

Level 2

11

Let U be the subspace V spanned by all characteristic vectors of T . Then $U = V$, i.e. the characteristic vectors of T span the whole space.

111 U^\perp is T -invariant.

112 Every nonzero T -invariant subspace of V contains a characteristic vector of T .

Proof. To prove $U = V$ we prove the equivalent statement $U^\perp = \{0\}$. This in turn will follow from 111 and 112. Since U^\perp cannot contain a characteristic vector of T (this would contradict $U \cap U^\perp = \{0\}$), we must have $U^\perp = \{0\}$, hence $U = V$. \square