# Level 2

**11**
*Let $U$ be the subspace $V$ spanned by all characteristic vectors of $T$. Then $U = V$, i.e.m the characteristic vectors of $T$ span the whole space.*

- **111** *$U^\perp$ is $T$-invariant.*
- **112** *Every nonzero $T$-invariant subspace of $V$ contains a characteristic vector of $T$.*

*Proof.* To prove $U = V$ we prove the equivalent statement $U^\perp = \{0\}$. This in turn will follow from 111 and 112. Since $U^\perp$ cannot contain a characteristic vector of $T$ (this would contradict $U \cap U^\perp = \{0\}$), we must have $U^\perp = \{0\}$, hence $U = V$. □