of works.

The licenses for most software and other practical works are designed to take away your freedom to share and change the works. By contrast, the GNU General Public License is intended to guarantee your freedom to share and change all versions of a program—to make sure it remains free software for all its users. We, the Free Software Foundation, use the GNU General Public License for most of our software; it applies also to any other work released this way by its authors. You can apply it to your programs, too.

When we speak of free software, we are referring to freedom, not price. Our General Public Licenses are designed to make sure that you have the freedom to distribute copies of free software (and charge for them if you wish), that you receive source code or can get it if you want it, that you can change the software or use pieces of it in new free programs, and that you know you can do these things.

To protect your rights, we need to prevent others from denying you these rights or asking you to surrender the rights. Therefore, you have certain responsibilities if you distribute copies of the software, or if you modify it: responsibilities to respect the freedom of others.

For example, if you distribute copies of such a program, whether gratis or for a fee, you must pass on to the recipients the same freedoms that you received. You must make sure that they, too, receive or can get the source code. And you must show them these terms so they know their rights.

Developers that use the GNU GPL protect your rights with two steps: (1) assert copyright on the software, and (2) offer you this License giving you legal permission to copy, distribute and/or modify it.

For the developers' and authors' protection, the GPL clearly explains that there is no warranty for this free software. For both users' and authors' sake, the GPL requires that modified versions be marked as changed, so that their problems will not be attributed erroneously to authors of previous versions.

Some devices are designed to deny users access to install or run modified versions of the software inside them, although the manufacturer can do so. This is fundamentally incompatible with the aim of protecting users' freedom to change the software. The systematic pattern of such abuse occurs in the area of products for individuals to use, which is precisely where it is most unacceptable. Therefore, we have designed this version of the GPL to prohibit the practice for those products. If such problems arise substantially in other domains, we stand ready to extend this provision to those domains in future versions of the GPL, as needed to protect the freedom of users.

Finally, every program is threatened constantly by software patents. States should not allow patents to restrict development and use of software on general-purpose computers, but in those that do, we wish to avoid the special danger that patents applied to a free program could make it effectively proprietary. To prevent this, the GPL assures that patents cannot be used to render the program non-free.

The precise terms and conditions for copying, distribution and modification follow. TERMS AND CONDITIONS 0. Definitions.

"This License" refers to version 3 of the GNU General Public License.

"Copyright" also means copyright-like laws that apply to other kinds of works, such as semiconductor masks.

"The Program" refers to any copyrightable work licensed under this License. Each licensee is addressed as "you". "Licensees" and "recipients" may be individuals or organizations.

To "modify" a work means to copy from or adapt all or part of the work in a fashion requiring copyright permission, other than the making of an exact copy. The resulting work is called a "modified version" of the earlier work or a work "based on" the earlier work.

A "covered work" means either the unmodified Program or a work based on the Program.

To "propagate" a work means to do anything with it that, without permission, would make you directly or secondarily liable for infringement under applicable copyright law, except executing it on a computer or displaying a private copy. Propagation includes copying, distribution (with or without modification), making available to the public, and in some countries other activities as well.

To "convey" a work means any kind of propagation that enables other parties to make or receive copies. Mere interaction with a user through a computer network, with no transfer of a copy, is not conveying.

An interactive user interface displays "Appropriate Legal Notices" to the extent that it includes a convenient and prominently visible feature that (1) displays an appropriate copyright notice, and (2) tells the user that there is no warranty for the work (except to the extent that warranties are provided), that licensees may convey the work under this License, and how to view a copy of this License. If the interface presents a list of user commands or options, such as a menu, a prominent item in the list meets this criterion. 1. Source Code.

The "source code" for a work means the preferred form of the work for making modifications to it. "Object code" means any non-source form of a work.

A "Standard Interface" means an interface that either is an official standard defined by a recognized standards body, or, in the case of interfaces specified for a particular programming language, one that is widely used among developers working in that language.

The "System Libraries" of an executable work include anything, other than the work as a whole, that (a) is included in the normal form of packaging a Major Component, but which is not part of that Major Component, and (b) serves only to enable use of the work with that Major Component, or to implement a Standard Interface for which an implementation is available to the public in source code form. A "Major Component", in this context, means a major operating component (kernel, window system, and so on) of the specific control system (if any) on which the executable work runs, or a compiler used to produce the work, or an object code interpreter used to run it.

The "Corresponding Source" for a work in object code form means all the source code needed to generate, install, and (for an executable work) run the object code and to modify the work, including scripts to control those activities. However, it does not include the work's System Libraries, or general-purpose tools or generally available free programs which are used unmodified in performing those activities but which are not part of the work. For example, Corresponding Source includes interface definition files associated with source files for the work, and the source code for shared libraries and dynamically linked subprograms that the work is specifically designed to require, such as by intimate data communication or control flow between those subprograms and other parts of the work.

The Corresponding Source need not include anything that users can regenerate automatically from other parts of the Corresponding Source.

The Corresponding Source for a work in source code form is that same work. 2. Basic Permissions.

All rights granted under this License are granted for the term of copyright on the Program, and are irrevocable provided the stated conditions are met. This License explicitly affirms your unlimited permission to run the unmodified Program. The output from running a covered work is covered by this License only if the output, given its content, constitutes a covered work. This License acknowledges your rights of fair use or other equivalent, as provided by copyright law.

You may make, run and propagate covered works that you do not convey, without conditions so long as your license otherwise remains in force. You may convey covered works to others for the sole purpose of having them make modifications exclusively for you, or provide you with facilities for running those works, provided that you comply with the terms of this License in conveying all material for which you do not control copyright. Those thus making or running the covered works for you must do so exclusively on your behalf, under your direction and control, on terms that prohibit them from making any copies of your copyrighted material outside their relationship with you.

Conveying under any other circumstances is permitted solely under the conditions stated below. Sublicensing is not allowed; section 10 makes it unnecessary. 3. Protecting Users' Legal Rights From Anti-Circumvention Law.

No covered work shall be deemed part of an effective technological measure under any applicable law fulfilling obligations under article 11 of the WIPO copyright treaty adopted on 20 December 1996, or similar laws prohibiting or restricting circumvention of such measures.

When you convey a covered work, you waive any legal power to forbid circumvention of technological measures to the extent such circumvention is effected by exercising rights under this License with respect to the covered work, and you disclaim any intention to limit operation or modification of the work as a means of enforcing, against the work's users, your or third parties' legal rights to forbid circumvention of technological measures. 4. Conveying Verbatim Copies.

You may convey verbatim copies of the Program's source code as you receive it, in any medium, provided that you conspicuously and appropriately publish on each copy an appropriate copyright notice; keep intact all notices stating that this License and any non-permissive terms added in accord with section 7 apply to the code; keep intact all notices of the absence of any warranty; and give all recipients a copy of this License along with the Program.

You may charge any price or no price for each copy that you convey, and you may offer support or warranty protection for a fee. 5. Conveying Modified Source Versions.

You may convey a work based on the Program, or the modifications to produce it from the Program, in the form of source code under the terms of section 4, provided that you also meet all of these conditions:

\* a) The work must carry prominent notices stating that you modified it, and giving a relevant date. \* b) The work must carry prominent notices stating that it is released under this License and any conditions added under section 7. This requirement modifies the requirement in section 4 to "keep intact all notices". \* c) You must license the entire work, as a whole, under this License to anyone who comes into possession of a copy. This License will therefore apply, along with any applicable section 7 additional terms, to the whole of the work, and all its parts, regardless of how they are packaged. This License gives no permission to license the work in any other way, but it does not invalidate such permission if you have separately received it. \* d) If the work has interactive user interfaces, each must display Appropriate Legal Notices; however, if the Program has interactive interfaces that do not display Appropriate Legal Notices, your work need not make them do so.

A compilation of a covered work with other separate and independent works, which are not by their nature extensions of the covered work, and which are not combined with it such as to form a larger program, in or on a volume of a storage or distribution medium, is called an "aggregate" if the compilation and its resulting copyright are not used to limit the access or legal rights of the compilation's users beyond what the individual works permit. Inclusion of a covered work in an aggregate does not cause this License to apply to the other parts of the aggregate. 6. Conveying Non-Source Forms.

You may convey a covered work in object code form under the terms of sections 4 and 5, provided that you also convey the machine-readable Corresponding Source under the terms of this License, in one of these ways:

\* a) Convey the object code in, or embodied in, a physical product (including a physical distribution medium), accompanied by the Corresponding Source fixed on a durable physical medium customarily used for software interchange. \* b) Convey the object code in, or embodied in, a physical product (including a physical distribution medium), accompanied by a written offer, valid for at least three years and valid for as long as you offer spare parts or customer support for that product model, to give anyone who possesses the object code either (1) a copy of the Corresponding Source for all the software in the product that is covered by this License, on a durable physical medium customarily used for software interchange, for a price no more than your reasonable cost of physically performing this conveying of source, or (2) access to copy the Corresponding Source from a network server at no charge. \* c) Convey individual copies of the object code with a copy of the written offer to provide the Corresponding Source. This alternative is allowed only occasionally and noncommercially, and only if you received the object code with such an offer, in accord with subsection 6b. \* d) Convey the object code by offering access from a designated place (gratis or for a charge), and offer equivalent access to the Corresponding Source in the same way through the same place at no further charge. You need not require recipients to copy the Corresponding Source along with the object code. If the place to copy the object code is a network server, the Corresponding Source may be on a

different server (operated by you or a third party) that supports equivalent copying facilities, provided you maintain clear directions next to the object code saying where to find the Corresponding Source. Regardless of what server hosts the Corresponding Source, you remain obligated to ensure that it is available for as long as needed to satisfy these requirements. \* e) Convey the object code using peer-to-peer transmission, provided you inform other peers where the object code and Corresponding Source of the work are being offered to the general public at no charge under subsection 6d.

A separable portion of the object code, whose source code is excluded from the Corresponding Source as a System Library, need not be included in conveying the object code work.

A "User Product" is either (1) a "consumer product", which means any tangible personal property which is normally used for personal, family, or household purposes, or (2) anything designed or sold for incorporation into a dwelling. In determining whether a product is a consumer product, doubtful cases shall be resolved in favor of coverage. For a particular product received by a particular user, "normally used" refers to a typical or common use of that class of product, regardless of the status of the particular user or of the way in which the particular user actually uses, or expects to be expected to use, the product. A product is a consumer product regardless of whether the product has substantial commercial, industrial or non-consumer uses, unless such uses represent the only significant mode of use of the product.

"Installation Information" for a User Product means any methods, procedures, authorization keys, or other information required to install and execute modified versions of a covered work in that User Product from a modified version of its Corresponding Source. The information must suffice to ensure that the continued functioning of the modified object code is in no case prevented or interfered with solely because modification has been made.

If you convey an object code work under this section in, or with, or specifically for use in, a User Product, and the conveying occurs as part of a transaction in which the right of possession and use of the User Product is transferred to the recipient in perpetuity or for a fixed term (regardless of how the transaction is characterized), the Corresponding Source conveyed under this section must be accompanied by the Installation Information. But this requirement does not apply if neither you nor any third party retains the ability to install modified object code on the User Product (for example, the work has been installed in ROM).

The requirement to provide Installation Information does not include a requirement to continue to provide support service, warranty, or updates for a work that has been modified or installed by the recipient, or for the User Product in which it has been modified or installed. Access to a network may be denied when the modification itself materially and adversely affects the operation of the network or violates the rules and protocols for communication across the network.

Corresponding Source conveyed, and Installation Information provided, in accord with this section must be in a format that is publicly documented (and with an implementation available to the public in source code form), and must require no special password or key for unpacking, reading or copying. 7. Additional Terms.

"Additional permissions" are terms that supplement the terms of this License by making exceptions from one or more of its conditions. Additional permissions that are applicable to the entire Program shall be treated as though they were included in this License, to the extent that they are valid under applicable law. If additional permissions apply only to part of the Program, that part may be used separately under those permissions, but the entire Program remains governed by this License without regard to the additional permissions.

When you convey a copy of a covered work, you may at your option remove additional permissions from that copy, or from any part of it. (Additional permissions may be written to require their own removal in certain cases when you modify the work.) You may place additional permissions on material, added by you to a covered work, for which you have or can give appropriate copyright permission.

Notwithstanding any other provision of this License, for material you add to a covered work, you may (if authorized by the copyright holders of that material) supplement the terms of this License with terms:

\* a) Disclaiming warranty or limiting liability differently from the terms of sections 15 and 16 of this License; or \* b) Requiring preservation of specified reasonable legal notices or author attributions in that material or in the Appropriate Legal Notices displayed by works containing it; or \* c) Prohibiting misrepresentation of the origin of that material, or requiring that modified versions of such material be marked in reasonable ways as different from the original version; or \* d) Limiting the use for publicity purposes of names of licensors or authors of the material; or \* e) Declining to grant rights under trademark law for use of some trade names, trademarks, or service marks; or \* f) Requiring indemnification of licensors and authors of that material by anyone who conveys the material (or modified versions of it) with contractual assumptions of liability to the recipient, for any liability that these contractual assumptions directly impose on those licensors and authors.

All other non-permissive additional terms are considered "further restrictions" within the meaning of section 10. If the Program as you received it, or any part of it, contains a notice stating that it is governed by this License along with a term that is a further restriction, you may remove that term. If a license document contains a further restriction but permits relicensing or conveying under this License, you may add to a covered work material governed by the terms of that license document, provided that the further restriction does not survive such relicensing or conveying.

If you add terms to a covered work in accord with this section, you must place, in the relevant source files, a statement of the additional terms that apply to those files, or a notice indicating where to find the applicable terms.

Additional terms, permissive or non-permissive, may be stated in the form of a separately written license, or stated as exceptions; the above requirements apply either way. 8. Termination.

You may not propagate or modify a covered work except as expressly provided under this License. Any attempt otherwise to propagate or modify it is void, and will automatically terminate your rights under this License (including any patent licenses granted under the third paragraph of section 11).

However, if you cease all violation of this License, then your license from a particular copyright holder is reinstated (a) provisionally, unless and until the copyright holder explicitly and finally terminates

your license, and (b) permanently, if the copyright holder fails to notify you of the violation by some reasonable means prior to 60 days after the cessation.

Moreover, your license from a particular copyright holder is reinstated permanently if the copyright holder notifies you of the violation by some reasonable means, this is the first time you have received notice of violation of this License (for any work) from that copyright holder, and you cure the violation prior to 30 days after your receipt of the notice.

Termination of your rights under this section does not terminate the licenses of parties who have received copies or rights from you under this License. If your rights have been terminated and not permanently reinstated, you do not qualify to receive new licenses for the same material under section 10. 9. Acceptance Not Required for Having Copies.

You are not required to accept this License in order to receive or run a copy of the Program. Ancillary propagation of a covered work occurring solely as a consequence of using peer-to-peer transmission to receive a copy likewise does not require acceptance. However, nothing other than this License grants you permission to propagate or modify any covered work. These actions infringe copyright if you do not accept this License. Therefore, by modifying or propagating a covered work, you indicate your acceptance of this License to do so. 10. Automatic Licensing of Downstream Recipients.

Each time you convey a covered work, the recipient automatically receives a license from the original licensors, to run, modify and propagate that work, subject to this License. You are not responsible for enforcing compliance by third parties with this License.

An "entity transaction" is a transaction transferring control of an organization, or substantially all assets of one, or subdividing an organization, or merging organizations. If propagation of a covered work results from an entity transaction, each party to that transaction who receives a copy of the work also receives whatever licenses to the work the party's predecessor in interest had or could give under the previous paragraph, plus a right to possession of the Corresponding Source of the work from the predecessor in interest, if the predecessor has it or can get it with reasonable efforts.

You may not impose any further restrictions on the exercise of the rights granted or affirmed under this License. For example, you may not impose a license fee, royalty, or other charge for exercise of rights granted under this License, and you may not initiate litigation (including a cross-claim or counterclaim in a lawsuit) alleging that any patent claim is infringed by making, using, selling, offering for sale, or importing the Program or any portion of it. 11. Patents.

A "contributor" is a copyright holder who authorizes use under this License of the Program or a work on which the Program is based. The work thus licensed is called the contributor's "contributor version".

A contributor's "essential patent claims" are all patent claims owned or controlled by the contributor, whether already acquired or hereafter acquired, that would be infringed by some manner, permitted by this License, of making, using, or selling its contributor version, but do not include claims that would be infringed only as a consequence of further modification of the contributor version. For purposes of this definition, "control" includes the right to grant patent sublicenses in a manner consistent with the requirements of this License.

Each contributor grants you a non-exclusive, worldwide, royalty-free patent license under the contributor's essential patent claims, to make, use, sell, offer for sale, import and otherwise run, modify and propagate the contents of its contributor version.

In the following three paragraphs, a "patent license" is any express agreement or commitment, however denominated, not to enforce a patent (such as an express permission to practice a patent or covenant not to sue for patent infringement). To "grant" such a patent license to a party means to make such an agreement or commitment not to enforce a patent against the party.

If you convey a covered work, knowingly relying on a patent license, and the Corresponding Source of the work is not available for anyone to copy, free of charge and under the terms of this License, through a publicly available network server or other readily accessible means, then you must either (1) cause the Corresponding Source to be so available, or (2) arrange to deprive yourself of the benefit of the patent license for this particular work, or (3) arrange, in a manner consistent with the requirements of this License, to extend the patent license to downstream recipients. "Knowingly relying" means you have actual knowledge that, but for the patent license, your conveying the covered work in a country, or your recipient's use of the covered work in a country, would infringe one or more identifiable patents in that country that you have reason to believe are valid.

If, pursuant to or in connection with a single transaction or arrangement, you convey, or propagate by procuring conveyance of, a covered work, and grant a patent license to some of the parties receiving the covered work authorizing them to use, propagate, modify or convey a specific copy of the covered work, then the patent license you grant is automatically extended to all recipients of the covered work and works based on it.

A patent license is "discriminatory" if it does not include within the scope of its coverage, prohibits the exercise of, or is conditioned on the non-exercise of one or more of the rights that are specifically granted under this License. You may not convey a covered work if you are a party to an arrangement with a third party that is in the business of distributing software, under which you make payment to the third party based on the extent of your activity of conveying the work, and under which the third party grants, to any of the parties who would receive the covered work from you, a discriminatory patent license (a) in connection with copies of the covered work conveyed by you (or copies made from those copies), or (b) primarily for and in connection with specific products or compilations that contain the covered work, unless you entered into that arrangement, or that patent license was granted, prior to 28 March 2007.

Nothing in this License shall be construed as excluding or limiting any implied license or other defenses to infringement that may otherwise be available to you under applicable patent law. 12. No Surrender of Others' Freedom.

If conditions are imposed on you (whether by court order, agreement or otherwise) that contradict the conditions of this License, they do not excuse you from the conditions of this License. If you cannot convey a covered work so as to satisfy simultaneously your obligations under this License and any other pertinent obligations, then as a consequence you may not convey it at all. For example, if you agree to terms that obligate you to collect a royalty for further conveying from those to whom you convey the Program, the only way you could satisfy

both those terms and this License would be to refrain entirely from conveying the Program. 13. Use with the GNU Affero General Public License.

Notwithstanding any other provision of this License, you have permission to link or combine any covered work with a work licensed under version 3 of the GNU Affero General Public License into a single combined work, and to convey the resulting work. The terms of this License will continue to apply to the part which is the covered work, but the special requirements of the GNU Affero General Public License, section 13, concerning interaction through a network will apply to the combination as such. 14. Revised Versions of this License.

The Free Software Foundation may publish revised and/or new versions of the GNU General Public License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns.

Each version is given a distinguishing version number. If the Program specifies that a certain numbered version of the GNU General Public License "or any later version" applies to it, you have the option of following the terms and conditions either of that numbered version or of any later version published by the Free Software Foundation. If the Program does not specify a version number of the GNU General Public License, you may choose any version ever published by the Free Software Foundation.

If the Program specifies that a proxy can decide which future versions of the GNU General Public License can be used, that proxy's public statement of acceptance of a version permanently authorizes you to choose that version for the Program.

## 3.2 GNU Free Documentation License

Version 1.3, 3 November 2008

Copyright (c) 2000, 2001, 2002, 2007, 2008 Free Software Foundation, Inc. <<http://fsf.org/>>

Everyone is permitted to copy and distribute verbatim copies of this license document, but changing it is not allowed. 0. PREAMBLE

The purpose of this License is to make a manual, textbook, or other functional and useful document "free" in the sense of freedom: to assure everyone the effective freedom to copy and redistribute it, with or without modifying it, either commercially or noncommercially. Secondly, this License preserves for the author and publisher a way to get credit for their work, while not being considered responsible for modifications made by others.

This License is a kind of "copyleft", which means that derivative works of the document must themselves be free in the same sense. It complements the GNU General Public License, which is a copyleft license designed for free software.

We have designed this License in order to use it for manuals for free software, because free software needs free documentation: a free program should come with manuals providing the same freedoms that the software does. But this License is not limited to software manuals; it can be used for any textual work, regardless of subject matter or whether it is published as a printed book. We recommend this License principally for works whose purpose is instruction or reference. 1. APPLICABILITY AND DEFINITIONS

This License applies to any manual or other work, in any medium, that contains a notice placed by the copyright holder saying it can be distributed under the terms of this License. Such a notice grants a world-wide, royalty-free license, unlimited in duration, to use that work under the conditions stated herein. The "Document", below, refers to any such manual or work. Any member of the public is a licensee, and is addressed as "you". You accept the license if you copy, modify or distribute the work in a way requiring permission under copyright law.

A "Modified Version" of the Document means any work containing the Document or a portion of it, either copied verbatim, or with modifications and/or translated into another language.

A "Secondary Section" is a named appendix or a front-matter section of the Document that deals exclusively with the relationship of the publishers or authors of the Document to the Document's overall subject (or to related matters) and contains nothing that could fall directly within that overall subject. (Thus, if the Document is in part a textbook of mathematics, a Secondary Section may not explain any mathematics.) The relationship could be a matter of historical connection with the subject or with related matters, or of legal, commercial, philosophical, ethical or political position regarding them.

The "Invariant Sections" are certain Secondary Sections whose titles are designated, as being those of Invariant Sections, in the notice that says that the Document is released under this License. If a section does not fit the above definition of Secondary then it is not allowed to be designated as Invariant. The Document may contain zero Invariant Sections. If the Document does not identify any Invariant Sections then there are none.

The "Cover Texts" are certain short passages of text that are listed, as Front-Cover Texts or Back-Cover Texts, in the notice that says that the Document is released under this License. A Front-Cover Text may be at most 5 words, and a Back-Cover Text may be at most 25 words.

A "Transparent" copy of the Document means a machine-readable copy, represented in a format whose specification is available to the general public, that is suitable for revising the document straightforwardly with generic text editors or (for images composed of pixels) generic paint programs or (for drawings) some widely available drawing editor, and that is suitable for input to text formatters or for automatic translation to a variety of formats suitable for input to text formatters. A copy made in an otherwise Transparent file format whose markup, or absence of markup, has been arranged to thwart or discourage subsequent modification by readers is not Transparent. An image format is not Transparent if used for any substantial amount of text. A copy that is not "Transparent" is called "Opaque".

Examples of suitable formats for Transparent copies include plain ASCII without markup, Texinfo input format, LaTeX input format, SGML or XML using a publicly available DTD, and standard-conforming simple HTML, PostScript or PDF designed for human modification. Examples of transparent image formats include PNG, XCF and JPG. Opaque formats include proprietary formats that can be read and edited only by proprietary word processors, SGML or XML for which the DTD and/or processing tools are not generally available, and the machine-generated HTML, PostScript or PDF produced by some word processors for output purposes only.

The "Title Page" means, for a printed book, the title page itself, plus such following pages as are needed to hold, legibly, the material this License requires to appear in the title page. For works in formats which do not have any title page as such, "Title Page" means the text near the most prominent appearance of the work's title, preceding the beginning of the body of the text.

The "publisher" means any person or entity that distributes copies of the Document to the public.

A section "Entitled XYZ" means a named subunit of the Document whose title either is precisely XYZ or contains XYZ in parentheses

Later license versions may give you additional or different permissions. However, no additional obligations are imposed on any author or copyright holder as a result of your choosing to follow a later version. 15. Disclaimer of Warranty.

THERE IS NO WARRANTY FOR THE PROGRAM, TO THE EXTENT PERMITTED BY APPLICABLE LAW. EXCEPT WHEN OTHERWISE STATED IN WRITING THE COPYRIGHT HOLDERS AND/OR OTHER PARTIES PROVIDE THE PROGRAM "AS IS" WITHOUT WARRANTY OF ANY KIND, EITHER EXPRESSED OR IMPLIED, INCLUDING, BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE. THE ENTIRE RISK AS TO THE QUALITY AND PERFORMANCE OF THE PROGRAM IS WITH YOU. SHOULD THE PROGRAM PROVE DEFECTIVE, YOU ASSUME THE COST OF ALL NECESSARY SERVICING, REPAIR OR CORRECTION. 16. Limitation of Liability.

IN NO EVENT UNLESS REQUIRED BY APPLICABLE LAW OR AGREED TO IN WRITING WILL ANY COPYRIGHT HOLDER, OR ANY OTHER PARTY WHO MODIFIES AND/OR CONVEYS THE PROGRAM AS PERMITTED ABOVE, BE LIABLE TO YOU FOR DAMAGES, INCLUDING ANY GENERAL, SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF THE USE OR INABILITY TO USE THE PROGRAM (INCLUDING BUT NOT LIMITED TO LOSS OF DATA OR DATA BEING RENDERED INACCURATE OR LOSSES SUSTAINED BY YOU OR THIRD PARTIES OR A FAILURE OF THE PROGRAM TO OPERATE WITH ANY OTHER PROGRAMS), EVEN IF SUCH HOLDER OR OTHER PARTY HAS BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES. 17. Interpretation of Sections 15 and 16.

following text that translates XYZ in another language. (Here XYZ stands for a specific section name mentioned below, such as "Acknowledgements", "Dedications", "Endorsements", or "History"). To "Preserve the Title" of such a section when you modify the Document means that it remains a section "Entitled XYZ" according to this definition.

The Document may include Warranty Disclaimers next to the notice which states that this License applies to the Document. These Warranty Disclaimers are considered to be included by reference in this License, but only as regards disclaiming warranties; any other implication that these Warranty Disclaimers may have is void and has no effect on the meaning of this License. 2. VERBATIM COPYING

You may copy and distribute the Document in any medium, either commercially or noncommercially, provided that this License, the copyright notices, and the license notice saying this License applies to the Document are reproduced in all copies, and that you add no other conditions whatsoever to those of this License. You may not use technical measures to obstruct or control the reading or further copying of the copies you make or distribute. However, you may accept compensation in exchange for copies. If you distribute a large enough number of copies you must also follow the conditions in section 3.

You may also lend copies, under the same conditions stated above, and you may publicly display copies. 3. COPYING IN QUANTITY

If you publish printed copies (or copies in media that commonly have printed covers) of the Document, numbering more than 100, and the Document's license notice requires Cover Texts, you must enclose the copies in covers that carry, clearly and legibly, all these Cover Texts: Front-Cover Texts on the front cover, and Back-Cover Texts on the back cover. Both covers must also clearly and legibly identify you as the publisher of these copies. The front cover must present the full title with all words of the title equally prominent and visible. You may add other material on the covers in addition. Copying with changes limited to the covers, as long as they preserve the title of the Document and satisfy these conditions, can be treated as verbatim copying in other respects.

If the required texts for either cover are too voluminous to fit legibly, you should put the first ones listed (as many as fit reasonably) on the actual cover, and continue the rest onto adjacent pages.

If you publish or distribute Opaque copies of the Document numbering more than 100, you must either include a machine-readable Transparent copy along with each Opaque copy, or state in or with each Opaque copy a computer-network location from which the general network-using public has access to download using public-standard network protocols a complete Transparent copy of the Document, free of added material. If you use the latter option, you must take reasonably prudent steps, when you begin distribution of Opaque copies in quantity, to ensure that this Transparent copy will remain thus accessible at the stated location until at least one year after the last time you distribute an Opaque copy (directly or through your agents or retailers) of that edition to the public.

It is requested, but not required, that you contact the authors of the Document well before redistributing any large number of copies, to give them a chance to provide you with an updated version of the Document. 4. MODIFICATIONS

You may copy and distribute a Modified Version of the Document under the conditions of section 2 above, provided that you release the Modified Version under precisely this License, with the Modified Version filling the role of the Document, this licensing distribution and modification of the Modified Version to whoever possesses a copy of it. In addition, you must do these things in the Modified Version:

\* A. Use in the Title Page (and on the covers, if any) a title distinct from that of the Document, and from those of previous versions (which should, if there were any, be listed in the History section of the Document). You may use the same title as a previous version if the original publisher of that version gives permission. \* B. List on the Title Page, as authors, one or more persons or entities responsible for authorship of the modifications in the Modified Version, together with at least five of the principal authors of the Document (all of its principal authors, if it has fewer than five), unless they release you from this requirement. \* C. State on the Title page the name of the publisher of the Modified Version, as the publisher. \* D. Preserve all the copyright notices of the Document. \* E. Add an appropriate copyright notice for your modifications adjacent to the other copyright notices. \* F. Include, immediately after the copyright notices, a license notice giving the public permission to use the Modified Version under the terms of this License, in the form shown below. \* G. Preserve in the license notice the full list of Invariant Sections and required Cover Texts given in the Document's license notice. \* H. Include an unaltered copy of this License. \* I. Preserve the section entitled "History", Preserve its Title, and add to it an item stating at least the title, year, new authors, and publisher of the Modified Version as given on the Title Page. If there is no section entitled "History" in the Document, create one stating the title, year, authors, and publisher of the Document as given on its Title Page, then add an item describing the Modified Version as stated in the previous sentence. \* J. Preserve the network location, if any, given in the Document for public access to a Transparent copy of the Document, and likewise the network locations given in the Document for previous versions it was based on. These may be placed in the "History" section. You may omit a network location for work that was published at least four years before the Document itself, or if the original publisher of the version it refers to gives permission. \* K. For a section entitled "Acknowledgements" or "Dedications", Preserve the Title of the section, and preserve in the section all the substance and tone of each of the contributor acknowledgements and/or dedications given therein. \* L. Preserve all the Invariant Sections of the Document, unaltered in their text and

If the disclaimer of warranty and limitation of liability provided above cannot be given local legal effect according to their terms, reviewing courts shall apply local law that most closely approximates an absolute waiver of all civil liability in connection with the Program, unless a warranty or assumption of liability accompanies a copy of the Program in return for a fee.

END OF TERMS AND CONDITIONS How to Apply These Terms to Your New Programs

If you develop a new program, and you want it to be of the greatest possible use to the public, the best way to achieve this is to make it free software which everyone can redistribute and change under these terms.

To do so, attach the following notices to the program. It is safest to attach them to the start of each source file to most effectively state the exclusion of warranty; and each file should have at least the "copyright" line and a pointer to where the full notice is found.

<one line to give the program's name and a brief idea of what it does.>  
Copyright (C) <year> <name of author>

This program is free software: you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation, either version 3 of the License, or (at your option) any later version.

This program is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public License for more details.

in their titles. Section numbers or the equivalent are not considered part of the section titles. \* M. Delete any section Entitled "Endorsements". Such a section may not be included in the Modified Version. \* N. Do not retile any existing section to be Entitled "Endorsements" or to conflict in title with any Invariant Section. \* O. Preserve any Warranty Disclaimers.

If the Modified Version includes new front-matter sections or appendices that qualify as Secondary Sections and contain no material copied from the Document, you may at your option designate some or all of these sections as invariant. To do this, add their titles to the list of Invariant Sections in the Modified Version's license notice. These titles must be distinct from any other section titles.

You may add a section Entitled "Endorsements", provided it contains nothing but endorsements of your Modified Version by various parties—for example, statements of peer review or of the text has been approved by an organization as the authoritative definition of a standard.

You may add a passage of up to five words as a Front-Cover Text, and a passage of up to 25 words as a Back-Cover Text, to the end of the list of Cover Texts in the Modified Version. Only one passage of Front-Cover Text and one of Back-Cover Text may be added (or through arrangements made by) any one entity. If the Document already includes a cover text for the same cover, previously added by you or by arrangement made by the same entity you are acting on behalf of, you may not add another; but you may replace the old one, on explicit permission from the previous publisher that added the old one.

The author(s) and publisher(s) of the Document do not by this License give permission to use their names for publicity for or to assert or imply endorsement of any Modified Version. 5. COMBINING DOCUMENTS

You may combine the Document with other documents released under this License, under the terms defined in section 4 above for modified versions, provided that you include in the combination all of the Invariant Sections of all of the original documents, unmodified, and list them all as Invariant Sections of your combined work in its license notice, and that you preserve all their Warranty Disclaimers.

The combined work need only contain one copy of this License, and multiple identical Invariant Sections may be replaced with a single copy. If there are multiple Invariant Sections with the same name but different contents, make the title of each such section unique by adding at the end of it, in parentheses, the name of the original author or publisher of that section if known, or else a unique number. Make the same adjustment to the section titles in the list of Invariant Sections in the license notice of the combined work.

In the combination, you must combine any sections Entitled "History" in the various original documents, forming one section Entitled "History"; likewise combine any sections Entitled "Acknowledgements", and any sections Entitled "Dedications". You must delete all sections Entitled "Endorsements". 6. COLLECTIONS OF DOCUMENTS

You may make a collection consisting of the Document and other documents released under this License, and replace the individual copies of this License in the various documents with a single copy that is included in the collection, provided that you follow the rules of this License for verbatim copying of each of the documents in all other respects.

You may extract a single document from such a collection, and distribute it individually under this License, provided you insert a copy of this License into the extracted document, and follow this License in all other respects regarding verbatim copying of that document. 7. AGGREGATION WITH INDEPENDENT WORKS

A compilation of the Document or its derivatives with other separate and independent documents or works, in or on a volume of a storage or distribution medium, is called an "aggregate" if the copyright resulting from the compilation is not used to limit the legal rights of the compilation's users beyond what the individual works permit. When the Document is included in an aggregate, this License does not apply to the other works in the aggregate which are not themselves derivative works of the Document.

If the Cover Text requirement of section 3 is applicable to these copies of the Document, then if the Document is less than one half of the entire aggregate, the Document's Cover Texts may be placed on covers that bracket the Document within the aggregate, or the electronic equivalent of covers if the Document is in electronic form. Otherwise they must appear on printed covers that bracket the whole aggregate. 8. TRANSLATION

Translation is considered a kind of modification, so you may distribute translations of the Document under the terms of section 4. Replacing Invariant Sections with translations requires special permission from their copyright holders, but you may include translations of some or all Invariant Sections in addition to the original versions of these Invariant Sections. You may include a translation of this License, and all the license notices in the Document, and any Warranty Disclaimers, provided that you also include the original English version of this License and the original versions of those notices and disclaimers. In case of a disagreement between the translation and the original version of this License or a notice or disclaimer, the original version will prevail.

If a section in the Document is Entitled "Acknowledgements", "Dedications", or "History", the requirement (section 4) to Preserve its Title

You should have received a copy of the GNU General Public License along with this program. If not, see <<http://www.gnu.org/licenses/>>.

Also add information on how to contact you by electronic and paper mail.

If the program does terminal interaction, make it output a short notice like this when it starts in an interactive mode:

<program> Copyright (C) <year> <name of author> This program comes with ABSOLUTELY NO WARRANTY; for details type 'show w'. This is free software, and you are welcome to redistribute it under certain conditions; type 'show c' for details.

The hypothetical commands 'show w' and 'show c' should show the appropriate parts of the General Public License. Of course, your program's commands might be different; for a GUI interface, you would use an "about box".

You should also get your employer (if you work as a programmer) or school, if any, to sign a "copyright disclaimer" for the program, if necessary. For more information on this, and how to apply and follow the GNU GPL, see <<http://www.gnu.org/licenses/>>.

The GNU General Public License does not permit incorporating your program into proprietary programs. If your program is a subroutine library, you may consider it more useful to permit linking proprietary applications with the library. If this is what you want to do, use the GNU Lesser General Public License instead of this License. But first, please read <<http://www.gnu.org/philosophy/why-not-lgpl.html>>.

(section 1) will typically require changing the actual title. 9. TERMINATION

You may not copy, modify, sublicense, or distribute the Document except as expressly provided under this License. Any attempt otherwise to copy, modify, sublicense, or distribute it is void, and will automatically terminate your rights under this License.

However, if you cease all violation of this License, then your license from a particular copyright holder is reinstated (a) provisionally, unless and until the copyright holder explicitly and finally terminates your license, and (b) permanently, if the copyright holder fails to notify you of the violation by some reasonable means prior to 60 days after the cessation.

Moreover, your license from a particular copyright holder is reinstated permanently if the copyright holder notifies you of the violation by some reasonable means, this is the first time you have received notice of violation of this License (for any work) from that copyright holder, and you cure the violation prior to 30 days after your receipt of the notice.

Termination of your rights under this section does not terminate the licenses of parties who have received copies or rights from you under this License. If your rights have been terminated and not permanently reinstated, receipt of a copy of some or all of the same material does not give you any rights to use it. 10. ALL FUTURE REVISIONS OF THIS LICENSE

The Free Software Foundation may publish new, revised versions of the GNU Free Documentation License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns. See <<http://www.gnu.org/copyleft/>>.

Each version of the License is given a distinguishing version number. If the Document specifies that a particular numbered version of this License "or any later version" applies to it, you have the option of following the terms and conditions either of that specified version or of any later version that has been published (not as a draft) by the Free Software Foundation. If the Document does not specify a version number of this License, you may choose any version ever published (not as a draft) by the Free Software Foundation. If the Document specifies that a proxy can decide which future versions of this License can be used, that proxy's public statement of acceptance of a version permanently authorizes you to choose that version for the Document. 11. RELICENSING

"Massive Multiauthor Collaboration Site" (or "MMC Site") means any World Wide Web server that publishes copyrightable works and also provides prominent facilities for anybody to edit those works. A public webkit that anybody can edit is an example of such a server. A "Massive Multiauthor Collaboration" (or "MMC") contained in the site means any set of copyrightable works thus published on the MMC site.

"CC-BY-SA" means the Creative Commons Attribution-Share Alike 3.0 license published by Creative Commons Corporation, a not-for-profit corporation with a principal place of business in San Francisco, California, as well as future copyleft versions of that license published by that same organization.

"Incorporate" means to publish or republish a Document, in whole or in part, as part of another Document.

An MMC is "eligible for relicensing" if it is licensed under this License, and if all works that were first published under this License somewhere other than this MMC, and subsequently incorporated in whole or in part into the MMC, (1) had no cover texts or invariant sections, and (2) were thus incorporated prior to November 1, 2008.

The operator of an MMC Site may republish an MMC contained in the site under CC-BY-SA on the same site at any time before August 1, 2009, provided the MMC is eligible for relicensing. ADDENDUM: How to use this License for your documents

To use this License in a document you have written, include a copy of the License in the document and put the following copyright and license notices just after the title page:

Copyright (C) YEAR YOUR NAME. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.3 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the License is included in the section entitled "GNU Free Documentation License".

If you have Invariant Sections, Front-Cover Texts and Back-Cover Texts, replace the "with ... Texts." line with this:

with the Invariant Sections being LIST THEIR TITLES, with the Front-Cover Texts being LIST, and with the Back-Cover Texts being LIST.

If you have Invariant Sections without Cover Texts, or some other combination of the three, merge those two alternatives to suit the situation.

If your document contains nontrivial examples of program code, we recommend releasing these examples in parallel under your choice of free software license, such as the GNU General Public License, to permit their use in free software.



# 3.3 GNU Lesser General Public License

GNU LESSER GENERAL PUBLIC LICENSE

Version 3, 29 June 2007

Copyright © 2007 Free Software Foundation, Inc. <<http://fsf.org/>>

Everyone is permitted to copy and distribute verbatim copies of this license document, but changing it is not allowed.

This version of the GNU Lesser General Public License incorporates the terms and conditions of version 3 of the GNU General Public License, supplemented by the additional permissions listed below.

0. Additional Definitions.

As used herein, “this License” refers to version 3 of the GNU Lesser General Public License, and the “GNU GPL” refers to version 3 of the GNU General Public License.

“The Library” refers to a covered work governed by this License, other than an Application or a Combined Work as defined below.

An “Application” is any work that makes use of an interface provided by the Library, but which is not otherwise based on the Library. Defining a subclass of a class defined by the Library is deemed a mode of using an interface provided by the Library.

A “Combined Work” is a work produced by combining or linking an Application with the Library. The particular version of the Library with which the Combined Work was made is also called the “Linked Version”.

The “Minimal Corresponding Source” for a Combined Work means the Corresponding Source for the Combined Work, excluding any source code for portions of the Combined Work that, considered in isolation, are based on the Application, and not on the Linked Version.

The “Corresponding Application Code” for a Combined Work means the object code and/or source code for the Application, including any data and utility programs needed for reproducing the Combined Work from the Application, but excluding the System Libraries of the Combined Work.

1. Exception to Section 3 of the GNU GPL.

You may convey a covered work under sections 3 and 4 of this License without being bound by section 3 of the GNU GPL.

2. Conveying Modified Versions.

If you modify a copy of the Library, and, in your modifications, a facility refers to a function or data to be supplied by an Application that uses the facility (other than as an argument passed when the facility is invoked), then you may convey a copy of the modified version:

\* a) under this License, provided that you make a good faith effort to ensure that, in the event an Application does not supply the function or data, the facility still operates, and performs whatever part of its purpose remains meaningful, or

\* b) under the GNU GPL, with none of the additional permissions of this License applicable to that copy.

3. Object Code Incorporating Material from Library Header Files.

The object code form of an Application may incorporate material from a header file that is part of the Library. You may convey such object code under terms of your choice, provided that, if the incorporated material is not limited to numerical parameters, data structure layouts and accessors, or small macros, inline functions and templates (ten or fewer lines in length), you do both of the following:

\* a) Give prominent notice with each copy of the object code that the Library is used in it and that the Library and its use are covered by this License.

\* b) Accompany the object code with a copy of the GNU GPL and this license document.

4. Combined Works.

You may convey a Combined Work under terms of your choice that, taken together, effectively do not restrict modification of the portions of the Library contained in the Combined Work and reverse engineering for debugging such modifications, if you also do each of the following:

\* a) Give prominent notice with each copy of the Combined Work that the Library is used in it and that the Library and its use are covered by this License.

\* b) Accompany the Combined Work with a copy of the GNU GPL and this license document.

\* c) For a Combined Work that displays copyright notices during execution, include the copyright notice for the Library among these notices, as well as a reference directing the user to the copies of the GNU GPL and this license document.

\* d) Do one of the following:

- o 0) Convey the Minimal Corresponding Source under the terms of this License, and the Corresponding Application Code in a form suitable for, and under terms that permit, the user to recombine or relink the Application with a modified version of the Linked Version to produce a modified Combined Work, in the manner specified by section 6 of the GNU GPL for conveying Corresponding Source.
- o 1) Use a suitable shared library mechanism for linking with the Library. A suitable mechanism is one that (a) uses at run time a copy of the Library already present on the user’s computer system, and (b) will operate properly with a modified version of the Library that is interface-compatible with the Linked Version.
- \* e) Provide Installation Information, but only if you would otherwise be required to provide such information under section 6 of the GNU GPL, and only to the extent that such information is necessary to install and execute a modified version of the Combined Work produced by recombining or relinking the Application with a modified version of the Linked Version. (If you use option 4d0, the Installation Information must accompany the Minimal Corresponding Source and Corresponding Application Code. If you use option 4d1, you must provide the Installation Information in the manner specified by section 6 of the GNU GPL for conveying Corresponding Source.)

5. Combined Libraries.

You may place library facilities that are a work based on the Library side by side in a single library together with other library facilities that are not Applications and are not covered by this License, and convey such a combined library under terms of your choice, if you do both of the following:

\* a) Accompany the combined library with a copy of the same work based on the Library, uncombined with any other library facilities, conveyed under the terms of this License.

\* b) Give prominent notice with the combined library that part of it is a work based on the Library, and explaining where to find the accompanying uncombined form of the same work.

6. Revised Versions of the GNU Lesser General Public License.

The Free Software Foundation may publish revised and/or new versions of the GNU Lesser General Public License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns.

Each version is given a distinguishing version number. If the Library as you received it specifies that a certain numbered version of the GNU Lesser General Public License “or any later version” applies to it, you have the option of following the terms and conditions either of that published version or of any later version published by the Free Software Foundation. If the Library as you received it does not specify a version number of the GNU Lesser General Public License, you may choose any version of the GNU Lesser General Public License ever published by the Free Software Foundation.

If the Library as you received it specifies that a proxy can decide whether future versions of the GNU Lesser General Public License shall apply, that proxy’s public statement of acceptance of any version is permanent authorization for you to choose that version for the Library.