TAIPEI TECH 國立臺北科技大學
National Taipei University of Technology

誠,樸,精,勤

Matlab HW5: Exercise from Chap 3

曾柏軒 (Po-Hsuan Tseng)
phtseng@ntut.edu.tw

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Matlab HW5

1. Nonlinear and Partially Linear Signals

Nonlinear and Partially Linear Signals

Example 1 (Least Squares Estimator for Partially Linear Signal)

Considered $p = 1$ in previous example, the sinusoids signal is given by

$$s[n] = \alpha_1 \cos(2\pi f_0 n) + \alpha_2 \sin(2\pi f_0 n)$$

Actual data $x[n]$ for $n = 0, 1, ..., N-1$ is available. Determine the unknown parameters $\{\alpha_{1_i}, \alpha_{2_i}, f_i\}$ to minimize the lease squares error

$$\begin{aligned} & J(\alpha_1, \alpha_2, f_0) \\ & = \sum_{n=0}^{N-1} \left(x[n] - [\alpha_1 \cos(2\pi f_0 n) + \alpha_2 \sin(2\pi f_0 n)] \right)^2 \end{aligned}$$

Nonlinear and Partially Linear Signals

- Written in the usual matrix/vector form as

$$\mathbf{s} = \underbrace{\begin{bmatrix} 1 & 0 \\ \cos(2\pi f_0) & \sin(2\pi f_0) \\ \vdots & \vdots \\ \cos[2\pi f_0(N-1)] & \sin[2\pi f_0(N-1)] \end{bmatrix}}_{\mathbf{H}(f_0)} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}}_{\boldsymbol{\theta}}$$

- $\mathbf{H}$ is no longer a known matrix since it depends upon the unknown parameter f_0
- The minimization of $J(\alpha_1, \alpha_2, f_0)$ can be carried out by performing a numerical maximization, by maximizing the function

$$J(f_0) = \mathbf{x}^T \mathbf{H}(f_0) \left(\mathbf{H}^T(f_0) \mathbf{H}(f_0) \right)^{-1} \mathbf{H}^T(f_0) \mathbf{x} \qquad (1)$$

over $0 < f_0 < 1/2$ to yield the maximizing value $\hat{f}_0$

Nonlinear and Partially Linear Signals

- Treating $\mathbf{H}(\hat{f}_0)$ as a known matrix, the remaining parameters are given by the least squares estimator

$$\begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix} = \left(\mathbf{H}^T(\hat{f}_0)\mathbf{H}(\hat{f}_0) \right)^{-1} \mathbf{H}^T(\hat{f}_0)\mathbf{x}$$

- The amplitude and phase estimates can now be found from

$$\hat{A} = \sqrt{\hat{\alpha}_1^2 + \hat{\alpha}_2^2}$$

$$\hat{\phi} = arctan(\frac{-\hat{\alpha}_2}{\hat{\alpha}_1})$$

- For p sinusoids the numerical maximization must be carried out over the p-dimensional space for $\{f_1, f_2, ..., f_p\}$

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Example 2 (Exercise 3.8: Numerical maximization)

Assume that the data is noiseless so that $x[n] = \cos(2\pi(0.12)n + \pi/4)$ for $n = 0, 1, ..., N-1$, where $N = 20$. Substitute $x[n]$ into (1) and then maximize $J(f_0)$ over $0 < f_0 < 1/2$ by computing the values of $J(f_0)$ explicitly for each value of f_0.

1. Choose the values of f_0 as $f_0 = 0.1, 0.2, 0.3, 0.4$.
2. Choose the values as $f_0 = 0.01, 0.02, ..., 0.49$.
3. Repeat the above test by modifying N. Does the correct value depend on the number of observation N?

Do you obtain the correct value of f_0?
[Hint: Modify the program **FSSP3exer3_8.m**]

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Example 3 (Modified of Exercise 3.8: Numerical maximization)

Redo the Exercise 3.8 by assuming the additive noise. Assume that the data is noiseless so that $x[n] = \cos(2\pi(0.12)n + \pi/4) + w[n]$ for $n = 0, 1, ..., N-1$, where $N = 20$ and $w[n] \sim \mathcal{N}(0, 1^2)$. Substitute $x[n]$ into (1) and then maximize $J(f_0)$ over $0 < f_0 < 1/2$ by computing the values of $J(f_0)$ explicitly for each value of f_0.

1. Choose the values of f_0 as $f_0 = 0.1, 0.2, 0.3, 0.4$.
2. Choose the values as $f_0 = 0.01, 0.02, ..., 0.49$.
3. Repeat the above test by modifying $w[n] \sim \mathcal{N}(0, 0.1^2)$
4. Repeat the above test by modifying N. Does the correct value depend on the number of observation N?

Do you obtain the correct value of f_0? [Hint: Modify the program **FSSP3exer3_8.m**]

Question & Answer