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# Matlab HW2: Exercise from Chap 2

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# Matlab HW2

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# Exercise 2.3

## Example 1 (Exercise 2.3: Distinguishability of sinusoids with different frequencies)

Assume that $s[n] = \cos(2\pi f_d n)$ for $n = 0, 1, \ldots, N-1$, where $N = 20$. Plot the magnitude of the Fourier transform, where the discrete-time Fourier transform is defined as

$$S(f) = \sum_{n=0}^{N-1} s[n] \exp(-j2\pi f n) \qquad -\frac{1}{2} \leq f \leq \frac{1}{2} \tag{1}$$

for $f_d = 0$ and compare it to the case when $f_d = 0.2$. Next let $f_d = 0$ and compare it to the case when $f_d = 0.01$. What can you say about the distinguishability of two different frequency sinusoids if they are much closer than $1/N$ in frequency?

# Exercise 2.3

%FSSP3exer2_3.m
%This program computes the magnitude of the discrete-time Fourier transform for a time truncated sinusoid having various frequencies. It illustrates the distinguishability of a sinusoid at $f_d = 0$ from a sinusoid at another frequency.
close all
clear all
$N = 20$; % number of data samples
$n = [0 : N - 1]'$; % time samples
$fd = 0.2$; % change to 0.01 to see a peak at $fd = 0$
$s1 = cos(2 * pi * (0) * n)$; % signal freq at zero
$s2 = cos(2 * pi * fd * n)$; % signal freq at $fd$
$f = ([0 : 1023]'/1024) - 0.5$; % frequencies from -0.5 to 0.5
for $i = 1 : 1024$
    $e = exp(j * 2 * pi * f(i) * n)$;
    $S1mag(i, 1) = abs(sum(e' * s1))$; % complex magnitude of Fourier transform
    $S2mag(i, 1) = abs(sum(e' * s2))$;
end
subplot(2,1,1)
plot(f,S1mag)
xlabel$('f')$
ylabel$('|S_1(f)|')$
grid
title$('f_d = 0')$
subplot$(2, 1, 2)$ plot$(f, S2mag)$
xlabel$('f')$
ylabel$('|S_2(f)|')$
grid
title$(['f_d =' \ num2str(fd)])$

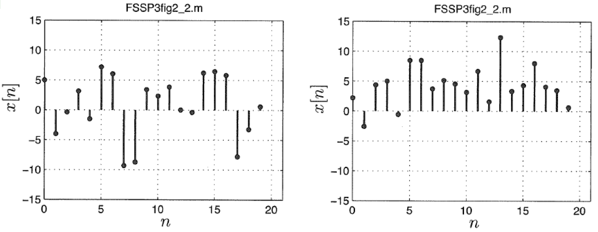
# Figure 2.2

## Example 2 (Plot Figure 2.2: Ref. Code in FSSP3exer2_4.m)

The clot detection problem is recognized as a hypothesis test between the two hypotheses $\mathcal{H}_0$ (no clot) and $\mathcal{H}_1$ (clot present)

$$\begin{aligned} \mathcal{H}_0 &: x[n] = A\cos[2\pi f_d n + \phi] + \omega[n] && \text{no clot} \\ \mathcal{H}_1 &: x[n] = A\cos[\phi] + \omega[n] && \text{clot present} \end{aligned} \tag{2}$$

for $n = 0, 1, ..., N-1$. Example of data sets: $A = \sqrt{15}$, $\sigma^2 = 15$, $f_d = 0.2$, $\phi = 0$, and $N = 20$. Please plot the figure of the $x[n]$ with no clot and $x[n]$ with clot present, as the reference figures shown below.




(a) Under $\mathcal{H}_0$ - no clot present, $f_d = 0.2$ (b) Under $\mathcal{H}_1$ - clot present, $f_d = 0$

Figure 1: Typical data sets observed under each hypothesis

# Exercise 2.4

## Example 3 (Exercise 2.4: Calculation of deflection coefficient)

For the parameters given, $A = \sqrt{15}$, $\sigma^2 = 15$, $f_d = 0.2$, $\phi = 0$, and $N = 20$, plot the *deflection coefficient* given by

$$d^2 = \frac{1}{\sigma^2} \sum_{n=0}^{N-1} \left(s_1[n] - s_0[n]\right)^2$$

versus $f_d$ for $0 \leq f_d \leq 1/2$. Interpret your results.

# Exercise 2.4

%FSSP3exer2_4.m
%This program calculates the deflection coefficient for two sinusoids at different frequencies, with one at $fd = 0$ and the other at various values of $fd$.
close all;
clear all;
$A = sqrt(15)$;$phi = 0$;$sig2 = 15$;$N = 20$; % set parameter values
$n = [0 : N - 1]'$; % set time samples
$s1 = A * cos(phi)$; % signal when clot is present
$fd = [0 : 1024]'/2048$; % signal frequencies from 0 to 0.5
for $i = 1 : length(fd)$
$s0 = A * cos(2 * pi * fd(i) * n + phi)$; % signal when no clot is present
$d2(i, 1) = (s1 - s0)' * (s1 - s0)/sig2$; % calculation of deflection coefficient
end
plot$(fd, d2)$
xlabel$('f_d')$
ylabel$('d2')$
grid

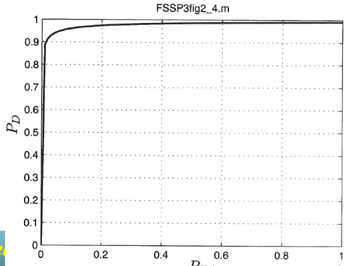
# Exercise 2.5

## Example 4 (Exercise 2.5: ENR required to meet spec)

The Neyman-Pearson (NP) performance bound is given by

$$P_D = Q\left(Q^{-1}(P_{FA}) - \sqrt{d^2}\right)$$

Figure 2.4 below uses the parameters $A = \sqrt{15}$, $\sigma^2 = 15$, $f_d = 0.2$, $\phi = 0$, and $N = 20$. Write a MATLAB code to generate Figure 2.4 but decrease the ENR to yield the spec of $P_D$ for $P_{FA} = 2 \times 10^{-4}$. You can do this by decreasing $A$. What value of $A$ is needed to just attain the spec? (use FSSP3exer2_5.m to find the detection bound)



# Exercise 2.5

%FSSP3exer2_5.m

%This program produces Figure 2.4. It uses the subprograms Q.m, which computes the Gaussian Q function and Qinv.m, which is the inverse function. Also, the subprogram plotlineroutine.m is used for plotting. All these programs are included on the CD in the folder "Utility_programs"..

close all;

clear all;

$A = sqrt(15)$;$phi = 0$;$sig2 = 15$;$N = 20$; % set parameter values

$s0 = A * cos(2 * pi * fd * [0 : N - 1]')$; % signal under H0

$s1 = A * ones(N, 1)$; % signal under H1

$d2 = (s1 - s0)' * (s1 - s0)/sig2$; % calculation of deflection coefficient

$Pfa = [0 : 0.00001 : 0.001]'$;

$Pd = Q(Qinv(Pfa) - sqrt(d2))$; % calculation of prob. of detection

plotlineroutine$(Pfa, Pd,' no',' o', 4, 4,' on',' on',' P_FA',' P_D','')$

# Question & Answer