

Level 2

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| 11 <i>Let U be the subspace V spanned by all characteristic vectors of T. Then $U = V$, i.e. the characteristic vectors of T span the whole space.</i> |
| 111 U^\perp is T -invariant. |
| 112 Every nonzero T -invariant subspace of V contains a characteristic vector of T . |

Proof. To prove $U = V$ we prove the equivalent statement $U^\perp = \{0\}$. This in turn will follow from 111 and 112. Since U^\perp cannot contain a characteristic vector of T (this would contradict $U \cap U^\perp = \{0\}$), we must have $U^\perp = \{0\}$, hence $U = V$. □