问题：

有两个天线，海拔、纬度相同，它们连线中心的座标是东经120度。它们之间的距离是100米。采集系统的观测频率范围是 50-200 MHz， 频率信道宽度是(200/8192) MHz，在50-200 MHz之间一共有6144个频率信道。也就是说每一个信道的中心频率值是(50+i*(200/8192))MHz。天空中在R.A.=0 deg, Dec=80 deg处有一个唯一的射电源，射电源的谱的斜率是0（也就是平谱），画出这两个天线在24小时内相关得到的visibility的实部、虚部的时频瀑布图。

射电干涉测量中Visibility函数的完整理论推导

1. 基本概念与定义

1.1 干涉测量基本原理

- **Visibility函数**：表示两个天线接收信号的互相关

$$V(u,v,w) = I \cdot e^{-2\pi i(ul+vm+wn)}$$

  - I: 源强度（平谱时设为1）
  - (u,v,w): 基线投影（波长单位）
  - (l,m,n): 源方向余弦

1.2 坐标系系统

坐标系	定义	关键参数
地球坐标系	原点在地心	(X, Y, Z)
天空坐标系	原点在观测站	(u, v, w)

2. 关键参数推导

2.1 基线矢量

$$\mathbf{B} = \mathbf{r}_2 - \mathbf{r}_1 = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

对于东西向基线（赤道）：

$$B_x = 100m,\ B_y = B_z = 0$$

2.2 方向余弦

对于R.A.=0°, Dec=80°：

$$\begin{cases} l = \cos 80^\circ \approx 0.1736 \\ m = 0 \\ n = \sin 80^\circ \approx 0.9848 \end{cases}$$

3. 坐标转换与投影

3.1 旋转矩阵

地球→天空坐标转换：

$$\mathbf{T} = \begin{bmatrix} -\sin H & \cos H & 0 \\ -\sin\phi\cos H & -\sin\phi\sin H & \cos\phi \\ \cos\phi\cos H & \cos\phi\sin H & \sin\phi \end{bmatrix}$$

3.2 基线投影

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda}\mathbf{T}\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

展开式：

$$\begin{cases} u = \frac{-B_x\sin H + B_y\cos H}{\lambda} \\ v = \frac{-B_x\sin\phi\cos H - B_y\sin\phi\sin H + B_z\cos\phi}{\lambda} \\ w = \frac{B_x\cos\phi\cos H + B_y\cos\phi\sin H + B_z\sin\phi}{\lambda} \end{cases}$$

4. Visibility函数计算

4.1 相位项

$$\phi = 2\pi(ul + wn) = \frac{2\pi B}{\lambda}(l \sin H + n \cos\phi \cos H)$$

4.2 实部/虚部分解

$$\begin{cases} Re(V) = \cos\phi \\ Im(V) = -\sin\phi \end{cases}$$

5. 时频特性分析

5.1 频率依赖

$$\phi \propto \frac{1}{\lambda} = \frac{\nu}{c}$$

- 高频→密集条纹
- 低频→稀疏条纹

5.2 时间演化

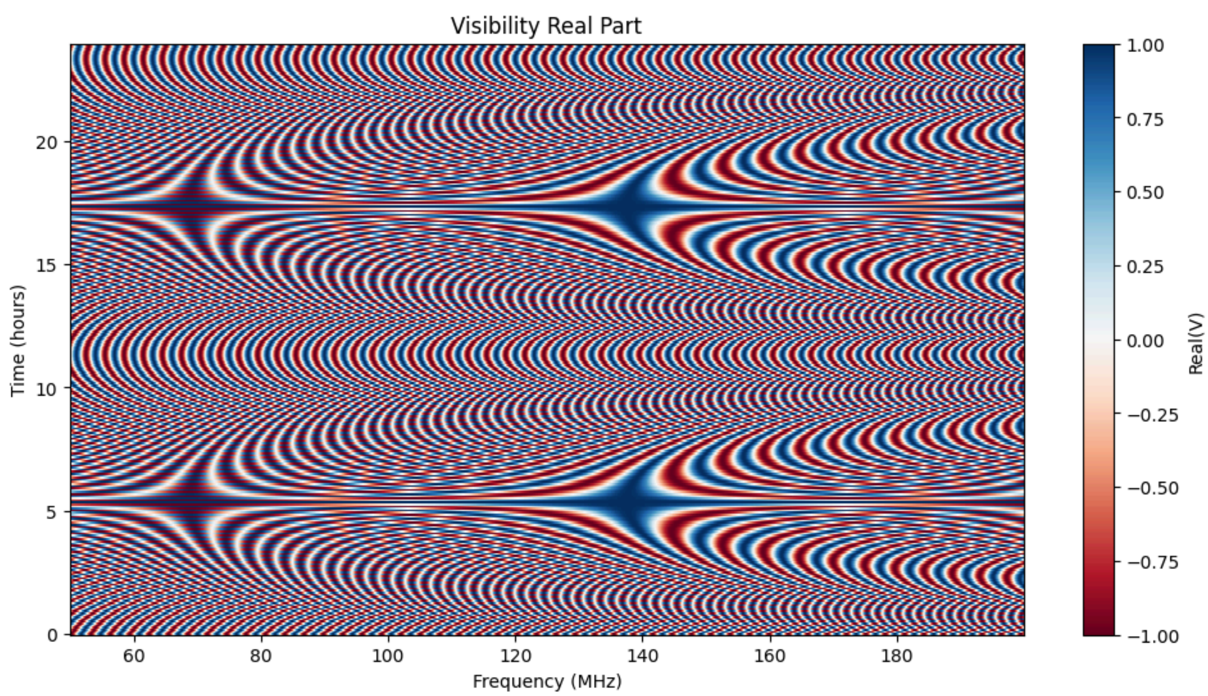
时角变化：

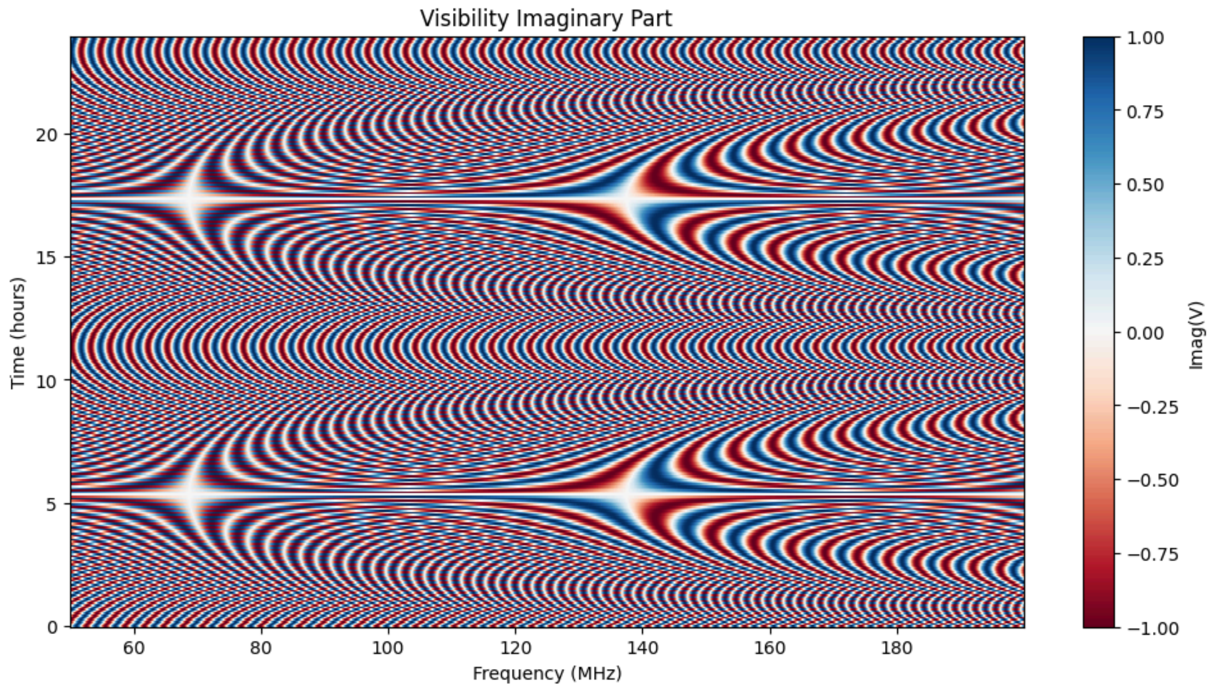
$$H(t) = \frac{2\pi}{24}t \quad (t\ in\ hours)$$

In [12]:

```
import numpy as np
import matplotlib.pyplot as plt

# Constants
c = 3e8  # Speed of light (m/s)
B = 100  # Baseline length (m)
dec_deg = 80  # Declination (degrees)
dec = np.deg2rad(dec_deg)

# Direction cosines (l, m, n)
l = np.cos(dec)  # l = cos(Dec) * cos(R.A.), R.A.=0
n = np.sin(dec)  # n = sin(Dec)

# Frequency setup (50-200 MHz, 6144 channels)
num_channels = 6144
frequencies = 50e6 + (200e6 / 8192) * np.arange(num_channels)  # Hz
wavelengths = c / frequencies  # Wavelengths (m)

# Time setup (24 hours, 5-minute resolution)
times_hours = np.arange(0, 24, 5/60)  # Time points (hours)
HA_rad = np.deg2rad(15 * times_hours)  # Hour angle (radians)

# Initialize visibility matrices
V_real = np.zeros((len(times_hours), num_channels))
```

```python
V_imag = np.zeros((len(times_hours), num_channels))

for i, H in enumerate(HA_rad):
    # Baseline projections (u, w) in wavelengths
    u = B * np.sin(H) / wavelengths  # East-West component
    w = B * np.cos(H) / wavelengths  # Vertical component

    # Phase: 2π (u*l + w*n)
    phi = 2 * np.pi * (u * l + w * n)

    # Visibility
    V_real[i, :] = np.cos(phi)
    V_imag[i, :] = -np.sin(phi)  # Negative sign for convention

# Plotting
plt.figure(figsize=(12, 6))
plt.pcolormesh(frequencies/1e6, times_hours, V_real,
               shading='auto', cmap='RdBu', vmin=-1, vmax=1)
plt.colorbar(label='Real(V)')
plt.xlabel('Frequency (MHz)')
plt.ylabel('Time (hours)')
plt.title('Visibility Real Part')

plt.figure(figsize=(12, 6))
plt.pcolormesh(frequencies/1e6, times_hours, V_imag,
               shading='auto', cmap='RdBu', vmin=-1, vmax=1)
plt.colorbar(label='Imag(V)')
plt.xlabel('Frequency (MHz)')
plt.ylabel('Time (hours)')
plt.title('Visibility Imaginary Part')

plt.show()
```




Visibility Imaginary Part
Time (hours)
0
5
10
15
20
Frequency (MHz)
60
80
100
120
140
160
180
Imag(V)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00