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Matlab HW6: Exercise from Chap 4

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Matlab HW5

1. Exercise 4.1: Generating white Gaussian noise
2. Exercise 4.2: Estimating the power
3. Exercise 4.4: AR(1) PSD - lowpass or highpass
4. Exercise 4.5: Filter frequency response

Generating white Gaussian noise

Example 1 (Exercise 4.1 Generating white Gaussian nosie)

Using the program **WGNgendata.m** generate a data record (also referred to as an outcome or as a realization) of WGN with variance $\sigma^2 = 4$. If $N = 1000$ samples are generated, how many exceed 3 in magnitude? How many exceed $3\sigma = 6$?

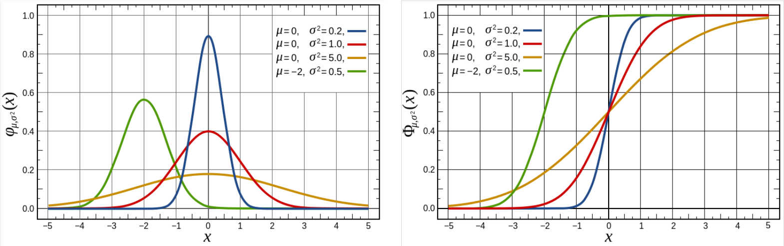
[hint: For $N = 1000$, we expect about 67 exceedances; **FSSP3exer4_1.m**]
The expected number is given by the total number of samples multiplied by the probability that a sample exceeds 3. Specifically, it is

$$N \cdot P[w[n] > 3] = N \cdot Q(\frac{3}{\sqrt{\sigma^2}}) = N \cdot Q(3/2) = 0.0668N$$

where
$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp(\frac{-t^2}{2})dt$$

is the right-tail probability for a $\mathcal{N}(0,1)$ random variable. It can be computed using the matlab program **Q.m** (Utility Program).

Review of the PDF and CDF



(a) PDF (b) CDF

Figure 1: PDF and CDF of Gaussian

Review of the PDF and CDF

- The probability density function (PDF) is Gaussian

$$p_W(w) = \frac{1}{\sqrt{2\pi\sigma^2}}\Big(-\frac{1}{2\sigma^2}w^2\Big) \quad -\infty < w < \infty$$

- The cumulative distribution function (CDF) of Gaussian is

$$\Phi(x) = \mathrm{P}(X \leq x) = \int_{-\infty}^{x} p_W(w)dw = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}}\Big(-\frac{1}{2\sigma^2}w^2\Big)dw$$

- Q function

$$Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(\frac{-t^2}{2})dt$$

- The relationship between the CDF of Gaussian and the Q function is

$$Q(\frac{x}{\sigma}) = 1 - \Phi(x)$$

Estimating the power

Example 2 (Exercise 4.2: Estimating the power)

Using the $N = 1000$ samples of the previous exercise the noise power by using

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{n=0}^{N-1} w^2[n]$$

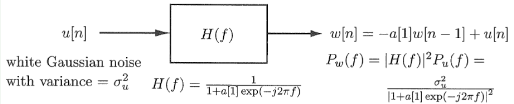
Is it close to the true value of $\sigma^2 = 4$? If not, try increasing N **[FSSP3exer4_2.m]**

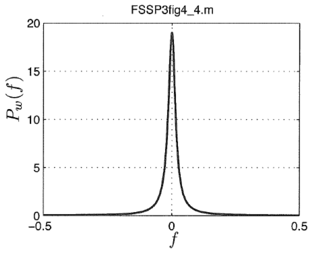
AR(1) PSD - lowpass or highpass

Example 3 (Exercise 4.4: AR(1) PSD - lowpass or highpass)

Choose for the AR parameters $a[1] = 0.9$ and $\sigma_u^2 = 0.19$.

1. Using **ARpsd.m** obtain values of the AR(1) PSD and the corresponding frequencies and plot the results.
2. Compare the PSD against the one $a[1] = -0.9$ and $\sigma_u^2 = 0.19$ as shown in the left figure below. What can you say about the effect of the sign of $a[1]$ upon the PSD?





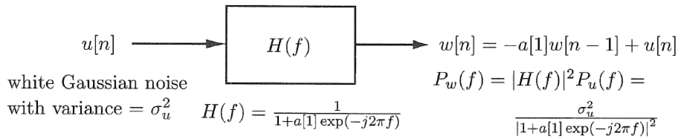
(b) $a[1] = -0.9$, $\sigma_u^2 = 0.19$

[FSSP3exer4_4.m]

Filter frequency response

Example 4 (Exercise 4.5: Filter frequency response)

Plot the magnitude of the filter frequency response given by (4.5) for $-1/2 \leq f \leq 1/2$ using $a[1] = -0.9$.



[FSSP3exer4_5.m]

Question & Answer