Homework11

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4.5.4

(a)

Denote *The* $\left(\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right)$ *as* B_v

The ith column of $[R]_{B_v}$ is $[Rv_i]_{B_v}$.

$$
\begin{aligned}
[Rv_1]_{B_v} &= \begin{bmatrix}1\\0\end{bmatrix}_{B_v} = \begin{bmatrix}1\\0\end{bmatrix}\\
[Rv_2]_{B_v} &= \begin{bmatrix}1\\-1\end{bmatrix}_{B_v} = \begin{bmatrix}2\\-1\end{bmatrix}\\
\Rightarrow &\begin{bmatrix}1 & 2\\0 & -1\end{bmatrix}
\end{aligned}
$$

(b)

Proof *By* ***Proposition 4.31*** *and* ***Definition of Orthogonal Matrix****, we know that if the columns are not* ***Othonomal****, then the matrix must not be orthogonal.*

$$\langle A_1, A_2\rangle = 2 \neq 0$$

Therefore, A is not orthogonal matrix.

(c)

Proof *Because* ***Proposition 4.30*** *says,* ***Suppose*** B_v, B_w ***are orthonormal basis of*** $\mathbb{V}, \mathbb{W}$. *But* $\left(\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right)$ *itself is not an orthonomal basis.*

4.5.6

Proof *Since* $\mathbb{C}_{2\pi}(\mathbb{R})$ *is equipped with innerporduct, so it is both an innerproduct space and a normed space.*
Suppose $\forall f(x) \in \mathbb{C}_{2\pi}(\mathbb{R})$*, which means* $f(x)$ *is a continuous* 2π *periodic function* $\Rightarrow \exists g(x) = f(x-t) \in \mathbb{C}_{2\pi}(\mathbb{R}), T(g)(x) = f(x+t)$. T *is a surjective lineaer map.*

Denote

$$\int_a^b f(x)\overline{f(x)}\,d(x) = F(b) - F(a) = \int_{a+2\pi}^{b+2\pi} f(x)\overline{f(x)}\,d(x) = F(b+2\pi) - F(a+2\pi)$$
$$\Rightarrow F(b+2\pi) - F(b) = F(a+2\pi) - F(a)$$

So we get

$$\begin{aligned}\|(Tf)(x)\| \int_0^{2\pi} (Tf)(x)\overline{(Tf)(x)}\,dx &= \int_0^{2\pi} f(x+t)\overline{f(x+t)}\,dx \\ &= \int_0^{2\pi} f(x+t)\overline{f(x+t)}\,d(x+t) \\ &= \int_t^{t+2\pi} f(x')\overline{f(x')}\,d(x') \\ &= F(t+2\pi) - F(t) \\ &= F(0+2\pi) - F(0) \\ &= \|(f)(x)\|\end{aligned}$$

So, it is an isometry.

4.5.8

4.5.10

4.5.14

(a)

Proof *By* ***Proposition 4.30****, we know that if U is unitary, then all columns of U is orthonomal. By* ***Theorem 4.3****, we know that all columns are independent. By* ***Theorem 3.28****, we know that the columns are orthonomal basis. By* ***Corrolary 4.30****, we Know that U is an isometry.*

$$\|U\|_{op} = \max_{\|v\|=1, v\in\mathbb{C}^n} \|Uv\| = \|v\| = 1$$

(b)

By **Proposition 4.30**, we know that if U is unitary, then all columns of U is orthonomal. By **Theorem 4.3**, we know that all columns are independent. By **Theorem 3.28**, we know that the columns are orthonomal basis. By **Corrolary 4.30**, we Know that U is an isometry.

$$\|U\|_F = \sqrt{trU^*U} = \sqrt{trI} = \sqrt{n}$$